Assignment → 4

Q1 → Find the rank of the matrix A by reducing in Row reduced echlon form.

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$$

Solving: Row operations:

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix} \xrightarrow[R_2 \to R_2 - 2R_1]{R_4 \to R_4 - 2R_3} \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 3 & 2 & 1 & 3 \\ 0 & 4 & 5 & -1 \end{bmatrix} \xrightarrow{R_3 \to R_3 - 3R_1} \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & 4 & 5 & -1 \end{bmatrix}$$

$$\xrightarrow{R_4 \to R_4 + R_3} \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -3 & 3 \\ 0 & 0 & -3 & 2 \end{bmatrix} \xrightarrow{R_4 \to R_4 - R_2} \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_2 \leftrightarrow R_3} \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Here, no. of non-zero rows = 3

$$\rho(A) = 3 \rightarrow \text{Ans}$$

Q2. Let W be the vector space of all symmetric 2×2 matrices and let $T: W \longrightarrow P_2$ be the linear transformation defined by $T\begin{bmatrix} a & b \\ c & d \end{bmatrix} = (a-b) + (b-c)x + (c-a)x^2$. Find the rank and nullity of T.

Soln: According to Question,

every matrix and vector space W is of the form $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

and

the linear transformation, $T: W \longrightarrow P_2$ defined by:

$$T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = (a-b) + (b-c)x + (c-a)x^2$$

Here we have three co-efficients corresponding to $1, x, x^2$ (R^2).

$\therefore$ Dimension $= 2+1 = 3$

Now, The standard basis for W is :

$$E = \left\{\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\right\}$$ ($\because$ W is 2x2 matrix space which is symmetric matrix)

Also T is defined as $T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \begin{bmatrix} a-b \\ b-c \\ c-a \end{bmatrix}$

So, we can represent T in terms of the standard basis element

$$T\left(\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \qquad T\left(\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$T\left(\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$$

Now, these column vectors form the matrix A:

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix}$$

Now, we have to convert this into the row-echlon form Applying row elementary operation.

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \xrightarrow{R_3 \to R_3 + R_1} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

Here $\boxed{\rho(A) = 3}$ linear independent.

Now $\rho(A) + N = D \Rightarrow 3 + N = 3$

$N = 3 - 3$

$N = 0$

Hence, Rank $\rho(A) = 3$ (Ans)

Nullity $(N) = 0$ (Ans)

Q3. Let $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$. Find the eigenvalues and eigenvectors of A^{-1} and $A + 4I$.

Solve To get the eigen values we need to solve characteristic equation:

$$\det(A - \lambda I) = 0$$

$$\Rightarrow \begin{bmatrix} 2-\lambda & -1 \\ -1 & 2-\lambda \end{bmatrix} = 0 \Rightarrow (2-\lambda)^2 - 1 = 0 \Rightarrow 4 + \lambda^2 - 4\lambda - 1 = 0$$

$$\lambda^2 - 4\lambda + 3 = 0$$

$$\Rightarrow \lambda^2 - 3\lambda - \lambda + 3$$

$$\Rightarrow \lambda(\lambda - 3) - 1(\lambda - 3) = 0$$

$\lambda = 1, 3$ (Ans) → Eigen Values

For $A^{-1} = \frac{1}{\lambda}$

$\therefore A^{-1} = 1, \frac{1}{3}$

For $\lambda = 1$, $[A - \lambda I]X = [0] \Rightarrow [A - I]X = [0]$

$$\Rightarrow \left(\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right) \begin{bmatrix} x \\ y \end{bmatrix} = [0]$$

$$\Rightarrow \left(\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \right) \begin{bmatrix} x \\ y \end{bmatrix} = [0]$$

Same for A^{-1}

$x - y = 0 \Rightarrow x = y$ ~~(Ans)~~

& $-x + y = 0$

So if $x = K$, then $\boxed{x = y} = K$

$\therefore$ Eigen Space $= \begin{bmatrix} K \\ K \end{bmatrix} = K \begin{bmatrix} 1 \\ 1 \end{bmatrix}, K \in R$

For $\lambda = 3$

$$[A - 3I][x] = [0]$$

$$\Rightarrow \begin{bmatrix} 2-3 & -1 \\ -1 & 2-3 \end{bmatrix} [x] = [0]$$

$$\Rightarrow \begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow -x - y = 0 \Rightarrow -x = y$$

