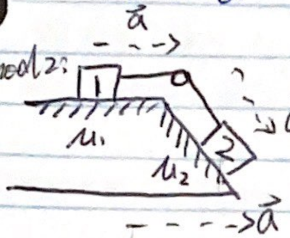
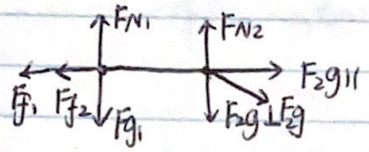
④ $\mu \neq 0$, $\vec{a} = \dfrac{m_2 g \sin\theta - \mu_2 m_2 g \cos\theta - \mu_1 m_1 g}{m_1 + m_2}$

method 2:



$\Sigma \vec{F} = m\vec{a}$

$F_{2g\parallel} - \vec{F}_{f_1} - \vec{F}_{f_2} = (m_1 + m_2)\vec{a}$

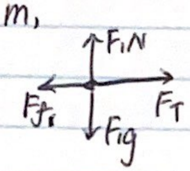


$m_2 \vec{g} \sin\theta - \mu_1 m_1 \vec{g} - \mu_2 m_2 g \cos\theta = (m_1 + m_2)\vec{a}$

$\dfrac{m_2 \vec{g} \sin\theta - \mu_1 m_1 g - \mu_2 m_2 g \cos\theta}{m_1 + m_2} = \vec{a}$

method 1:

m_1



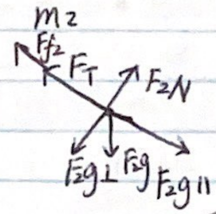
~~m~~

$\Sigma \vec{F} = m\vec{a}$

$F_T - F_{f_1} = m\vec{a}$

$F_T = m_1\vec{a} + F_{f_1}$

m_2



$\Sigma \vec{F} = m\vec{a}$ $- F_{f_2}$

$F_{2g\parallel} - F_T = m\vec{a}$

$F_{2g\parallel} - (m_1\vec{a} + F_f) - F_{f_2} = m_2\vec{a}$

$m_2 g \sin\theta - \mu_1 m_1 g - \mu_2 m_2 g \cos\theta = (m_1 + m_2)\vec{a}$

$\dfrac{m_2 g \sin\theta - \mu_1 m_1 g - \mu_2 m_2 g \cos\theta}{m_1 + m_2} = \vec{a}$

Equalibrium dynames. / not accelerating

① Static Equalibrium

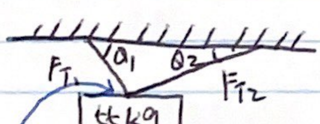
T: Translational Equalibrium

$\Sigma \vec{F} = 0$

R: Rotational Equalibrium

$\Sigma \tau = 0$

ex.



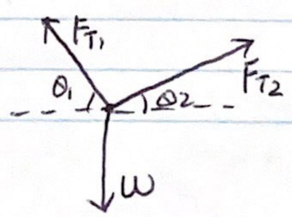
FBD where forces meet.

$\theta_1 = 50°$

$\theta_2 = 30°$

What is F_{T_1} / T_{T_2}

more vertical cable having more work.



• method 1:

x: $\Sigma \vec{F}_x = 0$

$F_{T_1x} - F_{T_2x} = 0$

$F_{T_1x} = F_{T_2x}$

$F_{T_1}\cos\theta_1 = F_{T_2}\cos\theta_2$

① $F_{T_1} = \dfrac{T_{T_2}\cos\theta_2}{\cos\theta_1}$

y: $\Sigma \vec{F}_y = 0$

$F_{T_1y} + F_{T_2y} = W$

$F_{T_1}\sin\theta_1 + F_{T_2}\sin\theta_2 = mg$

$\therefore$ ② $\dfrac{F_{T_2}\cos\theta_2}{\cos\theta_1} \times \sin\theta_1 + F_{T_2}\sin\theta_2 = mg$

$F_{T_2}\cos\theta_2\tan\theta_1 + F_{T_2}\sin\theta_2 = mg$

$F_{T_2}(\cos\theta_2\tan\theta_1 + \sin\theta_2) = mg$

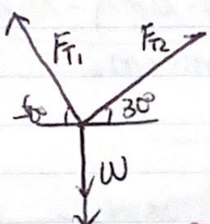
$\therefore F_{T_2} = \dfrac{mg}{\cos\theta_2 \cdot \tan\theta_1 + \sin\theta_2}$

