SUMMER INTERNSHIP REPORT

Data-driven modelling of Vortex-Induced Vibration Systems with Sparse Identification of Non-Linear Dynamics (SINDy)

Submitted by

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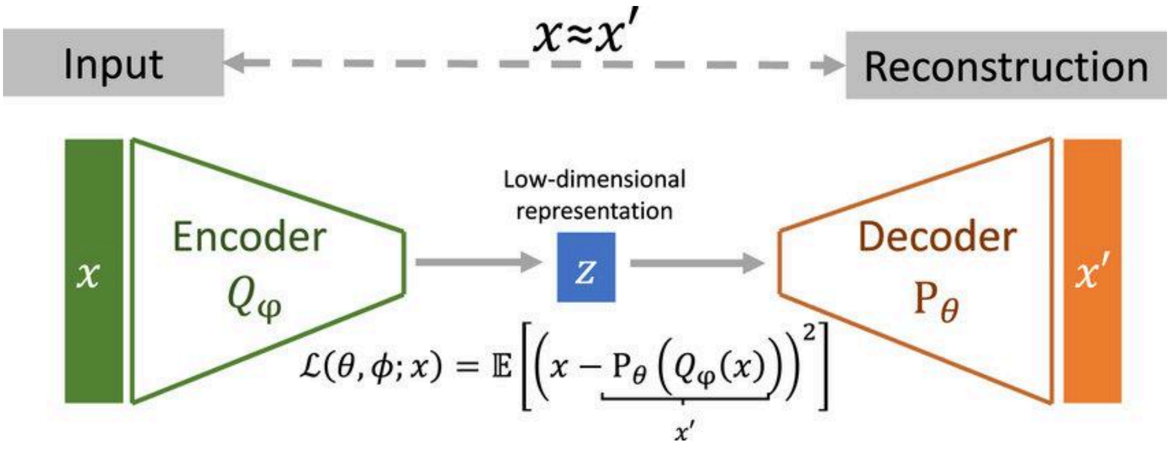
ABSTRACT

This study investigates the use of a data-driven model of Sparse Identification of Non-Linear Dynamics (SINDy), for Vortex Induced Vibration Systems. SINDy is primarily used to discover governing equations from the time histories of state variables of the system. The Lower-order models for Vortex-Induced Vibration systems typically involve a coupled Ordinary Differential equation-based formulation of the structural and the fluid oscillator system. One describes the dynamics of the structural oscillator and coupled with it another describes the dynamics, in most cases non-linear, of the fluctuating vortex street. In this study this framework has been applied to two different models of Vortex Induced Vibration of a cylinder, to discover sparse ODEs of the parameters governed by the fluctuating vortex street. Two Lower-order models for Vortex-induced vibrations, the lift oscillator model and the wake oscillator model have been investigated, through SINDy. Investigations on the data-driven sparse approximations of the non-linear nature of the non-dimensional wake (q) and unsteady lift coefficient (C_L), variables governed by the non-linear Vortex dynamics have been carried out in this report. Primarily 2 types of data were considered for the training data of SINDy, high-quality data from direct solving of the system and Computational Fluid Dynamics simulation data and the accuracies of the model responses with the discovered equations from both sets of data are examined.

1. Literature Review

The advent of Machine learning (ML) in recent years has paved the way for its use in the field of Computational Physics. The review paper by Burnton, SL., and Vinuesa, R., [1], thoroughly investigates the possibilities of Machine Learning in the critical field of Computational fluid dynamics (CFD). Data-driven models have the potential to ease up the computational efforts in fluid flow modelling in several ways. Machine Learning can potentially find its application in domains like Direct Numerical Simulation (DNS) in turbulence modelling, to Reduced Order Models (ROMs) in the vast subject of CFD.

In this literature review, the topic of Reduced Order Modeling will be investigated precisely. The very concept of ROM is widely used to tackle problems in areas of CFD and other problems in computational Physics, especially where there is a very high dimensionality to deal with. Modeling of systems with a very high degree of freedom (dof), requires extensive use of Reduced Order Models. Typically, Principle Component Analysis is carried out to map models in a lower-dimensional space. This is what a single layer of an Autoencoder network does. Deep Learning has severely enhanced and impacted these methods. This enabled us to create deep autoencoder networks that map down higher-order models to lower dimensional latent spaces, the activation functions also control the nature of the system in the latent space and reconstruction of the original parametric space on the other side of the network as given in the figure 1, constraints the approximation by accounting for the reconstruction loss.



This can effectively reduce higher order flow fields, in fluid flow as demonstrated by Wang, J., He, C., Li, R., Chen, H., Zhai, C., Zhang, M., [2], with an allowed margin of error. Such techniques can be very useful to tackle problems involving high-dimensional spaces. Deep Autoencoder networks, can be effective in tackling many problems in dynamical with higher dimensionality. However, there exists the need for a governing equation, which represents the temporal evolution of the state variable(s). This is especially required in cases where due to the complex nature of the phenomena, the governing equations are too complicated or yet to be discovered. On the other hand simplified sparse models are needed to effectively ease up the computations in many problems.

One of the very first and groundbreaking works that effectively tackled the aforementioned issues was done by Burnton, SL ., Kurtz, N ., Proctor, JL., [3], where the idea of model discovery from time histories of state variables was introduced. A Lorenz system and a flow past a cylinder problem were studied. The idea was to have a sparse representation of the dynamics, which in real-world cases, turned out to be non-linear. Hence the Sparse Identification of Non-Linear Dynamics (SINDy) framework is proposed, which was used on the data sets concerning the problem and it showed that SINDy can be very useful in sparse modelling of complex dynamical systems like Lorenz system and flow fields, if provided with the time histories of the state variables.

Subsequent works like, [4], showed more applications of SINDy in discovering governing Partial Differential Equations (PDEs). In a paper by Champion, K., Lusch, B., Kurtz, NJ., and Burnton, SL., [5], a hybrid framework of SINDy along with a deep autoencoder network has been proposed to tackle problems involving complex dynamics and higher dimensionality. In the above research, the aforementioned framework, of SINDy, coupled with a deep Autoencoder, was made to carry out a data-driven discovery of 3 dynamical systems, namely, the Lorenz system, reaction-diffusion and Non-Linear Pendulum. The results showed good agreement with the actual dynamics of the studied systems and uncovered the potential of the proposed framework of SINDy coupled with a deep Autoencoder.

A work by Lelkes, J., Horváth, DA., Lendvai, B., Farkas, B., Bak, BD., Kalmár-Nagy, T., [6], shows the application of the SINDy framework in Aeroelastic simulations. In this work, a sparse model was discovered from CFD data, on which further studies have been carried out. The time histories, for different parameters of the unsteady lift have been used as the training data for the SINDy model. The studied model is a 2-dof system, represented by coupled Ordinary Differential Equations (ODEs). The results of the SINDy model agreed with the CFD simulation results. The primary aim of the sparse approximation models in such problems is to come up with a simple representation of the dynamical system, by fitting a regression with the pre-defined library of functions (more commonly polynomial and Fourier), over the time series data of the state variables.

Vortex-induced vibrations (VIV) have long been a huge interest of study for engineers due to their vast application and their effect on structures under VIV. In 1970, Hartlan and Currie [14], proposed a lift oscillator-based model for Vortex Induced Vibration system. The work, one of the first of its kind, provided a comprehensive understanding of the ODE-based formulation of the oscillating lift and the structural system. In a study by Facchinetti, ML., Langre, E., Biolley, F., [7], a reduced order coupling model of structural and wake oscillators is studied. A van der Pol equation models the near wake dynamics, which describes the fluctuating nature of vortex shedding. This elementary wake oscillator was coupled with the motion of 1 dof elastically supported rigid structure, namely a structural oscillator. Qualitative and quantitative analysis of some of the major features of VIV, such as the lock-in behaviour at different values of Reduced Stream Velocities is conducted. Different models of coupling of the structural and wake oscillator, and their varying nature of results were studied. The paper by Aswathy, MS ., and Sarkar, S ., [8], studies the same VIV problem, except this time there is a noise component in the incoming stream velocity. In the paper, time histories of amplitudes of the structural oscillator for different values of Reduced Velocities, with and without noise are studied.

