Photoelectric Effect (Weight 2)

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1 Abstract

In this experiment, we find that the stopping voltage scales linearly with frequency and estimate the following parameters: h = 5.4 x 10^{-34} ± 2.3 x 10^{-52} m^2kg/s, E_0 = 1.7 x 10^{-19} ± 8.0 x 10^{-23} J and f_0 = 3.2 x 10^{14} ± 1.5 x 10^{11} Hz. The relationship between intensity of light and photocurrent is also found to be linear while the stopping voltage does not change with intensity. Then, the ideal time needed by an electron to absorb enough energy to escape a photocathode is computed to be approximately 1.0 x 10^{-2} seconds. This is compared directly with the experimental time constant which was found to be 150 microseconds.

2 Introduction

We attempt to verify our findings with previous results and theoretical predictions of the photoelectric effect. The effect itself describes the emission of electrons when an incident light, with sufficient energy per photon, shines on the material. Theoretically, there exists a linear relationship between the maximum kinetic energy of a photoelectron and the electron energy inside the metal:

$$K_{max} = E_{elec} - E_0, \ \ with \ V_{stop} = \frac{K_{max}}{e}$$

Where E_0 is the work function, V_{stop} is the stopping voltage and e is the energy of an electron (1.6 x 10^{-19} J). Later Einstein would postulate the quantization of electromagnetic radiation such that, $E_{elec} = hf$, with frequency f and h being Planck's constant (6.6 × 10^{-34} m^2kg /s). This simplifies the above equation:

$$eV_{stop} = hf - E_0 \ \ \Longrightarrow \ \ V_{stop} = \frac{h}{e}(f - \ f_0)$$

Where f_0 is defined to be the cut-off frequency. Throughout the experiment we will specifically attempt to analyze the relationship between the stopping voltage and frequency of light (which we expect to have a linear relationship) and from there estimate h, E_0 and f_0. Additionally, the relationship between intensity of light versus photocurrent and stopping voltage is also considered; theoretically the photocurrent should scale linearly with intensity while the stopping voltage is not affected. Finally, the time needed for an electron to escape the photocathode is computed and compared with the experimental time constant.

3 Methods and Materials

The instruments used for this experiment are: a power supply, 8 different LEDs (varying from 390nm to 935nm), a phototube, two multimeters, an oscilloscope and a wave generator. The Jupyter Notebooks software was used to analyze the data.

4 Experimental Procedure

Exercise 1: The UV LED (390nm) was inserted into the power supply socket and the diode head was aligned with the phototube window. Both multimeters are connected as voltmeters to measure photocurrent and stopping voltage. The power supply and potentiometer are turned on and the photocurrent is decreases to approximately zero. At this point the stopping voltage, V_{stop}, is measured. The errors are estimated for both parameters and the entire process is repeated for the remaining seven LEDs.

Exercise 2: The variable intensity LED was inserted into the power supply socket and the stopping voltage is measured similar to exercise 1 for all the different intensity settings of the LED. Photocurrent is also measured independently with the potentiometer turned off for all the different intensity settings.

Exercise 3: The phototube was connected to Ch1 of the oscilloscope through a rectifying adaptor and the power supply is not turned on. The wave generator was then connected to the oscillator-driven LED as well as Ch2 of the oscilloscope. The wave generator was set to a square-wave frequency (kHz) and amplitude in the middle range. The oscilloscope is adjusted to correctly view 1-2 periods of the oscillation. From this setup, the transient photocurrent was measured as a function of time and the time constant was estimated.

5 Results

The acquired data from exercise 1 gives wavelength, λ, in nanometers and stopping voltage, V_{stop}, in volts. Plotting V_{stop} vs. frequency of light allows us to study the relationship as well as compute useful results. This relationship ideally follows:

$$K_{max} = E_{elec} - E_0 \quad \longrightarrow \quad eV_{stop} = hf - E_0$$

Here frequency is plotted against the stopping voltage for each of the LEDs as well as a linear regression performed on the set of data. From this plot we find that there is a general linear trend within the data set, however many of the points do not fit the linear regression within their margin of error.

Additionally, it should be noted that the infrared LED is not included in the data set as it did not have enough energy to generate a photocurrent and thus the stopping voltage could not be measured.

