BT6270: Computational Neuroscience

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1 Assignment 2: Introduction

The assignment aims to

- simulate the **FitzHugh-Nagumo Model** and understand the implementation
- analyze the variation of voltage spiking frequency and analyze the phase plane.

2 The FitzHugh-Nagumo Model

The FH model derives its principal equations from the Hodgkin-Huxley model by simplifying the four differential equations of the latter into just two. The FH model equations are dimensionless -

$$\frac{dv}{dt} = v(a - v)(v - 1) - w + I_m$$

$$\frac{dw}{dt} = bv - rw$$

where v relates to membrane voltage, w relates to all gating variables combined, and b, r and v are model parameters.

2.1 Implementation

The equations are solved in Python using a simple Forward Euler approach. The script can be found in `EE20B018_A2.py` attached in the zipped folder. I use an object-oriented approach wherein separate methods are called on the instance of the model that implement the phase plane analysis, time analysis, and plotting. This reduces a lot of repetitive code, simplifying the overall understanding.

3 The Assignment

The assignment deals with four cases by varying the system behaviour caused by specific values of I_m. The following sections discuss each of those.

3.1 Case: $I_{ext} = 0$ Excitability

Let's start with the phase plane analysis in Figure 1.

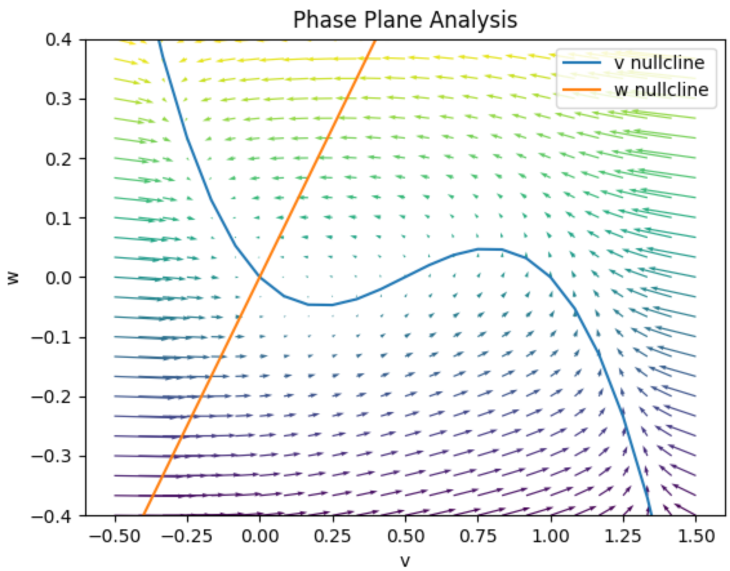

Figure 1: Phase plot for $I_{ext} = 0$.

The two lines in the plot are nullclines, as indicated in the legend. There is only **one stationary point** at the origin. Figures 2 and 3 shows the time variation of v and w for the following sub-cases:

1. $V(0) = 0.25 < a$ and $W(0) = 0$
2. $V(0) = 1.00 > a$ and $W(0) = 0$

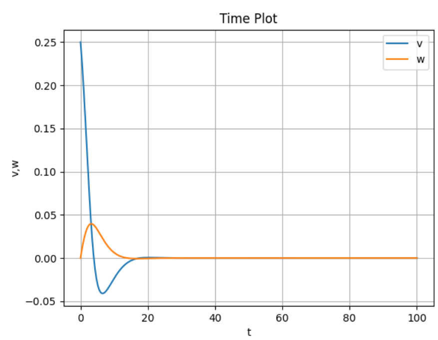

Figure 2: Case-I(i): $V(0) < a$ and $W(0) = 0$

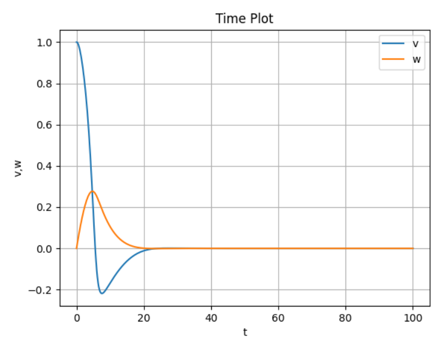

Figure 3: Case-I(ii): $V(0) < a$ and $W(0) = 0$

Furthermore, I plot the trajectory (Figure 4) of the two timed events on the phase plot.

