The critical region

The critical region is the region of values that corresponds to the rejection of the null hypothesis at some chosen probability level. The shaded area under the *Student's t* distribution curve is equal to the level of significance. The critical values are tabulated and thus obtained from the *Student's t* table or anther appropriate table. If the absolute value of the *t* statistic is larger than the tabulated value, then *t* is in the critical region.

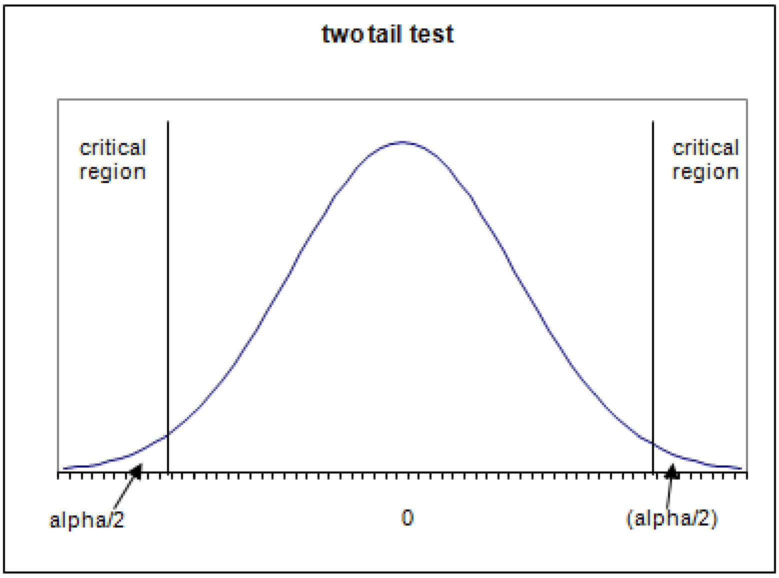
1. One tailed and two tailed tests

The statistical tests used will be **one tailed** or **two tailed** depending on the nature of the null hypothesis and the alternative hypothesis.

The following hypothesis applies to test for the mean:

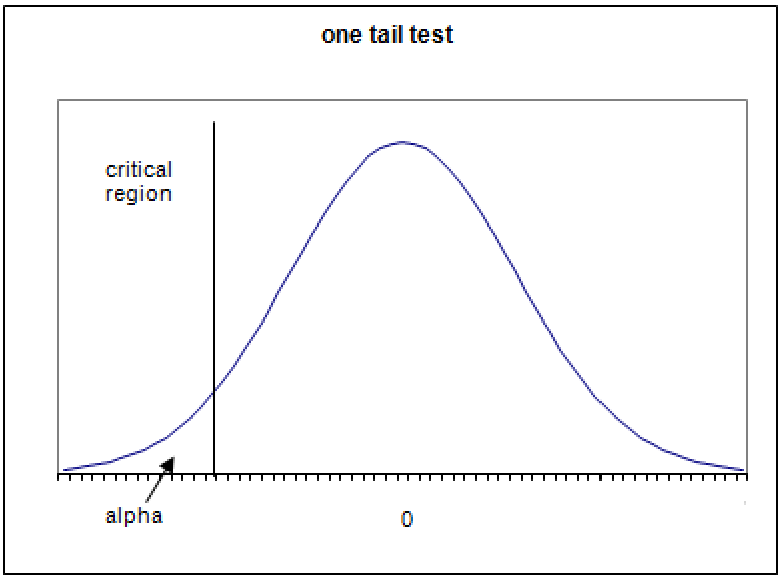
- **two tailed test:**

$\mathbf{H_0} : \mu = \mu_0$ $\qquad\qquad$ $\mathbf{H_1} : \mu \neq \mu_0$;



- **One tail tests:**

H_0 : $\mu = \mu_0$ **H_1** : $\mu < \mu_0$;



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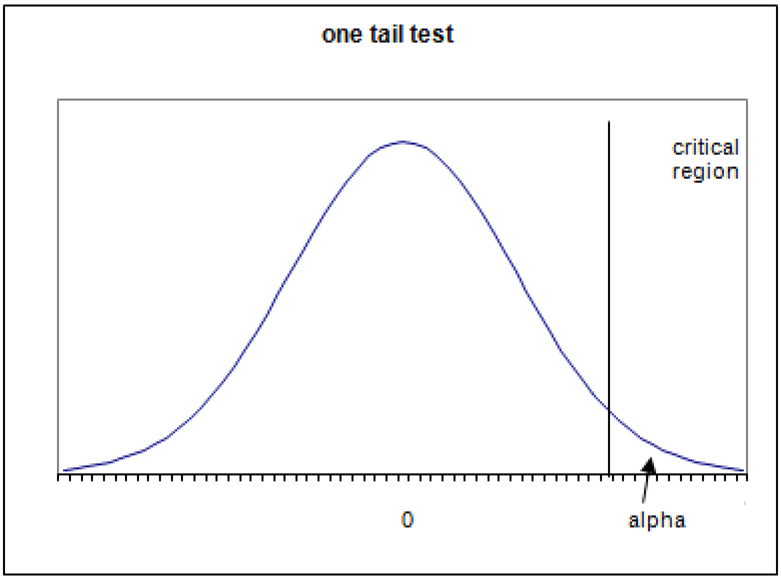


When we are interested only in the extreme values that are *greater than or less than a comparative value* (μ_0) we use a one tailed test to test for significance. When we are interested in determining that things are different or not equal, we use a two tailed.

