1.6.17

AI25BTECH11002 - Ayush Sunil Labhade

Question:

Using vectors, find the value of k such that the points (k, -10, 3), $\mathbf{B}(1, -1, 3)$ and (3, 5, 3) are collinear. **Solution:** Given:

Point	Vector
a	$\begin{pmatrix} k \\ -10 \\ 3 \end{pmatrix}$
b	$\begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}$
c	$\begin{pmatrix} 3 \\ 5 \\ 3 \end{pmatrix}$

TABLE I: Given data

Since the points are collinear, we can use

$$rank \left(\mathbf{B} - \mathbf{A} \quad \mathbf{C} - \mathbf{B} \right)_{T} = 1 \tag{1}$$

Therefore,

$$(\mathbf{B} - \mathbf{A} \quad \mathbf{C} - \mathbf{B})_{T} = \begin{pmatrix} 1 - k & 3 - 1 \\ -1 - (-10) & 5 - (-1) \\ 3 - 3 & 3 - 3 \end{pmatrix}_{T}$$
 (2)

$$\begin{pmatrix} 1-k & 9 & 0 \\ 2 & 6 & 0 \end{pmatrix} \xrightarrow{C_1 \leftrightarrow C_2} \begin{pmatrix} 9 & 1-k & 0 \\ 6 & 2 & 0 \end{pmatrix}$$
 (3)

$$\begin{pmatrix} 9 & 1-k & 0 \\ 6 & 2 & 0 \end{pmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{pmatrix} 6 & 2 & 0 \\ 9 & 1-k & 0 \end{pmatrix} \tag{4}$$

$$\begin{pmatrix}
9 & 1 - k & 0 \\
6 & 2 & 0
\end{pmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{pmatrix}
6 & 2 & 0 \\
9 & 1 - k & 0
\end{pmatrix}$$

$$\begin{pmatrix}
6 & 2 & 0 \\
9 & 1 - k & 0
\end{pmatrix} \xrightarrow{R_2 = R_2 - \frac{3}{2}R_1} \begin{pmatrix}
6 & 2 & 0 \\
0 & 1 - k - 3 & 0
\end{pmatrix}$$
(5)

The rank of the matrix will be 1 when

$$-k - 2 = 0 \tag{6}$$

$$\Rightarrow k = -2 \tag{7}$$

Graph:

Points A, B, C and the line passing through

