

# **Nuclear and Particle Physics (PH505)**

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Course Project Report

## 1 Team details

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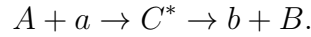
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## 2 Aim and Theory

The experiment aims to test the validity of Bohr's compound nucleus assumption for nuclear reactions.

According to this assumption a nuclear reaction proceeds in two stages:

- The target nucleus absorbs the incident nucleus to form a quasistable compound nucleus.
- The compound nucleus disintegrates by the emission of either the original incident particle or another particle or photon



The intermediate compound state for heavy nuclei ( $Z > 30$ ) has a long mean life compared to the time a nucleon takes to cross the nucleus  $\approx 10^{21}$  to  $10^{22}$  s [1]. When an intermediate state has a long mean life, the second process is independent of the first. Hence we can express the cross section of the above reaction type as

$$\sigma(a, b) = \sigma_a(\epsilon)\eta(E) \quad (1)$$

where

$\sigma(a, b)$  is the cross section of the reaction type  $A + a \rightarrow b + B$ ,

$\epsilon$  is the kinetic energy of the incident particle,

$\sigma_a(\epsilon)$  is the cross section for absorption of particle of kinetic energy  $\epsilon$  by the target nucleus  $A$  to form the compound nucleus  $C^*$ ,

$E = \epsilon + B_a$  is the excitation energy of the compound state  $C^*$ ,

$B_a$  is the binding energy of the particle  $a$  to the target nucleus  $A$ , and

$\eta_b(E)$  is the probability of disintegration of  $C^*$  into the final state  $B + b$ .

If the compound nucleus  $C^*$  is now formed in the same state of excitation by another process  $A' + a'$ , the cross section for disintegration into the same final state,  $B + b$  is given by  $\sigma(a', b) = \sigma_a(\epsilon')\eta_b(E)$ , where  $\epsilon'$  is the kinetic

energy of the incident particle  $a'$ . The binding energies between the two cases are different, thus  $\epsilon'$  will be different from  $\epsilon$ , i.e.  $\epsilon' = E - B_{a'}$ .  $\eta_b(E)$  will be the same in the two cases, because of the basic assumption that the mode of decay of the compound nucleus  $C^*$  is independent of the mode of its formation. If  $C^*$  decays into  $D + d$ , the corresponding cross sections will be given by

$$\sigma(a, d) = \sigma_a(\epsilon)\eta_d(E)\sigma(a', d) = \sigma'_a(\epsilon)\eta_d(E) \implies \frac{\sigma(a, b)}{\sigma(a, d)} = \frac{\eta_b(E)}{\eta_d(E)} = \frac{\sigma(a', b)}{\sigma(a, d)} \quad (2)$$

By testing the last of the above relations, Bohr's theory of the compound nucleus can be verified.

### 3 Reactions Used

The following six reactions and their excitation curves were studied by Ghoshal[1] to test Bohr's theory of the compound nucleus.

- $\text{Ni}^{60}(\alpha, n)\text{Zn}^{63}$
- $\text{Cu}^{63}(p, n)\text{Zn}^{63}$
- $\text{Ni}^{60}(\alpha, 2n)\text{Zn}^{62}$
- $\text{Cu}^{63}(p, 2n)\text{Zn}^{62}$
- $\text{Ni}^{60}(\alpha, 2n)\text{Zn}^{62}$
- $\text{Cu}^{63}(n, pn)\text{Cu}^{62}$

### 4 Experimental Results From Fig. 1

The observed cross sections for  $(\alpha, n)$ ,  $(\alpha, 2n)$ ,  $(\alpha, pn)$  reactions on  $\text{Ni}^{60}$  and  $(p, n)$ ,  $(p, 2n)$ ,  $(p, pn)$  reactions on the  $\text{Cu}^{63}$  are plotted as functions of the kinetic energy of the  $\alpha$  particles ( $\epsilon_\alpha$ ) and protons ( $\epsilon_p$ ) respectively. The proton energy scale has been shifted to bring the peaks of the proton curves into correspondence with those of the alpha curves.

