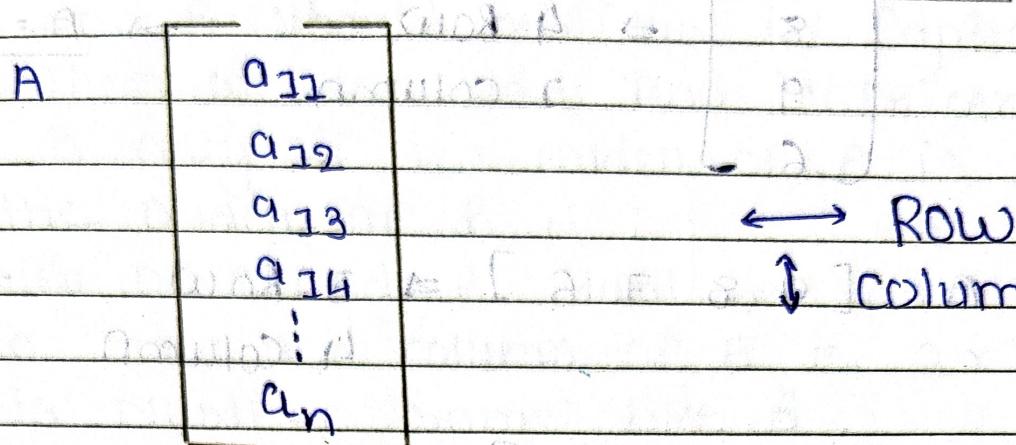


UNIT : I

- * Basic ideas about Matrices :-



→ I is Number of Row then Multiply by Number of column like 3×3

- * Examples :-

- * find out the order of following matrices

1) $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \Rightarrow A = [2 \text{ Row}] \Rightarrow A = 2 \times 3 \text{ column}$

2) $A = \begin{bmatrix} 1 & 2 \\ 4 & 5 \\ 6 & 7 \end{bmatrix} \Rightarrow 3 \text{ Row} \times 2 \text{ Column} \Rightarrow A = 3 \times 2$

3) $A = \begin{bmatrix} 1 & 9 & 4 \\ 7 & 8 & 8 \\ 5 & 6 & 0 \end{bmatrix} \Rightarrow 3 \text{ Row} \times 3 \text{ Column} \Rightarrow A = 3 \times 3$

4) $A = \begin{bmatrix} -1 & 4 & 5 \\ 6 & 8 & -7 \end{bmatrix} \Rightarrow 2 \text{ Row} \quad \Rightarrow A = 2 \times 3$
 3 column

5) $A = \begin{bmatrix} -1 \\ 8 \\ 9 \\ 6 \end{bmatrix} \Rightarrow 4 \text{ Row} \quad \Rightarrow A = 4 \times 1$
 1 column

6) $A = [9 8 7 6] \Rightarrow 1 \text{ Row} \quad \Rightarrow A = 1 \times 4$
 4 column

7) $A = [0 0 0 0 0] \Rightarrow 1 \text{ Row} \quad \Rightarrow A = 1 \times 5$

प्रतिक्रिया करने की विधि का समानांग है।

8) $A = \begin{bmatrix} 9 & 4 & 7 & 1 & 0 \\ 8 & 6 & 8 & 0 & 0 \end{bmatrix} \Rightarrow 2 \text{ Row} \quad \Rightarrow A = 2 \times 5$

* Addition of Two Matrices

2) $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 0 & 0 \\ 8 & 9 & 2 \end{bmatrix} = A + B$

$A + B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}_{2 \times 3} + \begin{bmatrix} 0 & 0 & 0 \\ 8 & 9 & 2 \end{bmatrix}_{2 \times 3} = \begin{bmatrix} 1 & 2 & 3 \\ 12 & 14 & 8 \end{bmatrix}_{2 \times 3}$

$B = \begin{bmatrix} 1 & 2 & 3 \\ 12 & 14 & 8 \end{bmatrix}_{2 \times 3}$

Conditions :-

- 1) Suppose we have Two Matrices Namely A and B
 (A and B should be Must in Capital)
 and Then we can add Two Matrices A and B only if the Order of A is same as the Order of B .
 → Order of A and B Must be same.
 like row and column of A is 2×3 so B is Must be same like A .

