

Portfolios for Investors Who Want to Reach Their Goals While Staying on the Mean–Variance Efficient Frontier

SANJIV DAS, HARRY MARKOWITZ, JONATHAN SCHEID,
AND MEIR STATMAN

SANJIV DAS

is a professor of finance at the Leavey School of Business, Santa Clara University, in Santa Clara, CA.
srdas@scu.edu

HARRY MARKOWITZ

is a professor of finance at the Rady School of Management, University of California, San Diego, in La Jolla, CA.
harryhmm@aol.com

JONATHAN SCHEID

is the president and CIO at Bellatore Financial, Inc., in San Jose, CA.
jscheid@bellatore.com

MEIR STATMAN

is Glenn Klimek Professor of Finance at the Leavey School of Business, Santa Clara University, in Santa Clara, CA, and a visiting professor at Tilburg University in the Netherlands.
mstatman@scu.edu

Mental-accounting portfolio theory is a goal-based portfolio theory, combining some of the most appealing features of Markowitz's [1952a] mean–variance portfolio theory and Shefrin and Statman's [2000] behavioral portfolio theory.

Mean–variance portfolio theory is a “production” theory. Investors in that theory produce portfolios that combine expected returns and standard deviations of returns at levels that are best for them. But what do mean–variance investors want to do with the money in their portfolios? Do their goals consist of funding a comfortable retirement and a modest bequest to children? Do they consist of a bare-bones retirement and a considerable bequest to charity? The production of mean–variance efficient portfolios is only a station on the way to investors' ultimate goals, yet mean–variance portfolio theory is silent about these goals. In contrast, behavioral portfolio theory combines the production of optimal portfolios for investors with the use of the money in investors' portfolios to reach their goals.

Mean–variance investors consider their portfolios as a whole, as depicted in Exhibit 1. In contrast, behavioral investors begin by dividing the whole of their portfolios into mental accounts, each dedicated to a particular goal and each with its own time horizon. These are depicted in Exhibit 2. For example,

a \$1 million portfolio might be divided into an \$800,000 mental account for a retirement goal that is 15 years away, a \$150,000 mental account for an education goal that is three years away, and a \$50,000 mental account for a bequest goal that is 25 years away. Next, behavioral investors specify threshold levels they want to reach or exceed for each goal when they arrive at its horizon. For example, investors might aim to reach or exceed a \$2 million threshold in the mental account dedicated to the retirement goal, \$180,000 in the mental account dedicated to the education goal, and \$850,000 in the mental account dedicated to the bequest goal.

Investors in mean–variance portfolio theory face a single efficient frontier, consisting of the best feasible combinations of expected returns and standard deviation of returns. In contrast, investors in behavioral portfolio theory face many efficient frontiers, one for each mental account. Moreover, while risk in mean–variance portfolio theory is measured by the standard deviation of return, risk in behavioral portfolio theory is measured by the probability of failing to reach the threshold level of a mental account. Investors in behavioral portfolio theory choose their best combination on the efficient frontier of expected returns and the probability of failing to reach the threshold of each mental account.

EXHIBIT 1

Mean-Variance Portfolio Theory

Investors consider the portfolio as a whole. They want to be on the mean-variance efficient frontier.

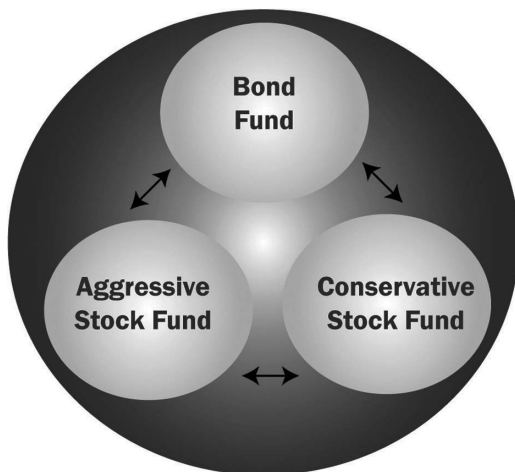
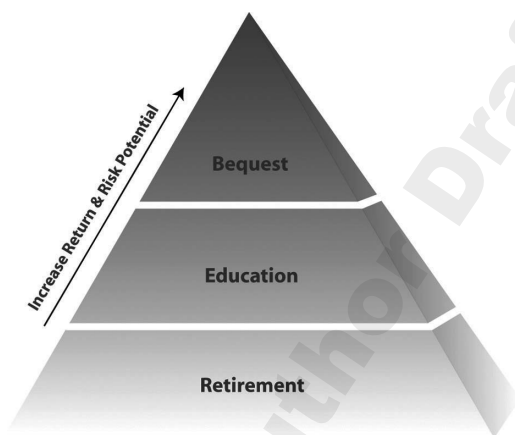


