Unit-II
Nonmonotonic Reasoning

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Syllabus

Unit-II Nonmonotonic Reasoning (05 Hours)

Types of Qualitative Nonmonotonic Reasoning, How Well Do Nonmonotonic Methods Work? Default Reasoning, Normal Default Theories, Other Modifications of Logic, Circumscription , Modal and Autoepistemic Logics , Rule Systems, Basic Concepts of Monotonic Systems, Forward versus Backward Chaining, Negation, Limiting the Effects of Contradictions, Nonmonotonicity, Semantic Nets, Frames, Manipulating Simple Inheritance Systems, Defeasible Reasoning, The Syntax of Defeasible Reasoning, The Laws of Defeasible Reasoning

Reasoning in AI

Reasoning (1)

- **Reasoning** is a way to infer facts from existing data. It is a general process of thinking rationally, to find valid conclusions. In AI, the reasoning is essential so that the machine can also think rationally as a human brain and can perform like a human.

- **Types of reasoning in AI:**

1. Deductive reasoning: It mostly starts from the general premises to the specific conclusion.
   - E.g., **Premise-1:** All the human eats veggies. **Premise-2:** Suresh is human. **Conclusion:** Suresh eats veggies.

2. Inductive reasoning: It uses historical data or various premises to generate a generic rule, for which premises support the conclusion.
   - E.g., **Premise:** All of the pigeons we have seen in the zoo are white. **Conclusion:** Therefore, we can expect all the pigeons to be white.

3. Common Sense Reasoning: It relies on good judgment rather than exact logic and operates on heuristic knowledge and heuristic rules.
   - E.g., One person can be at one place at a time.

Reasoning (2)

- **Types of reasoning in AI:** (cont..)

4. Abductive reasoning: It is an extension of deductive reasoning, but in abductive reasoning, the premises do not guarantee the conclusion. It starts with single or multiple observations then seeks to find the most likely explanation or conclusion for the observation. It involves generating hypotheses to explain observed phenomena. It's particularly useful when there are multiple possible explanations for a given observation, allowing us to generate the most likely explanations given the available evidence. However, these hypotheses may need to be revised as new evidence is obtained.
   – E.g., **Implication:** Cricket ground is wet if it is raining. **Axiom:** Cricket ground is wet. **Conclusion:** It is raining.

5. Causal Reasoning: It helps us understand the relationships between causes and effects. However, these inferences may need to be revised if additional information is provided, such as discovering exceptions to the causal relationship.
   – E.g., If a major road closure leads to increased traffic congestion in surrounding areas, we infer a causal relationship between the road closure and the congestion.

Reasoning (3)

- **Types of reasoning in AI:** (cont..)

6. Probabilistic Reasoning: It deals with uncertainty by assigning probabilities to different hypotheses. It allows us to make decisions even when we lack complete information, but these decisions may change as new evidence becomes available.
   – E.g., The forecaster uses probabilistic reasoning to estimate the likelihood of rain occurring tomorrow based on current atmospheric conditions, historical weather patterns, and forecast models. It understands that weather prediction involves uncertainty due to the complexity of atmospheric processes.

7. Analogical Reasoning: It involves drawing parallels between different situations or objects based on their similarities. It's useful for making inferences when direct evidence is lacking, but the conclusions drawn may need to be revised if additional information contradicts the analogy.
   – E.g., One person can be at one place at a time.

Reasoning (2)

- **Types of reasoning in AI:** (cont..)

8. Monotonic Reasoning: In monotonic reasoning, once the conclusion is taken, then it will remain the same even if we add some other information to existing information in our knowledge base.
   - In monotonic reasoning, adding knowledge does not decrease the set of prepositions that can be derived.
   - To solve monotonic problems, we can derive the valid conclusion from the available facts only, and it will not be affected by new facts.
   - Monotonic reasoning is not useful for the real-time systems, as in real time, facts get changed, so we cannot use monotonic reasoning.
   - Monotonic reasoning is used in conventional reasoning systems, and a logic-based system is monotonic.
   - Any theorem proving is an example of monotonic reasoning.
   - E.g., **Earth revolves around the Sun.** It is a true fact, and it cannot be changed even if we add another sentence in knowledge base like, "The moon revolves around the earth" Or "Earth is not round," etc.
   - **Advantages of Monotonic Reasoning:**
     - In monotonic reasoning, each old proof will always remain valid.
     - If we deduce some facts from available facts, then it will remain valid for always.
   - **Disadvantages of Monotonic Reasoning:**
     - We cannot represent the real-world scenarios using monotonic reasoning.
     - Hypothesis knowledge cannot be expressed with monotonic reasoning, which means facts should be true.
     - Since we can only derive conclusions from the old proofs, so new knowledge from the real world cannot be added.

