International Journal of Bifurcation and Chaos, Vol. 26, No. 7 (2016) 1650120 (10 pages)

DOI: 10.1142/S0218127416501200



# Emergence of Chimera in Multiplex Network

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Received December 16, 2015; Revised March 1, 2016

Chimera is a relatively new emerging phenomenon where coexistence of synchronous and asynchronous states is observed in symmetrically coupled dynamical units. We report the observation of the chimera state in multiplex networks where individual layer is represented by 1-d lattice with nonlocal interactions. While, multiplexing does not change the type of the chimera state and retains the multi-chimera state displayed by the isolated networks, it changes the regions of the incoherence. We investigate the emergence of coherent–incoherent bifurcation upon varying the control parameters, namely, the coupling strength and the network size. Additionally, we investigate the effect of initial condition on the dynamics of the chimera state. Using a measure based on the differences between the neighboring nodes which distinguishes smooth and non-smooth spatial profiles, we find the critical coupling strength for the transition to the chimera state. Observing chimera in a multiplex network with one-to-one inter layer coupling is important to gain insight to many real world complex systems which inherently posses multilayer architecture.

*Keywords*: Chimera; multiplex network; coupled map lattice.

## 1. Introduction

In the past few decades, network science has discovered a plethora of novel phenomena while trying to mimic real world systems in a better manner. One such discovery is an observation of the chimera state. It was first reported by Kuramato *et al.* in 2002 while investigating nonlocally coupled identical oscillators in a ring network [Kuramoto & Battogtokh, 2002]. Later, it was analyzed and christened by Abrams and Strogatz in 2004 as chimera state [Abrams & Strogatz, 2004]. Like, its counterpart in Greek mythology, a chimera state has come to be referred to as a mathematical hybrid state in which coherent and incoherent dynamics coexist in nonlocally coupled identical oscillators in a structurally symmetric network.

Chimera has been extensively investigated both theoretically [Sethia *et al.*, 2008; Laing, 2009; Omel'chenko *et al.*, 2011] and experimentally [Hagerstrom *et al.*, 2012; Larger *et al.*, 2013]. It has been observed in plenty of networks including phase oscillators [Abrams & Strogatz, 2004; Maistrenko *et al.*, 2014; Omel'chenko *et al.*, 2012b; Sethia *et al.*, 2008; Laing, 2009], chemical

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[Tinsley *et al.*, 2012; Nkomo *et al.*, 2013], mechanical oscillators [Martens *et al.*, 2013], neuron models [Hizanidis *et al.*, 2014], planar oscillators [Laing, 2010], boolean networks [Rosin *et al.*, 2014; Rosin, 2015], 1D superconducting meta material [Lazarides *et al.*, 2015], etc. Chimera was originally reported for nonlocally coupled oscillators, recently it has been reported in feed back delayed networks [Omel'chenko *et al.*, 2008; Sheeba *et al.*, 2010], globally coupled networks [Schmidt *et al.*, 2014; Yeldesbay *et al.*, 2014], time varying networks [Buscarino *et al.*, 2015] and networks with purely local coupling [Laing, 2015]. Moreover, different types of chimera have been reported including multicluster chimera [Xie *et al.*, 2014; Omel'chenko *et al.*, 2013], virtual [Larger *et al.*, 2013], breathing [Abrams *et al.*, 2008] and two-dimensional chimera [Omel'chenko *et al.*, 2012b; Panaggio & Abrams, 2015a]. A recent work suggests the emergence of chimera, dependent on nonhyperbolicity of dynamical systems for both the time-discrete and time continuous cases [Semenova *et al.*, 2015].

The chimera state has also been reported for various real world network models such as Rosenzweig–MacArthur oscillators for ecological networks [Dutta & Banerjee, 2015]. Chimera has also been characterized by the state of the dynamical evolution of the network. Type-I chimera is characteristic of the hyper chaotic behavior with many positive Lyapunov exponents [Wolfrum *et al.*, 2011]. This type of chimera has primarily been observed for time-continuous systems like complex Ginzburg–Landau oscillators or Kuramoto oscillators. In Type-II chimera, only spatial chaos has been observed with a rather simple temporal behavior (mostly periodic). Though, this type of chimera has been reported mainly for the time-discrete systems (maps) [Omel'chenko *et al.*, 2011; Omel'chenko *et al.*, 2012a; Hagerstrom *et al.*, 2012], it has recently been observed for the time-continuous Stuart–Landau oscillators as well [Zakharova *et al.*, 2014].

