9.2.35

AI25BTECH11002 - Ayush Sunil Labhade

Question : A circle S passes through the point (0, 1) and is orthogonal to the circle $(x-1)^2+y^2=16$ and $x^2+y^2=1$.

Solution :

The general equation of the circle is

$$x^2+y^2+dx+ey+f=0 \tag{1}$$

The condition for passing through $(0,1)$ is

$$e+f=-1 \tag{2}$$

The orthogonality condition is d_1 d + e_1 e = 2 $(f+f_1)$

For the first circle, d_1 = -2, e_1 = 0, f_1 = -15, yielding

$$-2d=2\,(f-15) \tag{3}$$

For the second circle, d_1 = 0, e_1 = 0, f_1 = -1, yielding

$$0=2\,(f-1) \tag{4}$$

The system of equations is

$$\left(\begin{array}{ccc|c} 0 & 1 & 1 & -1 \\ -2 & 0 & -2 & -30 \\ 0 & 0 & -2 & -2 \end{array}\right) \tag{5}$$

Converting in its RREF, we get,

$$\left(\begin{array}{ccc|c} 0 & 1 & 1 & -1 \\ -2 & 0 & -2 & -30 \\ 0 & 0 & -2 & -2 \end{array}\right) \tag{6}$$

Using RREF or solving, f = 1, d = 14, e = -2.

Thus, the equation is

$$x^2+y^2+14x-2y+1=0 \tag{7}$$

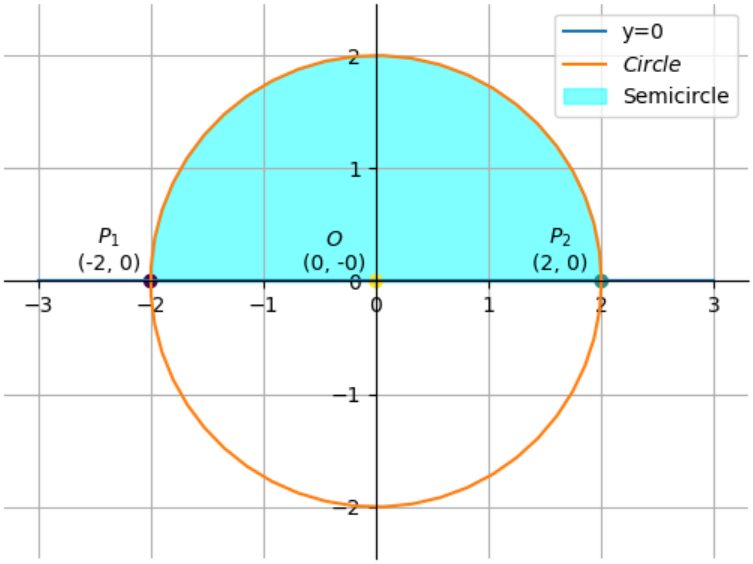

Fig : Circle