2. Sparse Identification of Non-Linear Dynamics (SINDy)

Sparse Identification of Non-Linear Dynamics (SINDy) is a novel framework proposed by Burnton, Kurtz and Proctor, [3], which facilitates the discovery of governing equations from dynamical systems, if provided with the time histories of the state variables. This method of Reduced order modelling can be beneficial in cases where the time series data of the state variables of the studied system are available, and a sparse form of the governing equations can be found. This data-driven discovery of governing equations of dynamical systems can find its application in cases where the original governing equations are too complex, in systems where any form of governing equations is yet to be discovered owing to its complexity, but the data (even noisy data) is available, from simulations or experiments.

A detailed overview of the SINDy framework is provided in [3], [4], [5], and [10], and a brief overview of its application in discovering governing equations in Aeroelastic problems, is given in [6], [11]. A brief overview of the SINDy framework is that, for a dynamical system of the form,

$$\frac{d}{dt}x(t) = f(x(t)) \text{ ,} \qquad \text{----(1)}$$

where, x is the state variable of the system, which is dependent on time, the function f, which represents the dynamic constraints that define the equations of motion of the system, has a sparse representation in the space of possible functions, thus a sparse regression can be used to discover f. As an input, we provide the model, the time histories of the state variable(s) of the system and the time derivatives of the state variables (if not provided, calculated numerically) and a sparse regression is carried out, with the terms defined in the library matrix given by θ. This library can contain any terms and cross terms between state variables, user-defined functions, polynomial and Fourier terms. This allows us to better identify the Non-Linear Dynamics of the system. Since it performs a regression, different optimizers can be used. In the present work, the Sequential threshold least square (STLSQ), optimizer has been used. It employs a Mean Squared Error (MSE) Loss with an L2 regularization.

$$min||Xw - y||_2^2 + \lambda||w||_2^2 \quad \text{———(2)}$$

where, X is the matrix of different terms from the defined library
w is the vector of weights of the model parameters.
y is the vector of observed responses.
λ is the regularization parameter.

In the present work, PySINDy library [12], [13], is used. The STLSQ optimizer has a hyperparameter, threshold, which is the regularization term given in the equation, and controls the sparsity of the discovered model. A very large value of the threshold results in the loss of essential information from the data and gives a very poor sparse identification of the actual system, which is far from the actual dynamical system. A very small value of the threshold will result in a system with, a lot of terms, essentially making the system more complex, to solve and analyse.

In this work, SINDy has been used in Flow-induced Vibration systems. A more detailed description is provided in the following sections of this report.

3. VORTEX INDUCED VIBRATIONS (VIV)

Vortex-induced vibration (VIV) occurs when a cylindrical structure (in this case) in a fluid flow experiences oscillations due to the periodic shedding of vortices from its surface. As vortices are shed alternately from either side of the cylinder, they induce fluctuating lift forces that can cause the structure to vibrate. The frequency and amplitude of these vibrations are influenced by factors such as the flow velocity, cylinder diameter, and Reynolds number. At some velocity of the flow, when the vortex shedding frequency equals the structural natural frequency the vibration amplitudes show a huge increase in its magnitude, possessing significant threats to the structural integrity. Marine Risers and Bridges are among many structures that experience VIV.

3.1. Wake Oscillator Model for VIV

A class of lower-order models has been used to study VIV, by Fachinetti et al [7] proposed a wake oscillator model for Vortex-Induced Vibration. A system of flow past a cylinder elastically mounted on a spring is considered. A diagrammatic representation of the system is given in Figure 1.

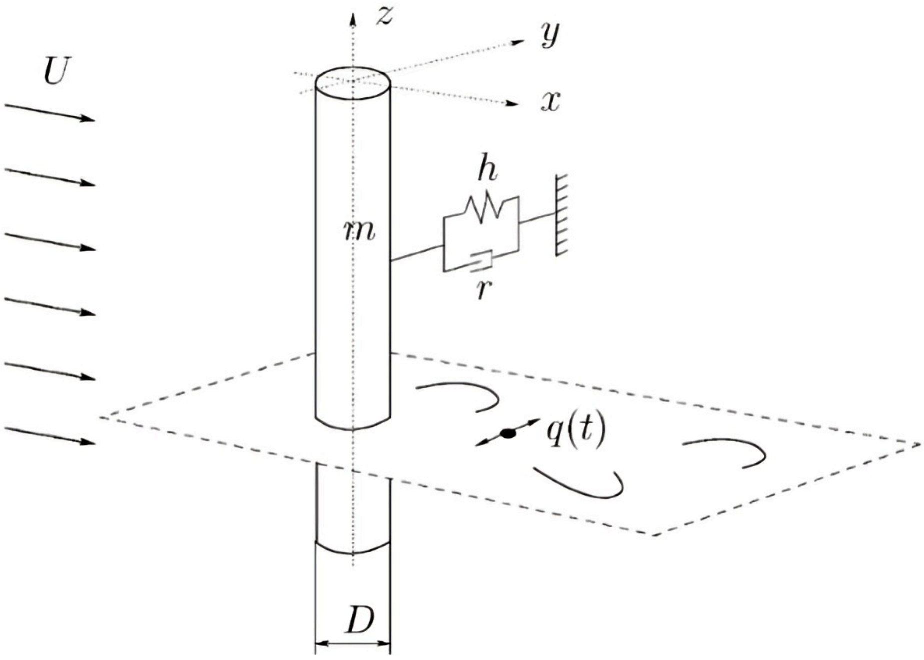


FIG. 1.

Here, a 1 dof elastically supported rigid circular cylinder of diameter D, constrained to oscillate transversely to a stationary and uniform flow of free stream velocity U. This is a coupled system given by,

$$y'' + (2\xi\delta + \frac{\gamma}{\mu})y' + \delta^2 y = s \quad \text{----(3)}$$

$$q'' + \epsilon(q^2 - 1)q' + q = f \quad \text{----(4)}$$

The equation 3 and 4 are non dimensionalized equations. Equation 3 represents the structural equation and equation 4 is the wake oscillator equation. The structural equation has a forcing term s, which models, the effect of the fluctuating wake near the cylinder and similarly, the wake oscillator equation has a coupling term f, which models the effect of the oscillating cylinder on the dynamics of the wakes. The fluctuating nature of the vortex street is modelled by a nonlinear oscillator satisfying the van der Pol equation. This is what Fachinetti et al., in [6], referred to as the wake oscillator. The parameters of the equations are as follows.

δ is the reduced frequency given by, $\frac{1}{St*Ur}$

Ur is reduced velocity, given by, $Ur = \frac{U}{f_n D}$, where, U is the free stream velocity, f_n is the structural natural frequency, D is the Diameter of the cylinder

γ is the dimensionless fluid-added damping coefficient

ξ is the dimensionless structural damping coefficient

μ non-diamesnional mass ratio

y non-dimensional displacement of the structural oscillator, $y = y^{\bullet}/D$

q is the non-dimensional wake variable

s, f are coupling terms, usually an acceleration-based coupling gives the best results for wake oscillator models

This q is described as the non-dimensional wake variable, and this can be associated with the fluctuating lift coefficient (C_L) on the cylinder, as for most of the models in the literature since the pioneering work on the lift oscillator model by Hartlen and Currie (1970), [14]. This approach is discussed in the next subsection. This system is later analysed by Aswathy and Sarkar, 2019 [8], in this case, the system is subjected to stochastic parametric noise. The Non-dimensionalization of the latter model was done slightly differently, although qualitatively and quantitatively, both models are found to be similar.

