JuliaOpt: Optimization packages for Julia

Iain Dunning and Joey Huchette

Mathematical Optimization

$$\begin{aligned} \min_{x} \quad & f(x) \\ \text{subject to} \quad & g_i(x) \leq 0 \quad \forall i \end{aligned}$$

- Machine learning, resource allocation, production planning, scheduling, control...

Mathematical Optimization

Many flavours with different approaches:

- **unconstrained optimization** *(machine learning, model fitting, ...)* versus **constrained problems** *(transportation, logistics, control systems, ...)*
- continuous, combinatorial, derivative-free, online, ...

Mathematical Optimization

- Need to be able to write code that is a **natural** and **maintainable** translation of the mathematics of our problem.
- Preferably loosely coupled to the specific solution method and data, but also...
- Need specialized solvers for linear problems, linear-integer, conic, semidefinite, ...

Structure of Talk

Part 1

What are JuliaOpt, `JuMP`, and `MathProgBase`?

Part 2

How Julia makes cool things possible

What is JuliaOpt?

- A collection of high-quality optimization-related packages
  - Documentation compulsory
  - Tests compulsory
  - Cross-platform binaries
  - BinDeps
- http://juliaopt.org
- http://github.com/JuliaOpt
- julia-opt mailing list

JuliaOpt Packages (June 2014)

- JuMP
- MathProgBase
- Optim
- NLopt
- LsqFit
- Mosek
- Clp/Cbc
- Gurobi
- Ipopt
- GLPK

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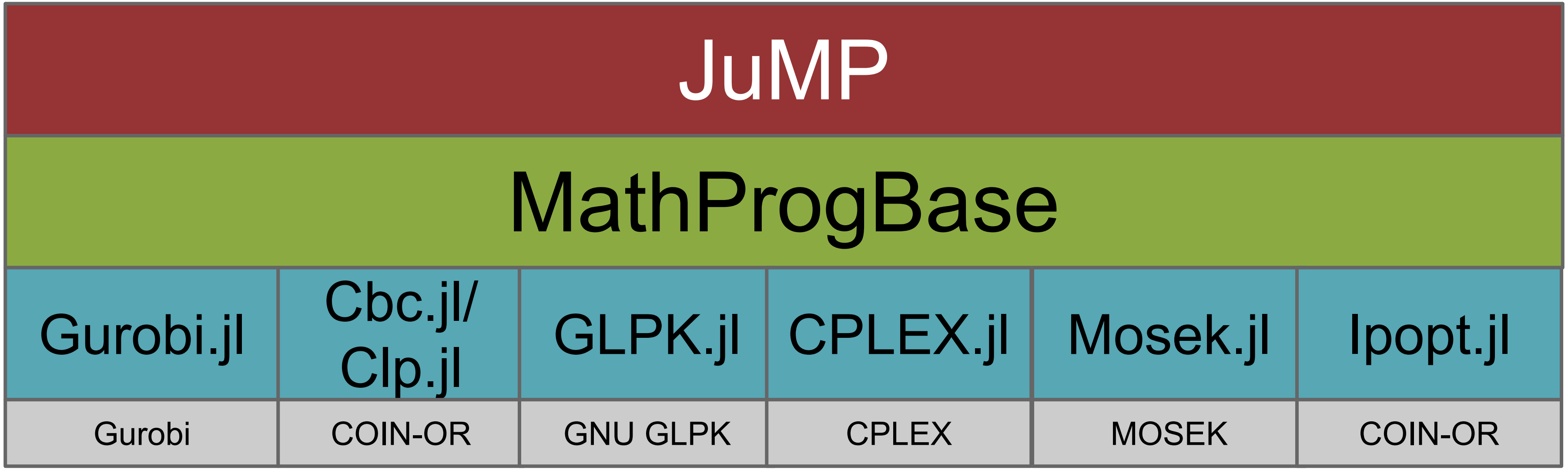
Main focus for today's talk

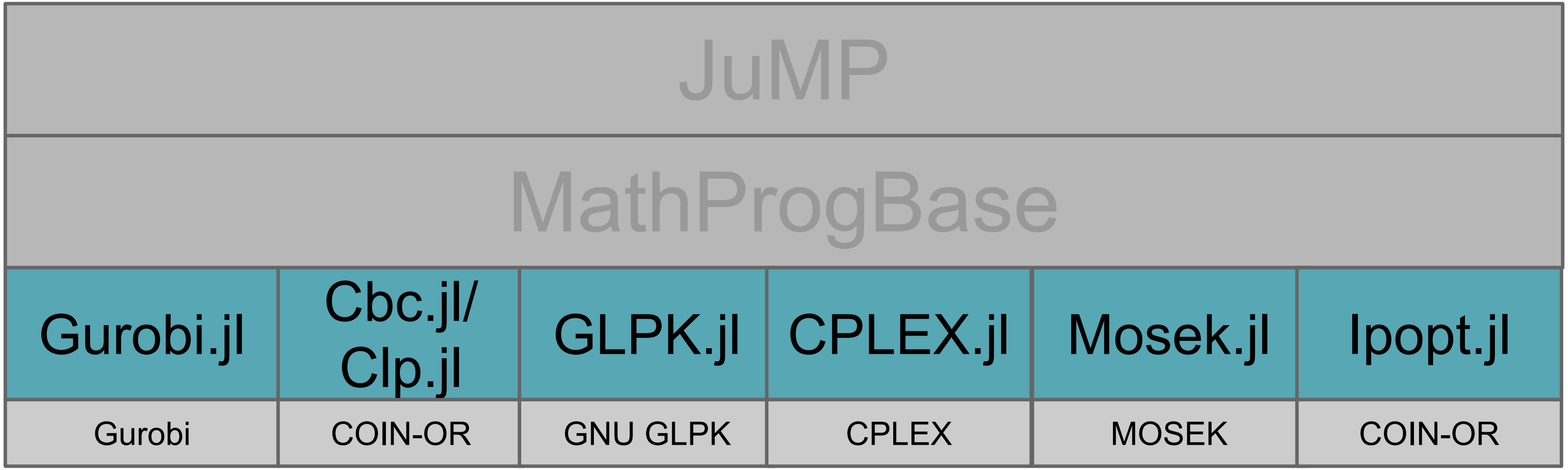
The JuliaOpt Stack

(Iain Dunning, Miles Lubin, Joey Huchette, Carlo Baldassi, Dahua Lin, Ulf Worsøe)



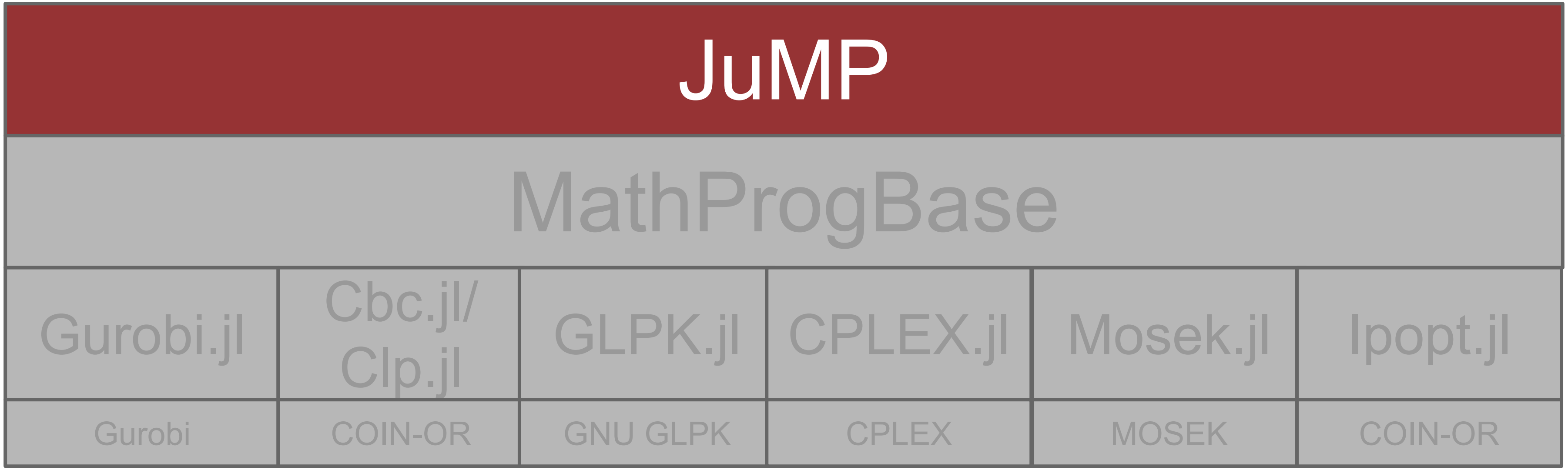
Light-weight **wrappers** around C interfaces of powerful commercial and open-source solvers

LPs, MILPs, SOCPs, nonlinear...



