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Sec - B

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Tutorial-2

1.
$$\left. \begin{array}{l} j=1 \\ j=2 \\ j=3 \\ \vdots \\ f(n) \end{array} \right\} \begin{array}{l} i=1 \\ i=1+2 \\ i=1+2+3 \\ \vdots \\ i=1+2+3+\dots < n \end{array} \quad \text{m-level}$$

$$1+2+3+\dots < n$$

$$1+2+3+\dots + m < n$$

$$\frac{m(m+1)}{2} < n$$

$$m \approx \sqrt{n}$$

By summation method

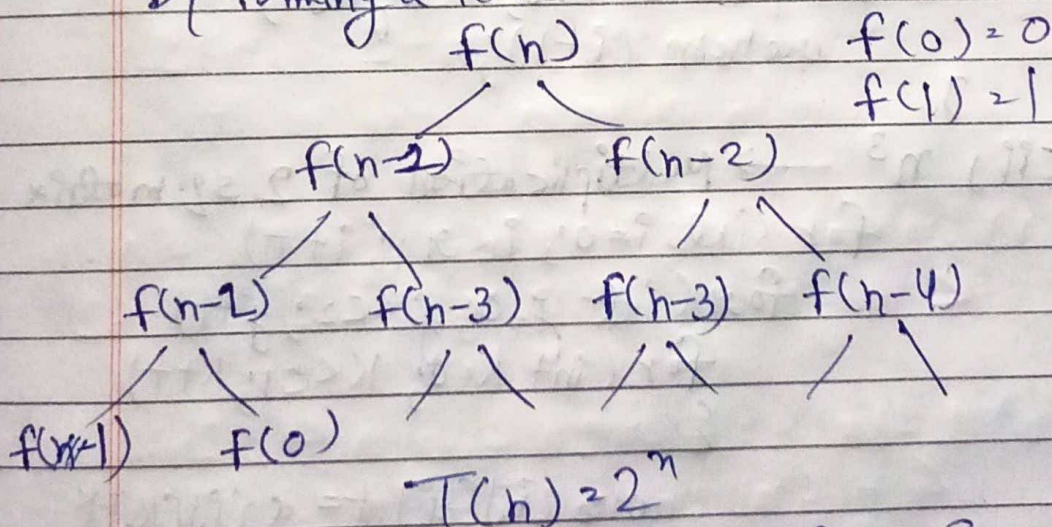
$$\Rightarrow \sum_{i=1}^n 1 \Rightarrow 1+1+\dots \sqrt{n}$$

$$T(n) = \sqrt{n}$$

2. Four Fibonacci Series

$$f(n) = f(n-1) + f(n-2)$$

By forming a tree



Maximum Space: Considering Recursion, $T(n) = O(n)$
Without considering Recursion, $T(n) = O(1)$

3. ci) $n \log n \rightarrow$ Quick Sort

```
void quicksort (int a[], int low, int high)
{
    if (low < high)
    {
        int pi = partition(a, low, high);
        quicksort(a, low, pi-1);
        quicksort(a, pi+1, high);
    }
}
```

```
int partition (int a[], int low, int high)
{
    int pivot = a[high];
    int i = (low-1);
    for (int j = low; j < high; j++)
    {
        if (a[j] < pivot) i++;
        swap(a[i], a[j]);
    }
    swap(a[i+1], a[high]);
    return (i+1);
}
```

cii) $n^3 \rightarrow$ Multiplication of 2 square matrices

```
for (int i = 0; i < n1; i++)
    for (int j = 0; j < n2; j++)
        for (int k = 0; k < n1; k++)
        {
            res[i][j] += a[i][k] *
                        b[k][j];
        }
```


Case 1) $\log(\log n)$

for $(i=2; i \leq n; i=i \times 2)$
 $\{$
 $\quad \text{count}++;$
 $\}$

4. At level $\rightarrow 0 \rightarrow Cn^2$

$$1 \rightarrow \frac{n^2}{4^2} + \frac{n^2}{2^2} \leq \frac{5n^2}{16}$$

$$2 \rightarrow \frac{n^2}{8^2} + \frac{n^2}{16^2} + \frac{n^2}{4^2} + \frac{n^2}{8^2} = \left(\frac{5}{16}\right)^2 n^2$$

$$\text{max level} = \frac{n}{2^k} = 1$$

$$= k = \log_2 n$$

$$T(n) = C \left(n^2 + \left(\frac{5}{16}\right)n^2 + \left(\frac{5}{16}\right)^2 n^2 + \dots + \left(\frac{5}{16}\right)^{\log_2 n} n^2 \right)$$

$$T(n) = Cn^2 \times \left[\frac{1 - \left(\frac{5}{16}\right)^{\log_2 n}}{1 - \left(\frac{5}{16}\right)} \right]$$

$$T(n) = O(n^2 C) = O(Cn^2)$$

5. $T(n) = \frac{(n-1)}{1} + \frac{(n-1)}{2} + \dots + \frac{(n-1)}{n}$

$$T(n) = n \left[1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n} \right] = n \left[1 + \frac{1}{n} \right]$$

$$= n \log n - \log n$$

$$T(n) = O(n \log n)$$

6. for $\frac{1}{2^1}$ where $2^{k^m} \leq n$
 $\frac{1}{2^{k^2}}$ $k^m = \log_2 n$
 $\frac{1}{2^{k^3}}$ $m = \log k \log_2 n$

$$\therefore \sum_{i=1}^m \frac{1}{i^2}$$

$$\Rightarrow 1 + 1 + 1 + \dots + m$$

$$T(n) = O(\log k \log n)$$

7. Given array algo divides array in 99% and 1% part

$$\therefore T(n) = T(n-1) + O(1)$$

$$T(n) = T(n-1) + T(n-2) + \dots + T(1) + O(1)$$

$$= n \times n$$

$$T(n) = O(n^2)$$

$$\text{Lowest height} = 2$$

$$\text{Highest height} = n$$

$$\therefore \text{difference} = n - 2, n > 1$$

The given algo produces linear result.

8. (a) $100 < \log \log n < \log n < (\log n)^2 < \sqrt{n} < n$
 $< n \log n < \log(n!) < n^2 < 2^n < 4^n < 2^{2^n}$

(b) $1 < \log \log n < \sqrt{\log n} < \log n < \log 2n < 2 \log n$
 $< n < n \log n < 2n < 4n < \log(n!) < n^2 < 2^{2^n}$

(c) $96 < \log n < \log 2n < 5n < n \log_2(n) < n \log_2 n$
 $< \log(n!) < 8n^2 < 7n^3 < n! < 8^{2^n}$