

ASSIGNMENT-2

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Download all python codes from

<https://github.com/behappy0604/Summer-Internship-IITH/tree/main/Assignment-2>

and latex-tikz codes from

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1 QUESTION No. 2.39

Construct a quadrilateral MORE where $MO = 6$, $OR = 4.5$, $\angle M = 60^\circ$, $\angle O = 105^\circ$ and $\angle R = 105^\circ$.

2 SOLUTION

1) Let us generalize the given data:

$$\angle M = 60^\circ = \theta \quad (2.0.1)$$

$$\angle O = 105^\circ = \alpha \quad (2.0.2)$$

$$\angle R = 105^\circ = \gamma \quad (2.0.3)$$

$$\|\mathbf{O} - \mathbf{M}\| = 6 = a, \quad (2.0.4)$$

$$\|\mathbf{R} - \mathbf{O}\| = 4.5 = b, \quad (2.0.5)$$

$$\mathbf{M} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{O} = \begin{pmatrix} 6 \\ 0 \end{pmatrix} \quad (2.0.6)$$

2) Also, Let us assume the other two sides as

$$\|\mathbf{R} - \mathbf{E}\| = c \quad (2.0.7)$$

$$\|\mathbf{M} - \mathbf{E}\| = d \quad (2.0.8)$$

$$\theta = \theta_1 + \theta_2 \quad (2.0.9)$$

$$\delta = 180^\circ - \alpha = 75^\circ \quad (2.0.10)$$

3) Now on calculating, we get

$$\Rightarrow \angle E + 270^\circ = 360^\circ, \quad (2.0.11)$$

$$\Rightarrow \angle E = 90^\circ \quad (2.0.12)$$

4) Now taking sum of all the angles given and (2.0.12) we get

$$\angle M + \angle O + \angle R + \angle E = 360^\circ \quad (2.0.13)$$

So construction of given quadrilateral is possible as sum of all the angles is equal to

360°.

5) Now, using cosine formula in $\triangle MOR$ we can find RM:

$$\begin{aligned} \Rightarrow \|\mathbf{R} - \mathbf{M}\|^2 &= \\ \|\mathbf{M} - \mathbf{O}\|^2 + \|\mathbf{O} - \mathbf{R}\|^2 - 2 \times \|\mathbf{M} - \mathbf{O}\| \times \|\mathbf{O} - \mathbf{R}\| \cos O & \quad (2.0.14) \end{aligned}$$

$$\Rightarrow RM = 8.38 \quad (2.0.15)$$

$$\Rightarrow \theta = \arcsin 31.24^\circ \quad (2.0.16)$$

6) Now in $\triangle MER$, we know

$$\angle M = 28.76^\circ, \angle E = 90^\circ. \quad (2.0.17)$$

We know that sum of the angles of a triangle is 180°

$$\Rightarrow \angle M + \angle E + \angle R = 180^\circ \quad (2.0.18)$$

$$\Rightarrow 28.76^\circ + 90^\circ + \angle R = 180^\circ \quad (2.0.19)$$

$$\Rightarrow \angle R = 61.24^\circ \quad (2.0.20)$$

7) Now applying sine law of triangle, we get $EM = 7.34 = d$

Lemma 2.1. Exact co-ordinates of the given R and E can be expressed as

$$\mathbf{R} = \mathbf{O} + b\mathbf{p} \quad (2.0.21)$$

$$\mathbf{E} = d\mathbf{q} \left(\because \mathbf{M} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right) \quad (2.0.22)$$

where,

$$b = \|\mathbf{R} - \mathbf{O}\|, d = \|\mathbf{E} - \mathbf{M}\|, \mathbf{p} = \begin{pmatrix} \cos O \\ \sin O \end{pmatrix}, \mathbf{q} = \begin{pmatrix} \cos M \\ \sin M \end{pmatrix} \quad (2.0.23)$$

Proof. • Calculating the co-ordinates of R:

Putting the values in the above equation we

get,

$$\Rightarrow \mathbf{R} = \begin{pmatrix} 6 \\ 0 \end{pmatrix} + 4.5 \begin{pmatrix} \cos 105 \\ \sin 105 \end{pmatrix} \quad (2.0.24)$$

$$\Rightarrow \mathbf{R} = \begin{pmatrix} 6 \\ 0 \end{pmatrix} + \begin{pmatrix} 1.16 \\ 4.35 \end{pmatrix} \quad (2.0.25)$$

$$\mathbf{R} = \begin{pmatrix} 7.16 \\ 4.35 \end{pmatrix} \quad (2.0.26)$$

- Calculating the co-ordinates of E:

Putting the values in the above equation we get,

$$\Rightarrow \mathbf{E} = 7.34 \begin{pmatrix} \cos 60 \\ \sin 60 \end{pmatrix} \quad (2.0.27)$$

$$\Rightarrow \mathbf{E} = \begin{pmatrix} 3.67 \\ 6.36 \end{pmatrix} \quad (2.0.28)$$

□

- 8) Now, we have the coordinate of vertices M,O,R,E as,

$$\mathbf{M} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{O} = \begin{pmatrix} 6 \\ 0 \end{pmatrix}, \mathbf{R} = \begin{pmatrix} 7.16 \\ 4.35 \end{pmatrix}, \mathbf{E} = \begin{pmatrix} 3.67 \\ 6.36 \end{pmatrix}. \quad (2.0.29)$$

- 9) On constructing the given quadrilateral on python and marking angle we get:

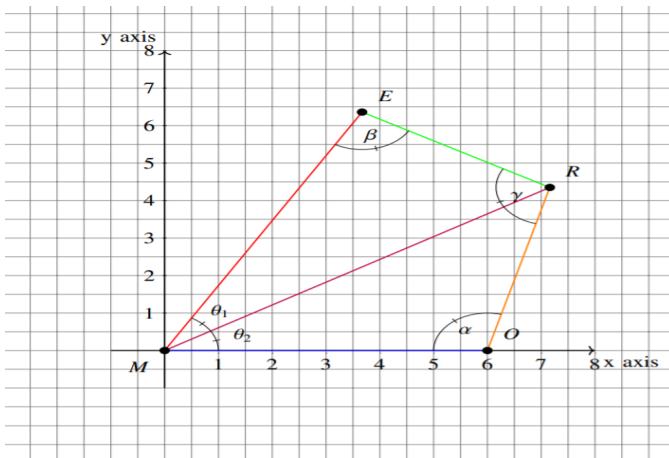


Fig. 2.1: Quadrilateral MORE