Electronic Devices- KEC 301

Syllabus

- <u>Unit I</u> Introduction to semiconductor physics: Review of quantum mechanics, electrons in periodic lattices, E-k diagrams.

- <u>Unit II</u> Energy bands in intrinsic and extrinsic silicon, carrier transport, diffusion current, drift current, mobility and resistivity, sheet resistance, design of resistors.

- <u>Unit III</u> Generation and recombination of carriers, Poisson and continuity equation P-N junction characteristics, I-V characteristics, and small signal switching models.

- <u>Unit IV</u> Avalanche breakdown, Zener diode, Schottky diode, Bipolar Junction Transistor, I-V characteristics, Ebers-Moll model.

- <u>Unit V</u> MOS capacitor, C-V characteristics, MOSFET, I-V characteristics, and small signal models of MOS transistor, LED, photodiode and solar cell.

Text/Reference Books:

1. G. Streetman, and S. K. Banerjee, "Solid State Electronic Devices," 7th edition, Pearson, 2014.

2. D. Neamen , D. Biswas, "Semiconductor Physics and Devices," McGraw-Hill Education.

3. S. M. Sze and K. N. Kwok, "Physics of Semiconductor Devices," 3rd edition, John Wiley &Sons, 2006.

4. C.T. Sah, "Fundamentals of Solid State Electronics," World Scientific Publishing Co. Inc, 1991.

5. Y. Tsividis and M. Colin, "Operation and Modeling of the MOS Transistor," Oxford univ. press, 2011.

6. Muhammad H. Rashid, "Electronic Devices and Circuits," Cengage publication, 2014.

Semiconductor Materials

Introduction to Semiconductors

Objective of the lecture:

1. Define a semiconductor – no. of electrons in outer shell, location on periodic table, most commonly used ones etc.

History

- Diode , in 1939 was using Ge
- Transistor, in 1947 was using Ge
- In 1954 Si was used in Transistor because Si is less temperature sensitive and abundantly available.
- High speed transistor was using GaAs in 1970 (which is 5 times faster compared to Si)
- Si, Ge and GaAs are the semiconductor of choice

Why semiconductors?

- **SEMICONDUCTORS: They are here, there, and everywhere**

- **Computers, palm pilots,** Silicon (Si) MOSFETs, ICs, CMOS **laptops, anything "intelligent"**
- **Cell phones, pagers** Si ICs, GaAs FETs, BJTs
- **CD players** AlGaAs and InGaP laser diodes, Si photodiodes
- **TV remotes, mobile terminals** Light emitting diodes (LEDs)
- **Satellite dishes** InGaAs MMICs (Monolithic Microwave ICs)
- **Fiber networks** InGaAsP laser diodes, pin photodiodes
- **Traffic signals, car** GaN LEDs (**green**, **blue**) **taillights** InGaAsP LEDs (**red**, **amber**)
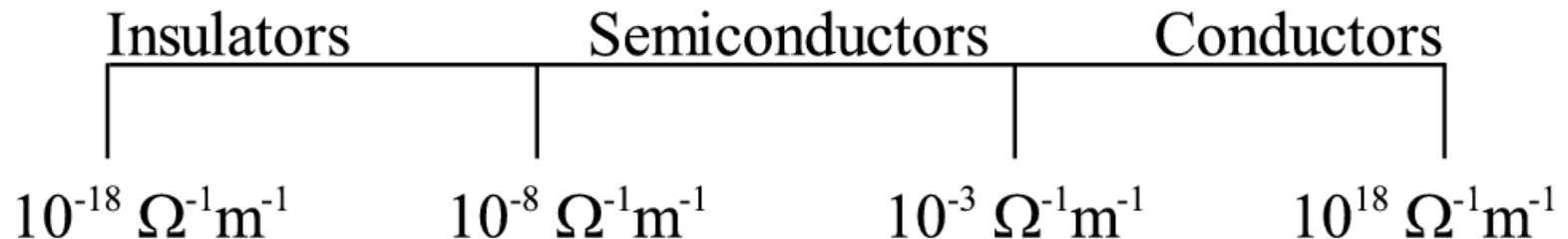- **Air bags** Si MEMs, Si ICs

Introduction

Semiconductors are materials whose electrical properties lie between Conductors and Insulators.

Ex : Silicon and Germanium

Give the examples of Conductors and Insulators!

Difference in conductivity

Insulators	Semiconductors		Conductors
$10^{-18}\ \Omega^{-1}m^{-1}$	$10^{-8}\ \Omega^{-1}m^{-1}$	$10^{-3}\ \Omega^{-1}m^{-1}$	$10^{18}\ \Omega^{-1}m^{-1}$

Semiconductor Materials

- **Elemental semiconductors** – Si and Ge (column IV of periodic table) –compose of single species of atoms

- **Compound semiconductors** – combinations of atoms of column III and column V and some atoms from column II and VI. (combination of two atoms results in binary compounds)

- There are also three-element (ternary) compounds (GaAsP) and four-elements (quaternary) compounds such as InGaAsP.

(a)	II	III	IV	V	VI
		B	C		
		Al	Si	P	S
	Zn	Ga	Ge	As	Se
	Cd	In		Sb	Te

(b)	Elemental	IV compounds	Binary III-V compounds	Binary II-VI compounds
	Si	SiC	AlP	ZnS
	Ge	SiGe	AlAs	ZnSe
			AlSb	ZnTe
			GaP	CdS
			GaAs	CdSe
			GaSb	CdTe
			InP	
			InAS	
			InSb	

Periodic Table of the Elements

IA																	0
1 H	IIA											IIIA	IVA	VA	VIA	VIIA	2 He
3 Li	4 Be											5 B	6 C	7 N	8 O	9 F	10 Ne
11 Na	12 Mg	IIIB	IVB	VB	VIB	VIIB	——— VIII ———			IB	IIB	13 Al	14 Si	15 P	16 S	17 Cl	18 Ar
19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
37 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
55 Cs	56 Ba	57 *La	72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn
87 Fr	88 Ra	89 +Ac	104 Rf	105 Ha	106 Sg	107 Ns	108 Hs	109 Mt	110 110	111 111	112 112	113 113					

Semiconductor materials

*Lanthanide Series	58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu
+Actinide Series	90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No	103 Lr

Semiconductor Materials

- The wide variety of electronic and optical properties of these semiconductors provides the device engineer with great flexibility in the design of electronic and opto-electronic functions.

- **Ge** was widely used in the early days of semiconductor development for transistors and diods.

- **Si** is now used for the majority of rectifiers, transistors and integrated circuits.

- **Compounds** are widely used in high-speed devices and devices requiring the emission or absorption of light.

- The electronic and optical properties of semiconductors are strongly affected by impurities, which may be added in precisely controlled amounts (e.g. an impurity concentration of one part per million can change a sample of Si from a poor conductor to a good conductor of electric current). This process called doping.

The Silicon Atomic Structure
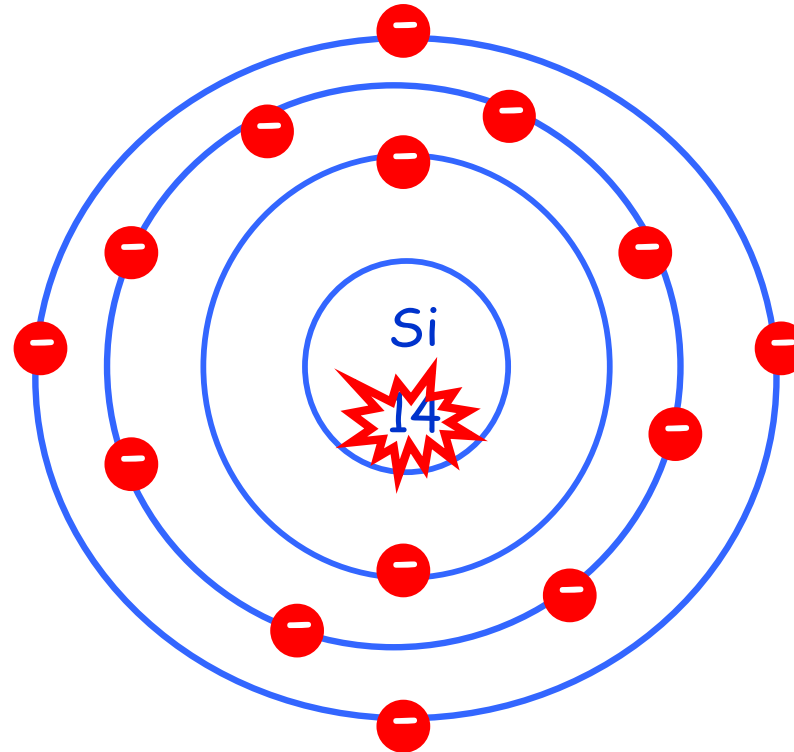
Silicon: our primary example and focus
Atomic no. 14
14 electrons in three shells: 2) 8) 4
i.e., 4 electrons in the outer "bonding" shell
Silicon forms strong covalent bonds with 4 neighbors

However, like all other elements it would prefer to have 8 electrons in its outer shell

- Valence electrons: electrons in the outermost shell.
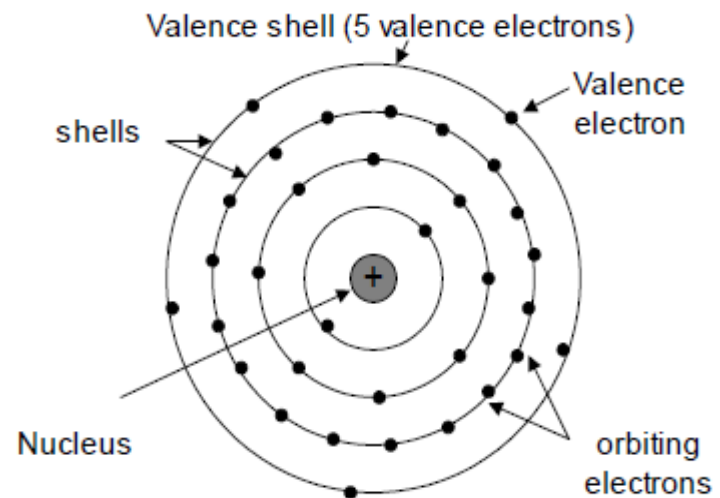- Atoms with four valence electrons are called tetravalent

