ASSIGNMENT-2

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Download all python codes from

```
https://github.com/behappy0604/Summer-Internship-IITH/tree/main/Assignment-2
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and latex-tikz codes from

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https://github.com/behappy0604/Summer-Internship-IITH/tree/main/Assignment-2
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1 Question No. 2.39

Construct a quadrilateral MORE where $MO = 6, OR = 4.5, \angle M = 60°, \angle O = 105°$ and $\angle R = 105°$.

2 Solution

1) Let us generalize the given data:

$$\angle M = 60° = \theta \quad (2.0.1)$$
$$\angle O = 105° = \alpha \quad (2.0.2)$$
$$\angle R = 105° = \gamma \quad (2.0.3)$$
$$\|\mathbf{O} - \mathbf{M}\| = 6 = a, \quad (2.0.4)$$
$$\|\mathbf{R} - \mathbf{O}\| = 4.5 = b, \quad (2.0.5)$$
$$\mathbf{M} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{O} = \begin{pmatrix} 6 \\ 0 \end{pmatrix} \quad (2.0.6)$$

2) Also, Let us assume the other two sides as

$$\|\mathbf{R} - \mathbf{E}\| = c \quad (2.0.7)$$
$$\|\mathbf{M} - \mathbf{E}\| = d \quad (2.0.8)$$
$$\theta = \theta_1 + \theta_2 \quad (2.0.9)$$
$$\delta = 180° - \alpha = 75° \quad (2.0.10)$$

3) Now on calculating, we get

$$\implies \angle E + 270° = 360°, \quad (2.0.11)$$
$$\implies \angle E = 90° \quad (2.0.12)$$

4) Now taking sum of all the angles given and (2.0.12) we get

$$\angle M + \angle O + \angle R + \angle E = 360° \quad (2.0.13)$$

So construction of given quadrilateral is possible as sum of all the angles is equal to 360°.

5) Now, using cosine formula in $\triangle MOR$ we can find RM:

$$\implies \|\mathbf{R} - \mathbf{M}\|^2 = \|\mathbf{M} - \mathbf{O}\|^2 + \|\mathbf{O} - \mathbf{R}\|^2 - 2 \times \|\mathbf{M} - \mathbf{O}\| \times \|\mathbf{O} - \mathbf{R}\| \cos O \quad (2.0.14)$$

$$\implies RM = 8.38 \quad (2.0.15)$$
$$\implies \theta = \arcsin 31.24° \quad (2.0.16)$$

6) Now in $\triangle MER$,we know

$$\angle M = 28.76°, \angle E = 90°. \quad (2.0.17)$$

We know that sum of the angles of a triangle is 180°

$$\implies \angle M + \angle E + \angle R = 180° \quad (2.0.18)$$

$$\implies 28.76° + 90° + \angle R = 180° \quad (2.0.19)$$

$$\implies \angle R = 61.24° \quad (2.0.20)$$

7) Now applying sine law of triangle, we get EM= 7.34=d

Lemma 2.1. *Exact co-ordinates of the given R and E can be expresssed as*

$$\mathbf{R} = \mathbf{O} + |b| \times \begin{pmatrix} \cos\delta \\ \sin\delta \end{pmatrix} \quad (2.0.21)$$

$$\mathbf{E} = \mathbf{M} + |d| \times \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} \quad (2.0.22)$$

Proof. • Calculating the co-ordinates of R:

The vector equation of a line can be given by as,

$$\implies \mathbf{R} = \mathbf{O} + |b| \times \begin{pmatrix} \cos 75° \\ \sin 75° \end{pmatrix} \quad (2.0.23)$$

$$\implies \mathbf{R} - \mathbf{O} = b \times \begin{pmatrix} \cos 75^\circ \\ \sin 75^\circ \end{pmatrix} \tag{2.0.24}$$

$$\implies \|\mathbf{R} - \mathbf{O}\| = |b| \times \|\begin{pmatrix} \cos 75^\circ \\ \sin 75^\circ \end{pmatrix}\| \tag{2.0.25}$$

$$\implies 4.5 = |b| \times 1 \tag{2.0.26}$$

$$\implies b = 4.5 \tag{2.0.27}$$

Putting the values in the above equation we get,

$$\implies \mathbf{R} = \begin{pmatrix} 6 \\ 0 \end{pmatrix} + 4.5 \times \begin{pmatrix} 0.259 \\ 0.965 \end{pmatrix} \tag{2.0.28}$$

$$\implies \mathbf{R} = \begin{pmatrix} 6 \\ 0 \end{pmatrix} + \begin{pmatrix} 1.16 \\ 4.35 \end{pmatrix} \tag{2.0.29}$$

$$\mathbf{R} = \begin{pmatrix} 7.16 \\ 4.35 \end{pmatrix} \tag{2.0.30}$$

- Calculating the co-ordinates of E:

The vector equation of a line can be given by as,

$$\implies \mathbf{E} - \mathbf{M} = d \times \begin{pmatrix} \cos 60^\circ \\ \sin 60^\circ \end{pmatrix} \tag{2.0.31}$$

$$\implies \|\mathbf{E} - \mathbf{M}\| = |d| \times \|\begin{pmatrix} 0.50 \\ 0.87 \end{pmatrix}\| \tag{2.0.32}$$

$$\implies 7.34 = |d| \times 1 \tag{2.0.33}$$

$$\implies d = 7.34 \tag{2.0.34}$$

Putting the values in the above equation we get,

$$\implies \mathbf{E} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + 7.34 \times \begin{pmatrix} 0.50 \\ 0.87 \end{pmatrix} \tag{2.0.35}$$

$$\implies \mathbf{E} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 3.67 \\ 6.36 \end{pmatrix} \tag{2.0.36}$$

$$\mathbf{E} = \begin{pmatrix} 3.67 \\ 6.36 \end{pmatrix} \tag{2.0.37}$$

□

8) Now, we have the coordinate of vertices M,O,R,E as,

$$\mathbf{M} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{O} = \begin{pmatrix} 6 \\ 0 \end{pmatrix}, \mathbf{R} = \begin{pmatrix} 7.16 \\ 4.35 \end{pmatrix}, \mathbf{E} = \begin{pmatrix} 3.67 \\ 6.36 \end{pmatrix}. \tag{2.0.38}$$

9) On constructing the given quadilateral on python and marking angle we get:

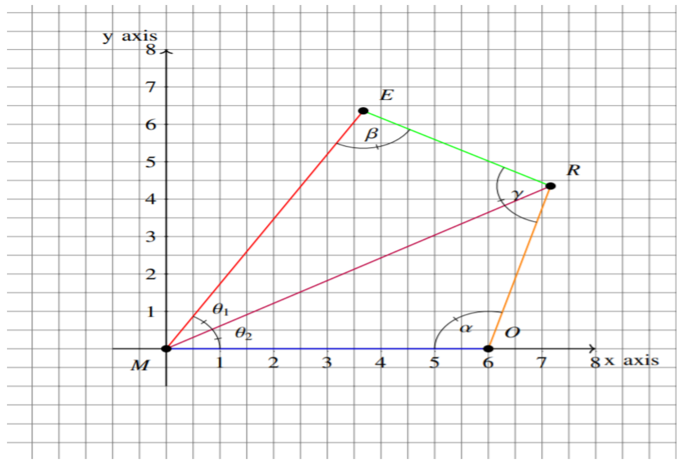

Fig. 2.1: Quadrilateral MORE