Find out the addition of Matrices :-

$$1) A = \begin{bmatrix} 1 \\ 9 \\ 8 \end{bmatrix}$$

$$B = \begin{bmatrix} 7 & 8 & 6 \\ 1 & 0 & 5 \end{bmatrix}$$

→ Matrices is Not possible because of order A and B is not same that's why Matrices of Addition is not possible.

$$2) A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 0 & 8 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 9 & 8 & 1 \\ 9 & 5 & 0 \\ -1 & -1 & 2 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 0 & 8 & 0 \end{bmatrix}_{3 \times 3} + \begin{bmatrix} 9 & 8 & 1 \\ 4 & 5 & 0 \\ -1 & -1 & 2 \end{bmatrix}_{3 \times 3}$$

$$A + B = \begin{bmatrix} 1 & 4 & 8 \\ 9 & 5 & 7 \end{bmatrix} + \begin{bmatrix} 7 & 8 & 6 \\ 4 & 0 & 0 \end{bmatrix}$$

2x3

$$= \begin{bmatrix} 8 & 12 & 14 \\ 13 & 5 & 7 \end{bmatrix}$$

2x3

4) $A = \begin{bmatrix} 9 \\ 8 \\ 7 \end{bmatrix}$ $B = \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix}$

$$C = A + B = \begin{bmatrix} 8 \\ 9 \\ 7 \end{bmatrix}$$

3x1

5) $A = \begin{bmatrix} 8 \\ 0 \\ 4 \end{bmatrix}$ $B = \begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix}$

$$A + B = \begin{bmatrix} 8 \\ 0 \\ 4 \end{bmatrix}_{3 \times 1} + \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix}_{3 \times 1} = \begin{bmatrix} 7 \\ 2 \\ 4 \end{bmatrix}_{3 \times 1}$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 8 \\ 8 & 0 & 0 \end{bmatrix}_{3 \times 3} \begin{bmatrix} 1 & 8 & 9 \\ 0 & 2 & 1 \\ 2 & 2 & 2 \end{bmatrix}_{3 \times 3} = \begin{bmatrix} 7 \\ 1 \\ 12 \end{bmatrix}_{3 \times 1}$$

$$6) A = \begin{bmatrix} -1 & -1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 7 & 8 & 0 \end{bmatrix}$$

$$A + B = \begin{bmatrix} -1 & -1 & 0 \end{bmatrix} + \begin{bmatrix} 7 & 8 & 0 \end{bmatrix} = \begin{bmatrix} 6 & 7 & 0 \end{bmatrix}_{1 \times 3}$$

$$7) A = \begin{bmatrix} 7 & 4 \\ 8 & 1 \\ 0 & 8 \end{bmatrix}_{3 \times 2} \quad B = \begin{bmatrix} 8 & 9 \\ 0 & 0 \\ 4 & 6 \end{bmatrix}_{3 \times 2}$$

$$A + B = \begin{bmatrix} 7 & 4 \\ 8 & 1 \\ 0 & 8 \end{bmatrix} + \begin{bmatrix} 8 & 9 \\ 0 & 0 \\ 4 & 6 \end{bmatrix}$$

Two matrices need same $n \times n$ to multiply and $n \times 1$ to add

$$\text{Matrix } A = \begin{bmatrix} 15 & 20 & 13 \end{bmatrix}_{3 \times 1} \text{ is a column vector}$$

$$\text{Matrix } B = \begin{bmatrix} 8 & 5 & 11 \end{bmatrix}_{3 \times 1} \text{ is a column vector}$$

$$\text{Matrix } C = \begin{bmatrix} 4 & 14 \end{bmatrix}_{2 \times 2} \text{ is a } 2 \times 2 \text{ matrix}$$

Two matrices can't multiply if one is $m \times n$ and other is $p \times q$

$$8) A = \begin{bmatrix} 1 & 9 & 8 \\ 4 & 5 & 6 \\ 0 & 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 9 & 8 & 0 \\ 0 & 0 & 4 \\ 5 & 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 8 & 1 \\ 4 & 6 & 7 \\ 5 & 6 & 6 \end{bmatrix} \quad D = \begin{bmatrix} 0 & 0 & 0 \\ 8 & 1 & 0 \\ 0 & 0 & -8 \end{bmatrix}$$

$$= A + B + C + D =$$

$$\begin{bmatrix} 1 & 9 & 8 \\ 4 & 5 & 6 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 9 & 8 & 0 \\ 0 & 0 & 4 \\ 5 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 8 & 1 \\ 4 & 6 & 7 \\ 5 & 6 & 6 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 8 & 1 & 0 \\ 0 & 0 & -8 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 25 & 9 \\ 16 & 12 & 17 \\ 10 & 6 & -9 \end{bmatrix} \quad 3 \times 3$$