Reasoning (3)

- **Types of reasoning in AI:** (cont..)

9. Nonmonotonic Reasoning: In nonmonotonic reasoning, some conclusions may be invalidated if we add some more information to our knowledge base.
   - Logic will be said as nonmonotonic if some conclusions can be invalidated by adding more knowledge into our knowledge base.
   - Nonmonotonic reasoning deals with incomplete and uncertain models.
   - "Human perceptions for various things in daily life" is a general example of non-monotonic reasoning.
   - E.g., **Premise-1:** Birds can fly. **Premise-2:** Penguins cannot fly. **Premise-3:** Tweety is a bird. **Conclusion:** Tweety can fly (From these 3 sentences). However, suppose we add one another sentence into knowledge base **Premise-4:** Tweety is a penguin. It gives **New Conclusion:** Tweety cannot fly. It invalidates the old conclusion.
   - **Advantages of Nonmonotonic Reasoning:**
     - For real-world systems such as Robot navigation, we can use non-monotonic reasoning.
     - In Non-monotonic reasoning, we can choose probabilistic facts or can make assumptions.
   - **Disadvantages of Nonmonotonic Reasoning:**
     - In non-monotonic reasoning, the old facts may be invalidated by adding new sentences.
     - It cannot be used for theorem proving.

Nonmonotonic Reasoning

Monotonic Vs Nonmonotonic Reasoning

Sr. No.	Monotonic Reasoning	Nonmonotonic Reasoning
1	Monotonic Reasoning is the process which does not change its direction or can say that it moves in the one direction.	Nonmonotonic Reasoning is the process which changes its direction or values as the knowledge base increases.
2	Monotonic Reasoning deals with very specific type of models, which has valid proofs.	Nonmonotonic reasoning deals with incomplete or not known facts.
3	The addition in knowledge won't change the result.	The addition in knowledge will invalidate the previous conclusions and change the result.
4	In monotonic reasoning, results are always true, therefore, set of prepositions will only increase.	In nonmonotonic reasoning, results and set of prepositions will increase and decrease based on condition of added knowledge.
5	Monotonic Reasoning is based on true facts.	Non-monotonic Reasoning is based on assumptions.
6	Deductive Reasoning is the type of monotonic reasoning.	Abductive Reasoning and Human Reasoning is a non-monotonic type of reasoning.

Nonmonotonic Reasoning

- By definition, nonmonotonic reasoning methods allow us to reach conclusions that may become invalid as we gain further knowledge. As a result: It is a priori necessary to examine the entire knowledge base before reaching a conclusion.
- This leads to a core problem for nonmonotonic methods whose solution is crucial for designing an algorithm that runs in a reasonable time:
  1. How can we limit our attention to a (small) part of the knowledge base and still guarantee that the conclusion reached will be correct?
  2. During the reasoning process, how do we detect missing knowledge and make reasonable assumptions about it?

- Inadequacy of classical logic:
- We cannot represent a rule such as "typically birds fly" as $\forall x(bird(x) \wedge \neg exception(x) \rightarrow fly(x))$ and then to add $\forall x(exception(x) \leftrightarrow penguin(x) \vee ostrich(x) \vee canary(x) \vee ...)$
- We do not know in advance all exceptions.
- In order to conclude that "Tweety flies" we should prove that "tweety is not an exception", that is: $\neg penguin(tweety), \neg ostrich(tweety), ...$
- On the contrary we would like to prove that Tweety flies because we cannot conclude that it is an exception, not because we can prove that it is not an exception.

Closed World Assumption

- A basic understanding of database logic, is that only positive information is represented explicitly.
- Negative information is not represented explicitly. If a positive fact is not present in the database (DB), it is assumed that its negation holds.
- This is called Closed World Assumption: the only true facts are the provable ones.
- If DB $\nvdash$ A then DB $\vdash_{CWA} \neg A$
- This inference is not valid in classical logic.

- Example: suppose a DB contains facts of the form "practice(person, sport)", for instance:
  - practice(anne, tennis)
  - practice(joe, tennis)
  - practice(anne, sky)
- Then we have DB $\vdash_{CWA}$ ¬practice(joe, sky)
- Trivially CWA is non-monotonic, since adding a fact may lead to withdraw the negative conclusion:
- DB ∪ {practice(joe, sky)} $\nvdash_{CWA}$ ¬practice(joe, sky)

Types of Qualitative Nonmonotonic Reasoning

Types of Qualitative Nonmonotonic Reasoning (1)

1. Default Logic:
   - It isolates beliefs in a new type of knowledge-base statement, the default rule, which is used to produce additional FOL statements (in other words, assumptions).
   - These assumptions supply information that is incomplete in the FOL part of the knowledge base. Thus, beliefs are contained in a part of the knowledge base that has limited interaction with the remainder and all reasoning is done in FOL.
   - The theory describes how to decide if a set of assumptions is acceptable, but it doesn't tell us how to find such sets. Which defaults should we assume? What if defaults contradict one another?
   - Default logic can express facts like "by default, something is true"; by contrast, standard logic can only express that "something is true" or that "something is false". This is a problem because reasoning often involves facts that are true in the majority of cases but not always.
   - A classical example is: "birds typically fly". This rule can be expressed in standard logic either by "all birds fly", which is inconsistent with the fact that penguins do not fly, or by "all birds that are not penguins and not ostriches and ... fly", which requires all exceptions to the rule to be specified.
   - Default logic aims at formalizing inference rules like this one without explicitly mentioning all their exceptions.

Types of Qualitative Nonmonotonic Reasoning (2)

2. Defeasible Reasoning:
   - It introduces new reasoning rules as well as new types of knowledge-base statements on which these rules act.
   - It is a particular kind of non-demonstrative reasoning, where the reasoning does not produce a full, complete, or final demonstration of a claim, i.e., where fallibility and corrigibility of a conclusion are acknowledged.
   - Superficially, defeasible reasoning is like default reasoning in that it augments FOL with rules of inference for dealing with rules that have exceptions.
   - There are important differences. Default logic emphasizes the logical concepts whereas defeasible reasoning emphasizes the reasoning methods.
   - Defeasible reasoning is used when there are specific conditions or exceptions that can invalidate the default conclusions. Defeated conclusions are replaced or revised based on the new evidence or exceptions.
   - The amount of FOL that is allowed depends on the default logic. Some limit the FOL formulas to Prolog-like statements.
   - E.g. “All experienced employees are competent.” This rule is defeasible because there may be exceptions; some experienced servers may not be competent due to lack of training, poor performance, etc.

Types of Qualitative Nonmonotonic Reasoning (3)

3. Autoepistemic Logic and Modal Logics:
   - Epistemology is the branch of philosophy that investigates the nature and limits of human knowing.
   - Autoepistemic logic is a form of non-classical logic that deals with self-knowledge or beliefs about one's own beliefs. It allows for reasoning about what an agent knows or believes, including reasoning about its own lack of knowledge or potential inconsistencies in its beliefs.
   - E.g., Suppose Alice knows that if it's raining, then the ground is wet. However, she's uncertain about whether it's currently raining or not. we can represent Alice's knowledge and uncertainty using modal operators.
     - We might denote "Alice knows that p" as K(p) and "Alice is uncertain about p" as U(p). Therefore, we can represent Alice's knowledge and uncertainty as follows:
       - Alice knows that if it's raining, then the ground is wet: K(Raining → WetGround)
       - Alice is uncertain about whether it's raining: U(Raining)
     - Using autoepistemic logic, we can reason about Alice's beliefs, including potential inconsistencies between what she knows and what she's uncertain about.
   - Autoepistemic logic is an example of modal logic, which is an extension of FOL that introduces one or more modal operators to indicate states of belief or (partial validity). Modal logics extend classical propositional logic by introducing modal operators that allow for reasoning about necessity, possibility, belief, knowledge, and other modalities. These modal operators modify propositions to express statements about what is necessary, possible, believed, known, etc.
   - Just as it is necessary to develop a new theory when we move from propositional logic to FOL, it’s necessary to develop a new theory when we move from FOL to autoepistemic logic.