- To determine the critical region for a normal distribution, we use the table for the standard normal distribution. If the level of significance is α = 0.10, then for a one tailed test the critical region is below z = -1.28 or above z = 1.28. For a two tailed test, use $\alpha/2$ = 0.05 and the critical region is below z = -1.645 and above z = 1.645. If the absolute value of the calculated statistics has a value equal to or greater than the critical value, then the null hypotheses, $\mathbf{H_0}$ should be rejected and the alternate hypotheses, $\mathbf{H_1}$.
- To determine the critical region for a t-distribution, we use the table of the t-distribution. (Assume for the moment that we use a t-distribution with 20 degrees of freedom). If the level of significance is α = .10, then for a one tailed test t = -1.325 or t = 1.325. For a two tailed test, use $\alpha/2$ = 0.05 and then t = -1.725 and t = 1.725. If the absolute value of the calculated statistics has a value equal to or greater than the critical value, then the null hypotheses, $\mathbf{H_0}$ will be rejected and the alternate hypotheses, $\mathbf{H_1}$, is assumed to be correct.

Example 1:

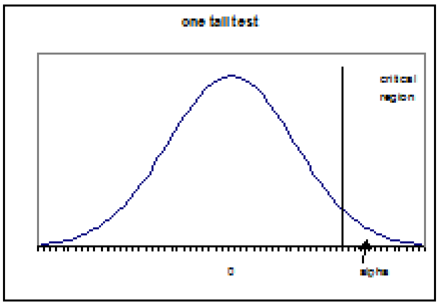
A tire manufacturing plant produces 15.2 tires per hour. This yield has an established variance of 2.5 (σ =1.58 tires/hour). New machines are recommended, but will be expensive to install. Before deciding to implement the change, 12 new machines are tested. They produce 16.8 tire per hour. Is it worth buying the new machines?

Before testing, verify that the data comes from a normal distribution (assumption of the test)

1. **Formulate hypotheses**:

H_0 : μ = 15.2
(ie. Mean yield of new machines is equal to 15.2 with a variance of 2.5)

H_1 : μ > 15.2
(ie. Mean yield of new machines is greater than 15.2)



2. **Choose α**

We choose α= 0.10.

3. **Select the statistic**

Here we must use the z statistic to test the null hypothesis since the variance is known.

4. **Find the critical region**:

The z-value obtained from **Table 1** for z is 1.282. Hence, the critical region for a one tailed test is: $z > 1.282$.

5. **Compute the statistic**:

Assume x (the *yield*) has a normal distribution with mean 15.2 and variance equal to 2.5 (N(15.2, 2.5)). Then, z can be calculated as following:

$$z = \frac{\bar{x}-\mu_0}{s/\sqrt{n}} = \frac{16.8-15.2}{1.58/3.4641} = 3.51.$$

1. **Draw conclusion:**

Since the calculated $z = 3.51 > 1.282$, we reject the $\mathbf{H_0}$ hypothesisthat the mean yield from the new machines equals 15.2. The mean yield of the new machine is greater than 15.2.

Example 2:

The manufacturing of rubber chemicals by a batch process, has a normal yield of 690 lbs per batch. A new process is tried experimentally on 12 batches with the following yields: 620, 590, 660, 620, 700, 710, 690, 720, 700, 690, 720 and 650 lbs. Is the yield of the new process a significantly different from that of the old process?

From the data we calculated the following:

$\bar{x} = 672.6\ lbs, s = 43.72\ lbs.$

Assume the data is approximately normal.

1. $\mathbf{H_0}$: $\mu = 690$

   That is 672.5 is not significantly different from 690

   $\mathbf{H_1}$: $\mu \neq 690$

   That is 672.5 is significantly different from 690

2. **Choose** $\alpha = 0.01$
3. **Select statistic** : test the $\mathbf{H_0}$ using the *Student's t* statistic.
4. **Critical region** (from **Table 2**, n=12-1=11):

   $|t| > t_{(.995)(11)} = 3.106$.
5. **Compute the value of the test statistic**:

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{672.5 - 690}{\frac{43.72}{3.464}} = -1.38.$$

6. Since the calculated $|t|$=1.38 < 3.106, the $\mathbf{H_0}$ that the new process does not differ from the old process with respect to batch yields can **not be rejected**. Conclude that the new process gives equal yields.

Example 3:

A drift test was run at San Angelo to compare Goodyear tires to competitor tires on a Ford Escort. Is there a significant difference in the precision of the results between suppliers?

	Goodyear	Competitor
No. of tests :	17	25
Mean	- 27.2	- 39.2
Variance s^2	132.25	207.36

Assume the observations come from two normal distributions.

1. **Formulate the hypotheses**:

H_0: $\sigma^2_{GDYR} = \sigma^2_{CMPT}$ vs **H_1** : $\sigma^2_{GDYR} \neq \sigma^2_{CMPT}$

2. **Choose** α=0.10.

3. **Select a test statistic:** *F* statistic with *F* distribution with (24, 16) degrees of freedom.

5. **The critical region** obtained from **Table 3**, is:

$$F_{\left(1-\frac{\alpha}{2}\right),(n_1-1,n_2-1)} = F_{(0.95),(24,16)} = 2.24.$$

6. **Compute the statistic**:

$$F = \frac{s^2_{CMPT}}{s^2_{GDYR}} = \frac{207.36}{132.25} = 1.57$$

Note: By convention, always put the larger variance in the numerator.

7. Since the calculated F value is smaller than the critical one (1.57 < 2.24), we accept **H_0** that the two population variances are equal. Thus, assume that the supplier does not affect the precision.