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Practical no: 1

Topic :- Limits & continuity

(1) $\lim_{x \to a} \left[\dfrac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \right]$

$$\lim_{x \to a} \left[\frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \times \frac{\sqrt{a+2x} + \sqrt{3x}}{\sqrt{a+2x} + \sqrt{3x}} \times \frac{\sqrt{3a+x} + 2\sqrt{x}}{\sqrt{3a+x} + 2\sqrt{x}} \right]$$

$$\lim_{x \to a} \frac{(a+2x-3x)(\sqrt{3a+x} + 2\sqrt{x})}{(3a+x-4x)(\sqrt{a+2x} + 3\sqrt{x})}$$

$$\lim_{x \to a} \frac{(a-x)(\sqrt{3a+x} + 2\sqrt{x})}{(3a-3x)(\sqrt{a+2x} + 3\sqrt{x})}$$

$$\frac{1}{3} \lim_{x \to a} \frac{(a-x)(\sqrt{3a+x} + 2\sqrt{x})}{(a-x)(\sqrt{a+2x} + 3\sqrt{x})}$$

$$\frac{1}{3} \times \frac{\sqrt{4a} + 2\sqrt{a}}{\sqrt{3a} + \sqrt{3a}}$$

$$\frac{1}{3} \times \frac{\sqrt{2a} + 2\sqrt{a}}{\sqrt{3a} + \sqrt{3a}}$$

$$\frac{1}{3} \times \frac{\cancel{4}^{2}\,\cancel{\sqrt{a}}}{\cancel{2}\,\cancel{\sqrt{a}}\,\sqrt{3}}$$

$$= \frac{2}{3\sqrt{3}}$$

2.] $\lim_{y \to 0} \left[\frac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}} \right]$

$\rightarrow \lim_{y \to 0} \left[\frac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}} \times \frac{\sqrt{a+y} + \sqrt{a}}{\sqrt{a+y} + \sqrt{a}} \right]$

$\lim_{y \to 0} \frac{a+y-a}{y\sqrt{a+y}(\sqrt{a+y} + \sqrt{a})}$

$\lim_{y \to 0} \frac{\cancel{y}}{\cancel{y}\sqrt{a+y}(\sqrt{a+y} + \sqrt{a})}$

$\frac{1}{\sqrt{a+0}(\sqrt{a+0} + \sqrt{a})}$

$\frac{1}{\sqrt{a}(\sqrt{a} + \sqrt{a})}$

$\frac{1}{\sqrt{a}(2\sqrt{a})} = \frac{1}{2a}$.

3.] $\lim_{x \to \pi/6} \frac{\cos x - \sqrt{3}\sin x}{\pi - 6x}$

By Substituting $x - \frac{\pi}{6} = h$

$x = \frac{\pi}{6} = h$

$x = h + \pi/$

$$\lim_{h \to 0} \frac{\cos\left(h+\frac{\pi}{6}\right) - \sqrt{3}\sin\left(h+\frac{\pi}{6}\right)}{\pi - 6\left(h+\frac{\pi}{6}\right)}$$

using
$\cos(A+B) = \cos A \cdot \cos B - \sin A B$
$\sin(A+B) = \sin A \cos B + \cos A \sin A \sin B$

$$\lim_{h \to 0} \frac{\cosh \cdot \cos \pi/6 - \sin h \sin \frac{\pi}{6} - \sqrt{3}\sin h \cos\frac{\pi}{6} + \cos h \sin\frac{\pi}{6}}{\pi - \cancel{6}\left(\frac{6h+\pi}{\cancel{6}}\right)}$$

$\cos\frac{\pi}{6} = \cos 30°$

$\sin\frac{\pi}{6} = \sin 30° = \frac{1}{\sqrt{2}}$

$$\lim_{h \to 0} \frac{\cosh \cdot \frac{\sqrt{3}}{2} - \sin h \frac{1}{\sqrt{2}} - \sqrt{3}\left(\sin h \cdot \frac{\sqrt{3}}{2} + \cos h \cdot \frac{1}{\sqrt{2}}\right)}{\cancel{\pi - 6\left(\frac{6h+\pi}{6}\right)}}$$

$\pi - 6h - \pi$

$$\lim_{h \to 0} \frac{\cos \cancel{h} \cancel{\frac{\sqrt{3}h}{2}} - \sin\frac{h}{2} - \sin\frac{3h}{2} - \cancel{\cos\frac{\sqrt{3}h}{2}}}{-6h}$$

$$\lim_{h \to 0} \frac{-\sin\frac{4h}{2}}{-6h}$$

$$\lim_{h \to 0} \frac{\sin 4h}{3\sqrt{2}h} \Rightarrow \frac{1}{3} \lim_{h \to 0} \frac{\sin h}{h} = \frac{1}{3} \times 1 = \frac{1}{3}$$

4.] $\lim_{x\to\infty}\left[\dfrac{\sqrt{x^2+5}-\sqrt{x^2-3}}{\sqrt{x^2+3}-\sqrt{x^2+1}}\right]$

By rationalizing Numerator and Denominator we

$$\lim_{x\to\infty}\left[\frac{\sqrt{x^2+5}-\sqrt{x^2-3}}{\sqrt{x^2+3}-\sqrt{x^2+1}}\times\frac{\sqrt{x^2+5}+\sqrt{x^2-3}}{\sqrt{x^2+5}+\sqrt{x^2-3}}\times\frac{\sqrt{x^2+3}+\sqrt{x^2+1}}{\sqrt{x^2+3}+\sqrt{x^2+1}}\right]$$

$$\lim_{x\to\infty}\left[\frac{(x^2+5-x^2+3)(\sqrt{x^2+3}+\sqrt{x^2+1})}{(x^2+3-x^2-1)(\sqrt{x^2+5}+\sqrt{x^2-3})}\right]$$

$$\lim_{x\to\infty}\frac{\overset{4}{\cancel{8}}(\sqrt{x^2+3}+\sqrt{x^2+1})}{\cancel{2}(\sqrt{x^2+5}+\sqrt{x^2-3})}$$

$$4\lim_{x\to\infty}\frac{\sqrt{x^2\left(1+\frac{3}{x^2}\right)}+\sqrt{x^2\left(1+\frac{1}{x^2}\right)}}{\sqrt{x^2\left(1+\frac{5}{x^2}\right)}+\sqrt{x^2\left(1-\frac{3}{x^2}\right)}}$$

After applying limit.
we get,

$=\underline{4}$

5.] $f(x)=\dfrac{\sin 2x}{\sqrt{1-\cos 2x}}$, for $0<x\leq \pi/2$

$=\dfrac{\cos x}{\pi-2x}$, for $\dfrac{\pi}{2}<x<\pi$

at $x=\pi/2$

$$f(\pi/2)=\frac{\sin 2(\pi/2)}{\sqrt{1-\cos 2(\pi/2)}}\qquad \therefore f(\pi/2)=0$$

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f at $x = \pi/2$ define

$$\lim_{x \to \pi/2} f(x) = \lim_{x \to \pi/2} \frac{\cos 2x}{\pi - 2x}$$

$$x - \frac{\pi}{2} = h$$

$$x = h + \pi/2.$$

where $h \to 0$

$$\lim_{h \to 0} \frac{\cos\left(h + \pi/2\right)}{\pi - 2\left(h + \frac{\pi}{2}\right)}$$

$$\lim_{h \to 0} \frac{\cos\left(h + \pi/2\right)}{\pi - \cancel{2}\left[\frac{2h+\pi}{\cancel{2}}\right)}$$

~~lim~~

$$\lim_{h \to 0} \frac{\cos\left[h + \pi/2\right]}{\cancel{\pi} - 2h - \cancel{\pi}}$$

$$\lim_{h \to 0} \frac{\cos\left[h + \frac{\pi}{2}\right]}{-2h}$$

$= \lim$ ~~cos~~ $\Big[$ ~~h+~~ $\cos h \cdot \cos \pi/2 - \sin h \cdot \sin \pi/2 \Big]$

$$\lim_{x \to \pi/2} Rf(x) = \lim_{x \to \pi/2} \frac{\sin 2x}{\sqrt{1-\cos 2x}}.$$

$$\lim_{x \to \pi/2} f(x) = \lim_{x \to \pi/2} \frac{2\sin x \cdot \cos x}{\sqrt{2\sin^2 x}}.$$

$$= \lim_{x \to \pi/2} \frac{2\cos \pi}{\sqrt{2}}$$

$$= \frac{2}{\sqrt{2}} \cdot \lim_{x \to \pi/2} \cos x.$$

L.H.S $\neq$ R.H.S

f is not continous at $x = \pi/2$

(ii) $f(x) = \frac{x^2-9}{x-3}$ $\quad 0 < x < 3$

$$\left.\begin{array}{ll} = x+3 & 0 < x < 3 \\ = \frac{x^2-9}{x+3} & 3 \le x < 6 \\ & 6 \le x < 9 \end{array}\right\} \text{at } x=3 \text{ \& } x=6.$$

at $x = 3$

$$f(3) = \frac{x^2-9}{x-3} = 0$$

f at $x=3$ define

1.] $f(3) \cancel{=} \frac{x^2-9}{x-3} = 0$

f at $x=3$. define

2.] $\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} x+3$

$f(3) = x+3 = 3+3 = 6$

f is define at $x=3$

$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} (x+3) = 6$

$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} \frac{x^2-9}{x-3} = \frac{\cancel{(x-3)}(x+3)}{\cancel{(x-3)}}$

$\therefore$ LHL = RHL

f is continous at $x=3$

For $x=6$

$f(6) = \frac{x^2-9}{x+3} = \frac{36-9}{6+3} = \frac{\cancel{27}3}{\cancel{9}} = 3$

6] (i) $f(x) = \frac{1-\cos 4x}{x^2}$ $x<0$ $\}$ at $x=0$

$= k$ $x=0$

Soln → f is continous at $x=0$

$\lim_{x \to 0} f(x) = f(0)$

$\lim_{x \to 0} \frac{1-\cos 4x}{x^2} = k$

$\lim_{x \to 0} \frac{2\sin^2 2x}{x^2} = k$

$2\lim_{x \to 0} \frac{\sin^2 2x}{x^2} = k$

$2\lim_{x \to 0} \left(\frac{\sin 2x}{x}\right)^2 = k \Rightarrow 2(2)^2 = k$

$k = 8$

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(iii) If $f(x) = (\sec^2 x)^{\cot^2 x}$ $\left.\begin{matrix} x \neq 0 \\ x = 0 \end{matrix}\right\}$ at $x = 0$

$= k$

Sol^n → $f(x) = (\sec^2 x)^{\cot^2 x}$

$$\lim_{x \to 0} (\sec^2 x)^{\cot^2 x}$$

$$\lim_{x \to 0} (1 + \tan^2 x)^{1/\tan^2 x}$$

we know that
$$\lim_{x \to 0} (1+px)^{1/px} = e$$

$\therefore = e$

$\therefore k = e.$

(i) $f(x) = \dfrac{\sqrt{3} - \tan x}{\pi - 3x}$ $\left.\begin{matrix} x \neq \pi/3 \\ x = \pi/3 \end{matrix}\right\}$ at $x = \pi/3$

$= k$

$x - \pi/3 = h$

$x = h + \pi/3$

where $h \to 0$.

