# ASSIGNMENT-2

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Download all python codes from

```
https://github.com/behappy0604/Summer−
    Internship−IITH/tree/main/Assignment−3
```

and latex-tikz codes from

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https://github.com/behappy0604/Summer−
    Internship−IITH/tree/main/Assignment−3
```

## 1 Question No. 2.60

Let ABC be a right triangle in which $a = 8, c = 6$ and $\angle B = 90^{\circ}$. $BD$ is the perpendicular from $\mathbf{B}$ on $AC$ (altitude). The circle through $\mathbf{B}, \mathbf{C}, \mathbf{D}$ (circumcircle of $\triangle BCD$) is drawn. Construct the tangents from $\mathbf{A}$ to this circle.

## 2 Solution

Data from the given question

| | Symbols | Circle |
|---|---|---|
| Centre | $\mathbf{E}$ | $\begin{pmatrix}4\\0\end{pmatrix}$ |
| Radius | $r$ | 4 |

1) Let us generalise the given data:

$$\mathbf{B} = \begin{pmatrix}0\\0\end{pmatrix}, \mathbf{A} = \begin{pmatrix}0\\6\end{pmatrix}, \mathbf{C} = \begin{pmatrix}8\\0\end{pmatrix} \tag{2.0.1}$$

$$\angle B = 90^\circ \tag{2.0.2}$$

$$\angle D = 90^\circ (\because BD \perp AC) \tag{2.0.3}$$

$$\mathbf{BC} = 8 \tag{2.0.4}$$

$$\mathbf{AC} = \mathbf{AD} + \mathbf{DC} = 10 (Using Pythagoras's Theorem) \tag{2.0.5}$$

$$\mathbf{AB} = 6 \tag{2.0.6}$$

2) Let $\mathbf{E}$ be the the midpoint of $\mathbf{BC}$, therefore

$$\mathbf{E} = \begin{pmatrix}4\\0\end{pmatrix} \tag{2.0.7}$$

3) Now taking $\mathbf{E}$ as center we will draw a circle of radius 4 which will circumscribe $\triangle BCD$.

4) Tangents to this circle from point $\mathbf{A}$ will be $\mathbf{AB}$ and $\mathbf{AP}$ as shown in the figure.

5) Using sine formula we get,

$$\angle BAE = 33.69^\circ \tag{2.0.8}$$

$$\angle BAC = 53.18^\circ \tag{2.0.9}$$

$$\mathbf{BD} = 4.8 \tag{2.0.10}$$

6) In $\triangle$BDC, using Pythagoras theorem we get:

$$\mathbf{DC} = 6.4 \tag{2.0.11}$$

therefore,

$$\mathbf{AD} = 3.6 (\because \mathbf{AC} = \mathbf{AD} + \mathbf{DC}) \tag{2.0.12}$$

7) Using section formula we will find the coordinates of $\mathbf{D}$ :

$$\mathbf{D} = \begin{pmatrix}\frac{ADx_2+DCy_2}{AD+DC}\\ \frac{ADx_1+DCy_1}{AD+DC}\end{pmatrix} \tag{2.0.13}$$

$$\mathbf{D} = \begin{pmatrix}2.88\\3.84\end{pmatrix} \tag{2.0.14}$$

8) Here,

$$\angle PEC = 2\angle BAE \tag{2.0.15}$$

$$\angle PEC = 2 \times 33.69^\circ \tag{2.0.16}$$

$$\angle PEC = 67.38^\circ \tag{2.0.17}$$

9) Now coordinates of $\mathbf{P}$ from center of circle $\mathbf{E}$

will be,

$$\mathbf{P} = \mathbf{E} + \begin{pmatrix} r\cos E \\ r\sin E \end{pmatrix} \tag{2.0.18}$$

$$\mathbf{P} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} + \begin{pmatrix} 1.53 \\ 3.69 \end{pmatrix} \tag{2.0.19}$$

$$\mathbf{P} = \begin{pmatrix} 5.53 \\ 3.69 \end{pmatrix} \tag{2.0.20}$$

10) Therefore, we have coordinates as,

$$\mathbf{A} = \begin{pmatrix} 0 \\ 6 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 8 \\ 0 \end{pmatrix}, \mathbf{E} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}, \mathbf{P} = \begin{pmatrix} 5.53 \\ 3.69 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} 2.88 \\ 3.84 \end{pmatrix} \tag{2.0.21}$$

11) On constructing the given figure we get:

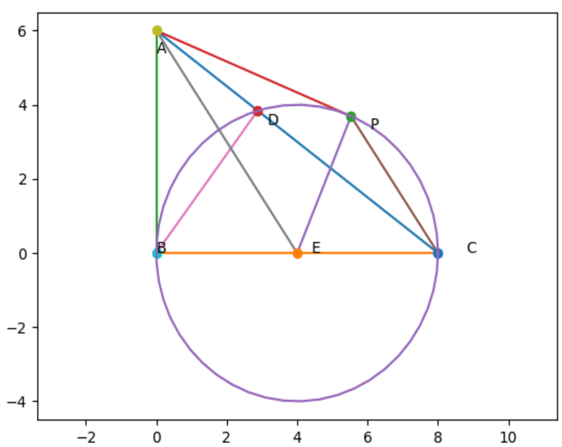


Fig. 2.1: Tangents to a Circle