

2.9.18

AI25BTECH11002 - Ayush Sunil Labhade

Question:

Find the volume of a cuboid whose edges are given by $-3\hat{i} + 7\hat{j} + 5\hat{k}$, $-5\hat{i} + 7\hat{j} - 3\hat{k}$ and $7\hat{i} - 5\hat{j} - 3\hat{k}$.

Solution: Given:

Point	Vector
a	$\begin{pmatrix} 3 \\ 7 \\ 5 \end{pmatrix}$
b	$\begin{pmatrix} -5 \\ 7 \\ -3 \end{pmatrix}$
c	$\begin{pmatrix} 7 \\ -5 \\ -3 \end{pmatrix}$

TABLE I: Given data

To find volume we need to compute $[a \ b \ c]$ We will compute it using Gram Matrix(**G**):

$$\mathbf{G} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} (a \ b \ c) \quad (1)$$

The Gram matrix is

$$\mathbf{G} = \begin{pmatrix} \mathbf{a}^T \mathbf{a} & \mathbf{a}^T \mathbf{b} & \mathbf{a}^T \mathbf{c} \\ \mathbf{b}^T \mathbf{a} & \mathbf{b}^T \mathbf{b} & \mathbf{b}^T \mathbf{c} \\ \mathbf{c}^T \mathbf{a} & \mathbf{c}^T \mathbf{b} & \mathbf{c}^T \mathbf{c} \end{pmatrix} \quad (2)$$

$$\mathbf{G} = \begin{pmatrix} 83 & 49 & -71 \\ 49 & 83 & -61 \\ -71 & -61 & 83 \end{pmatrix} \quad (3)$$

On computing,

$$\det(\mathbf{G}) = 69696 \quad (4)$$

The volume will be the squareroot of the $\det(\mathbf{G})$

$$volume = [a \ b \ c] = \sqrt{69696} = 264 \quad (5)$$

$$\therefore [a \ b \ c] = 264 \quad (6)$$

Graph:

