Electron Charge to Mass Ratio (Weight 1)

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1 Abstract

In this experiment, we find that the magnetic field scales linearly with $1/r$ and estimate the following parameters: B_e = 5.4 x 10^{-4} ± 5.5 x 10^{-5} teslas and e/m = -9.4 x 10^{12} ± 2.8 x 10^{13} C/kg. Additionally, it is found that the magnetic field is affected by external sources such as ferromagnetic materials. However, this effect falls off quickly with radius and is negligible when computing the results.

2 Introduction

The charge-to-mass experiment is also known as the J.J. Thomson experiment. It uses the basics of Lorentz force law and uniform circular motion to determine the charge-mass relation because charged particles follow a circular path in a constant external magnetic field if injected into the field at right angle to the field's direction. By relating the electron's kinetic energy with the potential that it has come across from, we will be able to conclude that how does its path vary under different voltage/current condition, which will enable us to determine the charge-to-mass ratio.

3 Methods and Materials

The instruments used for this experiment are: two Helmholtz coils (130 turns and 15 cm radius), a glass bulb with an electron gun and hydrogen gas, a 300V power supply, two multimeters, a rheostat and a self-illuminated scale. The Jupyter Notebooks software was used to analyze the data.

4 Experimental Procedure

The circuit system is set up as described in Figure 1 and the electrons that are emitted by a hot filament are shaped into a beam and accelerated through the anode. This anode voltage is supplied by a 300V power supply and the coil current is supplied by an 8V power supply. Once the electrons have sufficient kinetic energy to excite the gas through collisions, the glass bulb is turned until the electrons create a visible, closed circular trajectory. The diameter of the closed path is measured with the self-illuminated scale for varying values of current. Other cases are also observed, for example for low accelerating voltage and high current. This data is recorded and analyzed in python.

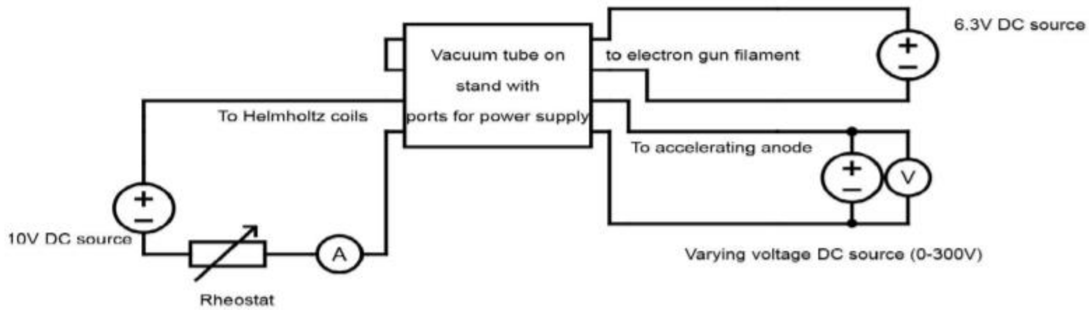

Figure 1: *A circuit diagram of the experimental setup.*

5 Results

The acquired data gives current, I, in amperes and the diameter of the closed path in centimeters. From the current, we are able to compute the magnetic field due to the coils, B_c, based on the relation:

$$B_c = \left(\frac{4}{5}\right)^{3/2} \frac{\mu_0 n I}{R}$$

However, there is an external field generated by the earth, building and other instruments/devices in the lab (B_e). Thus, the total magnetic field is given as $B = B_c + B_e$. Furthermore, we define the following terms:

$$k = \frac{1}{\sqrt{2}}\left(\frac{4}{5}\right)^{3/2} \frac{\mu_0 n}{R} \quad and \quad I_0 = \frac{B_e}{k}$$

This gives us the final relationship between radius and magnetic field which we can use to solve for the charge to mass ratio:

$$\frac{\sqrt{V}}{r} = \sqrt{\frac{e}{m}}k(I - I_0)$$

We then plot B_c against $1/r$ with a linear regression. The estimated y-intercept represents the excess magnetic field, B_e. From the plot we see a strong linear relationship with all of the data points fitting the regression to within their error margin. The estimated parameter from this fit is determined to be, B_e = $5.4 \times 10^{-4} \pm 5.5 \times 10^{-5}$ teslas. Using this with the equation above gives a charge to mass ratio of: e/m = $-9.4 \times 10^{12} \pm 2.8 \times 10^{13}$ C/kg.

