Nozzle and Injector Design Documentation

Engine Design Tool

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1 Introduction

The purpose of this document is to go over the key equations for nozzle design used for the **Engine Design Tool (EDT)**. The geometry of the nozzle is fundamental to determining the performance and size characteristics such as Thrust, Specific Impulse, Characteristic Velocity, and overall efficiency of a rocket engine. This document provides a clear guide for the initial design of a liquid rocket engine (LRE) nozzle.

In this report, we will be going over conical nozzle design. Conical nozzles were some of the first nozzles to be designed, and their ease of manufacturability makes them a strong contender for hobbyist LREs. The governing equations for nozzle theory begin with isentropic equations. These equations assume adiabatic conditions (no heat transfer) and reversible fluid flow (constant entropy). With this, we can simplify analysis for an initial design of a nozzle until computational fluid dynamics and live testing can confirm the validity of the engine design. The primary injector type we shall be going over are impinging injectors. Impinging injectors have fuel and oxidizer impinge onto each other to atomize and combust in the combustion chamber. Due to their ease of manufacturing and simplicity with analysis, they are one of the primary injector types for hobbyist rocket engines.

The goal of this document is to provide the theoretical background and practical implementation of the isentropic equations along with the nozzle and injector sizing parameters, allowing for users to understand, verify, and improve the EDT's LRE and injector sizing methods. Key references include Huzel and Huang's *Design of Liquid Propellant Rocket Engines*, George Sutton's *Rocket Propulsion Elements*, and multiple NASA and private research papers.

2 Nozzle Theory

2.1 Isentropic Equations

This section will lay out the governing equations for isentropic flow that we shall be using. The initial variables required for this section come from Chemical Equilibrium Analysis (CEA). The recommended CEA software is NASA CEARUN rev4. **PROMPT requires SI units!**

The main parameters required are as follows:

1. OF Ratio
2. Density ρ in kg/m^3
3. Gamma γ
4. Chamber Pressure p_c in Bar
5. Chamber Temperature T_c in K
6. Molecular Weight MW in g/mol
7. Specific Heat C_p in $kJ/kg*K$

NASA CEARUN will give outputs in the SI unit system.

The first equation that is required is to convert specific heat to the specific gas constant. This can be done with the equation:

$$R = C_p \cdot 1000 \cdot \left(1 - \frac{1}{\gamma}\right) \tag{1}$$

where C_p is in $kJ/kg*K$, and the factor of 1000 converts it into $J/kg*K$. The exit Mach number can then be computed from the chamber and exit pressures:

$$M_e = \sqrt{\frac{2}{\gamma - 1}\left[\left(\frac{p_c}{p_e}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]} \tag{2}$$

The exit temperature in Kelvin is found using chamber temperature, γ, and exit Mach number:

$$T_e = T_c\left(1 + \frac{\gamma - 1}{2}M_e^2\right)^{-1} \tag{3}$$

Exit pressure is useful to determine whether the exit conditions meet ambient conditions and if a nozzle is underexpanded or overexpanded. The equation for exit pressure in Bar is given by:

$$p_e = p_c \left(1 + \frac{\gamma - 1}{2} M_e^2\right)^{\frac{-\gamma}{\gamma-1}} \tag{4}$$

Exit Velocity can also be calculated using γ, R, T_e, and exit Mach in m/s as:

$$v_e = M_e \sqrt{\gamma R T_e} \tag{5}$$

Finally, the expansion ratio ϵ can be computed using the Mach number and γ as:

$$\varepsilon = \frac{A_e}{A_t} = \left(\frac{\gamma + 1}{2}\right)^{\frac{1-\gamma}{2(\gamma-1)}} \cdot \frac{\left(1 + \frac{\gamma-1}{2} M_e^2\right)^{\frac{\gamma+1}{2(\gamma-1)}}}{M_e} \tag{6}$$

With these equations, we can begin to compute the performance characteristics and size our nozzles.

2.2 Engine Performance Characteristics

It is important for a designer to be able to understand the performance of the engine they are designing. Some parameters for analyzing an engine are specific impulse I_{sp}, characteristic velocity C^*, and mass flow rate $\dot{m}$.

To compute these values, PROMPT requires engine thrust to be provided by the user in Newtons.

The general thrust equation for a rocket nozzle is:

$$T = \dot{m}v_e + (p_e - p_a)A_e \tag{7}$$

where $\dot{m}$ is the mass flow rate, v_e is the exhaust velocity, p_e is the nozzle exit pressure, p_a is the ambient pressure, and A_e is the nozzle exit area.

