Signed Distances

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A line is represented by the equation $\mathbf{n}^T\mathbf{x} = c$ where n is the normal vector, x is a point on the line and c is an arbitrary constant. The normal vector is perpendicular to the line and for more intuitive understanding it can be represented as:

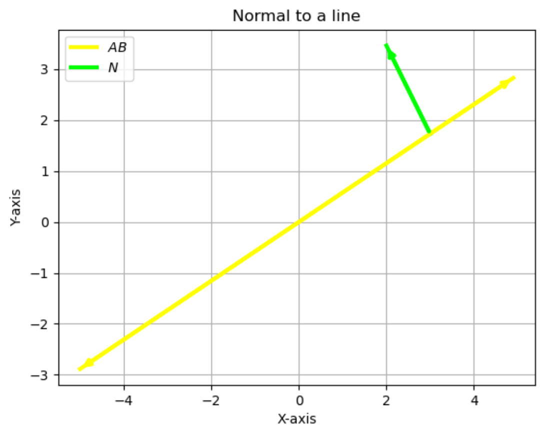

Fig. 1

The direction of the normal has only two possiblities as shown:

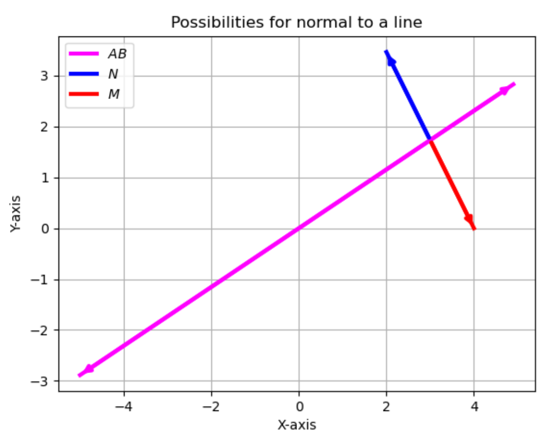

Fig. 2

The exact direction of the normal can be determined by the equation of line and substituting a known point $\left(usually\ \mathbf{x} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}\right)$ and using the concept of signed distance which is discussed further.

$$\mathbf{n}^T\mathbf{x} - c = 0 \tag{1}$$

$$\left(\mathbf{n}^T\mathbf{0} - c\right) > 0\ or\ \left(\mathbf{n}^T\mathbf{0} - c\right) < 0 \tag{2}$$

Respective normals in both the cases:

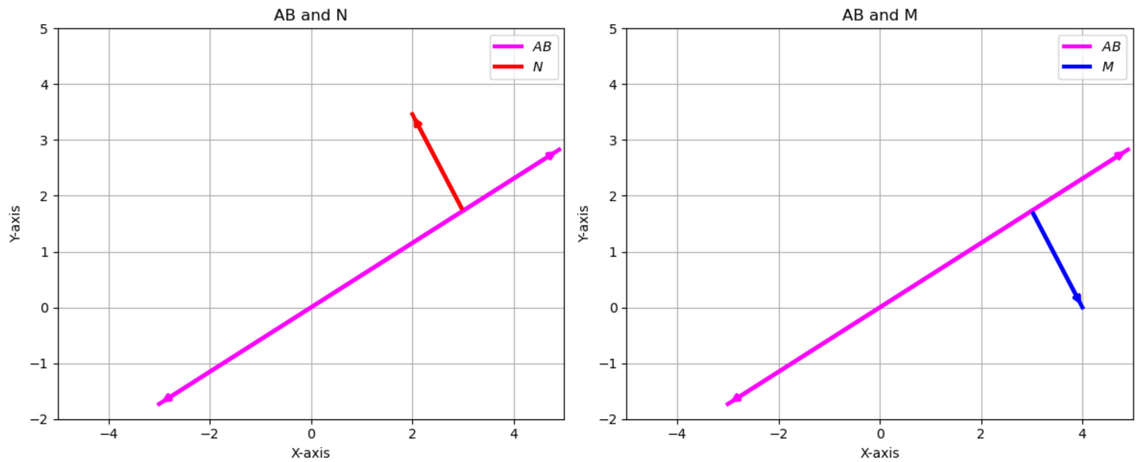

Fig. 3

Suppose one of the cases, the direction of normal would be fixed. Here the term $\mathbf{n}^T x - c$ is basically the dot product of any arbitrary $\mathbf{x}$ with the normal vector.(c is just a constant representing shifting) Here two cases arise:

1) When the point x is on the same side as normal, the dot product is positive, and hence the term

$$\left(\mathbf{n}^T\mathbf{x} - c\right) > 0$$

2) When the point x is on the opposite side of normal, the dot product is negative, and hence the term

$$\left(\mathbf{n}^T\mathbf{x} - c\right) < 0$$

Hence when two vectors $\mathbf{A}$ *and* $\mathbf{B}$ are on opposite side, the condition

$$\left(\mathbf{n}^T\mathbf{A} - c\right)\left(\mathbf{n}^T\mathbf{B} - c\right) < 0$$

is sufficient and necessary.