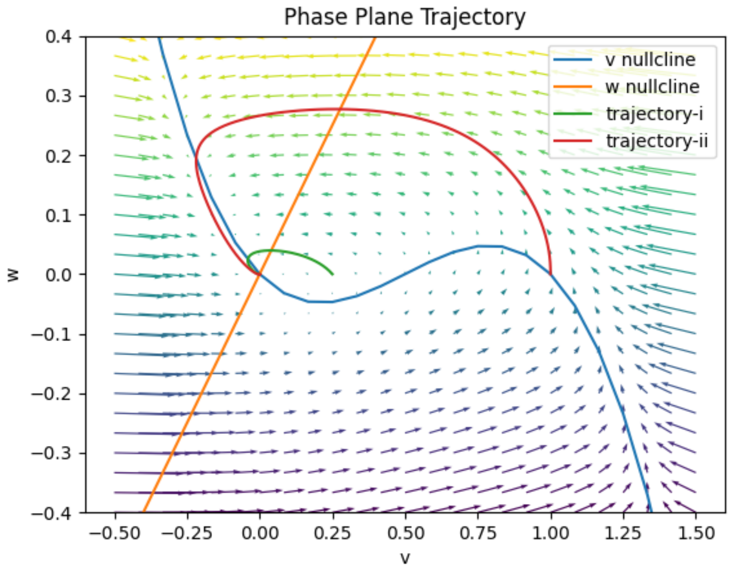

Figure 4: Trajectory on the Phase Plot for Case-1. The start position for case (i) is at 0.25 and 1.00 for (ii). One may observe how both these starting points evolve towards the stationary point at the origin. The vector field's direction helps understand the direction of trajectory evolution.

3.2 Case: $I_1 < I_{ext} < I_2$ Oscillations

First I must find the values of I_1 and I_2. The method used is by finding the current for which peaks start to appear, indicating sustained oscillations. I will skip describing the code here with a remark that roughly heights of peaks and non-decreasing amplitude was taken as a sign of limit cycle exhibition. The cutoffs were found empirically.

$$I_1 = 0.32A$$

$$I_2 = 0.68A$$

When we choose a current as a mean of these two values, within this range, we can expect limit cycle behaviour. I use $V(0) = 0.4$. Before that, let's see the phase plot first, in Figure 5.

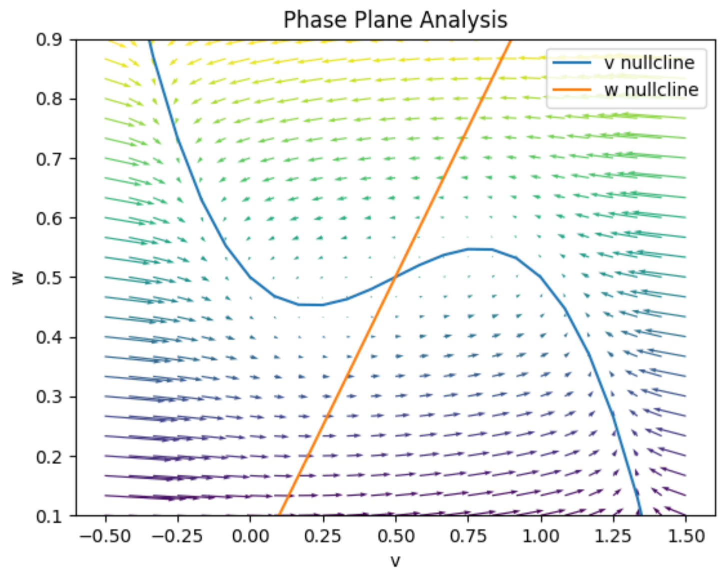

Figure 5: Phase plot for Case-2. The intersection point is between the two extrema of the v-nullcline.

The fixed point is unstable which is evident from the vector field in Figure 6. The trajectory shows a limit cycle behaviour.

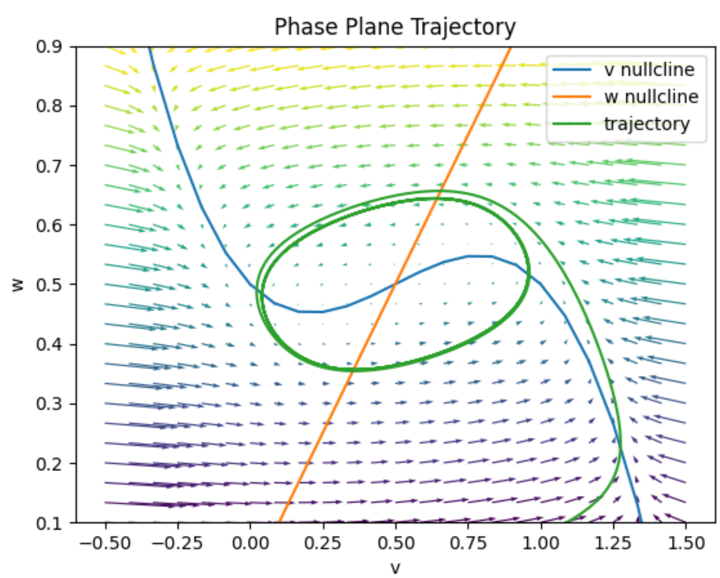

Figure 6: Trajectory for Case-2. The starting point is v=0.4 and w=0. The limit cycle is clear.

The time plot for v and w is shown in Figure 7.

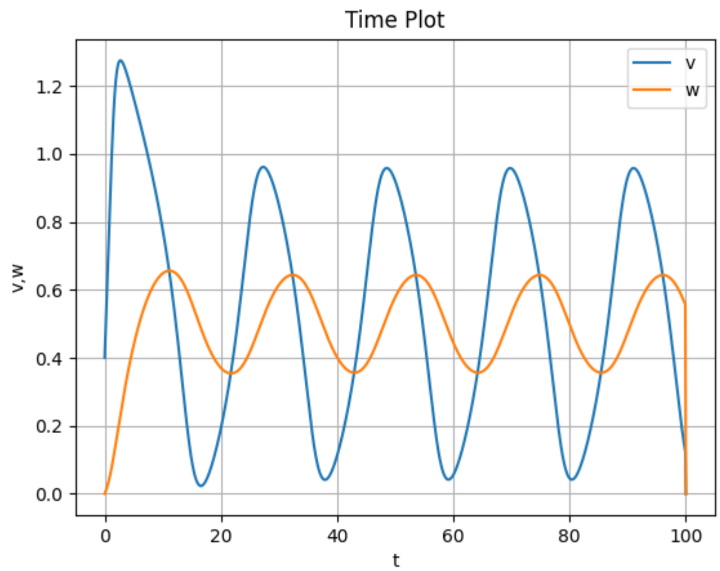

Figure 7: $v(t)$ and $w(t)$. Limit cycle behaviour.

3.3 Case: $I_{ext} > I_2$ Depolarization

I use $V(0) = 0.4$ and $I_{ext} = I_2 + 0.1$ Again, I start with the phase plot, in Figure 8.

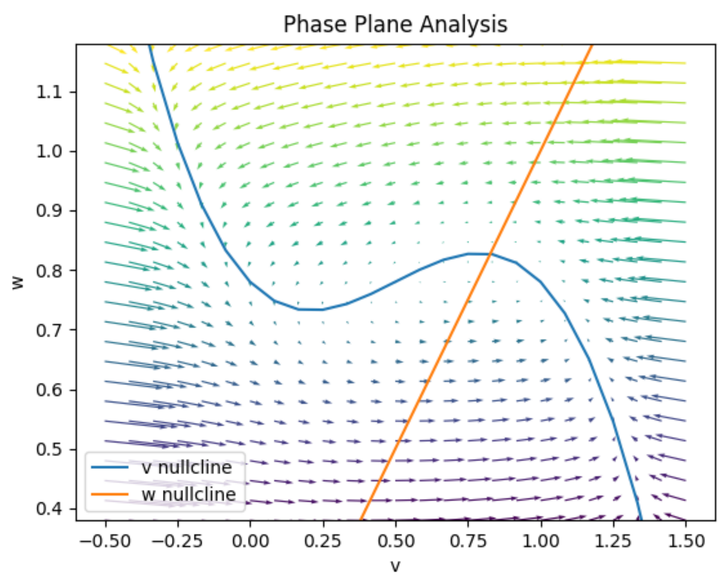

Figure 8: Phase plot for $I_{ext} > I_2$

The stationary point is stable as the vector field converges towards it. Figure 9 shows the trajectory starting from (0.4, 0) towards the stationary point, where the spiral indicated stability of that point. Figure 10 shows the time plot. The limit cycle has died down and the neuron is depolarized.

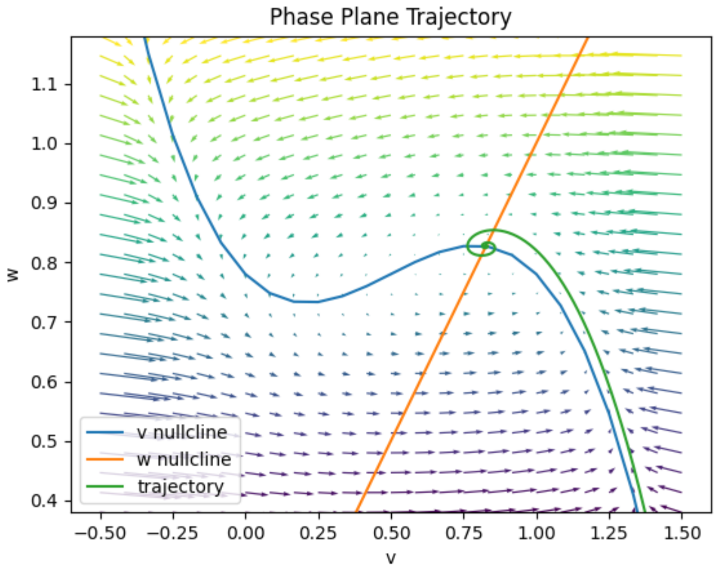

Figure 9: Trajectory for Case-3. The point is stable because the trajectory converges there.

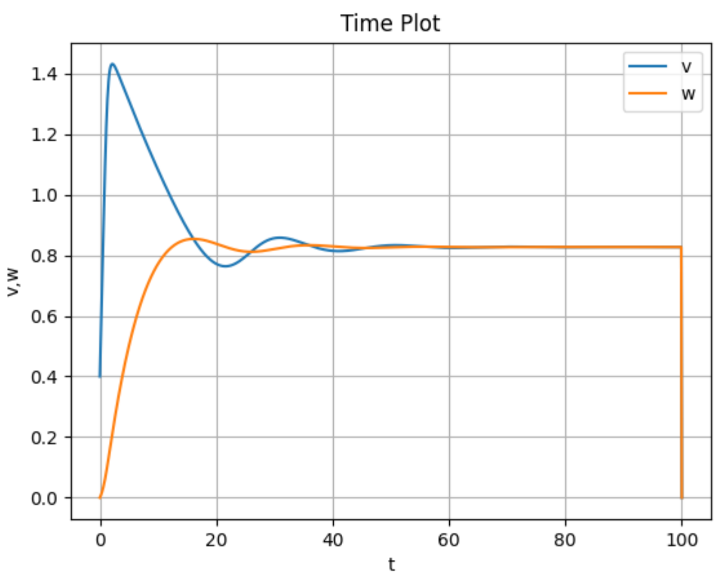

Figure 10: Case-3 Depolarization.

3.4 Case: Bistability

First we need to change b/r such that the nullclines intersect thrice, with three stationary points. The current and parameter ratio values were found by trial and error.

$$I_{ext} = 0.02$$

$$b = 0.02$$

$$r = 0.5$$

The phase plot is shown in Figure 11.

