Task: Discrete-Time-Fourier Transform and Hilbert-transform

1.

(a) Write a MATLAB script to estimate the pre-envelope of the modulated signal in amplitude modulation with graphical demonstration , where $m(t) = 2\sin(\omega t)$ and $c(t) = 0.5\cos(\omega_c t)$ are respectively, the baseband/message and carrier signal. Assume that the frequency of the baseband and carrier signals are 10Hz and 200Hz, respectively. Consider the value of sampling frequency (Fs) is 1000Hz.

(b) Write a Matlab script to generate the Single side-band using the concept of Hilbert-Transform.

Hints: you are allowed to use the MATLAB inbuild functions FFT and Hilbert-transform

2. Write a MATLAB script to evaluate the Discrete-Time Fourier-transform (DTFT) of signal $x(n) = (0.6)^{|n|}[u(n+10) - u(n)]$ and investigate its periodicity in frequency domain.

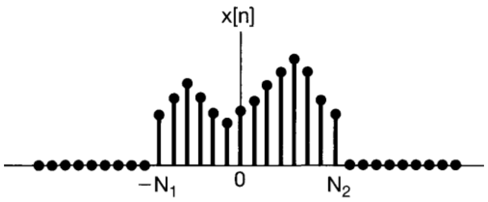
Hints: consider frequency range [-$2\pi, 2\pi$]

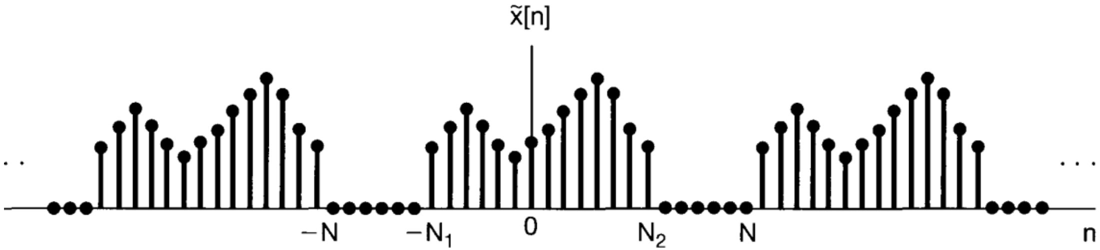
Supplementary materials

Discrete-time Fourier Transform (DTFT)

Let's consider an aperiodic Discrete-time signal

we can **construct a periodic signal** $\tilde{\boldsymbol{x}}(\boldsymbol{n})$ for which $x(n)$ *is* ***one period***





1. $x(n) = \begin{cases} \tilde{x}(n), & -N_1 \leq n \leq N_1 \\ 0, else \end{cases}$

2. $x(n) = \tilde{x}(t)$ when $N \rightarrow \infty$

Using the concept of **Fourier-series** **(FS)** **for periodic signal** $\tilde{x}(n)$

$$\tilde{x}(n) = \sum_{k=-N_1}^{N_2} a_k e^{jk\omega_0\, n} = \sum_{k=-N_1}^{N_2} a_k e^{jk\left(\frac{2\pi}{N}\right) n}$$

Fourier-series (FS) for periodic signal

$$x(n) = \sum_{k=0}^{N-1} a_k e^{jk\omega_0 n}$$

(synthesis equation)

$$a_k = \frac{1}{N}\sum_{n=0}^{N-1} x(n)\, e^{-jk\frac{2\pi}{N}n}$$

(Analysis equation of DTFS)

$$\tilde{x}(n) = \sum_{k=-N_1}^{N_1} a_k e^{jk\left(\frac{2\pi}{N}\right)n} \Rightarrow$$

$$a_k = \frac{1}{N}\sum_{n=-N_1}^{N_2} \tilde{x}(n)\, e^{-jk\frac{2\pi}{N}n}$$

$$\rightarrow a_k = \frac{1}{N}\sum_{n=-N_1}^{N_2} x(n)\, e^{-jk\frac{2\pi}{N}n}$$

$$since \quad x(n) = \begin{cases} \tilde{x}(n), & -N_1 \le n \le N_1 \\ 0, else & \end{cases}$$

$$\rightarrow a_k = \frac{1}{N}\sum_{n=-\infty}^{\infty} x(n)\, e^{-jk\frac{2\pi}{N}n}$$

