# Radius of the Earth (Weight 2)

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## 1 Abstract

In this experiment we estimate the radius of the earth using a gravity meter and measure the change in $g$ at different floors of Burton Tower in Toronto. We find that there is a roughly linear relationship between $\Delta g$ and $\Delta R$ and using a linear regression, the radius of the Earth is estimated to be $R$ = 6612.8 ± 0.1 km (floors 3 to 13) and $R$ = 6594.5 ± 0.1 km (including basement and 14$^{th}$ floor). The expected value of $R$ is 6371 km which does not match too well with the results of this experiment but it is reasoned that this discrepancy is due to the sources of error in the experimental setup and challenges of creating an accurate linear regression for a small data set.

## 2 Introduction

In this experiment, we are trying to determine the radius of the Earth using a method that was first developed to detect underground oil deposits. Since oil deposits have a lower density than their surroundings, there will be a slight change in the value of g over the area above them. We simulate this similar effect by going up the stairs of Burton Tower; as we go up more mass is shifted from above us to below us and thus the measurement of the $g$ value will change. Using the relationship between $\Delta g$ and $\Delta R$ we will then attempt to estimate the radius of the Earth through our $g$ data gathered on each floor. For this experiment, we consider floors 3 to 13 to be optimal for measurements as they are roughly consistent in regards to a height and mass for each floor. The basement and 14$^{th}$ floor are slight anomalies due to missing mass, non-standard ceilings and because the roof is slightly lighter than the other floors. Additionally, the third floor is set as a reference point at which the $g$ value is measured every 30 minutes to measure the change in our measurements due to the position of the Sun. From this we conclude that the variation of the $g$ value due to the Sun is deemed negligible as individual data sets were gathered in under half an hour each and the change in the reference floor (floor 3) measurements for this period of time is extremely small.

## 3 Methods and Materials

The only instrument used for this experiment is the Sodin Gravity Meter (Prospector 410 model). The data is analyzed using the Jupyter Notebooks software.

## 4 Experimental Procedure

For this experiment, the third floor was designated as the reference point and measurements are taken on each floor between the third and thirteenth floors. At each floor the meter is setup in the stair well by turning the light on and adjusting the levels such that they are centered as accurately as possible. Then the counter setting is calibrated to align beam with the long center line. The counter reading is then multiplied by the meter constant and the value is recorded in milligals. This process is repeated at each floor and every half an hour a measurement is done on the third floor to serve as a reference point for how the reading changes with the Sun's position in the sky.

## 5 Results

The acquired data is of milligals and the corresponding floor number which serves as a form of height spaced at even intervals. Converting to $g$ (m/s$^2$) and measuring the change in this parameter, $\Delta g$, allows us to examine the relationship between $\Delta g$ and $\Delta R$ (change in height). This relationship follows:

$$\frac{\Delta g}{g} = -2\frac{\Delta R}{R}$$

Here we plot floor numbers ($F$) as the independent variable instead of $\Delta R$ directly but since each floor's height is taken to be 3.95m, the relationship does not change except that it is multiplied by an additional factor of 3.95:

$$\frac{\Delta g}{g} = -7.9\frac{F}{R}$$

The data set used for plotting and analysis was taken to be the mean values for each floor from all three experimental data sets and the uncertainty was taken to be the maximum of the standard deviation between each floor's measurement and the error of accuracy on the instrument. Plotting the data and applying a linear regression we find that the linear approximation seems to fit the data decently well by visual inspection. There are no clear outliers in the data set and the linear fit is within the error margins of the individual data points.

