Blind Equalizers

ALBERT BENVENISTE, MEMBER, IEEE, AND MAURICE GOURSAT

***Abstract*—Blind equalizers do not require any known training sequence for the startup period, but can rather perform at any time the equalization directly on the data stream. In this paper, a general approach is presented for designing efficient blind equalizers for one and two independent carrier transmission systems; a special algorithm is given for the CCITT V29 constellation.**

Introduction

CONVENTIONAL equalization and carrier recovery algorithms for minimizing mean-squared error in digital communication systems generally require an initial training period during which a known data sequence is transmitted and synchronized at the receiver [1]–[4].

Equalizers for which such an initial training period can be avoided are referred to as *blind equalizers*. The need for blind equalizers in the field of data communication is widely discussed by Godard in [5], and we refer the reader to this paper for such a discussion.

The class of blind equalization algorithms we shall present in this paper has the following properties:

- In the case of a two carrier transmission system, they perform joint equalization and carrier recovery.
- The only change with respect to the classical algorithms lies in a modification of the standard error signal (i.e., difference between the output of the {equalizer + phase estimator} and the decoded data) which is used in such algorithms; hence, there is no increase in the computational complexity in comparison with the classical algorithms; furthermore, any classical structure may be used for the equalizer (transversal or recursive, with a stochastic gradient or a self-orthogonalizing type of adjustment).
- There is only a small increase in the duration of the blind startup period in comparison with classical startup periods involving a known training sequence.
- These blind equalizers are provided with a (smooth) automatic switching from the startup period to the standard transmission mode, and an automatic switching back to the blind startup period when an abrupt change occurs in the characteristics of the channel, without the need of any special testing procedure.

Hence, the blind equalizers we present in this paper can be considered as extensions of classical adaptive equalizers.

Some modems which perform the blind equalization are presently available, however, this problem has not received much attention in the open literature. Let us mention the pioneering work of Sato [6] which first designed such an equalizer for one carrier transmission systems, without any theoretical justification. Godard [5] designed blind equalizers for two-carrier transmission systems, by decoupling the phase estimation and the removal of the intersymbol interference (ISI), thus, resulting in a severe degradation with respect to the classical systems. The present authors have designed blind equalizers for one-carrier [7], [8], [9], and two-carrier [7], [10] transmission systems, without taking into account the problem of phase recovery. The present algorithms are based upon the theoretical analysis developed in [9] [10].

The paper is organized as follows. In Section I the basic problem is stated. Section II is devoted to a (sketchy) theoretical derivation of the special cost-functions which have to be used instead of the classical mean-squared error; both real case (for one carrier systems) and complex case (for two carrier systems) are investigated; the reader who is not interested in the theoretical analysis of the problem may skip this section. Blind equalization algorithms are presented in Sections III (one-carrier transmission systems) and Section IV (two-carrier transmission systems). Experimental results are reported in Section V. (Additional results are available in [10].)

Paper approved by the Editor for Communication Theory of the IEEE Communications Society for publicaton without oral presentation. Manuscript received April 4, 1983; revised January 5, 1984.

A. Benveniste is with IRISA/INRIA, Campus de Beaulieu, F-35042 Rennes Cédex, France.

M. Goursat is with INRIA, Domaine de Voluceau, 78153 Le Chesnay Cédex, France.

I. Statement of the Problem

Fig. 1 and Fig. 2(a) show the general form of one-carrier amplitude modulation systems and two-carrier phase and amplitude modulation systems respectively, whereas the Fig. 2(b) shows the complex baseband equivalent form of Fig. 2(a).

For deriving standard equalization algorithms, a classical method is the following:

1) During the startup period, adjust the equalizer (and the phase estimator) in order to minimize the mean-squared error

$$\mathbb{E}\,|c_t - a_t|^2,$$

where $\mathbb{E}$ denotes the expectation over noise and data sequences, and (a_t) is a *known* training sequence. Standard stochastic gradient (or "decision-directed") algorithms are used for this purpose.

2) For the transmission period, use the same algorithm as before, but replacing a_t by its estimate $\hat{a}_t$. The resulting algorithms require a low error rate, and cannot be used during a starting period.

What we need for designing a blind equalizer is to *recover the transmitted message (a_t) from the received one (x_t) only, without any preamble for identification of the unknown channel.* This requires that the input c_t of the decision circuit be close to the true data a_t, which is unknown: this is the problem we shall solve in the sequel.

Let us point out that, for the purpose of the theoretical analysis, where the characteristics of the unknown channel (including its phase) are assumed to be time-invariant, the equalizer and phase estimator of Fig. 2(b) are known to be redundant [12], [13], so that the latter will be dropped for the theoretical analysis in Section II. Recall that splitting the two tasks of carrier recovery and removal of ISI is needed only because of the different time-scales of the time variations of the phase (fast) and the ISI (slow).

II. Gain and Phase Identification of Nonminimum Phase Systems

A. The Problem

As shown in Section I, the theoretical problem we have to solve is the following: *we observe the output x_t of an unknown*

0090-6778/84/0800-0871$01.00 © 1984 IEEE

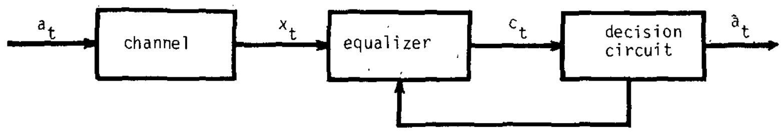
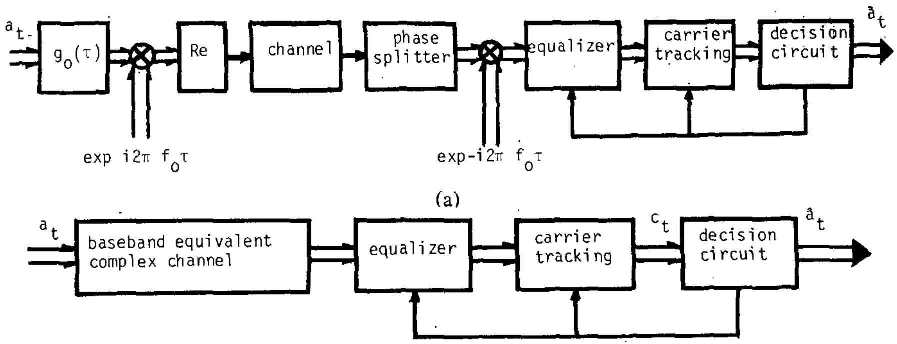


Fig. 1.



(a)

(b)

Fig. 2.

time-invariant linear system S admitting a_t as input, where a_t is an unobserved white (independent, identically distributed) *random sequence with known symmetric distribution ν of* finite variance (Fig. 3).

Problem: Reconstruct the message a_t, or, equivalently, identify the inverse S^{-1} of the unknown channel.

The following remarks are fundamental:

1) There is no solution for this problem when the input (a_t) is Gaussian (unless the channel S is known to be of minimal -or maximal- phase), since it is well known that only the attenuation of S can be recovered in this case, but not the group delay distorsion (i.e., phase of the linear system S). As a consequence, *no solution can be found, which would be based upon second order statistics only.*

2) Since S *is assumed to be nonminimum phase,* S^{-1} *is noncausal.* As a consequence the theoretical form of S^{-1} will be, using the z-transform notation,

$$S^{-1}(z^{-1}) = \sum_{k\in Z} \tilde{s}_k z^{-k}, \text{ where}$$
$$S^{-1}(z^{-1}) \circ S(z^{-1}) = z^{-N} \text{ for some } N, \tag{1}$$

i.e., S^{-1} is defined up to a time shift; on the other hand, (1) implies that the message (a_t) can be reconstructed in an off-line way only. Obviously, in practical situations, S^{-1} will be truncated, which will allow the reconstruction of (a_t) in real time, but with a (constant and finite) delay.

In the sequel, unless mentioned to the contrary, (a_t), (x_t), and S will assumed to be *real.*

B. A Unicity Result

According to Theorem 22 of [8], provided ν is non-Gaussian with finite variance, *if an equalizer* (i.e., adjustable filter) *is placed after the channel according to Fig. 4, it is sufficient to obtain that $c_t(\theta)$ admits ν as distribution, for ensuring that θ is, up to a sign and an unknown delay, the desired inverse denoted by S^{-1}* (the delay cannot be removed in view of (1); the sign cannot be recovered since ν was assumed to be symmetric, so that nothing can distinguish the true message (a_t) from the opposite one $(-a_t)$ from a statistical point of view). This unicity result is a very strong one, since knowledge about the *joint* distribution of the sequence (c_t) is not required. Not surprisingly, the theorem is false when ν is Gaussian!

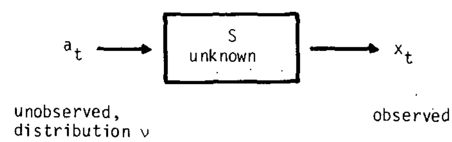


Fig. 3.

C. Gain-and-Phase Recovering Cost Functions

We were not able to design such cost functions for arbitrary ν, let us define the family of distributions for which we will give the theoretical results.

Definition: The distribution ν is said to be sub-Gaussian in one of the following cases:

$$\left.\begin{aligned} &\nu \text{ is uniform over } [-d, +d], \\ &\nu(dx) = K \exp(-g(x))\, dx, \end{aligned}\right\} \tag{2}$$

where k is a constant, and g is an even function, differentiable except possibly at the origin, such that $g(x)$ and $g'(x)/x$ are strictly increasing over R_+.

