

1.6.17

AI25BTECH11002 - Ayush Sunil Labhade

Question:

Using vectors, find the value of k such that the points $(k, -10, 3)$, $\mathbf{B}(1, -1, 3)$ and $(3, 5, 3)$ are collinear.

Solution: Given:

Point	Vector
a	$\begin{pmatrix} k \\ -10 \\ 3 \end{pmatrix}$
b	$\begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}$
c	$\begin{pmatrix} 3 \\ 5 \\ 3 \end{pmatrix}$

TABLE I: Given data

Since the points are collinear, we can use

$$\text{rank}(\mathbf{B} - \mathbf{A} \quad \mathbf{C} - \mathbf{B})^T = 1 \quad (1)$$

Therefore,

$$(\mathbf{B} - \mathbf{A} \quad \mathbf{C} - \mathbf{B})^T = \begin{pmatrix} 1 - k & 3 - 1 \\ -1 - (-10) & 5 - (-1) \\ 3 - 3 & 3 - 3 \end{pmatrix}_T \quad (2)$$

$$\begin{pmatrix} 1 - k & 9 & 0 \\ 2 & 6 & 0 \end{pmatrix} \xrightarrow{C_1 \leftrightarrow C_2} \begin{pmatrix} 9 & 1 - k & 0 \\ 6 & 2 & 0 \end{pmatrix} \quad (3)$$

$$\begin{pmatrix} 9 & 1 - k & 0 \\ 6 & 2 & 0 \end{pmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{pmatrix} 6 & 2 & 0 \\ 9 & 1 - k & 0 \end{pmatrix} \quad (4)$$

$$\begin{pmatrix} 6 & 2 & 0 \\ 9 & 1 - k & 0 \end{pmatrix} \xrightarrow{R_2 = R_2 - \frac{3}{2}R_1} \begin{pmatrix} 6 & 2 & 0 \\ 0 & 1 - k - 3 & 0 \end{pmatrix} \quad (5)$$

The rank of the matrix will be 1 when

$$-k - 2 = 0 \quad (6)$$

$$\Rightarrow k = -2 \quad (7)$$

Graph:

