APPLICATIONS OF CALCULUS

A PROJECT REPORT

SUBMITTED IN COMPLETE FULFILLMENT OF THE REQUIREMNETS

FOR THE AWARD OF THE DEGREE

OF

BACHELOR OF TECHNOLOGY

IN

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CANDIDATE'S DECLARATION

We, are the students of B. Tech. (Computer Engineering) hereby declare that the project Dissertation titled "Application of Calculus" which is submitted by us to the Department of Applied Mathematics, Delhi Technological University, Delhi in partial fulfilment of the requirement for the award of the degree of Bachelor of Technology, is original and not copied from any source without proper citation. This work has not previously formed the basis for the award of any Degree, Diploma Associateship, Fellowship or other similar title or recognition.

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CERTIFICATE

I hereby certify that the project Dissertation titled "Application of Calculus" which is submitted by Nitesh Kumar (2k20/B8/70), Ayush Jaiswal(2k20/B8/21) [Computer Engineering], Delhi Technological University, Delhi in complete fulfilment of the requirement for the award of the degree of the Bachelor of Technology, is a record of the project work carried out by the students under my supervision. To the best of my knowledge this work has not been submitted in part or full for any Degree or Diploma to this University or elsewhere.

Place: Delhi

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ABSTRACT

Calculus is about comparing quantities which vary in a non-linear way. It is used in science and engineering since many of the things we are studying (like velocity, acceleration, current in a circuit) do not behave in a simple, linear fashion. If quantities are continually changing, we need calculus to study what is going on.

This project is based on Application of Calculus in which we have be discussed about what is Calculus, it's history, types of Calculus and relation between them. We have also talked about its different mathematical concepts as well as its applications in various other fields.

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2. History

3. Types of Calculus

    - Differential Calculus

    - Integral Calculus

4. Applications of Calculus

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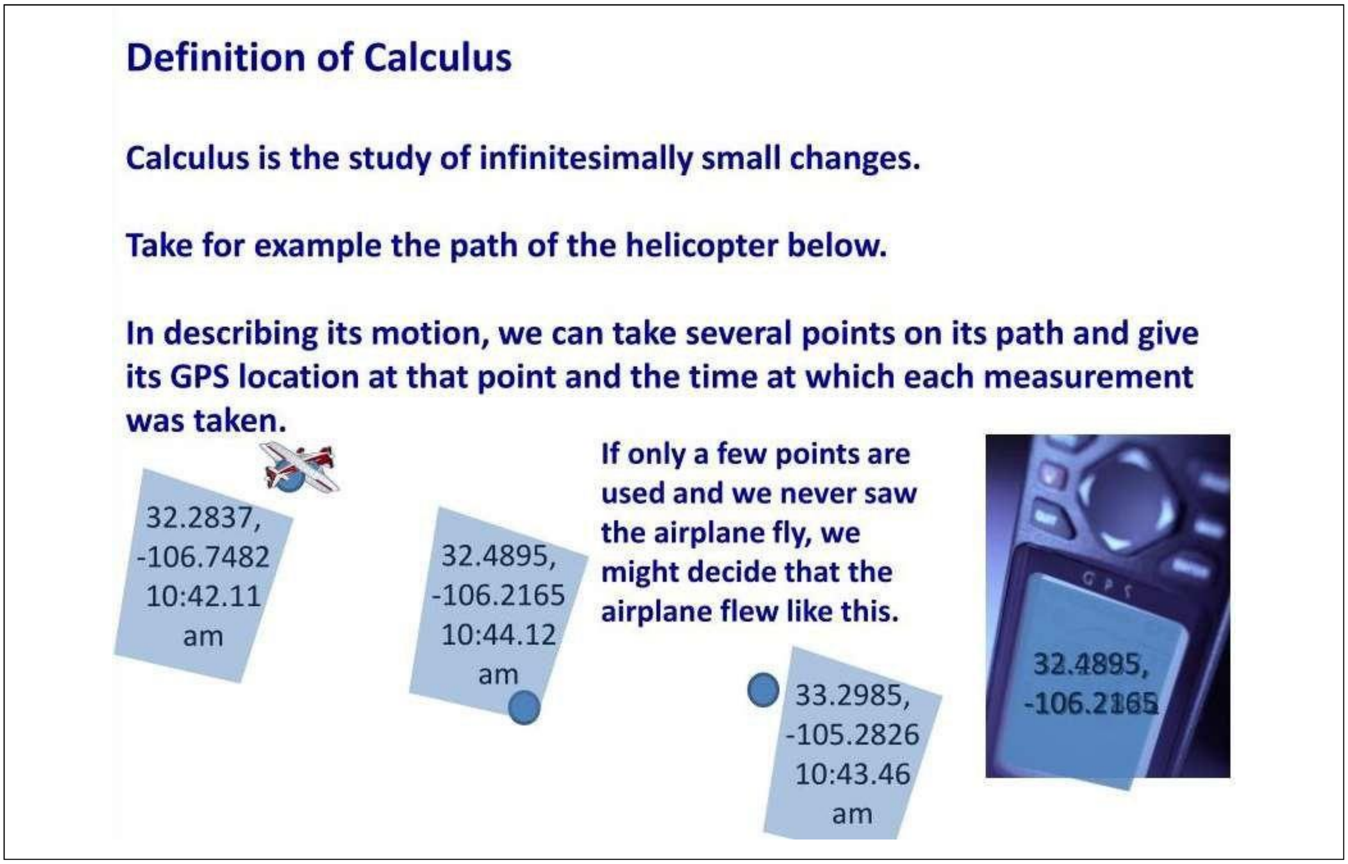
INTRODUCTION