Definition of Matrices :-

→ The system of $M \times N$ numbers arranged with the form of an order set of m defined line horizontal line called rows, and N vertical line called columns is called an $m \times n$ matrices this order is called matrices.

$$\begin{matrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ \vdots & \vdots & \vdots & & \vdots \end{matrix}$$

$$a_{m1} \ a_{m2} \ a_{m3} \ \dots \ a_{mn}$$

Rules :-

Matrices are generally denoted by capital letters.

Elements are generally denoted by corresponding small letters.

Order of Matrices :

We say a Matrix A is of order $m \times n$ if the matrix A has "m" number of rows and "n" numbers of columns,

Example :-

$$A = \begin{bmatrix} 1 & 8 & 9 \\ 7 & 6 & 4 \end{bmatrix}, \text{ order of } A = \underline{2 \times 3}$$

$$A = \begin{bmatrix} 1 & 8 \\ 8 & 9 \\ 0 \end{bmatrix}, \text{ order of } A = \underline{3 \times 1}$$

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}, \text{ order of } A = \underline{1 \times 4}$$

* Types of Matrices :-

ii) Rectangular Matrix :-

- Any $m \times n$ matrix where $m \neq n$ is called a Rectangular Matrix.
- m value and n value is not same that's called A Rectangular Matrix.

For example; and A relation with

$$A = \begin{bmatrix} 1 & 8 & 9 \\ 7 & 6 & 4 \end{bmatrix} \text{ Order of } A = 2 \times 3$$

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix} \text{ Order of } A = 1 \times 4 = A$$

$$A = \begin{bmatrix} 1 & 1 & 1 & 2 \\ 1 & 4 & 2 & 5 \\ 1 & 1 & 1 & 2 \end{bmatrix} \text{ Order of } A = 3 \times 4 = A$$

$$A = \begin{bmatrix} 1 \\ 8 \\ -7 \end{bmatrix} \text{ Order of } A = 3 \times 1$$

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix} = A$$

2) Column matrices :-

→ A matrix A is said to be column matrix if the matrix A has only 1 column.

Example :

$$A = \begin{bmatrix} 1 \\ 8 \end{bmatrix} \rightarrow A = \underline{2 \times 1}$$

$$A = \begin{bmatrix} 1 \\ -1 \\ 1 \\ 2 \end{bmatrix} \rightarrow A = \underline{4 \times 1}$$

$$A = \begin{bmatrix} 1 \\ 4 \\ 9 \end{bmatrix} \rightarrow A = \underline{3 \times 1}$$

$$\begin{bmatrix} 2 & 0 & 1 \\ 2 & 0 & 1 \\ 2 & 0 & 1 \end{bmatrix} \rightarrow A = \underline{3 \times 3}$$

3) Row Matrices :-

A matrix A is said to be row matrix if the matrix A has only 1 row.

Example :

$$A = [1 \ 9 \ 4 \ 2] \rightarrow A = \underline{1 \times 4}$$

$$A = [1 \ 2 \ 3] \rightarrow A = \underline{1 \times 3}$$

$$A = [2 \ 5 \ 6 \ 7 \ 8] \rightarrow A = \underline{1 \times 5}$$

$$A = [2 \ 3] \rightarrow A = \underline{1 \times 2}$$

4) Square Matrices :-

→ we say a matrix A is of type square m if the number of rows and column both are equal is called square matrices.