Germanium
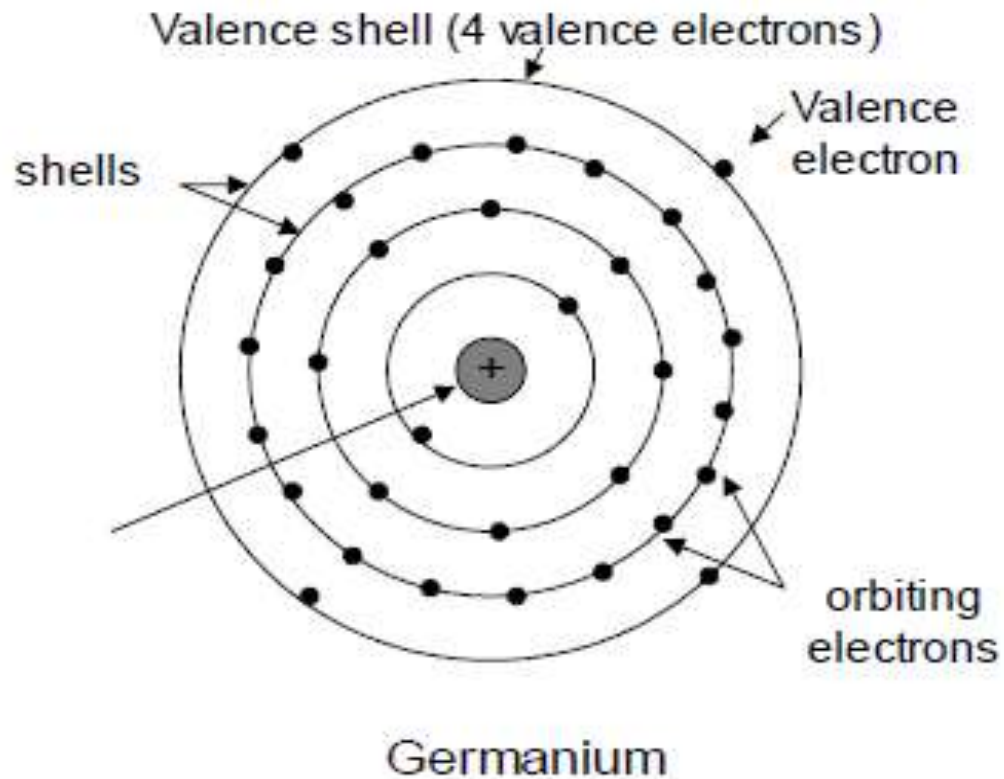
32 orbiting electrons
(tetravalent)

Atomic Structure

Valence shell (3 valence electrons)

Valence electron

shells

Nucleus

orbiting electrons
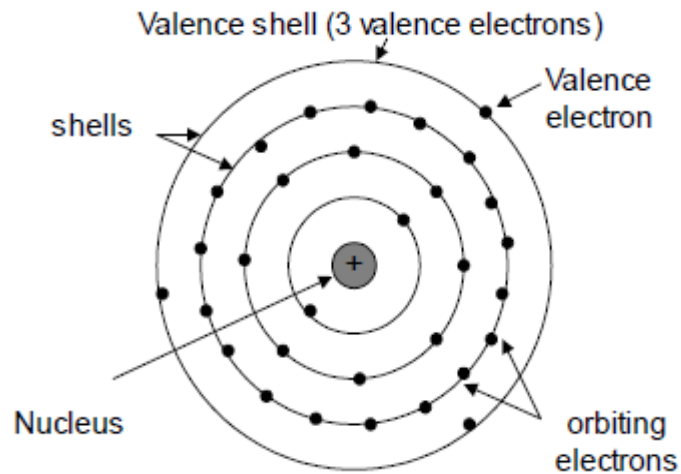
Gallium

31 orbiting electrons (trivalent)

Valence shell (5 valence electrons)

Valence electron

shells

Nucleus

orbiting electrons

Arsenic

33 orbiting electrons (pentavalent)

Atoms with three valence electrons are called trivalent, and those with five are called pentavalent.

Possible Semiconductor Materials

Carbon	*C*	*6*	1. Very Expensive 2. Band Gap Large: 6eV 3. Difficult to produce without high contamination
Silicon	**Si**	**14**	1. Cheap 2. Ultra High Purity
Germanium	**Ge**	**32**	1. High Mobility 2. High Purity Material
Tin	**Sn**	**50**	1. Only "White Tin" is semiconductor 2. Converts to metallic form under moderate heat
Lead	**Pb**	**82**	1. Only "White Lead" is semiconductor 2. Converts to metallic form under moderate heat

Bonding of Si atoms

This results in the covalent bonding of Si atoms in the crystal matrix

Two electrons per bond
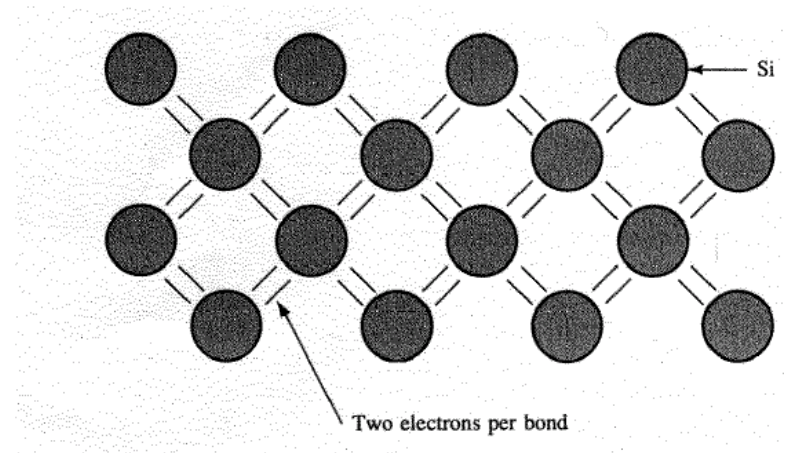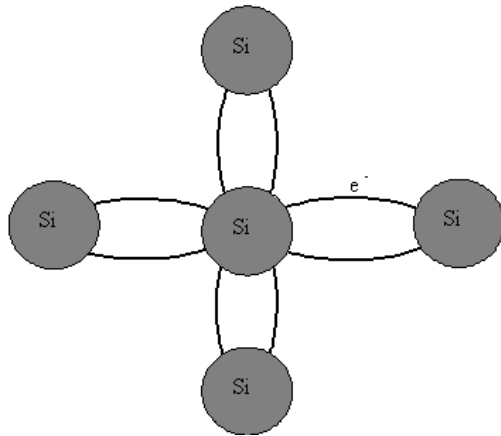
A Covalent Bond Formed by the Sharing of Electrons in an Outer Energy Level

Solid State Structures

A crystalline solid is distinguished by the fact that atoms making the crystal are arranged in a periodic fashion. That is, there is some basic arrangement of atoms that is repeated throughout the entire solid.

However, not all solids are crystals ; some have no periodic structure at all (*amorphous* solids),

and other are composed of many small regions of single-crystal material (*polycrystalline* solids).
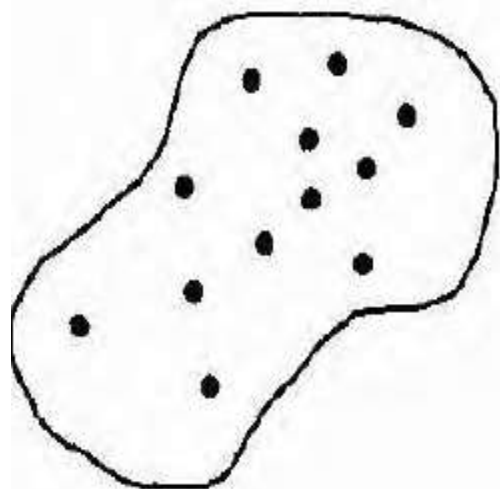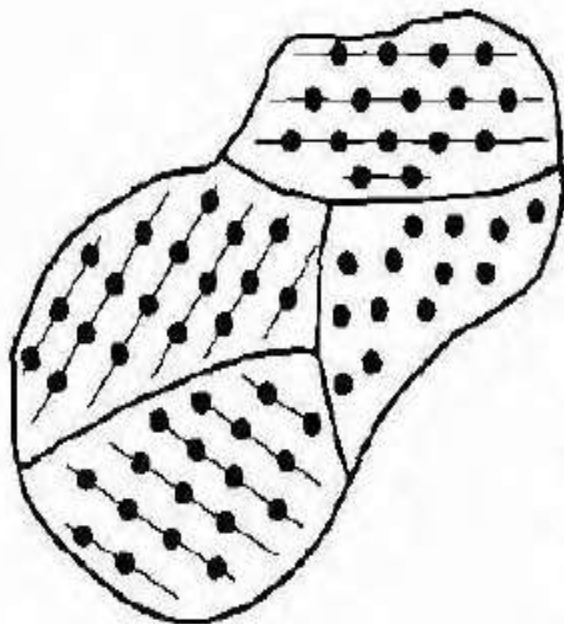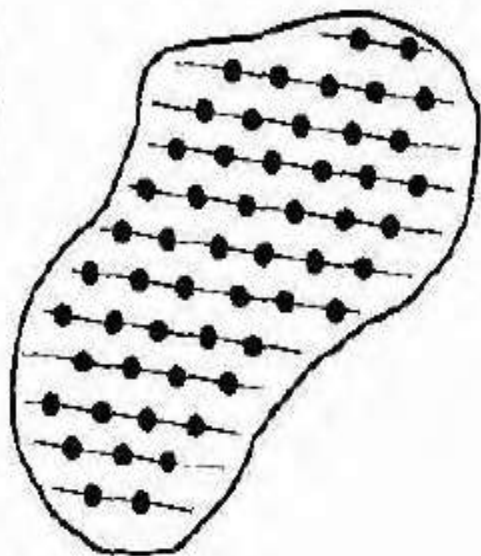
(a)   (b)   (c)

Unit Cell, Lattice and Primitive Cell

- The smallest repeating unit of the crystal lattice is the unit cell, the building block of a crystal.

