DAA ASSIGNMENT 5
Group No 19

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Problem Statement

In this report we designed a Dynamic Programming algorithm to find the number of subsets of given array of n elements arr[] having XOR of elements as a given number K.

Introduction

Given an array arr[] of size n ,we will use dynamic programming approach to find the number to elements having XOR value as K.Here in dp[i][j] we keep a count of number of sets from 0 to i-1 having XOR value as j. At the end the program we will give dp[n][k] as output.

Dynamic Programming is mainly an optimization over plainrecursion. Wherever we see a recursive solution that hasrepeated calls for same inputs, we can optimize it using Dynamic Programming. The idea is to simply store the results of sub-problems, so that we do not have to re-compute them when needed later. This simple optimization reduces time complexities from exponential to polynomial.

ALGORITHMIC DESIGN

Following is Dynamic Programming algorithm

- We initialize all values of dp[i][j] as 0.
- Set value of dp[0][0] = 1 since XOR of an empty set is 0.
- Iterate over all the values of arr[i] from left to right and

for each arr[i],iterate over all the possible values of XOR

i.e. from 0 to m (both inclusive).

Here m is maximum possible value for XOR of any of possible subsets. To

calculate m, we take maximum element from array and

```
m = (1 << (int)(log2(max) + 1)) − 1;
```

- Fill the dp array as following:

```
for i = 1 to n:
        for j = 0 to m:
                dp[i][j] = dp[i − 1][j] + dp[i − 1][j⊕arr[i − 1]]
```

- This can be explained as, if there is a subset arr[0. . . i-2] with XOR value j, then there also exists a subset arr[0. . . i-1] with XOR value j also if there exists a subset arr[0. . . .i-2] with XOR value $j \oplus arr[i]$ then clearly there exist a subset arr[0. . . i-1] with XOR value j, as:

$j \oplus arr[i - 1] \oplus arr[i - 1] = j$.

- Counting the number of subsets with XOR value k: Since dp[i][j] is the number of subsets having j as XOR value from the subsets of arr[0..i-1], then the number of subsets from set arr[0..n] having XOR value as K will be dp[n][K]

Illustration

Let’s take a array arr = [1, 2, 3, 4] and K=6

Firstly we initialize dp array as 0.

dp[i][j] = 0(for all i<n && j<n)

here in dp[i][j] we keep a count of number of subsets from 0 to i-1 having XOR value as j

n = sizeof(arr)

dp[0][0] = 1(Empty set)

Now take max element from array and use it to calculate maximum possible XOR value(m).

max=4

m = (1<<(int)(log(4)+1))-1

m = 7

Now we Iterate over all values of arr[i] from i=1 to i=n-1.Then

for each iteration ,we also iterate over all values of XOR.

dp(After 1st Iteration):

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \end{bmatrix}$$

dp(After 2nd Iteration):

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \end{bmatrix}$$

dp(After 3rd Iteration):

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 2 & 2 & 2 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \end{bmatrix}$$

dp(After 4th Iteration):

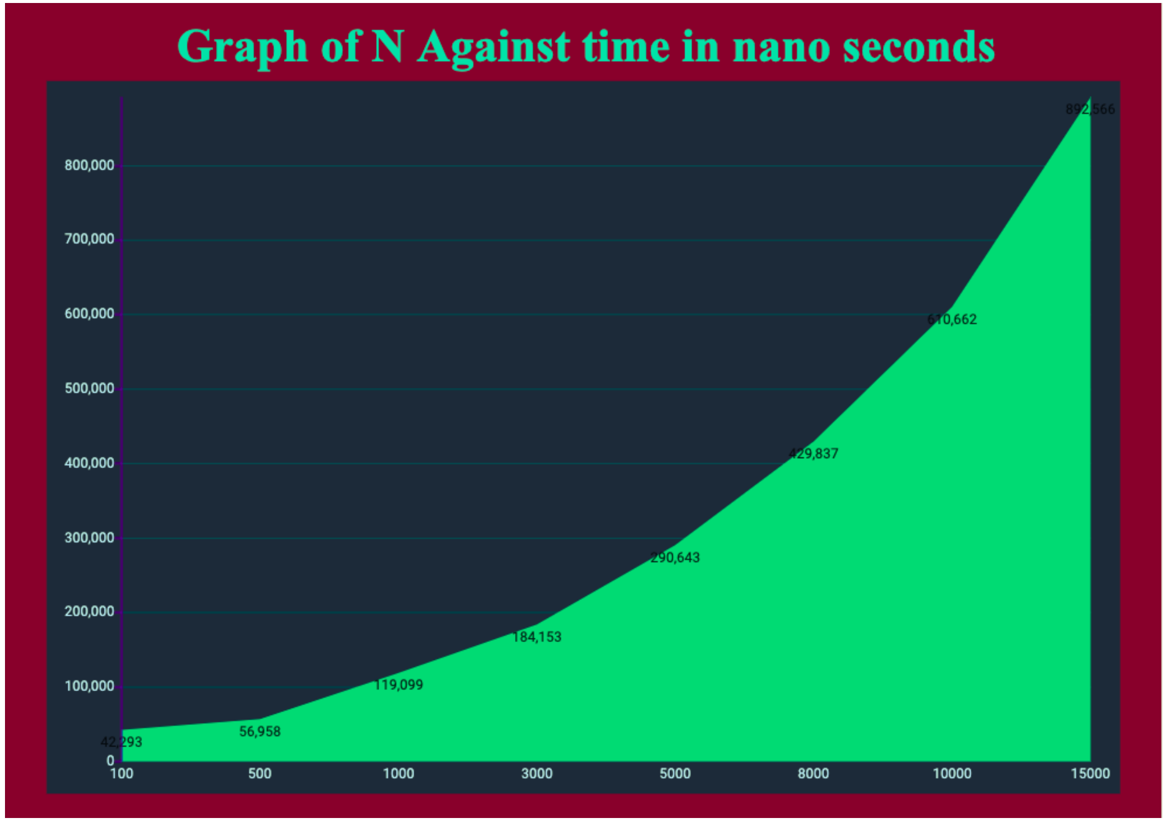
$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 2 & 2 & 2 & 2 & 0 & 0 & 0 & 0 \\ 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \\ \end{bmatrix}$$