If the nozzle is ideally expanded so that exit pressure is equivalent to the ambient pressure, mass flow rate in kg/s can be computed as:

$$\dot{m} = \frac{T}{v_e} \tag{8}$$

From here, you can calculate specific impulse in seconds as:

$$I_{sp} = \frac{T}{\dot{m}g_o} \tag{9}$$

where $g_o = 9.80665\ m/s^2$. Along with this, you can compute characteristic velocity, which is a measure of the energetic properties of the combustion chamber and propellants. Characteristic velocity can be computed as:

$$C^* = \frac{P_c A_t}{\dot{m}} \tag{10}$$

where A_t is the throat area of the nozzle, which will be computed in section 3.1. Using these equations, a designer can characterize and select an O/F ratio for their engine by considering chamber temperature, I_{sp}, and C^*.

3 Nozzle Sizing and Design

3.1 General Design Equations

To size a LRE nozzle, a few common parameters must be established. The two most critical geometric parameters we will cover in this section are the throat area (A_t) and exit area (A_e).

The throat area can be found with the equation

$$A_t = \frac{\dot{m}\sqrt{T_c}}{p_c}\left(\sqrt{\frac{\gamma}{R}} M_t \left(1 + \frac{\gamma - 1}{2} M_t^2\right)^{-\frac{\gamma+1}{2(\gamma-1)}}\right)^{-1} \tag{11}$$

As M_t is equal to 1 when flow is choked, the equation can be simplified into

$$A_t = \frac{\dot{m}\sqrt{T_c}}{p_c}\sqrt{\frac{R}{\gamma}}\left(\frac{\gamma + 1}{2}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \tag{12}$$

To find our exit area, we can use ϵ from equation 6

$$A_e = \epsilon A_t \tag{13}$$

From here, radii and diameter can be calculated, and the conical and bell nozzle design can begin in sections 3.2 and 3.3.

3.2 Conical Nozzle Design

Conical nozzles are some of the easiest nozzles to manufacture. Their simple angles make them suitable for machining on a lathe. The primary parameters in a conical nozzle design are the chamber length and diameter, throat length, and the converging and diverging angles. These are illustrated in

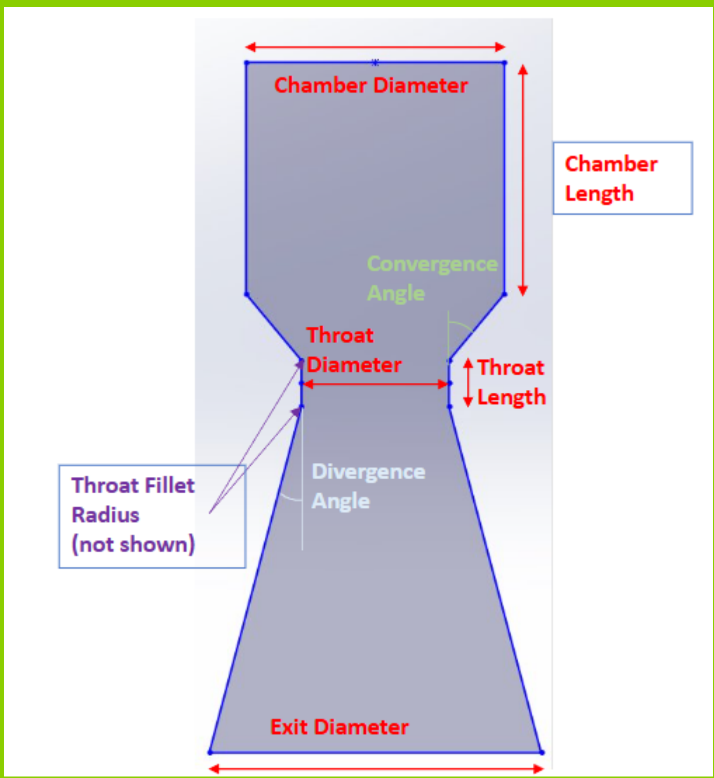

Figure 1: Conical Nozzle Geometry

Exit and throat diameter are already computable from equations 12 and 13 in section 3.1.

