MATH 213: Logical Equivalences, Rules of Inference and Examples

Tables of Logical Equivalences

Note: In this handout the symbol $\equiv$ is used the tables instead of $\iff$ to help clarify where one statement ends and the other begins, particularly in those that have a biconditional as part of the statement. The abbreviations are not universal.

Equivalence	**Name**	**Abbr.**
$p \wedge \mathrm{T} \equiv p$	Identity / Idempotent (Conjunction)	IdC
$p \vee \mathrm{F} \equiv p$	Identity / Idempotent (Disjunction)	IdD
$p \wedge \mathrm{F} \equiv \mathrm{F}$	Domination (Conjunction)	DomC
$p \vee \mathrm{T} \equiv \mathrm{T}$	Domination (Disjunction)	DomD
$\neg(\neg p) \equiv p$	Double Negation	DN
$p \wedge q \equiv q \wedge p$	Commutative (Conjunction)	CC
$p \vee q \equiv q \vee p$	Commutative (Disjunction)	CD
$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative (Conjunction)	AC
$(p \vee q) \vee r \equiv p \vee (q \vee r)$	Associative (Disjunction)	AD
$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive (Conjunction)	DC
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$	Distributive (Disjunction)	DD
$\neg(p \wedge q) \equiv \neg p \vee \neg q$	DeMorgan's Law (Conjunction)	DMC
$\neg(p \vee q) \equiv \neg p \wedge \neg q$	DeMorgan's Law (Disjunction)	DMD
$p \wedge (p \vee q) \equiv p$	Absorption (Conjunction)	AbC
$p \vee (p \wedge q) \equiv p$	Absorption (Disjunction)	AbD
$p \wedge \neg p \equiv \mathrm{F}$	Negation (Conjunction)	NegC
$p \vee \neg p \equiv \mathrm{T}$	Negation (Disjunction)	NegD

Table 1: Logical Equivalences

Equivalence	**Name**	**Abbr**
$\neg(p \implies q) \equiv p \wedge \neg q$	Negation of Implication	NI
$p \implies q \equiv \neg p \vee q$	Implication to Disjunction	ID
$p \implies q \equiv \neg q \implies \neg p$	Contrapositive	C
$p \vee q \equiv \neg p \implies q$		
$p \wedge q \equiv \neg(p \implies \neg q)$		
$(p \implies q) \wedge (p \implies r) \equiv p \implies (q \wedge r)$		
$(p \implies r) \wedge (q \implies r) \equiv (p \vee q) \implies r$		
$(p \implies q) \vee (p \implies r) \equiv p \implies (q \vee r)$		
$(p \implies r) \vee (q \implies r) \equiv (p \wedge q) \implies r$		

Table 2: Logical Equivalences Involving Implications

Equivalence	**Name**	**Abbr.**
$\neg(p \iff q) \equiv \neg p \iff q$	Negation of Biconditional	NB
$\neg(p \iff q) \equiv p \iff \neg q$	Negation of Biconditional (alternative)	NB
$p \iff q \equiv (p \implies q) \wedge (q \implies p)$	Biconditional	B
$p \iff q \equiv \neg p \iff \neg q$	Contrapositive of Biconditional	
$p \iff q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$		

Table 3: Logical Equivalences Involving Biconditionals

Tautology (so these will be true for an	**Name**	**Abbr.**
$p \vee \neg p$	Excluded Middle	EM
$(p \wedge q) \implies p$	Simplification	S
$p \implies (p \vee q)$	Addition	A
$(p \wedge (p \implies q)) \implies q$	Modus Ponens	MP
$((p \implies q) \wedge (q \implies r)) \implies (p \implies r)$	Hypothetical Syllogism	HS
$((p \vee q) \wedge \neg q) \implies p$	Disjunctive Syllogism	DS
$(\neg q \wedge (p \implies q)) \implies \neg p$	Modus Tollens	MT
$((p \vee r) \wedge ((p \implies q) \wedge (r \implies s))) \implies (q \vee s)$	Constructive Dilemma	CDL
$((\neg q \vee \neg s) \wedge ((p \implies q) \wedge (r \implies s))) \implies (\neg p \vee \neg r)$	Destructive Dilemma	DDL
$(p \vee p) \implies p$	Idempotent	IM

Table 4: Additional Tautologies

(Remember, *tautology* means these will always be true for any values of p, q, r, and s.)

Standard Rules of Inference

Each of the following is based on a tautology.

- Modus Ponens $$\begin{array}{rl} & p \\ & p \implies q \\ \hline \therefore & q \end{array}$$

- Modus Tollens $$\begin{array}{rl} & \neg q \\ & p \implies q \\ \hline \therefore & \neg p \end{array}$$

- Conjunctive Simplification $$\begin{array}{rl} & p \\ & q \\ \hline \therefore & p \end{array}$$

- Disjunctive Syllogism $$\begin{array}{rl} & p \vee q \\ & \neg p \\ \hline \therefore & q \end{array}$$

- Hypothetical Syllogism $$\begin{array}{rl} & p \implies q \\ & q \implies r \\ \hline \therefore & p \implies r \end{array}$$

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Others not give in the book:

- Addition $$\begin{array}{rl} & p \\ \hline \therefore & p \vee q \end{array}$$

- Conjunctive Simplification (alternate version) $$\begin{array}{rl} & p \wedge q \\ \hline \therefore & p \end{array}$$

- Resolution $$\begin{array}{rl} & p \vee q \\ & \neg p \vee r \\ \hline \therefore & q \vee r \end{array}$$

Example 1. Identify the rules of inference used in each of the following arguments.

(a) Alice is a math major. Therefore, Alice is either a math major or a c.s. major.

(b) If it snows today, the college will close. The college is not closed today. Therefore it did not snow today.

(c) If I go swimming, then I will stay in the sun too long. If I stay in the sun too long, then I will sunburn. Therefore, if I go swimming, then I will get sunburn.

Example 2. Use rule of inference to show that the premises "Henry works hard", "If Henry works hard then he is a dull boy", and "If Henry is a dull boy then he will not get the job" imply the conclusion "Henry will not get the job."

Symbolic Proofs using Rules of Inference

Example 6. Give an argument (based on rules of inference) to show that the hypotheses/premises

$$(\neg p \land q) \implies (r \lor s), \quad \neg p \implies (r \implies w), \quad (s \implies t) \lor p, \quad \neg p \land q$$

lead to the conclusion $w \lor t$.

Line	**Step**	**Reason**
(1)		
(2)		
(3)		
(4)		
(5)		

Example 7. Give an argument (based on rules of inference) to show that the hypotheses/premises

$$p \implies q, \quad \neg q \lor r, \quad r \implies (t \lor s), \quad \neg s \land p$$

lead to the conclusion t.

Line	**Step**	**Reason**
(1)		
(2)		
(3)		
(4)		
(5)		

Example 8. Give an argument (based on rules of inference) to show that the hypotheses/premises

$$p \wedge q,\ p \implies (\neg q \vee r),\ r \implies s$$

lead to the conclusion s.

Line	**Step**	**Reason**
(1)		
(2)		
(3)		
(4)		
(5)		