

Prerequisite

- In statistics "population" refers to the total set of observations that can be made.
 - e.g., if we want to calculate average height of humans present on the earth, "population" will be the "total number of people actually present on the earth".
- A **sample,** on the other hand, is a set of data collected/selected from a pre-defined procedure.
- For our example above, it will be a small group of people selected randomly from some parts of the earth.

Prerequisite

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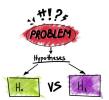
- To draw inferences from a sample by validating a hypothesis it is necessary that the sample is **random**.
- For instance, in our example above if we select people randomly from all regions(Asia, America, Europe, Africa etc.) on earth, our estimate will be close to the actual estimate and can be assumed as a sample mean, whereas if we make selection let's say only from the United States, then our average height estimate will not be accurate but would only represent the data of a particular region (United States).
- Such a sample is then called a biased sample and is not a representative of "population".

Prerequisite

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- Another important aspect to understand in statistics is "distribution".
- When "population" is infinitely large it is improbable to validate any hypothesis by calculating the mean value or test parameters on the entire population.
- In such cases, a population is assumed to be of some type of a distribution.

Hypothesis Testing



Hypothesis testing is a critical tool in inferential statistics, for determining what the value of a *population parameter* could be.

We often draw this conclusion based on a *sample data analysis*.

Hypothesis Testing

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- \bullet The basis of hypothesis testing has two attributes:
- (a) Null Hypothesis (H₀)and
- (b) Alternative Hypothesis (H₁or H_A)
- The null hypothesis is, in general, the **boring stuff** i.e. it assumes that nothing interesting happens/happened.
- The alternative hypothesis is, where the action is i.e. some observation/ phenomenon is real (i.e. not a fluke) and statistical analysis will give us more insights on that.

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What is the Process?

- Statisticians take a pessimistic sort of view and start with the Null hypothesis, and compute some sort of test-statistic in the sample data.
- It is given by, $\frac{\textit{Best Estimate} \textit{Hypothesized Estimate}}{\textit{Standard Error of Estimate}}$
- \bullet Here, the 'best estimate' comes from the sample e.g. sample mean or proportion of some data in the sample.
- Standard error represents the variability in this estimate and often depends on the variance and sample size.

Hypothesis Testing and p-value

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- In Null hypothesis significance testing, the p-value is the probability of obtaining test results at least as extreme as the result actually observed, under the assumption that the null hypothesis is correct.
- This chance probability value of observing the test-statistic is the so-called $\mbox{{\bf p-value}}.$
- The smaller the **p-value**, the stronger the evidence against the Null hypothesis. If p-value is less than the level of significance (α) , we should **reject the Null hypothesis**.
- p-values are expressed as decimals although it may be easier to understand
 what they are if you convert them to a percentage. For example, a p value of
 0.0254 is 2.54%.

Critical Value

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The p-value is probability of observing the test-statistic, as is, given the Null hypothesis is true. This probability calculated under the assumption of a certain probability distribution (that the test statistic is generated from).



If this particular value is very small (less than a pre-determined **Critical Value**), we can reject the Null hypothesis in favour of the alternative.

Important:

Pr (observation | hypothesis) = Pr (hypothesis | observation)

The probability of deserving a result spins that corresponds to its true rate equivalent to the probability that a hypothesis is true given that some result has been observed.

Using the p-value as a "order" is correctioning an eary egious logical error:

The transposed conditional fallacy.

More Blady deservation

A p-value of shaded green a real is the probability of an observed (or more extreme) result assuming that the null hypothesis is true.

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Critical Value

- A critical value is a point (or points) on the scale of the test statistic beyond which we reject the null hypothesis, and, is derived from the level of significance α of the test.
- Critical value can tell us, what is the probability of two sample means belonging to the same distribution.
- Higher, the critical value means lower the probability of two samples belonging to same distribution.
- The general critical value for a two-tailed test is **1.96**, which is based on the fact that **95%** of the area of a normal distribution is within 1.96 standard deviations of the mean.

Critical Value

- Critical values can be used to do hypothesis testing in following way
- 1. Calculate test statistic
- 2. Calculate critical values based on significance level alpha (α)
- 3. Compare test statistic with critical values.
- 4. If the test statistic is falling in the critical region, we reject the Null Hypothesis in favour of the Alternate Hypothesis.

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One-tailed / Two-tailed Tests

 $\begin{array}{c} \text{sigh-tail-test} \\ H_0: \mu > \text{value} \\ \\ \hline \\ 0 \\ \\ 1, \text{in more probable} \\ \\ \hline \\ 0 \\ \\ 1, \text{in more probable} \\ \\ \text{Two-tail-test} \\ \\ \text{How } \mu < \text{value} \\ \\ \text{Two-tail-test} \\ \\ H_0: \mu \neq \text{value} \\ \\ \text{Two-tail-test} \\ \\ \text{Two-t$

In some situations, the hypothesis deals with questions in the form of "x is greater than y" or "x is lesser than y".