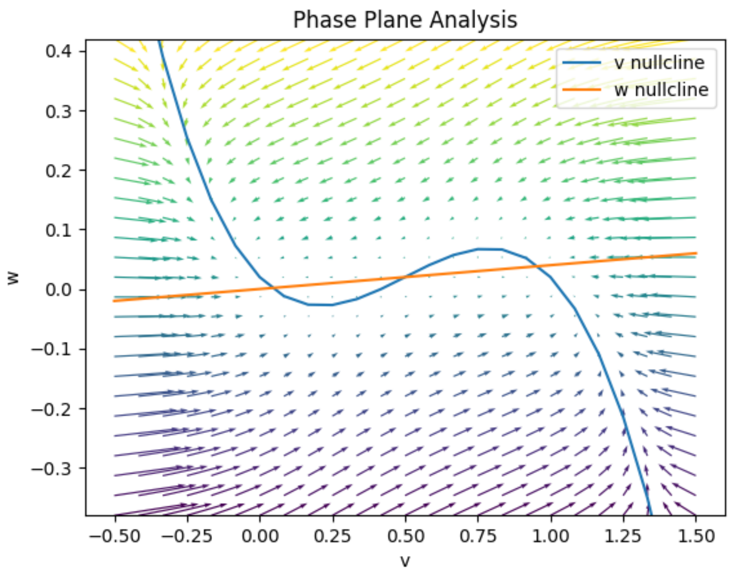

Figure 11: Phase plot for Case-4.

This type of a situation is where the neuron exhibits bistability - tonically high or tonically low ON-OFF - states. Figure 12.

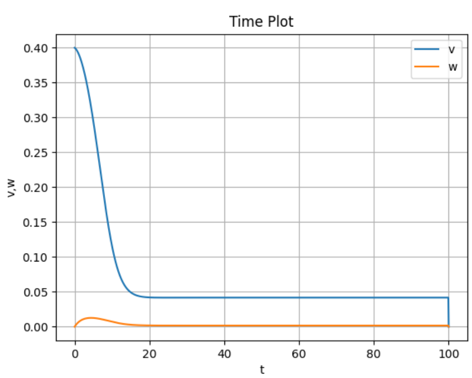

Figure 12: Case-4: Tonically Down (OFF)

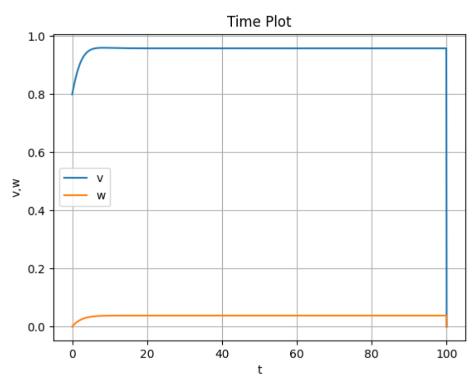

Figure 13: Case-4: Tonically Up (ON)

The trajectory is shown in Figure 13.

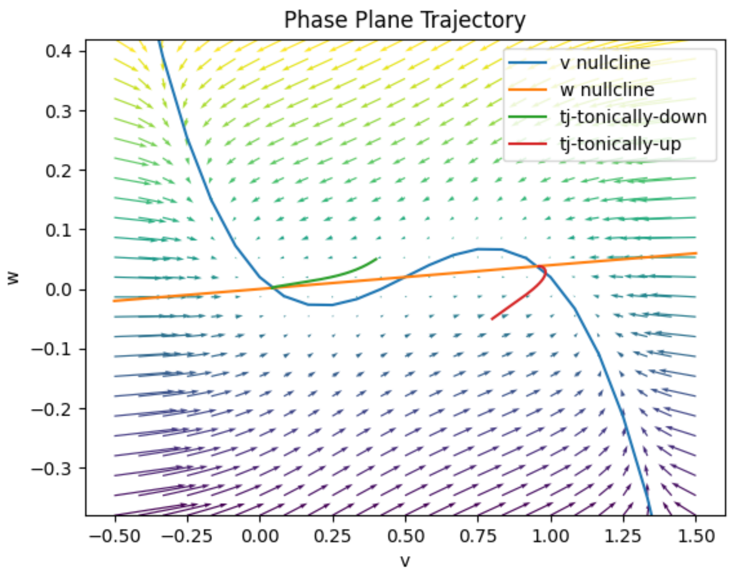

Figure 14: Trajectory for Case-4: One may observe from the vector field that the first and last points are stable (because the trajectories converge to those points) but the middle point is a saddle.

4 Conclusion

The aims planned at the beginning of the assignment are now seem to have been accomplished. We analysed the FitzHugh-Nagumo model in the phase and time domain. Unlike the HH model, here we were able to observe the stability of points.