$$f(\pi/3 + h) = \frac{\sqrt{3} - \tan(\pi/3 + h)}{\pi - 3(\pi/3 + h)}$$

$$\lim \sqrt{3} - \tan(\pi/3 + h)$$

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$$\lim_{h\to 0} \frac{\dfrac{\sqrt{3} - \tan\pi/3 + \tan h}{1-\tan\pi/3 \cdot \tan h}}{\cancel{\pi} - \cancel{\pi} - 3h}$$

$$\lim_{h\to 0} \frac{\dfrac{\sqrt{3}\left(1-\tan\frac{\pi}{3}\cdot\tan h\right)\cdot\left(\tan\frac{\pi}{3}+\tan h\right)}{1-\tan\frac{\pi}{3}\cdot\tan h}}{-3h}$$

$$\lim_{h\to 0} \frac{\dfrac{(\sqrt{3}-\sqrt{3}\times\sqrt{3}\cdot\tan h)-(\sqrt{3}+\tan h)}{1-\tan\pi/3\cdot\tan h}}{-3h}$$

$$\lim_{h\to 0} \frac{\dfrac{\sqrt{3}-3-\tan h)-(\sqrt{3}+\tan h)}{1-\sqrt{3}\cdot\tan h}}{-3h}$$

$$\lim_{h\to 0} \frac{\dfrac{(\sqrt{3}-3\tan h-\sqrt{3}-\tan h)}{1-\sqrt{3}\cdot\tan h}}{-3h}$$

$$\lim_{h\to 0} \frac{-4\tan h}{-3h(1-\sqrt{3}\tan h)}$$

$$\lim_{h\to 0} \frac{4\tan h}{3h(1-\sqrt{3}\tan h)}$$

$$\frac{4}{3}\lim_{h\to 0}\frac{\tan h}{h} \quad \lim_{h\to 0}\frac{2}{(1-\sqrt{3}\tan h}$$

$$\frac{4}{3}\cdot\frac{1}{(1-\sqrt{3}(0))} = \frac{4}{3}\left(\frac{1}{1}\right) = \frac{4}{3}$$

1) $f(x) = \left.\begin{matrix} \frac{1-\cos 3x}{x\tan x} & x \neq 0 \\ = 9 & x = 0 \end{matrix}\right\}$ at $x=0$ 7}

$f(x) = \frac{1-\cos 3x}{x\tan x}$

$\lim_{x \to 0} = \frac{2\sin^2 3/2\, x}{x\tan x}$

$\lim_{x \to 0} = \frac{\frac{2\sin^2 \frac{3x}{2}}{x^2} \times x^2}{\frac{x \cdot \tan x}{x^2} \times x^2}$

$= 2 \lim_{x \to 0} \frac{(3/2)^2}{1} = 2 \times \frac{9}{\cancel{4}\, 2} = \frac{9}{2}$

$\lim_{x \to 0} f(x) = \frac{9}{2}$ $\quad 9 = f(0)$

$\therefore$ f is not continous at $x=0$
Redefine function.

$f(x) = \left\{ \begin{matrix} \frac{1-\cos 3x}{x\tan x} & x \neq 0 \\ 9/2 & x = 0 \end{matrix} \right\}$

Now $\lim_{x \to 0} f(x) = f(0)$

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f has removable discontinuity at $x=0$

7)(ii) $f(x) = \frac{(e^{3x}-1)\sin x^\circ}{x^2} \quad x \neq 0$
$= \pi/60 \quad x=0$ } at $x=0$

$$\lim_{x\to 0} \frac{(e^{3x}-1)}{x} \frac{\sin\left(\frac{\pi x}{180}\right)}{x}$$

$$\lim_{x\to 0} \frac{3 \cdot e^{x}-1}{3x} \lim_{x\to 0} \sin\left(\frac{\pi x}{180}\right)$$

$$3 \lim_{x\to 0} \frac{e^{x}-1}{3x} \lim_{x\to 0} \frac{\sin\left(\frac{\pi x}{180}\right)}{x}$$

$$\lim_{x\to 0} \frac{3 \cdot e^{3x}-1}{3x} \lim_{x\to 0} \frac{\sin\left(\frac{\pi/x}{180}\right)}{x}$$

$$3 \lim_{x\to 0} \frac{e^{3x}-1}{3x} \lim_{x\to 0} \sin\left(\frac{\pi/x}{180}\right)$$

$$\cancel{3 \log e} \, \pi/\frac{180}{60} = \pi/60 = f(0)$$

f is continous at $x=0$

8.] $F(x) = \dfrac{e^{x^2} - \cos x}{x^2}$ $x=0$

is continous at $x=0$

given :→

f is continous at $x=0$

$$\lim_{x\to 0} F(x) = F(0)$$

$$\lim_{x\to 0} \frac{e^{x^2} - \cos x}{x^2} = f(0)$$

$$\lim_{x\to 0} \frac{e^{x^2} - \cos x - 1 + 1}{x^2}$$

$$\lim_{x\to 0} \frac{(e^{x^2} - 1)}{x^2} + \lim_{x\to 0} \frac{1 - \cos x}{x^2}$$

$$\log e + \lim_{x\to 0} \frac{2\sin^2 x/2}{x^2}$$

$$\log e + 2 \lim_{x\to 0} \left(\frac{\sin x/2}{x} \right)^2$$

Multiply with 2 on Num and Denominator

$$= 1 + \cancel{2} \times \frac{1}{\cancel{4}_2} = \frac{3}{2} = F(0)$$

9.]

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9] $f(x) = \dfrac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x}$, $x = \pi/2$.

$f(0)$ is continous at $x = \pi/2$

$\lim_{x \to \pi/2} \dfrac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x} \times \dfrac{\sqrt{2} + \sqrt{1+\sin x}}{\sqrt{2} + \sqrt{1+\sin x}}$

$\lim_{x \to \pi/2} \dfrac{2 - 1 + \sin x}{\cos^2 x\,(\sqrt{2} + \sqrt{1+\sin x}}$

$= \lim_{x \to \pi/2} \dfrac{1+\sin x}{1-\sin^2 x\,(\sqrt{2} + \sqrt{1+\sin x}}$

$= \lim_{x \to \pi/2} \dfrac{(1-\sin)\ \cancel{1+\sin x}}{(1 \cancel{+\sin x})(\sqrt{2} + \sqrt{1+\sin x})}$

$= \lim_{x \to \pi/2} \dfrac{1}{(1-\sin x)(\sqrt{2} \cdot \sqrt{1+\sin x})}$

$= \dfrac{1}{2(\sqrt{2}+\sqrt{2})}$

$= \dfrac{1}{2(2\sqrt{2}}$ $= \dfrac{1}{4\sqrt{2}}$

$\therefore f(\pi/2) = \dfrac{1}{4\sqrt{2}}$

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Practical no: 2

Derivatives

Show that following function defined from R are differentiable.

(i) cot x

$f(x) = \cot(x)$

$$Df(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{x \to a} \frac{\cot x - \cot a}{x - a}$$

$$= \lim_{x \to a} \frac{\frac{1}{\tan x} - \frac{1}{\tan a}}{x - a}$$

$$= \lim_{x \to a} \frac{\tan a - \tan x}{(x - a)\tan a \cdot \tan x}$$

Put $x - a = h$

$x = a + h$

as $x \to a$, $h \to 0$

$$Df(h) = \lim_{h \to 0} \frac{\tan a - \tan(a+h)}{(a+h-a)\tan(a+h)\tan a}$$

$$= \lim_{h \to 0} \frac{\tan a - \tan(a+h)}{h \times \tan(a+h)\tan a}$$

Formula: $\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \cdot \tan B}$

$\tan A - \tan B = \tan(A - B)(1 + \tan A \cdot \tan B)$

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$$= \lim_{h\to 0} \frac{\tan(a-a-h) - (1+\tan A) + \tan(a+h))}{h \times \tan(a+h)\tan a}$$

$$= \lim_{h\to 0} -\frac{\tan h}{h} \times \frac{1+\tan a \tan(a+h)}{\tan(a+h)\cdot \tan a}$$

$$= -1 \times \frac{1-\tan^2 a}{\tan^2 a}$$

$$= \frac{-\sec^2 a}{\tan^2 a} = \frac{-1}{\cancel{\cos^2 a}} \times \frac{\cancel{\cos^2 a}}{\sin^2 a} = \underline{\operatorname{cosec}^2 a}$$

$Df(a) = -\cos^2 a$

$\therefore$ f is differentialise $\forall a \in R$

(ii) $\operatorname{cosec} x$

$f(x) = \operatorname{cosec} x$

$$Df(a) = \lim_{x\to 0} \frac{f(x)-f(a)}{x-a}$$

$$= \lim_{x\to a} \frac{\operatorname{cosec} x - \operatorname{cosec} a}{x-a}$$

$$= \lim_{x\to a} \frac{\frac{1}{\sin x} - \frac{1}{\sin a}}{x-a}$$

$$= \lim_{x\to a} \frac{\sin a - \sin x}{(x-a)\sin a \sin x}$$

Put $x-a=h$

$x = a+h$

as $x \to a$, $h \to 0$

$$Df(h) = \lim_{h\to 0} \frac{\sin a - \sin(a+h)}{(a+h-a)\sin a \cdot \sin(a+h)}$$

Formula: $\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\cdot \sin\left(\frac{C-D}{2}\right)$

$$\lim_{h\to 0} \frac{2\cos\left(\frac{a+a+h}{2}\right)\cdot\sin\left(\frac{a-a+h}{2}\right)}{h\times\sin a - \sin(a+h)}$$

$$= \lim_{h\to 0} \frac{-\sin h/2}{h/2} \times \frac{1}{2} \times \frac{2\cos\left(\frac{2a+h}{2}\right)}{\sin a\cdot\sin(a+h)}$$

$$= \frac{-\cos a}{\sin^2 a} = \cot a\cdot\operatorname{cosec} a.$$

(iii) $\sec^2 x$

$\to f(x) = \sec x$

$$Df(a) = \lim_{x\to 0} \frac{f(x)-f(a)}{x\to a}$$

$$= \lim_{x\to a} \frac{\sec x - \sec a}{x\to a}$$

$$= \lim_{x\to a} \frac{1/\cos x - 1/\cos a}{x\to a}$$

$$= \lim_{x\to a} \frac{\cos a - \cos x}{\cos a\cdot\cos x\ (x-a)}$$

Put $x - a = h$

$x = a+h$

as $x\to a$, $h\to 0$

$$Df(h) = \lim_{h\to 0} \frac{\cos a - \cos(a+h)}{h\times\cos a\cdot\cos(a+h)}$$

Formula:- $2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

$$\lim_{h\to 0} \frac{-2\sin\left(\frac{a+a+h}{2}\right)\cdot\sin\left(\frac{a-a-h}{2}\right)}{h\times\cos a\cdot\cos(a+h)}$$

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$$\lim_{h \to 0} \frac{-2\sin\left(\frac{2a+h}{2}\right) \cdot \sin\left(-h/2\right)}{\cos a \cdot \cos(a+h) \times \left(-h/2\right)} \cdot -1/2$$

$$= \frac{-1}{2} \times \frac{-2\sin\left(\frac{2a+0}{2}\right)}{\cos a \cdot \cos(a+0)}$$

$$= \frac{\cancel{-}1}{\cancel{2}} \times \cancel{-}2 \frac{\sin a}{\cos a \times \cos a}$$

$$= \tan a \cdot \sec a$$

Q.2] If $f(x) = 4x+1 \quad x \leq 2$

$= x^2+5 \quad x > 0$ at $x = 2$, then

find function is differentiable or not.