In those cases, only one side of the probability distributions have to be checked for and we call them 'one-sided' or 'one-tailed' test

In other situation, we have to use both sides of the probability distribution.

That is called 'two-sided' or 'two-tailed'

In these situations, the alternative hypothesis is generally expressed in the form "x is not equal to y".

*Hypothesis tests are based on the notion of critical regions.

*The Null Hypothesis is rejected in favour of the alternate if the test satistic falls in the critical region.

*Otherwise, we reject the alternate hypothesis in favour of the Null hypothesis.

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test.

Steps - Hypothesis Testing

- Step 1 State the hypotheses and identify the claim.
- Step 2 Find the critical value(s) from the appropriate table.
- Step 3 Compute the test value.
- Step 4 Make the decision to reject or not reject the null hypothesis.
- Step 5 Summarize the results.

7-test

In a z-test, the sample is assumed to be normally distributed.

A z-score is calculated with population parameters such as "population mean" and "population standard deviation" and is used to validate a hypothesis that the sample drawn belongs to the same population.

Null: Sample mean is same as the population mean

Alternate: Sample mean is not same as the population mean

The statistics used for this hypothesis testing is called z-statistic, the score for which is calculated as follows:

 $Z = \frac{\bar{X} - \mu_0}{\sqrt[3]{\eta}}$ Population Manual Population Number of standard deviation samples

If the test statistic is lower than the critical value, we fail to reject the Null hypothesis or else reject it.

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T-test

In statistics, the term "t-test" refers to the hypothesis test in which the test statistic follows a Student's t-distribution.

A t-test is used to compare the mean of two given samples.

Like a z-test, a t-test also assumes a normal distribution of the sample.

A t-test is used when the population parameters (mean and standard deviation) are not known.

There are three versions of t-test

- $1. \ \mbox{Independent samples t-test}$ which compares mean for two groups
- 2. Paired sample t-test which compares means from the same group at different times $% \left(1\right) =\left(1\right) +\left(1\right)$
- $3. \, \mbox{One}$ sample t-test which tests the mean of a single group against a known mean.

T-test

T-test is used to check whether two data sets are significantly different from each other or not.

One of the variants of the t-test is the one-sample t-test which is used to determine if the sample is significantly different from the population.

The formula for a one-sample t-test is expressed using the observed sample mean, the theoretical population means, sample standard deviation, and sample size. Mathematically, it is represented as,

t = (x-μ)/(s/√n)

where

• x'= Observed Mean of the Sample

• μ = Theoretical Mean of the Population

• s = Standard Deviation of the Sample

• n = Sample Size

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T-test

• In case statistics of two samples are to be compared, then a two-sample t-test is to be used and its formula is expressed using respective sample means, sample standard deviations,

and sample sizes. · Mathematically, it is represented as,

• n 1 = Size of 1st Sample • n 2 = Size of 2nd Sample

 $t = (x_1 - x_2) / \sqrt{(s_1^2 / n_1) + (s_2^2 / n_2)}$

 x₁ = Observed Mean of 1st Sample • x₂ = Observed Mean of 2nd Sample

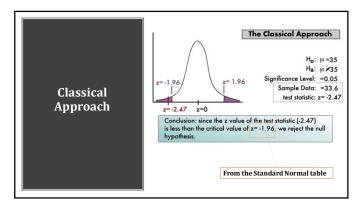
• s₁ = Standard Deviation of 1st Sample

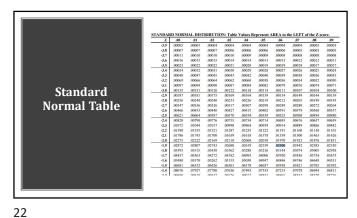
• s₂= Standard Deviation of 2nd Sample

If the test statistic is lower than the critical value, we fail to reject the Null hypothesis or else reject it.

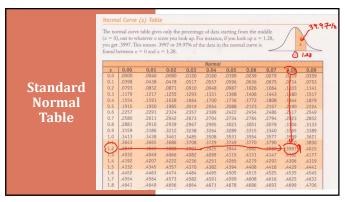
The P-Value Approach $H_0: \mu = 35$ Ha: u≠35 ance Level: =0.05 Sample Data: =33.6 a =0.025 $\alpha = 0.025$ p-value test statistic: z= -2.47 Approach P-value = 0.0068 times 2 (for a 2-sided test) = 0.0136 Conclusion: since the P-value of 0.0136 is less than the significance level of α =0.05, we reject the null hypothesis From the Standard Normal table for z=-2.47

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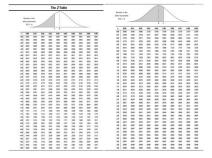
How to Find Probabilities for Z with the Z-Table?