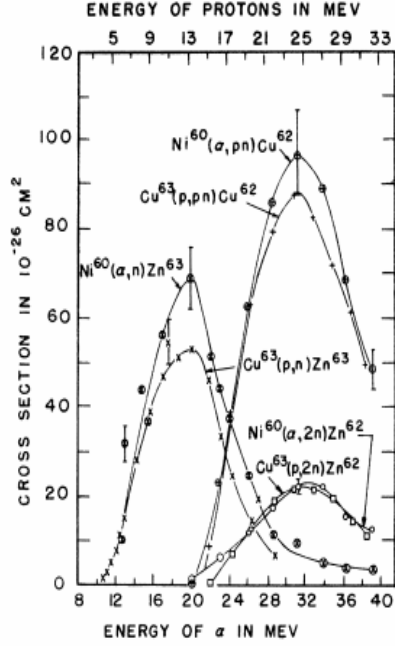


FIG. 1. Experimental cross sections for  $(p, n)$ ,  $(p, 2n)$ ,  $(p, pn)$  reactions on  $\text{Cu}^{63}$  and for  $(\alpha, n)$ ,  $(\alpha, 2n)$ ,  $(\alpha, pn)$  reactions on  $\text{Ni}^{60}$  plotted against  $\epsilon_p$  and  $\epsilon_\alpha$  respectively. The scale of  $\epsilon_p$  has been shifted by 7 Mev with respect to the scale of  $\epsilon_\alpha$ .

The above figure[1] plots the observations (cross section in barns) against the kinetic energies of the incident protons and alpha particles. The following results can be deduced

- The kinetic energy of the proton required to produce a given excitation  $E$  of the compound nucleus  $\text{Zn}^{64}$  will be different from the kinetic energy  $\epsilon_\alpha$  of the  $\alpha$  particle to produce the same excitation  $E$  in the  $\text{Zn}^{64}$ . This difference is due to the difference in the masses (or binding energies) of  $\text{Cu}^{63} + \text{H}^1$  and  $\text{Ni}^{60} + \text{He}^4$ . From Fig. 1, the difference is  $7 \pm 1$  MeV.
- It is evident that the ratios  $\sigma(\alpha, n) : \sigma(\alpha, 2n) : \sigma(\alpha, pn)$  for  $\text{Ni}^{60}$  and the ratios  $\sigma(p, n) : \sigma(p, 2n) : \sigma(p, pn)$  for  $\text{Cu}^{63}$  agree with each other within the limits of experimental error. This agreement, according to relationship (1), directly proves that **Bohr's compound nucleus assumption is indeed valid.**

## 5 Glas Mosel equation

In our code we have first calculated the fusion cross section, and then we calculated the cross section of residual reactions.

This section is aimed at explaining our calculation of the total fusion cross section using Glas-Mosel formula discussed in [2] and comparing the theoretical results with the experimental results obtained by Ghoshal. Consider this : if  $\text{Cu}^{63}(\text{p},\text{n}) \text{Zn}^{63}$ ,  $\text{Cu}^{63}(\text{p},2\text{n}) \text{Zn}^{62}$ ,  $\text{Cu}^{63}(\text{p},\text{pn}) \text{Cu}^{62}$  were the only reactions which take place when  $\text{Cu}^{63}$  is bombarded with protons then the sum of the observed cross sections should give  $\sigma_p(\epsilon_p)$ , the cross section for the absorption of a proton by the  $\text{Cu}^{63}$  nucleus to form  $\text{Zn}^{64}$ . The sum of the observed alpha-cross sections should similarly give  $\sigma_\alpha(\epsilon_\alpha)$ .

In Galin *et al.*[4], we see that not a critical angular momentum but instead a critical distance of approach may be the relevant quantity limiting complete fusion during a collision between two complex nuclei. The authors of [4] proceed in their analysis in three steps :

1. Calculate critical angular momentum  $l(E)$  using a sharp cutoff approximation from the measured cross sections for complete fusion

$$\sigma_{\text{fusion}}(E) = \pi \lambda^2 l_C(E)^2 \quad (3)$$

2. Calculate ion-ion potential using energy density method
3. Calculate classical turning point for partial waves with  $l = l(E)$  at the energy  $E$ .