Equation 4 which describes the fluctuating nature of the vortex street, which is a non-linear equation (having a square non-linearity) is very significant in this present study. The SINDy framework described in section 2, is employed to check whether it can capture the non-linear dynamics of the wake oscillator in the Results section.

3.2 Lift Oscillator Model for VIV

In 1970, a lift oscillator model for Vortex Induced Vibrations (VIV) was proposed by Hartlen and Currie [14]. This was a lower-order model where the same problem was analysed by a coupled ODE system. Here the second ODE models the fluctuating Lift coefficient (Cl) experienced by the cylinder, due to the fluctuating vortex street near the cylinder. The ODEs are given by,

$$x_r'' + 2\zeta x_r' + x_r = a w_0^2 C_L \quad \text{----(5)}$$
$$C_L'' - \alpha w_0 C_L' + \frac{\gamma}{w_0}(C_L')^3 + w_0^2 C_L = b x'_r \quad \text{-----(6)}$$

The equation parameters are as follows,
x_r is the dimensionless displacement given by, $x_r = x/D$
ζ is the dimensionless damping coefficient
γ is the fluid added damping
w_0 is the reduced frequency given by, $w_0 = \frac{f_s}{f_n} = St\frac{U}{f_n D} = StUr$, Ur is the reduced velocity.
St is Strouhal Number
C_L is the lift coefficient
Similar to the wake oscillator, the equation for the fluctuating lift coefficient C_L is non-linear, the damping term having a cubic non-linearity.

4. SINDy for VIV systems (the approach taken)

In the two lower-order ODE-based Vortex-Induced Vibration systems, the wake oscillator and the lift oscillator respectively, the dynamics of the fluctuations of the wake and lift, which is the result of the fluctuations of the vortex street, are non-linear.

$$q'' + \epsilon(q^2 - 1)q' + q = f$$
$$C_L'' - \alpha w_0 C_L' + \frac{\gamma}{w_0}(C_L')^3 + w_0^2 C_L = bx'_r$$

In this study, Sparse Identification of Non-Linear Dynamics (SINDy), is tasked to identify the non-linear wake and oscillator equation, from 2 different datasets. This serves as an attempt to validate whether or not a Data-Driven modelling framework like SINDy can be used to model VIV systems and the type of data acquisition needed to be carried out to get a reasonably accurate sparse model for the lift and wake oscillators. This analysis is done in 2 main steps;

1. Validation of SINDy with the wake oscillator model.
2. Using CFD data to solve the lift oscillator model.

1) **Wake-Oscillator with Simulation Data**- The time series data q and $q' = u$ used for the SINDy model is collected from the numerical solution of the wake oscillator-based coupled ODE system given by Aswathy and Sarkar, (2019) [8]. The q and u are the state variables for the SINDy. STLSQ optimizer with a threshold of 0.1 and a polynomial library up to 3rd-degree terms have been used. The same SINDy approach is repeated but this time, the Reduced velocity (Ur) is taken as an input in the SINDy, and a sparse representation of q'' having q , u, and Ur is obtained. The results are in the following section.

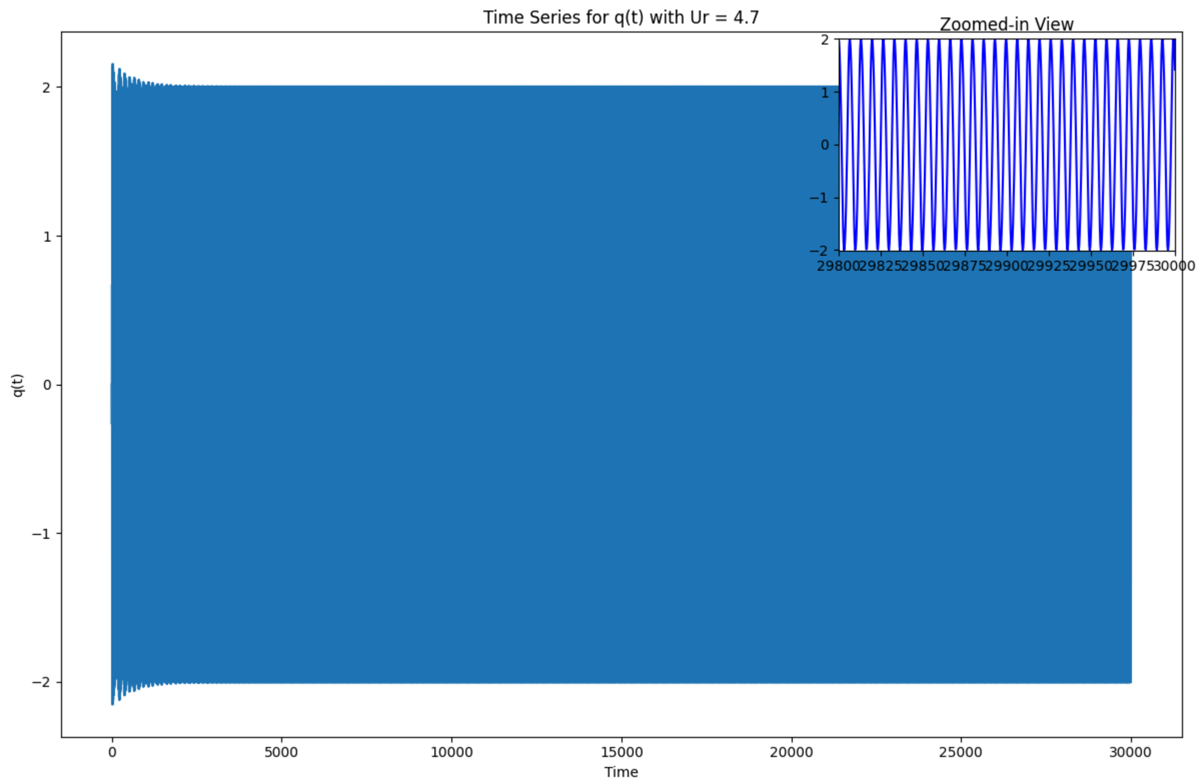


FIG. 2. - training data for the SINDy, $q(t)$

2) **Lift Oscillator with CFD data-** The same approach is undertaken, this time with the lift oscillator model, and more importantly the data this time was C_L data from a CFD simulation of flow past a static cylinder at various Reynold's Numbers. The sparse ODEs given by SINDy were then solved and the original response from the original lift oscillator model and the SINDy-derived model were compared.

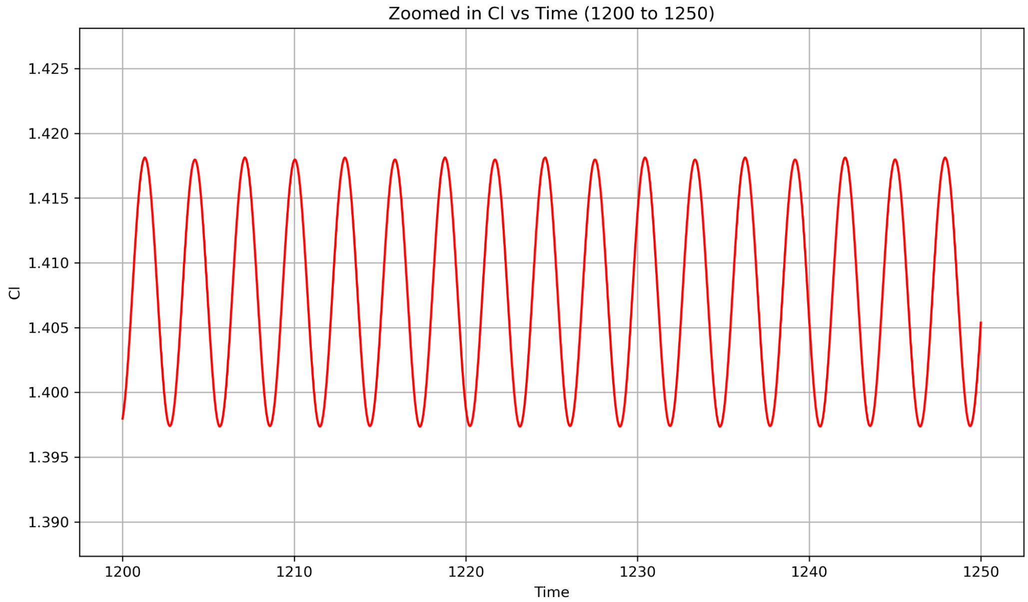


FIG. 3. - The data (zoomed in) for the fluctuating lift coefficient (C_L) for a flow past a static cylinder at Re = 150, obtained from a CFD simulation (later Normalized)

5. RESULTS and DISCUSSIONS

5.1. Wake-Oscillator with simulation Data

The present work starts by validating the solver with the results of Aswathy and Sarkar (2019) [8].

