Discrete Mathematics (BCA-401)

Unit IV Questions

Q:1) Use known logical equivalences to do each of the following:

(a) Show $p \to (q \vee r) \Leftrightarrow (p \wedge \neg q) \to r$.
(b) Show $\neg(p \vee q) \vee (\neg p \wedge q) \vee \neg(\neg p \vee \neg q) \Leftrightarrow \neg(p \wedge \neg q)$.
(c) Find an expression logically equivalent to $\neg(p \leftrightarrow q)$ that involves only $\neg$ and $\vee$.

Q:2) Write the argument below in symbolic form. If the argument is valid, prove it. If the argument is not valid, give a counter example:

If I watch football, then I don't do mathematics
If I do mathematics, then I watch hockey

∴ If I don't watch hockey, then I watch football **(Answer: Valid)**

Q:3) (a) Show that $p \to (q \to r)$ is logically equivalent to $(p \wedge q) \to r$.
(b) Establish the validity of the argument

$u \to r$
$(r \wedge s) \to (p \vee t)$
$q \to (u \wedge s)$
$\neg t$
q

$\therefore p$ **(Answer: Valid)**

Q:4) Let p : Jupiter is a planet and q : India is an island be any two simple statements. Give verbal sentence describing each of the following statements.

(i) $\neg p$
(ii) $p \vee \neg q$
(iii) $\neg p \vee q$
(iv) $p \to \neg q$
(v) $p \leftrightarrow q$

Q:5)Verify whether the following compound propositions are tautologies or contradictions or Contingency:

(i) $(p \wedge q) \wedge \neg(p \vee q)$ **(Answer: Contradiction)**
(ii) $((p \vee q) \wedge \neg p) \to q$ **(Answer: Tautology)**
(iii) $(p \to q) \leftrightarrow (\neg p \to q)$ **(Answer: Contingency)**
(iv) $((p \to q) \wedge (q \to r)) \to (p \to r)$ **(Answer: Tautology)**

Q:6) Is $(p \rightarrow q) \rightarrow [(\sim p \rightarrow q) \rightarrow q]$ a tautology or not?

ans. Tautology

Q:7) Show that $[(p \vee q) \wedge (r \vee \sim q)] \rightarrow (p \vee r)]$ is a tautology.

ans. Tautology

Q:8) Show that the two statements $(p \wedge q) \rightarrow r$ and $(p \rightarrow r) \wedge (q \rightarrow r)$ are not logically equivalent.

ans. Not equivalent

Q:9) (a) Using laws of logic show that $\sim(p \rightarrow q) \leftrightarrow (p \wedge \sim q)$ is a tautology.

ans. Tautology

(b) Use known logical equivalences to show that $\sim(p \leftrightarrow q) \equiv (p \vee q) \wedge (\sim p \vee \sim q)$.

ans. Equivalent

Q:10) Write the argument below in symbolic form. If the argument is valid, prove it. If you are pregnant or have a heart condition, then you can not use the hot tub.

You do not have a heart condition.
You can use the hot tub.
∴You are not pregnant.

ans. Argument is valid.

Q:11) Prove or disprove the equivalence of the propositions: $(p \wedge q) \rightarrow r$and $(p \rightarrow r) \vee (q \rightarrow r)$.

Q:12) Prove that $(p \vee q) \wedge (\neg p \vee q)$ is equivalent to q.

Q:13) Prove that: $(p \rightarrow q) \vee (q \rightarrow r) \rightarrow (p \rightarrow r)$ is a tautology.

Q:14) Prove that $(p \rightarrow q) \rightarrow ((p \rightarrow r) \vee (q \rightarrow r))$ is a tautology.

Q:15) Investigate the relationship between the propositions $p \rightarrow q$ and $\sim q \rightarrow \sim p$ by constructing truth tables and logical arguments.

Q:16) Using De Morgan's laws, demonstrate that $\sim(p \wedge q) = \sim p \vee \sim q$.

Q:17) Prove De Morgan's laws for propositions: ~(p ∧q) is equivalent to~pV~q, and ~(pVq) is equivalent to ~p ∧ ∼q.

Q:18) Construct truth table for Conjunction (^), Disjunction (v), Conditional (->) and biconditional (<->) using 2(p & q) and 3(p, q & r) variables.And write short notes on contingency, satisfiability, tautology, falsifiable.

Q:19) Show that ((P∧Q)V(~P∧R))^((P∧-R)V(~Q∧R)) is tautology or not using truth table.

Ans: it is not tautology.

Q:20) Show that ((P→Q) ∧ (Q→R))→ (P→R) is tautology. By using laws of logic and by truth table method also.

Q:21) Show that (((P∧Q)→R)↔ ((P→R) v (Q→R))) is tautology. By using laws of logic and by truth table method also.

Q:22) Prove by transformation, using proper logical equivalences that:

(a)~(~A U ~(B→~C)) = (A⋂~(B ∩ C))

(b) (-A⋂ (AUB)) = (-AU (-A⋂B))

Q:23) Consider this argument.

If Pat goes to the store, Pat will buy $1,000,000 worth of food.Pat goes to the store. Therefore, Pat buys $1,000,000 worth of food. This is a valid argument (you can test it on a truth table).However, even though Pat goes to the store, Pat does not buy $1,000,000 worth of food. The conclusion is false.How can the conclusion of a valid argument be false?