PROJECT SUMMARY REPORT

Modeling and Forecasting USD/INR Volatility Using a HAR-RV-X Linear Econometric Framework

1. Introduction and Theoretical Motivation

In financial markets, **volatility represents uncertainty and risk**, and in foreign exchange (FX) markets it is often more predictable than price direction. Unlike equity markets, FX returns are close to a martingale process, making return prediction unreliable. However, FX volatility exhibits **strong persistence, clustering, and long-memory behavior**, making it a suitable target for forecasting.

The USD/INR exchange rate is particularly relevant due to:

- India's dependence on imported commodities (especially crude oil),
- Global USD liquidity cycles,
- Risk-on / risk-off dynamics in international markets,
- Central bank interventions that dampen price trends but not volatility.

Therefore, this project focuses on **forecasting realized volatility**, not prices or returns.

2. Measuring Volatility: Realized Variance Theory

Instead of model-implied volatility, this project uses **realized volatility (RV)**, which is an observable, data-driven measure.

Parkinson (1980) High-Low Estimator

The realized variance is computed as:

$$RV_t = \frac{1}{4\ln(2)}\left(\ln\left(\frac{H_t}{L_t}\right)\right)^2$$

where:

- HtH_tHt = daily high price
- LtL_tLt = daily low price

Why Parkinson RV?

- More efficient than close-to-close variance
- Incorporates intraday price range
- Particularly suitable for FX markets with low microstructure noise

All volatility series are **log-transformed**: **log(RVt)**

3. Baseline Linear Regression and Its Failure

The project initially applies a **basic OLS regression**:

$$\log(RV_t) = \alpha + \beta \log(RV_{t-1}) + \varepsilon_t$$

Despite volatility persistence, this model performs poorly due to:

- Single-horizon memory assumption

- Inability to capture long-term dependence

- Ignoring heterogeneous trader behavior

Empirically, this results in **low or negative R^2**, confirming that naïve linear models are structurally inadequate.

4. Volatility Persistence and Long Memory

Empirical finance literature shows that volatility exhibits **long memory**, meaning shocks decay slowly over time. This behavior cannot be captured by a single lag but emerges from the aggregation of multiple trading horizons.

This motivates the **Heterogeneous Market Hypothesis**, which states that:

- Different market participants operate at different time scales,

- Volatility is an aggregation of these heterogeneous behaviors.

5. HAR-RV Model: Core Theory

The **Heterogeneous Autoregressive (HAR) model**, introduced by **Corsi (2009)**, approximates long-memory processes using simple linear averages.

HAR-RV Model:

$$\log(RV_t) = \beta_0 + \beta_d \log(RV_{t-1}) + \beta_w \log(\overline{RV}_{t-5:t-1}) + \beta_m \log(\overline{RV}_{t-22:t-1}) + \varepsilon_t$$

Where:

- Daily component → short-term traders
- Weekly component → medium-term investors
- Monthly component → long-term institutions

This formulation captures volatility persistence **without non-linear models**.

6. Extending to HAR-RV-X: Spillover Theory

FX volatility is not isolated. It is affected by global and domestic financial markets.

The **HAR-RV-X model** extends HAR-RV by adding exogenous realized volatility:

$$\log(RV_t^{FX}) = HAR\ terms + \sum_{i=1}^{N} \gamma_i \log(RV_t^{(i)}) + \varepsilon_t$$

Where exogenous variables include:

- DXY → USD strength
- VIX → Global risk sentiment
- Brent Crude → Trade balance risk
- S&P 500 & NIFTY → Equity market stress

This captures **volatility spillovers**, a well-documented phenomenon in international finance.

7. Estimation and Model Choice

- Estimation method: **Ordinary Least Squares (OLS)**

- Advantages:
  - Transparency
  - Interpretability
  - Statistical inference (t-stats, significance)

- No distributional assumptions on returns

This aligns with academic practice in volatility modeling.

8. Results and Empirical Performance

The final HAR-RV-X model achieves:

- **$R^2 \approx 0.44$**
- Significant improvement over baseline regression

Why This Matters

- FX volatility studies typically report R^2 between **0.15–0.30**
- USD/INR is a managed currency, making volatility harder to explain
- Coefficients show economically intuitive signs and magnitudes

9. Practical Implications

The model can be applied to:

- FX options pricing
- Dynamic hedging strategies
- Treasury risk forecasting
  Stress testing currency exposure

Because the model is linear and interpretable, it is suitable for **real-world deployment**.

10. Conclusion

This project demonstrates that **model structure matters more than model complexity**. By combining realized volatility theory with heterogeneous autoregressive dynamics and macro-financial spillovers, a purely linear model can achieve strong explanatory power for FX volatility.

11. Key References

1. Corsi, F. (2009). *A Simple Approximate Long-Memory Model of Realized Volatility*. Journal of Financial Econometrics.
2. Parkinson, M. (1980). *The Extreme Value Method for Estimating the Variance of the Rate of Return*. Journal of Business.
3. Andersen, T. G., Bollerslev, T., Diebold, F. X. (2007). *Roughing It Up*. Review of Economics and Statistics.
4. Degiannakis, S., Floros, C. (2015). *Modeling Volatility Spillovers Between Stock and FX Markets*.