Q2] European Call option

Current stock price - S_0 = 100

Strike price = 105

Time to maturity = 10

A part

a) Each stock move

After 10 days

$$S_T = 100 + 2k - 10$$

NOW $S_T > 105$

$$P = P(k=8) + P(k=9) + P(k=10)$$

$$= \sum_{k=8}^{10} \binom{10}{k} \left(\frac{1}{2}\right)^{10} = 56/1024$$

$$= 0.054$$

(b) Expected pay off :

$k = 8$: $S_T = 106$ ⇒ Payoff = 1

$k = 9$: $S_T = 100$ ⇒ Payoff = 3

$k = 10$: $S_T = 110$ ⇒ Payoff = 5

Prob = 45/1024

Prob = 10/1024

Prob = 1/1024

Expected payoff = $\frac{1 \times 45 + 3 \times 10 + 5 \times 1}{1024} \times 80/1024$

≈ 0.078

(c) Assuming no discounting

Fair value = expected Payoff

= 0.078

Part B

a) Mean = 0 ⇒ Daily move has expected absolute value = 1

$E[|x|] = \sigma\sqrt{2/\pi} = 1 \Rightarrow \sigma = \sqrt{\pi/2}$

$\sigma \cong 1.2533$

For 10 Days $\sigma_{10} = \sqrt{10} \cdot \sigma = 3.963$

Expected Payoff as Integral

$E[\max(S_T - K, 0)] = \int_K^{\infty} (S-K) f_{S_T}(S)\, dS$

Here, $S_T \sim N(100, \sigma^2 10)$ ⇒ $S_T \sim N(100, 15.7)$

$E[\max(S_T - 105, 0)] = \int_{105}^{\infty} (S-105) f_S(S)\, dS$

Expected Pay off = $0.196

Question 1:

(a) IV calculation from Black-Scholes call formula

Call price $C = 4.2$ Stock price $S = 38$

Strike price $K = 35$ Time $= 4/12 = 1/3$ yr

Risk free rate $= 6\%$

$$C = SN(d_1) - Ke^{-rT}N(d_2)$$

$$d_1 = \frac{\ln(S/K) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}, \quad d_2 = d_1 - \sigma\sqrt{T}$$

$\sigma = 0.2 \Rightarrow C = 3.62$

$\sigma = 0.28 \Rightarrow C \approx 4.21$

$\sigma = 0.35 \Rightarrow C \approx 4.71$

$IV \approx 0.28$ (28%)

(b) European put price using put-call parity

$$P = C - S + Ke^{-rT}$$

$$P = 4.2 - 38 + 35e^{-0.06 \times 0.333}$$

$$P \approx 0.51$$

European put price $\approx \$0.51$

(c) Stock - project NPV = \$38M

Strike - Cost to launch = \$35M

Time - 4 months - 1/3 yrs.

$r = 6\%$, $\sigma =$

From (a), the va

call option i

the firm should launch if option value > 0

Hence, Launch the drug.