4.3.58

AI25BTECH11002 - Ayush Sunil Labhade

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Question: Show that the lines

$$\frac{x+3}{-3} = \frac{y-1}{1} = \frac{z-5}{5} \tag{0.1}$$

$$\frac{x+1}{-1} = \frac{y-2}{2} = \frac{z-5}{5} \tag{0.2}$$

are coplanar.

Solution:

The given lines are

$$L_1 : \frac{x+3}{-3} = \frac{y-1}{1} = \frac{z-5}{5} \tag{0.3}$$

$$L_2 : \frac{x+1}{-1} = \frac{y-2}{2} = \frac{z-5}{5} \tag{0.4}$$

In vector form:

$$\mathbf{L_1} = \begin{pmatrix} -3 \\ 1 \\ 5 \end{pmatrix} \lambda + \begin{pmatrix} -3 \\ 1 \\ 5 \end{pmatrix} \tag{0.5}$$

$$\mathbf{L_2} = \begin{pmatrix} -1 \\ 2 \\ 5 \end{pmatrix} \mu + \begin{pmatrix} -1 \\ 2 \\ 5 \end{pmatrix} \tag{0.6}$$

So the direction vectors are

$$\mathbf{m_1} = \begin{pmatrix} -3 \\ 1 \\ 5 \end{pmatrix} \quad \mathbf{m_2} \qquad\qquad = \begin{pmatrix} -1 \\ 2 \\ 5 \end{pmatrix} \tag{0.7}$$

and the vector between points on the two lines is

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} -1 \\ 2 \\ 5 \end{pmatrix} - \begin{pmatrix} -3 \\ 1 \\ 5 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} \tag{0.8}$$

For coplanarity, we check

$$rank\begin{pmatrix}\mathbf{m_1} & \mathbf{m_2} & \mathbf{B}-\mathbf{A}\end{pmatrix} \leq 2 \tag{0.9}$$

That is,

$$\mathbf{M} = \begin{pmatrix} -3 & -1 & 2 \\ 1 & 2 & 1 \\ 5 & 5 & 0 \end{pmatrix}. \tag{0.10}$$

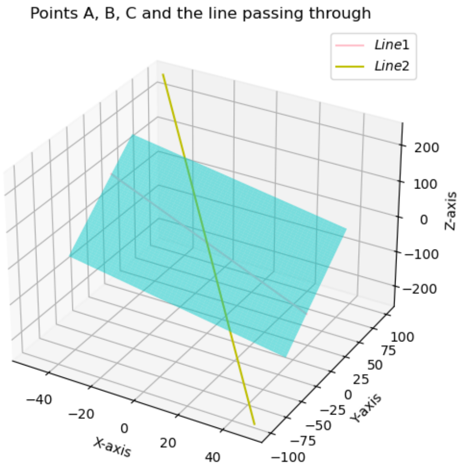
Perform row reduction:

$$\begin{pmatrix} -3 & -1 & 2 \\ 1 & 2 & 1 \\ 5 & 5 & 0 \end{pmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{pmatrix} 1 & 2 & 1 \\ -3 & -1 & 2 \\ 5 & 5 & 0 \end{pmatrix} \tag{0.11}$$

$$\xrightarrow{R_2 \rightarrow R_2 + 3R_1,\ R_3 \rightarrow R_3 - 5R_1} \begin{pmatrix} 1 & 2 & 1 \\ 0 & 5 & 5 \\ 0 & -5 & -5 \end{pmatrix} \tag{0.12}$$

$$\xrightarrow{R_3 \to R_3 + R_2} \begin{pmatrix} 1 & 2 & 1 \\ 0 & 5 & 5 \\ 0 & 0 & 0 \end{pmatrix} \tag{0.13}$$

Now the matrix is in echelon form.
The rank of the matrix is 2.
$\therefore$ the given lines are coplanar.
Graph:


Points A, B, C and the line passing through
Line1
Line2
200
100
0
−100
−200
Z-axis
−40
−20
0
20
40
X-axis
100
75
50
25
0
−25
−50
−75
−100
Y-axis

Figure