This point stays remarkably constant as a function of energy. All partial waves with  $l \ll l_c$  contribute to fusion, hence it can be inferred that the point found as outlined above acts as a critical distance for fusion. Its value is found to be:

$$R_C = r_C(A_1^{1/3} + A_2^{1/3}), r_C = 1.0 \pm 0.07 fm \quad (4)$$

At the same time, however, it is found that this critical distance also coincides with the minimum of the nuclear potential above. This minimum appears at a point where a core in the nuclear potential due to compression of nuclear matter sets in which is a consequence of the "sudden approximation" used in the calculation of the ion-ion potential. A sizable compression can't be expected in the range of energies at which the analyzed data were taken, which indicates that the finding of a critical distance is a consequence of the special potential used and thus, in view of the criticisms of the potential model, unphysical. Hence we understand that the analysis in [4] can be performed in a way that is independent of the special model used for the calculation of the potential. The procedure used in [4] actually corresponds exactly to the classical expression for the cross section for reaching the critical distance  $R_C$ :

$$\sigma = \pi R_C^2 \left( 1 - \frac{V}{E} \right) \quad (5)$$

The special nuclear model enters into this expression only through the value of  $V(R_C)$ .

To test the assumption of the existence of a critical distance that two heavy ions have to pass in order to fuse we can plot  $\sigma_{fusion}$  as a function of  $1/E$ . If  $\sigma_{fusion}$  is linearly dependent on  $V(R_C)$  within experimental limits then this establishes the existence of a critical distance independent of the special model used to calculate  $V(R)$ .

The method used in Galin *et al.* to extract critical distance is equivalent to the one used by Gutbrod *et al.* in [3]. However, from their analysis of low energy fusion data, the fusion distance obtained is

$$R_B = r_B(A_1^{1/3} + A_2^{1/3}), r_B = 1.4 fm \quad (6)$$

$R_B$  is 40% larger than  $R_C$  and corresponds to the distance of ions at the fusion barrier. The second point can be achieved by setting :

$$\sigma_{fusion}(E) = \pi \lambda^2 \sum_{l=0}^{\infty} (2l+1) T_l P_l \quad (7)$$

where  $T_l$  is the penetration probability through the interaction barrier whereas  $P_l$  is the probability for fusion to take place once the barrier has been passed. Following Glas and Mosel,  $T_l$  is approximated by the penetration factors of a parabolic barrier with frequency  $\hbar\omega_l$ , while for  $P_l$  sharp cutoff model was used.

$$P_l = \begin{cases} 1, & l \leq l_c \\ 0, & \text{otherwise} \end{cases}$$

Glas and Mosel then approximate the frequencies  $\hbar\omega_l$ , and the position of the interaction barrier  $R_{bl}$ , by constant values  $\hbar\omega$  and  $R_b$ , respectively, which are independent of  $l$ . Replacing the summation in Eq. (2) by an integral, the final equation of  $\sigma_{fusion}$  as calculated in [2] is:

$$\sigma_{fus}(E) = \frac{\hbar\omega}{2E} R_b^2 \ln \frac{1 + \exp\left(2\pi \frac{E-V(R_b)}{\hbar\omega}\right)}{1 + \exp\left(2\pi \frac{E-V(R_b) - \left(\frac{R_c}{R_b}\right)^2 (E-V(R_c))}{\hbar\omega}\right)} \quad (8)$$

where

$E$  is the incident energy of incoming particle

$\omega$  is an approximation of  $\omega_l$  which is used to approximate penetration probability,  $T_l$  by a parabolic barrier

$V(R_B)$  and  $V(R_C)$  are values of total potential calculated at  $R_B$  and  $R_C$

Using a diffuse potential as a guide, the effective potential for the reaction is [from [7]]

$$V(r) = -V_0 \left[ \frac{1}{1 + \exp\left(\frac{r-R_1-R_2}{a}\right)} \right] + \frac{Z_1 Z_2 e^2}{r} \quad (9)$$

Here, the first term in the above expression is the nuclear potential and the second term represents coulomb potential. We have ignored the centrifugal part in this expression. A possible explanation for this is because the authors of [2] have ignored all the  $l$  dependence coming in the final equation by approximating  $\omega_l$  by  $\omega$ . The same procedure was followed by Yulianto and Su'udb in their work where they have applied the Glas-Mosel formula for calculating fusion cross section of  $O^{16}$ - $O^{16}$  where the authors used only the nuclear and coulombic potential in their study.