Examples: $\nu(dx) = K \exp(-|x|^\alpha)dx$ for $\alpha > 2$, the limiting case $\alpha = +\infty$ corresponds to the uniform distribution.

In view of Section II-B, we look for cost functions of the form:

$$J(\theta) = E(\psi(c_t(\theta))), \tag{3}$$

where E denotes the expectation over all possible sequences of data; the problem is to choose the function ψ. The following theorem is stated in a more general and precise way, and proved in [7] [8].

Theorem 1: Assume ν is sub-Gaussian. Then the Sato cost function, which is defined as follows,

$$\left.\begin{aligned} &J(\theta) = E\left(\frac{1}{2}\, c_t^2(\theta) - \alpha |\, c_t(\theta)|\right) \\ &\alpha = \frac{Ea_t^2}{E|a_t|} \end{aligned}\right\} \tag{4}$$

admits as only local (and also global) *minima the inverse channels $\pm S^{-1}$, as defined in Section II-B.*

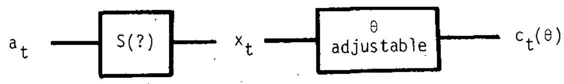


Fig. 4.

Comments: 1) This cost function was first used by Sato [6] without theoretical justification. It is very interesting, since it needs the knowledge of only the first and second order moments of ν, and leads to very simple algorithms. See [8] and [14] for further details.

2) *Finding the Good Sign in (1):* Fig. 5 shows the steepest descent lines of $J(\theta)$ for the case where the global channel $T = \theta \cdot S$ is subject to have only two nonzero coefficients (t_1, t_2).

It appears clearly that, when using a steepest descent method for minimizing J, it is sufficient to initialize the algorithm in the correct half space for θ in order to recover the correct sign; and it turns out that the very coarse information on S which is needed for a good initialization is always available (in practice $\theta = +$ Identity is always a good initialization).

3) *Use on Discrete Distributions:* The theorem we have given is not entirely satisfactory in our case, since the distributions ν encountered in data transmission are not subgaussian in our sense. Such typical distributions are uniform distributions over finite sets of the form $\{\pm 1, \pm 3, \cdots, \pm(2K + 1)\}$, such distributions are "close" to the uniform distribution, at least for K not too small. See [8] and [14] for further discussion on this point.

Rather than being considered as a comprehensive theoretical treatment of our problem, the results we have presented in this section have to be considered as a guide for designing gain and phase recovering cost functions.

D. Extension to the Complex Case

Here we consider the case where the signals a_t, x_t, c_t and the channel S and equalizer θ are complex.

Theorem 2: Assume we have the case of two independent carriers, namely the message (a_t) is an independent identically distributed (i.i.d.) *sequence such that* Re a_t *and* Im a_t *be independent, with same sub-Gaussian distribution ν. Then the complex Sato cost function*

$$\left.\begin{aligned} J(\theta) &= E(\psi(\operatorname{Re} c_t(\theta)) + \psi(\operatorname{Im} c_t(\theta))) \\ \psi(x) &= \frac{1}{2} x^2 - \alpha |x| \text{ (Sato function)} \\ \alpha &= \left(\int x^2 \nu(dx)\right)\left(\int |x| \nu(dx)\right)^{-1}, \end{aligned}\right\} \tag{5}$$

admits $\pm S^{-1}$ as the only local minima.

Proof: Define for $t \in Z$

$$b_{2t} = \operatorname{Re} a_t,\ b_{2t+1} = \operatorname{Im} a_t. \tag{6}$$

Then (b_t) is a real valued i.i.d. sequence with subgaussian distribution ν, and the random variable Re $c_o(\theta)$ is of the form

$$\operatorname{Re} c_o(\theta) = \sum_{k \in Z} t_k b_{-k} \tag{7}$$

for some real coefficients (t_k) depending upon θ. By Theorem 1, the local minima of the cost function

$$\mathbb{E}(\psi(\operatorname{Re} c_o(\theta))) \tag{8}$$

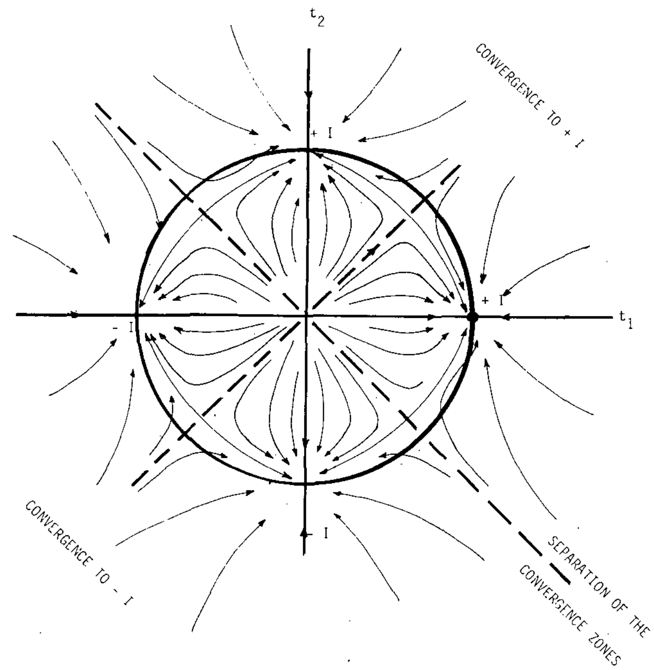


Fig. 5.

correspond to those $T(z^{-1}) = \sum_t t_k z^{-k}$ equal to $\pm z^N$ for some delay N.

Hence, according N be odd or even,

$$\left.\begin{aligned} &\mathbb{E}(\psi(\operatorname{Re} c_o(\theta))) \text{ is minimum for} \\ &\operatorname{Re} c_t(\theta) = \begin{cases} \pm\operatorname{Re} a_t \\ \text{or} \\ \pm\operatorname{Im} a_t \end{cases}, \text{ up to a delay,} \end{aligned}\right\} \tag{9}$$

which characterizes the desired inverse S^{-1} in the complex case. The same as in (9) holds with Im $c_o(\theta)$ instead of Re $c_o(\theta)$, thus leading to another convenient cost function. Add both of them for a better efficiency. This results in the Sato cost function of the (5), which proves the theorem.

This result will be extremely useful for designing blind equalizers in the two carrier case. The same remarks hold as in the real case. Unfortunately, *we have no similar result for the case of two dependent carriers* (as is the case when the 16-point V29 CCITT-constellation is used, see Fig. 17), which will result in great difficulties in designing blind equalizers in this case. (Recall the weaker results of [5] do not suffer from this restriction.)

III. Design of Blind Equalizers for One-Carrier Transmission Systems

We shall use the Sato cost function (and modifications of them) for the case of (a_t) being uniformly distributed over the set $\{\pm 1, 3, \cdots, \pm(2K + 1)\}$, which corresponds to amplitude modulation schemes.

A. General Form of the Algorithm

We shall use stochastic gradient methods, and self orthogonalizing algorithms for minimizing the cost functions we have defined; a theoretical analysis of the convergence of such adaptive schemes is given in [11].

The general form of the cost function we shall use is the following

$$J(\theta) = \mathbb{E}(\psi(c_t(\theta))); \tag{10}$$

for example, the cost function of Theorem 1 corresponds to the choice of the Sato function (5) for ψ.

Defining

$$\left.\begin{aligned}\frac{\partial}{\partial\theta} c_t(\theta) &= X_t(\theta)\\ \epsilon_t(\theta) &= -\psi'(c_t(\theta))\end{aligned}\right\} \tag{11}$$

(ψ' denoting the derivative of ψ), where $\epsilon_t(\theta)$ will be referred to as the *pseudo-error signal,* we get:

$$\operatorname{grad} J(\theta) = -\mathbb{E}(X_t(\theta)\epsilon_t(\theta)). \tag{12}$$

These considerations lead to the following general form for the corresponding stochastic gradient algorithm ([8], [11]):

$$\theta_{t+1} = \theta_t + \gamma X_t(\theta_t)\epsilon_t(\theta_t), \tag{13}$$

where γ is some small gain. Self-orthogonalizing procedures can also be designed according to the particular cases.

The following points have now to be investigated.

a) *How to synthesize the equalizer* θ? The theoretical form $\theta(z^{-1}) = \Sigma_{k\in Z}\, \theta_k z^{-k}$ cannot be used in practical algorithms, since only finitely many parameters can be used. The effect of such truncation on the recovery of S^{-1} is investigated in [7], [8]. The two classical *transversal* and *recursive* structures will be used for synthesizing θ.

b) *How to choose the pseudo-error signal* $\epsilon_t(\theta)$? The Sato cost function suggests a choice for ϵ_t, but others will be investigated, in order to improve the properties of the blind equalizer.

Note that both points a) and b) are in fact disconnected: any relevant pseudo-error signal can be used with any of the two considered structures. Let us now investigate these points in more detail.

B. Some Pseudo-Error Signals

Let us explain the Sato pseudo-error signal, which is obtained using (11) with ψ given in (5):

$$\epsilon_t^S(\theta) = c_t(\theta) - \alpha \cdot \operatorname{sgn} c_t(\theta), \alpha = \frac{Ea_t^2}{E|a_t|}. \tag{14}$$

Here, $\alpha \cdot \operatorname{sgn} c_t$ plays the role of a coarse estimate of the true signal a_t, hence the name of "pseudo-error". Unfortunately, the behavior of this pseudo-error signal around the solution ($\theta \approx S^{-1}$) is very noisy (unless a_t takes only the values of ± 1), since, although being zero-mean, the signal $\epsilon_t^S(\theta)$ is still nonzero for $\theta = S^{-1}$.

