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* Basic Probability, Conditional Probability and Baye's Theorem Question / Answers

A1 $S = \{H, T\}$

P(getting a head) = $\frac{\text{No. of favourable outcomes}}{\text{Total no. of outcomes}}$

$\therefore \boxed{P(H) = \frac{1}{2}}$

A2 $S = \{1, 2, 3, 4, 5, 6\}$

$\boxed{P(>4) = \frac{2}{6} = \frac{1}{3}}$

A3 $S = \{HH, HT, TH, TT\}$

Given that the second coin has Tail ::

$S' = \{HT, TT\}$

Now, $\boxed{P(\text{getting head on first coin}) = \frac{1}{2}}$

A4. Given that the card is red, therefore our sample space become all cards that are red {diamonds & Hearts}

$\therefore$ $\boxed{P(\text{card is heart}) = \frac{13}{26} = \frac{1}{2}}$

A5. Possible even outcomes :- {2, 4, 6}

$\therefore$ $\boxed{P(\text{number} = 2) = \frac{1}{3}}$

A6. $P(B_1) = P(B_2) = 1/2$ {bags are chosen randomly}

$P(H|B_1) = 1/2$ {Fair coin}

$P(H|B_2) = 1$ {double-headed coin}

Using Bayes' Theorem :-

$$P(B_2|H) = \frac{P(H|B_2) \cdot P(B_2)}{P(H|B_1) \cdot P(B_1) + P(H|B_2) \cdot P(B_2)}$$

$$= \frac{1 \times 1/2}{1/4 + 1/2}$$

$$= \frac{1/2}{3/4}$$

$\therefore$ $\boxed{P(B_2|H) = \frac{2}{3}}$

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A7 Let $P(D) = 0.001$ (probability of disease)
$P(\sim D) = 0.999$ (probability of no disease)
$P(T^+|D) = 0.99$ (true positive)
$P(T^+|\sim D) = 0.01$ (false positive)

Using Bayes' Theorem :-

$$P(D|T^+) = \frac{P(T^+|D) \cdot P(D)}{P(T^+|D) \cdot P(D) + P(T^+|\sim D) \cdot P(\sim D)}$$

$$= \frac{0.99 \times 0.001}{(0.99 \times 0.001) + (0.01 \times 0.999)}$$

$\therefore \boxed{P(D|T^+) = 0.0902}$ ($\approx 9.02\%$)

A8 Let $P(R) = 0.7$ (prior probability of rain)
$P(F|R) = 0.9$ (probability forecast says rain given
$P(F|\sim R) = 0.2$ (" " " given no rain

We want $P(R|F)$, the probability it rained given the forecast said rain.

Using Bayes' Theorem :-

$$P(R|F) = \frac{P(F|R) \cdot P(R)}{P(F|R) \cdot P(R) + P(F|\sim R) \cdot P(\sim R)}$$

$$= \frac{0.9 \times 0.7}{(0.9 \times 0.7) + (0.2)(0.3)}$$

$\therefore \boxed{P(R|F) = 0.913}$ $\{\approx 91.3\%\}$

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A9 Let $P(prep) = 0.7$ (students who prepared)

$P(\sim prep) = 0.3$ (students who do not prepare)

$P(Pass \mid Prep) = 0.9$ (students who pass given they prepared)

$P(Pass \mid \sim Prep) = 0.3$ (" " " they do not prepare)

Using Bayes' Theorem :-

$$P(Prep \mid Pass) = \frac{P(Pass \mid Prep) \cdot P(prep)}{P(Pass \mid Prep) \cdot P(prep) + P(Pass \mid \sim Prep) \cdot P(\sim prep)}$$

$$= \frac{0.9 \times 0.7}{(0.9 \times 0.7) + (0.3 \times 0.3)}$$

$$\therefore \boxed{P(Prep \mid Pass) = 0.875} \quad \{\approx 87.5\ \%\}$$

A10 $P(Boy) = 0.6$

$P(Girl) = 0.4$

$P(Math \mid Boy) = 0.7$

$P(Math \mid Girl) = 0.5$

Using Bayes' Theorem :-

$$P(Boy \mid Math) = \frac{P(Math \mid Boy) \cdot P(Boy)}{P(Math \mid Boy) \cdot P(Boy) + P(Math \mid Girl) \cdot P(Girl)}$$

$$= \frac{(0.7 \times 0.6)}{(0.7 \times 0.6) + (0.5 \times 0.4)}$$

$$\therefore \boxed{P(Boy \mid Math) \approx 0.6774} \quad \{\approx 67.74\ \%\}$$