Name – Ayush Chauhan
Section – B
Roll No. – 37

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Tutorial-1

1: Asymptotic notations are used to represent the complexities of algorithms for asymptotic analysis.

(i) Big Oh: $f(n) \Rightarrow O \log(n)$

if $f(n) \leq g(n) \times c \; \forall n \geq n_0$

for some constant, $c > 0$

$g(n)$ is tight upper bound of $f(n)$.

(ii) Big Omega: $f(n) = \Omega g(n)$, means $g(n)$ is tight lower bound of $f(n)$ i.e; $f(n)$ can go beyond $g(n)$

$f(x) = \Omega g(x)$

if and only if $f(n) \geq c \cdot g(n)$

(iii) Big Theta (θ): $f(n) = \theta(g(n))$, gives the tight upper bound and lower bound both

$f(x) = \theta \, g(x)$

if and only if $c_1 \times g(n_1) \leq f(n) \leq c_2 \times g(n_2)$

(iv) Small Oh: when $f(n) = o \, g(n)$ gives the upper bound i.e; $f(x) = o \, g(x)$

if and only if, $f(x) < c \times g(x) \; \forall n > n_0, c > 0$

(v) Small Omega: It gives the lower bound i.e; $f(n) = \omega g(n)$ where $g(n)$ is lower bound of $f(n)$ if and only if $f(n) > c * g(n)$

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2. for $i = 1, 2, 4, 8$ --- n times

i.e; series is a G.P

So $a = 1, r = \frac{2}{1} = 2$

K^{th} value of G.P, $t_k = ar^{k-1}$

$= 1(2)^{k-1}$

$2x = 2^k$

$\log_2(2x) = k\log 2$

$\log_2 2 + \log_2 x = k$

$\log_2 x + 1 = k$

So, T.C, $T(n) = O(\log_2 n)$

3. $T(n) = 3T(n-1)$ — (1)

$T(n) = 1$

put $n = n-1$ in (1)

$T(n-1) = 3T(n-2)$ — (2)

put (2) in (1)

$T(n) = 3 \times 3T(n-2)$

$T(n) = 9T(n-2)$ — (3)

put $n = n-2$ in (1)

$T(n-2) = 3T(n-3)$

put in (3)

$T(n) = 27T(n-3)$ — (4)

Generalizing series,

$T(k) = 3^k T(n-k)$ — (5)

for k^{th} terms, Let $n-k = 1$

$k = n-1$ put in (5)

$T(n) = 3^{n-1} T(1)$

$T(n) = O(3^n)$

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4. $T(n) = 2T(n-1) - 1$ —— (1)

put $n = n-1$

$T(n-2) = 2T(n-2) - 1$ —— (2)

put in (1)

$T(n) = 2\times(2T(n-2)-1) - 1$

$= 4T(n-2) - 2 - 1$ —— (3)

put $n = n-2$ in (1)

$T(n-2) = 2T(n-3) - 1$

put in (1)

$T(n) = 8T(n-3) - 4 - 2 - 1$ —— (4)

Generalizing series

$$T(n) = 2^k T(n-k) - 2^{k-1} - 2^{k-2} \ldots 2^0$$

k^{th} term, let $n-k = 1$

$k = n-1$

$$T(n) = 2^{n-1} T(1) - 2^k \left(\frac{1}{2} + \frac{1}{2^2} + \ldots \frac{1}{2^k}\right)$$

i.e, series in G.P, $a = \frac{1}{2}$, $r = \frac{1}{2}$.

So, $T(n) = 2^{n-1}\left(1 - \left(\frac{\frac{1}{2}\left(1-\left(\frac{1}{2}\right)^{k-1}\right)}{1-\frac{1}{2}}\right)\right)$

$= 2^{n-1}\left(1 - 1 + \left(\frac{1}{2}\right)^{n-1}\right) = \frac{2^{n-1}}{2^{n-1}}$

$T(n) = O(1)$

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5. $i = 1, 2, 3, \_\_\_$

$s = 1 + 3 + 6 + 10 + \_\_\_$

Sum of $s = 1 + 3 + 6 \_\_\_ + n$ — (1)

Also $s = 1 + 3 + 6 \_\_\_ T_{n-1} + T_n$ — (2)

$0 = 1 + 2 + 3 \_\_\_ n - T_n$

$T_k = 1 + 2 + 3 \_\_\_ k$

$T_k = \frac{1}{2} k(k+1)$

for k iterations, $1 + 2 + 3 \_\_\_ k, <= n$

$$\frac{k(k+1)}{2} <= n$$

$$\frac{k^2 + k}{2} <= n$$

$$O(k^2) \leq n$$

$$k = O(\sqrt{n})$$

$$T(n) = O(\sqrt{n})$$

6. As, $i^2 = n$, $i = \sqrt{n}$

$i = 1, 2, 3 \_\_\_ \sqrt{n}$

$\sum_{i=1} 1 + 2 + 3 \_\_\_ \sqrt{n}$

$$T(n) = \frac{\sqrt{n} + (\sqrt{n} + 1)}{2}$$

$$T(n) = \frac{n + \sqrt{n}}{2}$$

$$T(n) = O(n)$$

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7. Since, for $K = K^2$

$K = 1, 2, 4, --- n$

Series is in G.P, $a = 1$, $r = 2$

$= \frac{a(r^n - 1)}{r - 1} = \frac{1(2^k - 1)}{1}$

$n = 2^k - 1$

$\log_2(n) = k$

i	j	K
1	log(n)	log(n) * log(n)
2	log(n)	log(n) * log(n)
⋮	⋮	⋮
n	log(n)	log(n) * log(n)

$T.C = O(n * \log n * \log n)$

$= O(n \log^2(n))$

8. for ($i = 1$ to n)

we get $j = n$ times every turn

$\therefore i * j = n^2$

K^{th}, $T(n) = n^2 + T(n-3)$

$T(n-3) = (n^2 3)^2 + T(n-6)$

$T(n-6) = (n^3 6)^3 + T(n-9)$

$T(1) = 1$

Now, substitute each value in $T(n)$

$T(n) = n^2 + (n-3)^2 + (n-6)^2 + --- + 1$

Let, $K^n - 3K = 1$, $K = (n-1)/3$

$T(n) = n^2 + (n-3)^2 + (n-6)^2 + --- 1$

So, $T(n) = O(n^3)$

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9. for $i=1$, $j = 1+2+$ ---- $(n \geq j+i)$
$i=2$, $j=1+3+5$ ---- $(n \geq j+i)$
$i=3$, $j=1+4+7$ ---- $(n \geq j+i)$

n^{th} term of AP is
$T(n) = a + d \times n$
$(n-1)/d = n$
for $i=1$, $(n-1)/1$ times
$i=2$, $(n-1)/2$ times

$T(n) = i_1 j_1 + i_2 j_2 + \text{----} \; i_{n-1} j_{n-1}$
$= \frac{(n-1)}{2} + \frac{(n-2)}{2} + \frac{(n-3)}{3} \text{----} 1$

$= n * \log n - n + 1$
$T(n) = O(n \log n)$

10. As given n^k and c^n
Relationship b/w n^k & c^n is
$n^k = O(c^n)$
$n^k \leq a(c^n) \;\; \forall n \geq n_0, \; a>0$

for $n_0 = 1$, $c=2$
$\Rightarrow 1^k < a^2$
$\Rightarrow n_0 = 1$ & $c = 2$