

ASSIGNMENT-2

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Download all python codes from

<https://github.com/behappy0604/Summer-Internship-IITH/tree/main/Assignment-3>

and latex-tikz codes from

<https://github.com/behappy0604/Summer-Internship-IITH/tree/main/Assignment-3>

1 QUESTION No. 2.60

Let ABC be a right triangle in which $a = 8, c = 6$ and $\angle B = 90^\circ$. BD is the perpendicular from B on AC (altitude). The circle through B, C, D (circum-circle of $\triangle BCD$) is drawn. Construct the tangents from A to this circle.

2 SOLUTION

Data from the given question

	Symbols	Circle
Centre	E	$\begin{pmatrix} 4 \\ 0 \end{pmatrix}$
Radius	r	4

1) Let us generalise the given data:

$$B = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, A = \begin{pmatrix} 0 \\ 6 \end{pmatrix}, C = \begin{pmatrix} 8 \\ 0 \end{pmatrix} \quad (2.0.1)$$

$$\angle B = 90^\circ \quad (2.0.2)$$

$$\angle D = 90^\circ (\because BD \perp AC) \quad (2.0.3)$$

$$BC = 8 \quad (2.0.4)$$

$$AC = 10 (\text{Using Pythagoras's Theorem}) \quad (2.0.5)$$

$$AB = 6 \quad (2.0.6)$$

2) Let E be the the midpoint of BC , therefore

$$E = \begin{pmatrix} 4 \\ 0 \end{pmatrix} \quad (2.0.7)$$

3) Now taking E as center we will draw a circle of radius 4 which will circumscribe $\triangle BCD$.

4) Tangents to this circle from point A will be AB and AP as shown in the figure.

5) Using sine formula we get,

$$\angle BAE = 33.69^\circ \quad (2.0.8)$$

$$\angle BAC = 53.18^\circ \quad (2.0.9)$$

$$BD = 4.8 \quad (2.0.10)$$

6) In $\triangle BDC$, using Pythagoras theorem we get:

$$DC = 6.4 \quad (2.0.11)$$

therefore,

$$AD = 3.6 (\because AC = AD + DC) \quad (2.0.12)$$

7) Using Sine formula we get:

$$\angle DEC = 106.48^\circ \quad (2.0.13)$$

therefore,

$$D = E + \begin{pmatrix} r \cos E \\ r \sin E \end{pmatrix} \quad (2.0.14)$$

$$D = \begin{pmatrix} 4 \\ 0 \end{pmatrix} + \begin{pmatrix} -1.12 \\ 3.84 \end{pmatrix} \quad (2.0.15)$$

$$D = \begin{pmatrix} 2.88 \\ 3.84 \end{pmatrix} \quad (2.0.16)$$

8) Here,

$$\angle PEC = 2\angle BAE \quad (2.0.17)$$

$$\angle PEC = 2 \times 33.69^\circ \quad (2.0.18)$$

$$\angle PEC = 67.38^\circ \quad (2.0.19)$$

9) Now coordinates of \mathbf{P} from center of circle \mathbf{E} will be,

$$\mathbf{P} = \mathbf{E} + \begin{pmatrix} r \cos 67.38^\circ \\ r \sin 67.38^\circ \end{pmatrix} \quad (2.0.20)$$

$$\mathbf{P} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} + \begin{pmatrix} 1.53 \\ 3.69 \end{pmatrix} \quad (2.0.21)$$

$$\mathbf{P} = \begin{pmatrix} 5.53 \\ 3.69 \end{pmatrix} \quad (2.0.22)$$

10) Therefore, we have coordinates as,

$$\mathbf{A} = \begin{pmatrix} 0 \\ 6 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 8 \\ 0 \end{pmatrix}, \mathbf{E} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}, \mathbf{P} = \begin{pmatrix} 5.53 \\ 3.69 \end{pmatrix},$$

$$\mathbf{D} = \begin{pmatrix} 2.88 \\ 3.84 \end{pmatrix}$$

11) On constructing the given figure we get:

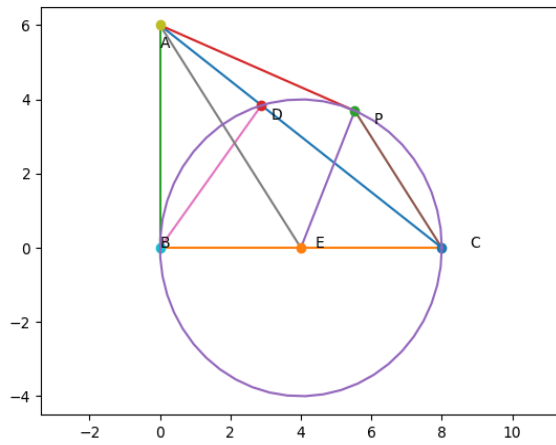


Fig. 2.1: Tangents to a Circle