$\therefore F_2 = \dfrac{55 \times 9.8}{\cos 30 \tan 50 + \sin 30}$

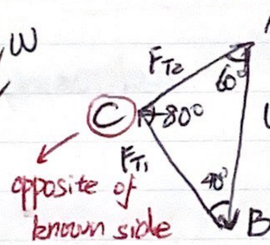
$\therefore F_2 = 352N$

$\therefore F_{T_1} = \dfrac{351.807N\cos 30}{\cos 50}$

$\therefore F_{T_1} = 474N$



• Method 2.



Use sine law.

$\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$

$\Sigma \vec{F} = 0$

$\vec{F}_{T_1} + \vec{F}_{T_2} + \vec{W} = 0$

$|F_{T_1}| = a = \dfrac{\sin A\, c}{\sin C}$

$a = \dfrac{\sin 60° \times mg}{\sin 80°}$

$a = \dfrac{\sin 60° \times 55 \times 9.8}{\sin 80°}$

$a = 474N$

$|F_{T_2}| = b = \dfrac{\sin B\, c}{\sin C}$

$b = \dfrac{\sin 40° \times mg}{\sin 80°}$

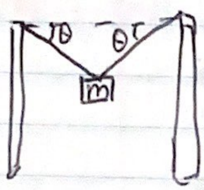
$b = 352N$

Facts to know:

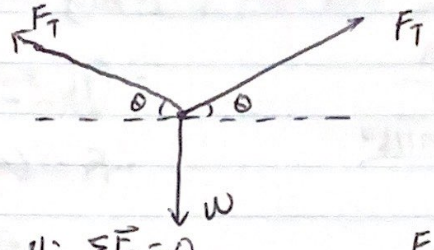
If $\theta_1 = \theta_2$, $\therefore F_{T1} = F_{T2}$

The more vertical cable have greater tension.

Determine F_T required to pull a cable mass m tight.



FBD



x: $\Sigma \vec{F} = 0$

$F_{Tx} = F_{Tx}$

$F_T \cos\theta = F_T \cos\theta$

y: $\Sigma \vec{F} = 0$

$F_{Ty} + F_{Ty} = W$

$2F_T \sin\theta = mg$

$F_T = \dfrac{mg}{2\sin\theta}$

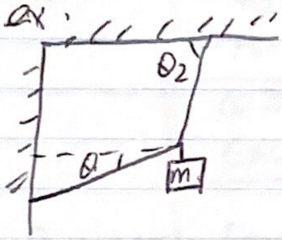
$F_T = \lim_{\theta \to 0} \dfrac{mg}{2\sin\theta}$

$\dfrac{1}{\sin 0.1} = 573N$

$\therefore F_T = \lim_{\theta \to 0^\circ} \dfrac{mg}{2\sin\theta} = \infty$

---

ex:

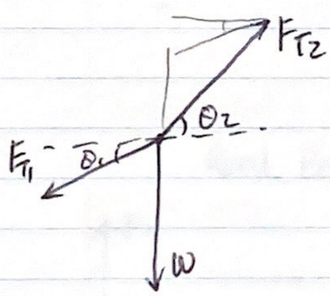


$\theta_1 = 25^\circ$

$\theta_2 = 60^\circ$

$m = 75.0kg$

Determine F_{T1}, F_{T2}



$\Sigma F_x = 0$

$\therefore F_{T1} \cos\theta_1 = F_{T2} \cos\theta_2$

$\therefore F_{T1} = \dfrac{F_{T2}\cos\theta_2}{\cos\theta_1}$

$\Sigma F_y = 0$

$W + F_{T1}\sin\theta_1 = F_{T2}\sin\theta_2$

$\therefore W + \dfrac{F_{T2}\cos\theta_2}{\cos\theta_1} = F_{T2}\sin\theta_2$

$\therefore mg + \dfrac{F_{T2}\cos 60^\circ}{\cos 25^\circ} = F_{T2}\sin 60^\circ$

