Q1)R = {(a, b) | (a - b) = km for some fixed integer m and a, b, k ∈ z}, then R is

Ans: An equivalence relation

Q2 Show that the given relation R is an equivalence relation, which is defined by (p, q) R (r, s) ⇒ (p+s)=(q+r)

Q3 If Q be the set of rational numbers and f / Q -> Q be defined by f(x) = 2x + 3 then prove that f is bijective. Also find f ^ - 1

Q4 If f / R -> R and g / R -> R be defined by f(x) = x - 1 and g(x) = x ^ 2 + 1 then find fog(1) , fog(2), gof(2) fof (2) and gog(2) .

Q5: For each of these relations on the set {1, 2, 3, 4}, decide whether it is reflexive, whether it is sym- metric, and whether it is transitive.

A {(2, 2), (2, 3), (2, 4), (3, 2), (3, 3), (3, 4)}

Answer

Not reflexive because we do not have (1, 1), (3, 3), and (4, 4).

Not symmetric because while we we have (3, 4), we do not have (4,3).

Transitive because if we have (a, b) in this relation, then a will be either 2 or 3. Then (2, c)and (3, c) are in the relation for all c≠ 1. Since whenever we have both (a, b) and (b, c), then we have (a, c) which makes this relation transitive.

B {(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (4, 4)}

Answer

Reflexive because (a, a) is in the relation for all a = 1, 2, 3, 4.

Symmetric because for every (a, b), we have a (b,a).

Transitive because while we have (1,2) and (2, 1), we also have (1,1) and (2,2) in the relation.

Q6: a) Let A, B and C be any three non empty sets. Prove that (A-B)-(B-C) = A-B.

Q7: Prove that A∩(B∪C)=(A∩B)∪(A∩C) for sets A, B, and C.

Q8: Prove that the power set of a set with n elements has 2^n elements.

Q9) Do the following relations represent functions? Why?

(a) f : R → R defined by f = { (x, √ x) : x ∈ R}

(b) f : R+ → R defined by f = {(x, ± √ x) : x ∈ R +}, where R+ is the set of all positive real numbers

Q10) Let f : X → Y be a function. Prove that f −1 : Y → X is a function if and only if f is a bijection.

Q11) A relation on a nonempty set may or may not be reflexive, symmetric, or transitive. Thus there are 8 types of relations. With X = {1, 2, 3, 4, 5}, give one example for each type of such relations.

Q 12) Define a relation R on a set A = {a, b, c} as R = {(a, b), (b, c), (b, b)}. Determine if R is a transitive relation.

Answer: R is not a transitive relation

Q13) Show that the relation R={ (a,a),(a,b),(b,a),(b,b)(c,c)} on A={a,b,c} is an equivalence relation and find A/R also find partitions of A.

Q14) Give an example of a relation which is symmetric but neither reflexive nor anti symmetric nor transitive.

Answer: Set A={1,2,3}, Relation R={(1,2),(2,1),(2,3),(3,2)}

Q15) Prove the Demorgan's first law if U = {11, 12, 13, 14, 15, 16, 17}, A = {11, 12, 13, 15}, and B = {13, 15, 16, 17}.

Q16) Let P & Q be two sets such that n(P) = 4 and n(Q) = 2. If in the Cartesian product we have (m,1), (n,-1), (x,1), (y, -1). Find P and Q, where m, n, x, and y are all distinct.

~~**Answer:** P = set~~ of first elements = {m, n, x, y} and Q = set of second elements = {1, -1}

Q17) If A = {2, 4, 6, 8) B = {5, 7, 1, 9}.
Let R be the relation 'is less than' from A to B. Find Domain (R) and Range (R).

Answer: Domain (R) = {2, 4, 6, 8} and Range (R) = {1, 5, 7, 9}

Q18. Consider the relation R = {(a, a), (b, b), (c, c), (d, d), (a, b), (b, a)} on set A = {a, b, c, d}.
Is R reflexive? Symmetric? Transitive? If a property does not hold, say why.
A1: It is reflexive, symmetric, transitive.

Q19. Consider the set f = {(x^2, x) : x ∈ R}. Is this a function from R to R? Explain.
A2. No, It is not a function.

Q20. Consider the cosine function cos : R → R. Decide whether this function is injective and whether it is surjective. What if it had been defined as cos : R → [−1, 1]?
A3. First Part, neither injective nor surjective, Second part, not injective but surjective.