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Tutorial – 4

1. $T(n) = 3T(n/2) + n^2$
   $T(n) = aT(n/b) + f(n^2)$

   $a \geq 1, b \geq 1$

   On comparing, $a = 3, b = 2, f(n) = n^2$

   Now, $c = \log_b a = \log_2 3 = 1.584$

   $n^2 = n^{1.58} < n^2$

   $\therefore f(n) > n^c$

   $T(n) = O(n^2)$

2. $T(n) = 4T(n/2) + n^2$

   $a \geq 1, b > 1$, $a = 4, b = 2, f(n) = n^2$

   $c = \log_2 4 = 2$

   $n^c = n^2 \Rightarrow f(n) = n^2$

   $T(n) = \theta(n^2 \log_2 n)$

3. $T(n) = T(n/2) + 2^n$

   $a = 1, b = 2$, $f(n) = 2^n$

   $c = \log_b a = \log_2 c = 0$

   $n^c = n^0 = 1$

   $f(n) > n^c$

   $T(n) = \theta(2^n)$

4. $T(n) = 2^n T(n/2) + n^n$

   $a = 2^n, b = 2, f(n) = n^2$

   $c = \log_b a = \log_2 2^n = n$

   $n^c = n^n \Rightarrow f(n) = n^c$

   $T(n) = \theta(n^2 \log_2 n)$

5. $T(n) = 16T(n/4) + n$

$a = 16, b = 4, f(n) = n$

$c = \log_4 16 = \log_4 (4)^2 = 2\log_4 4 = 2$

$n^c \Rightarrow n^2$

$f(n) < n^c$

$T(n) = \theta(n^2)$

6. $T(n) = 2T(n/2) + n\log n$

$a = 2, b = 2, f(n) = n\log n$

$c = \log_2 2 = 1$

$n^c = n^1 = n$

$n\log n > n$

$f(n) > n^c$

$T(n) = \theta(n\log n)$

7. $T(n) = 2T(n/2) + n\log n$

$a = 2, b = 2, f(n) = n\log n$

$c = \log_2 2 = 1$

$n^c = n^1 = n$

$\frac{n}{\log n} < n$

$f(n) < n^c \Rightarrow T(n) = \theta(n)$

8. $T(n) = 2T(n/4) + n^{0.51}$

$a = 2, b = 4, f(n) = n^{0.51}$

$c = \log_b a = \log_4 2 = 0.5$

$n^c = n^{0.5} \Rightarrow n^{0.5} < n^{0.5}$

$f(n) > n^c$

$T(n) = \theta(n^{0.51})$

9. $T(n) = 0.5T(n/2) + 1/n$

$a = 0.5$, $b = 2$, $a \geq 1$ but here a is 0.5, So we cannot apply Master's Theorem.

10. $T(n) = 16T(n/4) + n!$

$a = 16$, $b = 4$, $f(n) = n!$

$c = \log_b a = \log_4 16 = 2$

$n^c = n^2$

As $n! > n^2$

$T(n) = \Theta(n!)$

11. $4T(n/2) + \log n$

$a = 4$, $b = 2$, $f(n) = \log n$

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2$

$f(n) = \log n$

$\log n < n^2$

$f(n) < n^c$

$T(n) = \Theta(n^c) = \Theta(n^2)$

12. $T(n) = sqrt(n)\, T(n/2) + \log n$

$a = \sqrt{n}$, $b = 2$

$c = \log_b a = \log_2 \sqrt{n} = \frac{1}{2} \log_2 n$

$\therefore \frac{1}{2} \log_2 n < \log(n)$

$f(n) > n^c$

$T(n) = \Theta f(n) = \Theta(\log(n))$

Date : ___/___/___

13. $T(n) = 3T(n/2) + n$

$a = 3,\ b = 2,\ f(n) = n$

$c = \log_b a = \log_2 3 = 1.5849$

$n^c = n^{1.58}$

$n < n^{1.58}$

$f(n) < n^c$

$T(n) = \Theta(n^{1.58})$

14. $T(n) = 3T(n/3) + \text{sqrt}(n)$

$a = 3,\ b = 3 \Rightarrow c = \log_b a = \log_3 3 = 1$

$n^c = n^1 = n$

$c = \log_b a = \log_3 3 = 1$

$\text{sqrt}(n) < n$

$f(n) < n^c$

$T(n) = \Theta(n)$

15. $T(n) = 4T(n/2) + n$

$a = 4,\ b = 2$

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2$

$n < n^2$

$f(n) < n^c \Rightarrow T(n) = \Theta(n^2)$

16. $T(n) = 3T(n/4) + n\log n$

$a = 3,\ b = 4,\ f(n) = n\log n$

$c = \log_b a = \log_4 3 = 0.792$

$n^c = n^{0.79}$

$n^{0.79} < n\log n$

$T(n) = \Theta(n\log n)$

18. $T(n) = 6T(n/3) + n^2 \log n$

$a = 6, b = 3$

$c = \log_b a = \log_3 6 = 1.63$

$n^c = n^{1.63}$

$n^{1.63} < n^2 \log n$

$T(n) = \Theta(n^2 \log n)$

19. $T(n) = 4T(n/2) + n/\log n$

$a = 4, b = 2, f(n) = n/\log n$

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2$

$n/\log n < n^2$

$T(n) = \Theta(n^2)$

21. $T(n) = 7T(n/3) + n^2$

$a = 7, b = 3, f(n) = n^2$

$c = \log_b a = \log_3 7 = 1.77$

$n^c = n^{1.77}$

$n^{1.77} < n^2$

$T(n) = \Theta(n^2)$

22. $T(n) = T(n/2) + n(2 - \cos n)$

$a = 1, b = 2$

$c = \log_b a = \log_2 1 = 0$

$n^c = n^0 = 1$

$n(2 - \cos n) > n^c$

$T(n) = \Theta(n(2 - \cos n))$