Nozzle Design Documentation

PROpulsion Modeling and Performance Tool
(PROMPT)

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September 1, 2025

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1 Introduction

The purpose of this document is to go over the key equations for nozzle design used for the **PROpulsion Modeling and Performance Tool (PROMPT)**. The geometry of the nozzle is fundamental to determining the performance and size characteristics such as Thrust, Specific Impulse, Characteristic Velocity, and overall efficiency of a rocket engine. This document provides a clear guide for the initial design of a liquid rocket engine (LRE) nozzle.

In this report, two types of nozzles are included: the **conical nozzle** and **bell nozzle**. Conical nozzles were some of the first nozzles to be designed, and their ease of manufacturability makes them a strong contender for hobbyist LREs. The Bell nozzle was developed to be lighter and more efficient, but was harder to characterize until G.V.R Rao developed a method for simplifying the design. We will be covering Rao's method of designing a bell nozzle.

The governing equations for nozzle theory begin with isentropic equations. These equations assume adiabatic conditions (no heat transfer) and reversible fluid flow (constant entropy). With this, we can simplify analysis for an initial design of a nozzle until computational fluid dynamics and live testing can confirm the validity of the engine design.

The goal of this document is to provide the theoretical background and practical implementation of the isentropic equations along with the nozzle sizing parameters, allowing for users to understand, verify, and improve the analysis methods for PROMPT's LRE sizing module. Key references include Huzel and Huang's *Design of Liquid Propellant Rocket Engines*, George Sutton's *Rocket Propulsion Elements*, and multiple NASA and private research papers.

2 Nozzle Theory

2.1 Isentropic Equations

This section will lay out the governing equations for isentropic flow that we shall be using. The initial variables required for this section come from Chemical Equilibrium Analysis (CEA). PROMPT uses RocketCEA, which is a wrapper of the original NASA FORTRAN CEA code.

The main parameters required are as follows:

1. OF Ratio
2. Density ρ in kg/m^3
3. Gamma γ
4. Chamber Pressure p_c in Bar
5. Chamber Temperature T_c in K
6. Molecular Weight MW in g/mol

NASA CEARUN will give outputs in the SI unit system.

The first equation that is required is to find the specific gas constant. This can be done with the equation:

$$R = \frac{R_{universal}}{MW} * 1000 \tag{1}$$

where $R_{universal}$ is 8.314 $J/kg * K$, and the factor of 1000 converts the MW from g to kg, with R in $J/kg * K$. The exit Mach number can then be computed from the chamber and exit pressures:

$$M_e = \sqrt{\frac{2}{\gamma - 1}\left[\left(\frac{p_c}{p_e}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]} \tag{2}$$

The exit temperature in Kelvin is found using chamber temperature, γ, and exit Mach number:

$$T_e = T_c \left(1 + \frac{\gamma - 1}{2} M_e^2\right)^{-1} \tag{3}$$

Exit pressure is useful to determine whether the exit conditions meet ambient conditions and if a nozzle is underexpanded or overexpanded. The equation for exit pressure in Bar is given by:

$$p_e = p_c \left(1 + \frac{\gamma - 1}{2} M_e^2\right)^{\frac{-\gamma}{\gamma - 1}} \tag{4}$$

Exit Velocity can also be calculated using γ, R, T_e, and exit Mach in m/s as:

$$v_e = M_e \sqrt{\gamma R T_e} \tag{5}$$

Finally, the expansion ratio ϵ can be computed using the Mach number and γ as:

$$\varepsilon = \frac{A_e}{A_t} = \left(\frac{\gamma + 1}{2}\right)^{-\frac{(1+\gamma)}{2(\gamma-1)}} \cdot \frac{\left(1 + \frac{\gamma-1}{2} M_e^2\right)^{\frac{\gamma+1}{2(\gamma-1)}}}{M_e} \tag{6}$$

Additionally, it would be prudent to also calculate energetic properties for the throat of a nozzle. As M_t is equal to 1 at the throat, throat pressure and temperature can be found using equations (3) and (4), substituting Me for Mt and resulting in the equations shown below.

$$T_t = T_c \left(1 + \frac{\gamma - 1}{2}\right)^{-1} \tag{7}$$

$$p_t = p_c \left(1 + \frac{\gamma - 1}{2}\right)^{\frac{-\gamma}{\gamma - 1}} \tag{8}$$

With these equations, we can begin to compute the performance characteristics and size our nozzles.

2.2 Engine Performance Characteristics

It is important for a designer to be able to understand the performance of the engine they are designing. Some parameters for analyzing an engine are specific impulse I_{sp}, characteristic velocity C^*, and mass flow rate $\dot{m}$.

To compute these values, PROMPT requires engine thrust to be provided by the user in Newtons.

The general thrust equation for a rocket nozzle is:

$$T = \dot{m}v_e + (p_e - p_a)A_e \tag{9}$$

where $\dot{m}$ is the mass flow rate, v_e is the exhaust velocity, p_e is the nozzle exit pressure, p_a is the ambient pressure, and A_e is the nozzle exit area.

If the nozzle is ideally expanded so that exit pressure is equivalent to the ambient pressure, mass flow rate in kg/s can be computed as:

$$\dot{m} = \frac{T}{v_e} \tag{10}$$

From here, you can calculate specific impulse in seconds as:

$$I_{sp} = \frac{T}{\dot{m}g_o} \tag{11}$$

where $g_o = 9.80665\ m/s^2$. Along with this, you can compute characteristic velocity, which is a measure of the energetic properties of the combustion chamber and propellants. Characteristic velocity can be computed as:

$$C^* = \frac{P_c A_t}{\dot{m}} \tag{12}$$

where A_t is the throat area of the nozzle, which will be computed in section 3.1. Using these equations, a designer can characterize and select an O/F ratio for their engine by considering chamber temperature, I_{sp}, and C^*.

3 Nozzle Sizing and Design

3.1 General Design Equations

To size a LRE nozzle, a few common parameters must be established. The two most critical geometric parameters we will cover in this section are the throat area (A_t) and exit area (A_e).

The throat area can be found with the equation

$$A_t = \frac{\dot{m}\sqrt{T_c}}{p_c}(\sqrt{\frac{\gamma}{R}}M_t(1+\frac{\gamma-1}{2}M_t^2)^{-\frac{\gamma+1}{2(\gamma-1)}})^{-1} \tag{13}$$

As M_t is equal to 1 when flow is choked, the equation can be simplified into

$$A_t = \frac{\dot{m}\sqrt{T_c}}{p_c}\sqrt{\frac{R}{\gamma}}(\frac{\gamma+1}{2})^{\frac{\gamma+1}{2(\gamma-1)}} \tag{14}$$

To find our exit area, we can use ϵ from equation 6

$$A_e = \epsilon A_t \tag{15}$$

From here, radii and diameter can be calculated, and the conical and bell nozzle design can begin in sections 3.2 and 3.3.

3.2 Conical Nozzle Design

Conical nozzles are some of the easiest nozzles to manufacture. Their simple angles make them suitable for machining on a lathe. The primary parameters in a conical nozzle design are the chamber length and diameter, throat length, and the converging and diverging angles. These are illustrated in

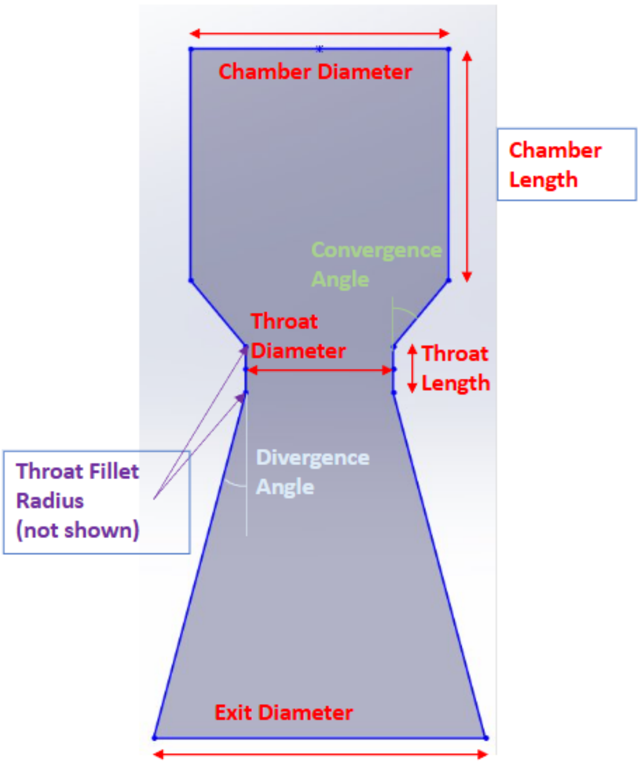

Figure 1: Conical Nozzle Geometry

Exit and throat diameter are already computable from equations 12 and 13 in section 3.1.