Calculus was developed by Newton and Leibniz. It deals with the study of the rate of change. It was initially called infinitesimal calculus or "the calculus of infinitesimals". Calculus is generally used in Mathematical models to obtain ideal solutions. It helps in understanding the changes between the values which are related by a function. It is concerned with comparing quantities which vary in a non-linear way.



The two major types of calculus are Derivatives and Integrals.

The main theorem in calculus is called the Fundamental Theorem of Calculus. This theorem explains the relationship between the derivative and the integral. It proves that the two calculus processes, differentiation and integration, are inverses of each other. Which means, one can use differentiation to undo an integration process. Also, can use integration to undo a differentiation.

HISTORY

Sir Isaac Newton Gottfried Leibniz

1642-1727 1646-1716

Calculus was developed separately by Sir Isaac Newton and Gottfried Leibniz. They were both working on issues of motion towards the end of the 17th century.

Newton wanted to find a new way to predict where to see planets in the sky,
because astronomy had always been a useful form of science and understanding more about the motions of the planets, stars etc. in the sky was important for navigation purposes.

Leibniz wanted to measure the area under a curve.

Later, the two men had a dispute over who discovered it first. Mathematicians from England supported Newton, but Mathematicians from the rest of Europe supported Leibniz.
Most scientists today accept that both men played important role in discovering it. Some parts of modern calculus came from Newton, like its uses in physics. While other parts come from Leibniz, such as the symbols used to write it.

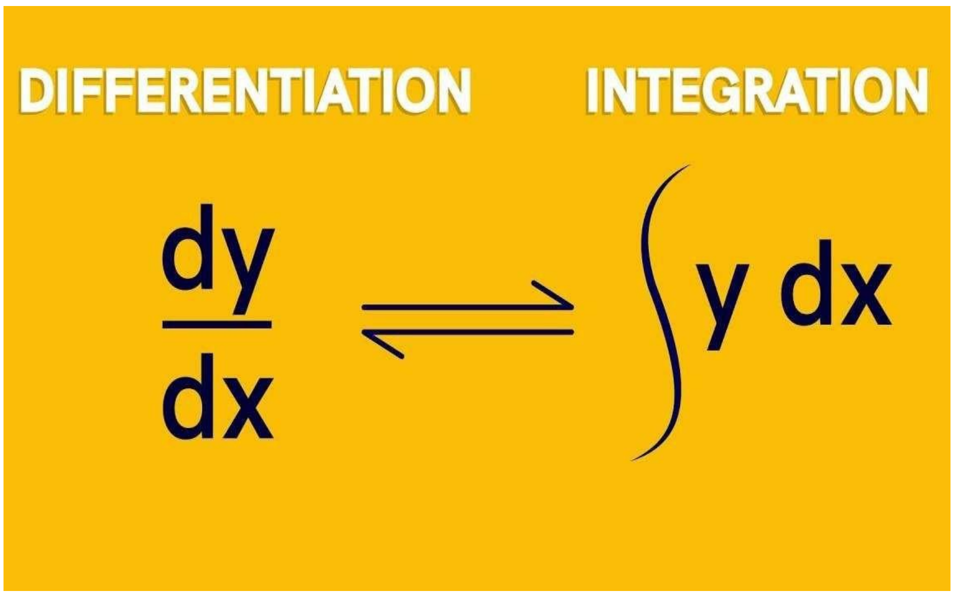
Differentiation		Integration	
Newton used a dot over the dependent variable to indicate a derivative, two dots for a second derivative, etc.	$\dot{y} = \frac{dy}{dt}$	Newton never used one consistent notation for integration, sometimes using a bar above a variable and sometimes putting the variable in a box.	$\bar{x}$ or $\boxed{x}$
Leibniz just used dx and dy to indicate infinitesimal increments in the independent and dependent variables.	$\frac{d(f(x))}{dx}$ or $\frac{dy}{dx}$	We have come to use Leibniz's notation, which uses an extended 'S' to indicate the sum of infinitely many infinitesimal quantities $f(x)$ for each infinitesimal increment dx, between the two stated limits a and b.	$\int_a^b f(x)\,dx$
The use of the prime mark for derivatives dates from Joseph Louis Lagrange's work on differential calculus in the late 18th / early 19th Century.	f' f''		

