# DETERMINATION OF THE NUCLEUS-NUCLEUS POTENTIAL FROM BOUNDARY CONDITIONS

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The nuclear potential acting between complex nuclei is determined assuming the boundary conditions following from the liquid-drop model. A convenient parametrization of the Coulomb interaction potential is proposed. Both nuclear and Coulomb potentials can be easily calculated for any target-projectile pair without free parameters.

The real part of the optical potential which can be determined from the analysis of heavy-ion elastic scattering data is very ambiguous because the scattering is mostly determined by the tail of the real potential. As has been pointed out by Satchler [1] and Glendening [2], the data do not necessarily determine the shape of the tail. Rather, what is determined is the value of the real potential at the distance $D$ at which many "equally good" potentials (of different slopes) cross each other. The nuclear interaction at the distance $D$ is quite weak, of the order of 1 MeV [3–6]. This is the only unambiguous information on the real potential one can get from the analysis of the elastic scattering data. However, for the dynamical description of the reactions between heavy ions it is necessary to know the real potential at much closer distances [7–9].

At closer distances there is another characteristic point where the nucleus-nucleus interaction seems to be reliably established. This is the distance corresponding to the sum of the half-density radii, $R_1 + R_2$, where the nuclear attractive force can be estimated from the liquid-drop model [10]:

$$F = 4\pi\gamma \frac{R_1 R_2}{R_1 + R_2}, \tag{1}$$

where $\gamma \approx 0.95\ \mathrm{MeV \cdot fm^{-2}}$ is the surface tension coefficient. The force (1) is supposed to be the maximum force acting between the two nuclei [10]. Therefore, assuming the Saxon-Woods form of the nucleus-nucleus potential, the radius parameter $R_0$ is automatically determined as $R_0 = R_1 + R_2$.

The distance $D$ and the value of the potential at this distance, $V_N(D)$, together with the value of the force (1),

$$\left(\frac{\mathrm{d}V_N}{\mathrm{d}r}\right)_{r=R_0} = \frac{V_0}{4a} = 4\pi\gamma \frac{R_1 R_2}{R_0}, \tag{2}$$

determine the two parameters: the diffuseness $a$ and the strength of the potential $V_0$. The third parameter, $R_0$, is the sum of the half-density radii and, according to Myers [11], can be expressed as

$$R_0 = R_1 + R_2 = 1.128\ \mathrm{fm} \cdot A_1^{1/3}(1 - 0.786 A_1^{-2/3}) + 1.128\ \mathrm{fm} \cdot A_2^{1/3}(1 - 0.786 A_2^{-2/3}). \tag{3}$$

The parameters of the potential determined in such a way are shown in table 1 for five different reactions. (The values of $D$ and $V_N(D)$ are well established for these reactions since the elastic scattering data have been analysed [1,3–6] with widely varying diffuseness parameter $a$.)

The lower rows (II) in table 1 show the parameters of the potential determined in a different way, using the liquid-drop boundary conditions exclusively. Again, the contact force (1) is assumed to act at $r = R_0$ and, additionally, the potential energy of the system at $r = 0 (V_N(0) \approx V_0)$ is postulated to be

$$V_0 = b_{\mathrm{surf}}[A_1^{2/3} + A_2^{2/3} - (A_1 + A_2)^{2/3}], \tag{4}$$

where $b_{\mathrm{surf}} \approx 17\ \mathrm{MeV}$ [12] is the surface energy param-

Table 1
Comparison of the parameters of the nuclear potential determined by the contact force (1) and the value of the potential $V_N(D)$ at the distance $D$, (I), with the parameters determined by the boundary conditions following from the liquid-drop model, (II).