Eigen Values for $A + \lambda I$

$\Rightarrow \lambda_1 + c, \ \lambda_2 + c$

$1 + 4, \ 3 + 4$

$5, 7$ (Ans)

if $x = K$, then $x = y = K$

$\therefore$ Eigen Space $= \begin{bmatrix} K \\ -K \end{bmatrix} = K \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ (Ans)

Q4 [illegible] iteration)

Solⁿ 3 Continues

We know if λ is eign values of matrix A then $1/\lambda$ $(\lambda \neq 0)$ is eign values of A^{-1}

$\therefore$ Eigen values $A^{-1} = 1, \frac{1}{3}$

For $\lambda = \frac{1}{3}$, $\left[A^{-1} - \frac{1}{3}I\right][x] = 0$

Here

$$|A| = \lambda_1 \lambda_2 = (1)(3) = 3 \neq 0 \quad \therefore A^{-1} \text{ exist}$$

and $\operatorname{adj}(A) = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$$\therefore A^{-1} = \frac{1}{|A|}(\operatorname{adj} A) = \frac{1}{3}\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{pmatrix} \text{ (Ans)}$$

~~For λ=1~~ $\therefore$ Eigen Vectors $= \begin{bmatrix} 29/7 & 9 & -3381/74 \\ 2/7 & -2 & 51/148 \\ 1 & 1 & 1 \end{bmatrix}$

For $\lambda = 1$ $\quad [A^{-1} - I][x] = [0]$

$$\Rightarrow \begin{bmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{bmatrix}[x] = 0$$

$$\Rightarrow \frac{2}{3}x + \frac{1}{3}y = 0 \Rightarrow x = -\frac{1}{3}y \times \frac{3}{2} = x = -\frac{1}{2}y$$

&

$$\frac{1}{3}x + \frac{2}{3}y = 0 \Rightarrow x = -\frac{2}{3}y \times 3 \Rightarrow x = -2y$$

This is not possible.

$$[A - 3I][x] = [0]$$

$$\Rightarrow \begin{bmatrix} 2-3 & -1 \\ -1 & 2-3 \end{bmatrix} [x] = [0]$$

$$\Rightarrow \begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow -x - y = 0 \Rightarrow -x = y$$

if $x = K$, then $x = y = K$

$$\therefore \text{Eigen Space} = \begin{bmatrix} K \\ -K \end{bmatrix} = K \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$ (Ans)

Eigen Values for $A + \lambda I$

$\Rightarrow \lambda_1 + c, \lambda_2 + c$

$1+4, 3+4$

$5, 7$ (Ans)

4. Solve by Gauss - Seidel Method (Take three iterations)

$$3x + 0.1y - 0.2z = 7.85$$
$$0.1x + 7y - 0.3z = -19.3$$
$$0.3x - 0.2y + 10z = 71.4$$

$$\Rightarrow \begin{bmatrix} 3 & 0.1 & -0.2 \\ 0.1 & 7 & -0.3 \\ 0.3 & -0.2 & 10 \end{bmatrix}$$

with the initial values $x(0) = 0, y(0) = 0, z(0) = 0$

Ans.

$$\begin{bmatrix} 3 & 0.1 & -0.2 \\ 0.1 & 7 & 0.3 \\ 0.3 & -0.2 & 10 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7.85 \\ -19.3 \\ 71.4 \end{bmatrix}$$

Now, Using Gauss Seidel Method, initial $K = 0$ ($\because K =$ iteration)

$$\therefore x^{(K+1)} = \frac{1}{a_{11}} \left(b_1 - a_{12} y^{(K)} - a_{13} z^{K} \right)$$

$$y^{(K+1)} = \frac{1}{a_{22}} \left(b_2 - a_{21} x^{(K+1)} - a_{23} z^{(K)} \right)$$

$$z^{(K+1)} = \frac{1}{a_{33}} \left(b_3 - a_{31} x^{(K+1)} - a_{32} y^{(K+1)} \right)$$

For $\lambda = 3$

$$[A - 3I][x] = [0]$$

$$\Rightarrow \begin{bmatrix} 2-3 & -1 \\ -1 & 2-3 \end{bmatrix} [x] = [0]$$

$$\Rightarrow \begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow -x - y = 0 \Rightarrow -x = y$$