The two main angles to note are the converging and diverging angles. The converging angle (θ_c) determines the angle of the converging section of the nozzle. The diverging angle (θ_d) determines the angle of the divergent section. The converging angle usually ranges from 20° to 45°, while the diverging angle varies from 12° to 18°. Most modern nozzles assume a 15° divergent angle due to its ability to compromise with weight, length, and performance. PROMPT can use any angles for θ_c and θ_d, but it is recommended to use a 45-15 nozzle, with θ_c being 45° and θ_d being 15°.

The length of the divergent section can be calculated using

$$l_d = \frac{(r_e - r_t)}{(\theta_d)} \tag{14}$$

where r_e and r_t are exit radius and throat radius. The throat length is also empirically found and can only be approximated until verified by CFD or live fire testing. The common ratio of throat length, τ, is 0.05 - 0.75 times the throat diameter. The equation for throat length is

$$l_t = \tau d_t \tag{15}$$

A common value for τ is 0.5.

Chamber diameter is found by the contraction ratio (β). This value is empirical and can be found using a graph from the MIT Rocketry Team. This graph is shown below:

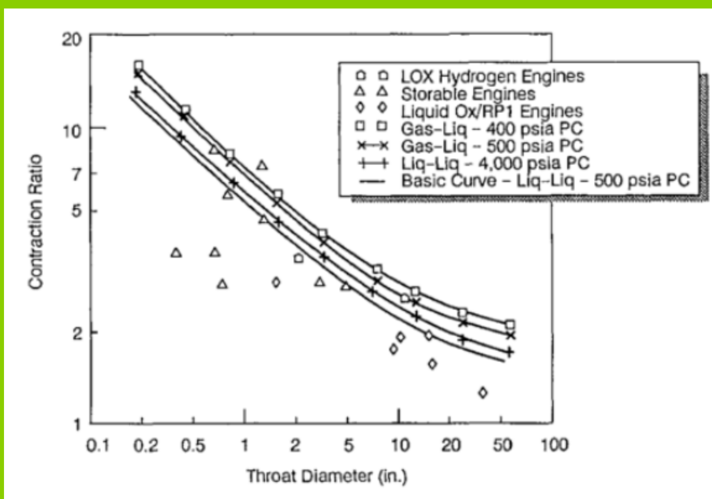

Figure 2: Contraction Ratio vs Throat Diameter

Using CR, the chamber area can be found in a similar manner to the exit area in the equation

$$A_c = \beta A_t \tag{16}$$

Using this equation, you can find your chamber area, and from there the chamber length needs to be computed. As the chamber is the primary area in a nozzle where the propellants are ignited, the chamber needs to be long enough to allow for combustion to occur. To characterise this, a useful parameter, *L**, is used to speficiy the propellant stay time. *L** is defined as the ratio of chamber volume to nozzle throat area, and can be expressed with the equation

$$L^* = \frac{V_c}{A_t} \tag{17}$$

To find *L**, tables from Huzel and Huang are provided. However, it is recommended to do research on combustion properties and find the stay time of the propellants. The tables are shown below:

Propellant combination	Combustion chamber characteristic length (L^*), in.
Chlorine trifluoride/hydrazine-base fuel	30-35
Liquid fluorine/hydrazine	24-28
Liquid fluorine/liquid hydrogen (GH_2 injection)	22-26
Liquid fluorine/liquid hydrogen (LH_2 injection)	25-30
Hydrogen peroxide/RP-1 (including catalyst bed)	60-70
Nitric acid/hydrazine-base fuel	30-35
Nitrogen tetroxide/hydrazine-base fuel	30-35
Liquid oxygen/ammonia	30-40
Liquid oxygen/liquid hydrogen (GH_2 injection)	22-28
Liquid oxygen/liquid hydrogen (LH_2 injection)	30-40
Liquid oxygen/RP-1	40-50

Figure 3: Characteristic Length Values for Common Propellant Combinations

As the volume would equal area multiplied by length, we can use the equation for chamber volume and solve it to find chamber length. The equation for chamber volume is shown below as

$$V_c = A_t(L_c\epsilon_c + \frac{1}{3}\sqrt{\frac{A_t}{\pi}}\;\theta_c(\epsilon_c^{\frac{1}{3}} - 1)) \tag{18}$$

We can use equation 17 and solve for L_c. With this, we get the equation

$$L_c = \frac{L^* - \frac{1}{3}\sqrt{\frac{A_t}{\pi}}\;\theta_c(\epsilon_c^{\frac{1}{3}} - 1)}{\epsilon_c} \tag{19}$$

With this, we have our chamber length and area. Using these equations, a simple conical nozzle can be designed and optimized for computational or live-fire testing.