Sol^n → L.H.S

$$Df(2^-) = \lim_{x \to 2^-} \frac{f(x) - f(2)}{x-2}$$

$$= \lim_{x \to 2^-} \frac{4x+1 - (4 \times 2 + 1)}{x-2}$$

$$= \lim_{x \to 2^-} \frac{4x+1-9}{x-2}$$

$$= \lim_{x \to 2^-} \frac{4x-8}{x-2}$$

$$= \lim_{x \to 2^-} \frac{4\cancel{(x-2)}}{\cancel{(x-2)}} = 4$$

$$Df(2^-) = 4$$

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R.H.D =

$$Df(2^+) = \lim_{x \to 2^+} \frac{x^2+5-9}{x-2}$$

$$= \lim_{x \to 2^+} \frac{x^2-4}{x-2}$$

$$= \lim_{x \to 2^+} \frac{(x+2)\cancel{(x-2)}}{\cancel{(x-2)}}$$

$$= 2+2$$

$$= \underline{\underline{4}}$$

$Df(2^+) = 4$

R.H.D = L.H.D

$\therefore$ F is differentiable at $x=2$.

If $f(x) = 4x+7 \quad , x<3$

$= x^2+3x+1 \quad , x \geq 3$ at $x=3$, then

F is ~~not~~

Find F is differentiable or not?

Solution →

RHD :

$$Df(3^+) = \lim_{x \to 3^+} \frac{f(x)-f(3)}{x-3}$$

$$= \lim_{x \to 3^+} \frac{x^2+3x+1-(3^2+3\times3\times1)}{x-3}$$

$$= \lim_{x \to 3^+} \frac{x^2+3x+1-19}{x-3}$$

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$$= \lim_{x \to 3^+} \frac{x^2 + 6x - 3x - 18}{x - 3}$$

$$= \lim_{x \to 3^+} \frac{x(x+6) - 3(x+6)}{x - 3}$$

$$= \lim_{x \to 3^+} \frac{(x+6)\cancel{(x-3)}}{\cancel{(x-3)}} = 3+6 = 9$$

$$DF(3^+) = 4$$

RHD $\neq$ LHD

f is not differentiable at $x=3$

Q.4] If $f(x) = 8x - 5 \quad , x \leq 2$

$= 3x^2 - 4x + 7 \quad , x > 2$ at $x = 2$, then

find f is differentiable or not.

Solution:

$$F(2) = 8 \times 2 - 3 = 16 - 3 = 11$$

R.H.D :

$$DF(2^+) = \lim_{x \to 2^+} \frac{f(x) - f(a)}{x - 2}$$

$$= \lim_{x \to 2^+} \frac{3x^2 - 4x + 7 - 11}{x - 2}$$

$$= \lim_{x \to 2^+} \frac{3x^2 - 4x + 4}{x - 2}$$

$$= \lim_{x \to 2^+} \frac{3x^2 - 6x + 2x - 4}{x - 2}$$

$$= \lim_{x \to 2^+} \frac{3x(x-2) + 2(x-2)}{x - 2}$$

$$= \lim_{x \to 2^+} \frac{(3x+2)\cancel{(x-2)}}{\cancel{(x-2)}}$$

$$= 3 \times 2 + 2 = 8$$

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$DF(2^+) = 8$

LHD:

$$DF(2^-) = \lim_{x \to 2^-} \frac{F(x) - F(2)}{x - 2}$$

$$= \lim_{x \to 2^-} \frac{8x - 5 - 11}{x - 2}$$

$$= \lim_{x \to 2^-} \frac{8x - 16}{x - 2}$$

$$= \lim_{x \to 2^-} \frac{8(x - 2)}{(x - 2)}$$

$$= 8$$

$DF(2^-) = 8$

LHD = RHD

F is differentiable at $x = 3$

Practical - 3

Topic :-> Application of Derivative

1] (i) $f(x) = x^3 - 5x - 11$

$\therefore f'(x) = 3x^2 - 5$

$\because f$ is increasing iff $f(x) > 0$

$3x^2 - 5 > 0$

$3(x^2 - 5) > 0$

$(x - \sqrt{5/3})(x + \sqrt{5/3}) > 0$

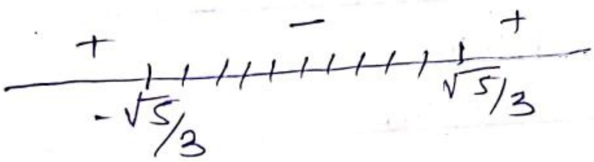
$x \in (-\infty, -\sqrt{5}/3) \cup (\sqrt{5}/3, \infty)$

and f is decreasing iff $f'(x) < 0$

$\therefore 3x^2 - 5 < 0$

$\therefore 3(x^2 - 5/3) < 0$

$\therefore (x - \sqrt{5/3})(x + \sqrt{5/3}) < 0$



$x \in (\sqrt{5}/3, \sqrt{5}/3)$

2] $f(x) = x^2 - 4x$

$f'(x) = 2x - 4$

$\because f(x)$ is increasing iff $f'(x) > 0$

$\therefore 2x - 4 > 0$

$\therefore 2(x - 2) > 0$

$\therefore x - 2 > 0$

$x \in (2, \infty)$

and f is decreasing iff $f'(x) < 0$

$\therefore 2x - 4 < 0$

$\therefore 2(x - 2) < 0$

$\therefore x - 2 < 0$

$x \in (-\infty, 2)$

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3] $f(x) = 2x^3 + x^2 - 20x + 4$

$\therefore f'(x) = 6x + 2x - 20$

$\because f(x)$ is increasing iff $f'(x) > 0$

$\therefore 6x^2 + 2x - 20 > 0$

$\therefore 2\ (3x^2 + x - 10) > 0$

$\therefore 3x^2 + x - 10 > 0$

$\therefore 3x^2 + 6x - 5x - 10 > 0$

$\therefore 3x \cdot (x+2) - 5\ (x+2) > 0$

$\therefore (x+2)\ (3x+5) > 0$

$x \in (-\infty, -2) \cup (5/3, \infty)$

4] $f(x) = x^3 - 27x + 5$

$f'(x) = 3x^2 - 27$

$\because f$ is increasing iff $f'(x) > 0$

$\therefore 3\ (x^2 - 9) > 0$

$\therefore (x-3)\ (x+3) > 0$

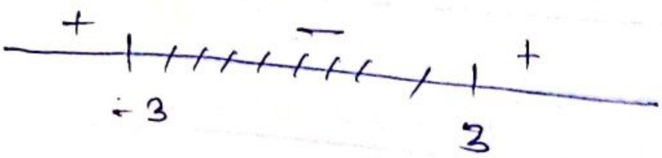
$\therefore x \in (-\infty, -3) \cup (3, \infty)$

and f is decreasing iff $f'(x) < 0$

$\therefore 3x^2 - 27 < 0$

$\therefore 3(x^2 - 9) < 0$

$\therefore (x-3)\ (x+3) < 0$



$\therefore x \in (-3, 3)$

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5] $F(x) = 2x^3 - 9x^2 - 24x + 69$

$F'(x) = 6x^2 - 18x - 24$

$\because$ F is increasing iff $F'(x) > 0$

$\therefore 6x^2 - 18x - 24 > 0$

$\therefore 6(x^2 - 3x - 4) > 0$

$\therefore x^2 - 4x + x - 4 > 0$

$\therefore x(x-4) + 1(x-4) > 0$

$\therefore (x-4)(x+1) > 0$

$\therefore x \in (-\infty, -1) \cup (4, \infty)$

and F is decreasing iff $F'(x) < 0$

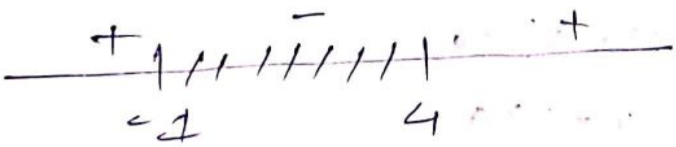
$\therefore 6x^2 - 18x - 24 < 0$

$\therefore 6(x^2 - 3x - 4) < 0$

$\therefore x^2 - 4x + x - 4 < 0$

$\therefore x(x-4) + 1(x-4) < 0$

$\therefore (x-4)(x+1) < 0$



$\therefore x \in (-1, 4)$

Q.2] 1] $y = 3x^2 - 2x^3$

$\therefore f(x) = 3x^2 - 2x^3$

$\therefore f'(x) = 6x - 6x^2$

$\therefore f''(x) = 6 - 12x$

f is concave upward if $f''(x) > 0$

$\therefore (6 - 12x) > 0$

$\therefore 12(6/12 - x) > 0$

$x - 1/2 > 0$

$x > 1/2$

$\therefore f''(x) > 0$

$\therefore x \in (1/2, \infty)$

2.] $y = x^4 - 6x^3 + 12x^2 + 5x + 7$

$f'(x) = 4x^3 - 18x^2 + 24x + 5$

$f''(x) = 12x^2 - 36x + 24$

f is concave upward is $f''(x) > 0$

$\therefore 12x^2 - 36x + 24 > 0$

$\therefore 12(x^2 - 3x + 2) > 0$

$\therefore x^2 - 2x - x + 2 > 0$

$\therefore x(x-2) - 1(x-2) > 0$

$\therefore (x-2)(x-1) > 0$

$+ \qquad - \qquad +$

$1 \qquad 2$

$x \in (-\infty, 1) \cup (2, \infty)$

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3) $y = x^3 - 27x + 5$

$f'(x) = 3x^2 - 27$

$f''(x) = 6x$

f is concave upward iff $f''(x) > 0$

$\therefore 6x > 0$

$\therefore x > 0$

$\therefore x \in (0, \infty)$

4) $y = 66 - 24x - 9x^2 + 2x^3$

$f(x) = 2x^3 - 9x^2 - 24x + 66$

$f'(x) = 6x^2 - 18x - 24$

$f''(x) = 12x - 18$

f is concave upward iff $f''(x) > 0$

$\therefore 12x - 18 > 0$

$\therefore 12(x - 18/12) > 0$

$\therefore x - 3/2 > 0 \quad \therefore x > 3/2 \quad \therefore x \in (3/2, \infty)$

5) $y = 2x^3 + x^2 - 20x + 4$

$f(x) = 2x^3 + x^2 - 20x + 4$

$f'(x) = 6x^2 + 2x - 20$

$f''(x) = 12x + 2$

f is concave upward iff $f''(x) > 0$

$\therefore f''(x) > 0$

$\therefore 12x + 2 > 0$

$12(x + 2/12) > 0$

$\therefore x + 1/6 > 0$

$\therefore x < -1/6$

$\therefore f''(x) > 0$

$\therefore$ There exists interval $(-1/6, \infty)$

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Topic :- App of Derivatives & Newton's Methods

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Practical 4

Q1] (i) $f(x) = x^2 + \frac{16}{x^2}$

$f'(x) = 2x - 32/x^3$

Now consider, $f'(x) = 0$

$\therefore 2x - 32/x^3 = 0$

$2x = 32/x^3$

$x^4 = 32/2$

$x^4 = 16$

$x = \pm 2$

$f''(x) = 2 + 96/x^4$

$f''(2) = 2 + \frac{96}{2^4}$

$= 2 + 96/16$

$= 2 + 6$

$= 8 > 0$

$\therefore f$ has minimum value at $x = 2$

$\therefore f(2) = 2^2 + 16/2^2$

$= 4 + 16/4$

$= 4 + 4$

$\therefore$ f''e $= 8$

$\therefore f''(-2) = 2 + 96/(-2)^4$

$= 2 + 96/16$

$= 2 + 6$

$= 8 > 0$

$\therefore f$ has minimum value at $x = -2$

$\therefore$ Function reaches minimum value at $x = 2$ and $x = -2$

b.] $f(x$

$\therefore$

con

b] $F(x) = 3 - 5x^3 + 3x^5$

$\therefore F'(x) = -15x^2 + 15x^4$

consider, $F'(x) = 0$

$\therefore -15x^2 + 15x^4 = 0$

$\therefore x^2 = 1$

$\therefore x = \pm 1$

$\therefore F''(x) = -30x + 60x^3$

$F(1) = -30 + 60$

$= 30 > 0$ $\therefore$ F has minimum value at $x = 1$

$\therefore F(1) = 3 - 5(1)^3 + 3(1)^5$

$= 6 - 5$

$= 1$

$\therefore F''(-1) = -30(-1) + 60(-1)^3$

$= 30 - 60$

$= 30$

$\therefore$ F has minimum value at $x = -1$.