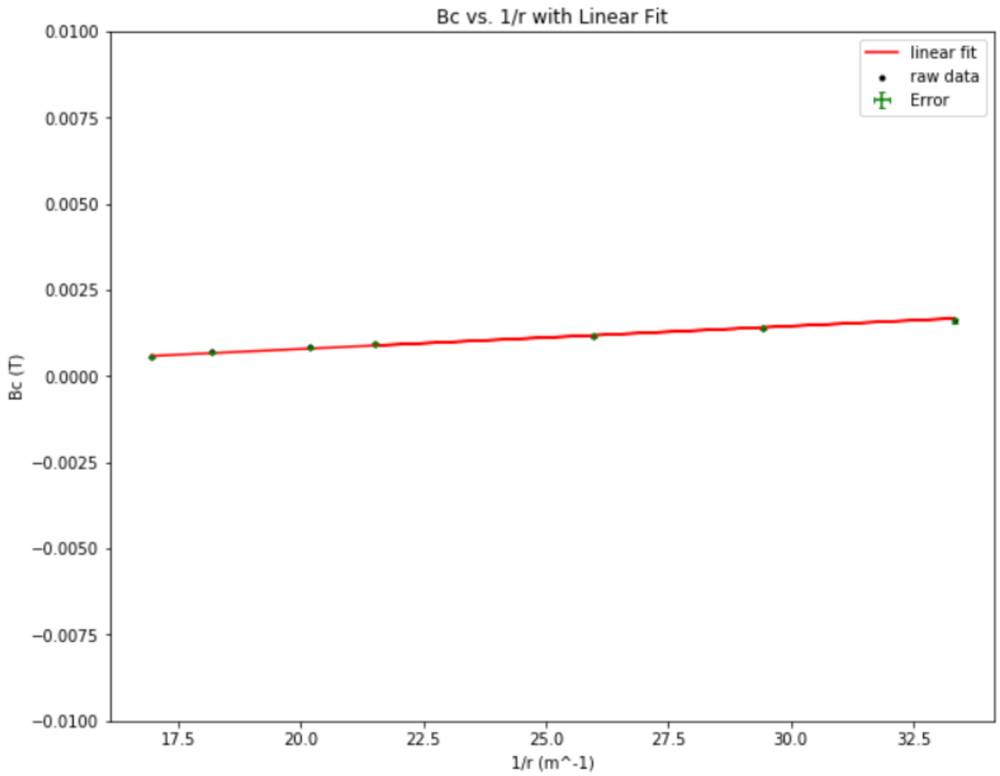

Figure 2: *Magnetic field due to Helmholtz coils plotted against the inverse of radius with a linear regression.*

Applying a statistical analysis on the linear regression, it was found that χ^2 = 2.1. This shows that the linear approximation is a good approximation for the B_c vs. $1/r$ relationship.

6 Discussion

The results of the analysis shows that the relationship between B_c and $1/r$ is well approximated by a linear approximation. The linear regression proved to be a very good fit with χ^2 = 2.1. From this fit, the y-intercept (B_e) is estimated as B_e = 5.4 x 10^{-4} ± 5.5 x 10^{-5} teslas and thus the charge to mass ration can be computed, e/m = -9.4 x 10^{12} ± 2.8 x 10^{13} C/kg. As expected, the excess magnetic field is very small (approximately a factor of 10 smaller than the mean value of B_c). Furthermore, the expected charge to mass ratio is approximately -1.76×10^{11} C/kg which lies within the margin of error of the experimental result. It should also be noted that the final propagated error for the e/m ratio is very large because there are very large initial errors from very rough measurements of diameter.