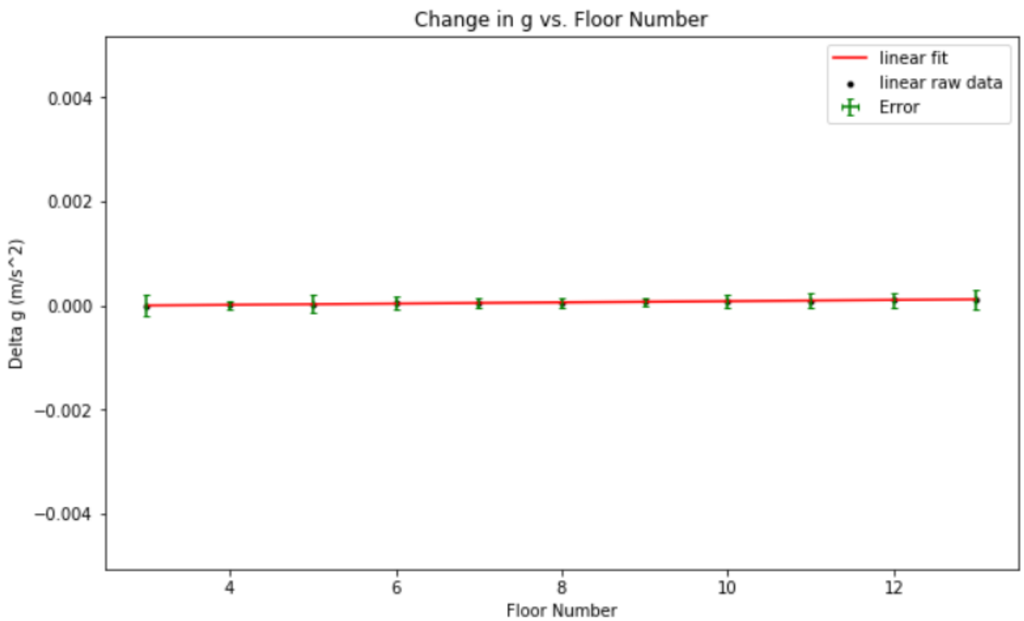


**Figure 1:** $\Delta g$ *plotted against floor number for floors 3 to 13 in Burton Tower with a linear regression.*

Applying a statistical analysis on the linear regression we find that $\chi^2$ = 2.2 x $10^5$. This suggests that the linear model is an over-fit for the data set which could be a result of a small number of data points.

The approximate radius of the Earth, $R$, is estimated from the slope of the linear regression and is found to be $R$ = 6612.8 ± 0.1 km. The expected value is 6371 km (+ 76 km if we consider Toronto's height above sea level), the discrepancy between the results and expected value is explained further in the next section.

An analogous analysis was done on a data set which includes measurements from the basement and fourteenth floor (which were deemed to be theoretical anomalies for the experiment earlier). The result is quite similar with $\chi^2$ = 3.8 x $10^5$ and $R$ = 6594.5 ± 0.1 km. Again, since the linear regression is a poor estimate of the linear relationship within the data, it is difficult to argue which of these results is more accurate. However, it seems that the results with both data sets are very similar.

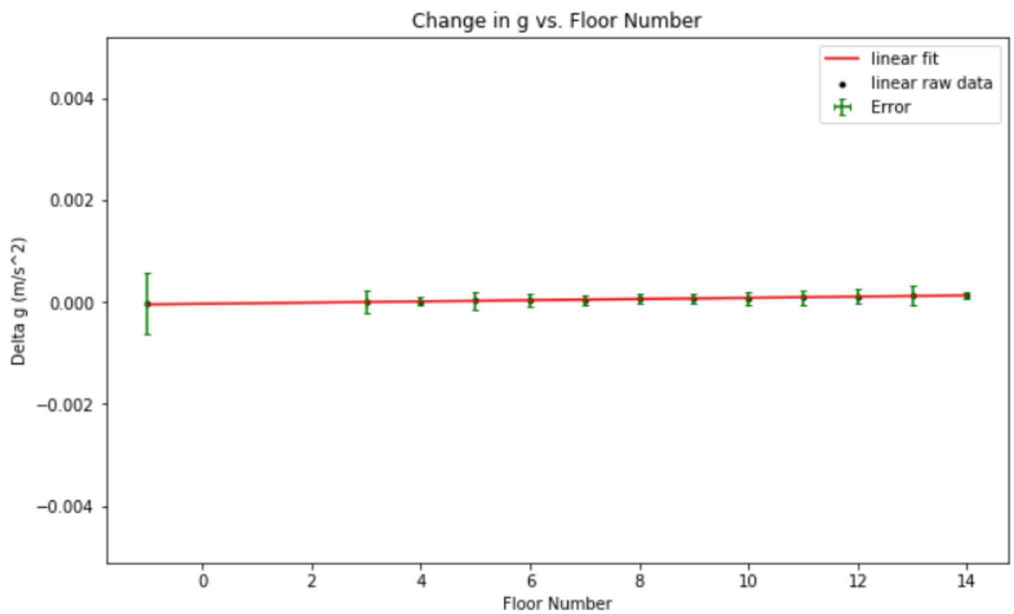


**Figure 2:** $\Delta g$ *plotted against floor number for floors 3 to 14 (and basement) in Burton Tower with a linear regression.*

# 6 Discussion

Figure 1 shows that the relationship between $\Delta g$ and $\Delta R$ is roughly linear, however the linear regression proved to be a poor fit to the data as the chi-squared analysis showed that $\chi^2$ = 2.2 x $10^5$. The estimated radius of the Earth from this fit was determined to be $R$ = 6612.8 ± 0.1 km. Similarly, using a data set which includes the basement and 14th floor measurements we found that: $\chi^2$ = 3.8 x $10^5$ and $R$ = 6594.5 ± 0.1 km. These estimated $R$ values are quite different from the expected value of 6371 km and we believe these stem from large errors and uncertainties in the experiment. Firstly, the instrument was calibrated on each floor manually and is therefore prone to variations. In addition, the mass and height of each floor of the Burton Tower is not necessarily consistent throughout all of the floors and could lead to slight variations in the data set. The experiment could be improved by automating the calibration system to be more precise and by performing the experiment in a more ideal location with a uniform distribution of mass.