4 Injector Design

4.1 Injector Theory

The injector plate is the second most critical piece of a liquid rocket engine. It is designed to inject the propellants from the feed system to the combustion chamber and make sure combustion is stable. The main principles required to allow for stable combustion are mixing and atomization. Mixing refers to the mixing of the fuel and oxidizer. Atomization is the breakup of the propellant stream into individual droplets so combustion can be more efficient. Droplet size is the main measure of atomization but this is hard to estimate. The best way to test injector designs are with CFD or live testing. The primary injector type calculated by EDT are impinging doublets. An impinging doublet is a impinging injector where the fuel and ox streams impinge on each other to atomize. A picture of an impinging injector is shown below.

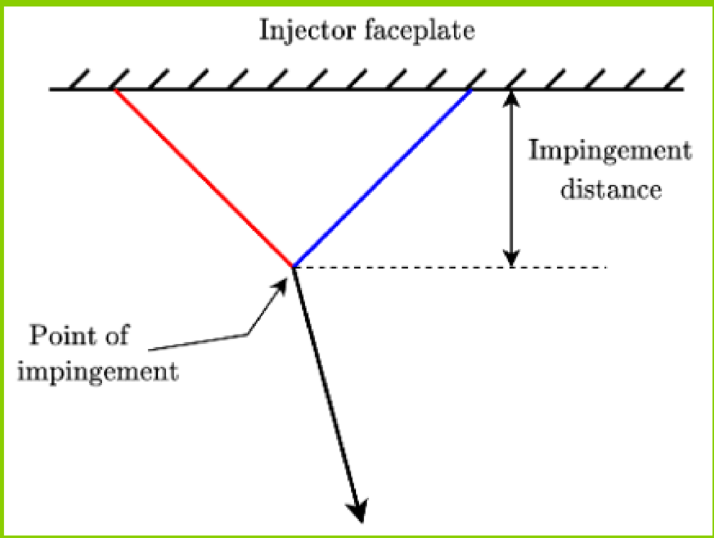

Figure 4: Impinging Injector Doublet

4.2 Impinging Injector Design

The first calculations that need to be made for an impinging injector are finding the fuel and oxidizer flow rates from the total mass flow rate and OF ratio.

The fuel flow rate can be calculated as

$$\dot{f} = \frac{\dot{m}}{1 + OF} \tag{20}$$

The ox flow rate can be calculated as

$$\dot{ox} = \dot{f}(OF) \tag{21}$$

From here, injector orfice sizing can be calculated. The primary equation used to characterize injector orfice sizing is the equation

$$\dot{m} = C_d A \sqrt{2\rho \Delta P} \tag{22}$$

where C_d is coefficient of discharge, A is area, ρ is density, and ΔP is the pressure drop. Coefficient of Discharge is a nondimensional number which represents the ratio of the actual flow rate of a fluid to the theoretical rate. The coefficient usually ranges from 0 to 1, but can be estimated as being from 0.6 to 0.75 for a first round of calculations. The pressure drop is through the injector plate and can be either calculated using the Darcy-Weisbach equation or estimated as being around a 25 percent drop throughout the orfice. Using this, you can solve for area for both the fuel and ox sides, resulting in the equation

$$A = \frac{\dot{m}}{C_d \sqrt{2\rho \Delta P}} \tag{23}$$

The area per each hole and diameter of each hole can be found by dividing the area by the number of holes. Diameter can be found by converting from area to diameter. From here, the angle of impingement and the impingement distance must be calculated. The equation for the resultant impingement angle is shown below as

$$\beta = \frac{\dot{m}_1 v_1 \ \alpha_1 - \dot{m}_2 v_2 \ \alpha_2}{\dot{m}_1 v_1 \ \alpha_1 + \dot{m}_2 v_2 \ \alpha_2} \tag{24}$$

where $\dot{m}$ is mass flow and v is velocity. Velocity can be calculated using flow rate, density, and orfice area. The equation for this is shown below

$$v = \frac{\dot{m}}{\rho A_{orfice}} \tag{25}$$

Finally, impingement distance can be calculated with trigonometry during the 3D modelling phase. The impingement angles can be set as desired by the user, but a 45-45 impingement seems to be the most efficient when compared with other common sizes. It is important to validate your injector with both CFD and live-testing using test fixtures. With all of this in mind, the user can now design an impinging injector for a rocket engine.

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