$\therefore F(-1) = 3 - 5(-1)^3 + 3(-1)^5$

$= 3 + 5 - 3 = 5$

$\therefore$ F has maximum value 5 at $x = -1$ and has the minimum value 1 at $x = 1$.

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82 c] $F(x) = x^3 - 3x^2 + 1$

$\therefore F'(x) = 3x^2 - 6x$

Consider, $F'(x) = 0$

$\therefore 3x^2 - 6x = 0$

$\therefore 3x(x-2) = 6$

$\therefore 3x = 0$ or $x - 2 = 0$

$\therefore x = 0$ or $x = 2$

$\therefore F''(x) = 6x - 6$

$\therefore F''(0) = 6(0) - 6$

$= -6 < 0$

$\therefore F$ has maximum value at $x = 0$

$\therefore F(0) = 0^3 - 3(0)^2 + 1 = 1$

$\therefore F''(2) = 6(2) - 6$

$= 12 - 6 = 6 > 0$

$\therefore F$ has minimum value at $x = 2$

$\therefore F(2) = 2^3 - 3(2)^2 + 1$

$= 8 - 3(4) + 1$

$= 9 - 12$

$= -3$

$\therefore F$ has maximum value 1 at $x = 0$ and has minimum value -3 at $x = 2$

d] $F(x$

$f(x) = 2x^3 - 3x^2 - 12x + 1$

$\therefore f'(x) = 6x^2 - 6x - 12$

Consider, $f'(x) = 0$

$\therefore 6x^2 - 6x - 12 = 0$

$\therefore 6(x^2 - x - 2) = 0$

$\therefore x^2 - x - 2 = 0$

$\therefore x^2 + x - 2x - 2 = 0$

$\therefore x(x+1) - 2(x+1) = 0$

$\therefore x = 2$ or $x = -1$

$\therefore f''(x) = 12x - 6$

$\therefore f''(2) = 12(2) - 6$

$= 24 - 6$

$= 18 > 0$

$\therefore$ f has minimum value at $x = 2$

$\therefore f(2) = 2(2)^3 - 3(2)^2 - 12(2) + 1$

$= 2(8) - 3(4) - 24 + 1$

$= 16 - 12 - 24 + 1$

$= -19$

$\therefore f''(-1) = 12(-1) - 6$

$= -12 - 6$

$= -18 < 0$

f has maximum value at $x = -1$

$\therefore f(-1) = 2(-1)^3 - 3(-1)^3 - 3(-1)^2 -$

$12(1) + 1$

$= -2 - 3 + 12 + 1$

$= 8$

$\therefore$ f has maximum value 8 at $x = -1$

and f has minimum value -19 at $x = 2$

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10.] $F($ $F'($ F

a.] $F(x) = x^3 - 3x^2 - 55x + 9.5$ $x = 0 \rightarrow$ given

$F'(x) = 3x^2 - 6x - 55$

By newton's method

$x_{n+1} = x_n - F(x_n) / F'(x_n)$

$\therefore x_1 = x_0 - F(x_0) / F'(x_0)$

$\therefore x_1 = 0 + 9.5 / 55$

$\therefore x_1 = 0.1727$

$\therefore F(x_1) = (0.1727)^3 - 3(0.1727)^2$
$55(0.1727) + 9.5$

$= 0.0051 - 0.0895 - 9.4985 + 9.5$

$= -0.0829$

$F'(x_1) = 3 (0.1727)^2 - 6(0.1727) - 55$

$= 0.0895 - 1.0362 - 55$

$= -55.9467$

$\therefore x_2 = x_1 - F(x_1) / f'(x_1)$

$= 0.1727 - 0.0829) \; 55.9467$

$= 0.1712$

$F(x_2) = (0.1712)^3 - 3(0.1712)^2 - 55 (0.1712)$
9.5

$= 0.0050 - 0.0879 - 9.416 + 9.5$

$= 0.0011$

$F'(x_2) = 3(0.1712)^2 - 6(0.1712) - 55$

$= 0.0879 - 1.0272 - 55$

$= -55.9392$

$\therefore x_3 = x_2 - F(x_2) / F'(x_2)$

$= 0.1712 - 0.0011 \; / 55.9393$

$= 0.1712$

$\therefore$ The root of the equation in
0.1712

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$f'(x) = 3x^2 - 4$

$f(2) = (2)^3 - 4(2) - 9$

$= 8 - 8 - 9$

$= -9$

$f(3) = 3^3 - 4(3) - 9$

$= 27 - 12 - 9$

$= 6$

let $x_0 = 3$ be the initial approximation

$\therefore$ By newton's method,

$x_{n+1} = x_n - f(x_n) / f'(x_n)$

$x_1 = x_0 - f(x_0) / f'(x_0)$

$= 3 - 6/23$

$= 2.7392$

$f(x_1) = (2.7392)^3 - 4(2.7392) - 9$

$= 20.5528 - 10.9568 - 9$

$= 0.596$

$f'(x_1) = 3(2.7392)^2 - 4$

$= 22.5096 - 4$

$= 18.5096$

$x_2 = x_1 - f(x_1) / f'(x_1)$

$= 2.7392 - 0.596 / 18.5096$

$= 2.7071$

$f(x_2) = (2.7071)^3 - 4(2.7071)$

$= 19.8386 - 10.8284$

$= 0.0102$

$f'(x_2) = 3(2.7071)^2 - 4$

$= 21.9859 - 4$

$= 17.9859$

$x_3 = x_2 - f(x_2) / f'(x_2)$

$= 2.7071 - 0.0109 / 17.9980$

$= 2.7071 - 0.0056$

$= 2.7015$

$f(x_3) = (2.7015)^3 - 4(2.7015) - 9$

$= 19.7158 - 10.806 - 9 = -0.0902$

$f'(x_3) = 3(2.7015)^2 - 4 = 21.8943 - 4 =$

17.8943

$x_4 = x_3 - f(x_3) / f'(x_3)$

$= 2.7015 + 0.0901 / 17.8943$

$= 2.7015 + 0.0050$

$= 2.7065$

c.] $f(x) = x^3 - 1.8x^2 - 10x + 17$ $[1, 2]$

$f'(x) = 3x^2 - 3.6x - 10$

$f(1) = 1^3 - 1.8(1)^2 - 10(1) + 17$

$= 1.8 - 10 + 17$

$= 6.2$

$f(2) = 2^3 - 1.8(2)^2 - 10(2) + 17$

$= 8 - 7.2 - 20 + 17 = \underline{-2.2}$

Let $x_0 = 2$ be initial approximation

By newton's method:

$x_{n+1} = x_n - f(x_n) / f'(x_n)$

$x_1 = x_0 - f(x_0) / f'(x_0)$

$= 2 - 2.2 / 5.2$

$= 2 - 0.4230 = 1.577$

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$f(x_1) = (1.577)^3 - 1.8(1.577)^2 - 10(1.577) + 17$

$= 3.9219 - 4.4764 - 15.77 + 17$

$= 0.6755$

$f'(x_1) = 3(1.577)^2 - 3.6(1.577) - 10$

$= -8.2164$

$\therefore x_2 = x_1 - f(x_1)/f'(x_1)$

$= 1.577 + 0.6755/8.2164$

$= 1.6592$

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Topic - Integration

Practical 5

$$\int \frac{1}{\sqrt{x^2-2x-3}}\,dx$$

$$= \int \frac{1}{\sqrt{x^2-2x+1-4}}\,dx$$

$(a+b)^2 = a^2+2ab+b^2$

$$= \int \frac{1}{\sqrt{(x+1)^2+4}}\,dx$$

substitute put $x+1=t$

$dx = \frac{1}{t} * dt$ where $t = 1,\ t = x+1$

$$\int \frac{1}{\sqrt{t^2-4}}\,dt$$

using

$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln(|x+\sqrt{x^2+a^2}|)$

$= \ln(|t+\sqrt{t^2-4}|)$

$t = x+1$

$= \ln(|x+1 + \sqrt{(x+1)^2-4}|)$

$= \ln(|x+1 + \sqrt{x^2+2x-3}|)$

$= \ln(|x+1+\sqrt{x^2-2x-3}|)+c$

2.] $\int (4e^{3x}+1)\,dx$

$= \int 4e^{3x}\,dx + \int 1\,dx$

$= 4 \frac{e^{3x}}{3} + x$

$\# \int e^{ax} = \frac{1}{a} \times e^{ax}$

$= \frac{4e^{3x}}{3} + x + c$

3.] $\int 2x^2 - 3\sin x + 5\sqrt{x}\, dx$

$= \int 2x^2 - 3\sin x + 5x^{1/2}\, dx$

$= \int 2x^2 dx - \int 3\sin x\, dx + \int 5(x)^{1/2} dx$

$= \frac{2x^3}{3} + 3\cos x + 10 \frac{x\sqrt{x}}{3} + c$

$= \frac{2x^3 + 10x\sqrt{x}}{3} + 3\cos x + c.$

4.] $\int \frac{x^3 + 3x + 4}{\sqrt{x}} dx$

$= \int \frac{x^3 + 3x + 4}{x^{1/2}} dx$

split the denominator

$= \int \frac{x^3}{x^{1/2}} + \frac{3x}{x^{1/2}} + \frac{4}{x^{1/2}} dx$

$= \int x^{5/2} + 3x^{1/2} + \frac{4}{x^{1/2}} dx$

$= \int x^{5/2} dx + \int 3x^{1/2} dx + \int \frac{4}{x^{1/2}} dx$

$= \frac{x^{5/2+1}}{5/2+1} = \frac{2x^3\sqrt{x}}{7} + 2x\sqrt{x} + 8\sqrt{x} + c$

