4.7.63

AI25BTECH11002 - Ayush Sunil Labhade

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Question: Find the equation of plane that contains the point $(1, -1, 2)$ and is perpendicular to each of the planes 2x+3y-2z=5 and x+2y-3z=8.

Solution:

Let the equation of plane be:

$$\mathbf{n}^T \mathbf{x} = 1 \tag{0.1}$$

Since the plane contains the point $\mathbf{A} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$,

$$\mathbf{n}^T \mathbf{A} = 1 \tag{0.2}$$

Also since the plane is perpendicular to the planes 2x+3y-2z=5 and x+2y-3z=8

$$\mathbf{n}^T \mathbf{B} = 0 \quad \mathbf{B} = \begin{pmatrix} 2 \\ 3 \\ -2 \end{pmatrix} \tag{0.3}$$

$$\mathbf{n}^T\mathbf{C} = 0 \quad \mathbf{C} = \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} \tag{0.4}$$

We can rewrite it as

$$(\mathbf{ABC})^T\mathbf{n} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \tag{0.5}$$

Forming the augmented matrix:

$$\left(\begin{array}{ccc|c} 1 & -1 & 2 & 1 \\ 2 & 3 & -2 & 0 \\ 1 & 2 & -3 & 0 \end{array} \right) \tag{0.6}$$

On row reducing we get,

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & -\frac{5}{7} \\ 0 & 1 & 0 & \frac{4}{7} \\ 0 & 0 & 1 & \frac{1}{7} \end{array} \right) \tag{0.7}$$
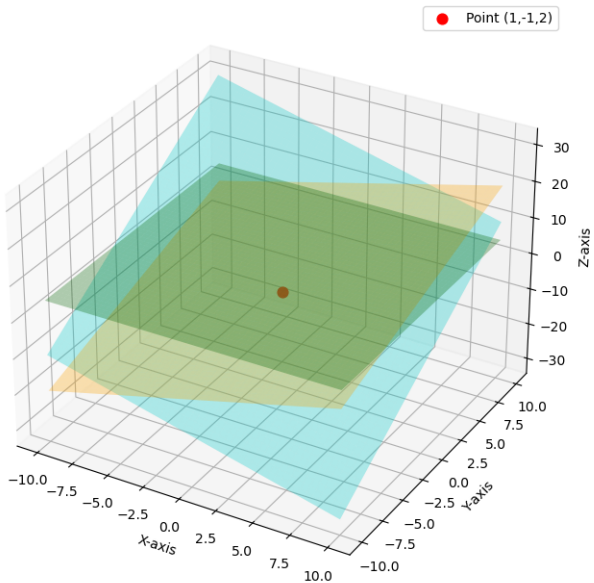
$$\mathbf{n} = \begin{pmatrix} -5 \\ 4 \\ 1 \end{pmatrix} \tag{0.8}$$

$\therefore$ the required equation is

$$\begin{pmatrix} -5 & 4 & 1 \end{pmatrix} \mathbf{x} = 7 \tag{0.9}$$

Graph:

Three Planes and a Point

Point (1,-1,2)