→ Example :-

$$1 \times 1 = A \quad 1 \times 2 = A \leftarrow \begin{bmatrix} 1 \end{bmatrix} = A$$
$$1 \times 1 = A \quad A = \begin{bmatrix} 1 & 9 \end{bmatrix} \rightarrow A = \underline{2 \times 2} \quad \begin{bmatrix} 8 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 & 5 \end{bmatrix} \quad 1 \times 8 = A \leftarrow \begin{bmatrix} 1 \end{bmatrix} = A$$
$$A = \begin{bmatrix} 4 & 3 & 6 \end{bmatrix} \rightarrow A = \underline{3 \times 3} \quad \begin{bmatrix} 8 \end{bmatrix}$$

$$A = \begin{bmatrix} 5 & 1 \end{bmatrix} \rightarrow A = \underline{2 \times 2}$$

definition of 2x2 matrix is a collection of more than one row and one column with 2 elements.

$$A = \begin{bmatrix} 1 & 9 & 4 & 2 \\ 5 & 8 & 4 & 2 \\ 4 & 1 & 2 & 3 \\ 9 & 4 & 6 & 2 \end{bmatrix} \rightarrow A = \underline{4 \times 4}$$

5) Diagonal Matrices :-

→ It is a square Matrix in which all non diagonal elements are 0 (zero)

Example :-

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \rightarrow A = 3 \times 3$$

jumlahan 1+0+0+3 di sekitar diagonal A

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \rightarrow A = 2 \times 2$$

jumlahan 1+0+0+0 di sekitar diagonal A

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \rightarrow A = 3 \times 3$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \rightarrow A = 3 \times 3$$

6) Zero Null Matrix :-

→ A matrix is said to be zero matrix if every elements of matrix is zero.

→ Example: $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$$\text{1) } \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \text{2) } \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \text{3) } \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$4) \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad 5) \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad 6) \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

7) Scalar Matrix:

- A Diagonal matrix in which all diagonal elements all are equal it's call a scalar matrix.
- In a scalar Matrix Diagonal values are equal (same).
- Example :

1) $\begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ 2) $\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$ 3) $\begin{bmatrix} -2 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & -2 \end{bmatrix}$

4) $\begin{bmatrix} 7 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 7 \end{bmatrix}$ 5) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

8) Identity or Unit Matrix:

- A scalar matrix , in which all diagonal elements are equal to 1 it's called Identity or Units Matrix.
- Example : $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

1) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ 2) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ 3) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

* Transpose of Matrix :-

→ The Matrix of $m \times n$ from any given matrix A by changing it's row into its corresponding column it's called the transpose Matrix. It's is denoted by (A') . If (A) is an $m \times n$ matrix, then (A') will be an $n \times m$ matrix.

→ In simple way changing Row into Column and column into row.

→ Here's an example :-

$$\text{Ex: } 1) A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}_{2 \times 3} \Rightarrow A' = \begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix}_{3 \times 2}$$

$$\text{Ex: } 2) A = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 0 \end{pmatrix}_{4 \times 1} \Rightarrow A' = \begin{pmatrix} 1 & 2 & 3 & 0 \end{pmatrix}_{1 \times 4}$$

$$3) A = \begin{pmatrix} 1 & 5 & 4 \\ 3 & 2 & 9 \\ 2 & 5 & 6 \\ 8 & 6 & 7 \end{pmatrix}_{4 \times 3} \Rightarrow A' = \begin{pmatrix} 1 & 3 & 2 & 8 \\ 5 & 2 & 5 & 6 \\ 4 & 9 & 6 & 7 \end{pmatrix}_{3 \times 4}$$

* Find the transpose cost following matrices.

$$\text{Ex: } A = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}_{3 \times 3} \Rightarrow A' = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}_{3 \times 3}$$

Example :-

$$\text{1) } A = [1 \ 8 \ 9 \ 4] \Rightarrow A' = \begin{bmatrix} 1 \\ 8 \\ 9 \\ 4 \end{bmatrix}$$

Observe that A is a 1×4 matrix and A' is a 4×1 matrix.

Similarly, if A is a $m \times n$ matrix, then A' is a $n \times m$ matrix.

Ex. 2) $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$

$$\Rightarrow A' = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$$

$$4) \quad A = \begin{bmatrix} 9 & 8 \\ 7 & 6 \\ 3 & 4 \end{bmatrix} \Rightarrow A' = \begin{bmatrix} 9 & 7 & 3 \\ 8 & 6 & 4 \end{bmatrix}$$

$$5) \quad A = [1 \ 1 \ -1 \ 8 \ 4 \ 9] \Rightarrow A' = \begin{bmatrix} 1 \\ -1 \\ 8 \\ 4 \\ 9 \end{bmatrix}$$

$$6) \quad A = [0 \ 0 \ 0 \ 0] \Rightarrow A' = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

