Indian Institute of Information Technology Sri City, Chittoor

(An Institute of National Importance under An Act of Parliament)

Name: **Probability and Statistics (Mathematics 2)** Duration: **1:30Hrs**

INSTRUCTIONS:

1. Please read all the questions carefully and there are no alternative options.
2. Clearly write your Roll Number and Name in capital letters on the top right corner of every page of the answer sheets. It is mandatory.
3. Marks are indicated in [ ] after each question.
4. The answer to the attempted question should be concluded by appropriate steps. Merely answering the question will not be entertained.
5. You are required to write the answers in A4 sheets.
6. Do not do any inappropriate activity as you are being recorded and please sit properly in front of your camera. For more information, please read all the guidelines given by the UG Exams Coordinator.
7. Submit a single PDF file containing all the answers to your attempted questions in the link to be provided during the exam.
8. Preferably use a ballpoint pen. The writing should be readable after scanning. (This is very important)
9. The naming conventions of the PDF file should be as follows: S2020xxxxx_NAME_M2_END_SET_1.pdf for SET-1.
10. The Exam's time duration is 1:30 Hrs. However, an additional 10 minutes will be given to scan and upload the answer sheets.

Not following instructions may lead to heavy penalty

SET-1

Q.1)

a) A box contains n different items. It is known that the probability of drawing an item A1 and then drawing another item A2 without replacement is 0.8%. What is the value of n? Explain. (nearest integer can be considered)

[3-Marks]

b) In a class room of 100 students 95 were promoted to higher class. If two people are chosen randomly from the entire class, find the probability that at least one of them did not get promoted.

[2-Marks]

c) A company uses three machines A, B and C to manufacture the products. Machine A produces 55% of the products, machine B produces 25% of the products and machine C produces 20% of the products. The defective products produced by the machines A, B and C respectively are 4%, 6% and 7% respectively. If a randomly chosen product is found to be defective, what is the probability that it is produced by machine B.

[2-Marks]

Q. 2)

a) Suppose there are 14 cars of a brand in a car showroom, out of which 7 are good (G), 3 have defective transmission (DT), and 4 have defective steering (DS). If 2 cars are selected at random, find $P(X < 2)$. (Assume X denotes the number of cars with defective transmission (DT) and Y denotes the number of cars with defective steering (DS))
[2-marks]

b) For the above random variables X and Y, find Cov(X,Y) and ρ_{XY}(coefficient of correlation)?
[3-marks]

Q. 3)

a) A coin is tossed 400 times. Find the approximate probability that the number of heads obtained is between 190 to 210. [1-mark]

b) Suppose that it is known that the number of items produced in a factory during a week is a random variable with mean 70. [2-marks]
I) What can be said about the probability that this week's production will exceed 90?
II) the variance of a week's production is known to equal 25, then what can be said about the probability that this week's production will be between 60 to 80?

c) Let $Y = X^2$, where X is a random variable. [2-marks]
Find $F_Y(y)$? (where F $is\ cumulative\ distribution\ function\ (CDF)$)

Q. 4)

a) Let X_1 and X_2 are two independent random samples taken from a population with mean μ and variance σ^2. Suppose that you have an estimator of μ: [2-marks]

$$\theta_1 = \frac{X_1 + 2X_2}{3}$$

Find bias and variance for θ_1.

b) Following i.i.d random samples were observed from a N(2μ, σ^2) population: [15.2, 11, 18, 25.8, 21]. σ^2 is known to be 16. Find MLE of μ.
[2-marks]

Q. 5)

a) A laptop manufacturing company is interested in the time (hours) that a laptop battery lasts after a full charge. This time is found to be normally distributed, with variance 0.0625. The company wants to perform a two-sided test on whether the expected battery time after charge (μ) is 5 hrs or not, using 9 samples. After sampling is done it was found that the sample mean is 4.85. What conclusion can be drawn at 5 % significance level? [2-marks]

b) In a medical study, 27 random samples of Covid survivors are investigated. It has been found that the amount spent in treatment for the randomly chosen people has a standard deviation of 5.86 (in thousands). Assume that medical expenditure of Covid survivors are normally distributed, develop 95 % confidence interval for the population variance of the medical expenditure. [2-marks]