Limitation on complete fusion during heavy-ion collisions*

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In a recent paper Lefort and collaborators have extracted a critical distance of approach as the mechanism limiting fusion of two heavy ions. In this comment it is shown that this analysis does not depend on the specific model used for the calculation of the ion-ion potential. Furthermore an expression is derived to analyze both high- and low-energy fusion data on the same basis. It is concluded that all existing data are compatible with the existence of a critical distance. However, data spanning a larger range of energies and being more accurate at high energies are needed for a final check of the critical distance hypothesis.

[NUCLEAR REACTIONS Heavy-ion complete fusion; critical distance of approach.]

In a recent paper[1] Lefort and his collaborators have shown that not a critical angular momentum but instead a critical distance of approach may be the relevant quantity limiting complete fusion during a collision between two complex nuclei. A similar observation is implicitly contained in the work of Natowitz, Chulick, and Namboodiri.[2]

The authors of Ref. 1 proceed in their analysis in three steps. They first calculate, from the measured cross sections for complete fusion $\sigma_{\text{fus}}(E)$, a critical angular momentum $l_c(E)$ using a sharp cutoff approximation:

$$\sigma_{\text{fus}}(E)=\pi\lambda\!\!\!^{-2}\sum_{l=0}^{l_c}(2l+1)\approx\pi\lambda\!\!\!^{-2}l_c(E)^2\,. \qquad (1)$$

In a second step, an ion-ion potential is calculated using an energy-density method as proposed by Brueckner, Buchler, and Kelly,[3] and in a third step the classical turning point for the partial waves with $l=l_c(E)$ at the energy E is calculated. It is found that this point stays remarkably constant as a function of energy. Since all partial waves with $l\leq l_c$ contribute to fusion this implies that the point found as outlined above acts as a critical distance for fusion. Its value is found to be:

$$R_c=r_c(A_1^{1/3}+A_2^{1/3});\quad r_c=1.0\pm0.07\text{ fm}\,. \qquad (2)$$

At the same time, however, it is found that this critical distance also coincides with the minimum of the nuclear potential above. This minimum appears at a point where a core in the nuclear potential due to compression of nuclear matter sets in. The compression is a consequence of the "sudden approximation" used in the calculation of the ion-ion potential. It is known, however, from recent studies on nuclear shock waves[4] that no sizeable compression can be expected in the range of energies at which the analyzed data were taken. This fact may seem to indicate that the finding of a critical distance is a consequence of the special potential used and thus, in view of the criticisms of the potential model, rather unphysical.

The purpose of this comment is to point out that the analysis of Galin *et al.* can be performed in a way that is independent of the special model used for the calculation of the potential.

One should first realize that the procedure of Ref. 1 corresponds exactly to the classical expression for the cross section for reaching the critical distance R_c:

$$\sigma=\pi R_c^2\left(1-\frac{V(R_c)}{E}\right)\,. \qquad (3)$$

This can be seen by writing down the equation for the classical turning point of particles with $l=l_c$ and energy E, solving this equation for l_c, and substituting that value into Eq. (1). The special nuclear model enters into this expression only through the value of $V(R_c)$.

A test of the assumption of the existence of a critical distance that two heavy ions have to pass in order to fuse can thus be obtained by plotting σ_{fus} as a function of $1/E$. A linear dependence with the value of $V(R)$ being in reasonable limits establishes the existence of a critical distance independent of the special model used to calculate $V(R)$.

It is thus seen that the method used in Ref. 1 to extract a critical distance is exactly equivalent to the one used by Gutbrod, Winn, and Blann[5] who, however, from their analysis of low-energy fusion data obtained a fusion distance:

$$R_B = r_B(A_1^{1/3} + A_2^{1/3}); \quad r_B \approx 1.4 \text{ fm}, \tag{4}$$

that is about 40% larger than the value of R_c and corresponds to the distance of the ions at the fusion barrier.

In order to substantiate the finding of a critical distance of approach, it is thus necessary first to check the linear dependence of σ_{fus} on $1/E$ in the high-energy range and second to understand the difference between the two different distances [Eqs. (2) and (4)] obtained from the data by using the same expression for their analysis.