To get exact expression for  $V(r)$ , we need the values of  $V_0$  and  $a$ , we used the work "Determination of the nucleus nucleus potential from boundary conditions"[6] by J. Wilczynski for calculating these parameters. The value of  $V_0$  was calculated using the equation

$$V_0 = b_{surf} [A_1^{2/3} + A_2^{2/3} - (A_1 + A_2)^{2/3}] \quad (10)$$

where  $b_{surf}$  is the surface energy parameter and has value 17 MeV. This expression represents the liquid-drop energy of the fused system  $A_1 + A_2$  (of spherical shape) calculated with respect to the energy of separated incident nuclei of the mass numbers  $A_1$  and  $A_2$ . The equation used for calculating

the diffuseness parameter,  $a$  is

$$a = 0.356 fm^2 \frac{R_0 [A_1^{2/3} + A_2^{2/3} - (A_1 + A_2)^{2/3}]}{R_1 R_2} \quad (11)$$

where

$$R_0 = 1.128 fm (A_1^{1/3} (1 - 0.786 A_1^{-2/3}) + A_2^{1/3} (1 - 0.786 A_2^{-2/3})) \quad (12)$$

After getting the expression for  $V(r)$ , we evaluate  $R_l$  as the separation at which derivative of  $V(r)$  vanishes.

$$\left. \frac{dV(r)}{dr} \right|_{r=R_l} = 0 \quad (13)$$

After calculating  $R_l$ , we use its value to evaluate  $\omega_l$

$$\omega_l = \left[ \frac{1}{\mu} \left. \frac{d^2V(r)}{dr^2} \right|_{r=R_l} \right]^{1/2} \quad (14)$$

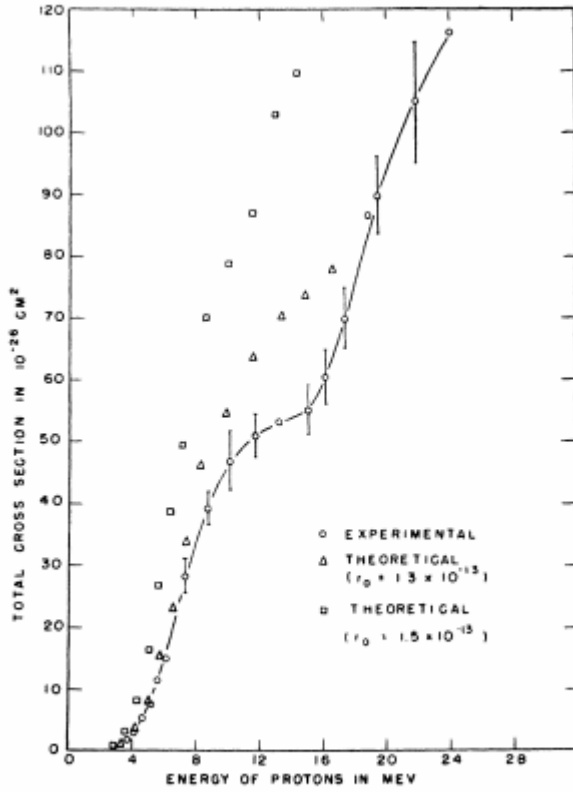


FIG. 2. Total cross section which is the sum of  $(p,n)$ ,  $(p,2n)$  and  $(p,pn)$  cross sections on  $\text{Cu}^{63}$  as determined experimentally is compared with theoretical  $\sigma_p$  which is the cross section for the absorption of a proton by  $\text{Cu}^{63}$  nucleus.

