2.9.18

AI25BTECH11002 - Ayush Sunil Labhade

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Question:

Find the volume of a cuboid whose edges are given by $-3\hat{\imath} + 7\hat{\jmath} + 5\hat{k}$, $-5\hat{\imath} + 7\hat{\jmath} - 3\hat{k}$ and $7\hat{\imath} - 5\hat{\jmath} - 3\hat{k}$.

Solution: Given:

Point	Vector
a	$\begin{pmatrix} 3 \\ 7 \\ 5 \end{pmatrix}$
b	$\begin{pmatrix} -5 \\ 7 \\ -3 \end{pmatrix}$
c	$\begin{pmatrix} 7 \\ -5 \\ -3 \end{pmatrix}$

Table: Given data

To find volume we need to compute $[a\ b\ c]$ We will compute it using Gram Matrix($\mathbf{G}$):

$$\mathbf{G} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \begin{pmatrix} a & b & c \end{pmatrix} \tag{0.1}$$

The Gram matrix is

$$\mathbf{G} = \begin{pmatrix} \mathbf{a}^T\mathbf{a} & \mathbf{a}^T\mathbf{b} & \mathbf{a}^T\mathbf{c} \\ \mathbf{b}^T\mathbf{a} & \mathbf{b}^T\mathbf{b} & \mathbf{b}^T\mathbf{c} \\ \mathbf{c}^T\mathbf{a} & \mathbf{c}^T\mathbf{b} & \mathbf{c}^T\mathbf{c} \end{pmatrix} \tag{0.2}$$

$$\mathbf{G} = \begin{pmatrix} 83 & 49 & -71 \\ 49 & 83 & -61 \\ -71 & -61 & 83 \end{pmatrix} \tag{0.3}$$

On computing,

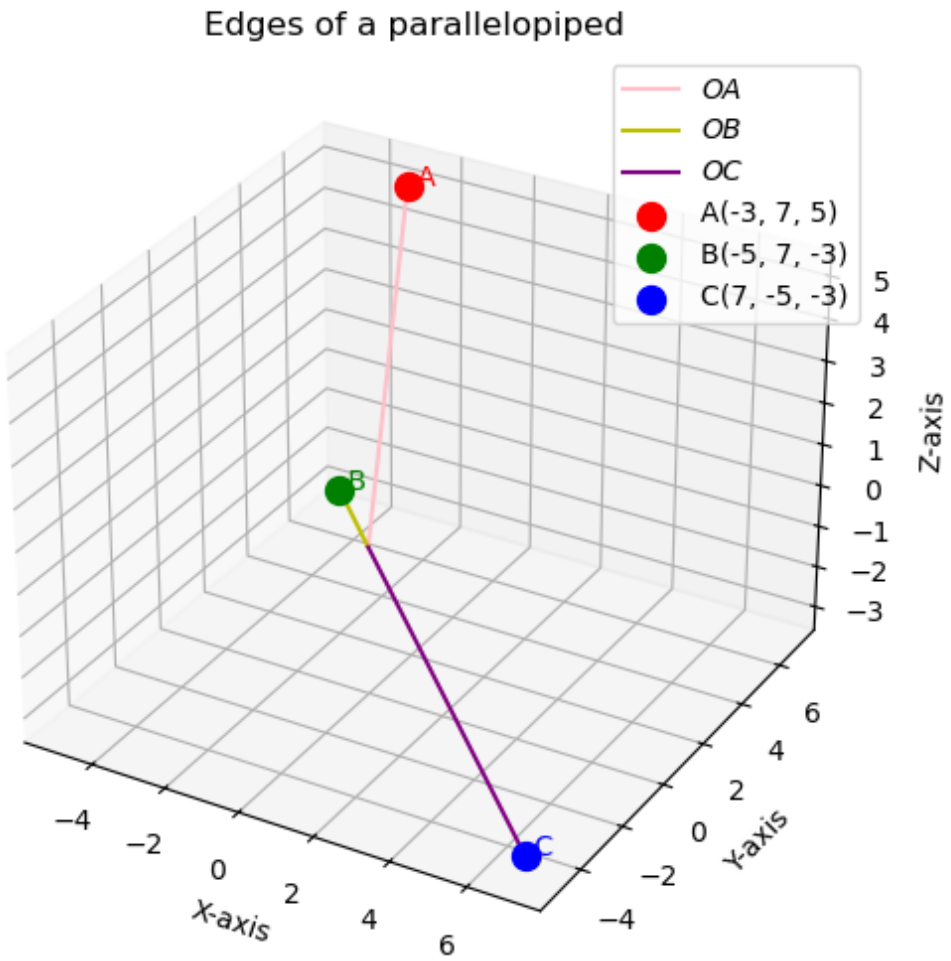
$$det(\mathbf{G}) = 69696 \tag{0.4}$$

The volume will be the squareroot of the det($\mathbf{G}$)

$$volume = [a\ b\ c] = \sqrt{69696} = 264 \tag{0.5}$$

$$\therefore [a\ b\ c] = 264 \tag{0.6}$$

Graph:


Edges of a parallelopiped
OA
OB
OC
A(-3, 7, 5)
B(-5, 7, -3)
C(7, -5, -3)
A
B
C
X-axis
Y-axis
Z-axis
−4
−2
0
2
4
6
5
4
3
2
1
0
−1
−2
−3