PyLab 2 – Introduction to Fitting Methods

October 9, 2018 – Ayush Pandhi (1003227457) and Xi Chen (1003146222)

1 Abstract

Ohm's Law is a fundamental concept that is commonly applied to various fields such as physics and engineering. In this experiment we aim to describe the relationship between current and voltage using a linear curve fitting model and test whether it is consistent with the theory of Ohm's Law. We find that indeed the linear fit provides a very accurate approximation of the theorized linear relationship between current and voltage for both the resistor and potentiometer.

2 Introduction

The purpose of this lab is to approximate the relationship between the experimental current and voltage parameters with a linear curve fitting function. Theoretically speaking, Ohm's law states that the relationship between current and voltage is indeed linear, with the slope being the reciprocal of resistance. Additionally, we compare the expected resistance to the resistance computed from the linear fitting and analyze the χ^2 values to determine if the fit is a reasonable approximation.

3 Methods and Materials

The materials utilized for this experiment were: a power supply, two multimeters, a board with electrical components (with resistors and a potentiometer), a sufficient amount of wires to create the required circuit, and the Jupyter Notebooks software.

4 Experimental Procedure

The first experiment was set up by connecting the two multimeters, power supply and a resistor as seen in Figure 1.

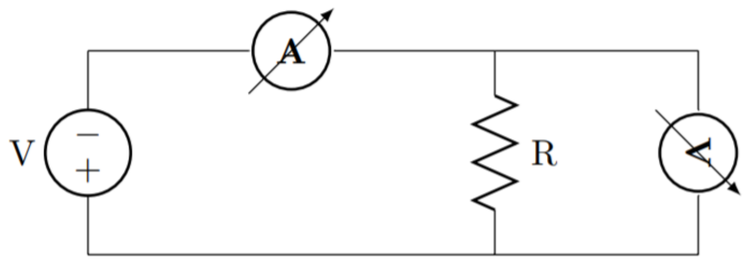

Figure 1: A circuit diagram for the experimental setup.

The power supply was set to the lowest setting and the voltage (V) and current (mA) was recorded from the multimeters. This process was repeated 10 times where the power supply output was increased by a small interval after each measurement. The voltage and current data was stored in a .txt file with the uncertainties of each measurement being computed in python by finding the maximum value between the Error of Accuracy and Error of Precision. After collecting the data, the power was disconnected from the system and the voltmeter was switched to measure the resistance as a reference value for the experiment.

The experiment was then repeated identically with a potentiometer instead of a resistor and the data was collected on a separate .txt file.

5 Results

Using the observational data gathered in both the resistor and potentiometer experiments, we generate a linear curve fit using scipy's "curve_fit()" module. The linear modelling is based on the function f, where f is defined as a general linear formula:

$$f = ax + b$$

For our curve fit the function f is the current, I (in amperes), the independent variable x is the voltage, V (in volts), and the slope a is thus $1/R$ because the theoretical relationship follows that $V = IR$. Therefore, we can rewrite the first equation to be representative of the linear model specifically applied to our data:

$$I = \frac{1}{R}V$$

Plotting the linear model fits for both collected data sets with the estimated parameters, (a = 0.01, b = 0.0) for the resistor and (a = 0.02, b = 0.0), shows that the fits are an accurate representation of the raw data.