Further, modeling real world complex systems under the multiplex network framework is one of the recent advancements in the network theory [Boccaletti *et al.*, 2014; Lee *et al.*, 2012; Wang *et al.*, 2015; Kivelä *et al.*, 2014]. We consider a multiplex network consisting of the same nodes across the layers (Fig. 1) and investigate the occurrence of chimera state in the multiplex ring networks. In this structure, each node has exactly the same connection architecture. We observe Type-II chimera with spatial chaos and periodic temporal behavior. Though, the chimera state, upon multiplexing, remains of the same type as observed for the isolated network, the multiplexing changes the region of incoherence. Dependence of the chimera state on initial conditions is observed for the multiplex networks as already observed for the isolated networks. Additionally we present a measure in terms of distance variable to distinguish between the coherent and the chimera states and to identify the transition point for the coherent–incoherent bifurcation. We also investigate the role of the size of the network in determining the critical coupling strength for the symmetry breaking and thus emergence of the chimera state.

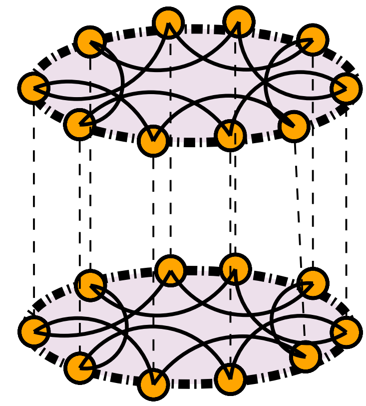

Fig. 1. Schematic diagram for multiplex network consisting of two layers. Each layer is represented by 1D lattice with nonlocal interaction. Each node (circle) has the same coupling architecture.

## 2. Model

We consider a multiplex $S^1$ ring network with $N$ nodes in each layer, where $S^1$ represents one-dimensional symmetric cyclic group with elements being invariant to the permutation operation [Jacobson, 2009]. Considering $z_t(i)$, $i = 1, \ldots, mN$ as a real dynamical variable at time $t$ for the $i$th node, the dynamics of the network can be described as,

$$z_{t+1}(i) = f(z_t(i)) + \frac{\varepsilon}{(2rN+1)} \sum_{j=1}^{mN} A_{ij} \times [f(z_t(j)) - f(z_t(i))] \quad (1)$$

where $\varepsilon$ represents the coupling strength, $m$ represents number of layers, $r$ represents the coupling radius defined by $r = P/N$, with $P$ signifying the number of neighbors in each direction in a layer. The elements of the adjacency matrix $A$ of a network is defined as $A_{ij} = 1$ or 0 depending upon whether $i$th and $j$th nodes are connected or not. The diagonal entries $A_{ij} = 0$ represent no self-connection in the network. The adjacency matrix $A$ for the multiplex network can be written as,

$$A = \begin{pmatrix} A^1 & I & \cdot & \cdot & \cdot & I \\ I & A^2 & I & \cdot & \cdot & I \\ \cdot & I & A^3 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ I & \cdot & \cdot & \cdot & \cdot & A^m \end{pmatrix},$$

where $A^1, A^2, \ldots, A^m$ represent the adjacency matrix of the first, second, $\ldots, m$th layer, respectively and $I$ is a unit $N \times N$ matrix. Note that the number of nodes in each layer of the multiplex network is the same. A mismatch in the network size of the layers will yield nodes in different layers having different interaction patterns and hence we cannot define the chimera state. We use logistic map $f(z) = \mu z(z - 1)$ with the bifurcation parameter $\mu = 3.8$ at which individual logistic map exhibits chaotic behavior. We consider coupling radius $r = 0.32$ for each layer indicating the degree of each node being 64. A state of the network is defined as spatially coherent if for any node $i \in S^1$, the spatial distance between the neighboring nodes approaches zero for $t \to \infty$

$$\lim_{t\to\infty} |z_t(i+1) - z_t(i)| \to 0, \quad \forall\, i \in S^1. \tag{2}$$

Geometrically, this signifies a smooth profile of the spatial curve. Smoothness of the curve, signifying the correlated spatial values of the neighboring nodes, is defined with the absence of any discontinuity in the spatial curve. Whereas, temporal coherence (synchronization) is defined as,

$$\lim_{t\to\infty} |z_t(j) - z_t(i)| \to 0 \quad \text{for } \forall\, i, j \in S^1.$$