7) $A = \begin{bmatrix} 1 & -18 & 0 & 4 \end{bmatrix} \Rightarrow A' = \begin{bmatrix} 1 & 18 & 0 & 4 \end{bmatrix}$
 condition 2×4 more to reduce -18
 transpose to convert the matrix $\begin{bmatrix} 1 & 18 & 0 & 4 \end{bmatrix} 4 \times 1$

8) transpose a row matrix into column matrix

8) $A = \begin{bmatrix} 9 \\ 8 \\ 0 \end{bmatrix} \Rightarrow A' = \begin{bmatrix} 9 & 8 & 0 \end{bmatrix} 1 \times 3$

9) $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \Rightarrow A' = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$
 3×3 3×3 3×3

10) $A = \begin{bmatrix} 7 & 8 & 4 \\ 5 & 9 & 8 \end{bmatrix} \Rightarrow A' = \begin{bmatrix} 7 & 5 \\ 8 & 9 \end{bmatrix}$
 2×3 2×3 3×2

* Matrix Operations :-

7) Addition of Two Matrices :-

Q1

8) Subtraction :-

Q1

→ The addition or subtraction of two matrices is possible only when these matrices have the same order.

→ That is Matrices must have the same number of rows & columns.

→ The sum or difference of Matrices is done by adding or subtracting element of the given matrices.

$$\begin{bmatrix} 0 & 1 & 8 & 10 \end{bmatrix} = A \quad \begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix} = B$$

Example of Addition :-

$$\begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix} = A + B$$

1) $A = \begin{bmatrix} 1 & 2 \\ 3 & 9 \end{bmatrix}_{2 \times 2} \quad \& \quad B = \begin{bmatrix} 9 & 8 \\ 7 & 4 \end{bmatrix}_{2 \times 2}$

Ans : $A + B = \begin{bmatrix} 10 & 10 \\ 10 & 13 \end{bmatrix}_{2 \times 2}$

2) $A = \begin{bmatrix} 9 & 8 & 7 & 10 \end{bmatrix}_{1 \times 4} \quad \& \quad B = \begin{bmatrix} 7 & 8 & 9 & 4 \end{bmatrix}_{1 \times 4}$

$A + B = [9+8+7+4] + [7+8+9+4]$

Ans = $A + B = \begin{bmatrix} 16 & 16 & 16 & 8 \end{bmatrix}_{1 \times 4}$

3) $A = \begin{bmatrix} 9 & 8 \end{bmatrix}_{1 \times 2} \quad \& \quad B = \begin{bmatrix} -1 & -1 \end{bmatrix}_{1 \times 2}$

$A + B = [9+8] + [-1-1]$

Ans = $A + B = [18+7+0]_{1 \times 2}$

Ans = $A + B = [18+7+0]_{1 \times 2}$

Example of Subtraction.

Ex) $A = \begin{bmatrix} 1 & 2 \\ 3 & 9 \end{bmatrix}$ & $B = \begin{bmatrix} 9 & 8 \\ 7 & 4 \end{bmatrix}$

$$A - B = \begin{bmatrix} 1 & 2 \\ 3 & 9 \end{bmatrix} - \begin{bmatrix} 9 & 8 \\ 7 & 4 \end{bmatrix}$$

Ans: $A - B = \begin{bmatrix} -8 & -6 \\ -4 & 5 \end{bmatrix}$

Ex) $A = \begin{bmatrix} 9 & 8 & 7 & 4 \end{bmatrix}$ & $B = \begin{bmatrix} 7 & 8 & 9 & 4 \end{bmatrix}$

$$A - B = \begin{bmatrix} 9 & 8 & 7 & 4 \end{bmatrix} - \begin{bmatrix} 7 & 8 & 9 & 4 \end{bmatrix}$$

Ans: $A - B = \begin{bmatrix} 2 & 0 & -2 & 0 \end{bmatrix}$

Ex) $A = \begin{bmatrix} 9 & 8 \end{bmatrix}$ & $B = \begin{bmatrix} -1 & -1 \end{bmatrix}$

$$A - B = \begin{bmatrix} 9 & 8 \end{bmatrix} - \begin{bmatrix} -1 & -1 \end{bmatrix}$$

