5.5.19

AI25BTECH11002 - Ayush Sunil Labhade

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Question: Using elementary row transformations, find the inverse of the matrix

$$\begin{pmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{pmatrix}$$

Solution:

Let $\mathbf{A} = \begin{pmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{pmatrix}$

Augment the matrix $\mathbf{A}$ with the identity

$$[\mathbf{A} \,|\, \mathbf{I}] = \left(\begin{array}{ccc|ccc} 2 & -3 & 5 & 1 & 0 & 0 \\ 3 & 2 & -4 & 0 & 1 & 0 \\ 1 & 1 & -2 & 0 & 0 & 1 \end{array}\right) \tag{0.1}$$

Row Transformation-1: $R_1 \rightarrow \frac{R_1}{2}$

$$\left(\begin{array}{ccc|ccc} 1 & -\frac{3}{2} & \frac{5}{2} & \frac{1}{2} & 0 & 0 \\ 3 & 2 & -4 & 0 & 1 & 0 \\ 1 & 1 & -2 & 0 & 0 & 1 \end{array}\right) \tag{0.2}$$

Row Transformation-2: $R_2 \to R_2 - 3R_1$

$$\left(\begin{array}{ccc|ccc} 1 & -\frac{3}{2} & \frac{5}{2} & \frac{1}{2} & 0 & 0 \\ 0 & \frac{11}{2} & -\frac{23}{2} & -\frac{3}{2} & 1 & 0 \\ 1 & 1 & -2 & 0 & 0 & 1 \end{array}\right) \tag{0.3}$$

Row Transformation-3: $R_3 \to R_3 - R_1$

$$\left(\begin{array}{ccc|ccc} 1 & -\frac{3}{2} & \frac{5}{2} & \frac{1}{2} & 0 & 0 \\ 0 & \frac{11}{2} & -\frac{23}{2} & -\frac{3}{2} & 1 & 0 \\ 0 & \frac{5}{2} & -\frac{9}{2} & -\frac{1}{2} & 0 & 1 \end{array}\right) \tag{0.4}$$

Row Transformation-4: $R_2 \to \frac{R_2}{\frac{11}{2}}$ (i.e., $R_2 \to \frac{2}{11}R_2$)

$$\left(\begin{array}{ccc|ccc} 1 & -\frac{3}{2} & \frac{5}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 1 & -\frac{23}{11} & -\frac{3}{11} & \frac{2}{11} & 0 \\ 0 & \frac{5}{2} & -\frac{9}{2} & -\frac{1}{2} & 0 & 1 \end{array}\right) \tag{0.5}$$

Row Transformation-5: $R_3 \to R_3 - \frac{5}{2}R_2$

$$\left(\begin{array}{ccc|ccc} 1 & -\frac{3}{2} & \frac{5}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 1 & -\frac{23}{11} & -\frac{3}{11} & \frac{2}{11} & 0 \\ 0 & 0 & \frac{11}{22} & \frac{1}{22} & -\frac{5}{11} & 1 \end{array}\right) \tag{0.6}$$

Row Transformation-6: $R_3 \to 22R_3$ (to clear the fraction)

$$\left(\begin{array}{ccc|ccc} 1 & -\frac{3}{2} & \frac{5}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 1 & -\frac{23}{11} & -\frac{3}{11} & \frac{2}{11} & 0 \\ 0 & 0 & 1 & \frac{1}{11} & -\frac{10}{11} & 22 \end{array}\right) \tag{0.7}$$

Row Transformation-7: $R_2 \to R_2 + \frac{23}{11}R_3$

$$\left(\begin{array}{ccc|ccc} 1 & -\frac{3}{2} & \frac{5}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 & \frac{4}{11} & -\frac{33}{121} & \frac{22}{11} \\ 0 & 0 & 1 & \frac{1}{11} & -\frac{10}{11} & 22 \end{array}\right) \tag{0.8}$$

Row Transformation-8: $R_1 \to R_1 + \frac{3}{2}R_2 - \frac{5}{2}R_3$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{7}{11} & -\frac{35}{121} & \frac{33}{11} \\ 0 & 1 & 0 & \frac{4}{11} & -\frac{33}{121} & \frac{22}{11} \\ 0 & 0 & 1 & \frac{1}{11} & -\frac{10}{11} & 22 \end{array}\right) \tag{0.9}$$

The Inverse Matrix of **A**:

$$\mathbf{A}^{-1} = \begin{pmatrix} \dfrac{7}{11} & -\dfrac{35}{121} & \dfrac{33}{11} \\ \dfrac{4}{11} & -\dfrac{33}{121} & \dfrac{22}{11} \\ \dfrac{1}{11} & -\dfrac{10}{11} & 22 \end{pmatrix} \tag{0.10}$$