EXHIBIT 2

Behavioral Portfolio Theory

Investors divide their portfolios into mental accounting layers or a portfolio pyramid, whereby each layer is associated with a particular goal and a particular attitude toward risk.



The road to behavioral portfolio theory started more than 60 years ago, when Friedman and Savage [1948] noted that hope for riches and protection from poverty share roles in our behavior; people who buy lottery tickets often buy insurance policies as well. So, people are risk seeking enough to buy lottery tickets and risk averse enough to buy insurance. Four years

later, Markowitz wrote two articles that reflect two very different views of behavior. In one [1952a], he created mean-variance theory, and in the other [1952b], he extended Friedman and Savage's insurance-lottery framework. People in mean-variance theory, unlike people in the insurance-lottery framework, never buy lottery tickets; they are always risk averse and never risk seeking.

Friedman and Savage observed that people buy lottery tickets because they aspire to reach higher social classes, whereas they buy insurance as protection against falling into lower social classes. Markowitz [1952b] clarified Friedman and Savage's observation by noting that people aspire to move up from their current social class or "customary wealth." So, people with \$10,000 might accept lottery-like odds in the hope of winning \$1 million, and people with \$1 million might accept lottery-like odds in the hope of winning \$100 million. Kahneman and Tversky [1979] extended the work of Markowitz [1952b] into prospect theory. Prospect theory describes the behavior of people who accept lottery-like odds when they are in the domain of losses, such as when they are below their aspiration levels, but reject such odds when they are in the domain of gains, such as when they are above their aspiration levels.

Behavioral portfolio theory's description of portfolios as collections of mental accounts, or a set of layers in portfolio pyramids, is part of common investment advice. The mental account or pyramid structure of behavioral portfolios is also reflected in the upside-potential and downside-protection layers of "core and satellite" portfolios.

One might argue that although the portfolios are described as layered pyramids, which are consistent with behavioral portfolio theory, investors actually consider them as a whole, which is consistent with mean-variance portfolio theory. But the evidence does not support such an argument. Consider, for example, Question 13 in the Asset Allocation Planner of Fidelity Investments [2003]:

If you could increase your chances of improving your returns by taking more risk, would you:

1. Be willing to take a *lot* more risk with *all* your money?
2. Be willing to take a *lot* more risk with *some* of your money?

3. Be willing to take a *little* more risk with *all* your money?
4. Be willing to take a *little* more risk with *some* of your money?
5. Be unlikely to take much more risk?

Answers 1 and 3 make sense within mean–variance theory. In that theory, only the risk of *all* the money in the overall portfolio matters. But Answers 2 and 4 make no sense within mean–variance theory because they assume a segmentation of the portfolio into mental accounts depending on where investors are willing to take more or less risk with *some* of their money. Mean–variance investors have a single attitude toward risk, not a set of attitudes mental account by mental account. In contrast, behavioral investors have many attitudes toward risk, one for each mental account, so they might be willing to take a lot more risk with some of their money. Statman [2004] found that the number of investors who were willing to take a *lot* more risk with *some* of their money exceeded the number of investors who were willing to take a *little* more risk with *all* their money by a ratio of approximately 10 to one. Yet taking a *lot* more risk with *some* of our money adds to our overall portfolio risk about the same as taking a *little* more risk with *all* our money.