Therefore, temporal coherence can be written as $z_t(1) = z_t(2) = \cdots = z_t$ which leads to a straight line for the spatial curve associated with the temporal coherence. Appearance of discontinuity in the smooth spatial curve implies coexistence of the coherence and incoherence regions. To demonstrate the absence of smoothness, we define a measure based on the spatial distance as follows,

$$d(i) = (z(i+1) - z(i)) - (z(i) - z(i-1)) \tag{3}$$

which captures the difference of the spatial distances between the neighboring nodes. For smooth spatial profile $d(i) = 0$, signifying a symmetric distribution of the distances between neighboring nodes, while discontinuity in the spatial profile, signifying the transition point, is indicated as kinks in the distribution. We use this measure to find the critical coupling strength for the symmetry breaking and thus resulting in the chimera state.

## 3. Coherent–Incoherent Bifurcation

We evolve Eq. (1) starting with a set of special initial conditions and after an initial transient, study the spatio-temporal patterns of the multiplex network. Note that uniform or the Gaussian distributed random initial conditions lead to either a completely coherent state, spanning all the nodes, or a completely incoherent state depending upon the coupling strength. When $0.28 \le \varepsilon \le 0.32$, it leads to the incoherent evolution of all the nodes. Motivated from [Kuramoto & Battogtokh, 2002], we use a hump back function to generate the initial conditions as follows. We choose a uniform random number $z_0(i)$ for the initial state for $i$th node within some interval which varies like a Gaussian function as:

$$z_0(i) = \exp\left[-\frac{\left(i - \frac{N}{2}\right)^2}{2\sigma^2}\right]. \tag{4}$$

The variance $\sigma$ is chosen depending upon the size of the network such that the random variable lies between 0 and 1. The same initial condition is used for both the layers in the multiplex network. A very narrow width of the function leads to almost very close value of the initial conditions for a fraction of nodes leading to the coherent state.

In the absence of any coupling between the nodes ($\varepsilon = 0$) or for weak couplings, all the nodes evolve independently and no spatial coherence is observed. For instance, as demonstrated in Fig. 2(a), for $\varepsilon = 0.1$, the evolution of the nodes in the multiplex network yields an incoherent state with no correlations in the neighboring nodes. As the coupling strength is increased, a partially coherent state emerges at $\varepsilon = 0.28$ with correlated spatial values of the neighboring nodes in the end and in

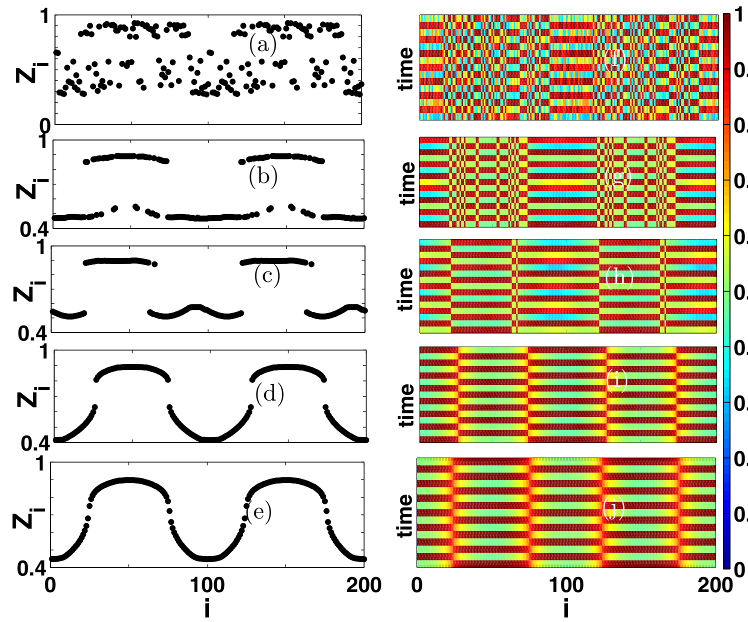


Fig. 2. Snapshots and spatio-temporal plots for multiplex $S^1$ ring network for (a) and (f) $\varepsilon = 0.1$, (b) and (g) $\varepsilon = 0.28$, (c) and (h) $\varepsilon = 0.30$, (d) and (i) $\varepsilon = 0.40$, (e) and (j) $\varepsilon = 0.44$. Number of nodes in each layer remains $N = 100$ and coupling radius $r = 0.32$. Initial transient is taken as 5000. Results are presented for time range 5000 to 5015.