| Reaction | $D$ (fm) | $V_N(D)$ (MeV) | ref. | $V_0$ (MeV) | $R_0$ (fm) | $a$ (fm) |
|---|---|---|---|---|---|---|
| $^{16}O + ^{40}Ca$ | 9.45 | − 1.1 | [3] | (I) 59.5<br>(II) 57.9 | 6.09<br>6.09 | 0.846<br>0.824 |
| $^{16}O + ^{96}Zr$ | 10.85 | − 1.4 | [3] | (I) 69.4<br>(II) 69.4 | 7.46<br>7.46 | 0.877<br>0.875 |
| $^{11}B + ^{208}Pb$ | 12.15 | − 1.0 | [1,4] | (I) 64.4<br>(II) 63.2 | 8.64<br>8.64 | 0.845<br>0.830 |
| $^{16}O + ^{120}Sn$ | 11.55 | − 0.9 | [5] | (I) 69.1<br>(II) 72.0 | 7.87<br>7.87 | 0.850<br>0.885 |
| $^{16}O + ^{150}Nd$ | 11.70 | − 1.6 | [6] | (I) 74.0<br>(II) 74.4 | 8.32<br>8.32 | 0.888<br>0.893 |

eter. This expression represents the liquid-drop energy of the fused system $A_1 + A_2$ (of spherical shape) calculated with respect to the energy of separated incident nuclei of the mass numbers $A_1$ and $A_2$. Combining (2) and (4), one can determine the diffuseness parameter $a$:

$$a = \frac{V_0 R_0}{16\pi\gamma R_1 R_2} = 0.356\,\mathrm{fm}^2 \cdot \frac{R_0[A_1^{2/3} + A_2^{2/3} - (A_1+A_2)^{2/3}]}{R_1 R_2} . \quad (5)$$

Using equations (3), (4) and (5) the nuclear potential

$$V_N(r) = -V_0 \left[1 + \exp\left(\frac{r - R_0}{a}\right)\right]^{-1} , \quad (6)$$

can be calculated for any target -projectile pair without free parameters. As is seen from table 1, this potential perfectly reproduces the outer region which is essentially for elastic scattering. (The parameters determined by $D$, $V_N(D)$, and the contact force $F$ are nearly identical with those determined by the liquid-drop boundary conditions.)

When the nuclei deeply penetrate each other the Coulomb interaction potential deviates significantly from the pointcharge approximation. Unfortunately, the numerical computations of the Coulomb energy are very time-consuming. The simple approximations used so far in optical model codes (e.g., in the ABACUS program [13]) and in the classical orbit calculations [7,8] are not accurate enough and, moreover, all have a break of the second derivative $d^2V_C/dr^2$ at an arbitrarily chosen distance which is called the Coulomb radius $R_C$.

Similarly as in the case of the nuclear potential, we assume that for $r = 0$ the colliding nuclei of atomic numbers $Z_1$ and $Z_2$ form a spherical compound nucleus of "normal" charge and density distribution. Therefore, the Coulomb potential energy of the system at $r = 0$ (calculated with respect to the Coulomb energy of the nuclei $Z_1$ and $Z_2$ at $r = \infty$) is

$$C_0 = \frac{3e^2}{5r_0}\left[\frac{(Z_1+Z_2)^2}{(A_1+A_2)^{1/3}} - \frac{Z_1^2}{A_1^{1/3}} - \frac{Z_2^2}{A_2^{1/3}}\right], \quad (7)$$

where $r_0 = 1.128$ fm is the parameter of the "equivalent sharp radius", defined by Myers [11].

An acceptable parametrization of the Coulomb energy $V_C(r)$ has to satisfy the following conditions:

(i) $V_C(0) = C_0$,

(ii) $V_C(r)$, $dV_C/dr$, and $d^2V_C/dr^2$ are continuous in the entire range of $r$,

(iii) $V_C(r)$ has a maximum at $r = 0$, i.e.,

$$\left(\frac{dV_C}{dr}\right)_{r=0} = 0 , \quad \text{and} \quad \left(\frac{d^2V_C}{dr^2}\right)_{r=0} \leqslant 0 , \quad (8)$$

(iv) $dV_C/dr$ reaches a maximum value for $r$ close to the touching distance $R_0$,

(v) for $r > R_0$, $V_C(r)$ approaches the point-charge approximation.