The two main angles to note are the converging and diverging angles. The converging angle (θ_c) determines the angle of the converging section of the nozzle. The diverging angle (θ_d) determines the angle of the divergent section. The converging angle usually ranges from 20° to 45°, while the diverging angle varies from 12° to 18°. Most modern nozzles assume a 15° divergent angle due to its ability to compromise with weight, length, and performance. PROMPT can use any angles for θ_c and θ_d, but it is recommended to use a 45-15 nozzle, with θ_c being 45° and θ_d being 15°.

The length of the divergent section can be calculated using

$$l_d = \frac{(r_e - r_t)}{\tan(\theta_d)} \tag{16}$$

where r_e and r_t are exit radius and throat radius. The throat length is also empirically found and can only be approximated until verified by CFD or live fire testing. The common ratio of throat length, τ, is 0.05 - 0.75 times the throat diameter. The equation for throat length is

$$l_t = \tau d_t \tag{17}$$

A common value for τ is 0.05.

Chamber diameter is found by the contraction ratio (β). This value is empirical and can be found using a graph from the MIT Rocketry Team. This graph is shown below:

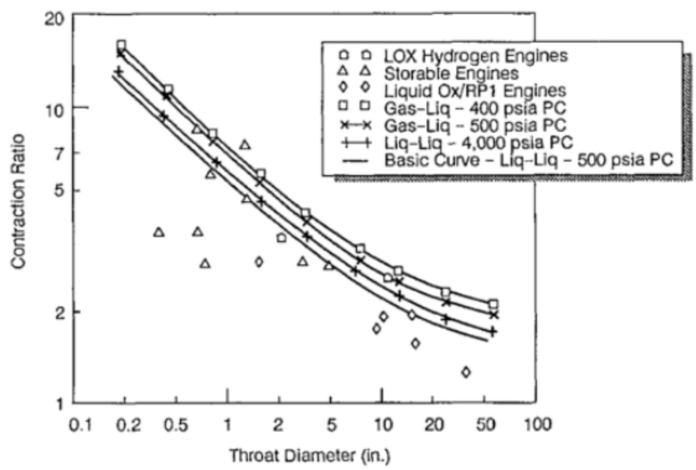

Figure 2: Contraction Ratio vs Throat Diameter

Using CR, the chamber area can be found in a similar manner to the exit area in the equation

$$A_c = \beta A_t \tag{18}$$

Using this equation, you can find your chamber area, and from there the chamber length needs to be computed. As the chamber is the primary area in a nozzle where the propellants are ignited, the chamber needs to be long enough to allow for combustion to occur. To characterise this, a useful parameter, $L*$, is used to speficiy the propellant stay time. $L*$ is defined as the ratio of chamber volume to nozzle throat area, and can be expressed with the equation

$$L^* = \frac{V_c}{A_t} \tag{19}$$

To find $L*$, tables from Huzel and Huang are provided. However, it is recommended to do research on combustion properties and find the stay time of the propellants. The tables are shown below:

Propellant combination	Combustion chamber characteristic length (L^*), in.
Chlorine trifluoride/hydrazine-base fuel	30-35
Liquid fluorine/hydrazine	24-28
Liquid fluorine/liquid hydrogen (GH_2 injection)	22-26
Liquid fluorine/liquid hydrogen (LH_2 injection)	25-30
Hydrogen peroxide/RP-1 (including catalyst bed)	60-70
Nitric acid/hydrazine-base fuel	30-35
Nitrogen tetroxide/hydrazine-base fuel	30-35
Liquid oxygen/ammonia	30-40
Liquid oxygen/liquid hydrogen (GH_2 injection)	22-28
Liquid oxygen/liquid hydrogen (LH_2 injection)	30-40
Liquid oxygen/RP-1	40-50

Figure 3: Characteristic Length Values for Common Propellant Combinations

As the volume would equal area multiplied by length, we can use the equation for chamber volume and solve it to find chamber length. The equation for chamber volume is shown below as

$$V_c = A_t(L_c\epsilon_c + \frac{1}{3}\sqrt{\frac{A_t}{\pi}}\cot\theta_c(\epsilon_c^{\frac{1}{3}} - 1)) \tag{20}$$

We can use equation 17 and solve for L_c. With this, we get the equation

$$L_c = \frac{L^* - \frac{1}{3}\sqrt{\frac{A_t}{\pi}}\cot\theta_c(\epsilon_c^{\frac{1}{3}} - 1)}{\epsilon_c} \tag{21}$$

With this, we have our chamber length and area. Using these equations, a simple conical nozzle can be designed and optimized for computational or live-fire testing.