On the other hand, the customary error signal

$$e_t(\theta) = c_t(\theta) - \hat{a}_t(\theta) \tag{15}$$

(where $\hat{a}_t$ is the estimate of a_t based upon c_t), which is used in the standard selfadaptative equalizers, is not robust, but enjoys the desirable property of being zero for $\theta = S^{-1}$.

Hence the idea of combining both signals results in the following *G-pseudo-error signal*

$$\epsilon_t^G(\theta) = k_1 e_t(\theta) + k_2 |e_t(\theta)| \cdot \epsilon_t^S(\theta), \tag{16}$$

where k_1 and k_2 are constants. The behavior of this G-pseudo-error signal is the following. For θ far from S^{-1}, $|e_t(\theta)|$ is large, and the second term ensures the robustness of the blind equalizer; on the other hand, for θ close to S^{-1}, the second term has the same order of magnitude as the first one and $\epsilon_t^G(\theta) = 0$ for $\theta = S^{-1}$, which ensures the removal of the noise due to ϵ_t^S.

Hence the G-pseudo-error signal provides us with a smooth automatic switching from a blind startup period to the conventional equalization mode; conversely, in case of an abrupt change in the characteristics of the unknown channel S, this signal goes automatically back to the blind startup mode. No further testing is needed for such switches. This is a highly interesting property of this kind of pseudo-error signal.

C. Structure of the Equalizer

Transversal Equalizer: Here θ is synthesized in the standard transversal form. The corresponding algorithm is:

$$\left.\begin{aligned}\theta_{t+1} &= \theta_t + \gamma X_t \cdot \epsilon_t(\theta)\\ X_t^T &= (x_{t+N}, \cdots, x_t, \cdots, x_{t-N})\\ c_t(\theta) &= \sum_{-N}^{+N} \theta_k \cdot x_{t-k} = X_t^T \cdot \theta,\end{aligned}\right\} \tag{17}$$

where $\epsilon_t(\theta)$ is one of the pseudo-error signals which was given in the preceding section.

A self-orthogonalizing procedure is suggested in [8], Section F], and was experimented in [7]. However multistep procedures are nearly as efficient, and require much less computational effort [10], even compared with fast algorithms in ladder form.

Recursive equalizer: analysis and results are found in [14].

IV. Design of Blind Equalizers for Two Carrier Transmission Systems

Here we are interested in *joint amplitude and phase* modulation schemes, the results on the case where $a_t = \pm 1$ (Section II-C) suggests that the classical error signals are suitable for blind equalization when pure phase modulation is used, which was also pointed out in [5], and is verified by the experiments.

Starting from the cost functions we have developed for the complex case, we shall now indicate how to achieve joint equalization and carrier recovery in the spirit of [12] [13]. This is useful in case the time variations of the phase of the channel (jitter, offset, $\cdots$) are fast compared to other time variations of the channel (corresponding to the intersymbol interference). The theoretical analysis we have done will provide us with blind equalizers which are much more efficient than those presented in [5].

The theoretical derivation of the algorithms will be done in the two independent carrier case only, since no similar theoretical approach is available in the general case; other examples, like the V.29 constellation of the CCITT will be considered via ad hoc techniques, without theoretical investigation. The corresponding scheme is given by the Fig. 6.

Without loss of generality, we can assume that demodulation by a suitable local carrier (without phase tracking !) is carried out before equalization and phase recovery, so that we can investigate the problem in the baseband equivalent form ([2], [3], [12]).

The equalizer is split in two parts, yielding

$$c_t(\theta, \phi) = c_t(\theta)e^{-i\phi}, \tag{18}$$

where the purpose of θ is to remove the intersymbol interference (which is assumed to be "slowly" varying), whereas the phase parameter ϕ will be devoted to the tracking of "fast" tap rotation effects (phase jitter, frequency offset, $\cdots$), as it has been pointed out by Falconer [12], these two parts are redundant in the theoretical case of a time invariant channel with time invariant phase.

With the cost function

$$J(\theta, \phi) \triangleq \mathbb{E}(\psi(\operatorname{Re} c_t(\theta, \phi)) + \psi(\operatorname{Im} c_t(\theta, \phi))). \tag{19}$$

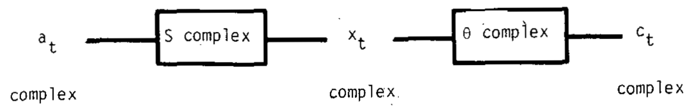


Fig. 6.

and setting

$$\operatorname{grad}_\theta J \triangleq \frac{\partial}{\partial(\operatorname{Re}\theta)} J + i \frac{\partial}{\partial(\operatorname{Im}\theta)} J, \tag{20}$$

we get by a straightforward calculation

$$\left.\begin{aligned} &\operatorname{grad}_\theta J(\theta,\phi) = -\mathbb{E}(X_t^*(\theta,\phi)\epsilon_t(\theta,\phi)), \\ &X_t(\theta,\phi) \triangleq \frac{\partial}{\partial(\operatorname{Re}\theta)}(\operatorname{Re} c_t(\theta,\phi)) \\ &\qquad + i\frac{\partial}{\partial(\operatorname{Im}\theta)}(\operatorname{Im} c_t(\theta,\phi)) \\ &\epsilon_t(\theta,\phi) = -(\psi'(\operatorname{Re} c_t(\theta,\phi)) + i\psi'(\operatorname{Im} c_t(\theta,\phi))), \end{aligned}\right\} \tag{21}$$

whereas

$$\frac{\partial}{\partial\phi} J(\theta,\phi) = -\mathbb{E}(\operatorname{Im}(c_t(\theta,\phi)\epsilon_t^*(\theta,\phi))). \tag{22}$$

Restricting ourselves to the transversal structure for the equalizer, we get the following algorithm:

$$\left.\begin{aligned} \theta_{t+1} &= \theta_t + \gamma X_t^* e^{+i\phi_t}\epsilon_t(\theta_t,\phi_t) \\ \phi_{t+1} &= \phi_t + \mu \operatorname{Im}(c_t(\theta_t,\phi_t)\cdot\epsilon_t^*(\theta_t,\phi_t)) \\ X_t^T &= (x_{t+N},\cdots,x_{t-N}) \\ c_t(\theta,\phi) &= X_t^T\cdot\theta e^{-i\phi}, \end{aligned}\right\} \tag{23}$$

where the pseudo-error signal ϵ_t has to be chosen.

Choice of the Pseudo-Error Signal: Here we shall choose the signal ϵ_t in order to ensure the robustness of the equalizer during a blind startup period. *We restrict ourselves to the case of two independent carriers*, where the real and imaginary parts of the message take the values $\{\pm 1, \pm 3, \cdots, \pm 2K+1\}$ with equal probability. Some pseudo-error signals for the 16-point V.29 constellation of the CCITT are given in Fig. 17.

Let us begin with the complex extension of the Sato cost function, which corresponds to (20) or (21) with ψ given in (5). This gives, after dropping the obvious dependence on θ and ϕ,

$$\left.\begin{aligned} &\epsilon_t^s = c_t - \hat{c}_t, \text{ where} \\ &\hat{c}_t \triangleq \alpha(\operatorname{sgn}(\operatorname{Re} c_t) + i\operatorname{sgn}(\operatorname{Im} c_t)) \\ &\alpha = \left(\int x^2\nu(dx)\right)\left(\int |x|\nu(dx)\right)^{-1}, \end{aligned}\right\} \tag{24}$$

where ν is the (common) distribution of $\operatorname{Re} a_t$ and $\operatorname{Im} a_t$. Note that, in the case of the 4-phase modulation scheme, $\hat{c}_t$ is nothing but the message reconstructed at the receiver, namely $\hat{a}_t$, so that ϵ_t^s is identical to the classical error signal, which turns out to be robust in this case. In the general case, it appears that $\hat{c}_t$ is nothing but the best estimate among the possible values of an equivalent 4-phase constellation. The Fig. 7 shows the resulting constellation for the case of a QAM-16 modulation scheme.

As in the one-carrier case, it is possible to remove the noise due to the use of ϵ_t^s even in the case of perfect equalization, since ϵ_t^s can never be zero. Denoting by $\hat{a}_t$ the signal which is reconstructed from c_t using the standard decision rules, let us introduce the modified pseudo-error signal

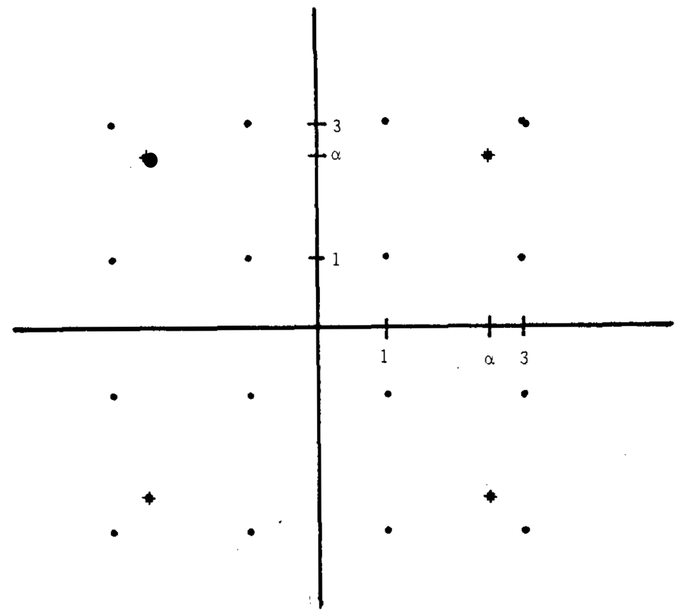


Fig. 7. Constellation for the QAM-16 modulation. $\alpha = 2.5$.

$$\left.\begin{aligned} &\epsilon_t^G = k_1 e_t + k_2|e_t|\epsilon_t^s, \text{ where} \\ &e_t \triangleq a_t - c_t. \end{aligned}\right\} \tag{25}$$

As discussed before, this pseudo-error signal is again robust, *allows the removal of the noise due to* ϵ_t^s *at the equilibrium, and provides us with a (smooth) automatic switching between the blind startup period and the normal receiving period* (and vice-versa in case of an abrupt change in the characteristics of the channel).