52. 5.] $\int t^7 \times \sin x \, (2t^4) \, dt$

put $u = 2t^4$

$du = 2 \times 4t^3$

$= \int t^7 \times \sin(2t^4) \times \frac{1}{2 \times 4t^3} \, du$

$= \int t^4 \sin(2t^4) \times \frac{1}{2 \times 4} \, du$

$= \int t^4 \sin(2t^4) \times \frac{1}{8} \, du$

$= \frac{t^4 \times \sin(2t^4) \, du}{8}$

Substitute t^4 with $\frac{u}{2}$

$= \int \frac{u/2 \times \sin(u) \, du}{8}$

$= \int \frac{u \times \sin(u)}{2} / 8 \, du$

$= \int \frac{u \times \sin(u)}{16} \, du$

\# $\int u \, dv = uv - \int v \, du$

where $u = v$

$dv = \sin(u) \times du$

$du = 1 \, du \quad v = -\cos(u)$

$= \frac{1}{16} (u \times (-\cos(u)) - \int -\cos(u) \, du$

$= \frac{1}{16} \times (u \times (-\cos(u)) + \int \cos(u) \, du$

\# $\int \cos x \, dx = \sin(x)$

$= \frac{1}{16} \times (u \times (-\cos(u)) + \sin(u))$

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Return the Substitution $u = 2t^4$

$= 1/16 \times (2t^4 \times (-\cos(2t^4)) + \sin(2t^4))$

$= -\frac{t^4 \times \cos(2t^4)}{8} + \frac{\sin(2t^4)}{16} + c$

(vi) $\int \sqrt{x}(x^2 - 1)\,dx$

$= \int \sqrt{x}\,x^2 - \sqrt{x}\,dx$

$= \int x^{1/2} \times x^2 - x^{1/2}\,dx$

$= \int x^{5/2} - x^{1/2}\,dx$

$= \int x^{5/2}\,dx - \int x^{1/2}\,dx$

$= I_1 = \frac{x^{5/2+1}}{5/2+1} = \frac{x^{7/2}}{7/2} = \frac{2x^{7/2}}{7} = \frac{2\sqrt{x^7}}{7} = 2x^3\ldots$

$= I_2 = \frac{x^{1/2+1}}{1/2+1} = \frac{x^{3/2}}{3/2} = \frac{2x^{3/2}}{3/2} = \frac{2\sqrt{x^3}}{3}$

$= \frac{2x^3\sqrt{x}}{7} + \frac{2\sqrt{x^3}}{3} + c$

(viii) $= \int \frac{\cos x}{3\sqrt{\sin(x^2)^2}}\,dx = \int \frac{\cos x}{\sin x^{2/3}}\,dx$

Put $t = \sin(x)$

$t = \cos x$

$$= \int \frac{\cos(x)}{\sin(x)^{3/2}} \times \frac{1}{\cos(x)}\,dt$$

$$= \frac{1}{\sin x^{3/2}}\,dt$$

$$= \frac{1}{t^{2/3}}\,dt$$

$$I = \int \frac{1}{t^{2/3}}\,dt = \frac{-1}{(2/3-1)}t^{2/3-1} =$$

$$\frac{-1}{-1/3\,t^{2/3-1}} = \frac{1}{1/3\,t^{-1/3}} = \frac{t^{1/3}}{1/3} = 3t^{1/3}$$

$$= 3\sqrt[3]{t}$$

Return Substitution $t = \sin(x)$

$$= 3\sqrt[3]{\sin(x)} + C$$

(x) $\displaystyle\int \frac{x^2 - 2x}{x^3 - 3x^2 + 1}\,dx$

Put $x^3 - 3x^2 + 1 = dt$

$$I = \int \frac{x^2 - 2x}{x^3 - 3x^2 + 1} \times \frac{1}{3x^2 - 3 \times 2x}\,dt$$

$$= \int \frac{x^2 - 2x}{x^3 - 3x^2 + 1} \times \frac{1}{3(x^2 \ldots}\,dt$$

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$$= \int \frac{x^2 - 2x}{x^3 - 3x^2 + 1} \times \frac{1}{3(x^2 - 2x)} dt$$

$$= \int \frac{1}{x^3 - 3x^2 + 1} \times 1/3 \, dt$$

$$= \int \frac{1}{3(x^3 - 3x^2 + 1)} dt = \int \frac{1}{3t} dt$$

$$= 1/3 \int 1/t \, dt \qquad \int 1/x \, dx = \ln | |x|$$

$$= 1/3 \int \ln |t| + c$$

$$= 1/3 \times \ln (|x^3 - 3x^2 + 1|) + c$$

Practical -6

Aim :-> Application of integration & Numerical Integration.

1.] $x = \sin t$, $y = 1 - \cos t$ $t \in [0, 2\pi]$

$$l = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$= \int_0^{2\pi} \sqrt{(1-(\cos t))^2 + (\sin t)^2}\, dt$$

$$= \int_0^{2\pi} \sqrt{1 - 2\cos t + \cos^2 t + \sin^2 t}\, dt$$

$$= \int_0^{2\pi} \sqrt{1 - 2\cos t + 1}\, dt$$

$$= \int_0^{2\pi} \sqrt{2 - 2\cos t}\, dt$$

$$= \int_0^{2\pi} 2\left|\sin \frac{t}{2}\right| dt \quad --- \quad \sin t/2 = 1 - \frac{\cos t}{2}$$

$$= \int_0^{2\pi} 2 \sin t/2\, dt.$$

$$\left[(-4\cos(t/2))\right]_0^{2\pi} = (-4\cos\pi) - (-4\cos 0)$$

$= 4 + 4$

$= 8$

2.] $y = \sqrt{}$

$\frac{dy}{dx}$

Q) $y = \sqrt{4 - x^2}$ $x \in [-2, 2]$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{4 - x^2}} \times (-2x)$$

$$= \frac{-2x}{\sqrt{4 - x^2}}$$

$$= \int_{-2}^{2} \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

$$= \int_{-2}^{2} \sqrt{1 + \left(\frac{-x}{\sqrt{4 - x}}\right)^2} dx$$

$$= \int_{-2}^{2} \sqrt{\frac{4 - x^2 + x^2}{4 - x^2}} dx$$

$$= \int_{-2}^{2} \sqrt{\frac{4}{4 - x^2}} dx$$

$$= \int_{-2}^{2} \frac{1}{\sqrt{(2)^2 - (x)^2}} dx$$

$$= 2\left[\sin^{-1}(x/2)\right]_{-2}^{2}$$

$$= \sin^{-1}(1) - \sin^{-1}(-1)$$

$$= 2[\pi/2) - (-\pi/2]$$

$$= 2[\pi/2 + \pi/2]$$

$$= 2\pi$$

$$y = x^{3/2} \quad , \quad x \ [0, 4]$$

$$\frac{dy}{dx} = 3/2 \cdot x^{3/2 - 1}$$

$$= \int_0^4 \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx$$

$$= \int_0^4 \sqrt{1 + \left(\frac{3\sqrt{x}}{2}\right)^2} \cdot dx$$

$$= \int_0^4 \sqrt{\left(1 + \frac{9x}{4}\right)} \, dx$$

$$= \int_0^4 \sqrt{\left(\frac{4 + 9x}{4}\right)} \, dx$$

$$= \frac{1}{2} \int_0^4 \sqrt{4 + 9x} \cdot dx$$

$$= \frac{1}{2} \left[\frac{(4 + 9x)^{1/2 + 1}}{1/2 + 1} \right]_0^4 dx$$

$$= \frac{1}{2} \left[\frac{(4 + 9x)^{3/2}}{3/2} \times \frac{1}{9} \right]_0^4$$

$$= \frac{1}{27} \left[(9x + 4)^{3/2} \right]^4$$

=

= -

(4)

$\frac{dx}{dt}$

$\frac{1}{27}\left[(4+0)^{3/2} - (4+36)^{3/2}\right]$

$= \frac{1}{27}\left[(4)^{3/2} - (40)^{3/2}\right]$

$= \frac{1}{27}\left[(4)^{3/2} - 8\right]$

(4) $x = 3\sin t$, $y = 3\cos t$, $t \in (0, 2\pi)$

$\frac{dx}{dt} = 3\cos t$ $\qquad \frac{dy}{dt} = -3\sin t$

$$L = \int_0^{2\pi} \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2}\, dt$$

$$= \int_0^{2\pi} \sqrt{(3\cos t)^2 + (-3\sin t)^2}\, dt$$

$$= \int_0^{2\pi} \sqrt{9\sin^2 t + 9\cos^2 t}\, dt$$

$$= \int_0^{2\pi} \sqrt{9\sin^2 t + 9\cos^2 t}\, dt$$

$$= \int_0^{2\pi} 3\sqrt{x}\, dt \qquad = 3\left[x\right]_0^{2\pi}$$

$$= 3(2\pi - 0)$$

$$= 3\int_0^{2\pi} dt \qquad L = 6\pi \text{ units.}$$

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Q5:] $x = \frac{1}{6}y^3 + \frac{1}{2y}$ on $y \in (1,2)$

Soln ->

$$\frac{dx}{dy} = \frac{y^2}{2} - \frac{1}{2y^2}$$

$$\frac{dx}{dy} = \frac{y^4 - 1}{2y^2}$$

$$L = \int_1^2 \sqrt{1 + \left(\frac{dy}{dy}\right)^2} \, dy$$

$$= \int_1^2 \sqrt{\frac{1 + (y^4 - 1)}{2y^2}} \, dy$$

$$= \int_1^2 \sqrt{\frac{(y^4+1)^2}{(2y^2)^2}} \, dy$$

$$= \int_1^2 \frac{y^4 + 1}{2y^2} \, dy$$

$$= \cancel{\int_1^2 dy} \; \frac{1}{2} \int y^2 \, dy + \frac{1}{2} \int_1^2 y^{-2} \, dy$$

$$= \frac{1}{2} \left[\frac{y^3}{3} - \frac{y^{-2}}{1} \right]_1^2$$

$$= \frac{1}{2} \left[\frac{1}{3} - \frac{1}{2} + \frac{1}{3} + 1 \right]$$

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$= \frac{1}{2}\left[\frac{7}{3} + \frac{1}{2}\right]$

$= \frac{1}{2}\left[\frac{17}{6}\right]$

$= \frac{17}{12}$ units.