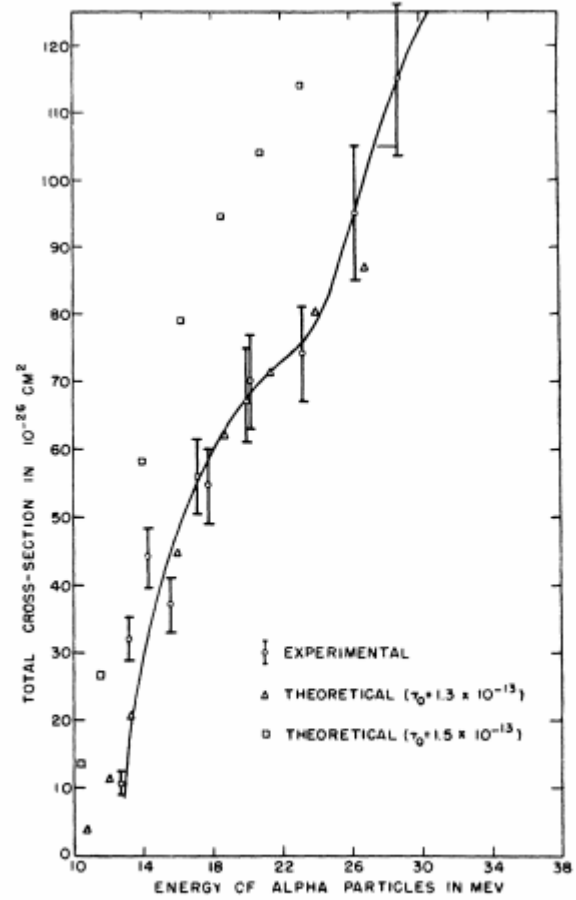


FIG. 3. Total cross section which is the sum of  $(\alpha,n)$ ,  $(\alpha,2n)$ ,  $(\alpha,pn)$  cross sections on  $\text{Ni}^{63}$  as determined experimentally is compared with theoretical  $\sigma_\alpha$  which is the cross section for the absorption of an  $\alpha$ -particle by  $\text{Ni}^{63}$  nucleus.

## 6 RESIDUES FORMED

The six major reactions involved in the experiment have been specified above. The compound nucleus  $\text{Zn}^{64}$  formed during the reactions disintegrates into proton and/or neutron(s) and the following daughter nuclei:



- $\text{Zn}^{63}$  [from the reactions  $\text{Ni60}(,n)$  and  $\text{Cu63}(p,n)$ ]
- $\text{Zn}^{62}$  [from the reactions  $\text{Ni60}(,2n)$  and  $\text{Cu63}(p,2n)$ ]
- $\text{Cu}^{62}$  [from the reactions  $\text{Ni60}(,pn)$  and  $\text{Cu63}(p,pn)$ ]

But these reactions are reversible to some extent, i.e., the compound nucleus  $\text{Zn64}$  can disintegrate back into the incident particle and the target nucleus. These reversible reactions have not been taken into consideration in the experiment by S. N. Ghoshal and thus in this report, but as a nod to these reverse reactions, following nuclei are included in the list of residues:

- $\text{Ni}^{60}$
- $\text{Cu}^{63}$

## 7 THEORETICAL ESTIMATION OF CROSS-SECTION FOR INDIVIDUAL RESIDUES

Above, we have estimated the total fusion cross-section for the formation of the compound nucleus ( $\sigma_{fusion}$ ). Bohr's compound nucleus assumption states that  $\sigma_{fusion}$  is independent of the entrance channel i.e., the mode of formation of the compound nucleus. We have three types of entrance channels and our compound nucleus decays into three residues-  $\text{Zn}^{63}$ ,  $\text{Zn}^{62}$  and  $\text{Cu}^{62}$ . Thus we have three cross-sections to evaluate.

The following is a general representation of the formation and the decay of the compound nucleus:



If  $a$  is the entrance channel and  $b$  is the exit channel, then cross section of the given exit channel ( $b + B$ ) is given by

$$\sigma_{ab} = \sigma_c P_c(b) \quad (15)$$

where,

$\sigma_c$  is the cross section of the fusion reaction (compound nucleus formation)

$P_c(b)$  is the probability that  $C$  will decay to form  $b+B$ .

As we can expect,  $\sum P_c(b) = 1$ , where the summation is over all the possible exit channels.

Mathematically,  $P_c(b) = \tau_b/\tau$  where  $\tau_b$  is the width of residual nucleus level for this exit channel and  $\tau$  is the sum of width of all the exit channels.

To calculate  $P_c(b)$  for all the exit channels, we use the equations given in lecture notes and [5].

$$\frac{\Gamma_x}{\Gamma_y} = \frac{g_x \mu_x}{g_y \mu_y} \frac{R_x a_x}{R_y a_y} \exp[2(a_x R_x)^{1/2} - 2(a_y R_y)^{1/2}] \quad (16)$$

Where,  $g_{(x/y)}$  is the spin multiplicity for the emitted particle given by

$$g_{x/y} = 2J_{x/y} + 1 \quad (17)$$

For multiple ejected particles, we just sum up the spin multiplicities for the particles.

For two neutrons,

$$g = [2^*(1/2) + 1] + [2^*(1/2) + 1] \quad (18)$$

Similarly for a proton and neutron,

$$g = [2^*(1/2) + 1] + [2^*(1/2) + 1] \quad (19)$$

where  $\mu_{x/y}$  is the reduced mass of the products of disintegration for a given exit channel given by

$$\frac{1}{\mu} = \frac{1}{m_1} + \frac{1}{m_2} + \dots \quad (20)$$

$a_i$  is the level density parameter given by the relation

$$E^* = aT^2 - T \quad (21)$$

where

$E^*$  is the excitation energy of the formed compound nucleus

T is the nuclear temperature

To calculate the excitation energy of the compound nucleus, we use the basic principle of momentum and energy conservation as discussed in lectures.