The uncertainty in experiment first arises from error in the experimental values of R, r, V and I. The uncertainty of R and r is taken as reading errors of a standard ruler (0.5 mm). For V and I the error is taken to be the maximum between the error of accuracy (0.1% and 0.075% of readings respectively) and error of precision (taken as the last digit available with value 1) for the multimeter. From the uncertainty in I, the uncertainty in B_c is propagated using quadrature:

$$\delta B_c = B_c \sqrt{\left(\frac{\delta I}{I}\right)^2 + \left(\frac{\delta R}{R}\right)^2}$$

Since B_e was estimated from a linear regression, the uncertainty is computed from the corresponding covariance matrix entry squared. Finally, the error on the e/m ratio is propagated using quadrature above as well as the propagation rules for addition and multiplication by a constant:

$$\delta Z = \sqrt{(\delta X)^2 + (\delta Y)^2} \;\; (propagation\, rule\; for\; addition)$$

$$\delta Z = |c|\delta X \;\; (propagation\, rule\; for\; multiplication\; by\; a\; constant, c)$$

The strength of this experimental design is that it is quite simple to set up and replicate the results. However, the uncertainties stem from large reading errors (especially in R and r) and cause the results to be quite imprecise. One clear improvement to the experiment is to use a digital scale to measure the diameter of the closed path. This would lead to more accurate measurements as well as a much smaller margin of error on the results.

7 Numbered Questions

Question 1:

There are clear problems with parallax when attempting to measure the diameter in the glass bulb. To eliminate these problems we use the self-illuminated scale and plastic reflector which are lined up with the center of the circular path. Additionally, we observe the scale from directly above the center to get the best possible measurement of diameter with little to no parallax bias.

Question 2:

When investigating the anomalous behaviour of the electron trajectory in the case of low accelerating voltage and high current, we find that the radius of the path shrinks and becomes very small. This agrees with the expected result as both $\sqrt{V}$ and B_c (and thus I) scales with $1/r$. Ideally, this would affect all parts of the trajectory equally however we know that the magnetic field from the coils decreases slightly as it moves away from the central axis. Up till $0.2R$ this effect is negligible but between $0.2R$ and $0.5R$ this effect can be determined as the ratio:

$$\frac{B(\rho)}{B(0)} = 1 - \frac{\rho^4}{R^4\left(0.6583 + 0.29\frac{\rho^2}{R^2}\right)^2}$$

Question 3:

The computed B_e from the y-intercept of the linear regression was found to be: B_e = 5.4 x 10^{-4} ± 5.5 x 10^{-5} teslas

Question 4:

Ferromagnetic materials and other sources of magnetic fields cause the electron trajectory to deform towards the material. This was tested using a cellphone in which case the path became irregular and stretched towards the cellphone. However, we do not consider this effect to be significant in the measurements because it scales inversely with radius. Thus, the effect very quickly becomes negligible when the material is moved away from the bulb.

Question 5:

Using the estimated B_e value from question 3, we compute the electron's charge to mass ratio to be: e/m = -9.4 x 10^{12} ± 2.8 x 10^{13} C/kg.

8 Conclusion

We have observed that when a charged particle moves in external magnetic field, if its initial velocity is perpendicular to the field's direction, it falls into uniform circular motion. The radius of the path closely follows the Lorentz force law and centripetal force. Current strength and voltage determine the radius of the circular motion. Furthermore, the result that is of most interest, the charge-to-mass ratio of electron, will follow from a relation Lorentz force (the centripetal force) with the kinetic energy (electron volt). The results of this experimental setup are such that: B_e = 5.4 x 10^{-4} ± 5.5 x 10^{-5} teslas and e/m = -9.4 x 10^{12} ± 2.8 x 10^{13} C/kg.