TYPES OF CALCULUS

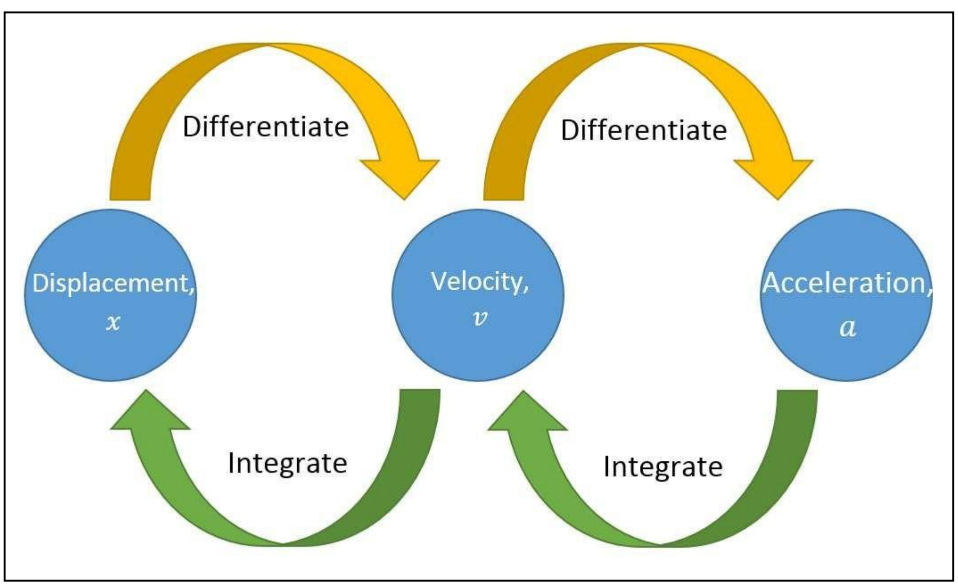
The two different types of calculus are :

1. Differential Calculus
2. Integral Calculus

Differential calculus divides things into tiny pieces and tells us how they change from one moment to the next, while integral calculus joins the tiny pieces together, and tells us how much of something is made, overall, by a series of changes.



Example of Application of relation between Differentiation and Integration



Differential Calculus

Differential calculus is the study of the definition, properties, and applications of the derivative of a function. The process of finding the derivative is called differentiation. Given a function and a point in the domain, the derivative at that point is a way of encoding the small-scale behaviour of the function near that point. By finding the derivative of a function at every point in its domain, it is possible to produce a new function, called the derivative function or just the derivative of the original function. In formal terms, the derivative is a linear operator which takes a function as its input and produces a second function as its output.

The most common symbol for a derivative is an apostrophe-like mark called prime. Thus, the derivative of a function called f is denoted by f′, pronounced "f prime".