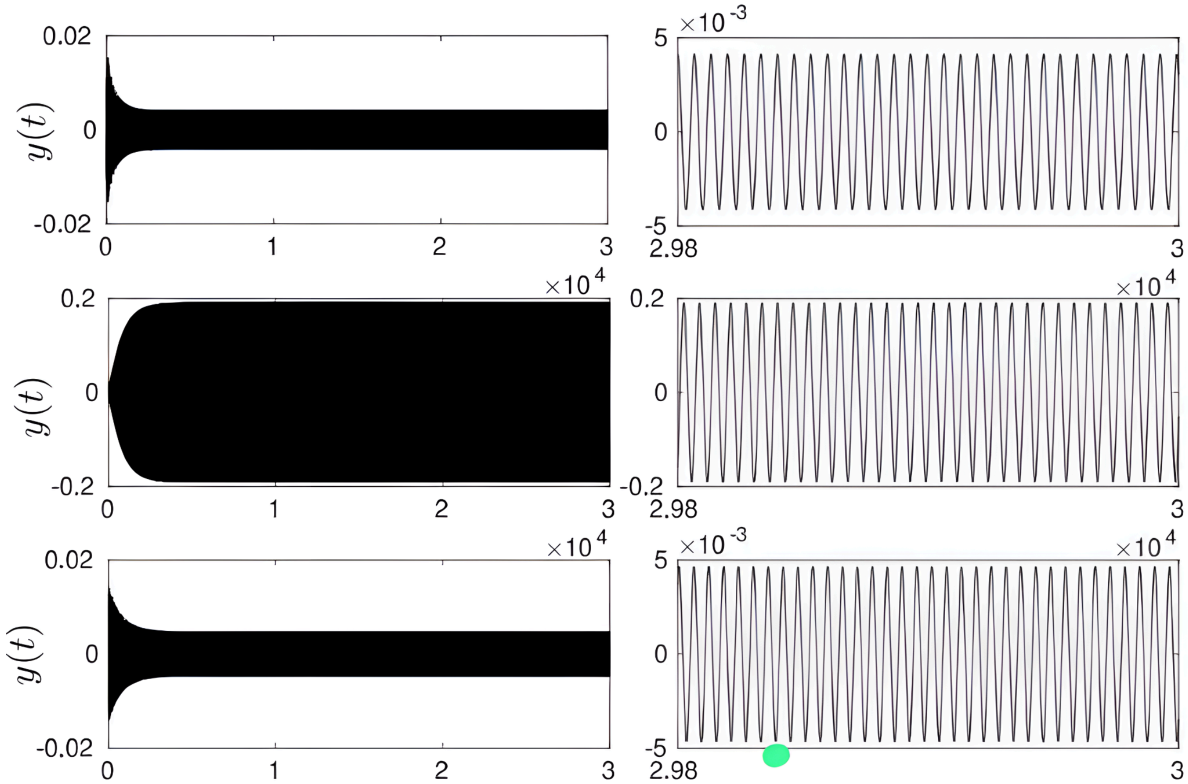


FIG. 4 - Aswathy and Sarkar [8] results for, time histories of amplitudes for, Ur values of 4.7, 4.8, 5.4. Lockin from 4.8 to 5.3.

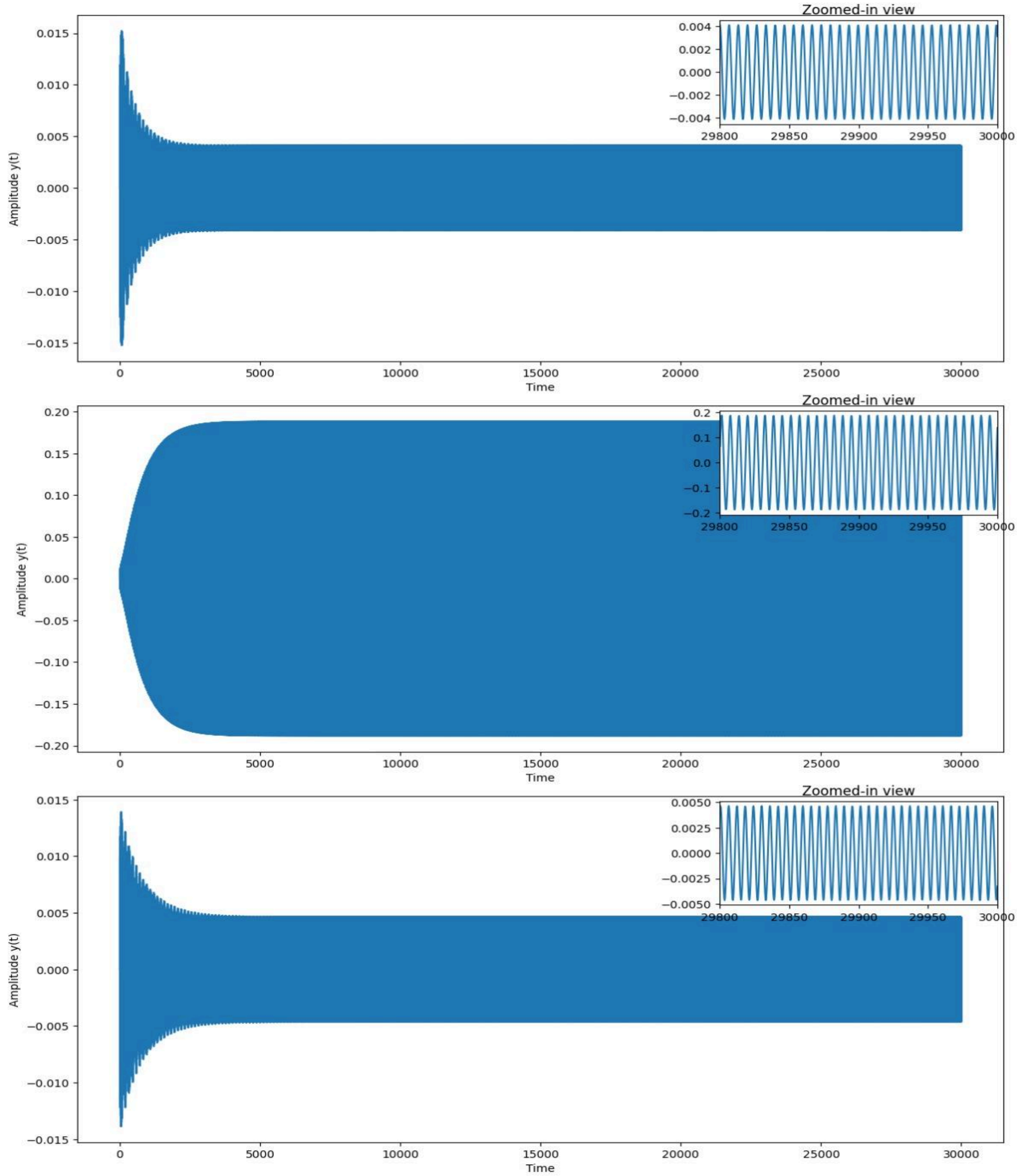


FIG. 5. - Results of the same system with the present solver for the Ur values of 4.7, 4.8, and 5.4. Lock-in behaviour was observed at the same range of Ur values. The solver used is a Python-based ODE solver, odeint, from the SciPy library. The above time histories of the amplitudes are in agreement with those of Aswathy and Sarkar [8]. At an Ur of 4.8, the amplitude increases 20 times. This lockin behaviour persists till Ur equals 5.3.