Ans: $A - B = \begin{bmatrix} 10 & 9 \end{bmatrix}$

$$A - B = \begin{bmatrix} 9 & 8 \end{bmatrix} - \begin{bmatrix} -1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 9 + 1 & 8 + 1 \end{bmatrix}$$

* Matrix Multiplication :-

→ Part I : Scalar Multiplication :

$$\text{if } A = [a_{ij}] \text{ then } kA = \begin{bmatrix} ka_{11} & ka_{12} & \dots & ka_{1n} \\ ka_{21} & ka_{22} & \dots & ka_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ ka_{m1} & ka_{m2} & \dots & ka_{mn} \end{bmatrix}$$

Explanation

If $A = [a_{ij}]$ is any Matrix of order $m \times n$ and k is any scalar then the multiplication KA is often by simply multiplying each element of A by the scalar kA that is $KA = [ka_{ij}]$.

$$[1 1 1 1] \cdot 2 = 2 \cdot [1 1 1 1] = [2 2 2 2]$$

Example 3

$$3) \text{ Let } A = \begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix} \text{ then } 5A = \begin{bmatrix} 10 & 20 \\ 30 & 40 \end{bmatrix}$$

$$2) \text{ Let } A = \begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix} \text{ then } 2A = \begin{bmatrix} 4 & 8 \\ 12 & 16 \end{bmatrix}$$

Part 2: Matrix Multiplication.

In order to multiply 2 matrices, the number of columns in the first (1) matrix must equal to (=) the number of rows in the second (2) matrix.

Order: $A_{m \times p}$ $B_{q \times 1}$
 Row column Row column (which)
 $p = q$ (must be same).

* Order is $AB_{m \times 1}$ (last matrix order)
 (from right to left)

Let AB a matrix of order $m \times p$ and B a matrix of order $p \times 1$ then the multiplication of 2 matrices A and B is possible and the final order of the matrix AB is $[m \times 1]$.

Example:

i) Let $A = \begin{bmatrix} 2 & 4 & 6 \\ 1 & 0 & 0 \\ 1 & 0 \end{bmatrix}_{3 \times 3}$ and $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}_{3 \times 2}$
 then multiplication of AB can be done in the following way.

$$\begin{aligned} & (2 \cdot 1 + 4 \cdot 0 + 6 \cdot 1) + (1 \cdot 0 + 0 \cdot 1 + 0 \cdot 1) = \\ & (2 + 0 + 6) + (0 + 0 + 0) = \end{aligned}$$

$$\begin{bmatrix} 2 & 4 & 6 \\ 1 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

Ques. Explain what is meant by Matrix?

(a) Explain what is meant by matrix?

$$\text{Ans} = (2(1) + 0(0) + 6(1)) \text{ or } 2(0) + 4(1) + 6(0)$$

(b) Explain what is meant by matrix?

$$(1(1) + 0(0) + 0(1)) \text{ or } 1(0) + 0(1) + 0(1)$$

$$\text{Order } 2 \ 4 \ 6 \rightarrow 1 \ 0 \ 1$$

$$\text{Order } 0 \ 0 \ 0 \rightarrow 1 \ 0 \ 1$$

$$2 \ 4 \ 6 \rightarrow 0 \ 0 \ 1$$

$$\text{Order } 1 \ 0 \ 0 \rightarrow 0 \ 1 \text{ is not } 0$$

Explanation of this Matrix

1) Ans = $\begin{bmatrix} 8 & 0 & 0 \end{bmatrix}$ because $A = 8A + 0I$

2) Order $3 \times 1 \times 3 = 3 \times 2$ (row \times column)

3) Order 3×3 is not 3×2 so it is not possible

Ans. In order to multiply two matrices we have to multiply the number of columns of first matrix with the number of rows of second matrix.