- The unit cells which are all identical are defined in such a way that they fill space without overlapping. The 3D arrangement of atoms, molecules or ions inside a crystal is called a **crystal lattice.** It is made up of numerous unit cells.

- A unit cell can either be *primitive cubic, body-centered cubic (BCC) or face-centered cubic (FCC).*

Miller Indices and Crystal Planes

- Miller Indices for Planes: Procedure

1. Identify the plane intercepts on the x, y and z-axes.
2. Specify intercepts in fractional coordinates.
3. Take the reciprocals of the fractional intercepts
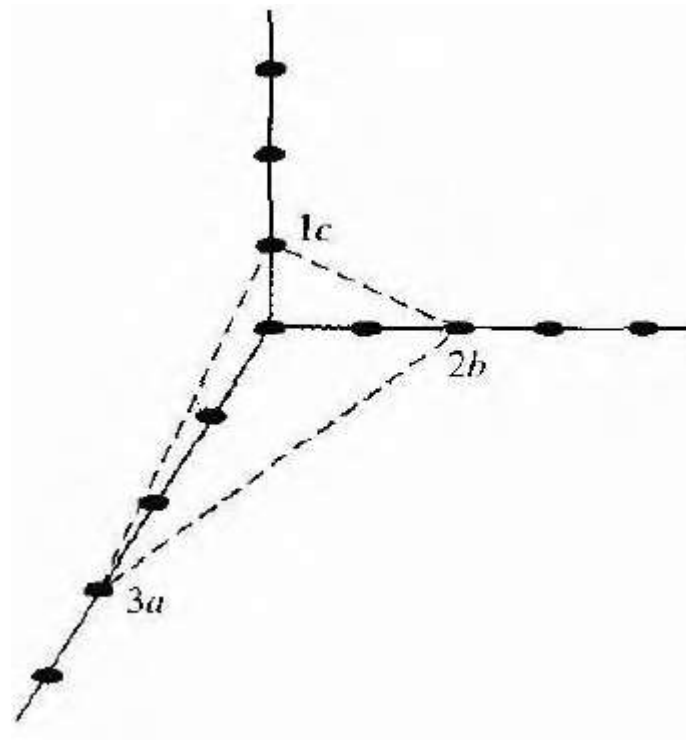
Intercepts: x=3, y= 2, and z=1

Take the reciprocal of the intercepts

h = 1/x, k=1/y, and l=1/z
h=1/3
k = 1/2
l = 1/1.
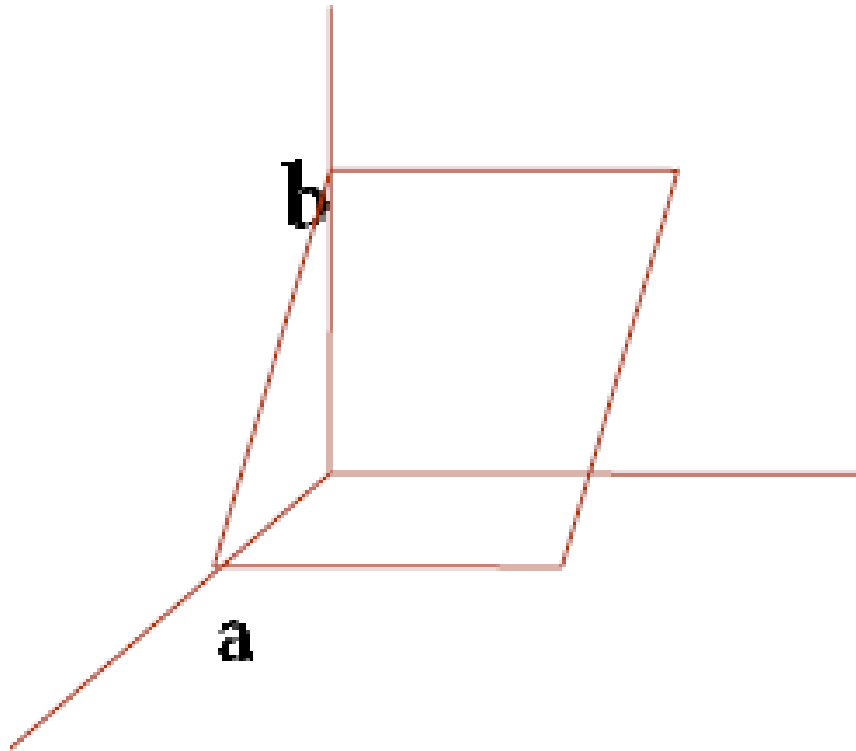Multiply h, k, and l by 6 to find the smallest integer.
(hkl) = (236)
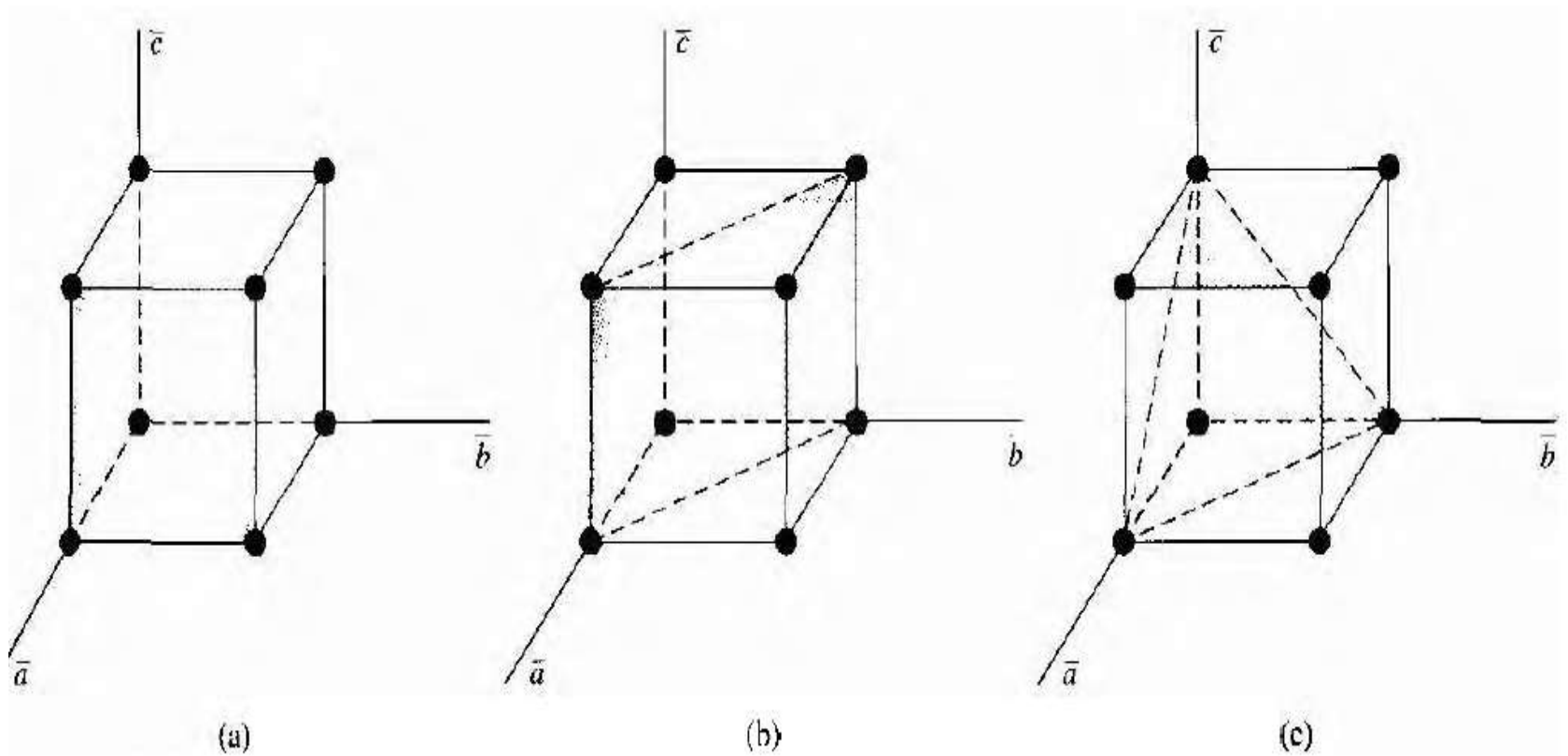
(hkl) – for single plane and {hkl} for a group of planes.