Now our solution is the value of $dp[n][k] = dp[4][5] = 2$.

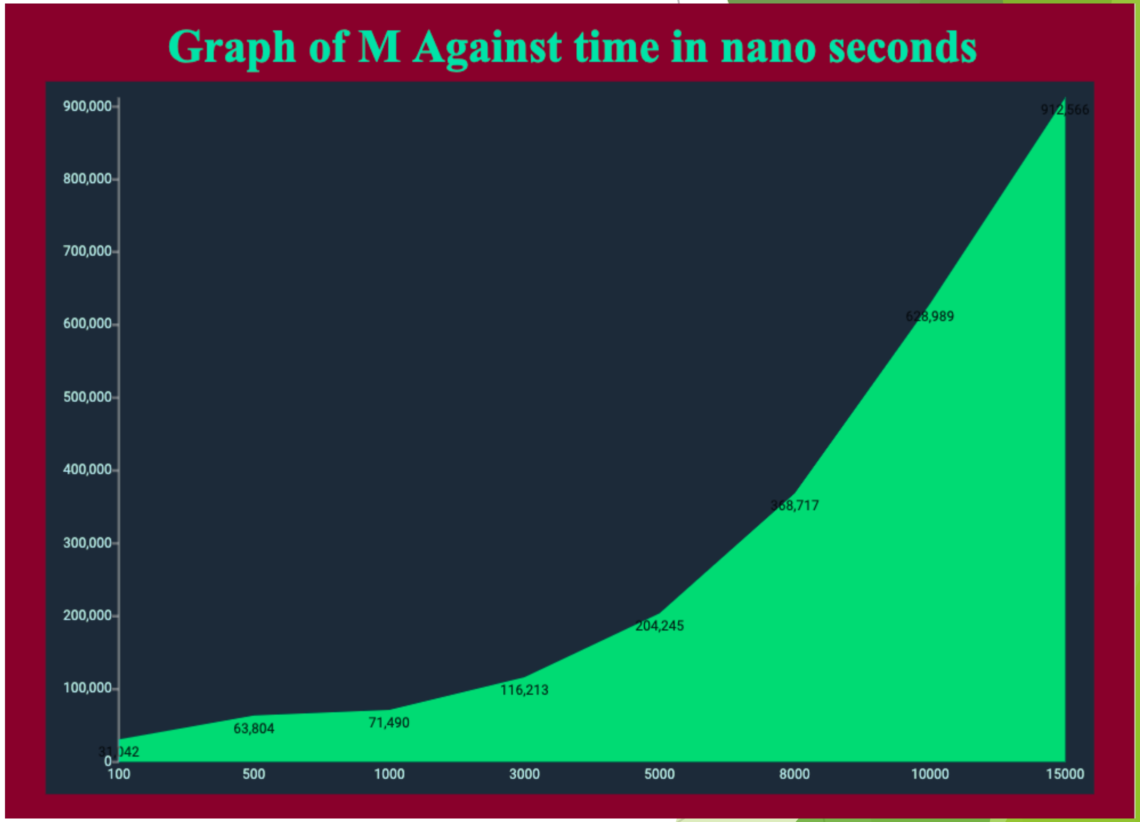
Time complexity

In this approach we iterate over whole array one by one finding and storing the possible subsets that generate a value p we say and store it in the dp[i][k] which will need time to iterate over the array and for each element a loop of time complexity m ,where m is the maximum possible value of XOR and could be found from the max value of element in array.

$m = 2[\log 2(max-element)]+1 - 1$



X-axis: N Y-axis: Time in ns



X-axis: M Y-axis: Time in ns

Space complexity

Brute-Force :

In Brute force we will be directly doing XOR operation on the variable or will be resetting the variable, no extra space other than input would be required . This will result in the space complexity of $O(1)$.

Dynamic-Programming :

In this approach we will have to store the values for all possible cases that are generated, as the dp[i][j] requires dp[i − 1][j] and hence a 2D array of size $n * m$ is required where m is same as mentioned above, that is:

$m = 2[\log2(max{-}element)]+1 - 1$

This will result in the space complexity of $O(n * m)$.

Conclusion

We can observe that In Dynamic Programming Approach it may consume more space but will have better time complexity than Brute force approach.

References

- 1. https://www.geeksforgeeks.org/calculate-xor-1-n/
- 2. https://www.geeksforgeeks.org/dynamic-programming/
- 3. Cormen, Leiserson, Rivest, and Stein (2009). Introduction to Algorithms, 3rd edition.