Types of Qualitative Nonmonotonic Reasoning (4)

4. Circumscription:
   - Circumscription extends the knowledge base rather than the language of FOL.
   - In circumscription, a circumscription formula is used to describe a minimal or simplest possible interpretation of a situation. The formula specifies what should be minimized or restricted to make the interpretation consistent with observed facts while avoiding unnecessary assumptions.
   - A circumscriptive approach might attempt to make the predicate “normal” apply to as many terms as possible. This sounds very much like default logic and, indeed, there are connections between the two approaches.
   - It introduces predicates for abnormality but makes no changes to FOL and introduces no new types of statements. Typically, we introduce one or more predicates whose truth indicates an abnormal situation.
   - Beliefs are expressed as predicate logic rules in the knowledge base by insisting that the abnormal predicate be false—the situation is not abnormal. If the abnormal predicates were always false, the knowledge base would be inconsistent.
   - To maintain consistency, circumscriptive reasoning adds new FOL statements to the knowledge base that assert that abnormality is true in certain situations. The number of additional statements(abnormalities) should be kept small.
   - In summary, circumscription provides a formal framework for reasoning about defaults and exceptions, allowing us to minimize assumptions and explain observed behavior in a consistent and concise manner.

Types of Qualitative Nonmonotonic Reasoning (5)

4. Circumscription (cont...):
   - E.g., Suppose we have the following facts: Tweety is a bird. Birds typically fly.
   - We want to formalize the concept that "Birds typically fly" while accounting for exceptions, such as flightless birds like penguins.
   - **Default Assumption**: We start with a default assumption that "Birds typically fly," which we can express as follows:
     - $\forall x(Bird(x) \rightarrow Fly(x))$
     - ∀x(Bird(x)→Fly(x))
   - **Observations:** We observe that Tweety is a bird (Bird(Tweety)). However, Tweety is not observed to fly (¬Fly(Tweety)).
   - **Circumscription:** Using circumscription, we aim to minimize the set of assumptions while still explaining the observed facts. We introduce a circumscription formula to restrict the extension of the predicate "Fly" to the minimal set necessary to make the observation consistent with the default assumption. The circumscription formula might look like this: $\exists x\ (Bird(x) \wedge (\neg \mathrm{Fly}(x))$.
   - This formula says that there exists at least one bird that does not fly. It allows for the existence of flightless birds like penguins without contradicting the default assumption that "Birds typically fly."
   - **Interpretation:** In this interpretation, the default assumption ("Birds typically fly") is upheld, but it's minimized by allowing for exceptions (flightless birds) to explain the observed fact that Tweety does not fly. The circumscription formula ensures that we make the fewest assumptions necessary to account for the observed facts while preserving the default assumption where applicable.

How Well Do Nonmonotonic Methods Work? (1)

- Goal:
  - The holy grail of nonmonotonic researchers is commonsense reasoning. An important facet of such reasoning is the ability to reach sensible conclusions when faced with incomplete and/or uncertain information. Hence, designing methods for doing so is a primary goal of AI research on nonmonotonic reasoning.
- Difficulties:
  - Nonmonotonic methods have been quite successful in some limited environments but seem far from being able to duplicate commonsense reasoning.
  - Core problems for nonmonotonic reasoning: problem complexity, limiting search of the knowledge base, detecting missing knowledge, and making reasonable assumptions.
  - Extending FOL makes these problems essentially intractable:
  - **Problem 1:** FOL is NP-hard: Since no efficient algorithm is known or likely to exist for FOL, the same is true for any extension of it. preparing a general FOL knowledge base for Horn clause resolution is NP-hard.