The uncertainty in the experiment first arises from error in the experimental values of $g$. This uncertainty is taken to be the maximum between the statistical uncertainty on each floor (standard deviation between data sets) and the error of accuracy of the instrument (0.1 counter div). Furthermore, when computing $R$, the error is propagated as:

$$Error_R = R \times \sqrt{\left(\frac{Error_g}{g}\right)^2 + \left(\frac{Error_a}{a}\right)^2}$$

Where $a$ is the estimated slope parameter from the linear regressions and its uncertainty, $Error_a$, is taken to be the square of the first entry in the resulting covariance matrix.

## 7 Numbered Questions

### Question 1:

We can compute the change in $g$ value due to the sun at noon versus midnight by doing:

$$|g_{noon} - g_{midnight}| = \frac{GM_{sun}}{R_{sun\ to\ earth}^2} - \frac{GM_{sun}}{(R_{sun\ to\ earth} + 2R_{earth})^2} \cong 5.0\ x\ 10^{-7}$$

Therefore we see that this effect is extremely miniscule.

### Question 2:

Going up one floor (~3.95m) removes approximately $10^6$ kg of mass from above you and places it below you. The magnitude of this effect on the $g$ value can be computed to be:

$$|g_1 - g_0| = \frac{G\left(M_{earth} + M_{floor}\right)}{\left(R_{earth} + R_{floor}\right)^2} - \frac{GM_{earth}}{R_{earth}^2} \cong 2.2\ x\ 10^{-5}$$

This effect is thus deemed small enough to be negligible in the data set.

### Question 3:

This deviation tells us that there are anomalies in terms of mass in these locations. As mentioned earlier, the mass of the roof is slightly less than other floors as well as the fact that the basement is missing mass and has a non-standard ceiling. These factors theoretically cause measurements of g at these locations to slightly deviate from the linear relationship we expect from the other floors (3 to 13).

### Question 4:

The centrifugal force due to the Earth's rotation will reduce the amount of gravitational force, but we can calculate the centripetal acceleration, hence the centrifugal acceleration of objects on the Earth due to its own rotation. We know that centrifugal force obeys inverse square law but since the height of the tower we climbed is so small compared to the radius of the earth, the deviation from this factor should almost be irrelevant.

**Question 5:**

The Earth's eccentricity refers to how much its orbit around the Sun deviates from a perfect circle. The Earth has an eccentricity about 0.0167, and thus the difference in $g$ values at perihelion versus aphelion can be computed to be:

$$|g_{aphelion} - g_{perihelion}| = \frac{GM_{sun}}{R_{aphelion}^2} - \frac{GM_{sun}}{R_{perihelion}^2} \cong 4.0\ x\ 10^{-4}$$

**Question 6:**

Our result ended up being less accurate than Eratosthenes prediction. However, as stated earlier we believe this is due to large sources of error in the experiment itself. Based on the expected accuracy from this instrument (1 or 2% uncertainty), the error from an ideal version of this experiment should be within 6 to 13 km of the true radius of the Earth. This would be a very large improvement over Eratosthenes' prediction.

## 8 Conclusion

The outcome of our experiment showed a very steadily varying value for $g$, which means that the value $\Delta g$ in our case is very small and almost constant throughout the 3rd to 13th floor. The radius of the Earth then follows from the slope of our linear fit and we find that $R$ = 6612.8 ± 0.1 km. Additionally, if the basement and 14th floor are included, the estimated parameter is then $R$ = 6594.5 ± 0.1 km. These values are quite a bit off the expected value of 6371 km. This is believed to be due to sources of error from the experimental setup as well as a poor linear regression due to too few data points.

## 9 References


[1] Pandhi, A. and Chen, X., University of Toronto, Toronto, ON. "PHY224 Laboratory Notes: Radius of the Earth", November 2018.