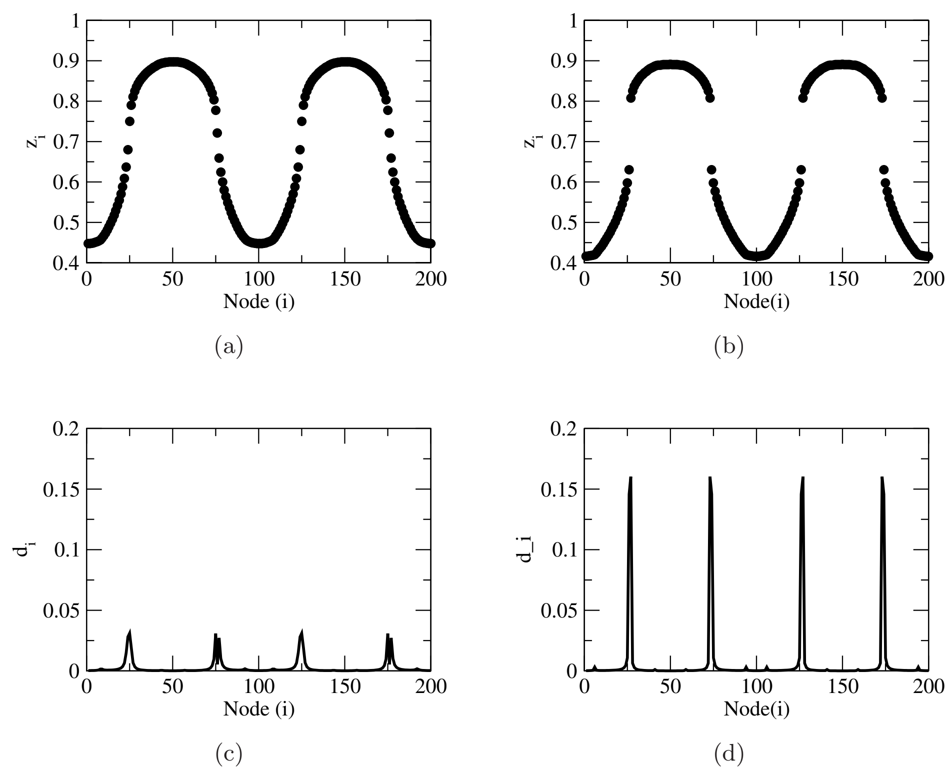


Fig. 3. Distance measure for multiplex $S^1$ ring network for (a), (c) $\varepsilon = 0.44$ and (b), (d) $\varepsilon = 0.4$. Network size is $N = 100$ in each layer and coupling range is $r = 0.32$.

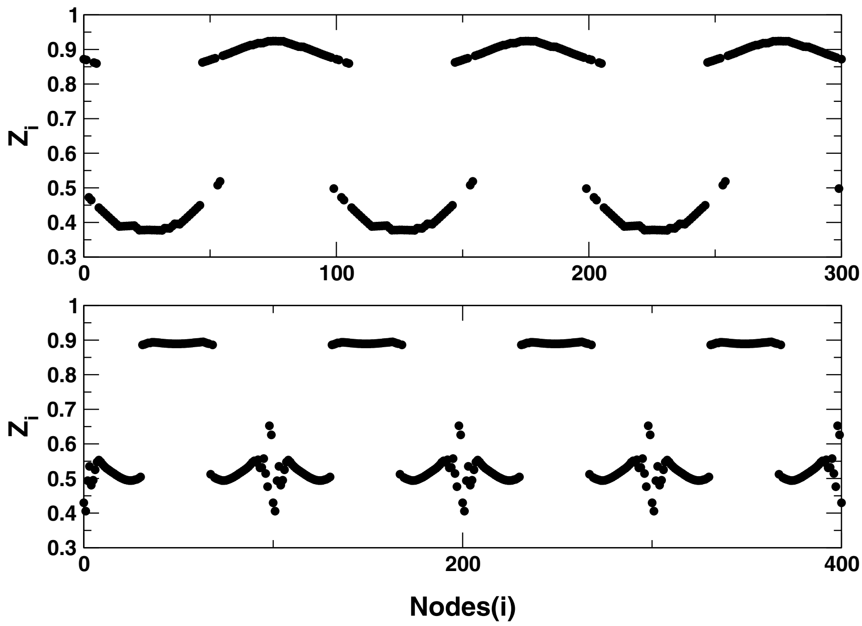


Fig. 4. Snapshots for (a) three-layer and (b) four-layer multiplex network. Parameters are $\varepsilon = 0.28$ and $r = 0.32$. Number of nodes $N = 100$ for each layer.



the middle regions of each layer, however, the spatial range of the incoherent region is more than the coherent region [Fig. 2(b)]. This coexistence of the coherent and incoherent dynamics corresponds to the chimera state in the multiplex network.