9 References

[1] Pandhi, A. and Chen, X., University of Toronto, Toronto, ON. "PHY224 Laboratory Notes: Electron Charge to Mass Ratio", November 2018.

10 Appendices

10.1 Python Code

The full code used for this lab can be found below as well as on the author's Github as "Photoelectric Effect.py":
https://github.com/AyushPandhi/Pandhi_Ayush_PHY224/tree/master/Electron%20Charge%20to%20Mass%20Ratio.

```
#Electron Charge to Mass Ratio
#Author: Ayush Pandhi (1003227457)
#Date: December 02, 2018

#Importing required modules
import numpy as np
import matplotlib.pyplot as plt
from scipy.optimize import curve_fit

#Defining the linear model function
def f(x, a, b):
    return a*x + b

#Loading Current vs. Diameter data
current = np.loadtxt('Current vs Diameter.txt', skiprows=2, usecols=(0,))
radius = (0.01*(np.loadtxt('Current vs Diameter.txt', skiprows=2, usecols=(1,))))/2

#Radius of coils and voltage in the system
R = 0.150
V = 181.0

#Defining errors in current, voltage and R
ierror = np.empty(len(current))
for i in range(len(current)):
    ierror[i] = max(current[i]*0.0075, 0.1/1000)
verror = max(V*0.001, 0.1/1000)
Rerror = 0.005
radiuserror = 0.005

#Computing Bc (magnetic field) from current
Bc = []
for i in current:
    Bc.append(((4/5)**(3/2))*(4*(np.pi)*10**(-7))*(130)*(1/R)*(i))
```

```
#Propagating error for Bc
Bcerror = Bc*(((ierror/current)**2 + (Rerror/R)**2)**0.5)
#Linear regression
p_opt, p_cov = curve_fit(f, 1/radius, Bc, (0,0), Bcerror, True)
output = f(1/radius, p_opt[0], p_opt[1])

#Calculating chi squared
chi_sq = (1/4)*(np.sum(((Bc - output)/Bcerror)**2))
print('Chi squared for linear regression: ', chi_sq)

#Outputting estimated Be (y-intercept)
print('Estimated Be: ', -p_opt[1])

#Plot of Bc vs. 1/r
plt.figure(figsize=(10,8))
plt.scatter(1/radius, Bc, label = 'raw data', marker = '.', color = 'k')
plt.plot(1/radius, output, 'r-', label = 'linear fit')
plt.title('Bc vs. 1/r with Linear Fit')
plt.xlabel('1/r (m^-1)')
plt.ylabel('Bc (T)')
plt.errorbar(1/radius, Bc, xerr=0, yerr=Bcerror, linestyle='none', ecolor='g', label='Error', capsize=2)
plt.ylim(-0.01, 0.01)
plt.legend()
plt.show()

#Defining k and I0
k = (((4/5)**(3/2))*(4*(np.pi)*10**(-7))*(130)*(1/R))/(2**0.5)
I0 = (-p_opt[1])/k

#Propagating errors for k and I0
kerror = (((4/5)**(3/2))*(4*(np.pi)*10**(-7))*(130)*(1/Rerror))/(2**0.5)
I0error = I0*(((Bcerror/Bc)**2 + (kerror/k)**2)**0.5)

#Computing the e/m ratio
emratio = -(((V**0.5)/(radius*k))*(1/(current - I0)))**2
print('Mean charge to mass ratio: ', np.mean(emratio))

sqrtVerror = (V**0.5)*0.5*(verror/V)
errorLHS = (V**0.5/radius)*(((sqrtVerror/V**0.5)**2 + (radiuserror/radius)**2)**0.5)
errorRHS = (ierror**2 + I0error**2)**0.5
errorRHS2 = (k*(current - I0))*(((errorRHS/(current - I0))**2 + (kerror/k)**2)**0.5)
erroremratio = emratio*(((errorLHS/(V**0.5/radius)**0.5)**2 + (errorRHS2/(k*(current - I0)))**2)**0.5)

print(np.mean(erroremratio))
```