3.3 Bell Nozzle Design

The bell nozzle was designed to be the culmination of years of research to find the 'perfect' nozzle: i.e. one that would cause the lowest thrust loss. The common method used to design a bell nozzle was the Method of Characteristics, which would relate the intersection of Mach waves from two points in a flow field. This method was complex and was unwieldy at large expansion ratios, leading to a new solution being required. The two people who found solutions to this issue are G.V.R Rao in the United States and Shmyglevsky in the former USSR. Both Rao and Shmyglevsky independently came to their conclusions, but we will be using Rao's method in PROMPT.

With Rao's Method, it was found that the length of a bell curve is equivalent to the length of an equivalent conical nozzle with a 15° divergent angle. Therefore, the length of a nozzle can be calculated by equation 14, with θ_d equalling 15°. However, empirical testing showed that after a nozzle reached 85% of the length of a bell nozzle, the performance was at 99% efficiency. Due to this, any additional increase in length would result in diminishing returns, especially when accounting for weight and size. Due to this, PROMPT currently only supports 80% bell nozzles. To account for this in length, the engine length equation is

$$l_d = 0.8\frac{(r_e - r_t)}{\tan(\theta_d)} \tag{22}$$

where 0.8 accounts for the 80% of length compared to a conical nozzle of the same angle. θ_d would equal 15 degrees in this instance.

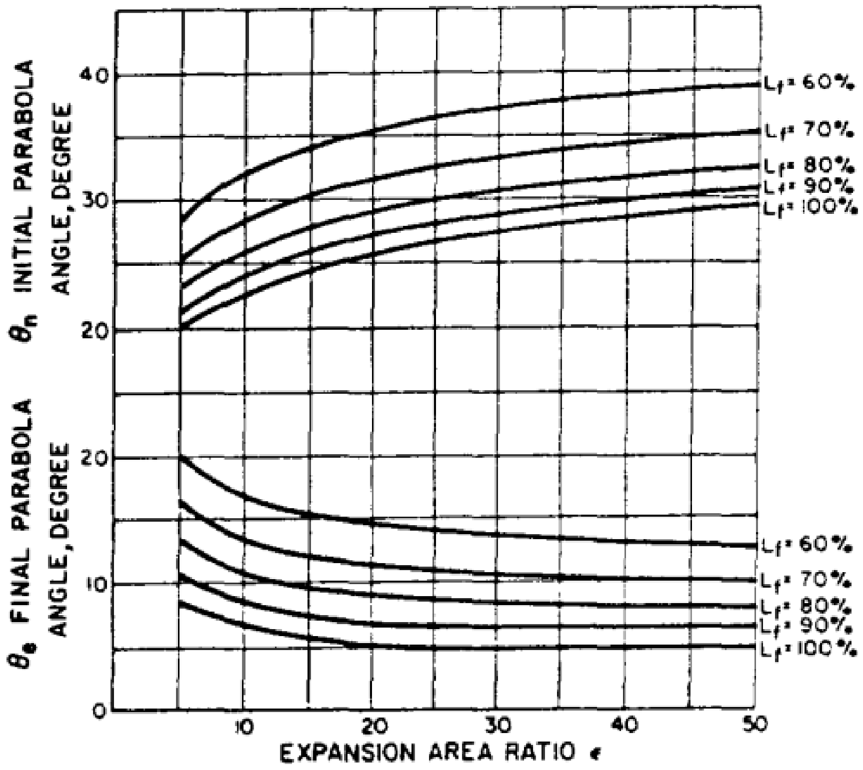

Figure 4: Curves to find Initial and Exit Angles

The first step to design a bell nozzle using the Rao Method is to find the initial and exit angles θ_n and θ_e using Figure 2. While it can be empirically estimated, PROMPT uses WebPlotDigitzer, a computer-vision-assisted tool, to extract the datapoints from the graph. After extracting points, a correlation is made using least square regression to find the data points. For θ_n, the curve is fitted with the equation

$$\theta_n = a + b\log(\epsilon) + c\log(\epsilon)^2 \tag{23}$$

and θ_e is fitted with the equation

$$\theta_e = a + be^{-c\epsilon} \tag{24}$$

From there, θ_n and θ_e can be given in degrees, and the bell nozzle can be modeled using common computer-aided design (CAD) tools.