Now, using the data in section 4, a SINDy model is trained, and the following is the output from the model;

$$(q)' = 1.052\, u$$
$$(u)' = -0.902\, q + 0.145\, u + -0.935\, q^2 u + -0.170\, q u^2 + 0.137\, u^3$$

It is observed that SINDy managed to get a sparse identification of the non-linear dynamics of the wake oscillator. The $q(t)$ data was for Ur = 4.7 (outside-lockin). The responses are now generated with the derived sparse equation.

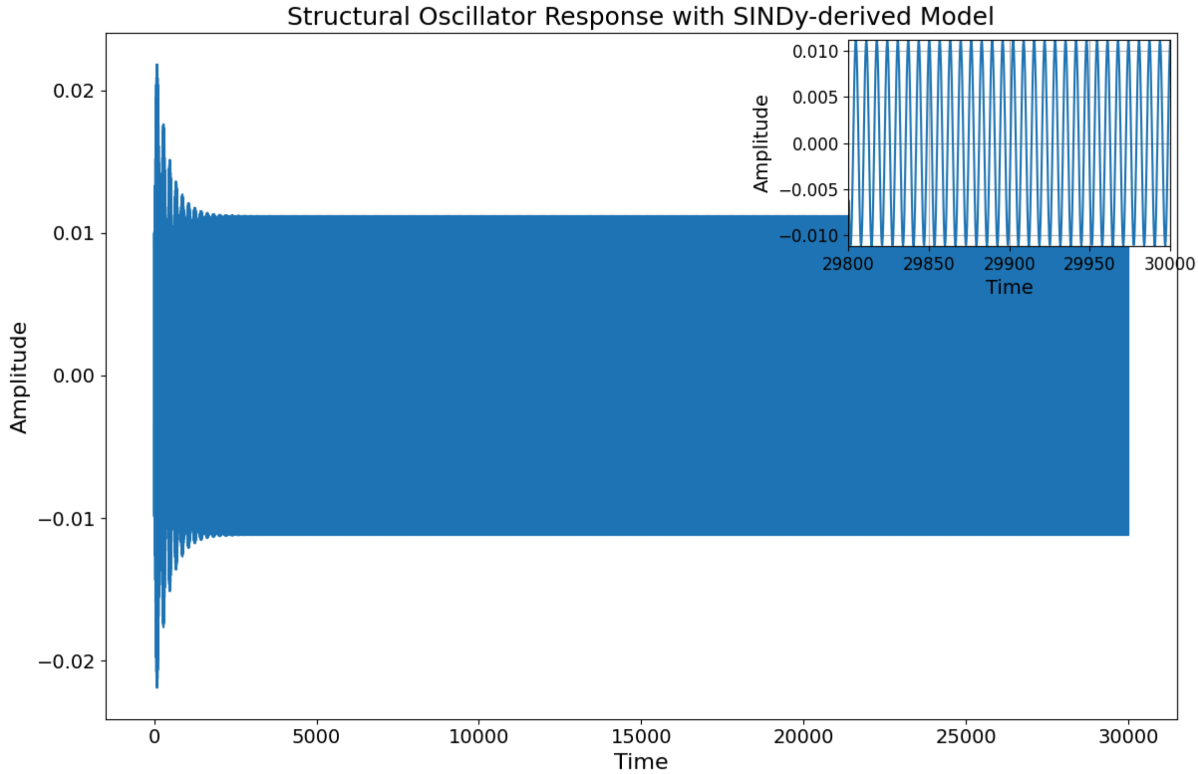


FIG. 6. - Structural Response with Ur = 4.7, derived from the SINDy model. The Responses show satisfactory agreement with those in FIG. 5.

Now, the model has a Ur variable included and the output is sought with a Ur term so that the responses can be obtained with the Reduced Velocity (Ur) specified. The responses for the same values of Ur, 4.7, 4.8 and 5.3 are obtained for better comparison.

The obtained model output is;
(q)' = -0.011 1 + -0.206 q + 1.025 u + 0.008 U_r + 0.082 q U_r + -0.013 u U_r + -0.002 U_r^2 + 0.002 q^3 + 0.007 q^2 u + -0.009 q u^2 + -0.008 q U_r^2 + 0.001 u U_r^2

(u)' = -0.024 1 + 2.845 q + -4.848 u + 0.016 U_r + -1.223 q U_r + 2.059 u U_r + -0.003 U_r^2 + 0.010 q^3 + -0.162 q^2 u + -0.060 q u^2 + 0.090 q U_r^2 + 0.026 u^3 + -0.209 u U_r^2

The system response is again simulated with the discovered equations for Ur = 4.7, 4.8 and 5.4.