Eigen Values for $A + \lambda I$

$\Rightarrow \lambda_1 + c, \ \lambda_2 + c$

$1+4, \ 3+4$

$5, 7$ (Ans)

if $x = K$, then $x = y = K$

$\therefore$ Eigen Space $= \begin{bmatrix} K \\ -K \end{bmatrix} = K \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ (Ans)

Q4. Solve by Gauss - Siedel Method (Take three iterations)

$$3x + 0.1y - 0.2z = 7.85$$
$$0.1x + 7y - 0.3z = -19.3$$
$$0.3x - 0.2y + 10z = 71.4$$

$$\Rightarrow \begin{bmatrix} 3 & 0.1 & -0.2 \\ 0.1 & 7 & -0.3 \\ 0.3 & -0.2 & 10 \end{bmatrix}$$

with the initial values $x(0) = 0, y(0) = 0, z(0) = 0$

Soln.

$$\begin{bmatrix} 3 & 0.1 & -0.2 \\ 0.1 & 7 & 0.3 \\ 0.3 & -0.2 & 10 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7.85 \\ -19.3 \\ 71.4 \end{bmatrix}$$

Now, Using Gauss Siedel Method, initial $K = 0$ ($\because K$ = iteration)

$$\therefore x^{(K+1)} = \frac{1}{a_{11}} \left(b_2 - a_{12} y^{(K)} - a_{13} z^{K} \right)$$

$$y^{(K+1)} = \frac{1}{a_{22}} \left(b_2 - a_{21} x^{(K+1)} - a_{23} z^{(K)} \right)$$

$$z^{(K+1)} = \frac{1}{a_{33}} \left(b_3 - a_{31} x^{(K+1)} - a_{23} y^{(K+1)} \right)$$

Iteration 1st

$$x' = \frac{1}{3}\left(7.85 + 0.1 \cdot 0 + 0.2 \cdot 0\right) = \frac{7.85}{3} = 2.6167$$

$$y^{(1)} = \frac{1}{7}\left(-19.3 - 0.1x' + 0.3 \cdot 0\right) = \frac{1}{7}\left(-19.3 - \frac{0.785}{3}\right) = -2.7571$$

$$z^{(1)} = \frac{1}{10}\left(-71.4 - 0.1x^{(1)} + 0.3 \cdot y^{(1)}\right) = \frac{1}{10}\left(-71.4 - \frac{0.785}{3} + \frac{16.8345}{3}\right)$$

$$= 6.7757$$

Iteration 2nd :-

$$x^{(2)} = \frac{1}{3}\left(-7.85 + 0.1 \cdot y^{(1)} + 0.2\, z^{(1)}\right) = \frac{1}{3}\left(-7.85 + 0.1\left(\frac{56.115}{21}\right) + 0.2\left(\frac{-198.150}{30}\right.\right.$$

$$y^{(2)} = \frac{1}{7}\left(-19.3 - 0.1\left(2.6167\right) + 0.3\left(6.7757\right)\right) = 1.7908$$

$$= -3.3377$$

$$z^{(2)} = \frac{1}{10}\left(71.4 - 0.3\left(1.7908\right) + 0.2\left(-3.3377\right)\right) = 7.2991$$

Iteration 3rd:

$$x^{(3)} = \frac{1}{3}\left(7.85 + 0.1\left(-3.3377\right) + 0.2\left(7.2991\right)\right) = 1.6792$$

$$y^{(3)} = \frac{1}{7}\left(-19.3 - 0.1\left(1.6792\right) + 0.3\left(7.2991\right)\right) = -3.2744$$

$$z^{(3)} = \frac{1}{10}\left(71.4 - 0.3\left(1.6793\right) + 0.2\left(-3.2744\right)\right) = 7.3651$$

Q5. Define consistent & inconsistent system of eqn. Hence sol following system of eqn. if consistent $x + 3y + 2z = 0$, $+ 3z = 0$, $3x - 5y + 4z = 0$, $x + 17y + 4z = 0$.