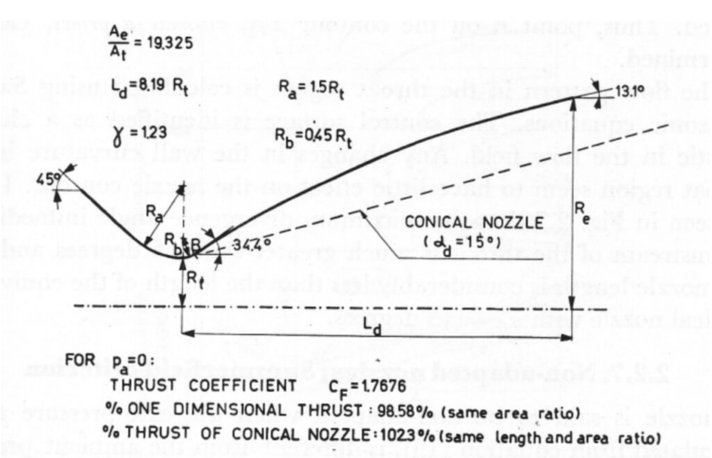

Figure 5: Rao Nozzle Engineering Drawing

However, most modern CAD tools like SolidWorks and Fusion 360 have APIs for scripting and methods to add datapoints to create shapes that are hard to design. Papers by the Reaction Rocket Society (RRS) and Rick Newlands from Aspire Space have derived methods to mathematically calculate these points to allow for quick output. PROMPT uses Rick Newlands' method of calculation by splitting the bell curve into 3 parts. The first part handles the entrant section, which is 1.5 times the throat radius. The second part covers the initial exit section, which is 0.382 times the throat radius. Finally, the bell is recreated using quadratic Bézier curves.

The first parametric equation is shown below as

$$x = 1.5R_t \cos(\theta) \tag{25}$$

$$y = 1.5R_t \sin(\theta) + 2.5R_t \tag{26}$$

from $-135 \leq \theta \leq -90$.

The second parametric equation is shown below as

$$x = 0.382R_t \cos(\theta) \tag{27}$$

$$y = 0.382R_t \sin(\theta) + 1.382R_t \tag{28}$$

from $-90 \leq \theta \leq (\theta_n - 90)$.

Finally, the bell curve equation is shown as

$$x(t) = (1 - t^2)N_x + 2(1 - t)tQ_x + t^2 E_x \tag{29}$$

$$y(t) = (1 - t^2)N_y + 2(1 - t)tQ_y + t^2 E_y \tag{30}$$

from $0 \leq t \leq 1$. Equations 24 and 25 require points N, Q, and E. Point N is found by setting equations 22 and 23 to $\theta = (\theta_n - 90)$. Point E_x is defined by equation 18 and point E_y is defined by the exit radius r_e. Point Q is defined by the equations

$$Q_x = \frac{C_2 - C_1}{m_1 - m_2} \tag{31}$$

$$Q_y = \frac{m_1 C_2 - m_2 C_1}{m_1 - m_2} \tag{32}$$

where

$$m_1 = \tan(\theta_n) \tag{33}$$

$$m_2 = \tan(\theta_e) \tag{34}$$

and

$$C_1 = N_y - m_1 N_x \tag{35}$$

$$C_2 = E_y - m_2 E_x \tag{36}$$

Using these equations, you can plot mathematical bell curves, of which a test case is shown below.

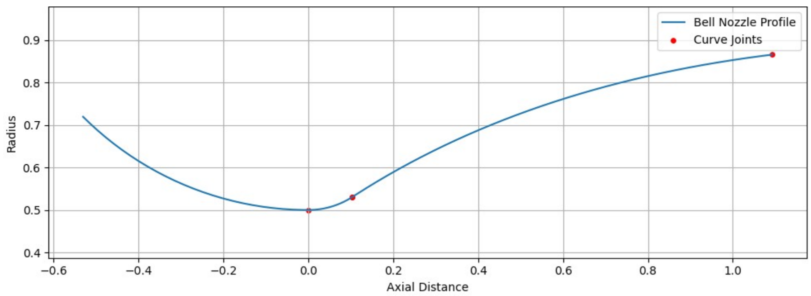

Figure 6: Test Case: $\epsilon = 3, \theta_n = 33, \theta_e = 7$

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