2) Let $A = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$

Find $AB = ?$

$A = 3 \times 3$ and $B = 3 \times 2$, so it is not possible to multiply them.

$$B = \begin{bmatrix} 0 & 1 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} \text{ or } B = \begin{bmatrix} 0 & 1 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$$

and $A = 3 \times 3$ and $B = 3 \times 2$, so it is not possible to multiply them.

$$= \begin{bmatrix} 8(1) + 0(0) + 0(1) \\ 0(1) + 1(0) + 1(1) \end{bmatrix} \quad \begin{bmatrix} 8(0) + 0(1) + 0(1) \\ 0(0) + 1(1) + 1(1) \end{bmatrix}$$

$$= \begin{bmatrix} 8+0+0 & 0+0+0 \\ 0+0+1 & 0+1+1 \end{bmatrix}$$

$$\text{Ans} = \begin{bmatrix} 8 & 0 \\ 1 & 8 \end{bmatrix}_{2 \times 2} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$3) A = \begin{bmatrix} 1 & 8 \\ 9 & 4 \\ 6 & 1 \end{bmatrix}, B = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 8 \\ 9 & 4 \\ 6 & 1 \end{bmatrix}_{3 \times 2} \cdot \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}_{2 \times 2}$$

$$= \begin{bmatrix} 1(2) + 8(2) & 1(2) + 8(2) \\ 9(2) + 4(2) & 9(2) + 4(2) \\ 6(2) + 1(2) & 6(2) + 1(2) \end{bmatrix}_{3 \times 2}$$

$$= \begin{bmatrix} 2 + 16 & 2 + 16 \\ 18 + 8 & 18 + 8 \\ 12 + 2 & 12 + 2 \end{bmatrix}_{3 \times 2}$$

S x S

S v S

S P

$$\text{(3) Ans (1)} \begin{bmatrix} 18 & 18 \\ 26 & 26 \\ 14 & 14 \end{bmatrix}_{3 \times 2} + \begin{bmatrix} 11 & 17 \\ 17 & 19 \\ 17 & 17 \end{bmatrix}_{3 \times 2}$$

$$4) A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}.$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2 \times 2} \cdot \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2 \times 2} = \dots$$

$$= \begin{bmatrix} 1(1) + 0(3) & 1(2) + 0(4) \\ 0(1) + 1(3) & 0(2) + 1(4) \end{bmatrix}$$

$$= \begin{bmatrix} 1+0 & 2+0 \\ 0+3 & 0+4 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}.$$

$$= \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2 \times 2}.$$

$$5) A = \begin{bmatrix} 1 & 0 \\ 8 & 4 \\ 9 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 4 \\ 8 & 6 \end{bmatrix}.$$

$$\begin{bmatrix} 1 & 0 \\ 8 & 4 \\ 9 & 2 \end{bmatrix}_{3 \times 2} \cdot \begin{bmatrix} 1 & 4 \\ 8 & 6 \end{bmatrix}_{2 \times 2} = \dots$$

$$= \begin{bmatrix} 1(1) + 0(8) & 1(4) + 0(6) \\ 8(1) + 4(8) & 8(4) + 4(6) \\ 9(1) + 2(8) & 9(4) + 2(6) \end{bmatrix}$$

$$B = 2 \begin{bmatrix} 1 + 0 & 24 + 0 \\ 8 + 32 & 32 + 24 \\ 9 + 16 & 36 + 12 \end{bmatrix} \quad (E)$$

$$\text{Ans} = \begin{bmatrix} 1 & 4 \\ 40 & 56 \\ 25 & 48 \end{bmatrix} \quad 3 \times 2$$

if $\begin{bmatrix} 1 & 2 & 1 \\ 1 & 0 & 2 \\ 1 & 1 & 0 \end{bmatrix}$

$$A = \begin{bmatrix} 1 & 2 & 1 \\ -2 & 4 & 0 \end{bmatrix} \quad 3 \times 3 \quad \text{and} \quad B = \begin{bmatrix} 2 & 4 & 1 \\ -2 & 4 & -2 \end{bmatrix}$$

$$(1)(1) + (2)(2) + (3)(1) = (1)(1) + (2)(2) + (3)(1) + (4)(2) + (5)(1) + (6)(2)$$

$$(1)(1) + (2)(2) + (3)(1) = (1)(1) + (2)(2) + (3)(1) + (4)(2) + (5)(1) + (6)(2)$$

$$(1)(1) + (2)(2) + (3)(1) = (1)(1) + (2)(2) + (3)(1) + (4)(2) + (5)(1) + (6)(2)$$

$$= (1)(1) + (2)(2) + (3)(1) + (4)(2) + (5)(1) + (6)(2)$$

$$= \begin{bmatrix} 1(1) + 1(-2) & 1(4) + 1(4) & 1(1) + 1(-2) \\ -2(2) + 4(-2) & -2(4) + 4(4) & -2(1) + 4(-2) \\ 3(2) + 2(-2) & 3(4) + 2(4) & 3(1) + 2(-2) \end{bmatrix}$$