An **algebraic modelling language** that lets you express optimization problems naturally and quickly in Julia programs: http://github.com/JuliaOpt/JuMP

c.f. YALMIP, PuLP, AMPL, ...



JuMP: Algebraic modeling

- AMLs allow us to translate mathematical statements of optimization problems into code.
- They efficiently generate the (sparse) data structures that solvers require as input.
- *Much* easier to reason about (and more generic) than hand-coding problem structure.

JuMP Philosophy

- Make modeling quick and painless for practitioners (a la AMPL)...
- ...while allowing experts to easily use integrate advanced features (usually only available through low-level solver interfaces)
- Make it as fast as possible!

Example 1: Sudoku

5	3			7				
6			1	9	5			
	9	8					6	
8				6				3
4			8		3			1
7				2				6
	6					2	8	
			4	1	9			5
				8			7	9

- 1 to 9 in each 3x3 square
- 1 to 9 in each row
- 1 to 9 in each column
- Each cell has 1 to 9 (implied)
- Define

  $x_{ijk} = 1$ iff cell $(i,j) = k$
- MILP!

Example 1: Sudoku

all numbers appear in each row

$$\sum_{j \in \{1,\ldots,9\}} x_{ijk} = 1$$

$$\forall i \in \{1,\ldots,9\}, k \in \{1,\ldots,9\}$$

all numbers appear in each subgrid

$$\sum_{i,j=1}^{3} x_{i+I,j+J,k} = 1$$

$$\forall I, J \in \{0,3,6\}, k \in \{1,\ldots,9\}$$

JuMP + Julia

```julia
@addConstraint(m,
    row[i=1:9,k=1:9],
    sum(x[i,:,k]) == 1)
```

Gurobi + C

```c
  for (v = 0; v < DIM; v++) {
    for (j = 0; j < DIM; j++) {
      for (i = 0; i < DIM; i++) {
        ind[i] = i*DIM*DIM + j*DIM + v;
        val[i] = 1.0;
      }
      error = GRBaddconstr(model, DIM,
ind, val, GRB_EQUAL, 1.0, NULL);
      if (error) goto QUIT;
    }
  }
```

JuMP + Julia

```julia
@addConstraint(m,
    subgrid[i=1:3:7,j=1:3:7,k=1:9],
    sum(x[i:i+2,j:j+2,k]) == 1)
```

Gurobi + C

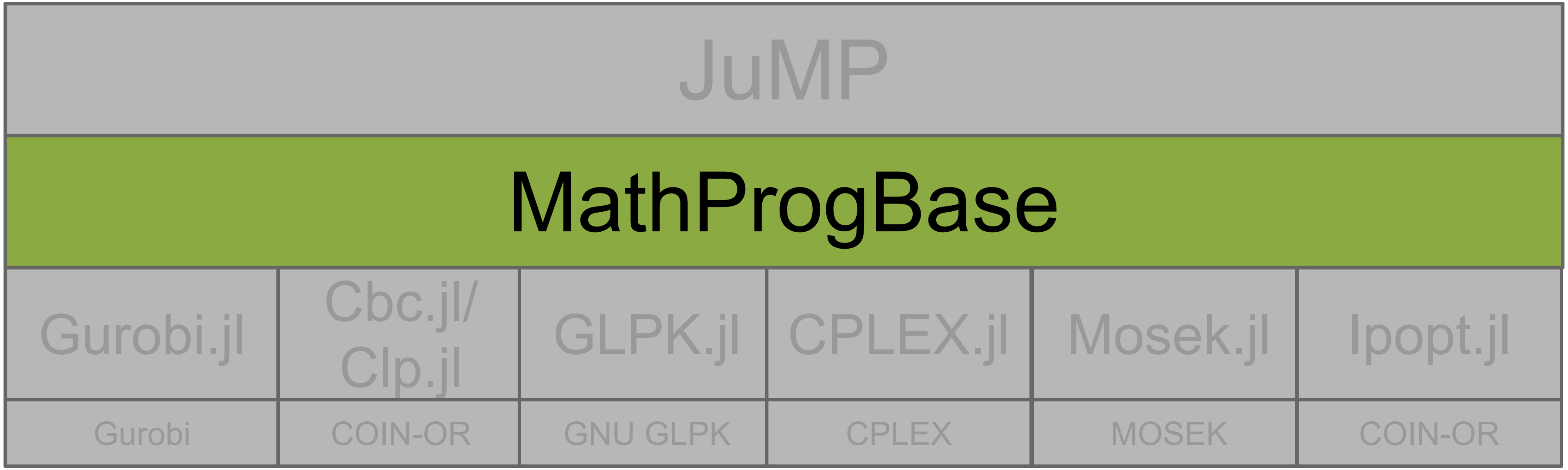
```c
for (v = 0; v < DIM; v++) {
  for (ig = 0; ig < SUBDIM; ig++) {
    for (jg = 0; jg < SUBDIM; jg++) {
      count = 0;
      for (i = ig*SUBDIM; i < (ig+1)*SUBDIM; i++) {
        for (j = jg*SUBDIM; j < (jg+1)*SUBDIM; j++) {
          ind[count] = i*DIM*DIM + j*DIM + v;
          val[count] = 1.0;
          count++;
        }
      }
      error = GRBaddconstr(model, DIM, ind, val,
GRB_EQUAL, 1.0, NULL);
      if (error) goto QUIT;
    }
  }
}
```