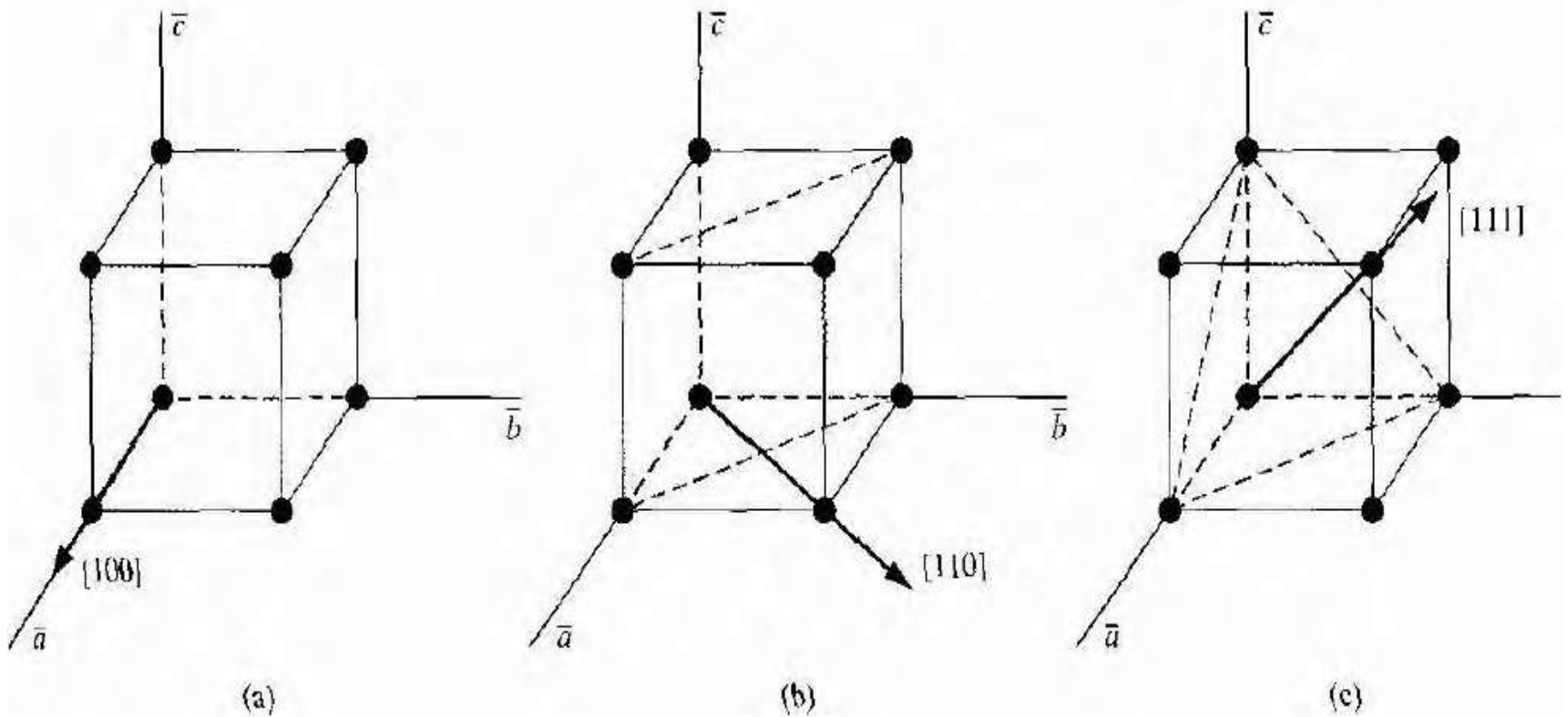Example 2:

Determine the miller indices of the given plane.

Notice that the plane does not intersect the y – axis. Therefore, take the intercept as ∞.

- Assume the intercepts of the x and z are 1 and 1.

- Take the reciprocals of the intercepts
- h = 1/1, k = 1/∞, and l= 1/1
- (hkl) = (101)

Three lattice planes (a) 100 plane (b) 110 plane (c) 111 plane

Directions

(a) (100) plane and [100] direction, (b) (110) plane and [110] direction. (c) (111) plane and [111] direction.

Review of Quantum Mechanics

PRINCIPLES OF QUANTUM MECHANICS

- the principle of energy quanta,
- the wave-particle duality principle,
- and the uncertainty principle.

Energy Quanta

- Based on the wave model of light, physicists predicted that increasing light amplitude would increase the kinetic energy of emitted photoelectrons, while increasing the frequency would increase measured current.

- Contrary to the predictions, experiments showed that increasing the light frequency increased the kinetic energy of the photoelectrons, and increasing the light amplitude increased the current.

- Based on these findings, Einstein proposed that light behaved like a stream of particles called *photons* with an energy of $E = h\nu$

- The work function, Φ, is the minimum amount of energy required to induce photoemission of electrons from a metal surface, and the value of Φ depends on the metal.

- The energy of the incident photon must be equal to the sum of the metal's work function and the photoelectron kinetic energy:

$$E_{photon} = KE_{electron} + \Phi$$

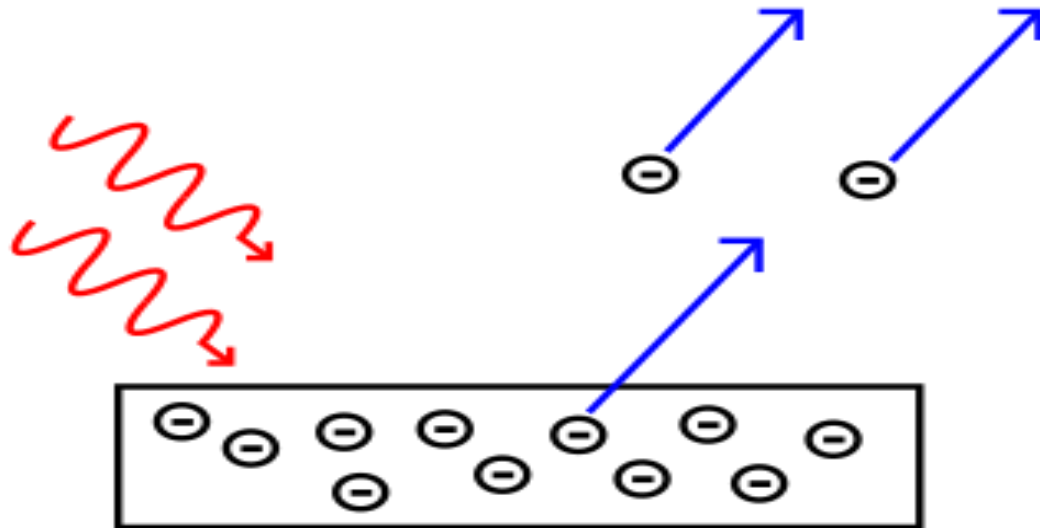$$E_{photon} = h\nu = KE_{electron} + \Phi$$

$$KE_{electron} = h\nu - \Phi$$

(work function, depends on material, but usually 2-5eV)

Photoelectric Effect

- When light shines on a metal, electrons can be ejected from the surface of the metal in a phenomenon known as the *photoelectric effect*. This process is also often referred to as *photoemission*, and the electrons that are ejected from the metal are called *photoelectrons*.

Example 1:

- The work function of copper metal is $\Phi=7.53\times10^{-19}$ J If we shine light with a frequency of 3.0×10^{16} Hz on copper metal, will the photoelectric effect be observed?

Solution: In order to eject electrons, we need the energy of the photons to be greater than the work function of copper. We can use Planck's equation to calculate the energy of the photon:

Ephoton=hv=$(6.626\times10^{-34}$ J·s$)(3.0\times10^{16}$ Hz$)$

$\qquad\qquad$ =2.0×10^{-17} J

- If we compare our calculated photon energy, Ephoton to copper's work function, we see that the photon energy is greater than Φ:

- 2.0×10^{-17} J > 7.53×10^{-19}

- Thus, we would expect to see photoelectrons ejected from the copper.

Compton Effect

- Compton (1923) measured intensity of scattered X-rays from solid target, as function of wavelength for different angles. He won the 1927 Nobel prize.

- **Classical picture**: oscillating electromagnetic field causes oscillations in positions of charged particles, which re-radiate in all directions at *same frequency and wavelength* as incident radiation.

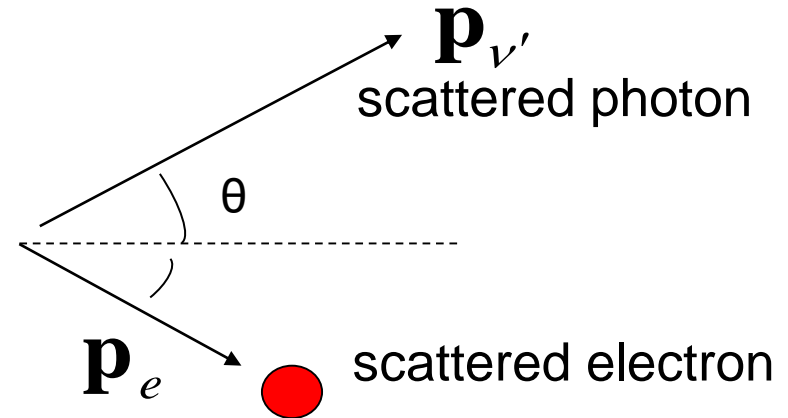- **Change in wavelength of scattered light is completely unexpected classically**

Incident light wave   Oscillating electron   Emitted light wave
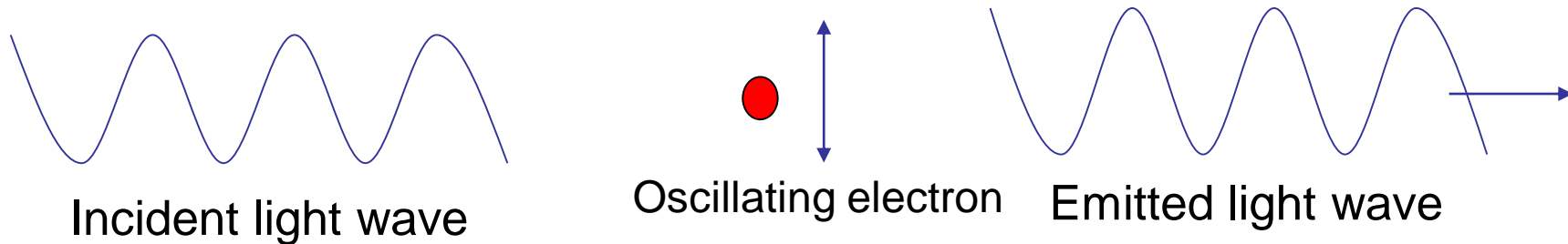
Compton's explanation: "billiard ball" collisions between particles of light (X-ray photons) and electrons in the material.

