Answer:

1. linear, partial, 2nd order
2. nonlinear, ordinary, 3rd order
3. linear, ordinary, 4th order
4. nonlinear, partial, 3rd order

Problem 1 [4pts] Classify the following equations as linear or nonlinear, ordinary or partial, and give the order.

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

$$\frac{d^3 y}{dx^3} + 17\, y\, \frac{dy}{dx} - 11 = 0$$

$$e^t \frac{d^4 y}{dt^4} + t^6 \frac{dy}{dt} - t^{27} y = \cos(t)$$

$$u_t + u_{xxx} + u u_x = 0$$

Answer:

y = Ce^-2t + C'e^-t

y = Ce^4t + C'e^t

y = Ce^-2t + C'e^2t

y = Ce^-t + C'e^4t

Problem 2 [3pts] For each of the following equations find a solution of the indicated form

$$\frac{d^2 y}{dt^2} + 3\frac{dy}{dt} + 2y = 0 \qquad\qquad y = e^{rt}$$

$$\frac{d^2 y}{dt^2} - 5\frac{dy}{dt} + 4y = 0 \qquad\qquad y = e^{rt}$$

$$t^2 \frac{d^2 y}{dt^2} + t\frac{dy}{dt} - 4y = 0 \qquad\qquad y = t^r$$

$$t^2 \frac{d^2 y}{dt^2} + 4t\frac{dy}{dt} - 4y = 0 \qquad\qquad y = t^r$$

Answer:

y(t)=sec(t)
y'(t)=sec(t)tan(t)
y''(t)= sec(t)tan^2(t) + sec^3(t)

2y^3 - y = 2sec^3(t) - sec(t)

y'' = 2y^3 - y
sec(t)tan^2(t) + sec^3(t)= 2sec^3(t) - sec(t)

sin^2(t)/cos^3(t) + 1/cos^3(t) =
2/cos^3(t) - 1/cos(t)

(sin^2(t) + 1)/cos^3(t) = (2 - cos^2(t))/cos^3(t)

(1 - cos^2(t) + 1)/cos^3(t) = (2 - cos^2(t))/
cos^3(t)

(2 - cos^2(t))/cos^3(t) = (2 - cos^2(t))/cos^3(t)

Problem 3 [3pts]

Verify that the function $y(t) = \frac{1}{\cos(t)}$ is a solution to the differential equation

$$\frac{d^2 y}{dt^2} = 2y^3 - y$$

IMPORTANT PLEASE READ: Your homework should be uploaded to the course Canvas space. Please make sure that your upload is no more than a couple of meg at most or it will crash the grading program and you will not get credit for the homework.

Please make sure that the first page of your upload is this sheet, with the answers filled in boxes above. You may attach as many additional sheets as is necessary to show your work.

Failure to follow these instructions will result in your not receiving credit for this assignment.

Problem 2 Working:

r^2e^rt + 3re^rt + 2e^rt = 0
(e^rt)(r^2+3r+2)=0
(e^rt)(r+2)(r+1)=0
r= -2 & -1
y = Ce^-2t + C'e^-t

r^2e^rt - 5re^rt + 4e^rt = 0
(e^rt)(r^2-5r+4)=0
(e^rt)(r-4)(r-1)=0
r= 4 & 1
y = Ce^4t + C'e^t

(t^2)(r(r-1)t^(r-2)) + t(rt^(r-1)) - 4t^r = 0
(r(r-1)t^(r)) + (rt^(r)) - 4t^r = 0
(r^2)(t^r) - 4t^r = 0
(t^r)(r^2 - 4) = 0, r = -2 & 2
y = Ce^-2t + C'e^2t

(t^2)(r(r-1)t^(r-2)) + 4t(rt^(r-1)) - 4t^r = 0
(r(r-1)t^(r)) + (4rt^(r)) - 4t^r = 0
(r^2)(t^r)+ 3rt^(r) - 4t^r = 0
(t^r)(r^2 - 3r - 4) = 0
(t^r)(r-4)(r+1) = 0
r = -1 & 4
y = Ce^-t + C'e^4t

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