Recently, Harry Markowitz joined Das et al. [2010] in the development of mental accounting portfolio theory, which combines mean–variance portfolio theory with several features of behavioral portfolio theory. These features include the mental accounting structure of portfolios and the definition of risk as the probability of failing to reach threshold levels of return. But mental accounting portfolio theory does not include two central features of behavioral portfolio theory. First, investors in behavioral portfolio theory can be risk seeking in some of their mental accounts, but risk-seeking preferences are excluded from both mean–variance portfolio theory and mental accounting portfolio theory. Second, the optimal securities in behavioral portfolio theory resemble call options with asymmetric distributions of returns rather than securities with symmetric distributions of returns, such as normal distributions. We hasten to add that mean–variance portfolio theory can accommodate many kinds of return distributions, including asymmetric return distributions.

INVESTOR GOALS AND MENTAL ACCOUNTS

Consider a 50-year-old investor with \$1 million in investable assets. She allocates her money to three mental account subportfolios, each associated with a goal. She allocates \$800,000 to the retirement account, \$150,000 to the education account, and \$50,000 to the bequest account that she hopes to leave for her children. Our investor has other assets, including a house and future Social Security payments. Moreover, our investor is still working and investing her savings. She intends to retire at 65 and estimates her life expectancy to be 75.

Our investor wants her \$800,000 retirement account to grow to \$1,917,247 by the time she is 65, which implies a compound annualized return of 6% during the remaining 15 years. Her horizon for the education account is shorter, only three years, and she wants that account to grow to \$188,957, which implies a compound annualized return of 8%. Our investor's horizon for the bequest account is long—25 years—but she has an ambitious aspiration; she wants her current account of \$50,000 to grow to \$850,003, which implies a compound annualized return of 12%.

The mean–variance efficient frontier faced by our investor is determined by the characteristics of available securities, namely, their expected returns, standard deviations, and correlations. She creates three mental account subportfolios associated with her three goals and will evaluate her progress toward her goals once a year, adjusting her goals and mental account subportfolios as necessary. Each mental account subportfolio is optimized by the mean–variance procedure. The optimal mean–variance subportfolio for the retirement goal is the one that corresponds to an expected return of 6%. The 6% expected return also determines the risk of this subportfolio. Mental account subportfolios are created similarly for the education goal, with its expected return of 8%, and the bequest goal, with its expected return of 12%. Exhibit 3 provides a summary of the mental account subportfolios and their related return and risk characteristics.

To keep our example simple, assume there are only three assets: a bond mutual fund or ETF with an expected return of 2% a year and a standard deviation of 5%, a conservative stock mutual fund or ETF with an expected return of 8% a year and a standard deviation of 20%, and an aggressive stock mutual fund or ETF with an expected return of 15% a year and a standard deviation

EXHIBIT 3

Mental Account Subportfolios and Goals

Mental Account Subportfolio & Goal	Allocation to the Subportfolio	Time Horizon	Desired Threshold Annualized Return	Desired Threshold Amount at the Conclusion of the Time Horizon
Bequest	\$50,000	25 years	12.00%	\$850,003
Education	\$150,000	3 years	8.00%	\$188,957
Retirement	\$800,000	15 years	6.00%	\$1,917,247

EXHIBIT 4

Mental Account Subportfolios and the Overall Portfolio

	Retirement	Education	Bequest	Overall
Composition of Subportfolios	\$800,000	\$150,000	\$50,000	\$1,000,000
<i>Bond Fund</i>	52.51%	29.40%	-16.80%	45.58%
<i>Conservative Stock Fund</i>	31.06%	45.40%	74.06%	35.36%
<i>Aggressive Stock Fund</i>	16.43%	25.20%	42.74%	19.06%
Total	100%	100%	100%	100%
Expected Return	6.00%	8.00%	12.00%	6.60%
Std. Deviation	10.45%	15.23%	25.28%	11.85%

of 40%. The returns of the bond fund are uncorrelated with the returns of the two stock funds. The returns of the two stock funds have a 25% correlation.