A possible analytical expression satisfying the conditions (i)–(v) is

Table 2
Comparison of the interaction barriers, $B_{exp}$, determined experimentally from the fusion-reaction-excitation functions with the barriers calculated with the proposed nucleus-nucleus potential.

| Reaction | $B_{exp}$ (MeV) | Ref. | $B_{calc}$ (MeV) |
|---|---|---|---|
| $^{24}Mg + ^{32}S$ | 28.3 ± 0.3 | [14] | 28.1 |
| $^{27}Al + ^{32}S$ | 29.7 ± 0.3 | [14] | 29.9 |
| $^{32}S + ^{40}Ca$ | 43.5 ± 0.3 | [14] | 44.0 |
| $^{12}C + ^{152}Sm$ | 46.8 ± 0.3 | [6] | 47.5 |
| $^{11}B + ^{238}U$ | 53 ± 1 | [15] | 54.5 |
| $^{16}O + ^{150}Nd$ | 58.3 ± 0.3 | [6] | 59.3 |
| $^{32}S + ^{58}Ni$ | 59.5 ± 0.5 | [14] | 58.7 |
| $^{12}C + ^{238}U$ | 63 ± 2 | [15] | 65.4 |
| $^{14}N + ^{238}U$ | 74 ± 2 | [15] | 75.1 |
| $^{16}O + ^{238}U$ | 83 ± 2 | [15] | 84.6 |
| $^{20}Ne + ^{238}U$ | 102 ± 2 | [15] | 103.4 |
| $^{40}Ar + ^{208}Pb$ | 157 ± 2 | [16] | 155.6 |
| $^{40}Ar + ^{238}U$ | 171 ± 2 | [16] | 171.1 |
| $^{52}Cr + ^{208}Pb$ | 203 ± 2 | [16] | 202.2 |

$$V_C(r) = \frac{Z_1 Z_2 e^2}{r+\delta} , \qquad (9)$$

where

$$\delta = \delta_0 \left[ 1 + \exp\left(\frac{4r}{\delta_0}\right) + \exp\left(\frac{4R_0}{\delta_0}\right) \exp\left(\frac{r-R_0}{a_C}\right) \right]^{-1} , \qquad (10)$$

with $\delta_0 = 2Z_1 Z_2 e^2 / C_0$.

This expression contains only one free parameter, $a_C$, which controls the speed of the approach $V_C(r) \to Z_1 Z_2 e^2/r$ outside the touching distance $R_0$. Obviously, $V_C(r)$ combined with the nuclear potential has to reproduce the fusion barriers observed experimentally. Following this requirement the value of $a_C$ was found to be $a_C = 1$ fm.

In table 2 the interaction barriers produced by the proposed nuclear and Coulomb potentials are compared with the available experimental data on the fusion barriers [6, 14–16]. Gutbrod, Winn and Blann [14] have proposed to define the height of the fusion barrier as the energy corresponding to the point where the linear extrapolation of the $\sigma_f$ versus $E^{-1}$ plot reaches $\sigma_f = 0$. All the experimental barriers listed in table 2 are determined in this way.

The calculated barrier heights fit the experimental values perfectly. It should be mentioned, however, that for very heavy colliding systems experimental determination of the interaction barrier is rather difficult and ambiguous because non-conservative interactions in the entrance channel become important. It is very likely that the colliding nuclei lose energy in irreversible processes before they come to the top of the barrier. Hence, the energy threshold for fusion reaction observed experimentally may be higher than the barrier of the conservative interaction potential. Perhaps, this effect is responsible for a considerable shift of the fusion barrier with respect to the threshold for inelastic direct processes (for $^{74}Ge + ^{232}Th$ and other heavy systems), reported by Oganessyan et al. [17], and by Lefort, Le Beyac and Péter [18].

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