Q2] Solve the following using simpson's rule:

1] $\int_0^2 e^{x} \, dx$ with $n = 4$

$\int_0^2 e^{x^2} \, dx = 16.4526$

$\Delta x = \frac{2-0}{4} = \frac{1}{2}$

By simpson's rule:

$\int_0^2 dx = \frac{1/2}{3} \left(y_0 + 4 \cdot \frac{1}{2} + 2\frac{1}{2} + 4\, \frac{1}{1} + \frac{1}{4}\right]$

$= \frac{1/2}{3} \cancel{\left(e^{0^2} + 4e^{(0.5)^2} + (2e)^{(1)^2} + 4e^{(1.5)^2} + e^{(1)^2}\right)}$

$\simeq \underline{17.3536}$

1] $\int_0^4 x^2 \, dx$: $n=4$

$\Delta x = \frac{4-0}{4} = 1.$

$\int_a^b f(x)\,dx = \frac{\Delta x}{3}\left[y_0 + 4y_1 + 2y_2 + 4y_3 + y_4 \right.$

$= 1/3 \left[f(0) \right] + 4f(1) + 2f(2) + 4f(3) + f(4)$

$= 1/3 \left[0^2 + 4^2 (1)^2 + 2(2)^2 + 4(3)^2 \right.$

$= \frac{64}{3} \approx 21.333$

2] $\int_0^{\pi/3} \sqrt{\sin x}$ on $n=6$

$\Delta x = \frac{b-a}{n} = \frac{\pi/3 - 0}{6} = \pi/18$

x	0	$\pi/18$	$2\pi/18$	$3\pi/18$	$4\pi/18$	$5\pi/18$
y	0	0.4167	0.584	0.707	0.801	0.87
	$\cancel{y_0}$	α_1	α_2	α_3	α_4	α_5

$\int_0^{\pi/3} \sqrt{\sin x}\, dx = \frac{\Delta x}{3}\left(y_0 + 4(y_1 + y_2 + y_5) \right) + 2($

$= \frac{\pi/18}{3} (0 + 4(0.4167 + 0.707$

$0.8715) +$

$2 + 2(0.$

0-687

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Practical No. 7

Topic :-> Differential Equation

Q.1] 1.] $x\dfrac{dy}{dx} + y = e^x$

$$\frac{dy}{dx} + \frac{1}{x}y = \frac{e^x}{x}$$

$$P(x) = 1/x \qquad Q(x) = \frac{e^x}{x}$$

$$I.F = e^{\int(1/x)\,dx}$$

$$= e^{\int 1/x\,dx}$$

$$= e^{\ln x}$$

$$I.F = x$$

$$y(I.F) = \int Q(x)(I.F)\,dx + C$$

$$= \int \frac{e^x}{x}\cdot x\cdot dx + C$$

$$= \int e^x\,dx + 1$$

$$xy = e^x + C$$

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$$e^x \frac{dy}{dx} + 2e^x y = 1$$

$$\frac{dy}{dx} + \frac{2e^x}{e^x} y = 1/e^x \qquad (\div \text{ by } e^x)$$

$$\frac{dy}{dx} + 2y = 1/e^x$$

$$\frac{dy}{dx} + 2y = e^{-x}$$

$P(x) = 2 \qquad Q(x) = e^{-x}$.

$$\int P(x)\,dx$$

$$I\cdot F = e^{\int 2\,dx}$$

$$= e^{2x}$$

$$y(I\cdot F) = \int Q(x)(I\cdot F)\,dx + C$$

$$y \cdot e^{2x} \int e^{-x+2x}\,dx + C$$

$$= \int e^x\,dx + C$$

$$y \cdot e^{2x} = \underline{e^x} + C$$

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$$x\frac{dy}{dx} = \frac{\cos x}{x} - 2y$$

$$x\frac{dy}{dx} = \frac{\cos x}{x} - 2y$$

$$\therefore \frac{dy}{dx} + \frac{2y}{x} = \frac{\cos x\, e}{x^2}$$

$P(x) = 2(x) \quad Q(x) = \cos x/x^2$

$$I.F = e^{\int Q(x)\,dx}$$

$$= e^{\int 2/x \, dx}$$

$$= e^{2/x\, x}$$

$$= \ln x^2$$

$$y(I.F) = \int Q(x)(I.F)\,dx + C$$

$$= \int \frac{\cos x}{x^2} - x^2\,dx + C$$

$$= \int \cos x + C$$

$$x^2 y = \sin x + C$$

$$x\frac{dy}{dx} + 2y = \frac{\sin x}{x^2}$$

$dy + 3y = \sin x \quad (\div$ by x on both side

$= e^{\int 3/x \, dx}$

$= e^{3/x}\, x$

$= e^{\ln x^3}$

$I.F = x^2$

$y(I.F) = \int Q(x)\,(I.F)\,dx + c$

$= \int \frac{\sin x}{x^3} \cdot x^3 \, dx + c$

$= \int \sin x \, dx + c$

$x^3 y = -\cos x + c$

5] $e^{2x} \frac{dy}{dx} + 2e^{2x} y = 2x$

$\frac{dy}{dx} + 2y = \frac{2x}{e^{2x}}$

$P(x) = 2 \qquad Q(x) = 2x / e^{2x} = 2x e^{-2x}$

$I.F = e^{\int P(x)\,dx}$

$= e^{\int 2\,dx}$

$= e^{2x}$

$y(I.F) = \int Q(x)\,(I.F)\,dx + c$

$= \int 2x e^{-2x} \cdot e^{2x} \, dx + c$

$= \int 2x \, dx + c$

$y e^{2x} = x^2 + c$

6.] $\sec^2 x \cdot \tan y + \sec^2 y \tan x \, dy = 0$

$\sec^2 x \cdot \tan y \, dx = -\sec^2 y \cdot \tan x \, dy$

$$\frac{\sec^2 x \, dx}{\tan x} = \frac{-\sec^2 y \, dy}{\tan y}$$

$$\int \frac{\sec^2 x \, dx}{\tan x} = -\int \frac{\sec^2 y \, dy}{\tan y}$$

$\therefore \log |\tan x| = -\log |\tan y| + c$

$\log |\tan x \cdot \tan y| = c$

$\tan x \cdot \tan y = e^c$

7.] $\frac{dy}{dx} = \sin^2 (x - y + 1)$

put $x - y + 1 = v$

differentiating on both sides

$x - y + 1 = v$

$$1 - \frac{dy}{dx} = \frac{dv}{dx}$$

$$1 - \frac{dv}{dx} = \frac{dy}{dx}$$

$$1 - \frac{dv}{dx} = \sin^2 v$$

$$\frac{dv}{dx} = 1 - \sin^2 v$$

$$\frac{dv}{dx} = \cos^2 v$$

$$\frac{dv}{\cos^2 v} = dx$$

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$$\int \sec^2 v \, dv = \int dx$$

$$\tan v = x + c$$

$$\tan(x+y-1) = x + c$$

5) $$\frac{dy}{dx} = \frac{2x+3y-1}{6x+9y+6}$$

Put $2x+3y = v$

$$2 + 3\frac{dy}{dx} = \frac{dv}{dx}$$

$$\frac{dy}{dx} = \frac{1}{3}\left(\frac{dv}{dx} - 2\right)$$

$$\frac{dy}{dx} = \frac{1}{3}\left(\frac{dy}{dx}\right.$$

$$\frac{1}{3}\left(\frac{dv}{dx} - 2\right) = \frac{1}{3}\frac{(v-1)}{(v+2)}$$

$$\frac{dv}{dx} = \frac{v-1}{v+2} + 2$$

$$\frac{dv}{dx} = \frac{v-1+2v+4}{v+2}$$

$$= \frac{3v+3}{v+2}$$

$$= 3\frac{(v+1)}{v+2} \qquad = \int \left(\frac{v+2}{v+1}\right) dv = 3 \, dx$$

$$= \int \frac{v+1}{v}\, dx + \int \frac{1}{v+1}\, dv = 3x$$

$$= \int \frac{v+1}{v}\, dx + \int \frac{1}{v+1}\, dv \;\; 3x$$

$$= v + \log|x| = 3x + C$$

$$= 2x + 3y + \log(2x + 3y + 1) = 3x + c$$

$$= 3y = x - \log|2x + 3y + 1| + C.$$

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Practical No 8

Aim -> Euler's method

1) Using Eulers method find the following

i) $\frac{dy}{dx} = y + e^{x} - 2$ $\quad y(0) = 2 \quad h = 0.5$
Find $y(2)$

Soln ->

$f(x) = y + e^{x} - 2$

$y(0) = 2$

$y(x_0) = y_0$

$\therefore x_0 = 0 \quad y_0 = 2 \quad h = 0.5$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	2	1	2.5
1	0.5	2.5	2.1487	3.5743
2	1	3.5743	4.2925	5.7205
3	1.5	5.7205	8.2021	9.8215
4	2	9.8215		

$y(2) = 9.8215$

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(ii) $\frac{dy}{dx} = 1+y^2$ $y(0)=0$ $h=0.2$ find $y(1)$

Soln →

$x_0 = 0$

$\therefore y(0) = 0$

$y(x_0) = y_0$

$x_0 = 0$

n	x_n	y_n	$F(x_n, y_n)$	y_{n+1}
0	0	0	1	0.2
1	0.2	0.2	1.04	0.408
2	0.4	0.408	1.1664	0.6412
3	0.6	0.6412	1.412	0.9234
4	0.8	0.9234	1.8526	1.293
5	0.1	1.2439		

$\therefore y(1) = 1.2439$

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(iii) $\frac{dy}{dx} = \sqrt{\frac{x}{y}}$, $y(0) = 1$, $h = 0.2$ find $y(1)$.

soln.->

$y(0) = 1$

$y(x_0) = y_0$

$\therefore x_0 = 0 \quad y_0 = 1 \quad h = 0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	1	0	1
1	0.2	1	0.4472	1.0894
2	0.4	1.0894	0.6059	1.2103
3	0.6	1.2105	0.7046	1.3513
4	0.8	1.3513	0.7649	1.5051
5	1	1.5051		

$y(1) = 1.5051$

) $\frac{dy}{dx} = 3x^2 + 1$, $y(1) = 2$ find $y(2)$
for $h = 0.5$ & $h = 0.25$

Soln.>
$y(1) = 2$ $h = 0.5$
$x_0 = 1$ $y_0 = 2$

n	x_n	y_n	$f(x_n\ y_n)$	y_{n+1}
0	1	2	4	4
1	1.5	4	7.75	1.875
2	2	7.875		

$y(2) = 7.875$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1.~~25~~	2	4	3
1	1.25	~~4.42183~~	5.6875	4.4218
2	1.5	4.42183 ~~19.3360~~	59.6569	19.3360
3	1.75	19.3360	1122.6426	299.9966
4	2	299.996		

$y(2) = 299.9966$

5) $\frac{dy}{dx} = \sqrt{xy} + 2$

$y(1) = 1$ find 0.2

Sol^n ->

$y(1) = 1$

$y(x_0) = y_0$

$\therefore x_0 = 1$ $y_0 = 1$ $h = 0.2$

$\therefore x_0 = 1$ $y_0 = 1$ $h = 0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	1	3	3.6
1	1.2	3.6		