# 10 Appendices

## 10.1 Python Code

The full code used for this lab can be found below as well as on the author's Github as "Radius of the Earth.py": https://github.com/AyushPandhi/Pandhi_Ayush_PHY224/tree/master/Radius%20of%20Earth.

```
#Radius of the Earth
#Author: Ayush Pandhi (1003227457)
#Date: 11/12/2018

#Importing required modules
import numpy as np
import matplotlib.pyplot as plt
from scipy.optimize import curve_fit

#Defining the linear model function
def f(x, a, b):
    return a*x + b

#Loading the data for all 3 runs
floor = np.loadtxt('Radius of Earth Set 1.txt', skiprows=2, usecols=(0,))
mgals1 = (1000)*(0.10023)*(np.loadtxt('Radius of Earth Set 1.txt', skiprows=2, usecols=(1,)))
mgals2 = (1000)*(0.10023)*(np.loadtxt('Radius of Earth Set 2.txt', skiprows=2, usecols=(1,)))
mgals3 = (1000)*(0.10023)*(np.loadtxt('Radius of Earth Set 3.txt', skiprows=2, usecols=(1,)))

#Getting a new data set of the averages and setting error to standard deviation
mgals_mean = np.empty(11,)
mgals_std = np.empty(11,)
for i in range(len(mgals_mean)):
    a = np.array([mgals1[i], mgals2[i], mgals3[i]])
    mgals_mean[i] = np.mean(a)
    mgals_std[i] = np.std(a)

g = 9.81 - mgals_mean/100000
delg = np.empty(11,)
for i in range(len(delg)):
    delg[i] = g[i] - g[0]
g_error = 1000*mgals_std*0.10023/100000

#Linear regression
p_opt_1, p_cov_1 = curve_fit(f, floor, delg, (0, 0), g_error, True)
lin_output = f(floor, p_opt_1[0], p_opt_1[1])
print('The two estimated parameters: ', p_opt_1[0],  p_opt_1[1])
```

```
#Plots of linear regression
plt.figure(figsize=(10,6))
plt.scatter(floor, delg, label = 'linear raw data', marker='.', color='k')
plt.plot(floor, lin_output, 'r-', label = 'linear fit')
plt.title('Change in g vs. Floor Number')
plt.xlabel('Floor Number')
plt.ylabel('Delta g (m/s^2)')
plt.errorbar(floor, delg, xerr=0, yerr=g_error, linestyle='none', ecolor='g', label='Error', capsize=2)
plt.legend()
plt.show()

#Calculating chi squared
chi_sq = (1/9)*(np.sum(((delg - lin_output) / g_error)**2))
print('Chi squared for linear regression: ', chi_sq)

#Estimating the radius of Earth using linear portion of data
r = 2*(np.mean(g))/p_opt_1[0]
r = 3.95*r/1000
print('Estimated Radius in km: ', r)
print('Error in Radius result: ', r*((p_cov_1[0,0]**2/p_opt_1[0])**2 + (np.max(g_error)/np.mean(g))**2)**0.5)

#Using measurements from the basement and 14th floor
mgals_anom = (1000)*(0.10023)*(np.loadtxt('Radius of Earth Anomalies Set.txt', skiprows=2, usecols=(1,)))
mgals_anom14, mgals_anomB = np.hsplit(mgals_anom, 2)
mgals_anom14_mean = np.mean(mgals_anom14)
mgals_anom14_std = np.std(mgals_anom14)
mgals_anomB_mean = np.mean(mgals_anomB)
mgals_anomB_std = np.std(mgals_anomB)

g_14 = 9.81 - mgals_anom14_mean/100000
delg_14 = g_14 - g[0]
g_14error = 1000*mgals_anom14_std*0.10023/100000

g_B = 9.81 - mgals_anomB_mean/100000
delg_B = g_B - g[0]
g_Berror = 1000*mgals_anomB_std*0.10023/100000

floor_anom = np.hstack((-1, floor, 14))
g_anom = np.hstack((g_B, g, g_14))
delg_anom = np.hstack((delg_B, delg, delg_14))
g_error_anom = np.hstack((g_Berror, g_error, g_14error))
```

```
#Linear regression
p_opt_1, p_cov_1 = curve_fit(f, floor_anom, delg_anom, (0, 0), g_error_anom, True)
lin_output = f(floor_anom, p_opt_1[0], p_opt_1[1])
print('The two estimated parameters: ', p_opt_1[0],  p_opt_1[1])

#Plots of linear regression
plt.figure(figsize=(10,6))
plt.scatter(floor_anom, delg_anom, label = 'linear raw data', marker='.', color='k')
plt.plot(floor_anom, lin_output, 'r-', label = 'linear fit')
plt.title('Change in g vs. Floor Number')
plt.xlabel('Floor Number')
plt.ylabel('Delta g (m/s^2)')
plt.errorbar(floor_anom, delg_anom, xerr=0, yerr=g_error_anom, linestyle='none', ecolor='g', label='Error', capsize=2)
plt.legend()
plt.show()

#Calculating chi squared
chi_sq = (1/11)*(np.sum(((delg_anom - lin_output) / g_error_anom)**2))
print('Chi squared for linear regression: ', chi_sq)

#Estimating the radius of Earth using linear portion of data
r = 2*(np.mean(g_anom))/p_opt_1[0]
r = 3.95*r/1000
print('Estimated Radius in km: ', r)
print('Error in Radius result: ', r*((p_cov_1[0,0]**2/p_opt_1[0])**2 + (np.max(g_error)/np.mean(g))**2)**0.5)
```