Remark: a simplification of the loop error signal. The two pseudo-error signals ϵ_t^s and ϵ_t^G have the property that

$$\left.\begin{aligned} \operatorname{Im}(c_t(\epsilon_t^s)^*) &= \operatorname{Im}(c_t\hat{c}_t^*) \\ \operatorname{Im}(c_t(\epsilon_t^G)^*) &= \operatorname{Im}(c_t(k_1\hat{a}_t^* + k_2|e_t|\hat{c}_t^*)), \end{aligned}\right\} \tag{26}$$

where $\hat{c}_t$ is defined in (24); hence, the loop error signals are are of the same form as in the case of a decision feedback loop for a 4-phase shift keying modulation. This strongly suggests a 4th power loop could be also used.

Summary of the Algorithms for Blind Joint Equalization and Carrier Recovery:

Using the Pseudo-Error Signal (24) *of Sato Type*—for the case of a transversal equalizer, the algorithm is the following (see (5) for the definition of α):

$$\left.\begin{aligned} &\theta_{t+1} = \theta_t + \gamma X_t^* e^{i\phi}\epsilon_t^s \\ &\phi_{t+1} = \phi_t + \mu\operatorname{Im}(c_t\hat{c}_t^*) \\ &\quad c_t = X_t^T\theta_t e^{-i\phi_t},\ \epsilon_t^s = \hat{c}_t - c_t \\ &\quad \hat{c}_t = \alpha(\operatorname{sgn}(\operatorname{Re} c_t) + i\operatorname{sgn}(\operatorname{Im} c_t)). \end{aligned}\right\} \tag{27}$$

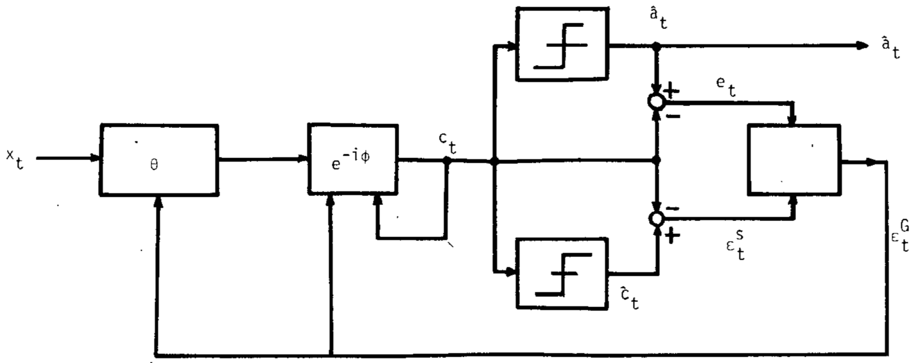


Fig. 8.

Using the Pseudo-Error Signal (25) with Smooth Switching Between the Blind Startup and Normal Periods—the algorithm is the following:

$$\left.\begin{aligned} \theta_{t+1} &= \theta_t + \gamma X^* e^{i\phi_t} \epsilon_t{}^G \\ \phi_{t+1} &= \phi_t + \mu \operatorname{Im}(c_t \epsilon_t{}^{G*}) \\ \epsilon_t{}^G &= k_1 e_t + k_2 |e_t| \epsilon_t{}^s, \end{aligned}\right\} \tag{28}$$

where e_t is the standard error signal defined in (25), whereas $\epsilon_t{}^s$ is defined in (27). The corresponding block-diagram is given in Fig. 8. The block-diagram for (27) is the same with the removal of e_t.

As a conclusion of this section, let us emphasize that the theoretical analysis we have done provides us with tools for deriving various forms for a blind equalizer; among them only the most illustrative were given here. Finally, let us mention the case of some schemes where the two carriers are dependent: in Fig. 17, a blind equalizer is given for the V.29 constellation of the CCITT, this equalizer was derived using ad hoc arguments, and is less efficient than those designed for QAM constellations, but still more efficient than those proposed in [5].

V. Computer Simulations

A. One Carrier

The system used is amplitude modulation. The transmitted data are equally distributed random variables on the 8 levels set $\{\pm1\pm3\pm5\pm7\}$. The impulse response of the channel is given in Fig. 9: the chosen sample is 1/3200 s giving an output of 9600 bits/s. The additive noise on the output is given by simulation of a Gaussian random variable $N(0, \sigma^2 = 0.02)$.

The number of the tap weights for the equalizer is $n = 21$. The algorithms are:

AN 1 given by (13) with the pseudo-error (14)

ASS 1 given by (13) with the pseudo-error (16).

For AN1 $\gamma = 10^{-4}$ and for ASS1 $\gamma = 2.10^{-4}$, $k_1 = 4$, $k_2 = 1$.

The Fig. 10 gives the evolution of the error rate (E.R.) and the corresponding mean squared error (M.S.E.) for AN1 and the Fig. 11 the same results for ASS1.

The duration of the blind training phase is here about 0.5 s. With a slightly modified algorithm (relaxation) this duration becomes 300 ms.

We can mention some other experiments with good results: different numbers of tap weights for the equalizer, insertion of a whitening filter before the equalizer, drastic changes of the impulse response (cf. [10]).

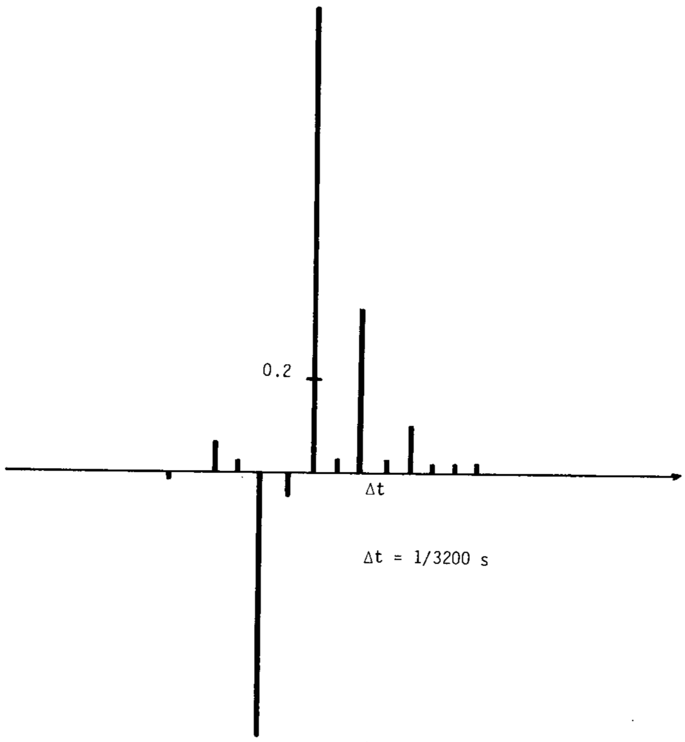


Fig. 9. Sampled impulse response.

B. Two Carriers

a) *With no Carrier Tracking:* in this case the algorithms are:

$$\left.\begin{array}{l} \text{AN 2 given by (27)} \\ \text{ASS 2 given by (28)} \end{array}\right\} \begin{array}{l} \text{with } \phi \text{ fixed.} \\ k_1 = 5, k_2 = 1. \end{array}$$

The impulse responses are given by in Fig. 12 corresponding to the amplitude and delay distortions given in Fig. 13. The output is 9600 bits/s. The additive white noise on the output (as for the following examples) is given by the simulation of two independent Gaussian random variables $N(0, \sigma^2 = 0.02)$. The equalizer uses a double sampling: $n = 21$ tap weights for the "main sample" and 11 tap weights for the shifted sample.

For the QAM transmission the results corresponding to AN2 and ASS2 are respectively given in Figs. 14 and 15.

The results of ASS2 for the amplitude and phase modulation corresponding to the 16 points V29 constellation are given in Fig. 16.

In this last case the decision areas for the received data are given in Fig. 17.