The dynamical behavior of two layers of a multiplex network is a replica of each other manifesting exactly the same spatio-temporal patterns (Fig. 2). Exactly same behavior is observed for multiplex networks having more than two layers (Fig. 4).

At the same coupling value, the spatio-temporal dynamics [Fig. 2(g)] reflects nonregular skeletal type pattern in the incoherent regions. This irregularity of the pattern suggests that, in the multiplex network framework, a node may get attracted to either of the upper or lower region depending on its initial value as reported for the isolated network [Omel'chenko *et al.*, 2011].

As we increase the coupling strength further, the range of the incoherent region decreases as depicted by Fig. 2(c) for $\varepsilon = 0.3$. At $\varepsilon = 0.4$, we observe a sharp discontinuity in the otherwise smooth profile of $z(j)$ and the incoherency appears at two distinct points in each layer. This is a bifurcation point for the coherent–incoherent transition. Above this coupling value, all the nodes in the multiplex network acquire the complete coherent state as indicated by the appearance of a smooth geometric profile at $\varepsilon = 0.44$ [Fig. 2(e)]. Figure 2 depicts spatial regions of incoherent nodes and thus indicates a nonzero spatial entropy with the periodic temporal dynamics, representing a Type II chimera state. Further, the regions of incoherence in the spatial profile continues to exist for narrower intervals with an increase in the coupling strength (Fig. 2).

Furthermore, in the chimera state, the time evolution of all the nodes in the network depict periodic behavior with the periodicity two depicting temporal regularity. The coupled dynamics displays

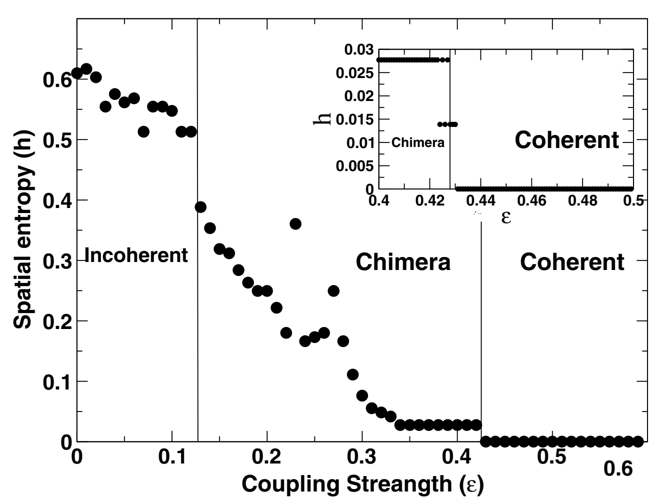


Fig. 5. Spatial entropy as a function of $\varepsilon$. Figure is plotted for $N = 100$ and $r = 0.32$.

the spatial chaos which is defined by the nonzero spatial entropy given by $h = d\log_e(2)$, where $d$ represents a fraction of the incoherent nodes in network [Omel'chenko *et al.*, 2011; Coullet *et al.*, 1987]. We show that the distance measure [Eq. (3)] is able to easily distinguish between coherent and chimera states. The discontinuous spatial profile [Fig. 3(b)] at $\varepsilon = 0.4$ gives rise to the kinks [Fig. 3(d)] in the distance measure distribution signifying transition to the chimera state. We calculate the spatial entropy as a function of the coupling strength ($\varepsilon$) in order to demonstrate the transition between the chimera to the coherent state. A transition from the chimera to coherent state is indicated by the discontinuous change in spatial entropy of the network (Fig. 5).

## 4. Multiplex Network Versus Isolated Network

In order to see the impact of multiplexity on the dynamical behavior of nodes, we compare the dynamical state of an isolated 1D lattice with that of the multiplexed with another 1D lattice. We find that while the chimera state is retained after multiplexing, the dynamical evolution of the network differs. The multiplexing may enhance or suppress the incoherency. For instance, at $\varepsilon = 0.30$, the isolated network displays the chimera state with incoherence in the middle regions of the spatio-temporal pattern [Figs. 6(a) and 6(c)]. After multiplexing, the region of incoherence shrinks to a point discontinuity in the middle and intermediate regions (Fig. 6). Thus, multiplexing here retains the chimera state as well as the type of the chimera state, but leads to a change in the region of incoherence as well as in the dynamical evolution.