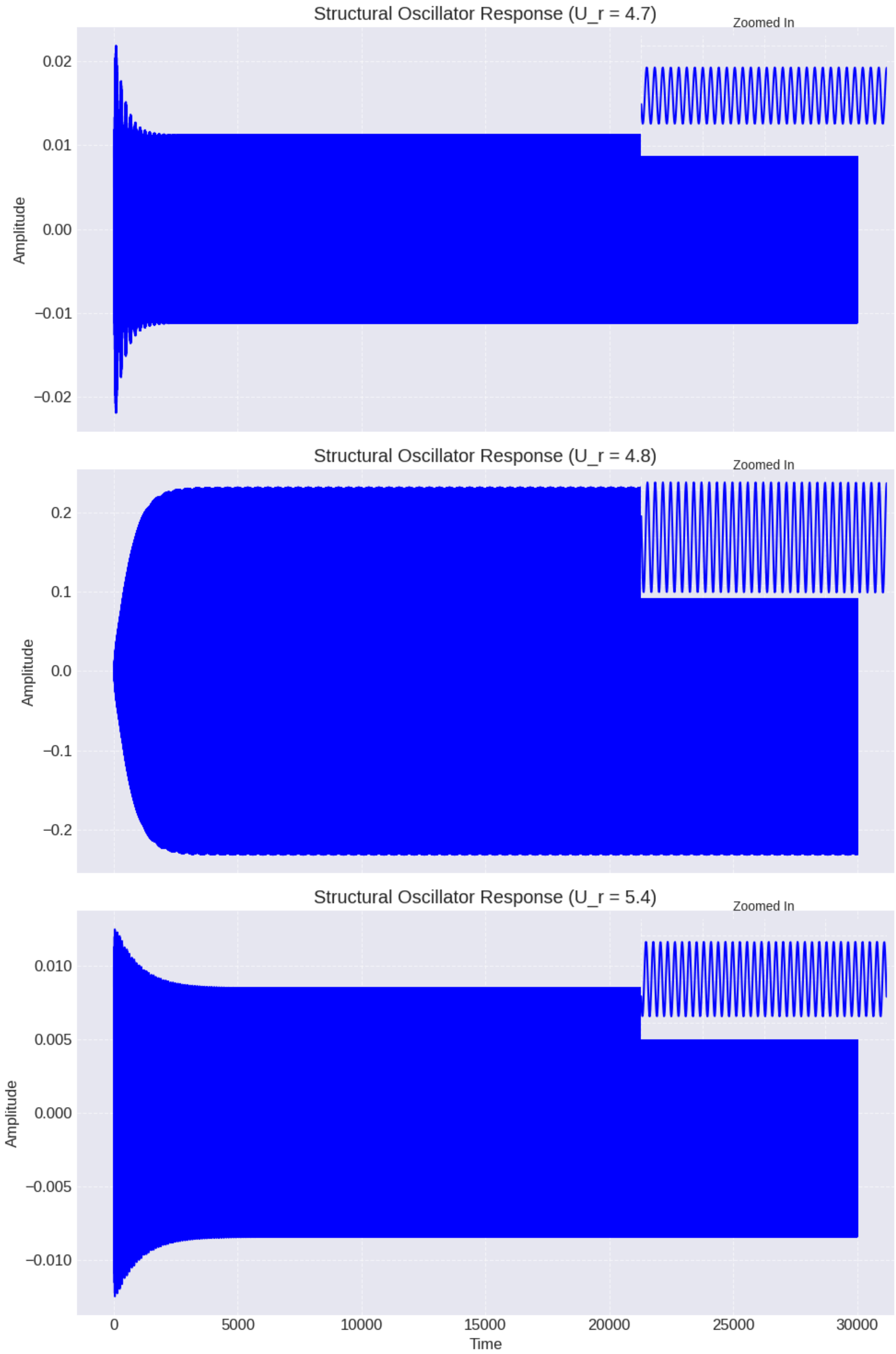
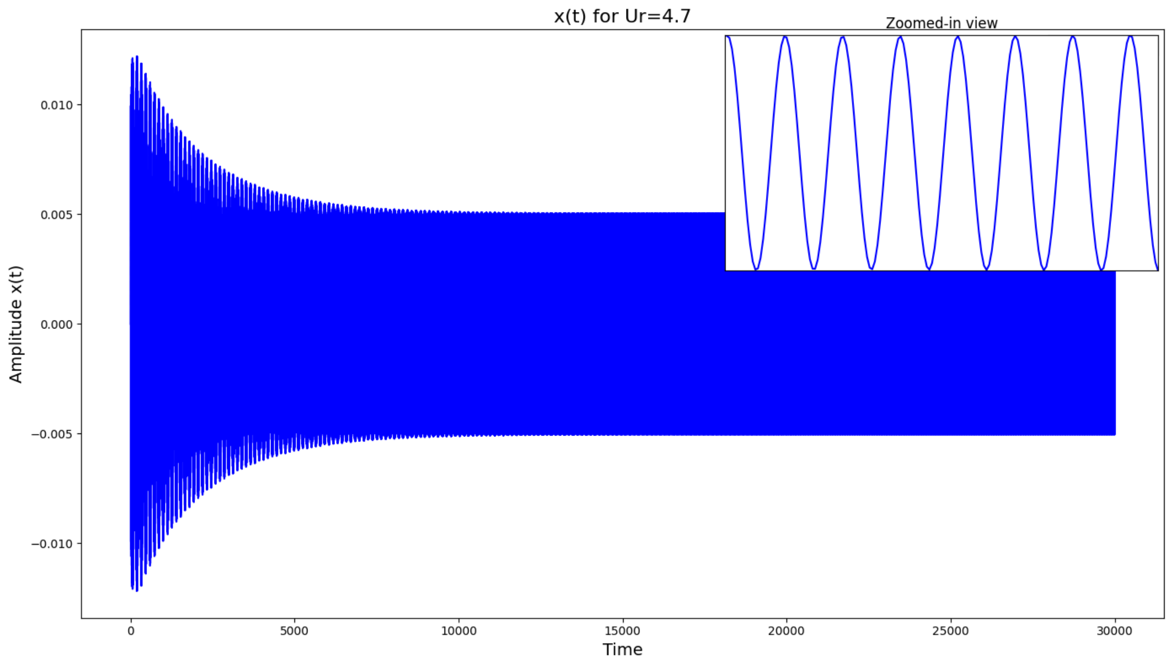


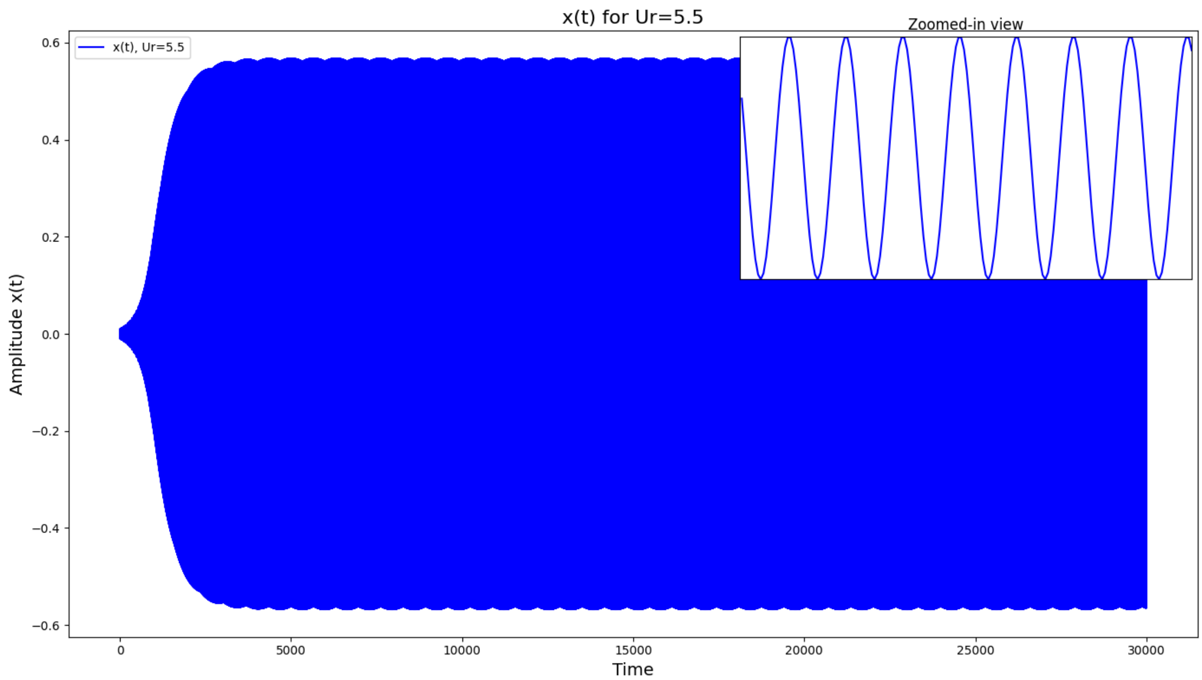
FIG. 7. - Results of the system with the SINDy identified model, for the same values of Reduced Velocity (Ur). The lock-in regime showed very good agreement with the original system and outside lockin showed satisfactory agreement. The above results show that

SINDy can identify sparse non-linear ODEs of the wake oscillator, with a very reasonable margin of error. Surprisingly the responses given in FIG. 7. show a very good agreement with the original responses, of FIG. 5., in the lockin region,
Here, to generate the values for multiple Ur values, it is used as a parameter in the SINDy framework, to capture the variation of the model, with the variation of the Ur. This approach, SINDy with control, is demonstrated by Burnton and Kurtz in [10]. This validation shows us that with good quality data, which capture the true dynamics of the system, SINDy can extract governing equations with a good amount of accuracy. This motivates the use of experimental or simulation data, in this case, CFD data in SINDy modelling. In the next section, CFD data of fluctuating lift coefficient (C_L), for a flow past a static cylinder is used and the responses are generated with the discovered models of the lift oscillator equation.

