12.256

AI25BTECH11002 - Ayush Sunil Labhade

November 3, 2025

Question:

Time series **P** and **Q** are given by

$$\mathbf{P} = \{1, -1, -2, 0, 1\}, \quad \mathbf{Q} = \{1, 0, -1\}.$$

Find their linear convolution using the Toeplitz matrix method.

Solution:

Represent the given sequences as:

$$\mathbf{P} = \begin{pmatrix} 1 \\ -1 \\ -2 \\ 0 \\ 1 \end{pmatrix}, \qquad \mathbf{Q} = \begin{pmatrix} 1 & 0 & -1 \end{pmatrix} \tag{0.1}$$

The Toeplitz matrix corresponding to $\mathbf{Q}$ is:

$$\mathbf{R} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{pmatrix} \tag{0.2}$$

Now, the convolution can be expressed as:

$$\mathbf{Y} = \mathbf{RP} \tag{0.3}$$

Performing the matrix multiplication gives:

$$\mathbf{Y} = \begin{pmatrix} 1 \\ -1 \\ -3 \\ 1 \\ 3 \\ 0 \\ -1 \end{pmatrix} \tag{0.4}$$

$$\therefore \ \mathbf{P} * \mathbf{Q} = \{1, -1, -3, 1, 3, 0, -1\}$$

Hence, the correct option is (b).