9.2.35

AI25BTECH11002 - Ayush Sunil Labhade

Question : Sketch the region $(x, 0)$: y = $\sqrt{4 - x^2}$ and x-axis. Find the area of the region using integration.

Solution :

Name	Value
Circle	$\mathbf{x}^\top \mathbf{x} - 4 = 0$
Line	$\mathbf{x} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \kappa \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

Table : Circle

The parameters for the circle are :

$$\mathbf{V} = \mathbf{I} \qquad \mathbf{u} = \mathbf{0} \qquad f = -4 \tag{1}$$

The parameters for the line are :

$$\mathbf{h} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \qquad \mathbf{m} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{2}$$

Substituting these in the below equation to find the intersection points :

$$\kappa_i = \frac{1}{\mathbf{m}^\top \mathbf{V} \mathbf{m}} \left(-\mathbf{m}^\top (\mathbf{V}\mathbf{h} + \mathbf{u}) \pm \sqrt{[\mathbf{m}^\top (\mathbf{V}\mathbf{h} + \mathbf{u})]^2 - g(\mathbf{h}) (\mathbf{m}^\top \mathbf{V} \mathbf{m})} \right) \tag{3}$$

$$g(\mathbf{x}) = \mathbf{x}^\top \mathbf{x} - 4 \tag{4}$$

$$g(\mathbf{h}) = \mathbf{h}^\top \mathbf{h} - 4 \tag{5}$$

$$\kappa_i = -\mathbf{m}^\top \mathbf{h} \pm \sqrt{4 - \mathbf{h}^\top \mathbf{h}} \tag{6}$$

$$\kappa_i = 2, -2 \tag{7}$$

Therefore the points of intersection are :

$$\mathbf{P_1} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \qquad \mathbf{P_2} = \begin{pmatrix} -2 \\ 0 \end{pmatrix} \tag{8}$$

Thus the area of the region is :

$$2 \int_0^2 \sqrt{4 - x^2}\, dx = 2\pi \tag{9}$$

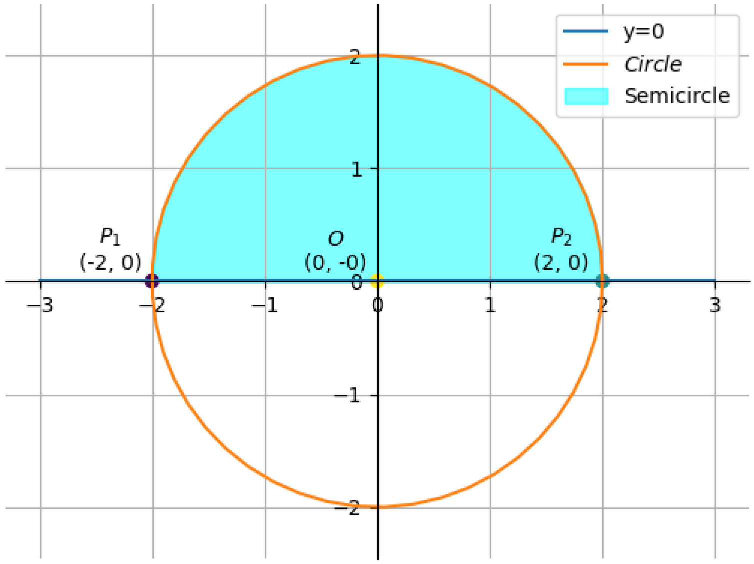

Fig : Circle