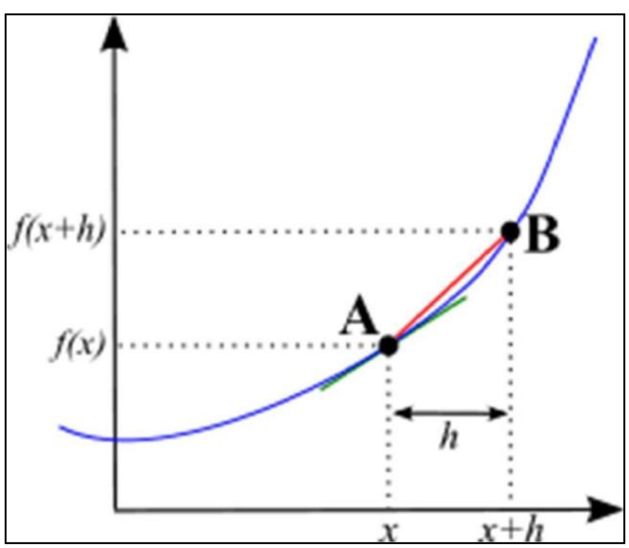
The way to write the derivative in mathematics is

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

In Leibniz notation:

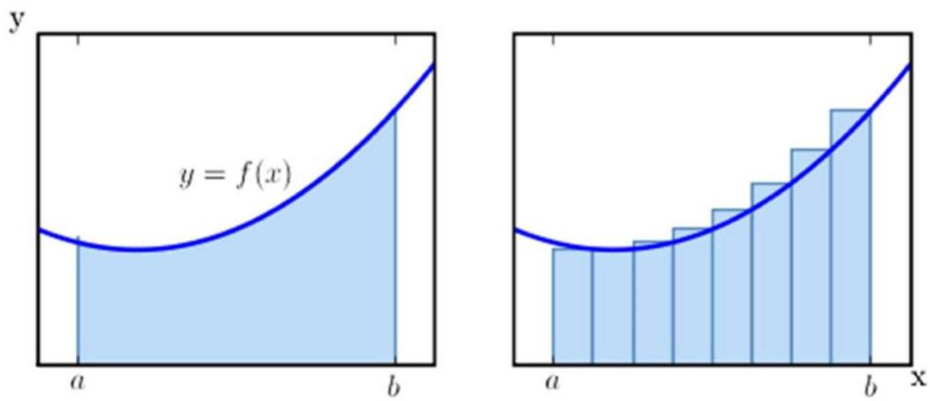
$$\frac{dy}{dx} = f'(x)$$

Differential calculus is useful for graphing. It can be used to find the slope of a curve, and the highest and lowest points of a curve (these are called the maximum and minimum, respectively), etc.



Integral Calculus

Integral calculus is the study of the definitions, properties, and applications of two related concepts, the indefinite integral and the definite integral. It is the process of calculating the area underneath a graph of a function. The way to do this is to divide the graph into many very small pieces, and then draw very thin rectangles under each piece. As the rectangles become thinner and thinner, the rectangles cover the area underneath the graph better and better. The area of a rectangle is easy to calculate, so we can calculate the total area of all the rectangles. For thinner rectangles, this total area value approaches the area underneath the graph. The final value of the area is called the integral of the function.



In technical language, integral calculus studies two related linear operators.

The indefinite integral, also known as the antiderivative, is the inverse operation to the derivative. F is an indefinite integral of f when f is a derivative of F. (This use of lower- and upper-case letters for a function and its indefinite integral is common in calculus.)

The indefinite integral, or antiderivative, is written as:

$$\int f(x)\,dx$$

The definite integral inputs a function and outputs a number, which gives the algebraic sum of areas between the graph of the input and the x-axis. The technical definition of the definite integral involves the limit of a sum of areas of rectangles, called a Riemann sum.

The definite integral is written as:

$$\int_a^b f(x)\,dx$$

APPLICATION OF CALCULUS

Calculus is a part of mathematics and also used in physics. Calculus helps us to find how the changing condition of a system affects us. Study of calculus helps us to learn how to control a system. Calculus is the language of engineers, economists, and scientists. From microwaves, cell phones, TV, and car to medicine, economy, and national defence all need calculus.