The second point can be achieved by setting

$$\sigma_{\text{fus}} = \pi\lambda\!\!\!^{-2} \sum_{l=0}^{\infty} (2l+1) T_l P_l, \tag{5}$$

where T_l is the penetration probability through the interaction barrier whereas P_l gives the probability for fusion to take place once the barrier has been passed. Following Thomas[6] we approximate T_l by the penetration factors of a parabolic barrier with frequency $\hbar\omega_l$. For P_l we still use a sharp cutoff model:

$$P_l = \begin{cases} 1 & l \leq l_c \\ 0 & l > l_c . \end{cases} \tag{6}$$

Approximating then the frequencies $\hbar\omega_l$ and the position of the interaction barrier R_{Bl} by constant values $\hbar\omega$ and R_B, respectively, and replacing the summation in Eq. (5) by an integration[7] one obtains:

$$\sigma_{\text{fus}} = \frac{\hbar\omega}{2} R_B^2 \frac{1}{E} \ln \frac{1+\exp\{2\pi[E - V(R_B)]/\hbar\omega\}}{1+\exp(2\pi\{E - V(R_B) - (R_c/R_B)^2[E - V(R_c)]\}/\hbar\omega)}. \tag{7}$$

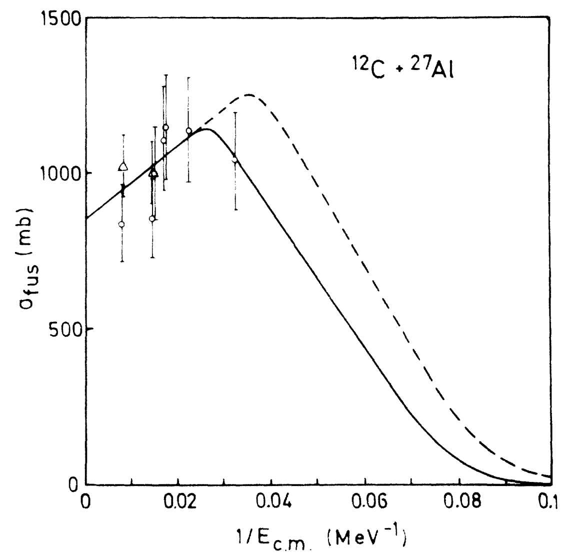

FIG. 1. Fusion cross section for the reaction $^{12}C + ^{27}Al$. The data marked by circles are taken from Ref. 2; these include only the evaporation residue cross sections, whereas the triangular points also contain the fission contribution (Ref. 8). The solid line shows the prediction for σ_{fus} obtained by using Eq. (7). The barrier parameters $R_B = 7.5$ fm and $V_B = 12.5$ MeV were obtained from a semiempirical formula in Ref. 5. The critical distance $R_c = 5.2$ fm and the potential $V_c = -14$ MeV were obtained from a calculation using the sudden approximation (Ref. 9). In the dashed line the values $R_B = 8.5$ fm and $V_B = 11.5$ MeV, as obtained from the potential calculation, were used. The oscillator frequency $\hbar\omega$ was taken to be 5 MeV.

