4.7.63

AI25BTECH11002 - Ayush Sunil Labhade

Question: Find the equation of plane that contains the point $(1,-1,2)$ and is perpendicular to each of the planes 2x+3y-2z=5 and x+2y-3z=8.

Solution:

Let the equation of plane be:

$$\mathbf{n}^T\mathbf{x} = 1 \tag{1}$$

Since the plane contains the point $\mathbf{A} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$,

$$\mathbf{n}^T\mathbf{A} = 1 \tag{2}$$

Also since the plane is perpendicular to the planes 2x+3y-2z=5 and x+2y-3z=8

$$\mathbf{n}^T\mathbf{B} = 0 \quad \mathbf{B} = \begin{pmatrix} 2 \\ 3 \\ -2 \end{pmatrix} \tag{3}$$

$$\mathbf{n}^T\mathbf{C} = 0 \quad \mathbf{C} = \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} \tag{4}$$

We can rewrite it as

$$(\mathbf{ABC})^T\,\mathbf{n} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \tag{5}$$

Forming the augmented matrix:

$$\left(\begin{array}{ccc|c} 1 & -1 & 2 & 1 \\ 2 & 3 & -2 & 0 \\ 1 & 2 & -3 & 0 \end{array}\right) \tag{6}$$

On row reducing we get,

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & -\frac{5}{7} \\ 0 & 1 & 0 & \frac{4}{7} \\ 0 & 0 & 1 & \frac{1}{7} \end{array}\right) \tag{7}$$

$$\mathbf{n} = \begin{pmatrix} -5 \\ 4 \\ 1 \end{pmatrix} \tag{8}$$

$\therefore$ the required equation is

$$\begin{pmatrix} -5 & 4 & 1 \end{pmatrix}\mathbf{x} = 7 \tag{9}$$

Graph:

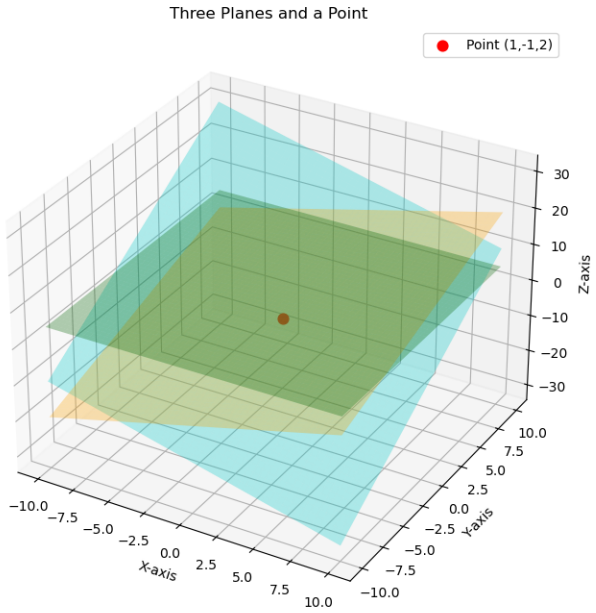
Three Planes and a Point
Point (1,-1,2)
X-axis
Y-axis
Z-axis
−10.0
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0
30
20
10
0
−10
−20
−30

Fig. 1