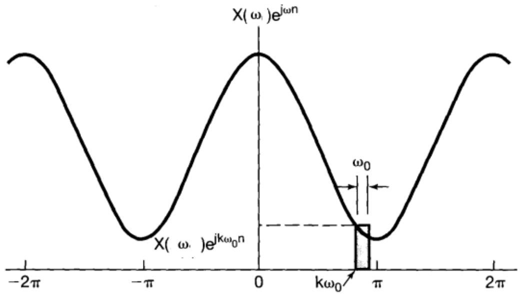
$Since\ x(n) = 0$ outside $-N_1 \le n \le N_2$

$$\rightarrow a_k = \frac{1}{N}\sum_{n=-\infty}^{\infty} x(n)\, e^{-jk\frac{2\pi}{N}n}$$

$$\rightarrow a_k = \frac{1}{N} X(k\omega_0)$$

Discrete-time Fourier-Transform (DTFT)

Where, $$X(\omega) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n}$$



Coefficient a_k are proportional to samples of $X(\omega)$

Discrete time Fourier-Transform (DTFT)

$$x(n) = \frac{1}{2\pi}\int_{2\pi} X(\omega) e^{j\omega n} d\omega$$

Synthesis equation

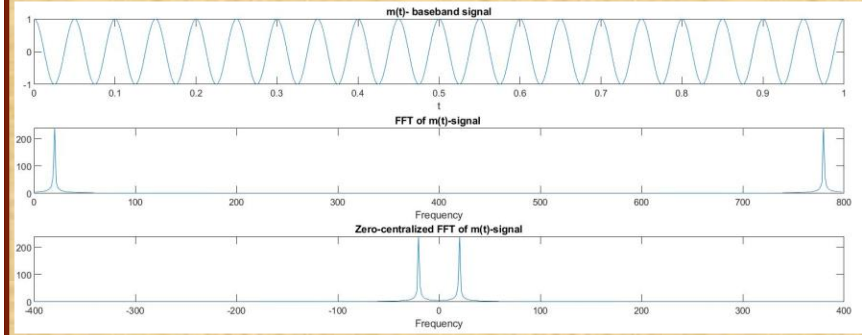
$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n}$$

Analysis equation

Example: DTFT using FFT algorithm

```
close all; clear all; clc

  fc=200; %Hz (carrier)
 fm=20 ; % HZ (baseband)- message signal

 Fs = 800; % sampling frequency i.e no of data/sample taken
per second to discrete-time a continuous signal
 t=0:1/Fs:1; % number of sample instant within 1sec
duration

 figure;
 %%%1. Baseband signal
 %% ===================
  mt=1*cos(2*pi*fm*t);

  subplot(4,1,1); plot(t, mt); %plot original time domain
baseband signal
  title('m(t)- baseband signal'); xlabel('t');

%DTFT using FFT algorithm
  NFFT= 512; % N-point DFT
  XMT = fft(mt, NFFT); % 0, ------ N-1 (points); -
frequency resolution - Fs/NFFT
 subplot(4,1,2); plot([0:NFFT-1]*(Fs/NFFT), abs(XMT));
%plot spectra of m(t) signal with standard FFT
  title('FFT of m(t)-signal '); xlabel('Frequency');
```

```
%with FFT with 'shiftfft'
  XMT = fftshift(fft(mt, NFFT)); % zero frequency
at centre [ --ne(freq.) --0 --+ve (value) ]

  % plot of zero frequency centralized FFF of m(t)
  subplot(4,1,3); plot([-NFFT/2:1:(NFFT/2)-
1]*(Fs/NFFT), abs(XMT)); % [ -NFFT/2:1:NFFT/2 -1] -
- negative to positive freq. range
  title('Zero-centralized FFT of m(t)-signal');
xlabel('Frequency');
```



Structure of lab report

a) Title of the experiment → “Creation a document using MS office”

b) Your name → XYZ, Roll-no: 1234

c) About the experiments →

d) Content of the experiment (diagram/programme source code/flowchart) →

e) Your observation/what you learned →

After complementation of the LAB, document has to be uploaded in Google classroom
filename: StudentName_rollNo

Thank you!