  - **Problem 2:** FOL is semidecidable: Missing information is detected by being unable to prove something: If the truth of ($\neg\alpha$) is not implied by the data, we may be allowed to assume that a is true. Unfortunately, FOL is semidecidable.
    - This means that although we can create algorithms (such as resolution) that allow us to prove any statement which is true in an FOL language, it is impossible to design an algorithm that will allow us to decide if statements are true or false.
    - What this means in practice is that there cannot be an algorithm to decide if statements are undecidable; that is, we can't create an algorithm to decide if additional information is needed to determine the truth or falsity of a statement.

How Well Do Nonmonotonic Methods Work? (2)

- Difficulties: (cont..)
  - **Problem 3:** Consistency is hard: Whatever assumptions are made should be consistent with one another and with the data base. It is often difficult to prove consistency.
  - **Problem 4:** Selecting assumptions is unclear: If the consistency problem is overcome, how should the reasoning system choose between competing consistent assumptions? The problem here is both conceptual (what should the decision criteria be) and procedural (how can they be implemented in a reasonable manner).
- Compromises:
  - The first two problems with FOL can be dealt with in at least three ways:
    1. The expressive capability of the language can be limited to the point where both problems disappear.
    2. The difficulty can be dumped onto the shoulders of the knowledge-base designer.
    3. They can be ignored.
  - Limiting the expressive capability of the language can make dealing with assumptions easier, too—particularly the problem of determining their consistency.
  - A conservative compromise is to make only those assumptions that, in some sense, must be true. Unfortunately, this approach may result in too few assumptions.
  - Another solution is to have some method for choosing one set of assumptions over another. For example, given two assumptions (“Birds fly” and “Penguins don’t fly”), we choose the less general one (“Penguins don’t fly”).

Default Reasoning

Default Reasoning (1)

- Introduced by R. Reiter in 1980.
- The semantic and syntactic definitions of **consistent:**
- Semantically, we say that β is consistent with S if there is an interpretation in which both β and the formulas in S are true.
- Syntactically, β is consistent with S if $S \nvdash (\neg \beta)$. These definitions are equivalent in FOL because of the soundness and completeness of the proof method for FOL. We'll extend the syntactic definition to default logic.

- Default logic extends classical logic by non-standard inference rules. These rules allows one to express default properties.
- Example: $$\frac{bird(x) \ : fly(x)}{fly(x)}$$

  that can be interpreted as:

  "if x is a bird and we can consistently assume that x flies then we can infer that x flies".

- Default logic extends FOL by allowing default rules in the knowledge base. An extension represents the set of plausible conclusions. A default-theory may have zero, one, or many extensions.

Default Reasoning (2)

- Default Rule:

If α, β, and γ are formulas in FOL with no free variables, then

$$\frac{\alpha : \beta}{\gamma} \quad \text{which we can also write as} \quad (\alpha : \beta) \rightharpoonup \gamma$$

is a ***default rule***, also called a ***default***. (The notation $(\alpha : \beta) \rightharpoonup \gamma$ is ***not*** standard.) This rule means "if α is true and β is consistent with what is true, then we may assume that γ is true." There are two special cases:

- The rule $(\alpha :) \rightharpoonup \gamma$ means "if α is true, then we *may* assume that γ is true." This is not the same as $\alpha \to \gamma$, which says that if α is true, then we *must* assume that γ is true.
- The rule $(: \beta) \rightharpoonup \gamma$ means "if β is consistent with our beliefs, then we may assume that γ is true."

Default Reasoning (3)

To indicate that $(\alpha(X) : \beta(X)) \to \gamma(X)$ is a default rule when any substitution is made for X, we write

$$\forall X \left(\frac{\alpha(X) : \beta(X)}{\gamma(X)} \right) \quad \text{or, more simply,} \quad \frac{\alpha(X) : \beta(X)}{\gamma(X)}. \tag{6.5}$$

The extension of this definition to more than one variable should be obvious.

Often, $\mathcal{D}$ and $\mathcal{W}$ denote the sets of default and FOL formulas, respectively.