- You can use the **Z-table** to find a full set of "less-than" probabilities for a wide range of z-values. To use the **Z-table** to find probabilities for a **statistical sample** with a standard normal (**Z-**) distribution, do the following:
- 1. Go to the row that represents the ones digit and the first digit after the decimal point (the tenths digit) of your z-value.
- $2.Go\ to\ the\ column\ that\ represents\ the\ second\ digit\ after\ the\ decimal\ point\ (the\ hundredths\ digit)\ of\ your\ z-value.$
- 3.Intersect the row and column from Steps 1 and 2. This result represents p(Z < z), the probability that the random variable Z is less than the value z (also known as the percentage of z-values that are less than the given z value).

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Computing p-Value

- For example, suppose you want to find *p*(*Z* < 2.13).
- Using the Z-table below, find the row for 2.1 and the column for 0.03.
- Intersect that row and column to find the probability: 0.9834.
- Therefore p(Z < 2.13) =



Computing p-Value

 \bullet Noting that the total area under any normal curve (including the standardized normal curve) is 1, it follows that p(Z < 2.13) + p(Z > 2.13) = 1. Therefore, p(Z > 2.13) = 1 - p(Z < 2.13) which equals 1 - 0.9834 which equals 0.0166.

- Suppose you want to look for p(Z < -2.13). You find the row for -2.1 and the column for 0.03. Intersect the row and column and you find 0.0166; that means p(Z < -2.13) = 0.0166. Observe that this happens to equal p(Z > +2.13).
- \bullet The reason for this is because the $normal\ distribution$ is symmetric.
- So the tail of the curve below -2.13 representing p(Z < -2.13) looks exactly like the tail above 2.13 representing p(Z > +2.13).

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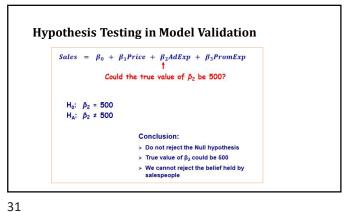
Significance Levels Significance Level 5% versus 10% $\alpha = 5\%$ The P-value is 7.8%. The P-value is greater than α . $\alpha = 10\%$ The P-value is 7.8%. The P-value is less than α Conclusion: Data is statistically significant. (Rare in random sampling if Ho is true.) Conclusion: Data is NOT statistically significant. (Not rare in random sampling if Ho is true.) sample proportions (if p=0.40) sample proportions (if p=0.40)

A (1 - α) confidence interval for the population mean... **Confidence Interval** $\overline{x} \, - \, \big| z_{\alpha/2} \big| \tfrac{\sigma}{\sqrt{n}} \ < \ \mu \ < \ \overline{x} \, + \, \big| z_{\alpha/2} \big| \tfrac{\sigma}{\sqrt{n}}$ When the population standard deviation (σ) is not known, we replace it by the sample standard deviation (s) $_{ extstyle e$ A (1 - α) confidence interval for the population mean... $\overline{x} - \left|t_{\alpha/2}\right| \frac{s}{\sqrt{n}} < \mu < \overline{x} + \left|t_{\alpha/2}\right| \frac{s}{\sqrt{n}}$

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Hypothesis Testing in Model Validation $Sales = \beta_0 + \beta_1 Price + \beta_2 AdExp + \beta_3 PromExp$ $Sales \,=\, -25096.83 - 5055.27 Price + \underline{648.61} AdExp + 1802.61 PromExp$ For every 1000 dollars increase in advertisement spending, the sales increase by 648.6 (~649) units, all other variables remaining at the same level. Belief held by Salespeople... For every 1000 dollars increase in advertising expenditure, the unit sales increase by 500 units.

Hypothesis Testing in Model Validation Hypothesis testing needed to test this belief. True beta values, fixed but unknown $Sales = \beta_0 + \beta_1 Price + \beta_2 AdExp + \beta_3 PromExp$ Sales = -25096.83 - 5055.27 Price + 648.61 AdExp + 1802.61 PromExpEstimates based on Sample data Hypothesis test needed to test whether β_2 is equal to 500



Confidence Interval Approach to Hypothesis Testing $Sales = \beta_0 + \beta_1 Price + \beta_2 AdExp + \beta_3 PromExp$ ANOVA
 SS
 MS
 F
 Significance F

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 1.0848E-08
 20 32509212.11 1625461 Residual Total 23 230308045
 Coefficients Standard Error
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 P-value
 Lower 95%
 Upper 95%

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 -1.009542
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 -76953.0734
 26759.408
 Intercept Price (\$) -5055.27 526.3995537 -9.603484 6.22E-09 -6152.22009 -3957.22 648.61214 209.0048787 3.103335 0.005602 212.635603 1084.5887 Adexp ('000\$) Promexp ('000\$) 1802.611 392.8485427 4.588565 0.000178 983.143256 2622.0787

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Confidence Interval Approach to Hypothesis Testing Step 1 : Formulate Hypothesis H_0 : $\beta_2 = 500$ H_A : $\beta_2 \neq 500$ Step 2 : Consider the 95% confidence interval for β_2 = [212.6, 1084.6] center point = 648.6 Conclusion: > Since 500 falls in the confidence interval, hence do not reject the Null hypothesis. > We cannot reject the H₀ for any value that is in the confidence interval.

