AE618 (2025-26-I): TERM PROJECT

Smooth Particle Hydrodynamics

What is SPH?

- **Definition :-** Smoothed Particle Hydrodynamics (SPH) is a computational method used for simulating fluid flows and solid mechanics, first developed in 1977 for astrophysics problems.
- **Core Philosophy:** It is a mesh-free, Lagrangian particle method

  Mesh-free: The simulation domain is represented by a set of discrete particles, not a connected grid or mesh.

  Lagrangian: The particles (which carry physical properties like mass, velocity, and pressure) physically move with the fluid flow.
- **Primary Aim:** To model physical phenomena that are extremely difficult for traditional grid-based methods, such as the Finite Element Method (FEM).

Limitations of Grid-Based Methods

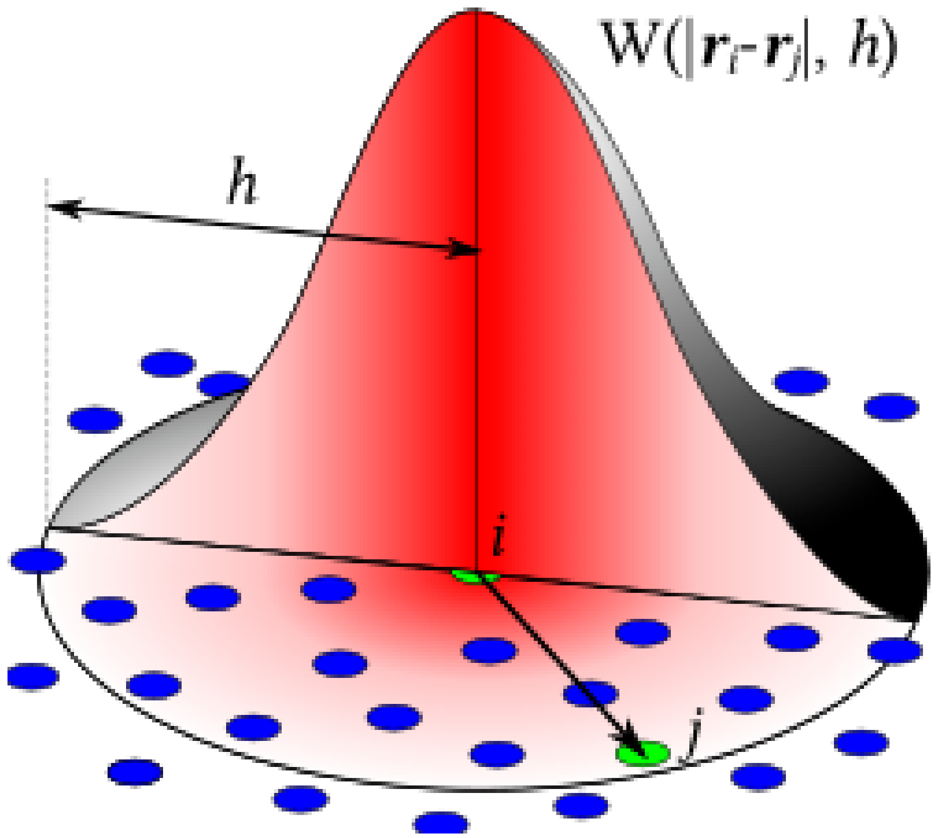
Traditional methods like FEM and the Finite Volume Method (FVM) are powerful but struggle with specific scenarios:

- **Mesh Tangling:** In Lagrangian (moving grid) simulations, extreme material deformation like in an explosion or high-velocity impact can cause the grid to tangle and stretch, leading to numerical errors and simulation failure.

- **Interface Complexity:** In Eulerian (fixed grid) simulations, tracking complex, fragmenting free surfaces (e.g., a splashing wave) requires computationally expensive and complex interface-tracking algorithms.

The core maths :- How SPH works

SPH uses a two-step mathematical process to approximate field variables at any particle location.



1. Kernel Approximation :

Kernel approximation in SPH means estimating a value at any point by smoothly averaging the values of nearby particles using a weighting function called the kernel function .

$$\int A(\mathbf{r}')W(\mathbf{r}-\mathbf{r}',h)d\mathbf{r}' = A(\mathbf{r})$$

2. Particle (Discrete) Approximation :

The integral is then discretized into a finite sum over all neighboring particles (indexed by j) .The value of a function A at particle i is the weighted sum of its neighbors' properties:

$$A_i \approx \sum_j m_j \frac{A_j}{\rho_j} W(\mathbf{r}_i - \mathbf{r}_j, h)$$

m_j is mass of particle

ρ_j is density of particle j

A_j is property A of particle j

W is the kernel function

Derivatives Approximation

$$\nabla_i A = \sum_j \frac{m_j}{\rho_j} \nabla W_{ij} A_{ij}, \qquad A_{ij} = A_i - A_j$$

$$\left(\frac{\nabla B}{\rho}\right)_i = \sum_j m_j \nabla W_{ij} \left(\frac{B_i}{\rho_i^2} + \frac{B_j}{\rho_j^2}\right)$$

$$\nabla_i \cdot \mathbf{A} = \sum_j \frac{m_j}{\rho_j} \nabla W_{ij} \cdot \mathbf{A}_{ij}$$