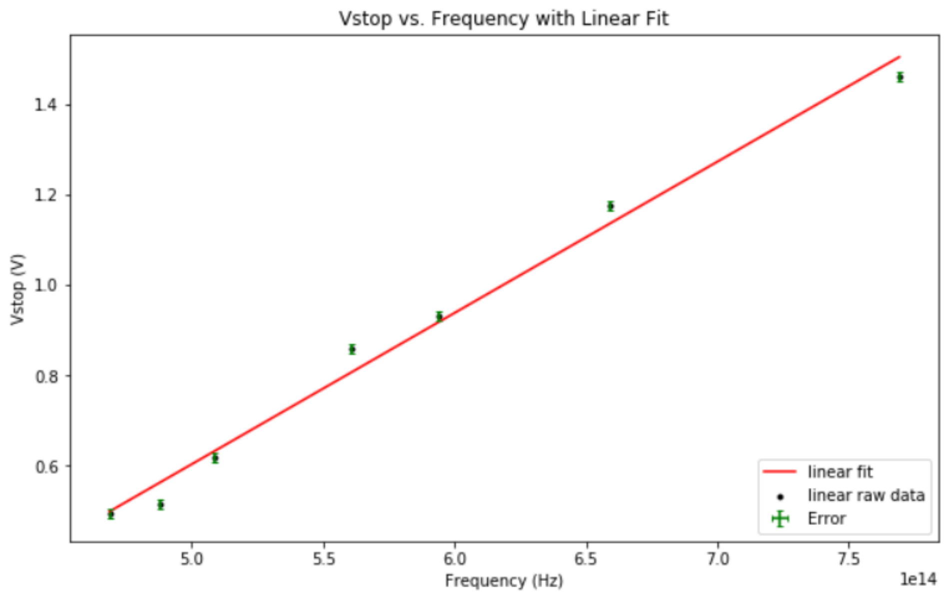

Figure 1: *Stopping voltage plotted against frequency of various LED lights with a linear regression.*

Applying a statistical analysis on this linear regression, it is found that χ^2 = 17.2. This suggests that the linear model is a fairly reasonable fit to the data which is perhaps a little incomplete. This could ideally be corrected by including more data points within the set.

From this linear regression, Planck's constant (h), the Work function (E_0) and Cut-off frequency (f_0) were estimated using the returned slope and y-intercept from the curve fit function. The results are as follows: h = 5.4 x 10^{-34} ± 2.3 x 10^{-52} m^2kg/s, E_0 = 1.7 x 10^{-19} ± 8.0 x 10^{-23} J and f_0 = 3.2 x 10^{14} ± 1.5 x 10^{11} Hz.

The data set for exercise 2 gives the photocurrent (amps) and stopping voltage (volts) individually with the four possible intensity levels of the variable intensity LED (lowest intensity is level 1 and highest is level 4). First, plotting the stopping voltage against intensity level shows a constant function; implying that the stopping voltage parameter does not change with intensity. Visually, the linear regression seems to agree well with the data; there is also no outlier points for the fit. The statistical analysis however outputs χ^2 = 0.02, which is primarily due to the fact that there are only four points in this data set and thus it is an over-fit.

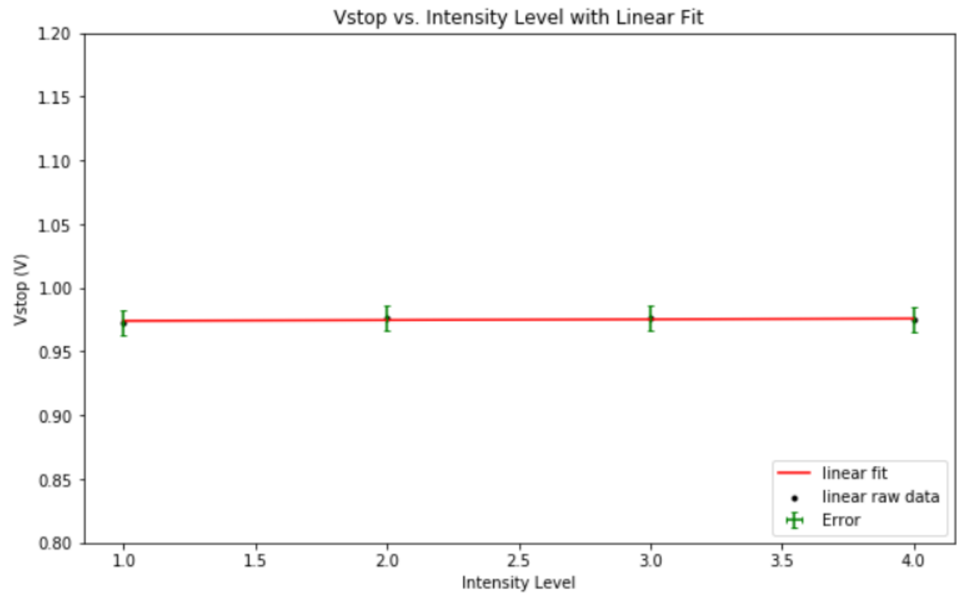

Figure 2: *Stopping voltage plotted against intensity level of the variable intensity LED with a linear regression.*