$$(1)(1) + (2)(2) + (3)(1) + (4)(2) + (5)(1) + (6)(2)$$

$$= \begin{bmatrix} 1 + (-2) & 4 + 4 + 0 & 1 + (-2) \\ (-4) + (-8) & (-8) + 16 & (-2) + (-8) \\ 6 + (-4) & 12 + 8 & 3 + (-4) \end{bmatrix}$$

$$\text{Ans} = \begin{bmatrix} 1 & 8 & -1 \\ -12 & 8 & -10 \\ -2 & 20 & -1 \end{bmatrix} \quad 3 \times 3$$

$$\exists) A = \begin{bmatrix} 9 & 4 & 5 \\ 12 & -12 & -1 \\ 8 & 1 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 3 & -2 & 4 \\ 2 & -2 & 1 \\ 4 & 5 & 2 \end{bmatrix}$$

Find AB & BA

$$AB = \begin{bmatrix} 2 & 4 & 5 \\ 2 & -1 & -1 \\ 3 & -1 & 2 \end{bmatrix} \begin{bmatrix} 3 & -2 & 4 \\ 2 & -2 & 1 \\ 4 & 5 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2(3) + 4(2) + 5(4) & 2(-2) + 4(-2) + 5(5) \\ 2(3) + -1(2) + -1(4) & 2(-2) + -1(-2) + -1(5) \\ 3(3) + -1(2) + 2(4) & 3(-2) + -1(-2) + 2(5) \end{bmatrix}$$

$$= \begin{bmatrix} 2(4) + 4(5) + 5(2) \\ 2(4) + -1(1) + -1(2) \\ 3(4) + -1(1) + 2(2) \end{bmatrix}$$

$$= \begin{bmatrix} 6 + 8 + 20 & (-4) + (-8) + 25 \\ 6 + (-2) + (-4) & (-4) + (-2) + (-5) \\ (-4) + 1 & (-6) + (-2) + 10 \\ (-8) + (-2) & (-8) + (-10) \\ (-4) + 2 & 8 + 4 + 10 \\ 8 + (-1) + (-2) & 12 + (-1) + 4 \end{bmatrix}$$

$$= \begin{bmatrix} 34 & 13 \\ 10 & -11 \\ 3 & 12 \\ -10 & -18 \\ 6 & 12 \\ 1 & 12 \end{bmatrix}$$

$$B \times A \quad B = \begin{bmatrix} 3 & -2 & 4 \\ 2 & -2 & 1 \\ 4 & 5 & 2 \end{bmatrix} \quad A = \begin{bmatrix} 2 & 4 & 5 \\ 2 & -1 & -1 \\ 3 & -1 & 2 \end{bmatrix}$$

$$BA = \begin{bmatrix} 3 & -2 & 4 \\ 2 & -2 & 1 \\ 4 & 5 & 2 \end{bmatrix} \quad \begin{bmatrix} 2 & 4 & 5 \\ 2 & -1 & -1 \\ 3 & -1 & 2 \end{bmatrix} \quad 3 \times 3$$

$$= \begin{bmatrix} 3(2) + -2(2) + 4(3) & 3(4) + -2(-1) + 4(-1) \\ 2(2) + -2(2) + 1(3) & 2(4) + -2(-1) + 1(-1) \\ 4(2) + 5(2) + 2(3) & 4(4) + 5(-1) + 2(-1) \end{bmatrix}$$

$$\begin{bmatrix} 3(5) + -2(-1) + 4(2) \\ 2(5) + -2(-1) + 1(2) \\ 4(5) + 5(-1) + 2(2) \end{bmatrix}$$

$$= \begin{bmatrix} 6 + (-4) + 12 & 12 + (-2) + (-4) & 15 + (-2) + 8 \\ 4 + (-4) + 3 & 8 + (-2) + (-1) & 10 + (-2) + 2 \\ 8 + 10 + 6 & 16 + (-5) + (-2) & 20 + (-5) + 6 \end{bmatrix}$$

$$Ans = \begin{bmatrix} 14 & 6 & 21 \\ 3 & 5 & 10 \\ 24 & 9 & 21 \end{bmatrix} \quad 3 \times 3$$