Before

After

Incoming photon $\mathbf{p}_\nu$

Electron

$\mathbf{p}_{\nu'}$
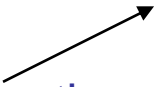scattered photon

θ

$\mathbf{p}_e$ scattered electron
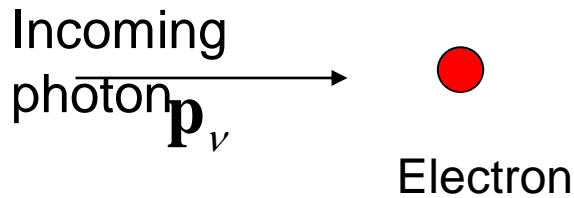
Wave particle Duality

- In 1924. de Broglie postulated the existence of matter waves. He suggested that since waves exhibit particle-like behavior, then panicles should he expected to show wave-like properties. The hypothesis of de Broglie was the existence of a wave-particle duality principle. The momentum of a photon is given by :

de Broglie wavelength

$$\lambda = \frac{h}{p}$$

- The Davisson-Germer experiment: scattering a beam of electrons from a Ni crystal. Davisson got the 1937 Nobel prize.

- This experiment also proposes wave nature of electrons.
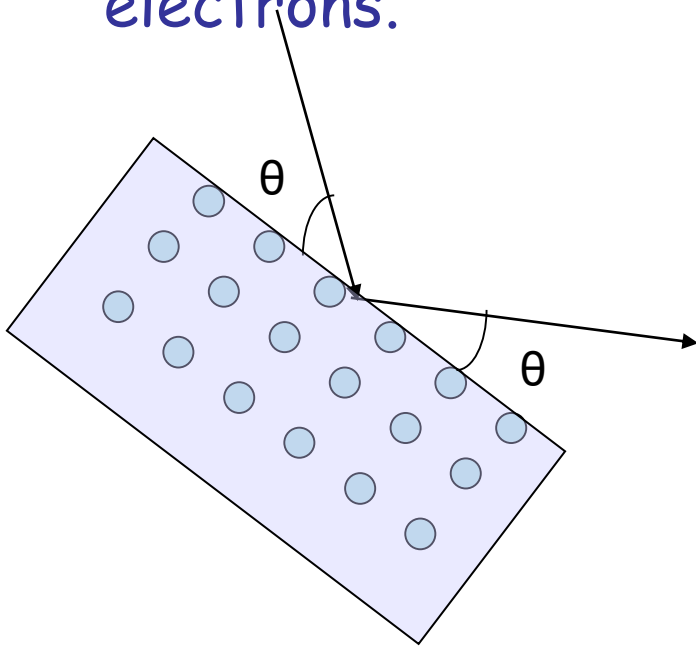
At fixed accelerating voltage (fixed electron energy) find a pattern of sharp reflected beams from the crystal

- This wave-particle duality applies primarily to small particles such as electrons, but it has also been shown to apply to protons and neutrons.
- The wave-particle duality principle is the basis on which we will use wave theory to describe the motion and behavior of electrons in a crystal.

Example

- Calculate the de broglie wavelength of a particle. Consider an electron travelling at a velocity of 10^7 cm/sec.

The Uncertainty Principle

- The first statement of the uncertainty principle is that it is impossible to simultaneously describe with absolute accuracy the position and momentum of a particle. If the uncertainty in the momentum is Δp and the uncertainty in the position is Δx, then the uncertainty principle is stated as

$$\Delta p \, \Delta x \geq \hbar$$

- Second statement of uncertainty principle is that it is impossible to simultaneously describe with absolute accuracy the energy of a particle and the instant of time the particle has this energy.

$$\Delta E\ \Delta t \geq \hbar$$

SCHRODINGER'S WAVE EQUATION

- Schrodinger in 1926. provided a formulation called wave mechanics, which incorporated the principles of quanta introduced by Planck, and the wave-particle duality principle introduced by de Broglie.

- Based on the wave-particle duality principle. we will describe the motion of electrons in a crystal by wave theory.

- This wave theory is described by Schrodinger's wave equation.

The Wave Equation

The one-dimensional, non relativistic Schrodinger's wave equation is given by:

$$\frac{-\hbar^2}{2m} \cdot \frac{\partial^2 \Psi(x, t)}{\partial x^2} + V(x)\Psi(x, t) = j\hbar\frac{\partial \Psi(x, t)}{\partial t}$$

…………………………….equation 1

where ψ(x, t) is the wave function, V(x) is the potential function assumed to be independent of time, m is the mass of the particle, and, j   is the imaginary constant

The wave function ψ(x, t) will be used to describe the behavior of the system and, mathematically, ψ(x, t) can be a complex quantity.

by using the technique of separation of variables:

$$\Psi(x, t) = \psi(x)\phi(t)$$

Substituting this form of the solution into Schrodinger's wave equation, we obtain

$$\frac{-\hbar^2}{2m}\phi(t)\frac{\partial^2\psi(x)}{\partial x^2} + V(x)\psi(x)\phi(t) = j\hbar\psi(x)\frac{\partial\phi(t)}{\partial t}$$

...........................equation 2

If we divide by the total wave function. Equation (2) becomes

$$\frac{-\hbar^2}{2m}\frac{1}{\psi(x)}\frac{\partial^2\psi(x)}{\partial x^2} + V(x) = j\hbar \cdot \frac{1}{\phi(t)} \cdot \frac{\partial\phi(t)}{\partial t}$$

.........................Equation 3

Since the left side of Equation (3) is a function of position x only and the right side of the equation is a function of time t only, each side of this equation must he equal to a constant. We will denote this separation of variables constant by η .

The time-dependent portion of Equation (3) is then written as:

$$\eta = j\hbar \cdot \frac{1}{\phi(t)} \cdot \frac{\partial \phi(t)}{\partial t}$$

η = called separation constant

$$\phi(t) = e^{-j(E/\hbar)t}$$

The time-independent portion of Schrodinger's wave equation can now be written from Equation (3) as:

$$\frac{-\hbar^2}{2m} \cdot \frac{1}{\psi(x)} \cdot \frac{\partial^2 \psi(x)}{\partial x^2} + V(x) = E$$

$$\frac{\partial^2 \psi(x)}{\partial x^2} + \frac{2m}{\hbar^2}(E - V(x))\psi(x) = 0$$

Physical Meaning and significance of the Wave Function

- A wave function is a mathematical description of a quantum state of a particle as a function of momentum, position, time and spin.

- Denoted by ψ

- By using a wave function, the probability of finding an electron within the matter wave can be explained.

We are ultimately trying to use the wave function ψ(x, t) to describe the behavior of an electron in a crystal. The function ψ(x, t) is a wave function. so it is reasonable to ask what the relation is between the function and the electron. The total wave function is the product of the position-dependent, or time-independent, function and the time-dependent function

$$\Psi(x, t) = \psi(x)\phi(t) = \psi(x)e^{-j(E/\hbar)t}$$

$|\psi(x. t)|^2$ is a probability density function. We have that

$$|\Psi(x,t)|^2 = \Psi(x,t) \cdot \Psi^*(x,t)$$

where $\psi^*(x, I)$ is the complex conjugate function. Therefore

$$\Psi^*(x,t) = \psi^*(x) \cdot e^{+j(E/\hbar)t}$$

Then the product of the total wave function and its complex conjugate is given by:

$$\Psi(x,t)\Psi^*(x,t) = \left[\psi(x)e^{-j(E/\hbar)t}\right]\left[\psi^*(x)e^{+j(E/\hbar)t}\right] = \psi(x)\psi^*(x)$$

$$|\Psi(x,t)|^2 = \psi(x)\psi^*(x) = |\psi(x)|^2$$

- is the probability density function and is independent of time. One major difference between classical and quantum mechanics is that in classical mechanics, the position of a particle or body can be determined precisely, whereas in quantum mechanics, the position of a particle is found in terms of a probability.

Boundary Conditions:

Condition 1. $\psi(x)$ must be finite, single-valued, and continuous.

Condition 2. $\delta \psi(x)/ \delta x$ must be finite, single-valued, and continuous.

Importance of Schrodinger Equation

- The electronic structure of atoms and molecules can be well explained using schrodinger's equation.

- The classification of solids (metals, semiconductors and insulators) can be well explained using band theory of solids.

- Most of the semiconductors properties are well explained such as : Origin of band gap, the behavior of dopants and their energy levels present

- The real materials are impure and they contain various impurities as well as defects. The ground state properties of theses defects can be well described by solving the schrodinger equation.

Application: A free Particle

- A particle is said to be free when no external force is acting on during its motion in the given region of space, and its potential energy *V* is constant.

- Let us consider an electron is freely moving in space in positive *x* direction and not acted by any force, there potential will be zero. The Schrodinger wave equation reduces to

$$\bigtriangledown^2 \psi + \frac{2m}{\hbar^2} E\psi = 0$$

……equation 1

Substituting, we get

$$\frac{2mE}{\hbar^2} = k^2$$

- As the electron is moving in one direction (say x axis), then the above equation can be written as:

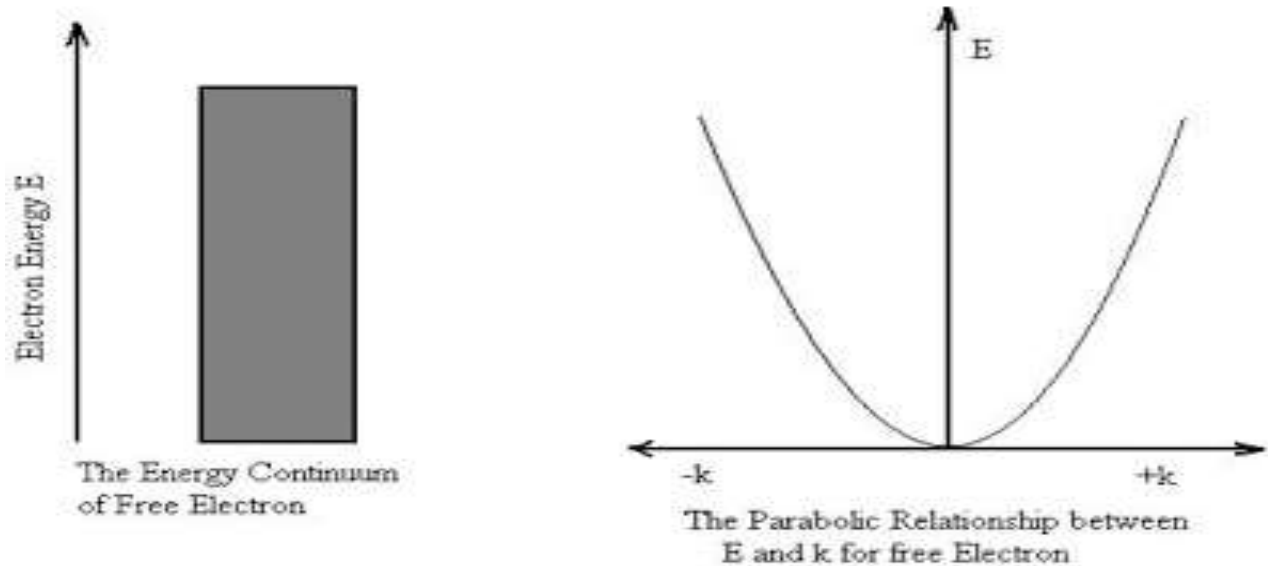$$\frac{\partial^2 \psi}{\partial x^2} + k^2 \psi = 0$$