From the conservation of mass-energy, we have

$$E^* = E_i - E_f + Q \quad (22)$$

From the conservation of momentum, we get

$$m_i E_i = m_f E_f \quad (23)$$

Using these equations, we get the expression for excitation energy of the compound nucleus:

$$E^* = Q + E_i \left[ 1 - \frac{m_i}{m_f} \right] \quad (24)$$

We calculate Q-value using the table of mass excesses.

So, we substitute  $E^*$  and T (from reference) in Eq.(19) to get the level density parameter,  $a_i$ .

In Eq.(16), the parameter R is the maximum excitation energy for the residual nucleus that results from the emission of the i-th particle.

Also,

$$R = E^* - S - \epsilon_s \quad (25)$$

Where,  $E^*$  is the excitation energy of the compound nucleus, S is the separation energy of the emitted particle(s) and  $\epsilon_s$  is the threshold for charged particle emission [0 for neutron].

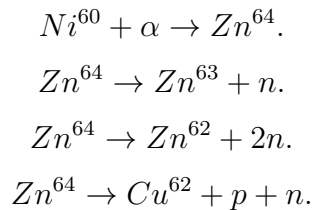
So, after substituting the parameters (g,  $\mu$ , R, a) in Eq.(16) we get the ratio of probabilities of reaction following a certain exit channel, i.e,  $\tau_1/\tau_2$ ,  $\tau_2/\tau_3$  and  $\tau_3/\tau_1$  as mentioned earlier, we are only considering the three exit channels considered in the paper. But as discussed in lectures, there is always some probability that the compound nucleus will disintegrate into the incoming particle and target nucleus.

Mathematically, reaction cross section for the exit channels are given by:

$$\begin{aligned} \sigma_1 &= \sigma_{fusion}(\tau_1/(\tau_1 + \tau_2 + \tau_3)) \\ \sigma_2 &= \sigma_{fusion}(\tau_2/(\tau_1 + \tau_2 + \tau_3)) \\ \sigma_3 &= \sigma_{fusion}(\tau_3/(\tau_1 + \tau_2 + \tau_3)) \end{aligned}$$

## 8 Some calculations directly used in the code

So, now we have all the parameters needed to calculate fusion cross section using Glas Mosel equation. This is our code for calculating reaction cross section for the reaction



Reduced mass for the three exit channels :

$$\begin{aligned}\mu_1 &= \frac{63 \times 1}{63 + 1} \\ &= 0.984375\end{aligned}$$

$$\begin{aligned}\mu_2 &= \frac{62 \times 1 \times 1}{62 \times 1 + 1 \times 1 + 1 \times 62} \\ &= 0.96875\end{aligned}$$

$$\begin{aligned}\mu_3 &= \frac{62 \times 1 \times 1}{62 \times 1 + 1 \times 1 + 1 \times 62} \\ &= 0.96875\end{aligned}$$

Q-values for the three exit channels :

$$\begin{aligned}Q_1 &= (-64473.1 + 2424.9) - (-62213.4 + 8071.3)keV \\ &= -7.906MeV\end{aligned}$$

$$\begin{aligned}Q_2 &= (-64473.1 + 2424.9) - (-61168 + 8071.3 + 8071.3)keV \\ &= -17.022MeV\end{aligned}$$

$$\begin{aligned}Q_3 &= (-64473.1 + 2424.9) - (-62787.4 + 8071.3 + 7288.9)keV \\ &= -14.621MeV\end{aligned}$$

Separation energy for three exit channels :

$$\begin{aligned}S_1 &= (-62213.4 + 8071.3) - (-66004)keV \\ &= 11.861MeV\end{aligned}$$

$$\begin{aligned}S_2 &= (-61168 + 8071.3 + 8071.3) - (-66004)keV \\ &= 20.978MeV\end{aligned}$$

$$\begin{aligned}S_3 &= (-62787.4 + 8071.3 + 7288.9) - (-66004)keV \\ &= 18.576MeV\end{aligned}$$

To calculate thresholds,  $\epsilon_s$ , we have approximated it by the coulomb barrier of the residual nucleus ( from MODERN NUCLEAR CHEMISTRY, by WALTER D. LOVELAND *et al.*). So, its value for first and second exit channels remain 0, since the ejected particles are only neutrons. For the third exit channel, we have calculated the potential barrier faced by a proton at the nuclear radius. This is an approximation we have used and the value we got for  $\epsilon_{s,3}$  is  $-9MeV$ .

