4.3.58

AI25BTECH11002 - Ayush Sunil Labhade

Question: Show that the lines

$$\frac{x+3}{-3} = \frac{y-1}{1} = \frac{z-5}{5} \tag{1}$$

$$\frac{x+1}{-1} = \frac{y-2}{2} = \frac{z-5}{5} \tag{2}$$

are coplanar.

Solution:

The given lines are

$$L_1 : \frac{x+3}{-3} = \frac{y-1}{1} = \frac{z-5}{5} \tag{3}$$

$$L_2 : \frac{x+1}{-1} = \frac{y-2}{2} = \frac{z-5}{5} \tag{4}$$

In vector form:

$$\mathbf{L_1} = \begin{pmatrix} -3 \\ 1 \\ 5 \end{pmatrix} \lambda + \begin{pmatrix} -3 \\ 1 \\ 5 \end{pmatrix} \tag{5}$$

$$\mathbf{L_2} = \begin{pmatrix} -1 \\ 2 \\ 5 \end{pmatrix} \mu + \begin{pmatrix} -1 \\ 2 \\ 5 \end{pmatrix} \tag{6}$$

So the direction vectors are

$$\mathbf{m_1} = \begin{pmatrix} -3 \\ 1 \\ 5 \end{pmatrix} \quad \mathbf{m_2} = \begin{pmatrix} -1 \\ 2 \\ 5 \end{pmatrix} \tag{7}$$

and the vector between points on the two lines is

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} -1 \\ 2 \\ 5 \end{pmatrix} - \begin{pmatrix} -3 \\ 1 \\ 5 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} \tag{8}$$

For coplanarity, we check

$$\text{rank} \begin{pmatrix} \mathbf{m_1} & \mathbf{m_2} & \mathbf{B} - \mathbf{A} \end{pmatrix} \leq 2 \tag{9}$$

That is,

$$\mathbf{M} = \begin{pmatrix} -3 & -1 & 2 \\ 1 & 2 & 1 \\ 5 & 5 & 0 \end{pmatrix}. \tag{10}$$

Perform row reduction:

$$\begin{pmatrix} -3 & -1 & 2 \\ 1 & 2 & 1 \\ 5 & 5 & 0 \end{pmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{pmatrix} 1 & 2 & 1 \\ -3 & -1 & 2 \\ 5 & 5 & 0 \end{pmatrix} \tag{11}$$

$$\xrightarrow{R_2 \to R_2 + 3R_1,\, R_3 \to R_3 - 5R_1} \begin{pmatrix} 1 & 2 & 1 \\ 0 & 5 & 5 \\ 0 & -5 & -5 \end{pmatrix} \tag{12}$$

$$\xrightarrow{R_3 \to R_3 + R_2} \begin{pmatrix} 1 & 2 & 1 \\ 0 & 5 & 5 \\ 0 & 0 & 0 \end{pmatrix} \tag{13}$$

Now the matrix is in echelon form.
The rank of the matrix is 2.
$\therefore$ the given lines are coplanar.
Graph:

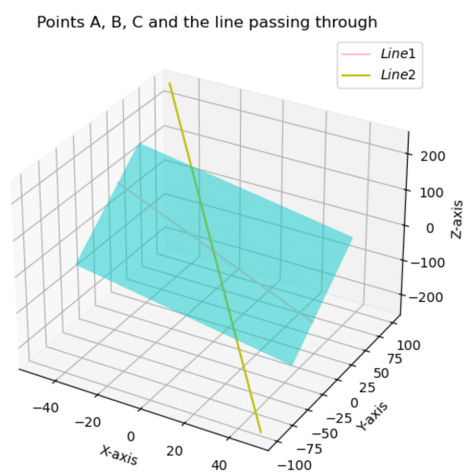

Fig. 1