Elements of Computer Science: Database Assignment 1

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1.

Implementing data duplication can be beneficial in use cases that require heavy GET requests, where data is only being fetched. For example, in big data warehouses or a user dashboard service that only needs to display data, multiple data servers with duplicate data might be implemented. However, write requests must still be managed carefully.

Advantages

1. Faster Requests: Processing time can be decreased greatly because multiple data servers are ready to handle the requests.
2. Scalability: Scalability is straightforward, as it can be achieved horizontally by adding more data servers.
3. Data Recovery: This provides another key advantage; in the case of some hazard or data loss, the data can be recovered easily from an adjacent data server.
4. Reduced Dependency on Primary Node: Dependency on the primary node is greatly reduced. Even in the case of primary node failure, the system will still function fine, as adjacent nodes are available to handle the requests.

Disadvantages

1. Difficulty Maintaining Integrity: In the case of a write request, an infrastructure must be implemented to modify the respective data across all data centres. In complicated systems, this also increases the risk of data mismatch, resulting in a flawed system.

2. Increased Cost: This operation might become costly as the system evolves. Additionally, maintaining these extra data servers can be expensive to implement.

2.

For a university, data can be huge and widespread, and quick indexing is required.

To implement a reliable database for this university, the three most important user groups would be:

User Groups

1. **Students:** This user group will contain various data about students studying, including information about their courses, grades, etc.

2. **Professors and Staff:** This user group will primarily focus on storing details of professors and staff working, e.g., in administration. It can include details like personal information, salary, subjects, courses, etc.

3. **Librarian:** This user group will focus on storing details about the librarian who has to manage a huge warehouse of books.

Advantages

1. **Secure Centralized Storage:** It implements a centre where data can be stored securely; it is safe, tamper-free, and can be backed up as well to mitigate data loss.

2. **Readily Available Data:** It makes any data readily available without going through any paper or books to retrieve the data; it is accessible by querying the data. This will greatly reduce time.

3. **Shift to Online Processes:** Implementing the database system allows shifting a lot of processes online, reducing human resources, wasted time, and making it faster, more proficient, and technologically advanced.

3.

Proof of Set Equality: Ma = Mb

To prove Ma = Mb, where:

- Ma = { x | x in N and x is odd }

- Mb = { x | x in N and y in N such that x + 1 = 2y }

and assuming N = {1, 2, 3, ...}

we show mutual inclusion: Ma subset Mb and Mb subset Ma.

Part 1: Ma subset Mb

Let x be in Ma. So by definition, x = 2k + 1 for some integer k >= 0.

Thus, x + 1 = 2k + 2 = 2(k + 1).

Let y = k + 1. Since k >= 0, y >= 1, so y in N.

Therefore, there exists y in N such that x + 1 = 2y, which means x in Mb.

Hence, Ma subset Mb.

Part 2: Mb subset Ma

Let x be in Mb. such that x + 1 = 2y.

x = 2y - 1.

Since y >= 1, x = 2(1) - 1 = 1, x = 2(2) - 1 = 3, etc., so x is odd (as it is one less than a multiple of 2).

Thus, x is odd and x in N, so x in Ma.

Hence, Mb subset Ma.

Since Ma subset Mb and Mb subset Ma, it follows that Ma = Mb.

Both sets consist precisely of the odd positive integers: {1, 3, 5, 7, ...}.

4

a. Cardinality of A = 4 Cardinality of B = 3

b. Elements are ordered pairs (x,y) where the first coordinate is x and the second is y. Order matters i.e. (1,2) != (2,1). A is an equivalence relation.

c. No, A is not an equivalence relation on $\mathbb{N}$.

To determine this, A must satisfy three properties: reflexivity, symmetry, and transitivity

Reflexivity: For every n belongs to N in A. This fails because A only contains (1,1) and (2,2) , but not (3,3) or any (n, n).

Symmetry: If (x, y) in A ,then (y, x) in A . This holds, as the pairs are symmetric: (1,2) and (2,1) both appear.

Transitivity: If (x, y) in A and (y, z) in A , then (x, z) n A . This holds on the elements present (e.g., (1,2) and (2,1) imply (1,1) , which is in A).

Since reflexivity fails, A is not an equivalence relation.

d. No, they are not equal.
M0: {((1,1),(1,2,3)), ((2,2),(1,2,3) ….}

M1: {((1,2,3),(1,1),((4,5,6),(1,1)……}
These are not equal as no elements in either sets matches to other

e. Elements of M2: (1,1,1,2,3), (1,1,4,5,6), (1,1,1,1,1), (2,2,1,2,3), (2,2,4,5,6), (2,2,1,1,1), (1,2,1,2,3), (1,2,4,5,6), (1,2,1,1,1), (2,1,1,2,3), (2,1,4,5,6), (2,1,1,1,1)

Elements of M3: ((1,1),(1,2,3)), ((2,2),(1,2,3)), ((1,2),(1,2,3)), ((2,1),(1,2,3)), ((1,1),(4,5,6)), ((2,2),(4,5,6)), ((1,2),(4,5,6)), ((2,1),(4,5,6)), ((1,1),(1,1,1)), ((2,2),(1,1,1)), ((1,2),(1,1,1)), ((2,1),(1,1,1))

They are not equal, as M2 has 12 elements with each element containing 5 natural numbers, but M3 has 12 elements with each element containing 2 tuples—one with 2 natural numbers and the other with 3 natural numbers.

f. Elements of M4: (1,1,1,2,3), (1,1,1,1,1), (1,2,1,2,3), (1,2,1,1,1)

g. Elements of M5: (1,1), (2,2)