Example 2: Wind power dispatch

- (Very) recent work with Argonne Lab
- Front-end to custom parallel stochastic interior point solver
- JuMP: ~70 LOC for modeling, ~500 for “glue”
- C++: ~2300 LOC
- Added bonuses: fast prototyping, *generic*

A **common interface** to solvers, as well as allow MATLAB-style optimization calls, e.g

`linprog(A,b,c...)` http://github.com/JuliaOpt/MathProgBase



MathProgBase

- Defines an `AbstractMathProgSolver`, documents interface, use multiple dispatch
- **For users**: Ignore solver interface details, decouple problem from solver
- **For package developers**: create 1:1 mapping to C interface -> build MPB interface -> instantly available to users

MPB-Supported Solver Types

- LP/MILP/MIO
  - Linear and mixed-integer linear optimization
- QCQP
  - Quadratically-constrained quadratic optimization
- SDP
  - Semidefinite programming
- Coming soon: constrained nonlinear, conic

Julia makes wrapping C simple

- e.g. Ipopt.jl is a wrapper for the nonlinear interior point solver Ipopt
- ~370 SLOC including MPB interface
- Compare with Python interface PyIpopt which ~1300 SLOC

Composability of Solvers

- JuliaOpt has solvers for MILP, and **convex** QCQP, but **nonconvex** QCQP is still quite new and quality of solvers is not there yet.
  - JuMP can model them though…
  - Can be solved as sequence of MILPs...
- MPB enabled creation of solver, Junquo, in ~400 SLOC including MPB interface

Other JuliaOpt packages

- `Optim.jl` - pure Julia implementations of the standard algorithms for unconstrained or box constrained problems.
- `NLopt.jl` - interface to the NLopt constrained solver by Steven G. Johnson
- `LsqFit.jl` - least-squares fitting functionality, formerly in `Optim.jl`

Why Julia?

Generation of Efficient Code

```
@addConstraint(mod, sum{x[i],i=1:9} + s == 10)
# Generates the following code
coefs, vars = Float64[], JuMP.Variable[]
len = length(1:9)
sizehint(coefs, len); sizehint(vars, len)
for i in 1:9
    push!(coefs, 1.0); push!(vars, x[i])
end
push!(coefs, 1.0); push!(vars, s)
addConstraint(mod, coefs, vars, :(==), 1.0)
```

Generation of efficient code

$$x_{i,j} \in \mathbb{Z}^+, \quad i \in \{3,4,5\}, \; j \in \{Red, Blue\}$$

```
@defVar(m, x[3:5,[:red,:blue]] >= 0, Int)
```

- Macro generates custom `JuMPDict` type with custom `Base.getindex` etc. to access elements, e.g. `getValue(x[4,:blue])`

Why go to all this trouble?

- Other languages use operator overloading
  - Problem: lots of memory allocation due to all the temporary variables
- JuMP can JIT-compile-time analyze statements and efficiently generated the sparse data structure
  - As fast as standalone commercial AMLs