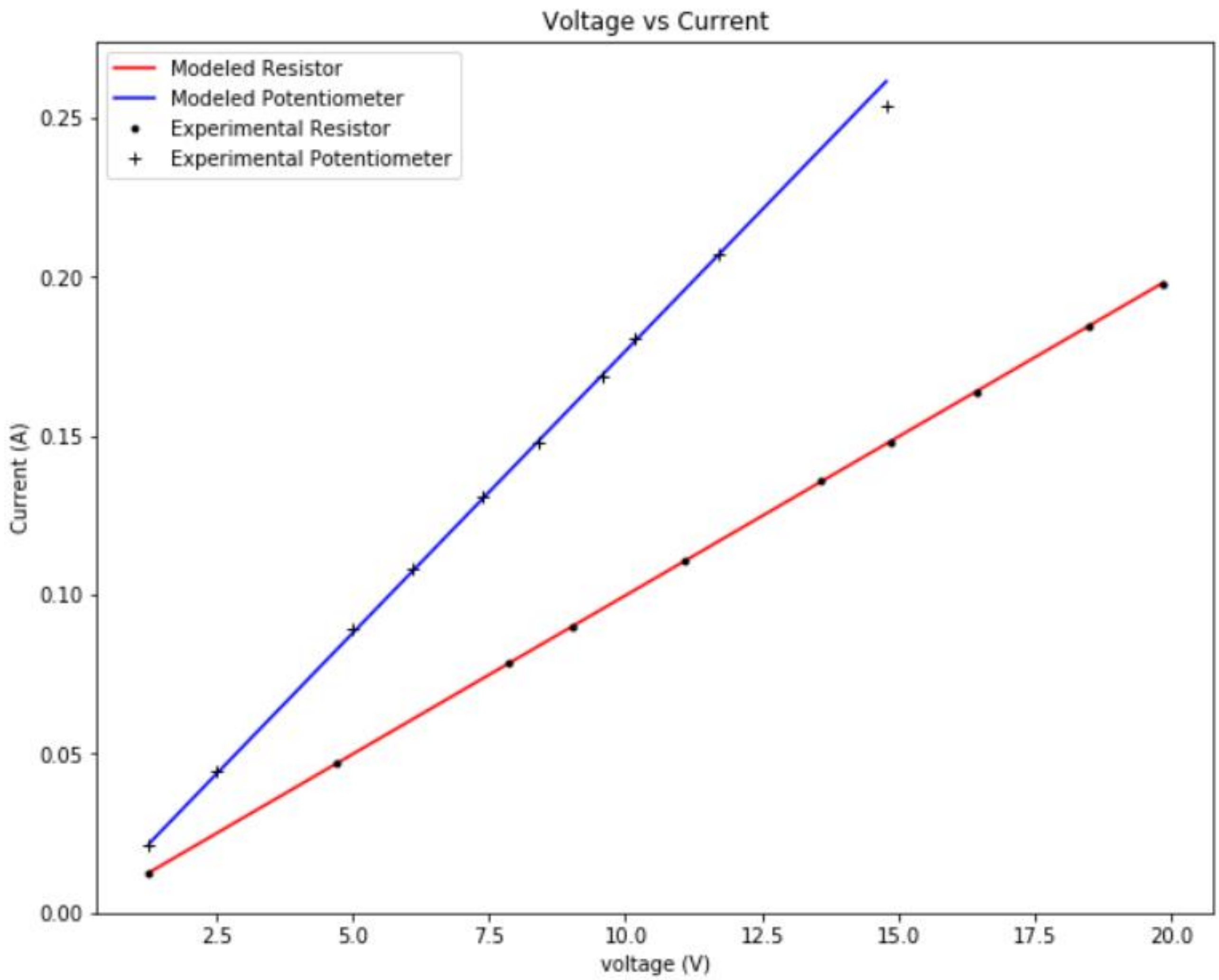

Figure 2: Linear model using curve_fit() over-plotted with the resistor and potentiometer observational data

Using the slopes of these lines we can compute the modelled resistance of both devices based on the previously mentioned relation that $a = 1/R$. With this we find that $I_{resistor}$ = 100 $\pm$ 0.3 ohms and $I_{potentiometer}$ = 56 $\pm$ 0.2 ohms. Comparing this with the mean resistance directly computed from the raw data ($V_{average}/I_{average}$), $I_{resistor}$ = 100 $\pm$ 0.3 ohms and $I_{potentiometer}$ = 57 $\pm$ 0.2 ohms, again we see that the linear model is an accurate fit for the data. These values also closely agree with the recorded experimental resistance values of 100 $\pm$ 0.03 ohms and 57 $\pm$ 0.03 ohms respectively.

Looking at the χ^2 values we see that $\chi^2_{resistor}$ = 0.1 and $\chi^2_{potentiometer}$ = 3.2 which suggests that the linear resistor model is over-fit due to an insufficient number of data points and the linear potentiometer model is an incomplete fit. These χ^2 values are somewhat expected as our data set is very small and the fits still look reasonable visually, so we conclude that the linear fits are fairly accurate.