## 5. Sensitivity to Initial Conditions

Furthermore, similar to the isolated networks, in the multiplex networks as well, the chimera exhibits dependency on the initial conditions. As already discussed in Sec. 3, the spatial profile displayed by the chimera state can be very different for different profiles of initial conditions for the same set of control parameters, namely $\mu$ and $r$. But interestingly, even for the initial condition given by the same profile as Eq. (4) with a constant value $\sigma$ for a given network size $N$, different realizations of the initial conditions can lead to different incoherent regions. For example, at $\varepsilon = 0.30$, for three different realizations of the initial conditions, all given by Eq. (4), three different spatio-temporal patterns are observed (Fig. 7). Though, the multichimera state is evident for all the realizations, the region of incoherence differs without any consistent behavior.

Moreover, we investigated the impact of network size on the emergence of chimera state as well as on the critical coupling strength below which one observes as incoherent–coherent regime. Figure 8 presents the chimera state for a multiplex ring

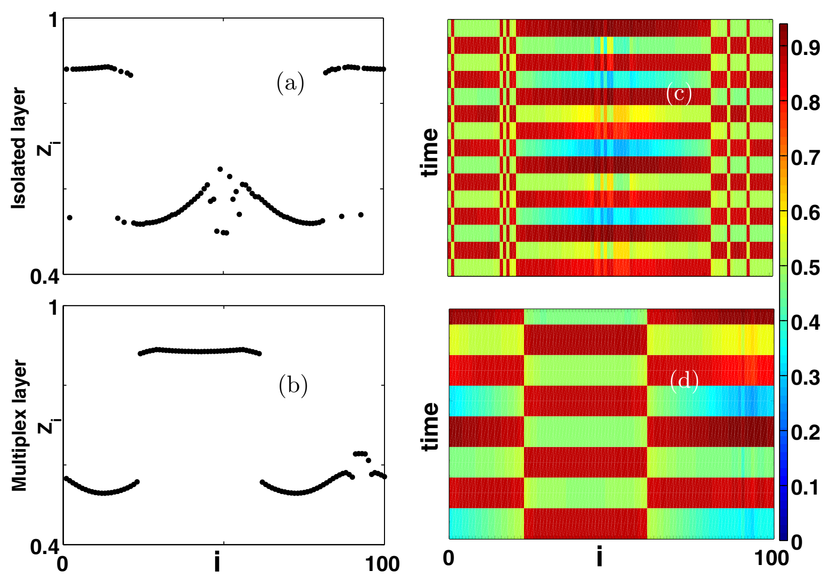


Fig. 6. Snapshot and spatio-temporal pattern for (a), (c) single and (b), (d) first $S^1$ ring of multiplexed network are shown. Parameters are $\varepsilon = 0.30$ and $r = 0.32$. Number of nodes $N = 100$ for each layer. Initial transient is taken as 5000. Results are presented for time range 5000 to 5015.