Furthermore, when plotting photocurrent against the intensity level, it is found that this parameter increases as intensity increases. The relationship seems to be roughly linear, although the linear fit seems to be a poor approximation overall as none of the points match the fit within their margin of error. Performing the same analysis as before provides χ^2 = 107.4, which again bolsters the idea that a linear relationship might be a poor fit for the data set. It is again hypothesized that this could be caused by a very small amount of data points in the set and a possible solution might be to increase the total number of sampled points.

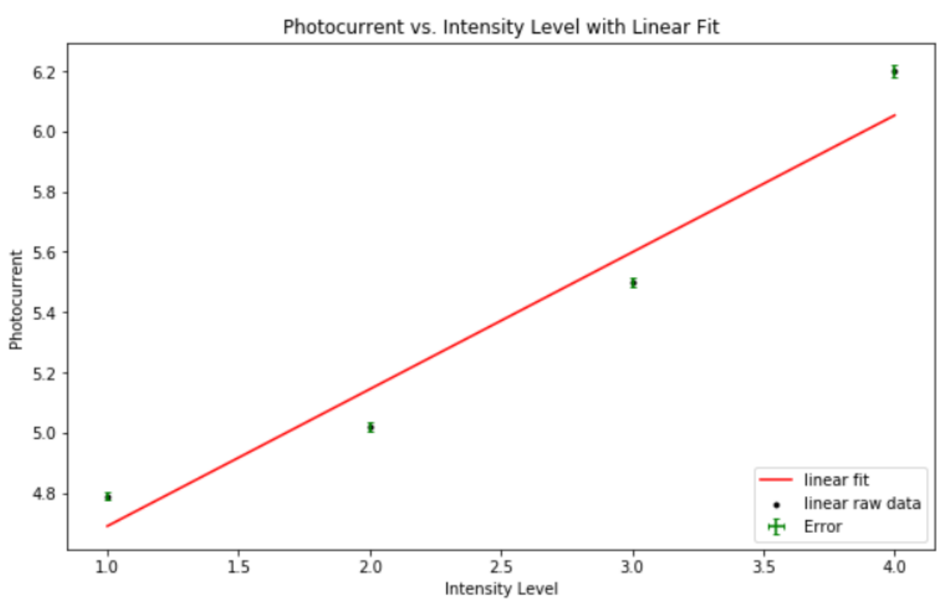

Figure 3: *Photocurrent plotted against intensity level of the variable intensity LED with a linear regression.*

In exercise 3, the transient photocurrent is measured as a function of time on an oscilloscope. The resulting time constant, which is found as the time taken for the amplitude of the transient photocurrent to drop to A/e, is approximately 150 microseconds. Additionally, the energy absorbed by each electron per second is given by:

$$P_e = P_{LED}\frac{A_e}{A_{PC}}$$

Where P_{LED} = 60 mW, A_{PC} = 3.23 cm^2, A_e ~ 0.09 nm^2. The result gives that P_e ~ 1.67 x 10^{-17} W, which means each electron absorbs 1.67 x 10^{-17} joules of energy per second. Using the work function, E_0, computed in exercise 1, the time needed by an electron to absorb enough energy to escape the photocathode is estimated to be approximately 1.0 x 10^{-2} seconds (10000 microseconds). This discrepancy between the observed and expected time constant values is discussed further in the next section.