6 Discussion

At the end of collecting each data set, the power was disconnected from the system and the voltmeter was switched to measure the resistance as a reference value for the experiment. These reference values of resistance were $I_{resistor}$ = 100 $\pm$ 0.3 ohms and $I_{potentiometer}$ = 57 $\pm$ 0.2 ohms which match exactly with the average resistance that was computed directly from the data set. This computation follows Ohm's Law which states:

$$V = IR$$

Where V is voltage (volts), I is current (amps) and R is the resistance (ohms) of the system. As we saw in the previous section, the linear modelling returned $I_{resistor}$ = 100 $\pm$ 0.3 ohms and $I_{potentiometer}$ = 56 $\pm$ 0.2 ohms which were computed from the fact that the resistance is the reciprocal of the slope of the linear fit. Overall, the modelled values line up very well with the computed and reference values for the resistance of each system and we conclude that the curve fit is an accurate representation of the experimental data.

The uncertainty in the experiment first arises from error in the experimental values of V and I, for which we take the error to be the maximum between the error of accuracy and error of precision. Additionally, for the reference resistance values measured from the multimeter, we apply this same rule. The individual uncertainty values for the resistor experiment are: for V (0.01, 0.01175, 0.019625, 0.02255, 0.027675, 0.033975, 0.03715, 0.0411, 0.0462, 0.049625) and for I (0.0001, 0.00035325, 0.0005895, 0.000675, 0.00083025, 0.0010185, 0.0011115, 0.001227, 0.00138225, 0.001482). From this the uncertainty in the mean of both parameters is found using the error propagation rule for sums and multiplying by a constant:

$$E_{mean\ V} = \left|\frac{1}{10}\right| \sum_{i=1}^{10} E_{V,i}^2$$

Where $E_{mean\ V}$ is the error of the mean of the V parameter data and $E_{V,i}$ is the uncertainty of the i^{th}data point in the voltage data set. The error of the mean of the I parameter is computed analogously. From this we are able to compute the error in our computed R value using the error propagation rule for multiplication and division:

$$E_R = |R| \sqrt{\left(\frac{E_{mean\ V}}{\bar{V}}\right)^2 + \left(\frac{E_{mean\ I}}{\bar{I}}\right)^2}$$

The strength of this experimental design is that it is quite simple to set up and replicate the results. Additionally, the uncertainties mostly stem from the error of accuracy and precision of the electronic equipment, which tend to be rather small. Thus the uncertainty on our final results is also small and we achieve accurate results compared to the theoretical predictions. Overall, the experiment was a replicable setup with minimal uncertainties and this reflected well in the highly accurate results; thus there are no obvious adjustments that should be made to improve the system.

7 Numbered Questions

Question 1:

Our linear fit has a y-intercept just below the origin, it is small enough to be negligible and might be caused by the inaccuracy of the voltmeter as there might be some leftover charges still remaining inside the circuit somewhere. This could perhaps cause the potential to be off from zero when the current is not running through the circuit.

Question 2:

Forcing the fit parameter b to be zero in our case only slightly changed the slope of our fit if we zoom in very close to our graph, since we had a very small y-intercept before. The line of fit still passed through most of the data points. This can also be seen from comparing the values of two different reduced-chi-square.

Question 3:

The reference resistance value measured on the multimeter was 100 $\pm$ 0.03 ohms and 57 $\pm$ 0.03 ohms for resistor and potentiometer respectively. The curve fitting produced: 100 $\pm$ 0.3 ohms and 56 $\pm$ 0.2 ohms for the modeled resistor and potentiometer respectively. Thus the results are extremely similar and we can conclude that the curve fit is a good estimation of the data. This was expected because linear fitting should be able to reflect the linear relationship between voltage and current from Ohm's Law.

Question 4:

The reduced-chi-square (χ^2) measures the squares of the quotient between current offsets and voltage offsets from experimental values to the fitted values. From the results section we find that: $\chi^2_{resistor}$ = 0.1 and $\chi^2_{potentiometer}$ = 3.2. This suggests that the linear resistor model is over-fit due to an insufficient number of data points and the linear potentiometer model is an incomplete fit. This is to be expected to some degree as the collected data only contains 10 data points respectively and the χ^2 would improve with more data points.