$$\left(\frac{\nabla \cdot \mathbf{B}}{\rho}\right)_i = \sum_j m_j \nabla W_{ij} \cdot \left(\frac{\mathbf{B}_i}{\rho_i^2} + \frac{\mathbf{B}_j}{\rho_j^2}\right)$$

$$(\nabla\nabla A)_i = \sum_j \frac{m_j}{\rho_j}(d+2)\frac{A_{ij}}{r_{ij}} \nabla W_{ij} \mathbf{e}_{ij} - \mathbf{I} \sum_j \frac{m_j}{\rho_j} \frac{A_{ij}}{r_{ij}} \nabla W_{ij} \cdot \mathbf{e}_{ij} \qquad \mathbf{e}_{ij} = \frac{\mathbf{r}_{ij}}{|\mathbf{r}_{ij}|}, \qquad \mathbf{r}_{ij} = \mathbf{r}_i - \mathbf{r}_j$$

$$\nabla_i^2 A = 2 \sum_j \frac{m_j}{\rho_j} \frac{A_{ij}}{r_{ij}} \nabla W_{ij} \cdot \mathbf{e}_{ij}$$

where d is dimension

Numerical Implementation

The SPH approximations are applied to the governing fluid dynamics equations (Navier-Stokes) to solve for particle motion.

- **Continuity Equation**

  (Density Summation)

  Density is not tracked over time, but rather calculated at each step by summing the mass of neighbors, weighted by the kernel.

$$\rho_i = \sum_j m_j W(\mathbf{r}_i - \mathbf{r}_j, h)$$

- **Momentum Equation**

  (Pressure and viscous Force)

  The pressure gradient is discretized into a symmetric particle-particle force to ensure momentum conservation (Newton's Third Law).

$$\frac{D\mathbf{v}_i}{Dt} = -\sum_j m_j \left(\frac{P_i}{\rho_i^2} + \frac{P_j}{\rho_j^2} \right) \nabla_i W_{ij} + \mathbf{g}$$

- **Equation of State**

  (Weakly Compressible)

  Pressure (P) is calculated directly from density (ρ) using an equation of state, avoiding a complex pressure-solver step.

$$P_i = c_s^2(\rho_i - \rho_0)$$

Numerical Implementation

Two other components are critical for a functioning simulation:

- **Time Integration :**

  An explicit integrator updates particle positions and velocities based on the forces calculated in the previous step.

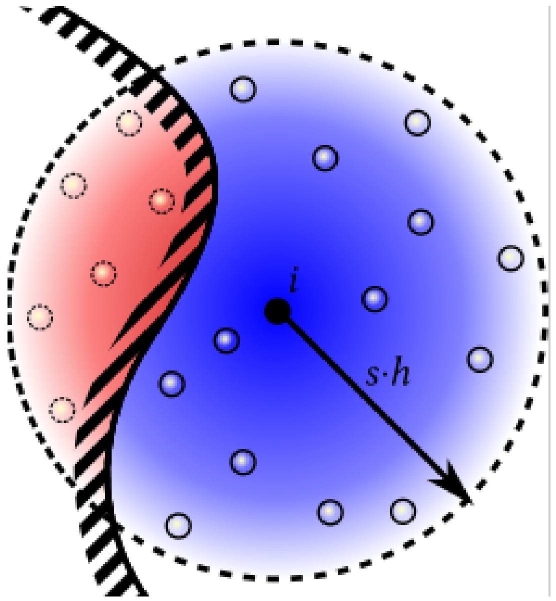
  Kick: $\mathbf{v}_i^{n+1/2} = \mathbf{v}_i^{n-1/2} + \mathbf{a}_i^n \Delta t$ **Drift:** $\mathbf{r}_i^{n+1} = \mathbf{r}_i^n + \mathbf{v}_i^{n+1/2} \Delta t$

- **Boundary Conditions :**

  Handling solid walls is the most difficult part of SPH because the kernel is truncated (cut off) at the boundary, leading to errors.

  Common solutions involve creating virtual particles to complete the kernel:

  Ghost / Dummy Particles: Virtual particles placed inside the wall that "mirror" the fluid's properties to create a realistic pressure force.



Applications & Modern Trends

- **Classical Applications:**

  Modeling astrophysical phenomena such as **star formation** and **supernovae**

  Its key applications include simulating fluid flows like **waves**, **splashes**, and **dam breaks**.

- **Modern & Emerging Applications:**

  Fluid-Structure Interaction (FSI): Coupling SPH (for the fluid) with FEM (for the solid) to model hydroelastic problems.

  Biomechanics: Modeling blood flow, tissue deformation, and injury.

Key Strengths v/s Drawbacks

Strengths	Drawbacks
Naturally handles large deformations, fragmentation, and splashing.	**Boundary condition implementation** is complex and less accurate than in grid methods.
Free surfaces and moving interfaces are handled automatically and accurately.	**High computational cost** per particle (though GPUs help this).
Inherently conserves mass perfectly, as particles carry a constant mass.	**Lower accuracy** than FEM/FVM for simple, non-deforming flows.
Algorithm is highly parallelizable, making it ideal for modern GPU acceleration.	Can suffer from **particle disorder** and **tensile instability** (clumping).

References

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Thank You !!!

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