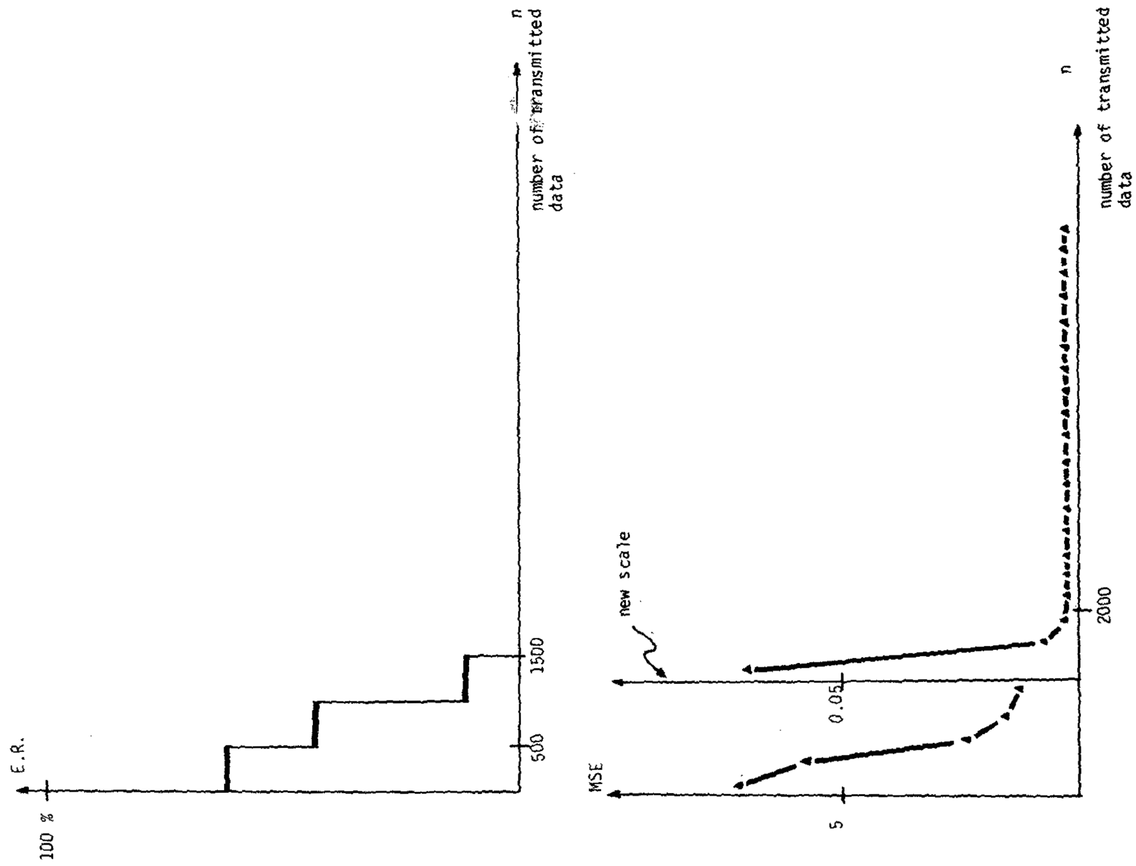


Fig. 11. Error rate and MSE for ASS1.

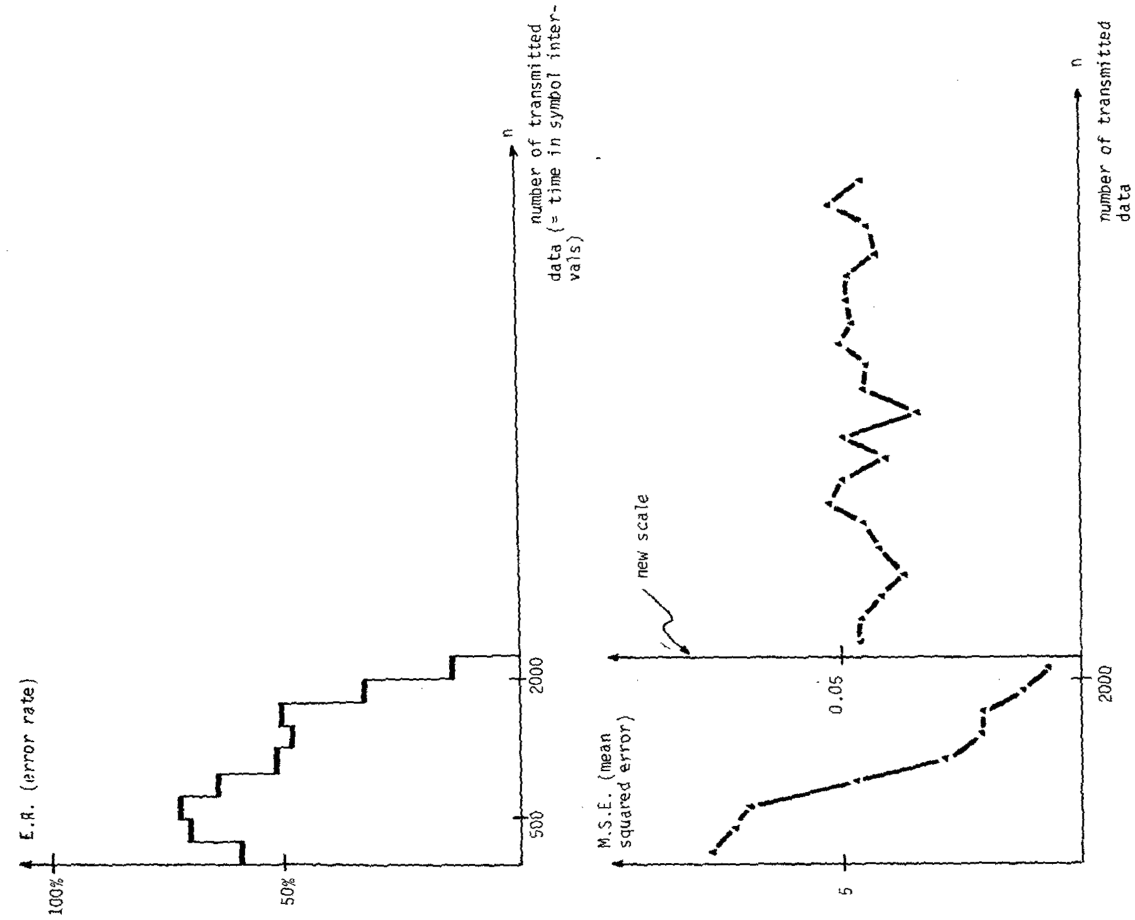


Fig. 10. Error rate and mean squared error for AN1.

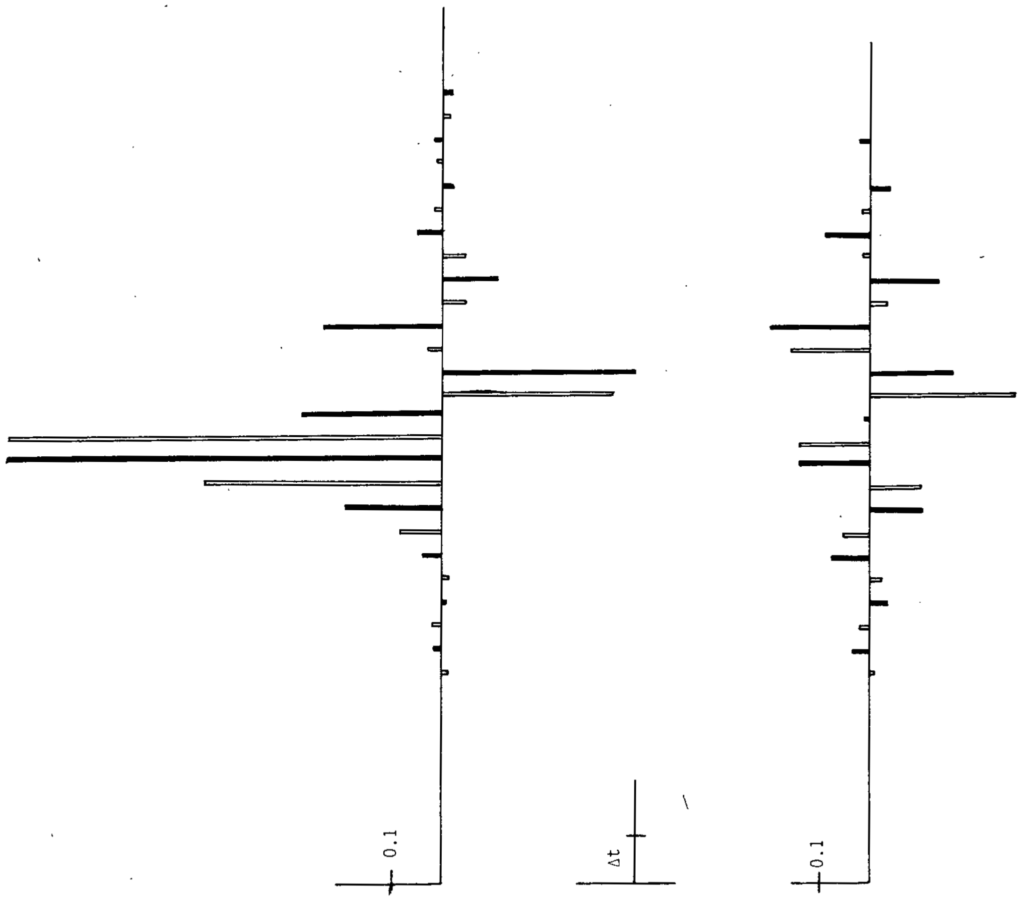


Fig. 12. Sampled impulse responses.