18/01/2020

Practical No. 9

1] $\lim_{(x,y)\to(-4,-1)} \dfrac{x^2-3y+y^2-1}{xy+5}$

$\Rightarrow \lim_{(x,y)\to(-4,-1)} \dfrac{x^2-3y+y^2-1}{xy+5}$

At $(-4,-1)$ Denominator $\neq 0$

$\therefore$ By appying limit

$= \dfrac{(-4)^2-3(-4)+(-4)^2-1}{(-4)(-1)+5}$

$= \dfrac{-61}{9}$

2.] $\lim_{(x,y)\to(-2,0)} \dfrac{(y+1)(x^2+y^2-4x)}{x+3y}$

$\lim_{(x,y)\to(-2,0)} \dfrac{(y+1)(x^2+y^2-4x)}{x+3y}$

At $(-2,0)$ Denominator $\neq 0$

$\therefore$ By appying Denominator

$= \dfrac{(0+1)(4+0-8)}{-2+3(0)}$

$= \dfrac{1(-4)}{2}$

$= \dfrac{4}{2} = 2$

3]

$$\lim_{(x,y,z) \to (1,1,1)} \frac{x^2 - y^2 z^2}{x^3 - x^2 y^2}$$

$\Rightarrow$ At $(1,1,1)$ Denominator $\neq 0$

$$= \lim_{(x,y,z) \to (1,1,1)} \frac{x^2 - y^2 z^2}{x^3 - x^2 y^2}$$

$$= \lim_{(x,y,z) \to (1,1,1)} \frac{(xy + yz)}{x^2}$$

on applying limit

$$= \frac{1 + 1(1)}{1^2}$$

$$= \underline{2}$$

Q.2] (i) $F(x,y) = xye^{x^2} + y^2$

$$\therefore F_x = \frac{\partial f}{\partial x}(x,y)$$

$$= \frac{\partial (xye^{x^2+y^2})}{\partial x}$$

$$= ye^{x^2} + y^2 (2x)$$

$$\therefore f_x = 2xye^{x^2+y^2}$$

$$f_y = \frac{\partial f}{\partial y}(x,y)$$

$$= \frac{\partial (xye^{x^2+y^2})}{\partial x}$$

$$= xe^{x^2+y^2}(2y)$$

$$\therefore f_y = 2yxe^{x^2+y^2}$$

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$F(x,y) = e^x \cos y$

$\cancel{f_x = \frac{\partial (x y e^{x^2+y^2})}{\partial x}}$

$\cancel{= x e^{x^2+y^2} (2x y)}$

$\cancel{\therefore f_y = 2y x e^{x^2+y^2}}$

$F(x) = \frac{\partial (e^x \cos y)}{\partial x}$

$= \frac{\partial (e^x \cdot \cos y)}{\partial x}$

$F_x = e^x \cdot \cos y$

$F_y = \frac{\partial f(x,y)}{\partial y}$

$\cancel{F_y =} \frac{\partial (e^x \cdot \cos y)}{\partial y}$

$F_y = e^x \cdot \sin y$

$F(x,y) = x^3 y^2 - 3x^2 y + y^3 + 1$

$F(x) = \frac{\partial f (x,y)}{\partial y}$

$= \partial f (x^3 y^2 - 3x^2 y + y^3 + 1)$

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$= 2xy - 3x^2 + 3y^2$

Q 2] (i) $f(x,y) = \frac{2x}{1+y^2}$

$$f(x) = \frac{\partial f(x,y)}{\partial x}$$

$$= \frac{\partial f(2x)(1+y^2)}{\partial x}$$

$$= 1+y^2 \frac{\partial(2x)}{\partial x} \cdot 2x \frac{(1+y^2)}{2x}$$

$$= \frac{2+2y^2}{(1+y^2)}$$

$$= 2/(1+y^2)$$

At $f(0,0)$

$$= \frac{2}{1+0} = 2$$

$$f(y) = \frac{\partial f(2x)/(1+y^2)}{\partial y}$$

$$= \frac{(1+y^2)(0) - 2x(2y)}{(1+y^2)^2}$$

$$= -\frac{4xy}{(1+y^2)^2}$$

At $(0,0)$

$$= \frac{-4(0)(0)}{(1+0)^2} = 0$$

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Q.4.](i) $f(x,y) = \frac{y^2 - xy}{x^2}$

$$f_x = \frac{x^2 \cdot \frac{\partial f(y^2 - xy)}{\partial x} - (y^2 - xy)\frac{d}{dx}(x^2)}{(x^2)^2}$$

$$= \frac{x^2(-y) - (y^2 - xy)(2x)}{x^4}$$

$$= \frac{-x^2y - 2x(y^2 - xy)}{x^4}$$

$$f_y = \frac{2y - x}{x^2}$$

$$f_{xx} = \frac{\partial}{\partial x}\frac{(-x^2y - 2x(y^2 - xy))}{x^4}$$

$$= x^4(\partial/dx(-x^2y - 2xy^2 + 2x^2y)) - (x^2y - 2xy + 2x^2y)\,\partial/\partial x(x^4)$$

→

$$f_{yx} = \frac{\partial}{\partial x}\left(\frac{2y}{} - \frac{x}{x^2}\right)$$

$$= \frac{x^2 \frac{\partial}{\partial x}(xy - x) - (xy - x)\frac{\partial}{\partial x}(x^2)}{(x^2)^2}$$

$$= \frac{x^2 - 4xy + 2x^2}{x^4} \rightarrow (4)$$

From (3) & (2)

$$f_{xy} = f_{yx}$$

(ii) $f(x, y) = x^3 + 3x^2 - \log(x^2 + 1)$

$$f(x) = \frac{\partial}{\partial x}(x^3 + 3x^2y^2 - \log(x^2+1))$$

$$= 3x^2 + 6xy^2 - \frac{2x}{x + 1}$$

$$f_y = \frac{\partial}{\partial y}(x^3 + 3x^2y^2 - \log(x^2+1))$$

$$= 6x^2y.$$

$$\cancel{f_{xx} = 6x + 6y^2 \; (x^2 + \frac{\partial}{\partial x}(2x) - 2x \left(\frac{\partial}{\partial x}(x^2 - 1)\right)}$$

$$\frac{}{(x^2+1)^2}$$

$$= 6x + 6y^2 - \left(\frac{2(x^2+1) - 4x^2}{(x^2+1)^2}\right.$$

$$f_{yy} = x^2 \frac{\partial}{\partial y}(6x^2y)$$

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$= 6x^2 \ldots (2)$

$$f_{xy} = \frac{\partial}{\partial y}\left(3x^2 + 6xy^2 \cdot \frac{2x}{x^2+1}\right)$$

$= 0 + 12xy - 0$

$= 12xy \ldots (3)$

$$f_{yx} = \frac{\partial}{\partial x}(6x^2 y)$$

$= 12xy \ldots (4)$

From (3) & (2)

$f_{xy} = f_{yx}$

) $f(x,y) = \sin(xy) + e^{x+y}$

$\Rightarrow f_x = y\cos(xy) + e^{x+y}(1)$

$= y\cos(xy) + e^{x+y}$

$f_y = x\cos(xy) + e^{x+y}(1)$

$= x\cos(xy) + e^{x+y}$

$$f_{xx} = \frac{\partial}{\partial x}(y\cos(xy) + e^{x+y})$$

$= -y\sin(xy)(y) + e^{x+y}(e)$

$= -y^2\sin(xy) + e^{x+y} \ldots (6)$

$$f_{yy} = \frac{\partial}{\partial y}(x\cos(xy) + e^{x+y})$$

$= -x\sin(xy)(x) \cdot 1\, e^{x+y}(1)$

$= -x^2 \cdot \sin(xy) + e^{x+y} \ldots (1)$

$$f_{xy} = \frac{\partial}{\partial y}(y\cos(xy)) + e^{x+y}$$

$= -y^2\sin(xy) - \cos(xy) + e^{x+y} \ldots (3)$

$f_{yx} = \frac{\partial}{\partial x}(x \cos(yx)) + e^{x+y}$

$= -x \sin(xy) + \cos(xy) + e^{x+y}$ .... (4) 68

From (3) & (4)

$f_{xy} \neq f_{yx}$

Q.5)(i) $f(x,y) = \sqrt{x^2+y^2}$ at $(1,1)$

$\Rightarrow f(1,1) = \sqrt{1^2+1^2} = \sqrt{2}$

$$f_x = \frac{1}{2\sqrt{x^2+y^2}} (2x)$$

$$= \frac{x}{\sqrt{x^2+y^2}}$$

$$f_y = \frac{1}{2\sqrt{x^2+y^2}} (2y)$$

$$= \frac{y}{\sqrt{x^2+y^2}}$$

f_x at $(1,1) = 1/\sqrt{2}$

f_y at $(1,1) = 1/\sqrt{2}$

$f(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$$= \sqrt{2} + \frac{1}{\sqrt{2}}(x-1) + \frac{1}{\sqrt{2}}(y-1)$$

$$= \sqrt{2} + \frac{1}{\sqrt{2}}((x-1) + (y-1))$$

$$= \sqrt{2} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}y - \frac{2}{\sqrt{2}}$$

$$= \frac{x+y}{\sqrt{2}}$$

(ii) $f(x,y) = 1-x+y\sin x$ at $x = \pi/2$

33 $f(\pi/2, 0) = 1 - \pi/2 + 0 + \sin(\pi/2)$

$f(\pi/2, 0) = 2 - \pi/2$

$f_x = -1 + y\cos x$ $f_y = \sin x$

$f(x) = (\pi/2, 0) = \sin(\pi/2)$

$(x,y) = f(\pi/2, 0) + f_x(\pi/2, 0)(x - \pi/2)$

$+ f_y(\pi/2, 0)(y - 0)$

$= 2 - \pi/x + (-1)(x - \pi/2) + e1(y)$

$L(x,y) = 1 - x + y$

(iii) $f(x,y) = \log x + \log y$ at $x = (1,1)$

$f(1,1) = \log 1 + \log 1$

$= 0$

$f(x) = 1/x$ $f_y = 1/y$

$f(x) = (1,1) = 1$ $f_y = (1,1) = 1$

$L(x,y) = f(1,1) + f_x(1,1)(x-1)$

$+ f_y(1,1)(y-1)$

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$= 0 + 1(x-1) + (y-1)$

$L(x,y) = x + y - 2.$

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Practical No: 10.

Q1] Find the directional derivative of the following function at given points & in the direction of given vector.

i) $F(x,y) = x + 2y - 3$ $\quad a(1,-1) \quad u = 3i - j$

~~Hence~~ Here, $u = 3i - j$ is not a unit vector

$$|\bar{u}| = \sqrt{(3)^2 + (-1)^2} = \sqrt{9+1} = \sqrt{10}$$

Unit vector along u is $\frac{u}{|u|} = \frac{1}{\sqrt{10}}(3,-1)$

$$= \left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$$

$$F(a+hu) = F\left[(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)\right]$$

$$F(a) = f(1,-1) = (1) + 2(-1) - 3 = 1 - 2 - 3 = -4$$

$$f(a+hu) = F(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{10}\right)$$

$$= f\left(1 + \frac{3}{\sqrt{10}}\right), \left(-1 - \frac{h}{\sqrt{10}}\right)$$

$$f(a+hu) = \left(1 + \frac{3}{\sqrt{10}}\right) + 2\left(-1 - \frac{h}{\sqrt{10}}\right) - 3$$

$$1 + \frac{3}{\sqrt{10}} - 2 - \frac{2h}{\sqrt{10}} - 3$$

$$f(a+hu) = -4 + \frac{h}{\sqrt{10}}$$

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$$D_uf(a) = \lim_{h \to 0} \frac{f(a+hu) - f(a)}{h}$$

$$= \lim_{h \to 0} \cancel{4} + \frac{\cancel{h}/\sqrt{10} + \cancel{4}}{\cancel{h}}$$

$$D_uf(a) = \frac{1}{\sqrt{10}}$$

(ii) $f(x) = y^2 - 4x + 1$ $\quad a = (3,4)$ $\quad u = i + \sqrt{J}$

Here $u = i + 5J$ is not a unit vector

$$|\bar{u}| = \sqrt{(1)^2 + (5)^2} = \sqrt{26}$$

Unit vector along u is $\frac{u}{|u|} = \frac{1}{\sqrt{26}} (1, 5)$

$$= \left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}} \right)$$

$$f(a) = f(3,4) = (4)^2 - 4(3) + 1 = 5$$

$$f(a+hu) = f(3,4) + h\left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}} \right)$$

$$= f\left(3 + \frac{h}{\sqrt{26}}, 4 + \frac{5h}{\sqrt{26}} \right)$$

$$f(a+hu) = \left(4 + \frac{5h}{\sqrt{26}} \right)^2 - 4\left(3 + \frac{h}{\sqrt{26}} \right) + 1$$

$$= 16 + \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - 12 - \frac{4h}{\sqrt{26}} + 1$$

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$= \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - \frac{4h}{\sqrt{26}} + 5$

$= \frac{25h^2}{26} + \frac{40h - 4h}{\sqrt{26}} + 5$

$= \frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + 5$

$$D_uF(a) = \lim_{h \to 0} \frac{\frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + \cancel{5} - \cancel{5}}{h}$$

$$\frac{\cancel{h}\left(\frac{25h}{26} + \frac{36}{\sqrt{26}}\right)}{\cancel{h}}$$

$$D_uF(a) = \frac{25h}{26} + \frac{36}{\sqrt{25}} //$$

(iii) $2x + 3y$ - $a = (1, 2)$, $u = (3i + 4j)$

Here $u = 3i + 4j$ is not a unit vector

$|\bar{u}| = \sqrt{(3)^2 + (4)^2} = \sqrt{25} = 5$

unit vector along u is $\frac{u}{|u|} = \frac{1}{5}(3, 4)$

$= \left(\frac{3}{5}, \frac{4}{5}\right)$

$F(a) = F(1, 2) = 2(1) + 3(2) = 8$

$F(a + hu) = F(1, 2) + h\,(3/5, 4/5)$

$= F\left(1 + \frac{3h}{5},\ 2 + \frac{4}{5}\right)$

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$$F(a+hu) = 2\left(1+\frac{3h}{5}\right)+3\left(2+\frac{4h}{5}\right)$$

Q7.

$$= 2+\frac{6h}{5}+6+\frac{12h}{5}$$

$$= \frac{18h}{5}+8$$

$$D_uF(a) = \lim_{h\to 0} \frac{\frac{18\cancel{h}}{5}+\cancel{8}-\cancel{8}}{\cancel{h}}$$

$$= \frac{18}{5}$$

Q.2] Find gradient vector for the following function at given points.