Definition 6.1 says that we *may* assume γ is true—not that we *must* assume it is true. Because of the may/must distinction, it's important to regard (6.5) as representing a collection of default rules instead of one single rule. If the substitution of t for X gives an inconsistent $\beta(t)$, then we cannot assume $\gamma(t)$. If the substitution of t for X gives a consistent $\beta(t)$ and a true $\alpha(t)$, then we may assume $\gamma(t)$ but need not do so. Consequently, we might assume some $\gamma(t)$'s and not others.

- Terminology: $\alpha(x)$: the prerequisite, $\beta(x)$: the justification, $\gamma(x)$: the consequent.

Default Reasoning (4)

- A default theory is a pair <D, W>, where *D* is a set of default rules and *W* is a set of first-order formulas.
- Example: let <*D, W*> be

$$D = \{\frac{bird(x) : fly(x)}{fly(x)}\}$$

$$\begin{aligned} W = &\{bird(tweety), \forall x(penguin(x) \rightarrow bird(x)), \\ &\forall x(penguin(x) \rightarrow \neg fly(x))\} \end{aligned}$$

- Intuitively, in a default theory <*D, W*>:
  - *W* represents the stable (but incomplete) knowledge of the world
  - *D* rules for extending the knowledge W by plausible (but defeasible) conclusions.
  - Notion of extension of a default theory: the theory (= deductively closed set of logical formulas) obtained by extending *W* by the rules in *D*.

- Example: Let $< D, W>$ be as in the previous example.
- Since $bird(tweety)$ is true, and it is consistent to assume $fly(tweety)$, then $fly(tweety)$ is true in the (unique) extension of $< D, W>$.
- Consider now the default theory $< D, W'>$, where $W' = W \cup \{penguin(tweety)\}$ then the assumption $fly(tweety)$ is no longer consistent, and the application of the default rule is blocked.

Default Reasoning (5)

- Interpretation of Defaults:

$A : B1, \ldots, Bn / C$

If A is currently known,
and if it is consistent to assume B1, . . ., Bn,
then conclude C.

$A : B1, \ldots, Bn / C$ is ***applicable*** to a deductively closed set of formulas E iff $A \in E$ and $\neg B1 \notin E, \ldots, \neg Bn \notin E$.

Extensions (1)

- Extensions are "world views" that are based on the given default theories or information (facts and defaults).
- Extensions are obtained from the application of some defaults in D. They include always the certain information W.
- They seek to extend the set of known facts with “reasonable” conjectures based on the available defaults.
- Desirable properties of extensions:
  - an extension should include the set W of facts—the certain information
  - an extension should be deductively closed (Keep classical reasoning! Derive more from the defaults.)
  - an extension should be closed under the application of the defaults. Apply defaults exhaustively.

Formally: if $\frac{\varphi : \psi_1, \ldots, \psi_n}{\chi} \in D, \varphi \in E$ and $\neg\psi_1 \notin E, \ldots, \neg\psi_n \notin E$ then $\chi \in E$.

Extensions(2)

Let a default theory $\Delta = (\mathcal{D}, \mathcal{W})$ be given and let $\mathcal{S}$ be any collection of FOL formulas. Define $\Gamma(\mathcal{S})$ to be the set of FOL formulas such that

(a) $\mathrm{Th}(\Gamma(\mathcal{S})) = \Gamma(\mathcal{S}) \supseteq \mathcal{W}$.

(b) If $\frac{\alpha:\beta}{\gamma} \in \mathcal{D}$, $\alpha \in \Gamma(\mathcal{S})$, and $\neg\beta \notin \mathcal{S}$, then $\gamma \in \Gamma(\mathcal{S})$.

(c) $\Gamma(\mathcal{S})$ is a minimum with respect to (a) and (b); that is, if Γ' satisfies (a) and (b), then $\Gamma' \supseteq \Gamma(\mathcal{S})$.

Finally, if $\Gamma(\mathcal{S}) = \mathcal{S}$, we call $\mathcal{S}$ an *extension* of Δ.

Extensions(3)

1. "Ungrounded" beliefs

An extension must not contain "ungrounded" beliefs, i.e. every formula in the extension must be derivable from W and the consequents of applied defaults. We require extensions to be minimal w.r.t. to these properties.