## 9 Code

```
# Import all the libraries

import math
import matplotlib.pyplot as plt
import numpy as np
import sympy as sp
import scipy

# define the parametrs used

A1 = 60
A2 = 4
A3 = 64
reduced_mass = A1*A2/(A1+A2)

Z1 = 28
Z2 = 2
Z3 = 30

pi = np.pi
e = np.sqrt(1.44) # sqrt(MeV-fm)
b_surf = 17 # MeV
h = 197.32 # Mev-fm
hcut = 1

# Defining function to solve with the Newton raphson method
# source for this code :
# https://stackoverflow.com/questions/42449242/newton-raphsons
#-method-user-input-and-numerical-output-problems

def f(symx):
    tmp = sp.sympify(symx)
    return tmp

def fprime(symx):
    tmp = sp.diff(f(symx), r)
    return tmp;

def newtons_method(symx):
    guess = sp.sympify(0.5) # Convert to an int immediately.
    div = f(symx)/fprime(symx)

    for i in range(1, 100):
        nextGuess = guess - div.subs(r, guess)
        guess = nextGuess
    return guess.evalf()
```

```

r = sp.Symbol('r')

V_coulomb = Z1*Z2*(e**2)/r    # MeV

R1 = (1.128)*(A1**(1/3))*(1 - 0.786*(A1**(-2/3))) # fm
R2 = (1.128)*(A2**(1/3))*(1 - 0.786*(A2**(-2/3))) # fm
V_0 = b_surf*(A1**(2/3) + A2**(2/3) - (A1+A2)**(2/3))
a = (0.356)*((R1+R2)*(A1**(2/3)+A2**(2/3)-(A1+A2)**(2/3)))/(R1*R2)

V_nuclear = -V_0/(1+sp.exp((r-R1-R2)/a))

V_net = V_coulomb + V_nuclear

dV_dr = sp.diff(V_net)

R_l = newtons_method(dV_dr)

d2V_dr2 = sp.diff(dV_dr)
omega = ((d2V_dr2.subs(r, R_l))/(reduced_mass))**(0.5)

r_B = 1.4
r_C = 1

R_B = r_B*((A1**(1/3)) + (A2**(1/3)))
R_C = r_C*((A1**(1/3)) + (A2**(1/3)))

V_B = V_net.subs(r, R_B)
V_C = V_net.subs(r, R_C)

E = sp.Symbol('E')

hcut_omega = hcut*omega

num = 1 + sp.exp(2*pi*(E-V_B)/hcut_omega)
den = 1 + sp.exp(2*pi*(E-V_B - ((R_C/R_B)**2)*(E-V_C))/hcut_omega)
sigma_fusion = (0.5)*(hcut_omega)*(R_B**2)*(1/E)*(sp.log(num/den))

E_values = np.linspace(10,38)

sigma_fusion_values = np.zeros(50)

for i in range(50):
    sigma_fusion_values[i] = sigma_fusion.subs(E, E_values[i])

reduced_masses = np.array([0.984375, 0.96875 , 0.96875])

spin_multiplicity = np.array([2, 4, 4])

```

```

Q_values = np.array([-7.906, -17.022, -14.621])    # in MeV

Separation_energy = np.array([11.861, 20.978, 18.576])    # in MeV

T = 1.255 # MeV,
# nuclear temperature for the compound nucleus Zn-64

E_excitation = np.zeros([50,3])
# number of incident energy values vs no of exit channels

for i in range(50):
    for j in range(3):
        E_excitation[i][j] = Q_values[j] + (E_values[i])*(15/16)

a_values = (E_excitation+T)/(T**2)

threshold = np.array([0, 0, -9]) #

R_values = E_excitation - Separation_energy - threshold

tau_values = np.zeros_like(E_excitation)

for i in range(tau_values.shape[0]):
    for j in range(tau_values.shape[1]):
        ax_Rx = (R_values[i][j])*(a_values[i][j])
        ax_Rx = np.abs(ax_Rx)
        tau_values[i][j] = (spin_multiplicity[j])*\
            (reduced_masses[j])*(ax_Rx)*(np.exp(2*((ax_Rx)**0.5)))

for i in range(tau_values.shape[0]):
    tau = tau_values[i][0]+tau_values[i][1]+tau_values[i][2]
    for j in range(tau_values.shape[1]):
        tau_values[i][j] = tau_values[i][j]/(tau)

residual_cross_sections = np.zeros_like(tau_values)

for i in range(residual_cross_sections.shape[0]):
    for j in range(residual_cross_sections.shape[1]):
        residual_cross_sections[i][j] = sigma_fusion_values[i]\
            *tau_values[i][j]

plt.plot(E_values, sigma_fusion_values, \
label='fusion_cross_section')
plt.plot(E_values, residual_cross_sections[:,0], \
label='first_cross_section')
plt.plot(E_values, residual_cross_sections[:,1], \
label='second_cross_section')
plt.plot(E_values, residual_cross_sections[:,2], \