APPLICATION OF DIFFERNTIAL CALCULUS

1. Rate of Change of a Quantity
2. Minimum and Maximum Values
3. Increasing and Decreasing Functions
4. Tangent and Normal to a Curve

RATE OF CHANGE OF QUANTITY:-

If $f(x)$ is a function defined on an interval $[a, a+h]$, then the amount of change of $f(x)$ over the interval is the change in the y values of the function over that interval and is given by $f(a+h) - f(a)$

Then average rate of change of the function f over that same interval is the ratio of the amount o changeover that interval to the corresponding change in the x values. It is given by $[f(a+h)-f(a)]/h$

As we already know, the instantaneous rate of change of $f(x)$ at a is its derivative

$$f'(a)=\lim_{h\to 0} (f(a+h)-f(a))/h$$

For small enough values of h, $f'(a) \approx f (a+-hf)(a)/h$. We can then solve fofr$(a+h)$ to get the amount of change formula:

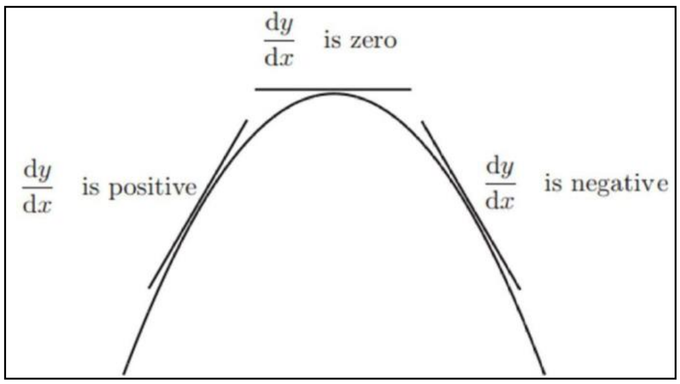
$$f(a+h) \approx f(a)+f'(a)h$$

We can use this formula, if we know only $f(a)$ and $f'(a)$ and wish to estimate the value of $f(a+h)$.

MAXIMA AND MINIMA OF A FUNCTION:

Maxima and minima are critical points on graphs which could be found by the first derivative and the second derivative.

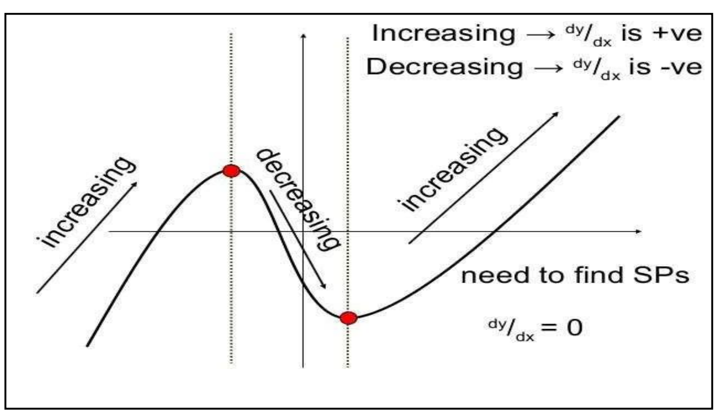
- to have a local (or relative) maximum point at the point X_{max} if there exists some $\epsilon > 0$ such that $f(X_{max}) \geq f(X)$ when $|X - X_{max}| < \epsilon$. The value of the function at this point is called maximum of the function
- to have a local (or relative) minimum point at the point X_{min} if $f(X_{min}) \leq f(X)$ when $|X - X_{min}| > \epsilon$. The value of the function at this point is called minimum of the function



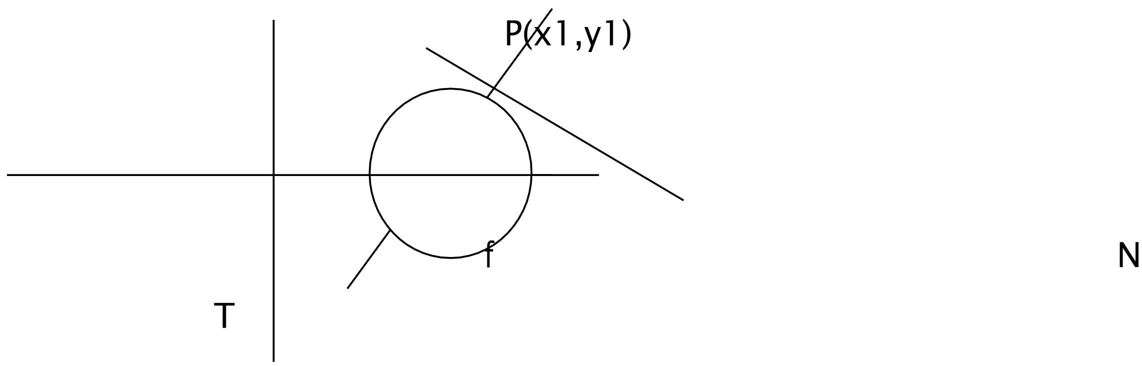
INCREASING OR DECREASING FUNCTION:-

To determine intervals where a function is increasing or decreasing, you have to first find domain values where all critical points will occur afterwards, test all intervals in the domain of the function to the left and to the right of these values to check if the derivative is positive or negative.