Our investor calculates three mean–variance efficient subportfolios corresponding to the three goals. They are displayed in Exhibit 4 along with the overall portfolio. The standard deviation of the returns of the retirement subportfolio is the lowest at 10.45%, followed by the 15.23% of the education subportfolio and the 25.28% of the bequest subportfolio. The 6.60% expected return of the overall portfolio is a weighted average of the returns of the three mental account subportfolios, but the 11.85% standard deviation of the overall portfolio is different from the weighted average of the standard deviations of the three mental account subportfolios.

Asking our investor to state her preferences for expected return and standard deviations for each goal is better than asking her to state her preferences for the expected return and standard deviation of the overall portfolio. The latter requires that she aggregate the three goals in her mind, which is difficult. It is easier to match low risk tolerance with the retirement subportfolio, medium risk tolerance with the education subportfolio,

and high risk tolerance with the bequest subportfolio than to match a weighted-average risk attitude toward the three goals.

Moreover, since our investor has little sense of her true attitude toward risk in the overall portfolio, asking her to state that risk attitude is likely to result in her choosing a portfolio on the efficient frontier that does not correspond well to her true risk attitude. There is a loss that comes from choosing the wrong portfolio on the efficient frontier, and that loss can range between a few basis points of annual return to several percentage points. See Exhibit 5 for a schematic representation of these losses.

Mental account subportfolios and the overall portfolio are always on the mean–variance efficient frontier when there are no constraints on allocation, such as a preclusion of short or leveraged positions (see Exhibit 6). Mental account subportfolios may be a few annual basis points below the mean–variance efficient frontier when such constraints are imposed. Yet even such small losses are rarely incurred in usual practice, as financial advisors rarely recommend overall portfolios with short or leveraged positions.

The proportion allocated to the bond fund is highest in the retirement subportfolio, lower in the education subportfolio, and lowest in the bequest portfolio. Arranging the overall portfolio as a set of three mental account subportfolios does not imply that we need three “real” accounts for each fund, or one for the bond fund in the retirement subportfolio, another for the bond fund in the education subportfolio, and a third for the bond fund in the bequest subportfolio. Instead, we have

EXHIBIT 5

Loss in Units of Expected Return from Misestimated Risk Attitudes

Misestimation of risk attitudes leads to suboptimal portfolios on the efficient frontier.

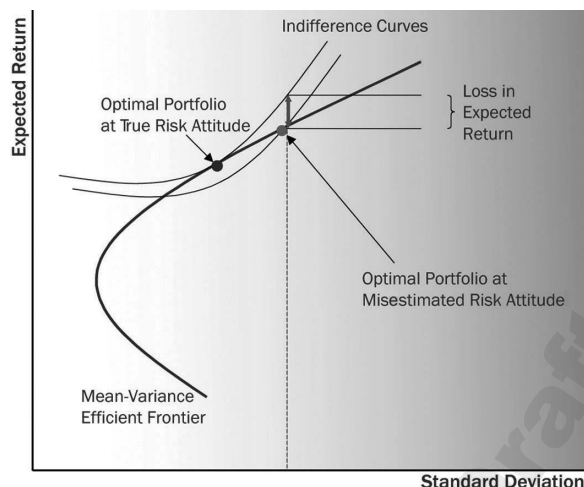
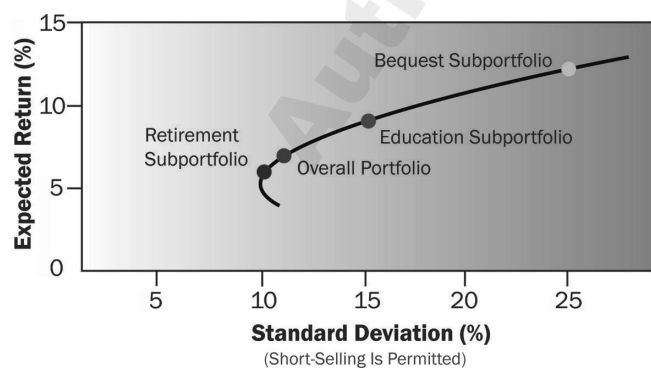


EXHIBIT 6

The Mean-Variance Efficient Frontier with Mental Account Subportfolios and the Overall Portfolio



one real bond fund account and three “virtual” bond accounts listing the allocation in the bond fund of each subportfolio. Financial advisors can provide portfolio reports to their clients in two formats—real account formats for the overall portfolio and virtual account formats for each of the mental account subportfolios (see Exhibit 7).