……equation 2

The general solution of the equation (2) is of the form

$$\psi = \psi_0 e^{-i\omega t}$$

The electron is not bounded and hence there are no restrictions on *k*. This implies that all the values of energy are allowed. The allowed energy values form a continuum and are given by:

$$E = \frac{\hbar^2 k^2}{2m}$$

……equation 3

- The wave vector *k* describes the wave properties of the electron. It is seen from the relation that $E \propto K^2$. Thus the plot of *E* as a function of *k* gives a parabola.

Electron Energy E

The Energy Continuum of Free Electron

E

−k   +k

The Parabolic Relationship between E and k for free Electron

The momentum is well defined.

Therefore, according to uncertainty principle it is difficult to assign a position to the electron.

2.   Infinite Square-Well Potential
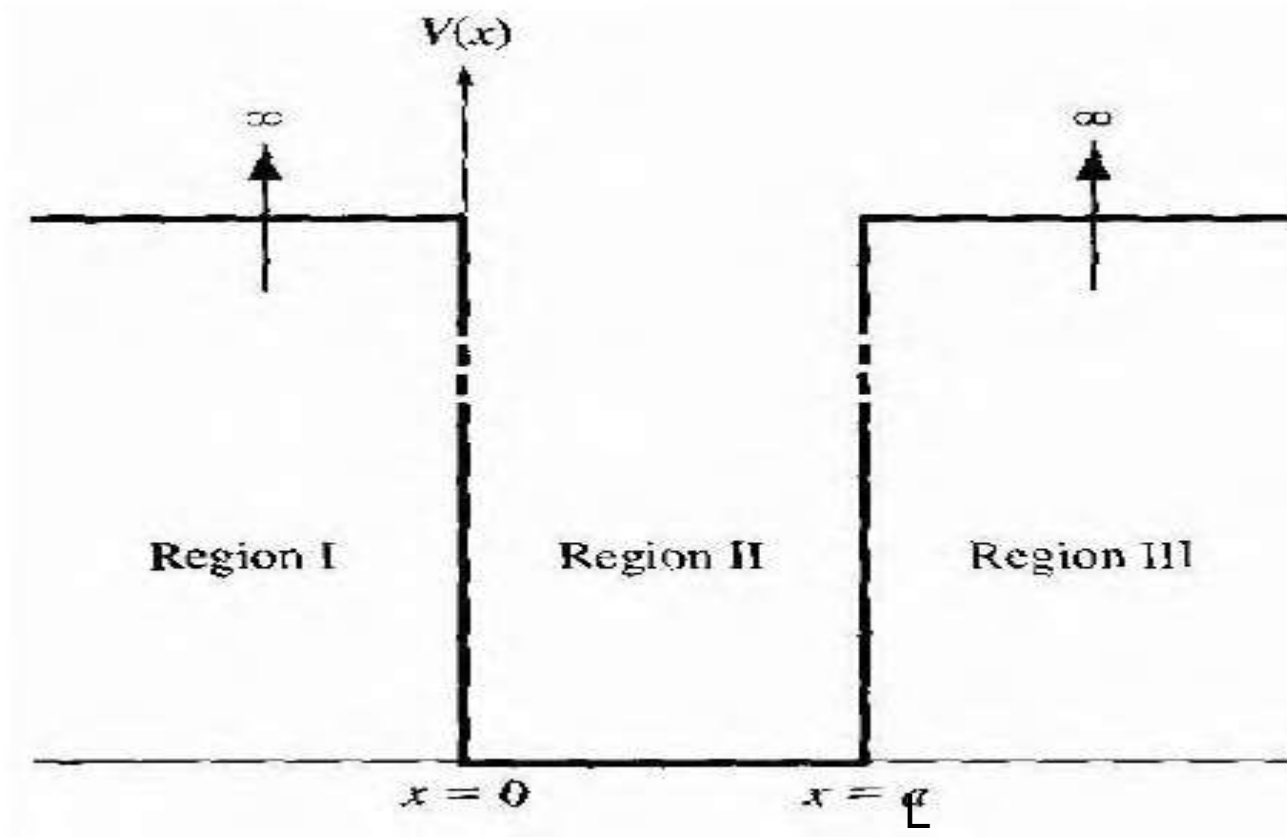
2. Infinite Square-Well Potential

- The simplest such system is that of a particle trapped in a box with infinitely hard walls that the particle cannot penetrate. This potential is called an infinite square well and is given by

$$V(x) = \begin{cases} \infty & x \leq 0, x \geq L \\ 0 & 0 < x < L \end{cases}$$

- Clearly the wave function must be zero where the potential is infinite.

- Where the potential is zero inside the box, the Schrödinger wave equation becomes   where

$$\frac{d^2\psi}{dx^2} = -\frac{2mE}{\hbar^2}\psi = -k^2\psi \qquad k = \sqrt{2mE/\hbar^2}$$

- The general solution is   .

$$\psi(x) = A\sin kx + B\cos kx$$

Boundary conditions of the potential dictate
that the wave function must be zero at $x = 0$
   and $x = L$. This yields valid solutions for
   integer values of n such that $kL = n\pi$.
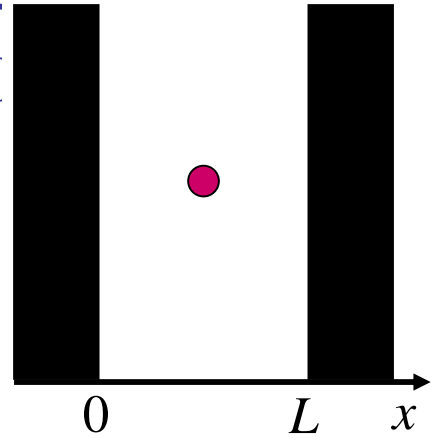
- The wave function is:

$$\psi_n(x) = A \sin\left(\frac{n\pi x}{L}\right)$$

$0 \qquad L \quad x$

- We normalize the wave function:

$$\tfrac{1}{2} - \tfrac{1}{2}\cos(2n\pi x/L)$$

$$\int_{-\infty}^{\infty} \psi_n^*(x)\psi_n(x)\,dx = 1 \quad\Rightarrow\quad A^2 \int_0^L \sin^2\left(\frac{n\pi x}{L}\right) dx = 1 \quad\Rightarrow\quad A = \sqrt{2/L}$$

- The normalized wave
function becomes:

$$\psi_n(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi x}{L}\right)$$

Quantized Energy

- The quantized wave number now becomes $k_n = \dfrac{n\pi}{L} = \sqrt{\dfrac{2mE_n}{\hbar^2}}$
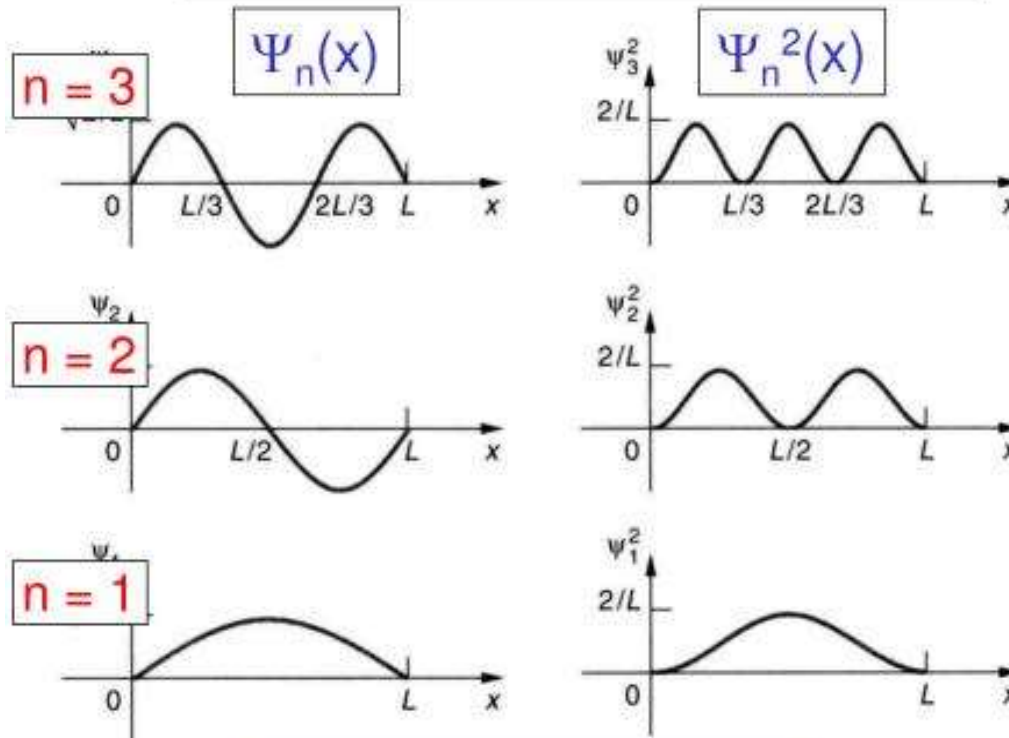- Solving for the energy yields