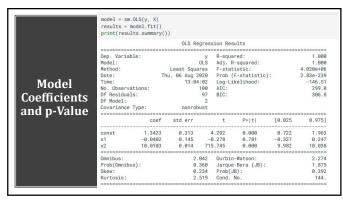
p-Value and Model Coefficients p-values and their importance in interpreting regression results $Sales = \beta_0 + \beta_1 Price + \beta_2 AdExp + \beta_3 PromExp$ ANOVA
 SS
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 Significance F

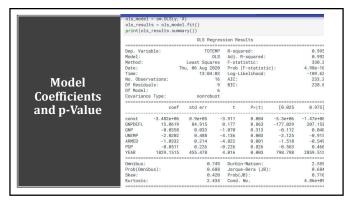
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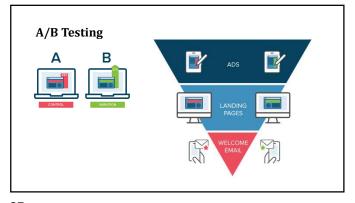
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 H_0 : $\beta_1 = 0$ H_a : $\beta_1 \neq 0$ Total > Reject the Null hypothesis that β_2 = 0. > Advertising expenditure is an important variable in explaining Sales.

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A/B Testing

- This method of introducing changes to a user experience also allows the experience to be optimized for a desired outcome and can make crucial steps in a marketing campaign more effective.
- A/B testing can also be used by product developers and designers to demonstrate the impact of new features or changes to a user experience.
 Product onboarding, user engagement, modals, and in-product experiences can all be optimized with A/B testing, so long as the goals are clearly defined, and you have a clear hypothesis.
- By testing ad copy, marketers can learn which version attracts more clicks. By testing the subsequent landing page, they can learn which layout converts visitors to customers best. The overall spend on a marketing campaign can actually be decreased if the elements of each step work as efficiently as possible to acquire customers.

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A/B Testing Process

- The following is an A/B testing framework you can use to start running tests:
- Collect Data: Your analytics will often provide insight into where you can begin optimizing. It helps to begin with high
 traffic areas of your site or app, as that will allow you to gather data faster. Look for pages with low conversion rates or
 high drop-off rates that can be improved.
- Identify Goals: Your conversion goals are the metrics that you are using to determine whether or not the variation is
 more successful than the original version. Goals can be anything from clicking a button or link to product purchases and e-mail signups.
- Generate Hypothesis: Once you've identified a goal you can begin generating A/B testing ideas and hypotheses for why you think they will be better than the current version. Once you have a list of ideas, prioritize them in terms of expected impact and difficulty of implementation.
- Create Variations: Using your A/B testing software (like Optimizely), make the desired changes to an element of your website or mobile app experience. This might be changing the color of a button, swapping the order of elements on the page, hiding navigation elements, or something entirely custom. Many leading A/B testing tools have a visual editor that will make these changes easy. Make sure to QA your experiment to make sure it works as expected.
- will make these changes easy. Make sure to QA your experiment to make sure it works as expected.

 Run Experiment: Kick off your experiment and wait for visitors to participated At this point, visitors to your site or app will be randomly assigned to either the control or variation of your experience. Their interaction with each experience is measured, counted, and compared to determine how each performe.

 Analyze Results: Once your experiment is complete, it's time to analyze the results. Your A/B testing software will persent the data from the experiment and show you the difference between how the two versions of your page performed, and whether there is a statistically significant difference.

A/B Testing

Seen by another user

TV series on Netflix: Users are subjected to a form of hypothesis testing called A/B testing.

Netflix shows the same show, differently designed, to different user groups. Responses of the users (click/no-click/browse/nobrowse/comes-back-to-watch-or-not) are recorded analyzed using the good old hypothesis testing method.

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Hypothesis Testing in Python

We can perform Hypothesis testing in Python (using functions from the Statsmodels package).

These four situations appear in a large fraction of statistical analyses,

- •One Population Proportion
- •A difference in Population Proportions
- One Population Mean
- •A difference in Population Means

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Go to the coding demo...

To be continued in the next session.....