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AI25BTECH11002 - Ayush Sunil Labhade

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Question : Let $T : R^4 \rightarrow R^4$ be the linear map satisfying

$$T(e_1) = e_2, \quad T(e_2) = e_3, \quad T(e_3) = 0, \quad T(e_4) = e_3,$$

where $\{e_1, e_2, e_3, e_4\}$ is the standard basis of R^4 . Then determine which of the following statements are true:

- (a) T is idempotent.
- (b) T is invertible.
- (c) $\text{rank } T = 3$.
- (d) T is nilpotent.

Solution:

The matrix of T has its j th column equal to the coordinates of $T(e_j)$.

Hence $[T]_{\{e_i\}}$

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (0.1)$$

Compute T^2 :

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (0.2)$$

So T^2 is not the zero.

Now compute T^3 :

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (0.3)$$

T^3 is a null matrix

$\therefore T$ is a nilpotent matrix.