ASSIGNMENT - 1

DISCRETE MATHEMATICS

BCA - 401

UNIVERSITY OF LUCKNOW

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ASSIGNMENT - 1

Que 1 → If R be a relation in the set of integer z defined by $R = \{(x, y) : x \in z, y \in z, (x-y)$ is divisible by $6\}$. Then prove that R is an equivalence relation.

Sol^n → Reflexive relation → need to prove $xRx \; \forall x \in z$

$xRx \Rightarrow x - x = 0$

0 is divisible by 6.

Hence, relation is reflexive.

Symmetric Relation → need to prove if xRy then yRx

let, $xRy \Rightarrow x - y = 6k$ (divisible by 6)

$-(y-x) = 6k$

$y - x = -6k$

$y - x = 6k'$

$\Rightarrow y - x$ is divisible by 6 means yRx.

Hence, relation is symmetric

Transitive Relation → need to prove if xRy & yRz then xRz

let, $xRy \Rightarrow x - y = 6k_1$ —— ①

$yRz \Rightarrow y - z = 6k_2$ —— ②

adding ① & ②

$x - y + y - z = 6k_1 + 6k_2$

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$(x-z) = 6(k_1 + k_2)$

$x - z = 6k$

$x - z$ is divisible by 6 means xRz.

Hence, relation is transitive

→ Relation is reflexive, symmetric and transitive means it is an equivalence relation

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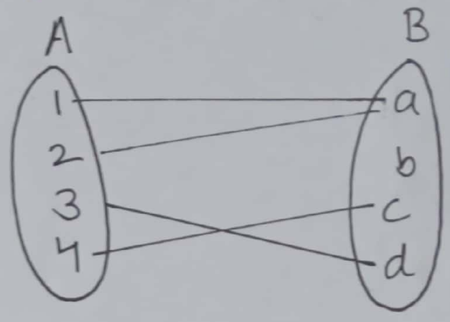
Que 2 → Let $A = \{1,2,3,4\}$ and $B = \{a,b,c,d\}$ and let $f = \{(1,a)(2,a)(3,d)(4,c)\}$ Show that f is a function but f^{-1} is not.

Solⁿ → $f = \{(1,a)(2,a)(3,d)(4,c)\}$



[f is a function]

Condition for function → every element in set A have 1 image in set B

$f(1) = a \Rightarrow f^{-1}(a) = 1$

$f(3) = d \Rightarrow f^{-1}(d) = 3$

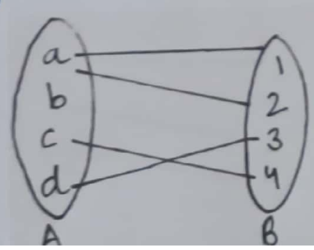
$f(4) = c \Rightarrow f^{-1}(c) = 4$

$f(2) = a$

$\Downarrow$

$f^{-1}(a) = 2$

$f^{-1} = \{(a,1)(a,2)(d,3)(c,4)\}$



→ b does not have its image.

→ a have 2 images.

- So, it does not satisfy the condition of being a function.

So, f is not a function.

Que-3 → Let R and S be relation from A to B, show that :

(i) If $R \subseteq S$, then $R^{-1} \subseteq S^{-1}$

(ii) If $(R \cap S)^{-1} = R^{-1} \cap S^{-1}$

(iii) $(R \cup S)^{-1} = R^{-1} \cup S^{-1}$

Solⁿ

(i) let $R \subseteq S$

$(x, y) \in R \implies (y, x) \in R^{-1}$

$(x, y) \in S \implies (y, x) \in S^{-1}$

Therefore, $R^{-1} \subseteq S^{-1}$

(ii) $(R \cap S)^{-1} = (b, a) \mid (a, b) \in (R \cap S)$

$= (b, a) \mid (a, b) \in R$ and $(a, b) \in S$

$= (b, a) \mid (a, b) \in R \cap (b, a) \mid (a, b) \in S$

$= R^{-1} \cap S^{-1} \Rightarrow$ proved.

(iii) $(R \cup S)^{-1} = (b, a) \mid (a, b) \in R \cup S$

$= (b, a) \mid (a, b) \in R$ or $(a, b) \in S$

$= (b, a) \mid (a, b) \in R \cup (b, a) \mid (a, b) \in S$

$= R^{-1} \cup S^{-1} \Rightarrow$ proved

Que - 4 → Let $A = B = C = R$. Consider the function $f : A \to B$ and $g : B \to C$ defined by $f(a) = 2a+1$, $g(b) = \frac{b}{3}$, verify $(gof)^{-1} = f^{-1} o g^{-1}$.

Solⁿ → $f : A \to B$ defined by $f(a) = 2a+1$
$g : B \to C$ defined by $g(b) = b/3$

$$gof(a) = g(f(a))$$
$$gof(a) = g(2a+1)$$
$$gof(a) = \frac{2a+1}{3}$$

inverse of gof

$$(gof)^{-1}(a) = f^{-1}(g^{-1}(a))$$
$$(gof)^{-1}(a) = f^{-1}(3a)$$
$$(gof)^{-1}(a) = \frac{3a-1}{2}$$

find f^{-1} if $f(a) = 2a+1$, then $f^{-1}(a) = \frac{a-1}{2}$

find g^{-1} if $g(b) = \frac{b}{3}$, then $g^{-1}(b) = 3b$

Now $(gof)^{-1} = f^{-1} o g^{-1} \Rightarrow f^{-1} o g^{-1}(a)$

$$\frac{3a-1}{2} = f^{-1}(3a) = \frac{3a-1}{2}$$

hence, verified ✓

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Que 5 → Let $A = \{1, 2, 3\}$ $B = \{a, b\}$
$C = \{5, 6, 7\}$

$f = \{(1, a) (2, a) (3, b)\}$ $f : A \to B$

$g = \{(a, 5) (b, 7\}$ $g : B \to C$

find $g \circ f$?

Solⁿ $g \circ f(x) = g(f(x))$

$f(1) = a$ $f(3) = b$

$f(2) = a$

$g(a) = 5$ $g(b) = 7$

$g \circ f(1) = g(f(1)) = g(a) = 5$

$g \circ f(2) = g(f(2)) = g(a) = 5$

$g \circ f(3) = g(f(3)) = g(b) = 7$

$g \circ f = \{(1, 5) (2, 5) (3, 7)\}$

Ans

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