$\therefore \cos 25^\circ mg + F_{T2}\cos 60^\circ = \cos 25^\circ F_{T2} \sin 60^\circ$

$\therefore \dfrac{\cos 25^\circ mg}{(\sin 60^\circ \cos 25^\circ - \cos 60^\circ)} = F_{T2}$

$\therefore F_{T2} = 2338.25$

$\therefore F_{T1} = \dfrac{F_{T2}\cos 60^\circ}{\cos 25^\circ}$

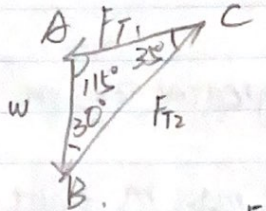
$\therefore F_{T1} = 1289.99$

0.284885567

Method 2.

$\Sigma \vec{F} = 0$

$F_{T1} + F_{T2} + W = 0$



F_{T1}: $\frac{\sin\angle C}{mg} = \frac{\sin\angle A}{F_{T2}}$

$\therefore F_{T2} = \frac{mg \times \sin\angle A}{\sin\angle C}$

$\therefore F_{T2} = \frac{9.8 \times 75 \times \sin 115°}{\sin 35°}$

$\therefore F_{T2} = 1161 N$

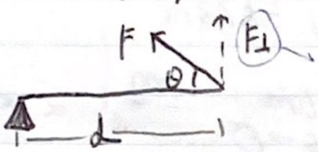
F_{T2}: $\frac{\sin\angle C}{mg} = \frac{\sin\angle B}{F_{T1}}$

$\therefore F_{T1} = \frac{mg \times \sin\angle B}{\sin\angle C}$

$\therefore F_T = 641 N$

Rotational Equalibrium:

Torque: is to rotation as Force is to translation.



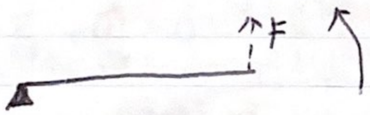
▲: Fulcrum point around which the object rotates.

d: lever distance or lever arm.

$\tau = F_\perp \cdot d = F\sin\theta\, d$

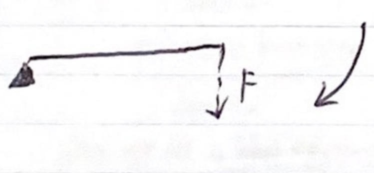
"tow" units = NM. (newtons/meters)

foot pounds / pound feet



counterclockwise

+ "τ"

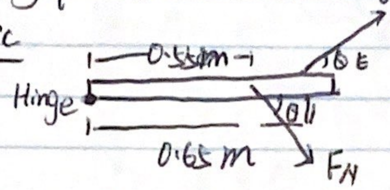


clockwise

- "τ"

ex. Two boys push as a bedroom door

Nik Eric



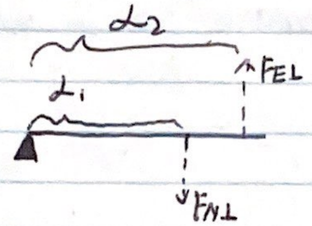
$F_N = 550N$ $\theta_N = 50.0°$

$F_E = 420N$ $\theta_E = 65.0°$

Determine $\Sigma\tau$

τ

FBD : must locate fulcrum ▲ $F_\perp$ / must have d



~~$\Sigma\tau = \tau_E\ \tau_{E\perp} - \tau_{N\perp}$~~

$\Sigma\tau = \tau_{F_{E\perp}} - \tau_{F_{N\perp}}$

$= F_{E\perp} \cdot d_2 - F_{N\perp} \cdot d_1$

$= F_E \sin\theta_E \cdot d_2 - F_N \sin\theta_N \cdot d_1$

$= 420 \sin 65° \cdot 0.65 - 550 \sin 50° \times 0.55$

$= 15.7 N \cdot m$

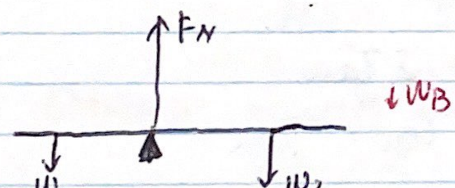
$\because$ 15.7N is "+"
$\therefore$ Eric wins.

Rotational Equalibrium condition : $\Sigma\tau = 0$.

or $\Sigma\tau = \Sigma\tau_{counter} - \Sigma\tau_{clock}$

$\therefore \Sigma\tau_{counter} = \Sigma\tau_{clock}$ / $\Sigma\tau_{cw} = \Sigma\tau_{ccw}$

Real Beams with mass

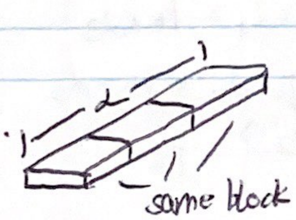


※ • uniform beam • Same density along its length.

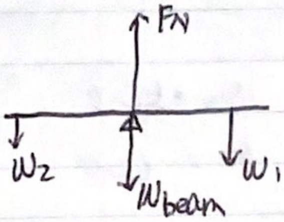
• Same cross-section along its length. (uniform by construction)

• Center of mass and therefore the weight is located in the geometric center

ex.

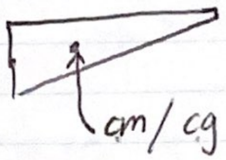


ex



← uniform beam

※ Inuniform beam :



center of mass is the intersection of the balance line in three dimentions

- the center of mass move in a smooth arc

ex. Any point can be fulcrum if a system's at rotational equalibrium.

Equivalent of fulcrum location:

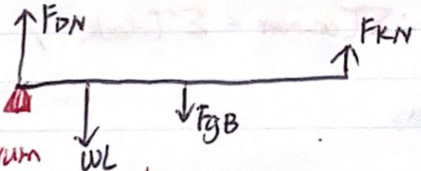
If an object is at rotational equlibrium, it is not rotating around any point. Therefore every point is an equally ~~balanced~~ valid location for fulcrum for analysis.

ex. A 2.0m long, 25.0kg uniform beam is being carried by team Fulop. A 75.0kg load is 0.5m from one end, Determine upward force on each end.

①

Beam
Load
Kid
Dad

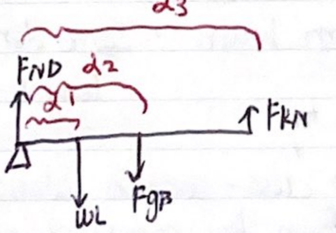
FBD ↳



$$\Sigma \vec{F}_y = 0$$

$$F_{ND} + F_{NK} = W_L + W_B$$

↓ τ FBD



$$\Sigma \tau_{cw} = \Sigma \tau_{ccw}$$

$$\tau_{FgB} + \tau_{FgL} = \tau_{FK}$$

$$W_B \times d_2 + W_L \times d_1 = F_K \times d_3$$

$$mg_b \times d_2 + mg_L \times d_1 = F_K \times d_3$$

$$\frac{g(m_b \times d_2 + m_L \times d_1)}{d_3} = F_K$$

$$\therefore F_K = 306.25N$$

plug F_K in to $\Sigma \vec{F}_y = 0$

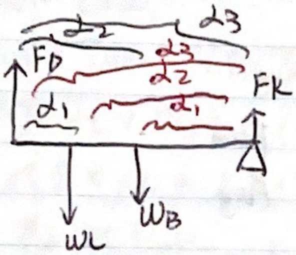
$\therefore F_K + F_D = W_L + W_B$

$\therefore F_D = W_L + W_B - F_K$

$\therefore F_D = 9.8(25+75) - 306.25$

$\therefore F_D = 674\,N$

(2)



$\tau_{cw} = \tau_{ccw}$

$\tau_{W_L} + \tau_{W_B} = \tau_{F_D}$

$\therefore W_L \times d_2 + W_B \times d_1 = F_D \times d_3$

$m_L g \times d_2 + m_B g \times d_1 = F_D \times d_3$

$\therefore \dfrac{g(m_L \times d_2 + m_B \times d_1)}{d_3} = F_D$

$\therefore F_D = \dfrac{9.8(75 \times 1.5 + 25 \times 1)}{2}$

$\therefore F_D = 674\,N$

$\Sigma F_y = 0$

$F_D + F_K = W_L + W_B$

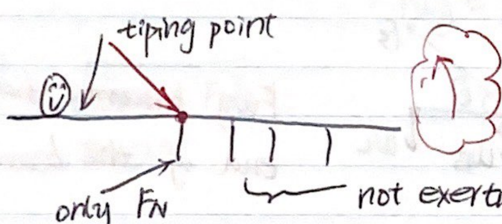
$F_K = W_L + W_B - F_D$

$F_K = mg_L + mg_B - F_D$

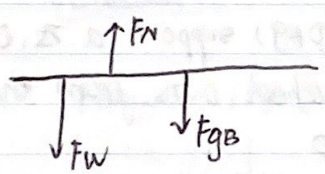
$\therefore F_K = (75+25)9.8 - F_D$

$\therefore F_K = 306\,N$

Tiping issue

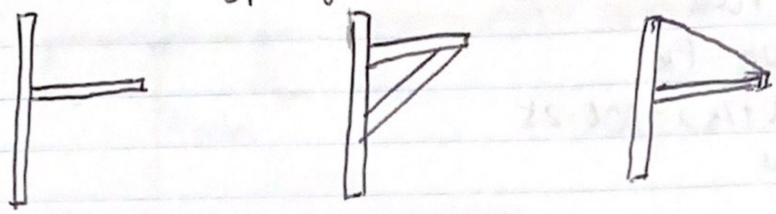


↓ FBD

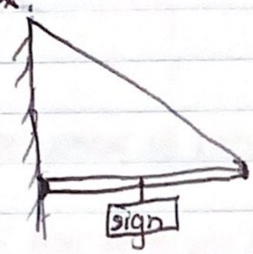


Cantilever: supporting a load by of torque.

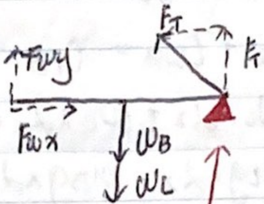
Types of cantilevers.



ex.



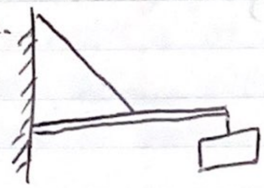
FBD:



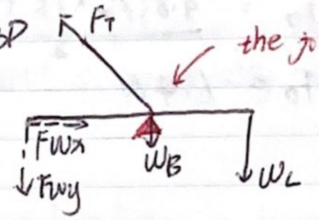
Tension F_{wx}
Beam F_{wy}
Load
Wall

unsure about wy? Temporary fulcrum

ex 2.

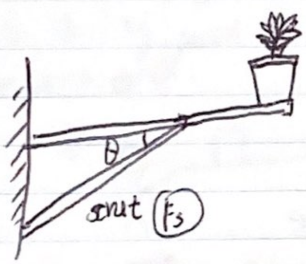


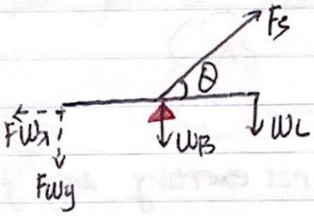
FBD



the joint of the cable / struct.

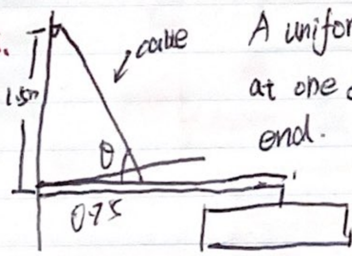
ex





Fwy prevents the end of the beam go down.

ex.

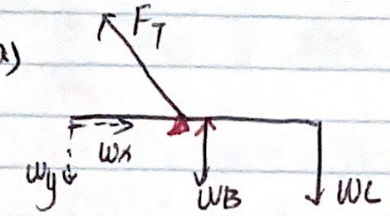


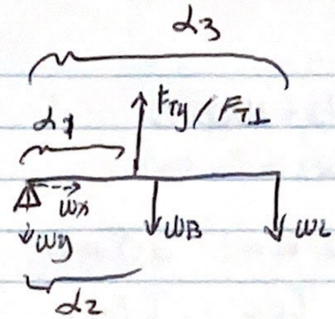
A uniform 2.0 m long beam (25.0 kg) support a 75.0 kg sign at one end. A cable is attached 0.75 from the other end.

a) Determine FBD
b) Draw T FBD. (fulcrum at wall)
c) Determine F_T
d) Determine the force of the wall.

