BT6270: Computational Neuroscience

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1 Assignment 1: The Hodgkin-Huxley Neuron Model

The assignment aims to

- simulate the HH Model in MATLAB and understand the implementation
- analyze the variation of voltage spiking frequency with change in input impulse current

To view only the numerical results required by the Assignment, please navigate to Section 4.1. The rest of the report describes the process and intuition.

2 Understanding the HH Model Implementation

The given MATLAB script demonstrates the Hodgkin-Huxley model in current clamp experiments and shows action potential propagation. It is known that the threshold value of current beyond which oscillatory behaviour begins.

The differential equations for model variables - V, m, n, and h - are solved numerically. Some results are displayed here (Figures 1 through 4).

2.1 Variation of Action Potentials

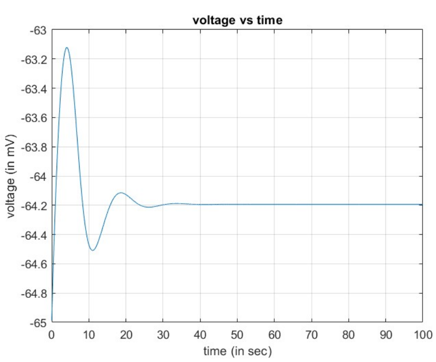

Figure 1: At $I_{ext} = 0.01\ \mu A$

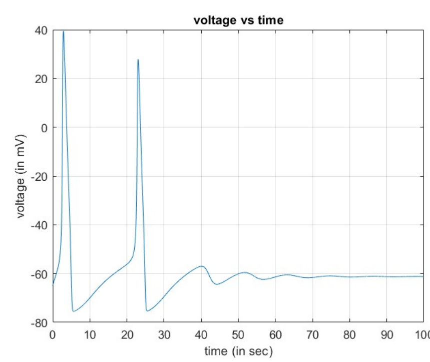

Figure 2: At $I_{ext} = 0.06\ \mu A$

3 AP Frequency Analysis

Now I attempt to analyse the action potential frequency. Being from an Electrical Engineering background, I have performed this part using the Fourier Transform. The voltage signal obtained for each of the input external currents is Fourier Transformed to search for constituent frequencies. Here is an example.

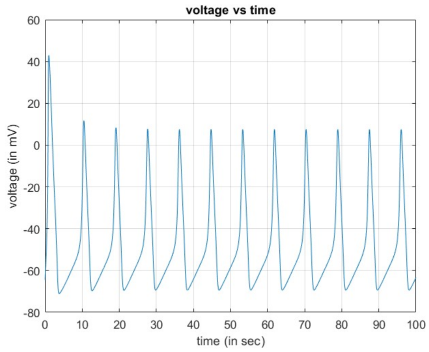

Figure 3: At $I_{ext} = 0.5\ \mu A$

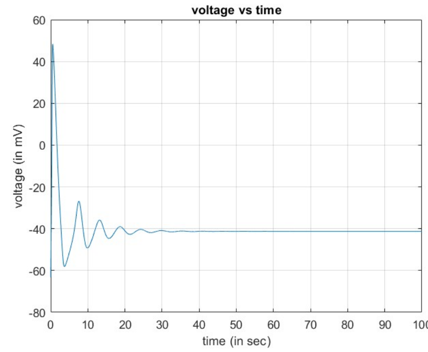

Figure 4: At $I_{ext} = 1.9\ \mu A$

3.1 Fourier Analysis

On operating a Discrete Fourier Transform on the Voltage array obtained from the HH Model, the following plot is obtained.

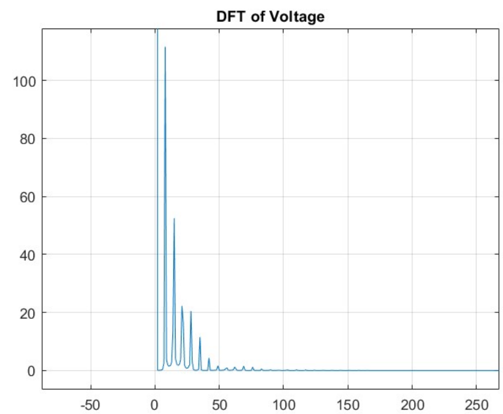

Figure 5: For $I_{ext} = 0.1\mu A$

It is obvious from the previuos plots that the signal has a non-zero DC component so I use the first peak after zero to find the fundamental oscillation frequency. Now this frequency is acceptable only when the signal is really periodic like Figure 3. However, even damped oscillations like Figure 1 and 4 will have frequency components.