(i) $f(x,y) = x^y + y^x \Rightarrow a(1,1)$

$$f_x = y \cdot x^{y-1} + y^x \log y$$

$$f_y = x^y \log x + xy^{x-1}$$

$$\nabla f(x,y) = (f_x, f_y)$$

$$= \left(yx^{y-1} + y^2 \log y,\ x^y \log x + xy^{x-1}\right)$$

$$f(1,1) = (1+0,\ 1+0)$$

$$= (1,1)$$

(ii) $f(x,y) = (\tan^{-1}x) \cdot y^2 \quad a = (1,-1)$

$$f_x = \frac{1}{1+x^2} \cdot y^2$$

$$f_y = 2y\tan^{-1}x$$

$$\nabla f(x,y) = (f_x, f_y)$$

$$= \left(\frac{y^2}{1+x^2},\ 2y\tan^{-1}x\right)$$

$f(1,-1) = \left(\frac{1}{2}, \tan^{-1}(1)(-2) \right)$

$= \left(\frac{1}{2}, \frac{\pi}{4}(-2) \right)$

$= \left(\frac{1}{2}, \frac{-\pi}{2} \right)$

(iii) $f(x,y,z) = xyz - e^{x+y+z}$, $a = (1,-1,0)$

$f_x = yz - e^{x+y+z}$

$f_y = xz - e^{x+y+z}$

$f_z = xy - e^{x+y+z}$

$\nabla f(x,y,z) = f_x, f_y, f_z$

$= yz - e^{x+y+z}, xz - e^{x+y+z}, xy - e^{x+y+z}$

$f(1,-1,0) = \left((-1)(0) - e^{(1+(-1)+0}, (1)(0) - e^{1+(-1)+0}, (1)(-1) - e^{1+(-1)+0} \right)$

$= (0 - e^0, 0 - e^0, -1 - e^0)$

$= (-1, -1, -2)$

Q.3] Find the equation of tangent & normal to each of the following using curves at given points.

(i) $x^2 \cos y + e^{xy} = 2$ at $(1,0)$

$f_x = \cos y \, 2x + e^{xy} y$

$f_y = x^2(-\sin y) + e^{xy} \cdot x$

$(x_0, y_0) = (1,0) = \therefore x_0 = 1, y_0 = 0$

eq^n of tangent

$f_x(x_0)(x - x_0) + f_y(y - y_0) = 0$

$f_x(x_0 y_0) = \cos 0 \, 2(1) + e^0 \, 0$

$= 1(2) + 0$

$= 2$

$Fy(x_0, y_0) = (1)^2(-\sin 0) + e^0 \cdot 1$

$= 0 + 1 \cdot 1$

$= 1$

$2(x-1) + 1(y-0) = 0$

$2x - 2 + y = 0$

$2x + y - 2 = 0$ $\rightarrow$ It is the Required eqⁿ of tangent.

eqⁿ of Normal

$= ax + by + c = 0$

$= bx + ay + d = 0$

$1(1) + 2(y) + d = 0$

$\therefore 1 + 2y + d = 0$ at $(1, 0)$

$= 1 + 2(0) + d = 0$

$d + 1 = 0$

$\therefore \boxed{d = -1}$

(i) $x^2 + y^2 - 2x + 3y + 2 = 0$ at $(2, -2)$

$fx = 2x + 0 - 2 + 0 + 0$

$= 2x - 2$

$fy = 0 + 2y - 0 + 3 + 0$

$= 2y + 3$

$(x_0, y_0) = (2, -2)$ $\therefore x_0 = 2, y_0 = -2$

$f_x(x_0, y_0) = 2(2) - 2 = 2$

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$2x - y - 4 = 0 \rightarrow$ It is required eqn of tangent

eqn of Normal

$= ax + by + c = 0$

$bx + ay + d = 0$

$= -1(x) + 2(y) + d = 0$

$-x + 2y + d = 0$ at $(2, -2)$

$-2 + 2(-2) + d = 0$

$-2 - 4 + d = 0$

$-6 + d = 6$

$\therefore d = 6$

Q.4.] Find the eqn of tangent & normal line to each of the following Surface

(i) $x^2 - 2yz + 3y + xz = 7$ at $(2, 1, 0)$

$Fx = 2x - 0 + 0 + z$

$Fx = 2x + z$

$Fy = 0 - 2z + 3 + 0.$

$= 2z + 3$

$Fz = 0 - 2y + 0 + x$

$= -2y + x$

$(x_0, y_0, z_0) = (2, 1, 0)$ $\therefore x_0 = 2, y_0 = 1, z_0 = 0$

$Fx(x_0, y_0, z_0) = 2(2) + 0 = 4$

$Fy(x_0, y_0, z_0) = 2(0) + 3 = 3$

$Fz(x_0, y_0, z_0) = -2(1) + 2 = 0$

eqn of tangent

$Fx(x_0 - x_0) + Fy(y_0 - y_0) + Fz(z_0 - z_0) = 0$

$= 4(x - 2) + 3(y - 1) + 0(z - 0) = 0$

$= 4x - 8 + 3y - 3 = 0$

$4x + 3y - 11 = 0 \rightarrow$ This is required eqn of tangent

Eqn of normal at (...)

55

$$\frac{x-x_0}{F_x} = \frac{y-y_0}{F_y} = \frac{z-z_0}{F_z}$$

$$= \frac{x-2}{4} = \frac{y-1}{3} = \frac{z+11}{6}$$

(ii) $3xyz - x - y + z = 4$

$3xy = -x - y + z + 4 = 0$

$F_x = 3yz - 1 - 0 + 0 + 0$

$= 3yz - 1$

$F_y = 3xz - 0 - 1 + 0 + 0$

$= 3xz - 1$

$F_z = 3xy - 0 - 0 + 1 + 0$

$= 3xy + 1$.

$(x_0, y_0, z_0) = (1, -1, 2)$ $\therefore x_0 = 1, y_0 = -1, z_0 = 2$

$F_x(x_0, y_0, z_0) = 3(-1)(2) - 1 = -7$

$F_y(x_0, y_0, z_0) = 3(1)\cdot(2) - 1 = 5$

$F_z(x_0, y_0, z_0) = 3(1)(-1) + 1 = -2$

eqn of tangent

$-7(x-1) + 5(y+1) - 2(z-2) = 0$

$-7x + 7 + 5y + 5 - 2z + 4 = 0$

$-7x + 5y - 2z + 16 = 0$

This is Required eqn of tangent.

eq^n of normal at $(-7, 5, -2)$

$$\frac{x - x_0}{F_x} = \frac{y - y_0}{F_y} = \frac{z - z_0}{F_z}$$

$$= \frac{x-7}{-7} = \frac{y+1}{5} = \frac{z-2}{-2}$$

Q5] Find the local maxima & minima for the following function

i) $f(x,y) = 3x^2 + y^2 - 3xy + 6x - 4y$

$$F_x = 6x + 0 - 3y + 6 - 0$$
$$= 6x - 3y + 6$$

$$F_y = 0 + 2y - 3x + 0 - 4$$
$$= 2y - 3x - 4$$

$F_x = 0$

$$6x - 3y + 6 = 0$$
$$3(2x - y + 2) = 0$$
$$2x - y + 2 = 0$$
$$2x - y = -2 \quad \text{— (1)}$$

$$2y - 3x - 4 = 0$$
$$2y - 3x = 4 \quad \text{— (2)}$$

Multiply eq^n with 2

$$4x - \cancel{2}y = -4$$
$$2y - 3x = 4$$
$$x = 0$$

Substitute value of x in eq^n (1)

$$2(0) - y = -2$$
$$-y = -2 \quad \boxed{y = 2}$$

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$\therefore$ Critical Points are $(0,2)$

$r = F_{xx} = 6$

$t = f_{yy} = 2$

$s = f_{xy} = -3$

Here $r > 0$

$= rt - s^2$

$= 6(2) - (-3)^2$

$= 12 - 9$

$= 3 > 0$

$\therefore$ f has maximum at $(0,2)$

$3x^2 + y^2 - 3xy + 6x - 4y$ at $(0,2)$

$3(0)^2 + (2)^2 - 3(0)(2) + 6(0) - 4(2)$

$0 + 4 - 0 + 0 - 8$

$= \underline{-4}$

(ii) $F(x,y) = 2x^4 + 3x^2 y - y^2$

$F_x = 8x^3 + 6xy$

$F_y = 3x^2 - 2y$

$F_x = 0$

$\therefore 8x^3 + 6xy = 0$

$2x(4x^2 + 3y) = 0$ — ①

$F_y = 0$

$3x^2 - 2y = 0 \rightarrow$ ②

Multiply eqn ① with 3

② with 4

$12x^2 + 9y = 0$

$-12x^2 - 8y = 0$

$+$

$1y = 0$

$\boxed{y = 0}$

Substitute value of y in eqn (1)

$$4x^2+3(0)=0$$
$$4x^2=0$$
$$x=0$$

Critical Point is (0,0)

$r=fxx=24x^2+6x$

$t=fyy=0-2=-2$

$s=fxy=6x-0=6x=6(0)=0$

r at (0,0)

$=24(0)+6(0)=0$

$\therefore r=0$

$rt-s^2=0(-2)-(s)^2$

$=0-0=0$

$r=0$ & $rt-s^2=0$

(nothing to say)

(iii) $f(x,y)=x^2-y^2+2x+8y-70$

$fx=2x+2$

$fy=-2y+8$

$fx=0$ $\therefore 2x+2=0$

$x=\frac{-2}{\cancel{2}}-1$ $\underline{x=-1}$

$fy=0$ - $-2y+8=0$

$y=\frac{-\cancel{8}}{-\cancel{2}}4$

$\therefore y=4$

Critical Point is (-1,4)

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