Exam.Code:0906
Sub. Code: 6660

2071

B.E. (Bio-Technology) Second Semester

ASM-201: Differential Equations and Transforms

(Common to all streams)

Time allowed: 3 Hours

Max. Marks: 50

NOTE: *Attempt five questions in all, including Question No. I which is compulsory and selecting two questions from each Part. Use of non-programmable calculator is allowed.*

x-x-x

1. (a) Find the general solution of the differential equation:

$$y(x^3 - y)dx - x(x^3 + y)\ dy = 0$$

(b) Verify that $\sigma(x) \doteq e^{\int P(x)dx}$ is an integrating factor of the following ordinary differential equation

$$\frac{dy}{dx} + P(x)y = Q(x)$$

and use it to find its general solution.

(c) State and prove second shifting property for Laplace transforms.

(d) Check whether the function $f(x) = \sin(2\pi x/k)$ is periodic or not. If it is periodic, find its fundamental period.

(e) Formulate the partial differential equation by eliminating the arbitrary functions:

$$z = f(x + ay) + g(x - ay)$$

$(5 \times 2 = 10)$

PART A

2. (a) Find the general solution of the following differential equation: (5)

$$(D^3 - 2D + 4)y = e^x \sin x + x^3$$

(b) Solve the following differential equation by the method of variation of parameters: (5)

$$(D^2 + 5D + 6)y = e^{-2x}\ \sec^2 x(1 + 2\tan x)$$

3. (a) Solve the given differential equation: (4)

$$(D^3 - D)y = 1 + x^5 + e^x$$

(b) Find the inverse Laplace transform of the given function: (3)

$$\frac{se^{-s/2} + \pi e^{-s}}{s^2 + \pi^2}$$

P.T.O.

(c) Find Laplace transform of $f(t) = e^{-3t} \int_0^t \frac{\sin 2u}{u}\, du.$ (3)

4. (a) Find the Laplace transform of $f(t) = \frac{t^{n-1}}{1-e^{-t}}.$ (2)

(b) Find the inverse Laplace transform of $\ln\left(1 - \frac{a^2}{s^2}\right).$ (3)

(c) Solve the following differential equation using Laplace transforms: (5)

$$y'' + y = \begin{cases} 1; & 0 < t < 1 \\ 0; & t > 1 \end{cases}, \quad y(0) = 1, \quad y'(0) = -1$$

PART B

5. (a) Find the Fourier series of the periodic function $f(t)$ with period $p = 2$: (5)

$$f(t) = \begin{cases} 0, & if \quad -1 \le t < -1/2 \\ \cos 3\pi t, & if \quad -1/2 \le t < 1/2 \\ 0, & if \quad 1/2 \le t < 1 \end{cases}$$

(b) Find the Fourier intergal representation of the following function: (5)

$$f(x) = \begin{cases} 0 & x < 0 \\ e^{-x} & x > 0 \end{cases}$$

and using Fourier integral theorem evaluate the value of the integral for all values of x.

6. (a) Find the integral surface of the following partial differential equation: (6)

$$x(z+2a)p + (xz + 2yz + 2ay)q = z(z+a)$$

Also find a particular solution passing through $y = 0$, $z^3 + x(z+a)^2 = 0$.

(b) Find the Fourier cosine series of the function $f(x) = x$, $0 \le x \le 2$ and hence find the sum of the series (4)

$$\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \cdots$$

7. Find the solution of the one dimensional heat equation (10)

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}, \qquad c^2 = \frac{K}{\sigma \rho}$$

where $u(x,t)$ represents the temperture of the bar of length L. Given that both ends of the bar are insulated and initial temperature of the bar is given by $f(x)$.

x-x-x