$$E_n = n^2 \dfrac{\pi^2 \hbar^2}{2mL^2} \qquad (n = 1, 2, 3, \ldots)$$

- Note that the energy depends on the integer values of *n*. Hence the energy is quantized and nonzero.
- The special case of *n* = 1 is called the ground state energy $E_1 = \dfrac{\pi^2 \hbar^2}{2mL^2}$
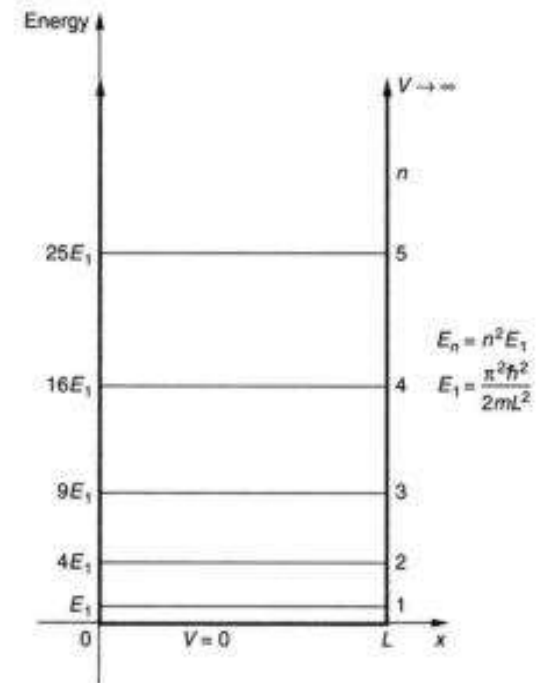

ψ_3

ψ_2

ψ_1

0   L

$|\psi_3|^2$

$|\psi_2|^2$

$|\psi_1|^2$

0   L

Position

∞   ∞

↑ Energy

25 E_1

16 E_1

9 E_1

4 E_1

E_1
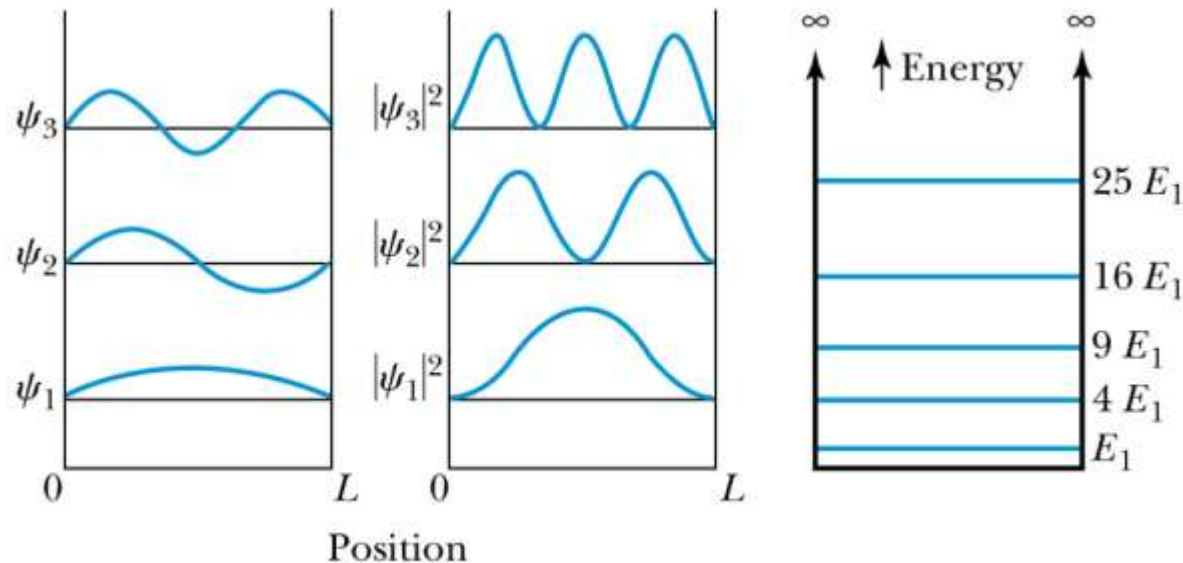
Infinite Square Well Potential: Visual Solutions

Wave and Probability Solutions

Energy Solutions

$\Psi_n(x)$

$\Psi_n^2(x)$

n = 3

n = 2

n = 1

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)$$

$$E_n = \frac{\hbar^2 k^2}{2m} = n^2 \left(\frac{\hbar^2 \pi^2}{2mL^2}\right)$$

$E_n = n^2 E_1$

$E_1 = \frac{\pi^2 \hbar^2}{2mL^2}$

# Electron in a periodic potential: Kronig Penney Model

- According to quantum free electron theory of metals, a conduction electron in a metal experiences constant (or zero) potential and free to move inside the crystal but will not come out of the metal because an infinite potential exists at the surface.

- This theory successfully explains electrical conductivity, specific heat, thermionic emission and paramagnetism.

- This theory is fails to explain many other physical properties, for example: (i) it fails to explain the difference between conductors, insulators and semiconductors, (ii) positive Hall coefficient of metals and (iii) lower conductivity of divalent metals than monovalent metals.

- To solve this problem, the basic assumption of free electron theory was modified by Bloch.
- Bloch proposed that the electron inside the material are not in constant potential but they are moving in periodic potential well.
- Kronig and Penney define the periodic one dimensional square potential well.

- To overcome the problems, the periodic potentials due to the positive ions in a metal have been considered. shown in Fig. (a), if an electron moves through these ions, it experiences varying potentials.

- The potential of an electron at the positive ion site is zero and is maximum in between two ions.

- The potential experienced by an electron, when it passes along a line through the positive ions is as shown in Fig. (b).
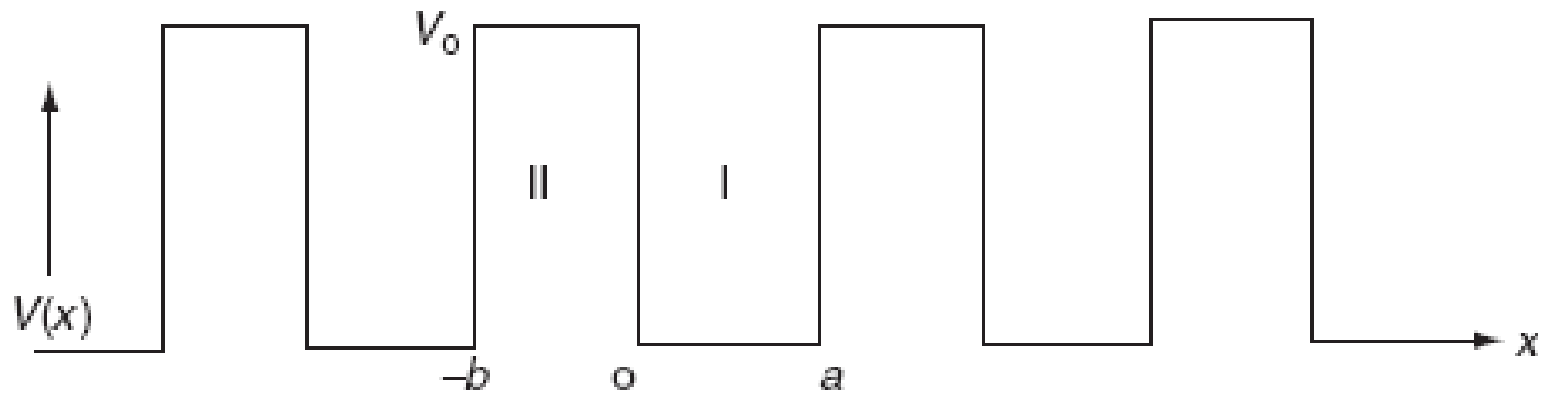
ions

e⁻

(a)

Ions

Distance through ions

$V(x)$ Potential

(b)