```

```
label='third_cross_section')

plt.legend(bbox_to_anchor=(1.05, 1), \
loc='upper left', borderaxespad=0.)
plt.show()
```

## 10 Results

Figure 1. is the plot that we got for fusion cross section vs incident energy values and Figure 2. shows the plot for residual cross sections and fusion cross section v/s incident nergy values.

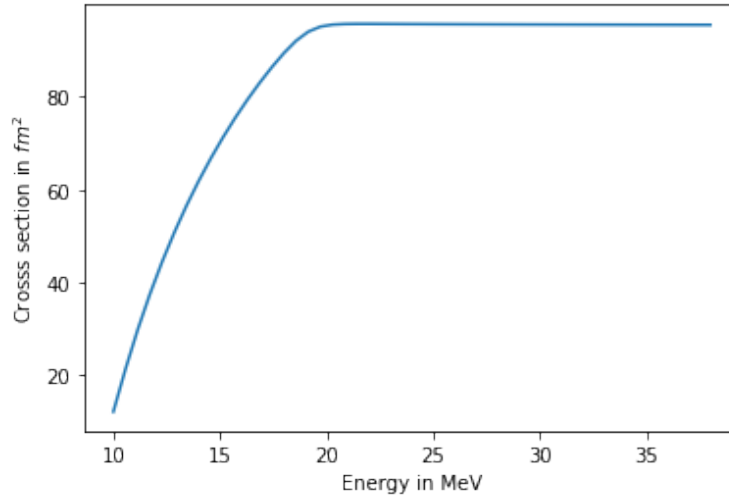


Figure 1:

## 11 Conclusion

As we can see in Figure 1., the value of fusion cross section calculated using the Glas-Mosel equation has the same trend as observed in the experiments for lower values of incident energy. For higher energy values, the cross section value gets saturated, which is contrary to what is observed. This short-coming of the Glas-Mosel equation has visible effects while calculating the residual cross section values as well. In Figure 2., we can note that our calculated values follow the same trend as observed in experiments for lower



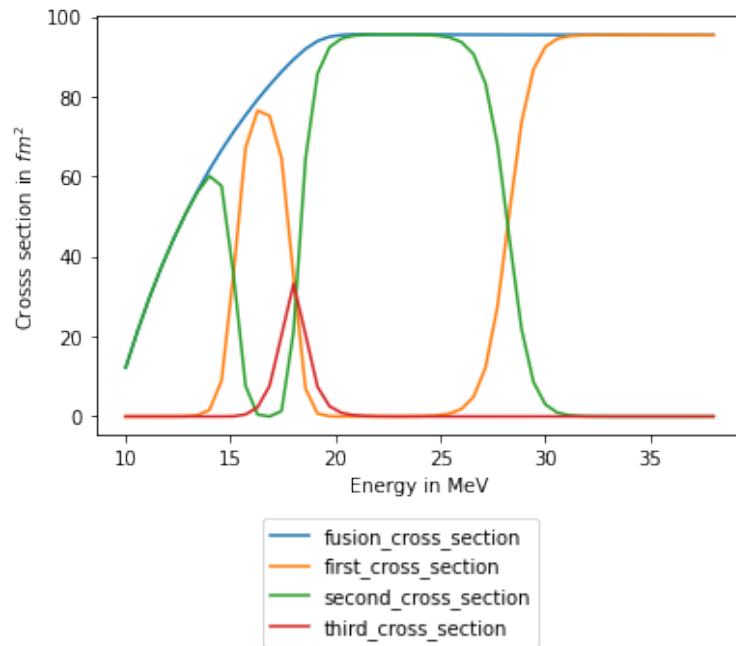


Figure 2:

values of energy. However, at higher values, we observe something completely unexpected.

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