PyLab 1 Responses to the Lab Questions

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The following are direct responses to the numbered questions provided throughout the experiment.

Question 1:
Using Newton's 2nd law of motion F = ma, and equating it with the Hooke's force, by doing this we are assuming ideal situation, i.e. no external resistance force on the moving body**.** Hence:

$$m\,\mathrm{a} = -ky$$

where m is the mass of the object and a is its acceleration, k is the spring constant and x is the mass' displacements from the equilibrium position. Now if we set x(t) as the mass' position of time, then:

$$a(t) = \frac{d^2y}{dt^t} = y''(t)$$

the first equation then becomes:

$$m\,y'' + ky = 0 \text{ (or } m\frac{d^2y}{dt^t}t\; + ky = 0)$$

Dividing through by m we get:

$$\frac{d^2y}{dt^t}\; + \frac{k}{m}y = 0$$

Question 2:

$$m\frac{d^2y}{dt^t}\; + ky = 0$$

$$\frac{dp}{dt} = m\frac{dv}{dt} = F$$

$$\frac{p(t+\Delta t)-p(t)}{\Delta t} = m\frac{v(t+\Delta t)-v(t)}{\Delta t} = \; m\frac{v(t_{i+1})-v(t_i)}{\Delta t} = F$$

$$m\frac{v(t_{i+1})-v(t_i)}{\Delta t} + ky_i = 0$$

$$\frac{v(t+\Delta t)-v(t)}{\Delta t} + \frac{k}{m}y_i = 0$$

$$v_{i+1} - v_i \;=\; (-\frac{k}{m}y_i)\Delta t \;=\; (-\omega_0^2 y_i)\Delta t$$

$$q(t+\Delta t) = \; q(t) + \;\frac{\Delta t}{m}p_i = q(t) + \;\frac{\Delta t}{m}mv_i = q(t) + \;\Delta t v_i$$

$$q(t_{i+1}) - \; q(t_i) = \Delta t v_i = q_{i+1} \;- q_i = \; y_{i+1} \;- y_i$$

Question 3:

Using the fact that $k = m\omega_0{}^2$, we can immediately solve for the spring constant in both experiments:

k_{damped} $= 15.4 \pm 0.12$ N/m

$k_{undamped}$ $= 14.8 \pm 0.12$ N/m

Question 4:

The amplitudes of the position vs time and velocity vs time graphs are increasing almost exponentially using the Forward Euler method, this is happening because the error in approximation accumulated. Since the method uses a time step Δt and so in the recursive formula we proved above, the t_{i+1} term increases at the rate of Δt^i. This matches what the graphs are showing.

Question 5:

The energy plot is again an exponential graph since we know that the energy of a system whose motion follows sine/cosine waves is proportional to the amplitude squared, so the exponentially increasing error in the position vs time graph will also have a similar impact on the energy plot.

Question 6:

The phase plot can be thought of the phase portrait of the solutions of a 2nd order ODE system.

$\begin{cases} x'' + k_1 y' = 0 \\ y'' + k_2 y = 0 \end{cases}$ and the solutions to these systems are complex exponentials. These linear systems usually only have purely imaginary eigenvalues, i.e. $\pm i\sqrt{k}$. Therefore, the phase plots are ellipses with semi major and semi minor axis k_1, k_2 depending on which one is larger. But with an exponentially increasing position/velocity function, the system's solution picks up a time dependent phase factor e^{rt} for some real r. Hence the phase portrait will still pertain its elliptic shape but its radius will grow outwards due to the exponential term. Which exactly matches the Forward Euler method plots.

Question 7:

$y(t + \Delta t)$ = $y(t)$ + $\Delta t \frac{dy}{dt}$ + $\frac{1}{2}\frac{d^2y}{dt^2}$, $\frac{1}{2}\frac{d^2y}{dt^2} = \frac{1}{2m}F(t)$

Since the last term is then the second derivative, which have yet another Δt in the F term. So, each time step actually accounts for 2 times of the increment in time.

Question 8:

The energy plot was close to being a straight line after we used symplectic method, and the phase plot appeared to be a small ellipse, which also follows from the discussion in question 6, the amplitudes have not shown exponential increase, hence the total energy is relatively constant.

Question 9:

Using the fact that $Re = \frac{\rho v l}{\eta}$, we can immediately solve for Reynold's Number in our system:

Re = 3050 $\pm$ 0.0025 $\approx$ 3050 with a negligible error margin. However this is with the initial (max) velocity of the system and Re drops to under 2300 quickly so we still assume the system is laminar.

Question 10:

Using the fact that $\gamma = \frac{2}{\tau}$, where we have defined τ as the time it takes for the amplitude to reach $\frac{A_0}{e}$, we can immediately solve for γ:

$\gamma = 0.025 \pm 0.0003\ s^{-1} \approx 0.025$ with negligible error.

Question 11:

We are assuming underdamped motion for the system; therefore, it must be the case that the position vs time graph be a sinusoidal wave again, but sandwiched between two exponential decay limit curves, $y = \pm A_0 e^{-\gamma t}$. Where A_0 is the undamped amplitude. Which again causes the energy to decrease exponentially.

Question 12:

Program plots use a much smaller time increment for numerical estimation, but our motion sensor only has 100 sample/s rate, which is much less than the amount of data that the program can generate. Additionally, experiment has more variables such as mass does not oscillate in straight line, apparatus has different sources of errors. Hence the graph from experimental data definitely does not look as smooth as the program generated one.