To understand when the periodicity arises, we must look at the power spectrum of the DFT. Only when the fundamental frequency component is strong, can we claim it to be the limit cycle frequency. This will be clear when we plot the power of fundamental frequency against input impulse current.

3.1.1 Power Spectrum

Here, for each value of input impulse current sampled from zero μA to 2 μA, we first perform an `fft(voltage)`. Next, square this Discrete Fourier Transform (DFT) to get the Power Spectrum. The following plot is obtained.

The falls at some input currents (corresponding to their fundamental frequencies) indicate that although the frequency increases with current somewhat linearly, the power content of the mode does not. This is a very interesting result because some AP frequencies are suddenly weaker than their neighbouring modes (and corresponding currents) implying that the solution to the differential equations selectively disfavours some frequencies.

Let's now analyze the oscillatory behaviour. A zoomed in plot is shown in Figure 7.

The following points deduced from this power spectrum are salient -

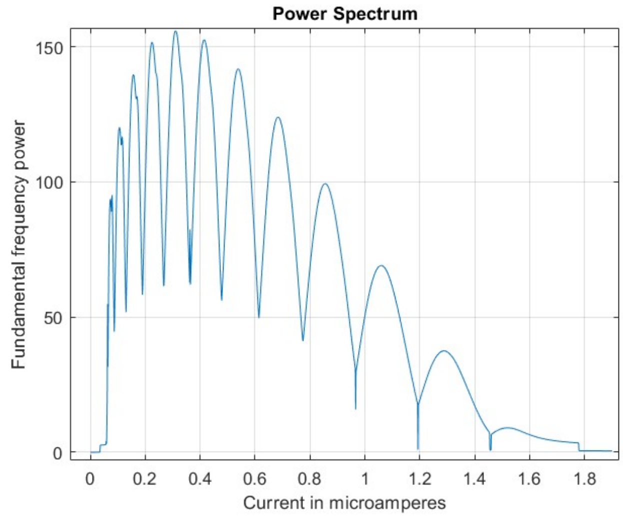

Figure 6: Note that the y-axis units are immaterial. One may observe the periodic rise and fall in power as current varies. There are several sharp valleys and intermediate smooth peaks. More discussion in text.

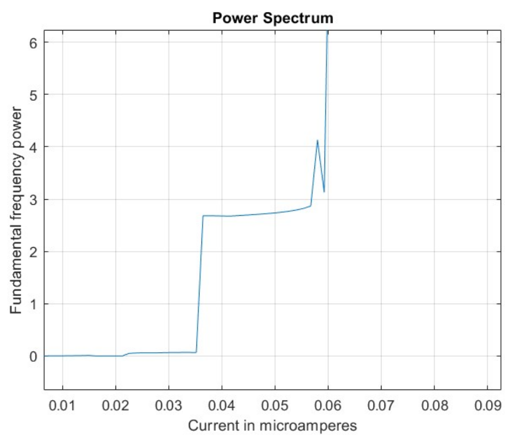

Figure 7: A magnified section of the previous figure.

- Below 0.0223 μA (AP threshold), a very weak (overdamped) oscillation begins. This is no Action Potential but a weak signal has emerged.
- After 0.0223 μA (AP threshold), the fundamental mode starts to grow stronger and Action Potentials start to appear. They are finite in number, as implied by the weakness of the signal.
- After around 0.07 μA, strong frequencies start, this indicates the presence of undamped oscillation in the signal. This region is where continuous neuron firing is observed.
- From 0.45 μA, the AP peak-peak voltage starts reducing. There is periodicity but the energy reduces.
- Around 1.5 μA the oscillation again weakens as further increase in current doesn't engender limit cycles. Roughly, hereon we can say that there are no more APs. (If the oscillation peak-to-peak reduces below $10mV$, it is not an AP. This is an arbitrary limit thus, possibly ambiguous.)

