a) Q5

Is the stock price recurrent?

We model this as a Markov chain with discrete steps

$$120 + 0.01 \times k \quad k \in \mathbb{Z}$$

1) 0.1 probability of moving up by 0.01
2) 0.85 probability of staying at current price
3) 0.05 probability of moving down by 0.01

This is a random walk with a bias upward, because the upward movement probability (0.10) is greater than downward movement probability (0.05). Since the walk is on an infinite countable state space with asymmetric transition probabilities

The chain is not symmetric

positive drift

Ans: Stock price is not recurrent.

It is transient

(b) Q5 A station

only if the

Since, c

to drift

Ans: "

not exist

(c) AMER

Stock tou

1:00 pm

[ 10:00 am

~~[illegible]~~

Time =

Step siz

Starting p

Target

Each step

(b) Q5 A stationary distribution for a Markov chain exists only if the chain is positive recurrent ~~(...)~~

Since, chain is "transient" the stock tend to drift upward indefinitely,

Ans: "NO, a stationary distribution does not exist

(c) AMERICAN OPTION SIMULATION

Stock touches Rs 130 $\left( \begin{array}{l} \text{Strike price} = 125 \\ \text{profit} = 5 \end{array} \right)$ before 1:00 pm

( 10:00 am to 1:00 pm)

~~...~~

Time = 3 hrs = 10800 seconds

Step size = 5 seconds $\Rightarrow$ No. of steps

= 10800/5 = 2160

Starting price = Rs 120

Target price = Rs 130

$\boxed{\text{Each step}}$ Up with prob = 0.1 (increase Rs 0.01)

Same with prob = 0.85

Down with prob = 0.05.

```
[2]: import numpy as np

def simulate_american_option(num_simulations=100000):
    tick_size = 0.01
    start_price = 120.0
    target_price = 121
    steps = 2160  # from 10:00 am to 1:00 pm (3 hours = 10800 seconds; every 5 sec = 10800 / 5 = 2160 steps)
    success = 0

    for _ in range(num_simulations):
        price = start_price
        for _ in range(steps):
            move = np.random.choice(
                [tick_size, 0, -tick_size],
                p=[0.1, 0.85, 0.05]
            )
            price += move
            if price >= target_price:
                success += 1
                break

    probability = success / num_simulations
    return probability
prob = simulate_american_option(num_simulations=1000)
print(f"Probability of earning Rs. 5 before 1:00 pm: {prob:.4f}")
```

```
Probability of earning Rs. 5 before 1:00 pm: 0.7110
```

Q4) Soln The king moves randomly in any legal directions (up to 8 directions)

Each move is equally likely

Total squares = 64

~~Type~~

1) CORNER     Legal moves     SQUARES

3     4

Total weight = 12

Total = 420 (64

Probability = $3/420$ = $1/140$

| TYPE | SQUARES | MOVES | TOTAL WEIGHT | Probability |
|---|---|---|---|---|
| Corner | 4 | 3 | 12 | $3/420$ |
| Edge | 24 | 5 | 120 | $5/420$ |
| Center | 36 | 8 | 288 | $8/420$ |
| Total | 64 | — | 420 | |

Ans : a) 4 Corners Squares = $\frac{3}{420}$

b) 24 edge squares = $5/420$

c) 36 Center Square = $8/420$

Q6 | Transition probability & Stationary distribution

a) for any 2 permutations g & h, the transition probability $q(g, h)$ is

$$q(g,h) = \begin{cases} 2/26\cdot 25 & \text{if } h \text{ is obtained from } g \text{ by swapping letters} \\ 0 & \text{otherwise} \end{cases}$$

because there are $\binom{26}{2} = 325$ possible ways.

The probability of picking any pair $(i,j)$ with $i \neq j$ is $2/26\cdot 25$

The distribution is uniform over all permutations as the chain is symmetric & all state are equally likely in the long run.

(b) The proposal probability is $q(g,h) = \frac{2}{26\cdot 25}$ if h can be reached by swapping two letters in g & zero otherwise

- the acceptance probability is

$$A(g \to h) = \begin{cases} 1 & \text{if } s(h) \geq s(g) \\ \frac{s(h)}{s(g)} & \text{if } s(h) \leq s(g) \end{cases}$$

Transition probability is $p(g,h) = q(g,h)\,A(g \to h)$

for reversibility

$S(g)q(g,h)A(g \to h) = S(h)q(h,g)A(h \to g)$

we have,

$S(g)q(g,h)A(g \to h) = S(h)q(h,g)A(h \to g)$

Thus, the chain is reversible with respect to Distribution

$\boxed{\pi(g) \propto S(g)}$ so Stationary Distribution is proportional to $S(g)$