- If $f'(x) > 0$, then f is increasing on the interval, and
- If $f'(x) < 0$, then f is decreasing on the interval.



<u>TANGENT AND NORMAL</u>:-



Tangent is the line that touches the curve at a point and does not cross it, whereas normal is t perpendicular to that tangent.

Let the tangent meet the curve at P(x1, y1).

Therefore, the straight-line equation which passes through a point having slope m can now be written as;

$$y - y1 = m(x - x1)$$

We can see from the equation, the slope of the tangent to the curve y = f(x) and at the point P(x1, y1), it is given by dy/dx at P(x1, y1) = f'(x).

Therefore, Equation of the tangent to the curve at P(x1, y1) can be written as:

$$y - y1 = f'(x1)(x - x1)$$

Equation of normal to the curve is given by;

$$y - y1 = [-1 / f'(x1)] (x - x1)$$

Or

$$(y - y1) f'(x1) + (x - x1) = 0$$

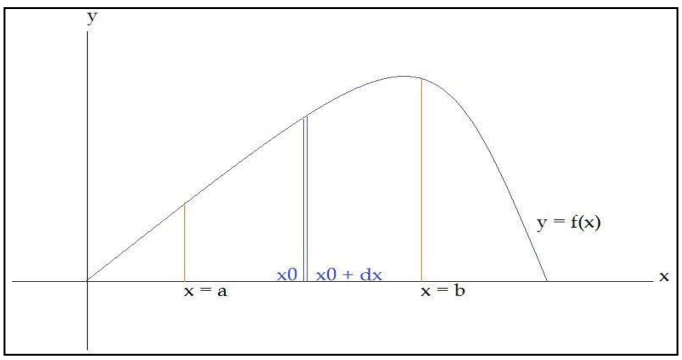
APPLICATION OF INTEGRAL CALCULUS:-

1. Area between curves
2. Volume
3. Arc length
4. Surface Area

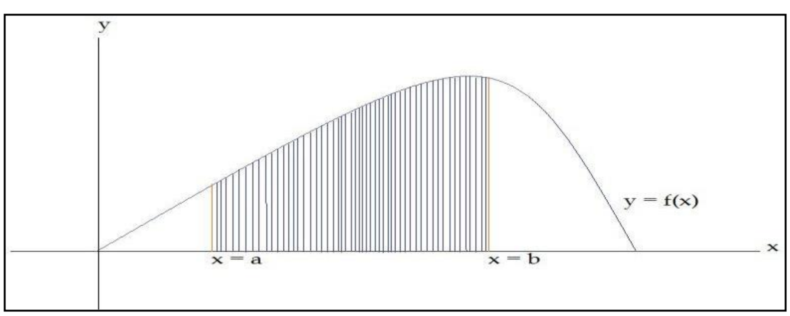
AREA BETWEEN CURVES:-

Step 1: Form a rectangular strip of height/length = $f(x_0)$ and breadth = dx as shown in the figure below.

- You can consider the rectangle to be centred at the value $x = x_0$
- dx is an infinitesimally small value which could be taken equal to the difference in the x-coordinates on which the sides of the rectangle are placed.



Step 2: Move the strip under the curve, beginning from the lower bound of x i.e. x = a; and terminating at the upper bound of x i.e. x = b, changing the value of x_0 at each point but retaining the same value of dx throughout. Place all of these strips adjacent to each other and get a resultant figure such as:



Step 3: The area dA of a single rectangular strip = length × breadth

$$dA=f(x0)\times dx$$

This is known as the Differential/Elementary Area.