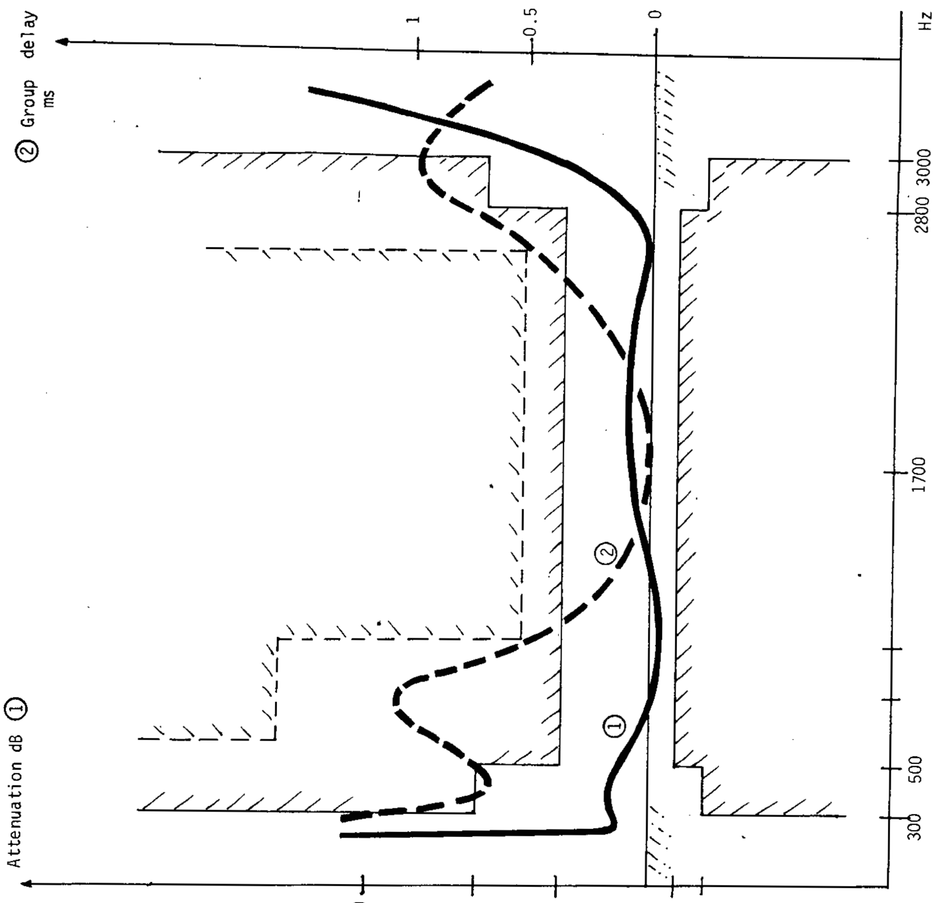
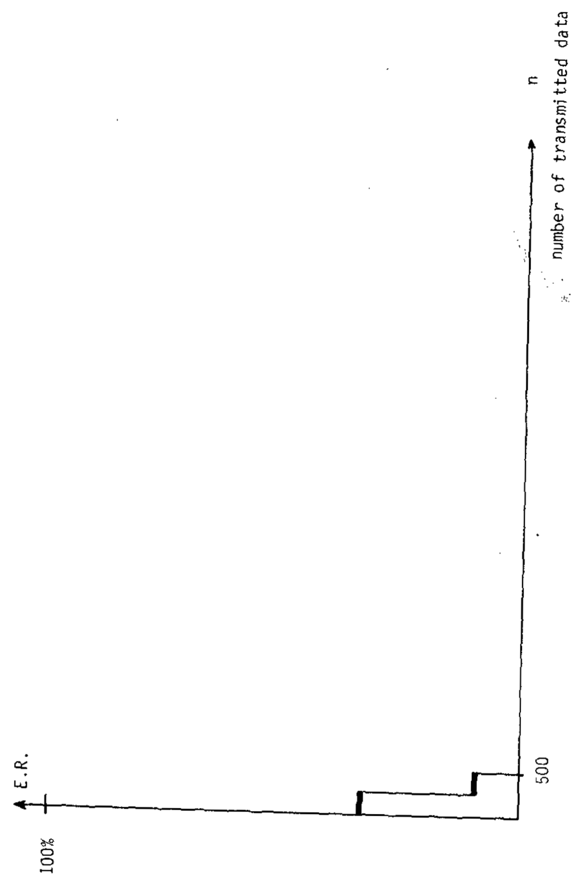
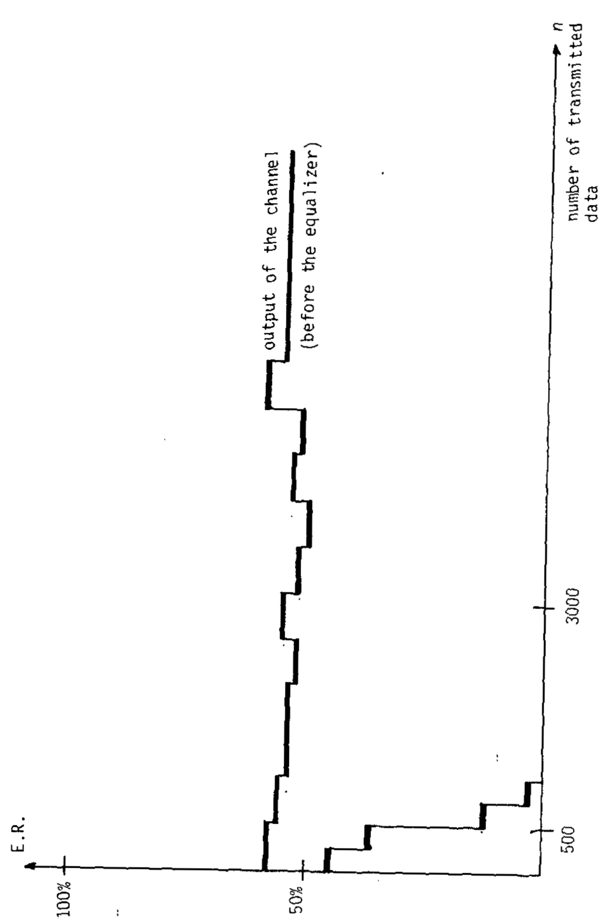


Fig. 13. Amplitude and delay distortions of the channel given by Fig. 8.





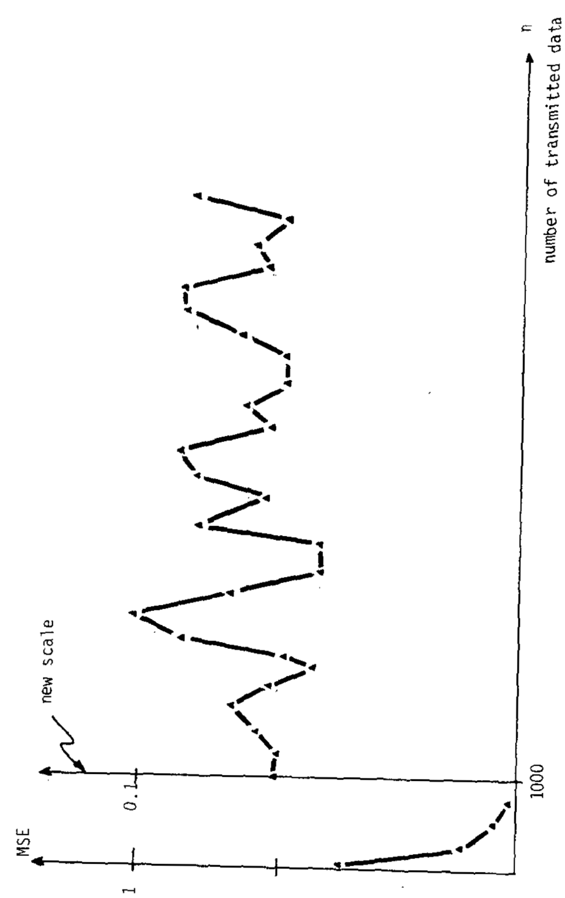


Fig. 14. Results for AN2 (MAQ transmission).

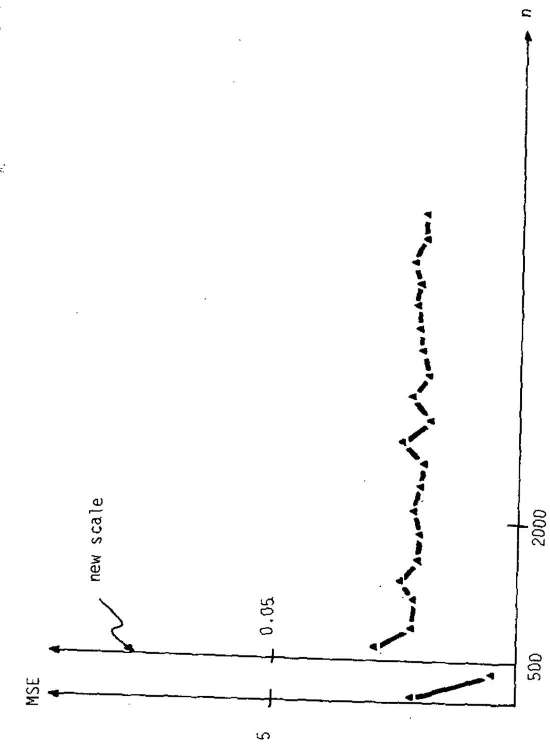


Fig. 15. Results for ASS2 (MAQ transmission).