Consider: $T = (W, D)$ with $W = \{german\}$ and $D = \left\{ \frac{german : drinksBeer}{drinksBeer} \right\}$

Now, $E = Th(\{german, \neg drinksBeer\})$ is minimal w.r.t. to the properties, but unintuitive.

2. Applications of defaults can contradict the application of an earlier default.

Consider:

$$\frac{true \ : \ creditworthy}{approveCredit}, \quad \frac{true \ : \ \neg creditworthy}{\neg creditworthy}$$

We apply the first default, since nothing contradicts the assumption *creditworthy*. We then apply the second, since $\neg creditworthy$ is consistent with the knowledge, $\neg creditworthy$ is derived.

Inclusion of $\neg creditworthy$ shows a-posteriori that, we should not have assumed *creditworthy*.

Normal Default Theories

Normal Default Theories (1)

- A default d is normal if its consequent is its only justification. They have the form: $\frac{\alpha : \beta}{\beta}$
- A normal default theory $\Delta = < W, D >$ is a default theory where all defaults in D are normal.
- A normal default theory has always an extension.
- It has limited expressivity: no interactions among defaults.
- A normal default theory draws the conclusion when is known and it is consistent to conclude .
- E.g., Statements such as:

  "Typically, birds fly"

  "Assume the accused is innocent unless you know otherwise"

  can be captured by normal defaults:

$$\frac{bird(x) \; : \; flies(x)}{flies(x)} \qquad \frac{accused(x) \; : \; innocent(x)}{innocent(x)}$$

Normal Default Theories (2)

- Limitations of normal default theories:
- Often a default rule on its own is normal, but problems arise when several defaults have to interact in a theory.
- Consider the example of a normal default theory:

$$T = \left(\{dropout(bill)\}, \left\{ \frac{dropout(x) \ : \ adult(x)}{adult(x)}, \frac{adult(x) \ : \ employed(x)}{empolyed(x)} \right\} \right)$$

- Th as the single extension Th({dropout(bill), adult(bill), employed(bill)}).
- But it is counterintuitive to assume that Bill is employed!
- How to prevent the application of the 2 default, if X is a dropout?

$$\frac{adult(x) \ : \ employed(x) \wedge \neg dropout(x)}{employed(x)}$$

- But this is no longer a normal default!

Normal Default Theories (3)

- Limitations of normal default theories: (cont..)

1) Unwanted Transitivity:

- Unwanted transitivity: let $\Delta = < W, D >$, where $W = \{student\}$ and

$$D = \{ d_1 = \frac{student : adult}{adult}, d_2 = \frac{adult : works}{works} \}$$

- it is easy to see that Δ has a unique extension including $\{student, works, adult\}$.
- it is rather unintuitive (as students usually do not work).
- if we add the default $\frac{student : \neg work}{\neg work}$, the theory has then two extensions:
  $E_1 = \{student, adult, works\}$
  $E_2 = \{student, adult, \neg works\}$
- But E_2 is more plausible than E_1

Normal Default Theories (4)

- Limitations of normal default theories: (cont..)

1) **Unwanted Transitivity:**

- Solution: replace d_2 by:

$$\frac{adult : works \wedge \neg student}{works}$$

- then the only extension is $E_2 = \{student, adult, \neg works\}$
- this default is not normal
- it is ***semi-normal***: the justification implies the consequent
- a semi-normal default theory (= a theory where all default are semi-normal) may have no extensions

- Do semi-normal default theories always have extensions? ---No.
- Only some classes of restricted semi-normal theories do always have extensions.

Normal Default Theories (5)

- Limitations of normal default theories: (cont..)

2) **Specificity:**

- Handling specificity: let $\Delta = < W, D >$, where $W = \{user, blacklisted\}$ and

$$D = \{d_1 = \frac{user : login}{login}, d_2 = \frac{user \wedge blacklisted : \neg login}{\neg login}\}$$

- the theory has then two extensions:
  $E_1 = \{user, blacklisted, login\}$
  $E_2 = \{user, blacklisted, \neg login\}$
- But of course only E_2 is the intended one.

- The problem of *specificity* can be handled by assigning a priority to defaults on the base of their specificity.
- The priority order is taken into account for calculating extensions.
- Reiter's Default logic has also other problems (e.g. cumulativity)

Thank you!