5.2. Lift Oscillator with CFD data

The Hartland Currie Lift Oscillator Model (1970), [14], is solved and the following are the responses from the model at Ur = 4.7, 5.5, 10





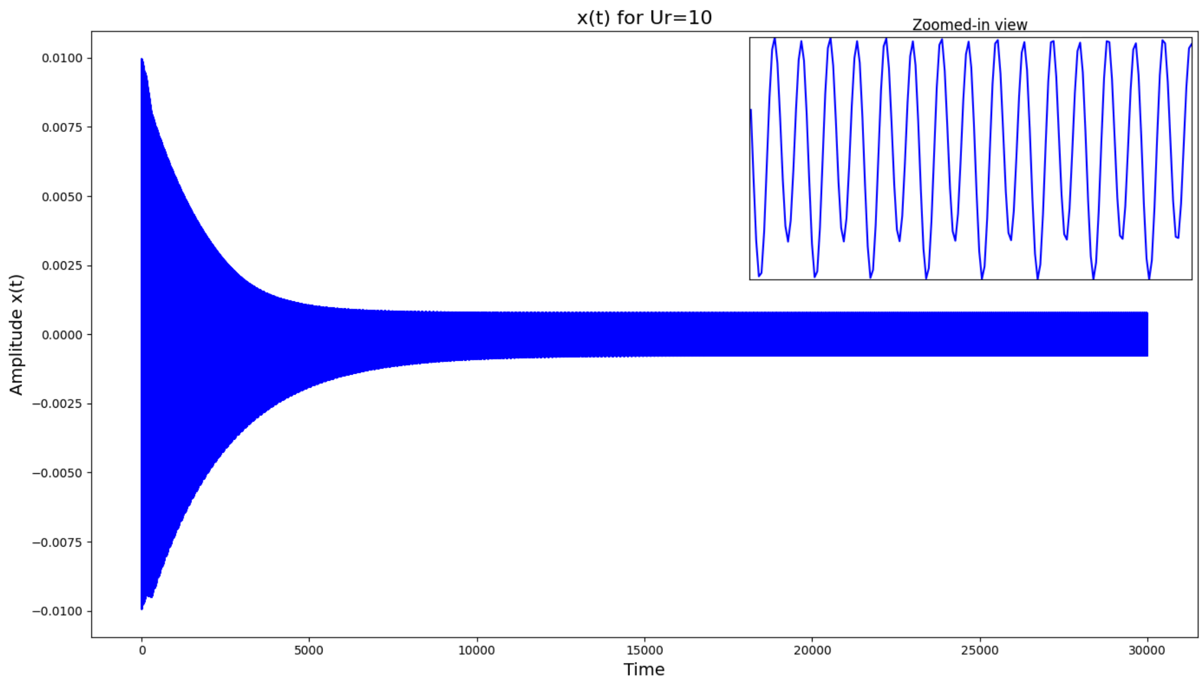


FIG. 8, 9, 10. - Responses of the structural oscillator from the lift Hartlan Curie Model (1970), [14] at Ur = 4.7 (outside lockin), 5.5 (inside lockin), 10 (outside lockin).

Now, with the fluctuating C_L data from the CFD simulation, shown in FIG. 3. for a flow past a cylinder at Re = 150, a SINDy model is trained. The data was normalized before it was fed to the model. The responses are then regenerated for the system, but this time with the sparse ODE discovered by SINDy.

The obtained model output from SINDy is;

(Cl)' = 0.239 Cl_dot
(Cl_dot)' = 0.169 1 + 0.119 Cl + 1.536 Cl_dot + -0.088 Cl^2 + -0.021 Cl Cl_dot + -0.064 Cl_dot^2 + -0.180 Cl^3 + -0.769 Cl^2 Cl_dot + -0.180 Cl Cl_dot^2 + -0.766 Cl_dot^3

The following are the system responses with the SINDy-derived model, and the original model with Ur = 4.5 and Ur = 2

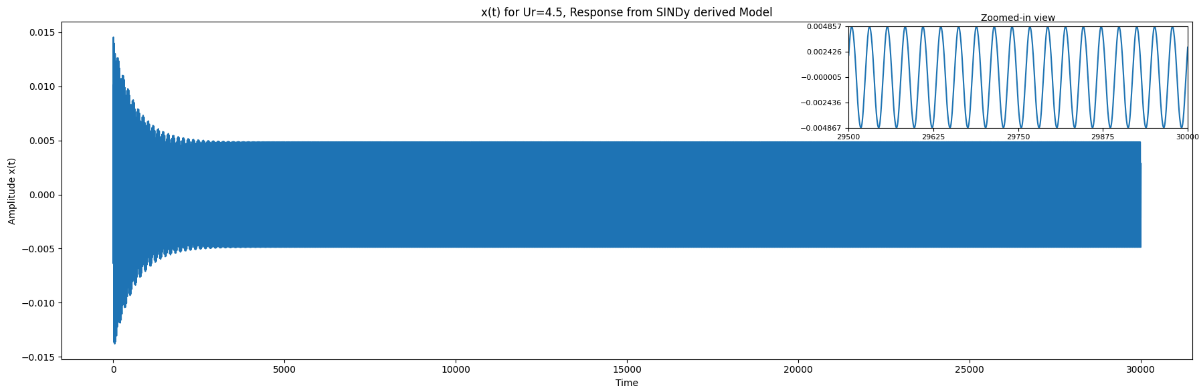


FIG.11.- Response of the system, with the SINDy derived lift oscillator model, at Ur = 4.5

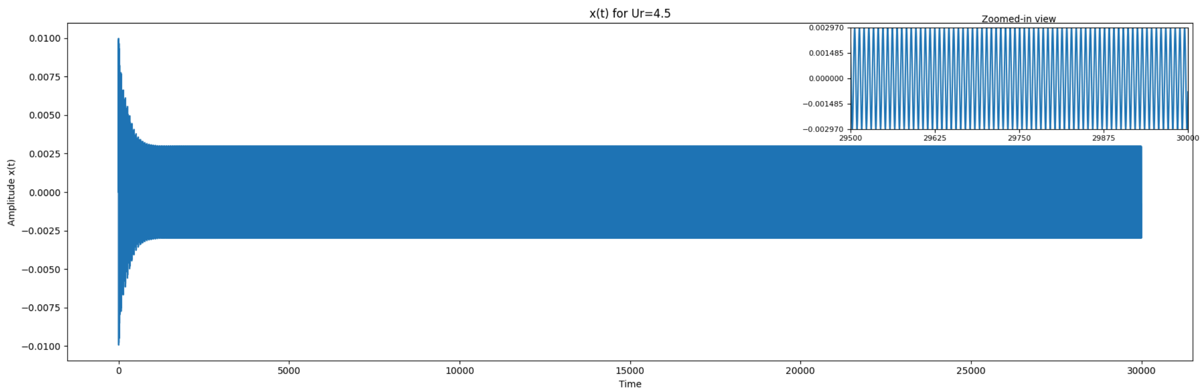


FIG. 12. - Response of the original Lift Oscillator based system at Ur =4.5

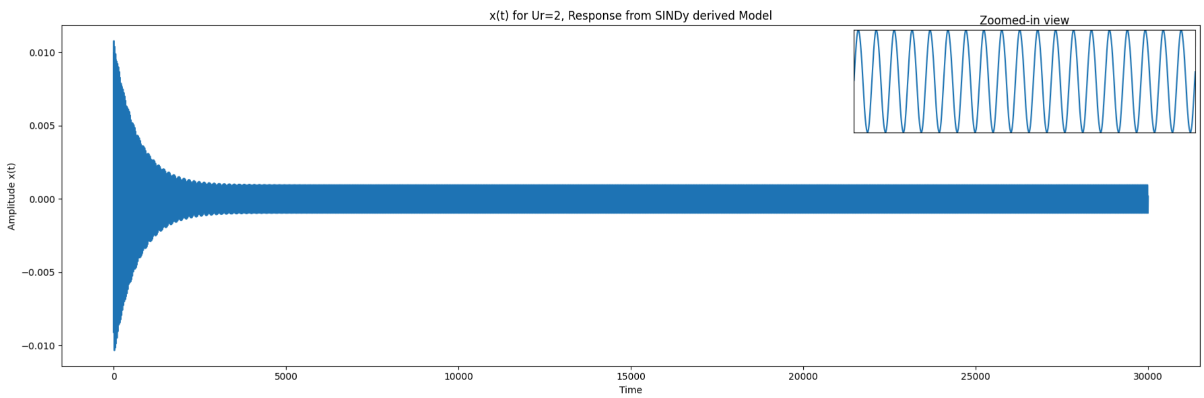


FIG. 13. - Response of the system, with the SINDy derived lift oscillator model, at Ur = 2