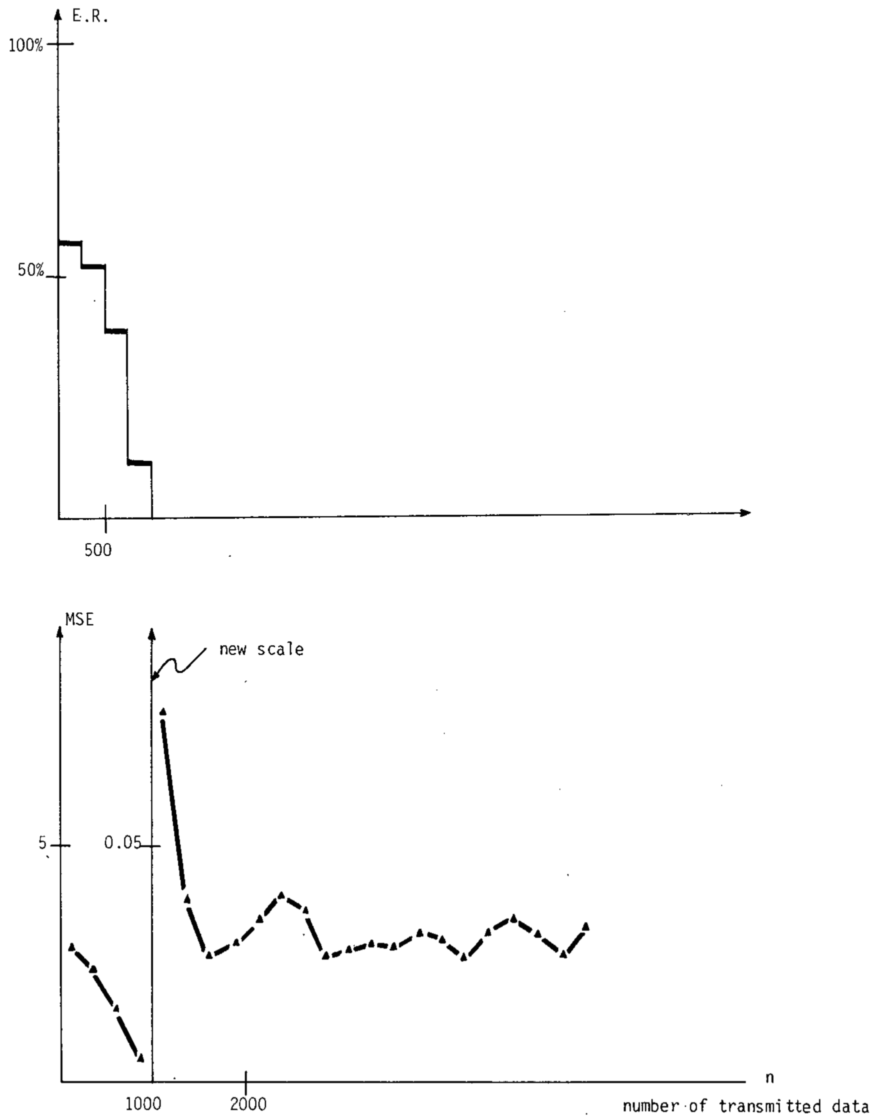


Fig. 16. Results of ASS2 for the 16 points V 29 constellation.

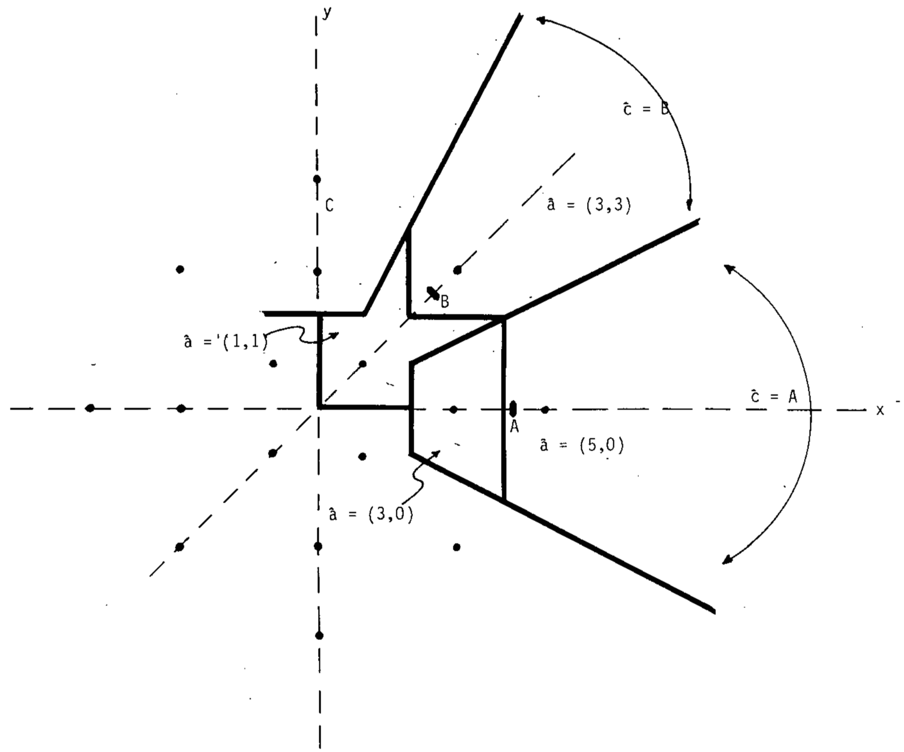


Fig. 17. Decision areas for the 29 CCITT constellation. Example:

$$D=\left\{P=(x,y);\, x\geq 2,\, y\leq \frac{1}{4}x,\, y\geq -\frac{1}{4}x\right\}$$

$D_1=\{P=(x,y),\, P\in D,\, x\leq 4\}$ $\quad D_2=\{P=(x,y);\, P\in D;\, x>4\}$

for $c\in D$, $\hat{c}=A=(4.25,0)$; if $c\in D_1$, $\hat{a}=(3,0)$;

if $c\in D_2$, $\hat{a}=(5,0)$.

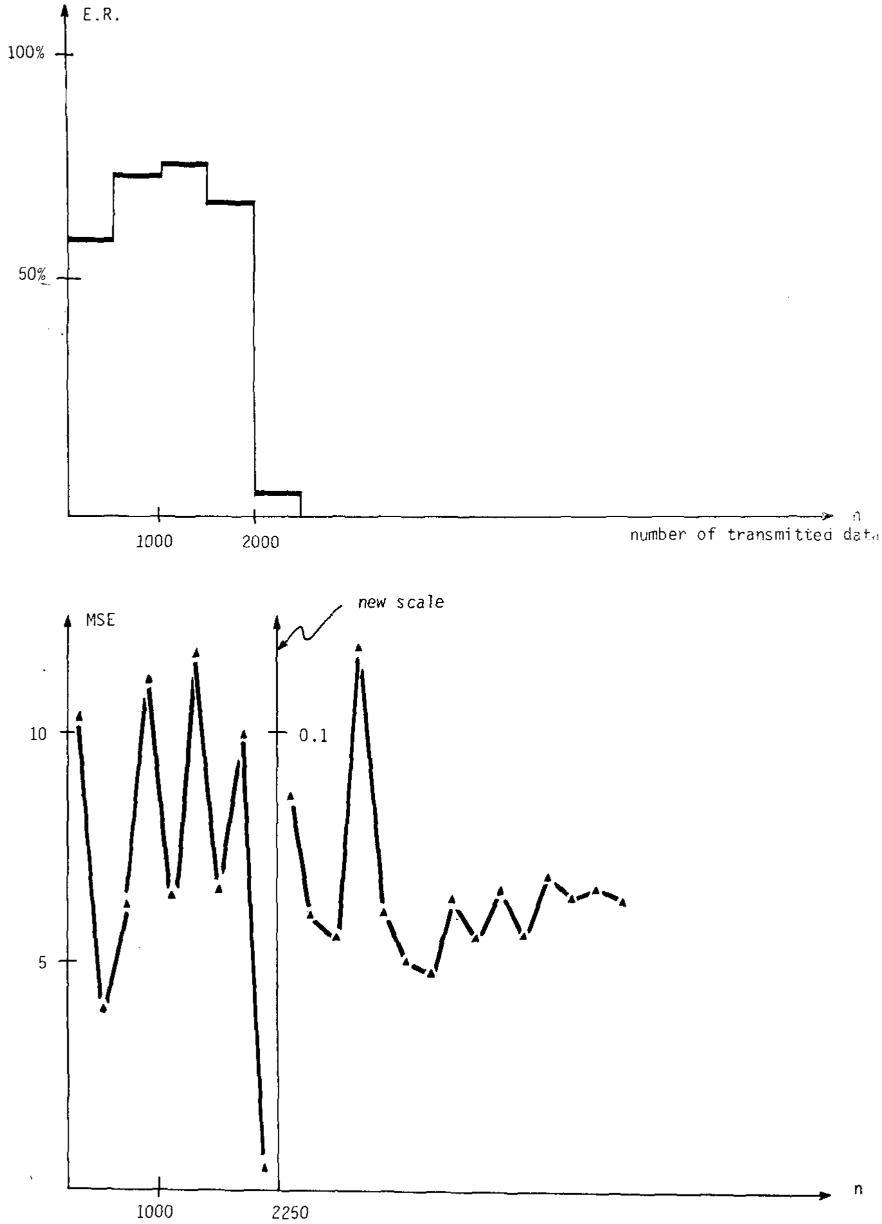


Fig. 18. Results of ASS2 with a 7 Hz frequency offset.

b) *Jointly Self Recovering Equalization and Carrier Tracking:*

Case 1, frequency offset: We consider the same channel as in the previous case (figures 12 and 13) with the QAM transmission and we add a 7 Hz frequency offset. The corresponding results for the algorithm ASS2 are shown on the figure 18.