8 Conclusion

From the experiment we conclude that the linear curve fitting method produced a very accurate approximation of the data which adheres to the linear relationship between current and voltage through Ohm's law. Additionally, the linear fitting was able to reproduce resistance values of both the resistor and potentiometer ($I_{resistor}$ = 100 $\pm$ 0.3 ohms and $I_{potentiometer}$ = 56 $\pm$ 0.2 ohms) that closely match the reference resistance values measured directly in the experiment ($I_{resistor}$ = 100 $\pm$ 0.03 ohms and $I_{potentiometer}$ = 57 $\pm$ 0.03 ohms). Although the χ^2 values suggest that it is not a great fit, this is thought to be due to the arbitrarily small number collected of data points and the χ^2 values should improve with a larger data set.

9 References

[1] Pandhi, A. and Chen, X., University of Toronto, Toronto, ON. "PHY224 Laboratory Notes: PyLab 2 – Introduction to Fitting Methods", October 2018.

10 Appendices

The full code used for this lab can be found below as well as on the author's Github as "PyLab 2 Code.py": https://github.com/AyushPandhi/Pandhi_Ayush_PHY224/tree/master/PyLab%202.

```
#PyLab 2: Introduction to Fitting Methods
#Author: Ayush Pandhi (1003227457)
#Date: 10/09/2018

#Importing required modules
import numpy as np
import math as m
import matplotlib.pyplot as plt
from scipy.optimize import curve_fit

#Defining the model function
def f(x, a, b):
   return a*x + b

#Loading the resistor data
voltage1 = np.loadtxt('resistor data.txt', skiprows=1, usecols=(0,))
current1 = (1/1000)*(np.loadtxt('resistor data.txt', skiprows=1, usecols=(1,)))

#Loading the potentiometer data
```

```
voltage2 = np.loadtxt('potentiometer data.txt', skiprows=1, usecols=(0,))
current2 = (1/1000)*(np.loadtxt('potentiometer data.txt', skiprows=1, usecols=(1,)))

#Finding max of precision and accuracy error for resistor data
v_error1 = np.empty(len(voltage1))
for i in range(len(voltage1)):
    v_error1[i] = max(voltage1[i]*0.0025, 0.01)

i_error1 = np.empty(len(current1))
for i in range(len(current1)):
    i_error1[i] = max(current1[i]*0.0075, 0.1/1000)

#Finding max of precision and accuracy error for potentiometer data
v_error2 = np.empty(len(voltage2))
for i in range(len(voltage2)):
    v_error2[i] = max(voltage2[i]*0.0025, 0.01)

i_error2 = np.empty(len(current2))
for i in range(len(current2)):
    i_error2[i] = max(current2[i]*0.0075, 0.1/1000)

#Calling curve_fit() for these data sets
p_opt1 , p_cov1 = curve_fit(f, voltage1, current1, (1/100, 0), i_error1, True)
p_opt2 , p_cov2 = curve_fit(f, voltage2, current2, (1/57, 0), i_error2, True)

#Outputs based on the model function
output1 = f(voltage1, p_opt1[0], p_opt1[1])
output2 = f(voltage2, p_opt2[0], p_opt2[1])

#Plotting Voltage vs Current
plt.figure(figsize=(10,8))
plt.plot(voltage1, output1, 'r-', label='Modeled Resistor')
plt.plot(voltage2, output2, 'b-', label='Modeled Potentiometer')
plt.plot(voltage1, current1, 'k.', label='Experimental Resistor')
plt.plot(voltage2, current2, 'k+', label='Experimental Potentiometer')
plt.xlabel('voltage (V)')
plt.ylabel('Current (A)')
plt.title('Voltage vs Current')
plt.legend(loc='upper left')
plt.show()

#Defining chi squared data
chisqr1 = sum(((current1-output1)/i_error1)**2)
chisqr1 = chisqr1/8
```

```
chisqr2 = sum(((current2-output2)/i_error2)**2)
chisqr2 = chisqr2/8

#Printing resulting parameters
print('Experimental Resistor Resistance: ', 1/p_opt1[0], 'Ohms')
print('Experimental Potentiometer Resistance: ', 1/p_opt2[0], 'Ohms')
print('Experimental Resistor Chi Squared Value: ', chisqr1)
print('Experimental Potentiometer Chi Squared Value ', chisqr2)
```