Soln: Here in the form of $AX = B$.

$$A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & -1 & 3 \\ 3 & -5 & 4 \\ 1 & 17 & 4 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Since $B=0$ $\therefore$ Our system of eq^n is consistent. (Ans)

(Homogeneous)

Now, lets convert this into row-echlon form

$$\begin{bmatrix} 1 & 3 & 2 \\ 2 & -1 & 3 \\ 3 & -5 & 4 \\ 1 & 17 & 4 \end{bmatrix} \xrightarrow[\substack{R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1}]{R_4 \to R_4 - R_1} \begin{bmatrix} 1 & 3 & 2 \\ 0 & -7 & -1 \\ 0 & -14 & -2 \\ 0 & 14 & 2 \end{bmatrix} \xrightarrow{R_4 \to R_4 + R_3} \begin{bmatrix} 1 & 3 & 2 \\ 0 & -7 & -1 \\ 0 & -14 & -2 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\xrightarrow{R_3 \to R_3 - 2R_2} \begin{bmatrix} 2 & 3 & 2 \\ 0 & -7 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Here, $(\rho(A) = 2) < (n=3)$

$\therefore$ This system of eq^n has infinite sol^n.

Now we have $x + 3y + 2z = 0 \to ①$

$-7y - z = 0$ $\quad z = 7y$

From ① $\quad x + 3y + 14y = 0$

$x + 17y = 0$

$x = -17y$

If $y = k$, $x = -17k$, $z = 7k$ $\Rightarrow \therefore X = \begin{bmatrix} -17k \\ k \\ 7k \end{bmatrix} = k \begin{bmatrix} -17 \\ 1 \\ 7 \end{bmatrix}$ (Ans)

- Determine whether the function $T: P_2 \to R_2$ is linear transformation or not, where $T(a + bx + cx^2) = (a+1) + (b+1)x + (c+1)x^2$

Ans:- To check if T is a linear Transformation, we need to verify two properties

Additivity: $T(u+v) = T(u) + T(v)$ $\quad (u, v \in \text{Polynomial})$

Homogeneity: $T(cu) = c\,T(u)$ (For any polynomial u and scalar c)

Let's Check

① Additivity : $T\left((a_1+b_1x+c_1x^2)+(a_2+b_2x+c_2x^2)\right)$

$T\left((a_1+a_2)+(b_1+b_2)x+(c_1+c_2)x^2\right)$

$=(a_1+a_2+1)+(b_1+b_2+1)x+(c_1+c_2+1)x^2 = L.H.S$

Now

$T(a_1+b_1x+b_2x^2)+T(a_2+b_2x+c_2x^2)$

$=(a_1+1)+(b_1+1)x+(c_1+1)x^2+(a_2+1)+(b_2+1)x+(c_2+1)x^2$

$=(a_1+a_2+2)+(b_1+b_2+2)x+(c_1+c_2+2)x^2 = R.H.S.$

$\because$ L.H.S. $\neq$ R.H.S is not additive.

② Homogeneity :- $T(cu)=T(c(a+bx+cx^2))=(ca+1)+(cb+1)+(cc+1)$ L.H.S

& RHS $= cT(u)=cT(a+bx+cx^2)=c((a+1)+(b+1)x+(c+1)x^2)$

$=(ca+c)+(cb+c)x+(cc+c)x^2$

$\because$ L.H.S $\neq$ R.H.S, T is not homogeneous.

Therefore, T is not linear Transformation. (Ans)

Q7: Determine whether the set $S=\{(1,2,3),(3,1,0),(-2,1,3)\}$ is basis of $V_3(R)$. In case S is not a basis determine the dimension and the basis of the subspace spanned by S.

Soln: We need to check 2 things in order to determine if set S is a basis for $V_3(R)$

(1) Linear Independence: confirm that none of these vectors in S can be written as a linear ~~transformation~~ combination of the other.

(2) Spanning: Verify that the set S spans $V_3(R)$, meaning that any vector in $V_3(R)$ can be expressed as a linear combination of vect

Now form a matrix with vectors of S as its columns & now reduce to check for linear independence.

$$\begin{bmatrix} 1 & 3 & -2 \\ 2 & 1 & 1 \\ 3 & 0 & 3 \end{bmatrix} \xrightarrow[R_2 \to R_2 - 2R_1]{R_3 \to R_3 - 3R_1} \begin{bmatrix} 1 & 3 & -2 \\ 0 & -5 & 5 \\ 0 & -9 & 9 \end{bmatrix} \xrightarrow{R_3 \to R_3 - 9/5 R_2} \begin{bmatrix} 1 & 3 & -2 \\ 0 & -5 & 5 \\ 0 & 0 & 0 \end{bmatrix}$$

Now, $\rho(A) = 2 < (n = 3)$

So, it is infinite, linearly dependent, hence don't span $V_3(\mathbb{R})$ and not basis.