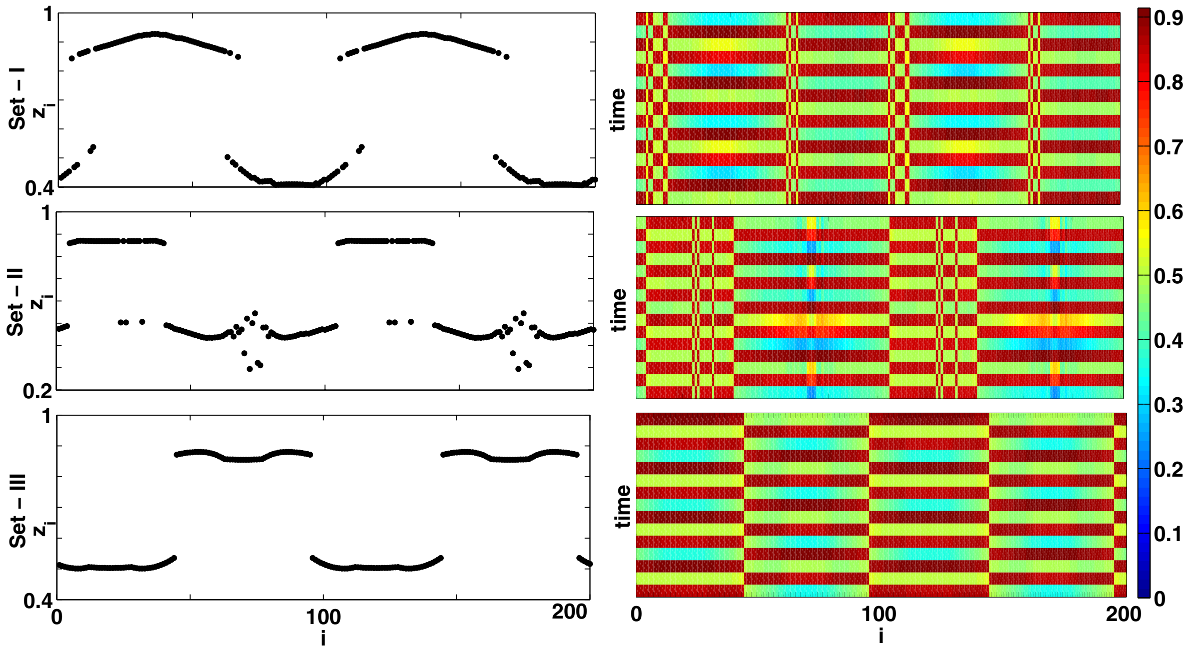


Fig. 7. Spatio-temporal pattern for multiplex $S^1$ rings for different realizations of the initial conditions. Parameters are $\varepsilon = 0.32$ and $r = 0.32$. Number of nodes $N = 100$ for each layer. Initial transient is taken as 5000. Results are presented for time range 5000 to 5015.

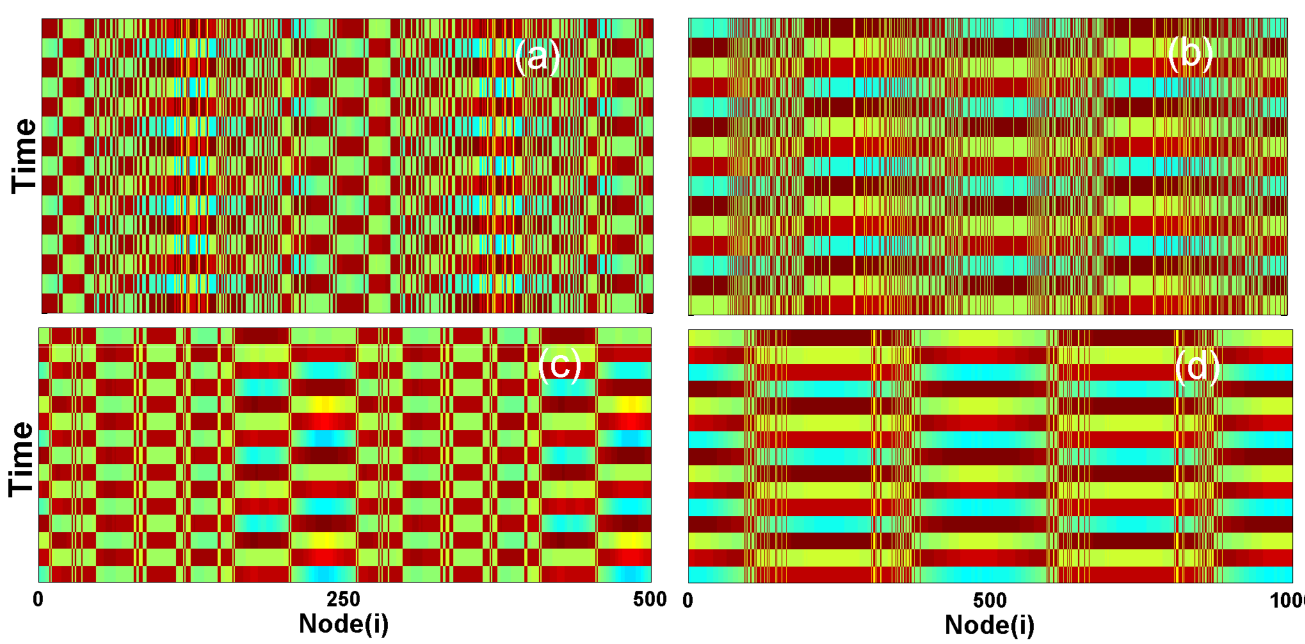


Fig. 8. Spatio-temporal pattern for multiplex $S^1$ rings with coupling strength (a) $\varepsilon = 0.21$ (b) $\varepsilon = 0.23$ (c) $\varepsilon = 0.28$ and (d) $\varepsilon = 0.27$. Coupling range $r = 0.32$ and network size for (a), (c) $N = 500$ and (b), (d) $N = 1000$. Initial transient is taken as 5000. Results are presented for time range 5000 to 5015.