The presentation of the overall portfolio as the sum of the three mental account subportfolios has an advantage over a sole presentation of the overall portfolio. The mental account subportfolios’ presentation speaks the language of investors. Investors want to reach their goals, not have portfolios only on the mean–variance efficient frontier. Goal-based mental account subportfolios let investors articulate each goal, the horizon of each goal, and the attitude toward risk for each goal. Telling the investor that the expected return of her overall portfolio is 6.60% and its standard deviation is 11.85% tells her little because the overall portfolio combines three very different goals. Telling her the expected returns and standard deviations of each subportfolio highlights her goals.

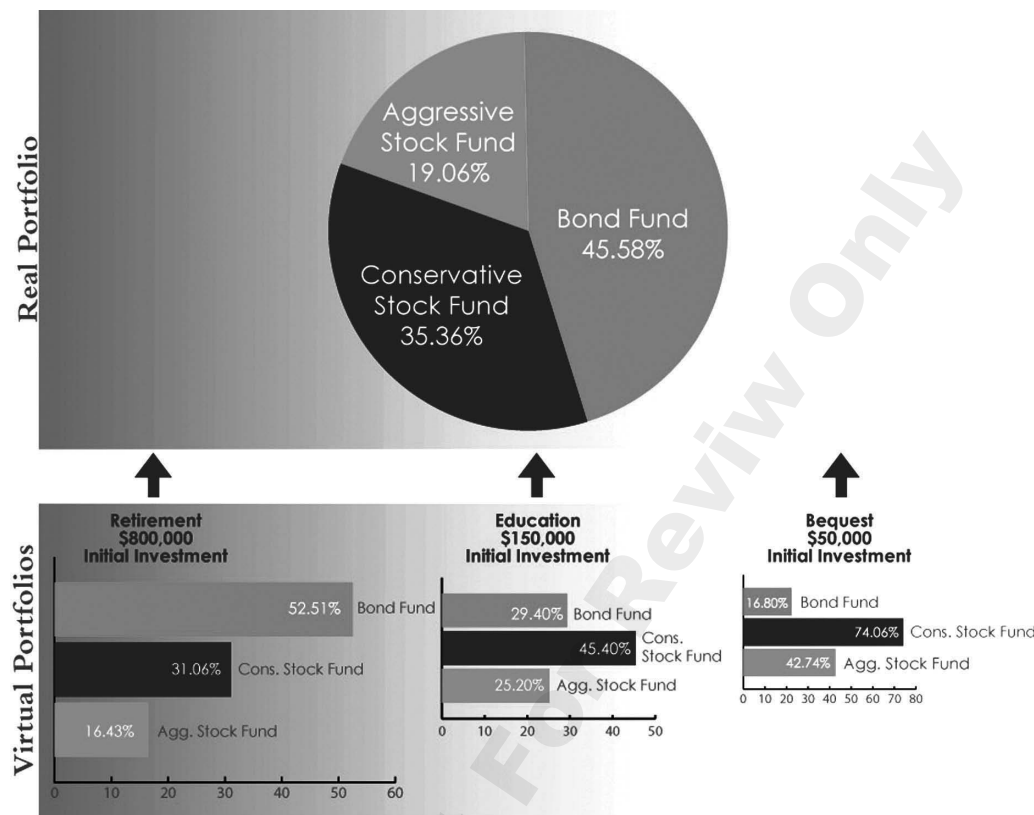
MEASURING RISK BY THE PROBABILITY OF LOSSES AND THEIR AMOUNTS

Our investor is better able to convey her preferences for expected returns and standard deviations associated with each goal than her preferences for expected returns and standard deviations associated with the overall portfolio. Yet standard deviations have little intuitive meaning to most investors who are not schooled in statistics. Standard deviations can be made more intuitive to understand. For example, we can tell our investor that the 10.45% standard deviation in her retirement subportfolio implies that she has approximately a two in three chance that her realized return would be higher than -4.45% yet lower than 16.45% . The first is the 6% expected return minus the 10.45% standard deviation, and the second is 6% plus 10.45% . This explanation helps convey the facts of risk somewhat more intuitively but not much more intuitively. (Indeed, standard deviations have little intuitive meaning even to investors schooled in statistics.)