$|\vec{F_W}|$ $\quad \Sigma F = 0$

$\Sigma \vec{F_W} = \vec{F_{wy}} + \vec{F_{wx}}$

a) 

b) 

c) $\tau_{cw} = \tau_{ccw}$

$\tau_{wB} + \tau_{wC} = \tau_{F_{Ty}/F_{T\perp}}$

$m_B g \times d_2 + m_C g \times d_3 = F_{Ty\perp} \times d_1$

$25 \times 9.8 \times 1.0 + 9.8 \times 75 \times 2.0 = 0.75 \times F_T \sin\theta$

$\frac{9.8(25+150)}{0.75} = F_T \sin 63.4°$

$\therefore F_T = 2556.57N$

$\theta = \tan^{-1}\left(\frac{1.5}{0.75}\right)$

$\theta = 63.43494882°$

d) $\vec{F_W} = \vec{F_{Wy}} + \vec{F_{Wx}}$

x: $\Sigma F_x = 0$

$F_{Wx} = F_{Tx} = F_T \cos\theta$

$\therefore F_{Wx} = 1145.33N$

y: $\Sigma F_y = 0$

$F_{Wy} + F_{gL} + F_{gB} = F_T \sin\theta$

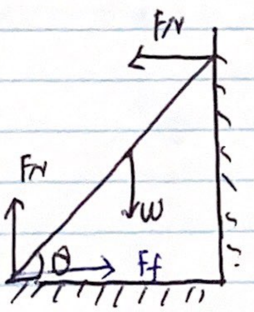
$F_{Wy} = F_T \sin\theta - F_{gL} - F_{gB}$

$\therefore F_{Wy} = 1306.66N$

$\therefore |\vec{F_W}| = \sqrt{F_{Wx}^2 + F_{Wy}^2}$

$\therefore |F_W| = 1740N$

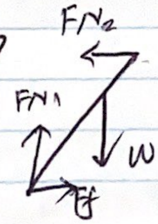
$\boxed{\text{Ladder}}$



$|F_{N_R}| = |F_f|$

$|W| = |F_N|$

Determine minimum θ for an empty uniform ladder if μ of ground/ladder is 0.25

FBD 

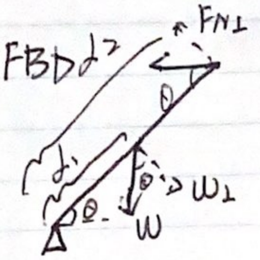
x: $\Sigma F_x = 0$

$\therefore F_f = F_{N2}$

y: $\Sigma \vec{F_y} = 0$

$F_{N1} = W$

τ FBD



$\because d_2 = 2d_1$

$W_\perp = W\cos\theta$

$F_{N\perp} = F_N \sin\theta$

$\Sigma \tau_{cw} = \Sigma \tau_{ccw}$

$\tau_{W\perp} = \tau_{F_{N2}\perp}$

$W_\perp \times d_1 = F_{N2\perp} \times d_2$

$W\cos\theta \times d_1 = F_N \sin\theta \times d_2$

$W\cos\theta = 2F_{N2}\sin\theta$

$\cancel{\therefore \theta = \tan^{-1}\left(\frac{2F_{N2}}{W}\right)}$

$\cancel{\therefore \theta = \tan^{-1}(2x}$

$\frac{W}{2F_{N2}} = \tan\theta$

$\frac{W}{2F_{N1}\mu} = \tan\theta$

$\frac{\cancel{W}1}{2\cancel{W}\mu} = \tan\theta$

$\tan\left(\frac{1}{2\mu}\right) = \theta$

$\theta = \cancel{\tan^{-1}}\ \cot^{-1}\left(\frac{W}{2F_{N2}}\right)$

$\theta = \cancel{\tan^{-1}}\ \cot^{-1}\left(\frac{mg}{2\times\mu F_{N1}}\right)$

$\theta = \cancel{\tan}\ \cot^{-1}(2\mu)$

$\therefore 2\mu = \frac{1}{\tan\theta}$

$\theta = \tan^{-1}\left(\frac{1}{2\mu}\right)$

$\theta = 63.4°$