The above deductions are solely based on this Power Spectrum and the exact currents at which AP behaviour changes must be verified from the HH Model simulation.

3.1.2 Frequency Plot

Finally, using the intuition from the Power Spectrum, we can discard the frequencies below 0.036 μA and above 1.6 μA since they do not contribute to limit cycle behaviour. Thus collecting the finite to continuous AP currents, I generate this plot of AP frequency against current.

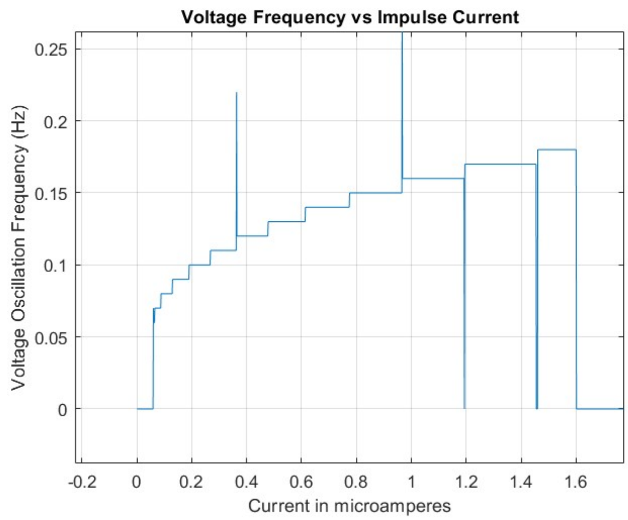

Figure 8: Spiking frequency vs I_{ext}. NOTE that the section of this plot where the continuous firing has not begun has been cut off. This plot only represents the firing rate after 0.07 because that's where it is meaningful. These cutoffs have been obtained from Figure 7. Also note that the sharp spikes can be ignored since they have appeared due to minor approximations in the Fourier Transform.

3.2 Number of APs

Another approach to the AP frequency plot is by simply counting the number of peaks. Recall that the simulation runs for hundred seconds of neural firing, hence the finite cardinality of peaks. The same is demonstrated below.

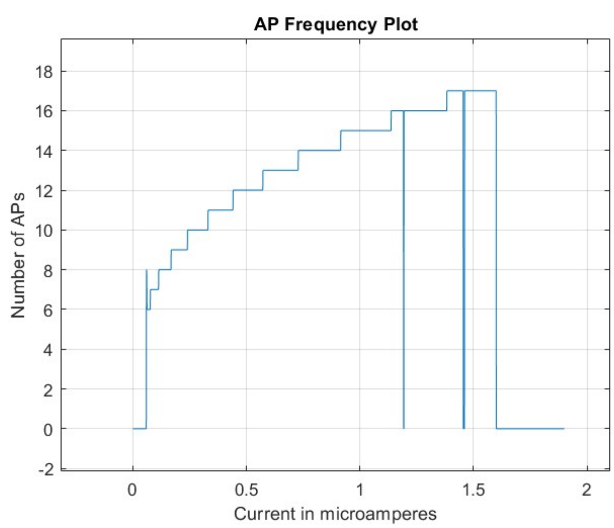

Figure 9: The comments from Figure 8 apply here too. Note that the sharp start and end is because of the AP voltage threshold obtained from the Power Spectrum.

4 Conclusion

The aims planned at the beginning of the assignment are now seen to have been accomplished. Overall, it can be inferred that the HH Model implementation very aptly demonstrates the point neuron function. The final firing rate plot illustrates how the behaviour evolves with input impulse current.

Fourier analysis provides a detailed understanding of signalling and how the energy of the fundamental mode tells us the presence of oscillations.

4.1 Numerical Assignment Results

- Threshold values of current that govern AP signalling - Section 3.1.1
- Plot of firing rate vs I external - Section 3.1.2 and 3.2