Our investor is likely to prefer to express her attitude toward risk by specifying a probability of incurring a loss and the amount of loss. Referring to the retirement mental account subportfolios, she might say that she

EXHIBIT 7

Real Portfolio and Virtual Portfolios



does not want too much risk. Specifically, she is willing to accept no more than a 6.28% probability of losing more than 10% of her retirement subportfolio during the coming year. Our investor is willing to accept considerably more risk in her bequest subportfolio. In that subportfolio, she is willing to accept a 14.27% probability of losing more than 15% of her funds during the coming year.

It turns out that the 10.45% standard deviation of the retirement subportfolio can be expressed more intuitively as a probability no higher than 6.28% of losing more than 10% of her funds in the retirement subportfolio during the coming year. Similarly, the 25.28% standard deviation of the bequest subportfolio is expressed more intuitively as a probability no higher than 14.27% of losing more than 15% of her funds in the bequest subportfolio during the coming year.

Exhibit 8 compares the mental account subportfolios' risk with the language of probability.

CONCLUSION

Investors are attracted to Markowitz's mean-variance portfolio theory by its logic and practical application. It seems logical to choose portfolios based on their overall expected returns and standard deviations of returns. And the mean-variance optimizer is a practical tool, quick at drawing the efficient frontier of expected returns and standard deviations of returns. Yet investors want more than portfolios on the mean-variance efficient frontier. Ultimately, investors want their portfolios to satisfy goals such as a secure retirement, college education for their grandchildren, or a bequest for children.

While Markowitz's mean-variance portfolio theory is silent about ultimate portfolio goals, these goals are central in Shefrin and Statman's behavioral portfolio theory. Among other things, investors in behavioral portfolio theory divide their portfolios into mental account layers of a portfolio pyramid, whereby each layer

EXHIBIT 8

Probability of Return Lower than Threshold Return

Threshold Return	Probability of Return Lower than Threshold Return in the Retirement Mental Account	Probability of Return Lower than Threshold Return in the Education Mental Account	Probability of Return Lower than Threshold Return in the Bequest Mental Account	Probability of Return Lower than Threshold Return in the Overall Portfolio
-25.00%	0.15%	1.51%	7.16%	0.38%
-20.00%	0.64%	3.30%	10.28%	1.24%
-15.00%	2.22%	6.55%	14.27%	3.41%
-10.00%	6.28%	11.86%	19.21%	8.06%
-5.00%	14.62%	19.67%	25.06%	16.37%
0.00%	28.28%	29.97%	31.75%	28.87%
5.00%	46.19%	42.19%	39.09%	44.63%
10.00%	64.91%	55.22%	46.85%	61.30%
15.00%	80.55%	67.71%	54.72%	76.09%
20.00%	90.99%	78.46%	62.42%	87.10%
25.00%	96.55%	86.78%	69.65%	93.98%

is associated with a particular goal and a particular attitude toward risk. Risk in behavioral portfolio theory is the probability of failing to reach the threshold of each goal, not the standard deviation of the returns of the overall portfolio. In this article, we combine appealing features of mean–variance portfolio theory and behavioral portfolio theory in a mental accounting portfolio theory. Investors divide their portfolios by goals into mental account layers of a portfolio pyramid, and risk is the probability of failing to reach the threshold return of each goal. Yet each mental account is optimized by the rules of mean–variance portfolio theory.

ENDNOTE

This article is an abbreviated version of “Portfolio Optimization with Mental Accounts” [Das et al. 2010].

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