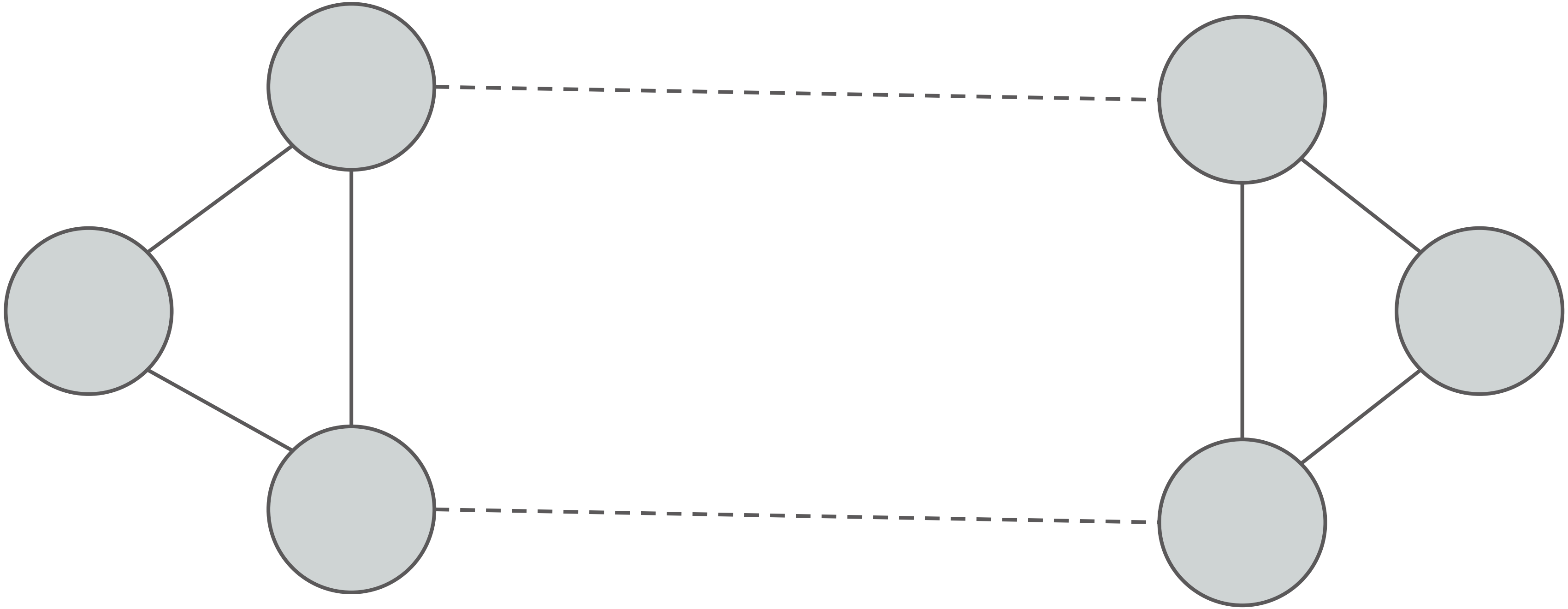
Callbacks

- Solvers allow experts to tinker with internals of optimization process with *callbacks*
- In e.g. Python, lots of work needs to be done
- With Julia, it's as simple as `cfunction` and `ccall`

Example: Lazy constraints

- Travelling Salesman Problem: find the shortest trip through N cities that visits them all once and ends up where you started.
- We can write down an optimization problem to solve it, but it has an exponential number of constraints that eliminate subtours.
- Add as needed: “lazy” constraints

Subtours in TSP



Solving TSP with JuMP

- Create JuMP model with only the constraints that ensure each city is visited once
- Write callback that analyses intermediate solutions, and finds the constraint to eliminate the current incorrect solution
- Julia advantage: low overhead C interop
- JuliaOpt/JuMP unique: solver-indep. callbacks

Derivatives

- Optimization methods perform better when derivatives available. Two options:

Provide derivatives	Estimate derivatives
• often complex • error-prone	• inefficient • inaccurate

JuliaDiff http://juliadiff.org

- Use type system, multiple dispatch, metaprogramming to get **exact derivatives**
  - `DualNumbers`,`HyperDualNumbers`, `PowerSeries`,`...` create new data types that are used in place of e.g. `Float64`
  - `ReverseDiff[Sparse|Source|Overload]` implement automatic differentiation, operate on AST (as does `Calculus`)

How are they used?

- `Optim` and `NLSolve` use `DualNumbers`
  - Just set `autodiff=true`
- `JuMP` uses `ReverseDiffSparse`
  - Compute gradients and sparse Hessians for efficient interior-point solvers
- `MCMC` uses `ReverseDiffSource`
  - Compute gradients of statistical models

What's next for JuliaOpt?

- Constraint programming with SCIP
- JuMP still adding features:
    - SDP modeling, presolve, reformulations...
- DCP via CVX.jl is under development @Stanford - talk to Madeleine
- Solvers written in pure Julia, esp. leveraging distributed computing?

Problem modification

- Solver-level model persists in memory after optimization
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