Case 2, frequency offset and change of the channel: Consider a line $S = \{s_i\}_{-N \leqslant i \leqslant N}$ and

$$t(S) = T = t_i \text{ with } \begin{cases} t_i = -0.5 \cdot s_i \text{ for } i \neq 0. \\ t_0 = s_0 \end{cases}$$

We start with $T = t(S)$, S given in Fig. 12 and a 2 Hz frequency offset; after 2000 data T switches to S while keeping the same offset. The results are on the Fig. 19 ($\gamma = 4.10^{-4}$, $k_1 = 3$, $k_2 = 1$).

Remark: The computer simulations for different other cases are given in [10]:

- Simplified pseudo-error signal $\epsilon_t \to \text{sign}(\epsilon_t)$.
- Relaxation over the tap weights in the stochastic approximation algorithm.
- Using a better integration method (multistep stochastic gradient).
- Drastic channel distortion.

VI. Conclusions

The algorithms which we have proposed here seem to have essential good properties:

- Same implementation and same computational cost as the classical stochastic gradient algorithm for minimizing the mean-squared error.
- Same form for all the different transmission systems: one or two carriers, QAM, V29...
- Convergence of the equalizer of the order of 1 s.
- Automatic smooth switching to an algorithm almost identical to the classical one with possible automatic restarting.
- Extreme robustness with respect to the number of digits

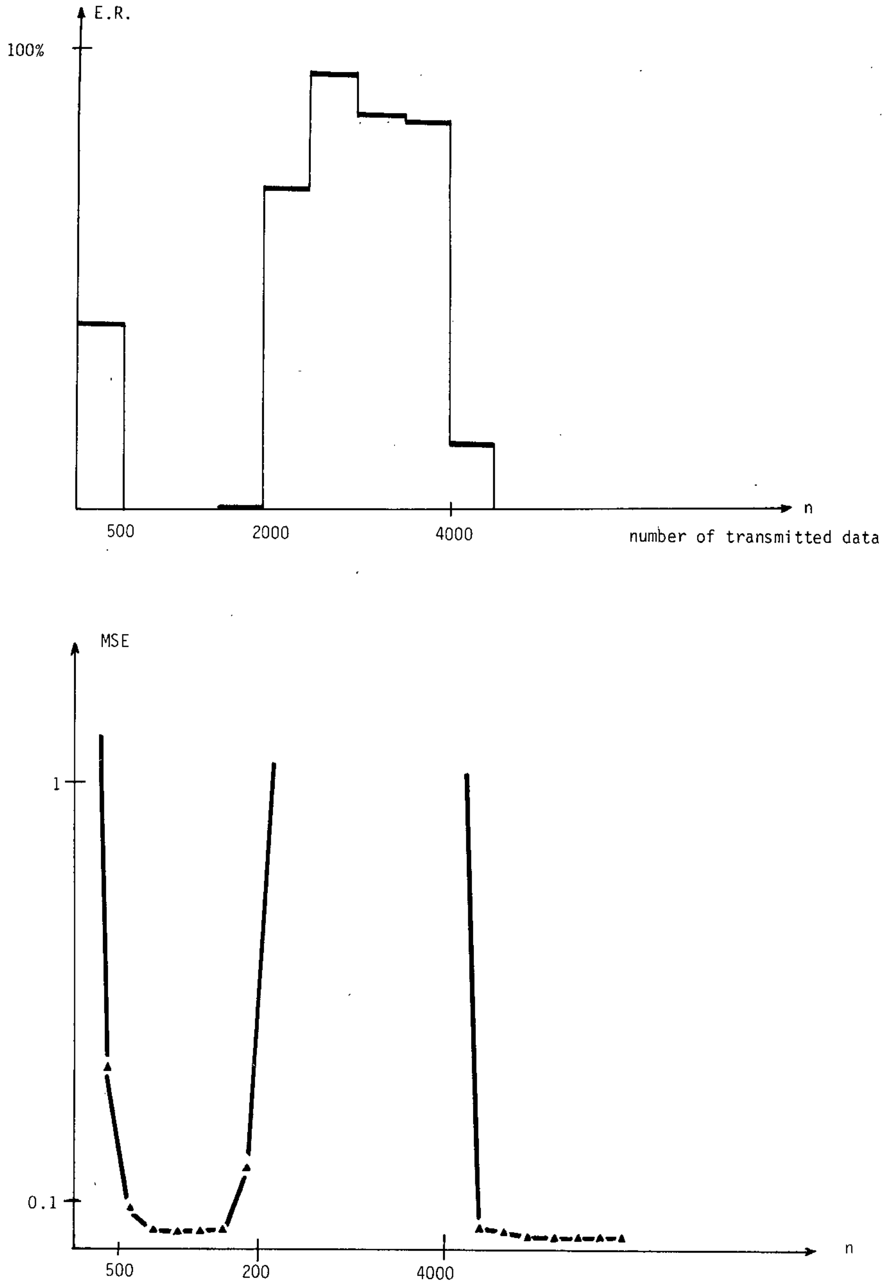


Fig. 19. Results of ASS2 (2 Hz offset + jump of the channel).

used in computing shown by the results given in [10] when ϵ_t or X_t are replaced by sign ϵ_t and sign X_t.

Possible modifications in order to increase the performance if necessary (case 12 000 bits/s): relaxation in the adjustment of the tap weights ($n \rightarrow 2n$ multiplications) and more precise integration method.

References

[1] R. W. Lucky, "A survey of the communication theory literature, 1968–1973," *IEEE Trans. Inform. Theory,* vol. IT-9, pp. 725–739, Nov. 1973.

[2] A. J. Viterbi, Ed. *Advances in Communications Systems*, vol. 4. New York: Academic, 1975.

[3] R. W. Lucky, J. Salz, E. J. Weldon, *Principles of Data Communication.* New York: McGraw-Hill, 1968.

[4] R. D. Gitlin and F. R. Magee, "Self orthogonalizing adaptive equalization algorithms," *IEEE Trans. Commun.*, vol. COM-25, pp. 666–672, July 1977.

[5] D. N. Godard, Self recovering equalization and carrier tracking in two-dimensional data communication systems," *IEEE Trans. Commun.* vol. COM-28, pp. 1867–1875, Nov. 1980.

[6] Y. Sato, "A method of self-recovering equlization for multilevel amplitude-modulation systems," *IEEE Trans. Commun.,* vol. COM-23, pp. 679–682, June 1975.

[7] A. Benveniste, M. Bonnet, M. Goursat, C. Macchi, G. Ruget, INRIA-Rep. 325, 1978.

[8] A. Benveniste, M. Goursat, and G. Ruget, "Robust identification of a nonminimum phase system : Blind adjustment of a linear equalizer in data communication," *IEEE Trans. Automat. Contr.*, vol. AC-25, pp. 385–399, June 1980.

[9] A. Benveniste and M. Goursat, "A gain and phase identification procedure: Blind adjustment of a recursive equalizer," in *Proc. IEEE Inform. Theory Symp.,* Grigagno, Italy, June 1980.

——, "Conception et expérimentation d'égaliseurs robustes," Contrat INRIA-CNET Rep., 1982.

[11] A. Benveniste, M. Goursat, and G. Ruget, "Analysis of stochastic approximation schemes with discontinuous and dependent forcing terms with applications to data communication algorithms," *IEEE Trans. Automat. Contr.*, vol. AC-25, pp. 1042–1058, Dec. 1980.
[12] D. D. Falconer, "Jointly adaptive equalization and carrier recovery in two-dimensional digital communication systems," *Bell Syst. Tech. J.*, vol. 55, pp. 316–334, 1976.
[13] D. D. Falconer, "Analysis of a gradient algorithm for simultaneous passband equalization and carrier phase recovery," *Bell Syst. Tech. J.*, pp. 409–428, Apr. 1976.
[14] A. Benveniste and M. Goursat, INRIA-Rep. 219, July 1983.
[15] A. Benveniste and G. Ruget, "A measure of the tracking capability of recursive stochastic algorithms with constant gains," *IEEE Trans. Automat. Contr.*, vol. AC-27, June 1982.
[16] D. Falconer and J. Salz, "Optimal reception of digital data over the gaussian channel with unknown delay and phase jitter," *IEEE Trans. Inform. Theory,* vol. IT-23, pp. 117–126, Jan. 1977.
[17] L. Ljung, "On positive real transfer functions and the convergence of some recursive schemes," *IEEE Trans. Automat. Contr.,* vol. AC-22, pp. 539–550, Aug. 1977.
[18] ——, "Analysis of recursive stochastic algorithms," *IEEE Trans. Automat. Contr.*, vol. AC-22, pp. 551–575, Aug. 1977.
[19] T. Soderstrom, Ljung, and I. Gustavsson, "A theoretical analysis of recursive identification methods," *Automatica*, vol. 14, pp. 231–244, May 1978.

★

Albert Benveniste (M'81), for a photograph and biography, see p. 728 of the June 1984 issue of this TRANSACTIONS.

★

Maurice Goursat was born in 1945. He received the 3rd cycle thesis degree from the University of Paris VI, Paris, France in 1971.

Since 1969, he has been with INRIA, Le Chesnay, France. His first interest in numerical analysis moved towards the solution of stochastic control problems. He is now with the System Theory Group, with present interests in stochastic control and signal processing.