V_0

$V(x)$

$v = 0$

a
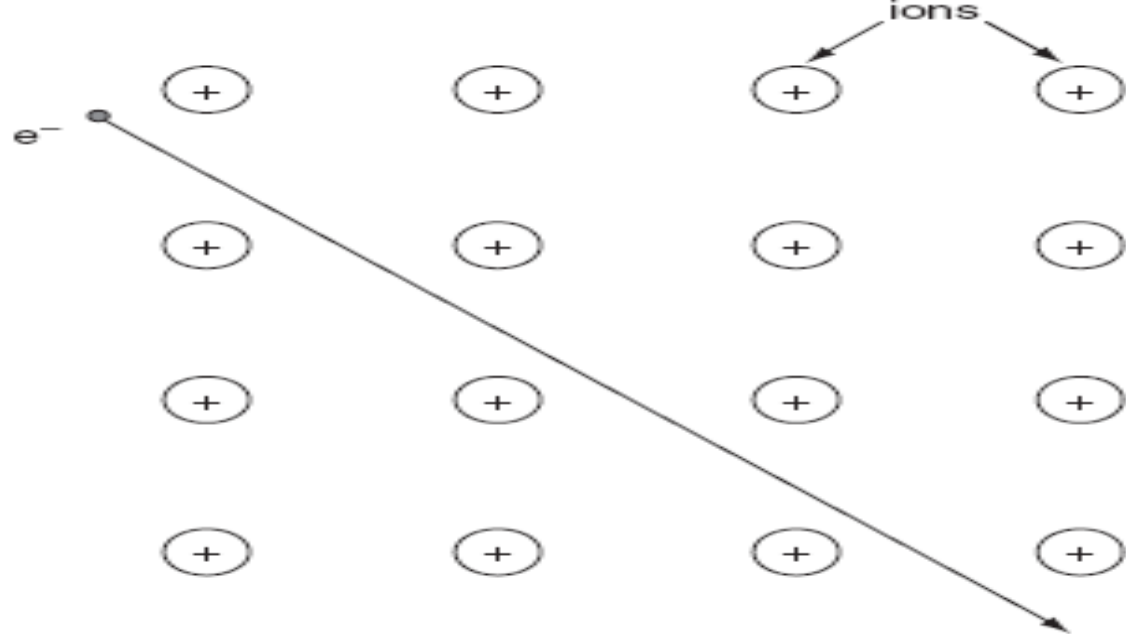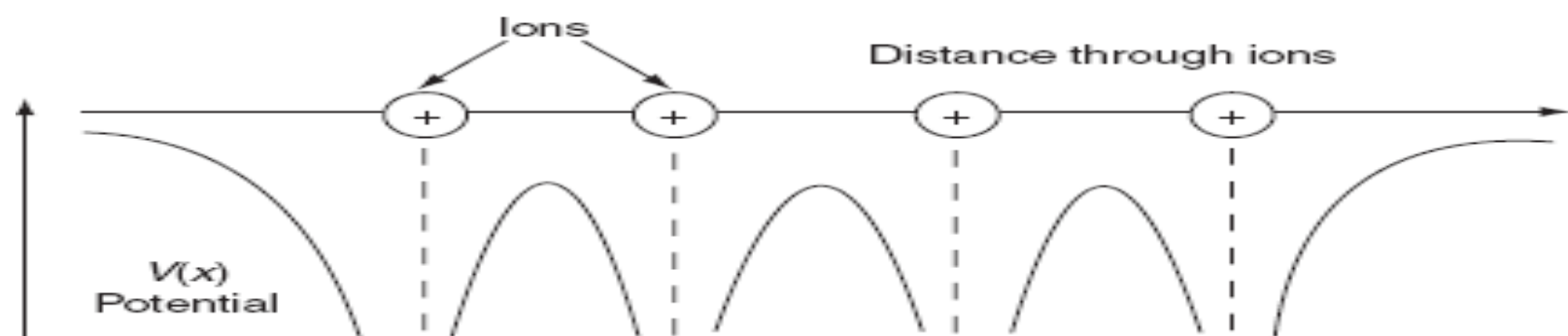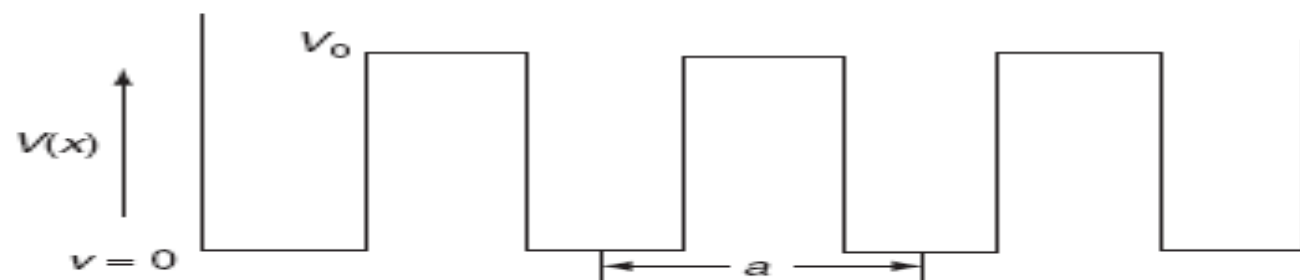
(c)

The energies of electrons can be known by solving Schrödinger's wave equation in such a lattice. The Schrödinger time-independent wave equation for the motion of an electron along X-direction is given by:

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}\left[E - V(x)\right]\psi = 0$$

<div style="text-align: right;">Equation………………1</div>

The energies and wave functions of electrons associated with this model can be calculated by solving time-independent one-dimensional Schrödinger's wave equations for the two regions I and II as shown in Fig

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}E\psi = 0$$

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}\left[E - V_o\right]\psi = 0$$

We define two real quantities (say) α and β such that:

$$\alpha^2 = \frac{2mE}{\hbar^2} \qquad \text{and} \qquad \beta^2 = \frac{2m}{\hbar^2}\left(V_0 - E\right)$$
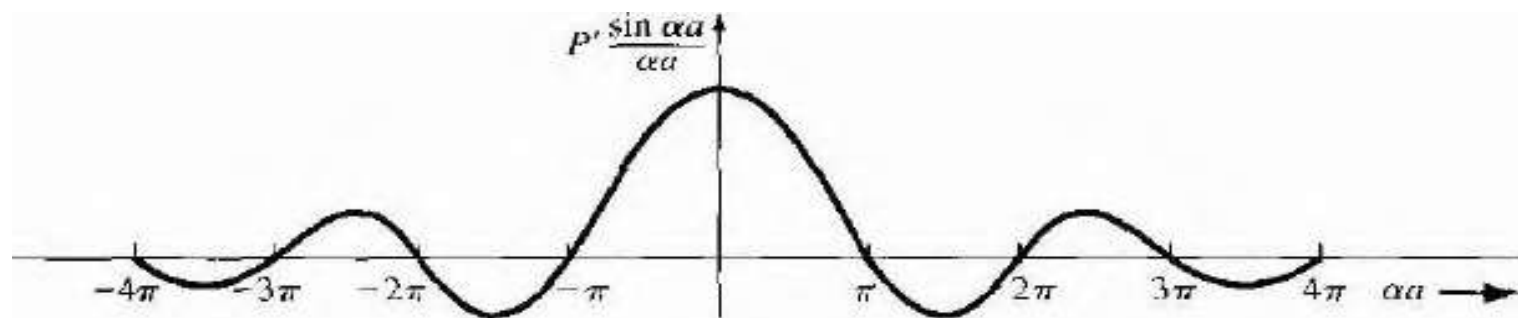
$$\frac{d^2\psi}{dx^2} + \alpha^2\psi = 0 \qquad \text{For } 0<x<a$$

$$\frac{d^2\psi}{dx^2} - \beta^2\psi = 0 \qquad \text{For } -b<x<0$$
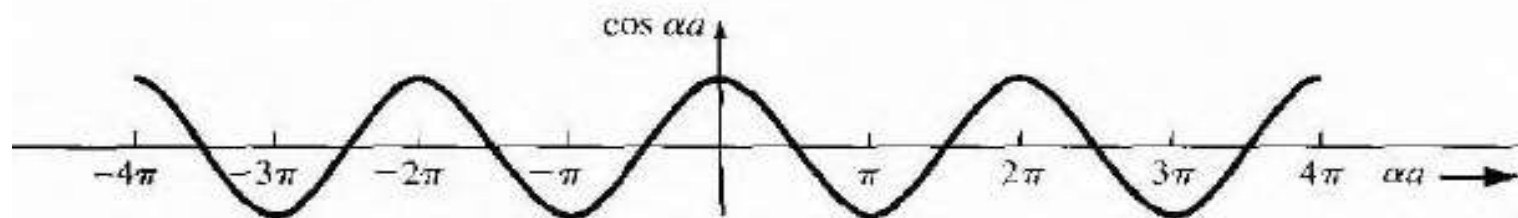
- Using Bloch theorem and all the boundary conditions for the continuity of the wave function the solution of Schrodinger wave equation obtained as
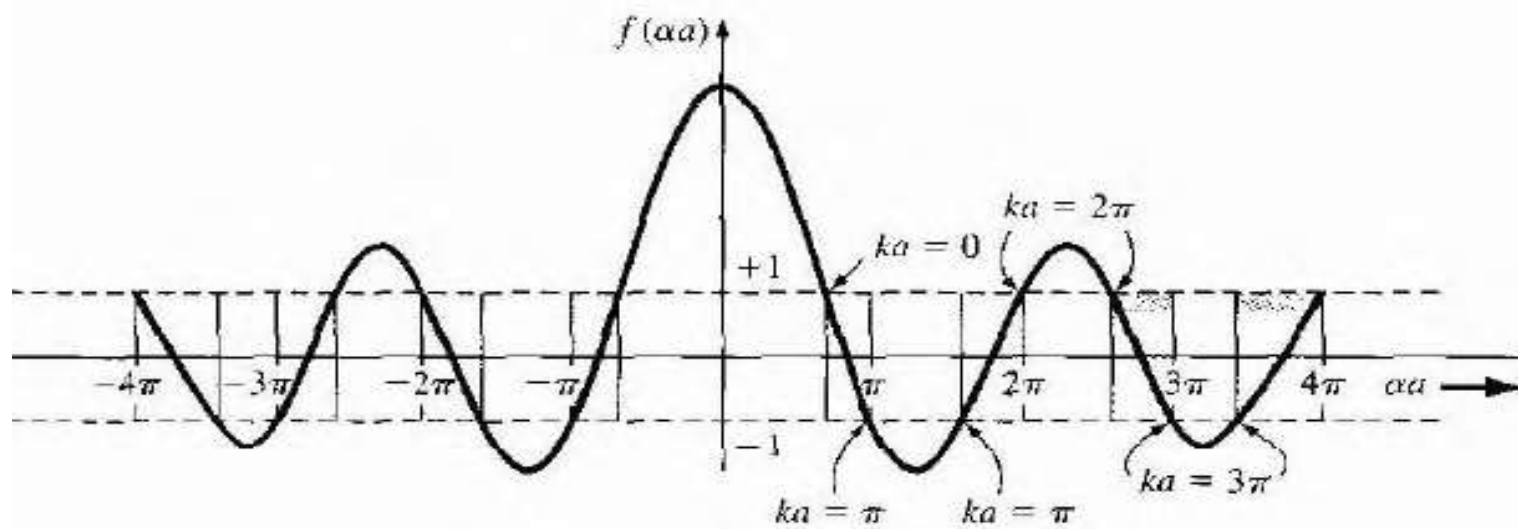
$$P\frac{sin\alpha a}{\alpha a} + cos\alpha a = cosKa$$

where

$$P = \frac{mV_o ab}{\hbar^2}$$

(a)

(b)

$ka = 0$

$ka = 2\pi$

$ka = \pi$ $ka = \pi$

$ka = 3\pi$

E vs K diagram: Allowed and Forbidden bands