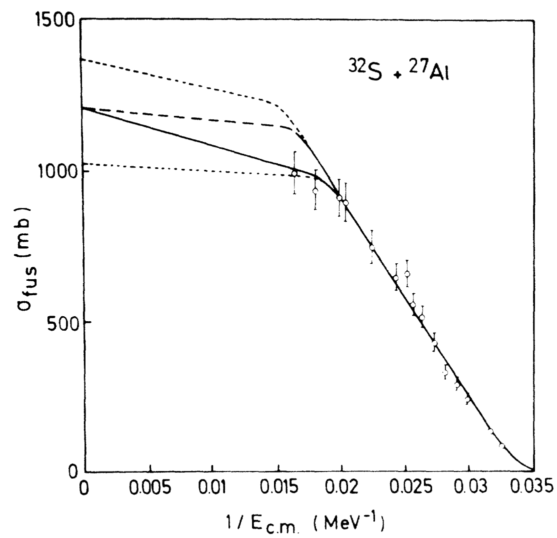

FIG. 2. Fusion cross section for $^{32}S + ^{27}Al$. The solid line gives the prediction of Eq. (7) with $R_B = 8.5$ fm, $V_B = 29.7$ MeV, $R_c = 6.2$ fm, $V_c = 10$ MeV. The barrier parameters were obtained from a semiempirical formula in Ref. 5; r_c was taken to be 1.0 fm and V_c was fitted to the data to reproduce the observed bend. For the dotted lines σ_{fus} was calculated with $r_c = 1.0 \pm 0.07$ fm with the corresponding values for V_c. The dashed line has the same parameters as the solid one except for $V_c = 3$ MeV as obtained from the potential calculation. The comparison of the solid with the dashed line illustrates the uncertainties connected with the use of the sudden approximation for calculating V_c. The data are taken from Table I in Ref. 5. The highest energy point may be somewhat too low because the fission contributions to σ_{fus} were not taken into account.

This expression gives the fusion cross section as a function of bombarding cm energy E. It contains both the effects of the fusion barrier and those of the critical distance and can thus be used to check the existence of the latter. Expression (7) has the interesting property that at small energies E (neglect exponential in denominator and 1 in numerator), it reduces analytically to expression (3) with $R=R_B$ and $V=V(R_B)$ whereas for large E (drop 1 both in numerator and denominator) it again assumes the same analytical form except that now $R=R_c$ and $V=V(R_c)$.[10]

This means that low-energy data are only sensitive to the interaction barrier. Only high-energy data can, therefore, be used to check the hypothesis of the existence of a critical distance for fusion. This explains the result of Gutbrod *et al.* who, by analyzing low-energy data, obtained a value for R corresponding to the barrier distance.

We have performed an analysis of all existing fusion data as a function of bombarding energy using expression (7) where the parameters R_B and $V(R_B)$ were usually obtained from an empirical fit to the data.[5] For R_c we used the value given by the authors of Ref. 1. The potentials $V(R_c)$ were obtained by modifying the values calculated by Galin *et al.* within reasonable limits. The detailed analysis will be published elsewhere.[11] Two of the results are shown in Figs. 1 and 2 as an example for the data analyzed by Galin *et al.*[1] and Gutbrod, Winn, and Blann,[5] respectively.

It is seen that the high-energy data (Fig. 1) can be described rather well by expression (7) although the large error bars prevent one from proving a linear dependence of σ_{fus} on $1/E$ in an unambigous way. On the other hand, Fig. 2 shows that the data of Ref. 5—although being more accurate—only test the barrier region except perhaps for the few highest energy points.

The arguments given above and the full analysis[11] allow us to draw the following conclusions:

(1) The observation of a critical distance made by Lefort and collaborators does not depend critically on the model used for the calculation of the ion-ion potential and in particular not on the sudden approximation.

(2) The accuracy of the presently available high-energy data does not allow any completely unambiguous quantitative comparison with the dependence of σ_{fus} on $1/E$ as given by Eq. (7) and thus any final conclusion on the existence of a critical distance of approach. However, all existing fusion data—including those of the Rochester group—can be well described by Eq. (7) with physically reasonable values for the parameters. Thus all data are compatible with the assumption of a critical distance for fusion.

(3) The existence of a critical distance expresses itself in Eq. (7) not only through the linear dependence of σ_{fus} on $1/E$ in the high-energy limit but also through the bend in σ_{fus} as a function of $1/E$ at a point where the fusion barrier loses its importance and the critical distance becomes effective. Although this bend seems to be seen in a few low-energy measurements,[5] data that span a larger range of energies with improved accuracy at high energies are necessary for a final check of the critical distance hypothesis.

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[9]We are grateful to Professor M. Lefort for making the program for the calculation of the ion-ion potential available to us.

[10]Expression (7) represents an extension of Wong's expression for the reaction cross section to the problem of fusion.

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