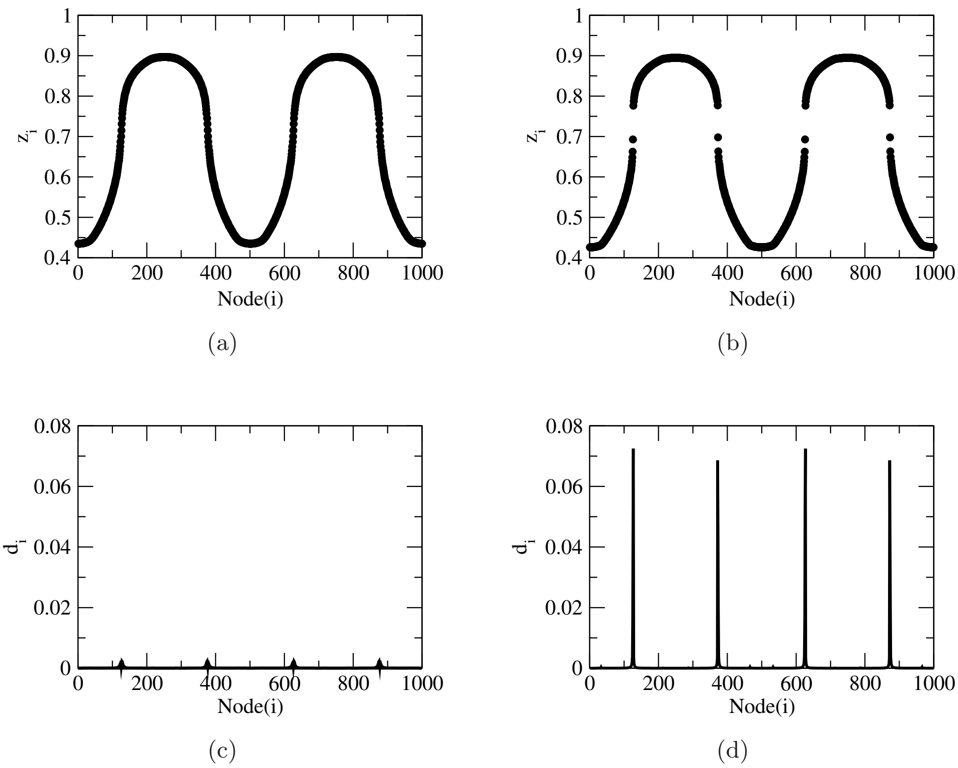


Fig. 9. Distance measure for multiplex $S^1$ ring network for (a), (c) $\varepsilon = 0.44$ and (b), (d) $\varepsilon = 0.41$. Network size $N = 500$ and coupling range $r = 0.32$ in each layer.

network with two different network sizes. For both the network sizes, the coupled evolution exhibits the coexistence of coherence–incoherence dynamics. However, the critical coupling strength for the coherent–incoherent bifurcation increases to $\varepsilon = 0.41$ (Fig. 9) as compared to $\varepsilon = 0.4$ for network size $N = 200$ as indicated by Fig. 2.

## 6. Conclusion

To summarize, we report an emergence of the chimera in the multiplex networks with the layers being represented by 1-d lattice architecture having nonlocal couplings. We find that an emergence of the chimera is identical in the mirror layers arising due to the underlying symmetry of the network. Furthermore, while the temporal behavior of the network remains periodic even after multiplexing, the range of the coupling strength for which chimera is observed changes. The chimera in the multiplex network is found to be sensitive to the changes in the initial conditions, which is revealed through the changes in the incoherent region of the dynamical evolution for different sets of initial conditions. We also show that the critical coupling strength increases with the size of the network. The results presented here may provide a better understanding to the peculiar nature of the chimera state observed in many natural systems like unihemispheric sleep, ventricular fibrillation, brain networks which incorporate multilayer network architecture [Panaggio & Abrams, 2015b].


## Acknowledgments

S. Jalan is grateful to Department of Science and Technology (DST), Government of India and Council of Scientific and Industrial Research, Government of India project grants EMR/2014/000368 and 25(0205)/12/EMR-II respectively for financial support. S. Ghosh acknowledges DST, Government of India, for the INSPIRE fellowship (IF150149) as well as the Complex Systems Lab members for timely help and useful discussions. Authors acknowledge the Conference on Nonlinear Systems and Dynamics 2015 held at IISER Mohali where this work was initiated.


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