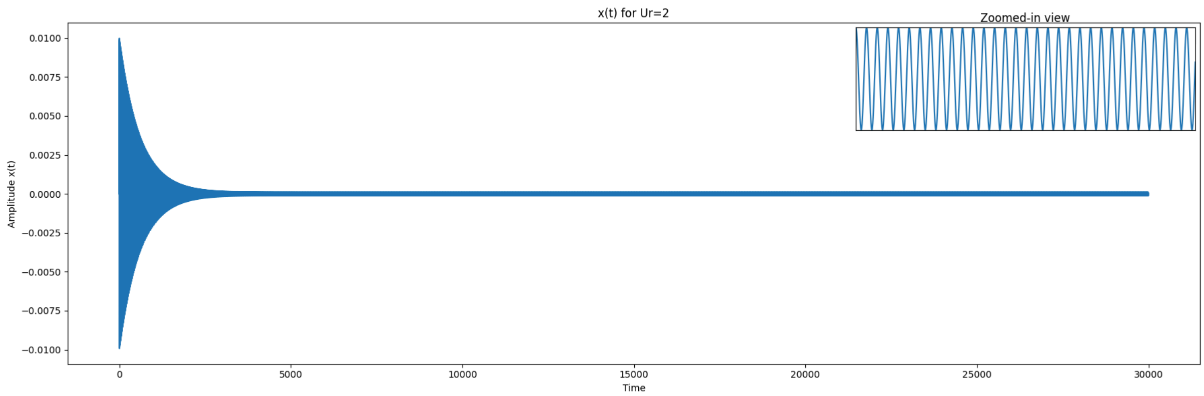


FIG. 14. - Response of the original Lift Oscillator based system at Ur =2

The above results show that, for lower velocities of Ur, that is the pre-lockin values, SINDy has been able to predict the C_L dynamics with a reasonable margin of error. The training data for SINDy was generated for a flow past a static cylinder at Re = 150. Back calculating, Ur was found to be in the same region, that is at the left-hand side of the lock-in region. The data was not available for higher Re values, so in a way, SINDy derived sparse ODE when used for higher Ur values, was generating responses, which was far away from the actual responses of the lift oscillator model.

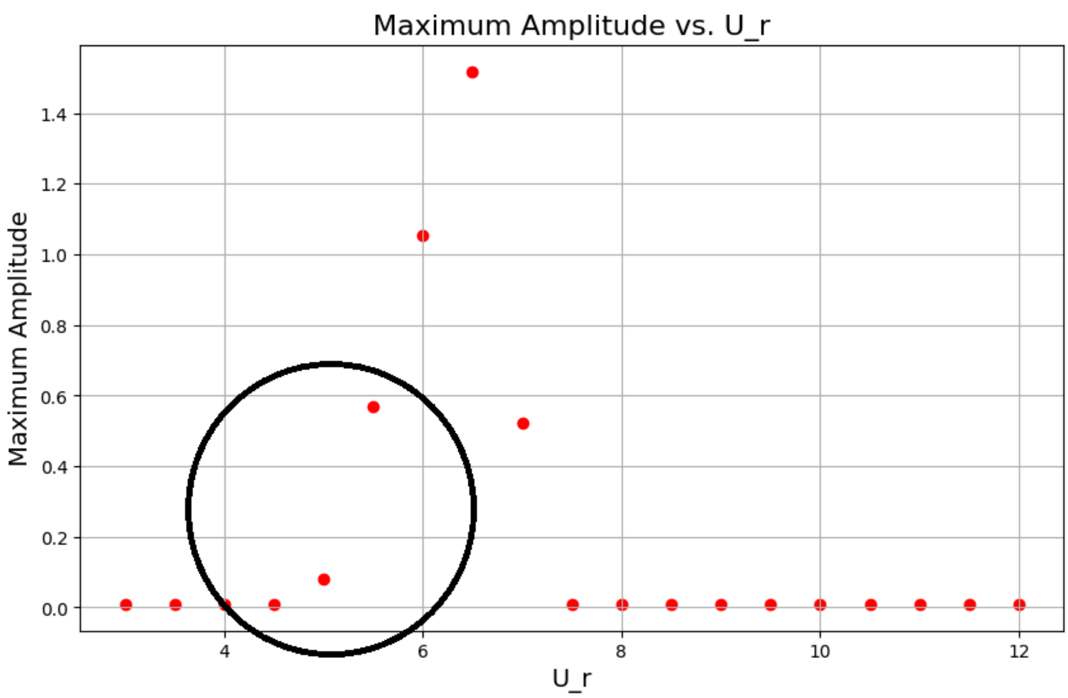


FIG. 15.- Maximum Amplitude of the structural oscillator for a range of Ur values. The encircled part is the region where SINDy performs the best in this case, with the available data. To its left the oscillations are too much less, with a very low amplitude and to the right of its peak, the model gives an erroneous response due to the unavailability of the data, at higher Re.

The fluctuating lift coefficient for a flow past a static cylinder at Re = 150, is used as the training data for the SINDy model. The core objective was to have a qualitative agreement in the responses from the original and the SINDy-discovered models, in which SINDy

succeeded. The original lift oscillator model, when solved for Ur = 4 to Ur = 4.9, had a fluctuating value uplift coefficient C_L, between -0.02 to +0.02, which is the closest to the C_L fluctuation of, -0.028 to 0.028 as seen in the CFD simulation data, for a flow past a static cylinder. When unsteady lift data is provided to SINDy, it discovers a sparse ODE representing the dynamics of the C_L and gives a satisfactory response, when compared to the original system, in that specific regime of Ur, shown in FIG. 15.. It can also be inferred that if C_L data for higher Re is provided to the model, the then derived model can predict the responses for the right-hand side of the lockin with the same accuracy as the model did with the left-hand side.
As for the performance of the model-derived sparse equations outside the lockin region is concerned, the ODEs approximated from static cylinder data fail to capture the lockin phenomena.

6. SUMMARY & CONCLUSIONS

The study presented here shows that Sparse Identification of non-linear Dynamics (SINDy) can serve as an effective way of data-driven modelling for Vortex-Induced Vibration systems. The non-linearities induced by the fluctuating vortex street in the overall dynamics of the system can be sparsely identified by SINDy if time histories of the state variables which is governed by the non-linear nature of the vortex street, in this case q, non-dimensional wake variable for the wake oscillator model and C_L, unsteady list coefficient, for the lift oscillator model, are provided. The lift oscillator model described here shows the need for complete data which describes the behaviour of the system throughout. The predictions for the higher ranges of Ur values show the lack of accuracy and the need for high Re CFD simulation data.

The modelling of the multiple Ur cases for the lift oscillator system required SINDy to capture the lockin phenomena, in a way that requires the data for higher Re so that the Ur gets in the lockin region. The wake oscillator model worked as the data set was generated for 3 different values of Ur, left of lock-in, within lockin and right of lockin. This data acquisition allowed the model to capture the true dynamics of the system and its behaviour. The lift oscillator model was being fitted with fluctuating C_L data from a flow past a static cylinder. The lock-in phenomena resulting in the sudden rise in the amplitude of oscillations is not possible as the cylinder is not elastically mounted on a spring. This essential loss of information from the training data resulted in poor SINDy outcomes within lock-in Ur values.

7. FUTURE SCOPE

The same framework of SINDy can be well extended to the lock-in region of the system. Since the fluctuating lift coefficient data was from a flow past a static cylinder, the lock-in behaviour could not be predicted by SINDy, qualitatively or quantitatively. Further works can be carried out, with coarse data but, for a cylinder elastically mounded, and then training the same SINDy model. This should help SINDy capture effectively the lock-in phenomena of the system resulting in the identification of a more robust sparse ODE, which incorporates the lockin behaviour at a given Range Ur.

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