Step 4: The total area A under the curve can be approximately obtained by summing over the areas of all the rectangular strips.

$$A=x0=a\Sigma x0=bdA$$

Using the value of dA from a previous step:

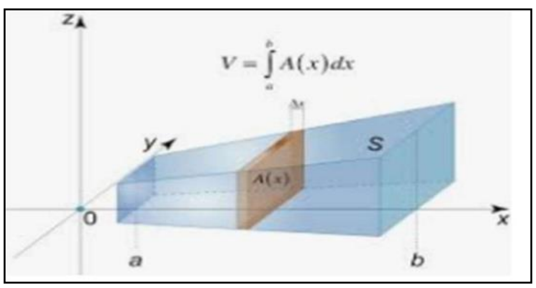
$$A=x0=a\Sigma x0=bf(x0)dx$$

Step 5: If $dx \rightarrow 0$, the summation can be converted to an integral. Then we have;

$$A=\int abf(x)\, dx$$

VOLUME:-

If a region in the plane is revolved about a given line, the resulting solid is a solid of revolution, and the line is called the axis of revolution. When calculating the volume of a solid generated by revolving a region bounded by a given function about an axis, follow the steps below:



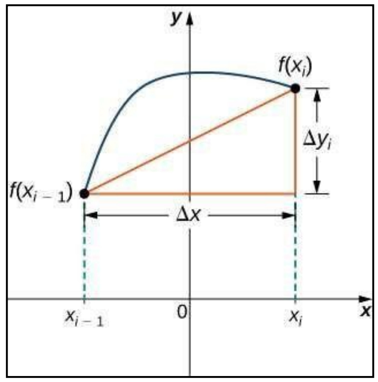
1. Sketch the area and determine the axis of revolution, (this determines the variable of integration)

2. Sketch the cross-section, (disk, shell, washer) and determine the appropriate formula.

3. Determine the boundaries of the solid.

4. Set up the definite integral and integrate.

<u>ARC LENGTH</u>:-



By the Pythagorean theorem, the length of the line segment is

$$\sqrt{.}\ (\Delta x)^2+(\Delta yi)2$$

We can also write this as

$$\Delta x\ \sqrt{.}1+((\Delta yi)/(\Delta x))^2$$

Now, by the Mean Value Theorem, there is a point $x{*}i\in[xi-1,xi]xi{*}\in[xi-1,xi]$ such that $f'(x{*}i)=(\Delta yi)/(\Delta x)f'(xi{*})=(\Delta yi)/(\Delta x)$. Then the length of the line segment is given by

$$\sqrt{.}\Delta x1+[f'(xi{*})]^2$$

Adding up the lengths of all the line segments, we get

$$\text{Arc Length}\approx\ \Sigma i=\ \sqrt{1}+[f'(x{*}i)]^2\ \Delta x$$

This is a Riemann sum. Taking the limit as $n\rightarrow\infty,n\rightarrow\infty$, we have

$$\text{Arc Length}=\lim n\rightarrow\infty\ \Sigma i=1n\sqrt{1}+[f'(x{*}i)]^2\ \Delta x\ =\ \int ba2\sqrt{1}+[f'(x)]dx$$

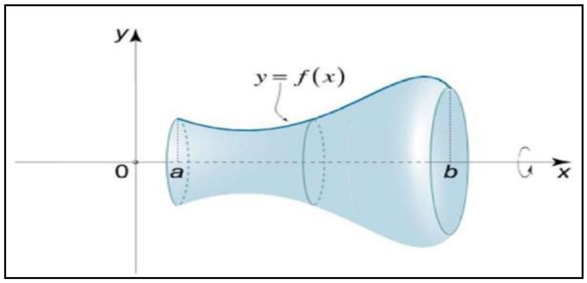
<u>SURFACE AREA</u>:-

We consider two cases - revolving about the x−axis and revolving about the y−axis.

Revolving about the x−axis

Suppose that y(x) is smooth non−negative functions on the given interval.

If the curve $y=f(x)$, $a\le x\le b$ is rotated about th−eaxis, then the surface area is given by