Next. Let's Determine dimension & basis of subspace spanned by S.

Dimension $\rightarrow$ no. of linearly independent vectors in S.

From above, we see that the first and second row are L.I. but row 3 not.

$\therefore$ Dimension $= 2$ and basis for the subspace spanned by S is given by L.I. vectors $= \{(1,2,3), (3,1,0)\}$ (Ans)

Q.8. Using Jacobi's method (perform 3 iterations), solve

$$3x - 6y + 2z = 23$$
$$-4x + y - z = -15$$
$$x - 3y + 7z = 16$$

$$\Rightarrow \begin{bmatrix} 3 & -6 & 2 \\ -4 & 1 & -1 \\ 1 & -3 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 23 \\ -15 \\ 16 \end{bmatrix}$$

with initial values $x_0 = 1, y_0 = 1, z_0 = 1$

Soln. Initialize $x^{(0)} = 1, y^{(0)} = 1, z^{(0)} = 1$

$$x^{K} = \frac{1}{a_{11}}\left(b_1 + a_{12} y^{(K-1)} + a_{13} z^{(K-1)}\right)$$

$$y^{K} = \frac{1}{a_{22}}\left(b_2 + a_{21} x^{(K-1)} + a_{23} z^{(K-1)}\right)$$

Similarly z

1st iteration

$$x^{(1)} = \frac{1}{3}\left(23 + 6\cdot 1 \cancel{+} - 2\cdot 1\right) = 9$$

$$y^{(1)} = \frac{1}{1}\left(-15 + 1 + 1\right) = -10$$

$$z^{(1)} = \frac{1}{7}\left(16 + 1 - 3 + 1\right) = 2$$

2nd iteration

$$x^{(2)} = \frac{1}{3}\left(23 + 6\cdot(10) - 2\cdot 2\right) = -36$$

$$y^{(2)} = \frac{1}{1}\left(-15 + 4(9) + 2\right) = 26$$

$$z^{(2)} = \frac{1}{7}\left(16 - (36) + 3(-10)\right) = 8$$

3rd iteration:

$$x^{(3)} = 1/3\left(23 + 6(26) - 2(8)\right) = 47$$

$$y^{(3)} = 1/1\left(-15 + 4(-36) + 8\right) = -163$$

$$z^{(3)} = \frac{1}{7}\left(16 - 47 + 3(26)\right) = 11$$

After 3 iteration $x = 47$, $y = -163$, $z = 11$ (Ans)

Q9. Explain an application of matrix operation in image processing with example.

Soln:- Application:- Image Transformation through affine Transform

Affine transformation involve translation rotation, scaling and shearing of image and these operations can be efficiently represented by using matrices.

Example:

Scaling Operation:-

Let's take a 2D image represented as $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

To scale the image by a factor of s in the x-direction and t in the y-direction and the transformation matrix T is

$$T = \begin{bmatrix} s & 0 \\ 0 & t \end{bmatrix}$$

The resulting transformed image I' is obtained by multiplying the original image matrix I with the transformation matrix T

$$I' = T.I.$$

This multiplication efficiently applies scaling to each pixel in the image.

For eg: we have image represented by $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

we want to scale it by a factor of 2 in x-direction and 3 in y-direction the transformation would be:

$$T = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$$

The resulting transformed image I' would be

$$T' = \begin{bmatrix} 2 & 4 \\ 9 & 12 \end{bmatrix}$$

This demonstrates how matrix operations facilitate efficient and systematic manipulation of images in various ways within the field of image processing.

Q10. Give a brief description of linear Transformation for computer Vision for rotating 2D image.

Soln: Linear transformation plays a crucial role in computer vision, particularly in the context of rotating 2D images. A linear transformation can be presented by a matrix that operates on the co-ordinates of each pixel in the image, producing a transformed image. For rotation specifically the rotation matrix is employed.

Rotation matrix For 2D ~~arrays~~ images :-

$$R = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & +\cos\theta \end{bmatrix}$$

If I = original image matrix after multiplying I with R, we obtain the rotated image I' reflecting the rotation.

This linear transformation is fundamental in computer vision application allowing for efficient manipulation and analysis of images. indicating rotation to correct orientation or align objects in the image.