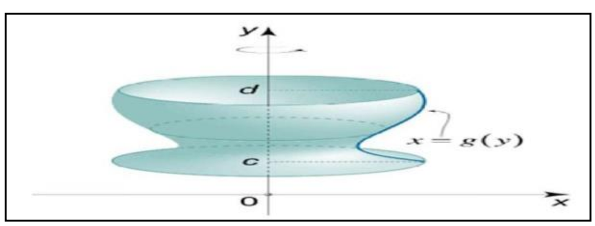


$$A=2\pi\int_a^b \left(f(x)\sqrt{1+[f'(x)]^2}\right)dx.$$

Revolving about the y−axis

The functions g(y) is supposed to be smooth and non−negative on the given interval. If the curve is described by the function $x=g(y)$, $c\le y\le d$, and rotated about the axis y, then the area of the surface of revolution is given by

$$A=2\pi\int_c^d \left(g(y)\sqrt{1+[g'(y)]^2}\right)dy$$

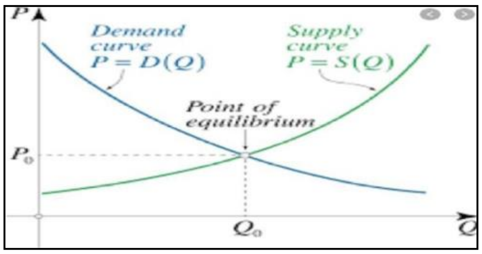


APPLICATION OF CALCULUS IN VARIOUS FIELDS

Finance

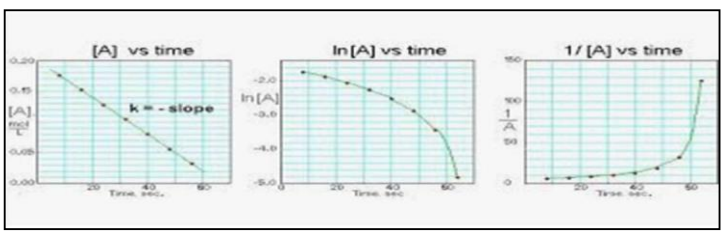
It is used for Portfolio Optimization i.e., how to choose the best stocks.

- Statisticians will use calculus to evaluate survey data to help develop business plans. A survey involves many different questions with a range of possible answers, calculus allows a more accurate prediction.
- Credit card companies use calculus to set the minimum payments due on credit card statements at the exact time the statement is processed.



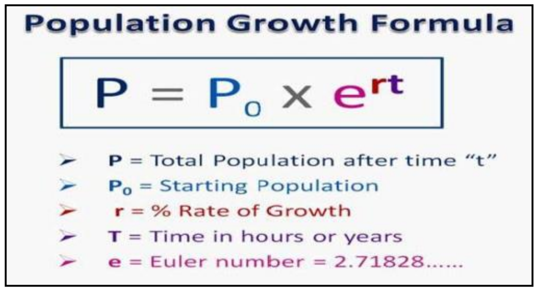
Chemistry

- Inorganic Chemistry: The Rate of Reaction i.e., How fast a reaction takes place. (Integration)



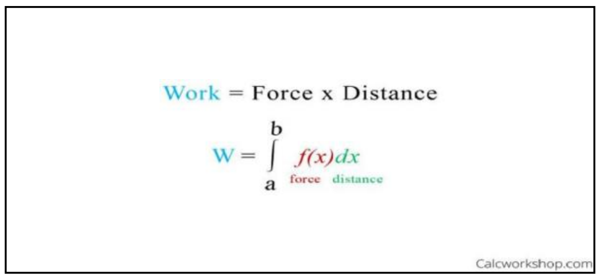
Biology

- Study of Population: Analysing how the population of predators and prey evolves over time. It is done using Differential Equation. Biologists use differential calculus to determine the exact rate of growth in a bacterial culture when different variables such as temperature.



Physics

- Velocity and acceleration all come from simple derivatives of the position function.
- To calculate work done, kinetic energy, centre of mass, etc.
- To calculate radioactivity



Other fields

1. Electrical Engineering: An electrical engineer uses integration to determine the exact length of power cable needed to connect two substations that are miles apart.

2. Architect: An architect will use integration to determine the number of materials necessary to construct a curved dome over a new sports arena.

3. Space flight engineers: Space flight engineers frequently use calculus when planning lengthy missions. To launch an exploratory probe, calculus allows each of those variables to accurately consider the orbiting velocities under the gravitational influences of the sun and the moon.

4. Graphic artist: A graphics artist uses calculus to determine how different three-dimensional models will behave when subjected to rapidly changing conditions.

CONCLUSION

Calculus plays very important role in various fields. Whether it is architecture i.e., in designing of bridges, buildings etc, in medical science, economy or chemical science. A wide range of careers uses calculus. Calculus helps to build discipline necessary for solving complex problems. Therefore, we can conclude that calculus is used almost everywhere and is inseparable from one's life.