6 Discussion

The results of exercise 1 show that the relationship between the stopping voltage and frequency of light is roughly linear. The linear regression proved to be a somewhat reasonable fit as well with χ^2 = 17.2. The estimated parameters that were computed from this regression include: h = 5.4 x 10^{-34} ± 2.3 x 10^{-52} m^{2}kg/s, E_0 = 1.7 x 10^{-19} ± 8.0 x 10^{-23} J and f_0 = 3.2 x 10^{14} ± 1.5 x 10^{11} Hz. Comparing to expected results we find that the estimated Planck constant is very close to the expected value of 6.6 × 10^{-34} m^{2}kg /s. Although there is no direct comparison for the other two parameters, they are of the correct magnitude for a work function and cut-off frequency in relation to this problem. Additionally, the fact that infrared light did not have sufficient energy to generate a photocurrent is also accurate with the expected results as seen by the cut-off frequency (f_{IR} ~ 3.21 x 10^{14} Hz > 3.2 x 10^{14} Hz).

The primary conclusions from exercise 2 are that: stopping voltage does not scale with intensity but photocurrent does seem to scale linearly with it. This follows the theoretical predictions as intensity increasing means more photons being emitted. However, stopping voltage depends on the energy of each individual photon (i.e. its wavelength/frequency) and thus it does not matter how many photons are emitted, rather that each one has sufficient energy to cause an electron emission. On the other hand the photocurrent increases because having more photons (of sufficient energy) results in more electrons being emitted in the system. The statistical analysis of these linear regressions showed: χ^2 = 0.02 (Figure 2) and χ^2 = 107.4 (Figure 3), which indicates that the linear model is not a particularly good fit for these data sets. This result is theorized to be due to the extremely small size of these data sets and that a poor χ^2 fit in this case is not necessarily indicative of the linear model being a poor fit for a much larger sample size.

In exercise 3 it was found that the energy absorbed by each electron per second is roughly P_e ~ 1.67 x 10^{-17} W. Combining this with the work function from earlier, the time needed by an electron to absorb enough energy to escape the photocathode is estimated to be approximately 1.0 x 10^{-2} seconds. In comparison, the experimental time constant is roughly 150 microseconds. There is a clear discrepancy between the experimental result and expected value in this case which is attributed to the experiment being non-ideal; since it is not realistic to expect all of the power consumed by the oscillator-driven LED to be converted to light.

The uncertainty in experiment first arises from error in the experimental values of V_{stop} and λ. The uncertainty of V_{stop} is taken to be the maximum between the error of accuracy (0.1% of reading) and error of precision (taken as the last digit available with value 1) for the multimeter. From the uncertainty in λ, the uncertainty in frequency is propagated using the rule for powers and multiplication by a constant:

$$\frac{\delta Q}{|Q|} = |n|\frac{\delta x}{|x|}\ (Rule\ for\ propagting\ error\ for\ power\ n)$$

$$\delta Q = |A|\delta x\ (Rule\ for\ propagating\ error\ for\ multiplication\ by\ some\ constant\ A)$$

Similarly for the computed results for h, E_0 and f_0, the errors originally stem from the covariance matrix values output by the curve fit for the slope and y-intercept parameters. The same rules as above are then used to propagate the errors correctly. The uncertainty in the experimental photocurrent for exercise 2 is dealt with in the same fashion as the experimental V_{stop} values. The only difference being that the error of accuracy in this case is 0.3% of the reading; the error of precision is the same as for V_{stop}.

The strength of this experimental design is that is it quite simple to set up and replicate the results. Additionally, the uncertainties mostly stem from the error of accuracy and precision of the electronic equipment, which tend to be rather small. One clear improvement to the experiment is to gather a much larger sample size for each respective data set to generate a more accurate linear fit and to overall better analyze the relationship between some of the previously mentioned parameters. This would be done by adding a larger variety of LED options as well as more variations in intensity level for the variable intensity LED.

7 Conclusion

The outcome of the experiment shows that the relationship between the stopping voltage and frequency of light is roughly linear and the estimated results from the linear relationship are: h = 5.4 x 10^{-34} ± 2.3 x 10^{-52} m^2kg/s, E_0 = 1.7 x 10^{-19} ± 8.0 x 10^{-23} J and f_0 = 3.2 x 10^{14} ± 1.5 x 10^{11} Hz, which agree with the expected outcomes. It was also found that photocurrent scales somewhat linearly with intensity while stopping voltage does not scale with intensity at all. Lastly, the experimental time constant was approximated to be 150 microseconds. In comparison the ideal time needed by an electron to escape the photocathode was computed to be roughly 1.0 x 10^{-2} seconds.

8 References

[1] Pandhi, A. and Chen, X., University of Toronto, Toronto, ON. "PHY224 Laboratory Notes: Photoelectric Effect, November 2018.

9 Appendices

9.1 Python Code

The full code used for this lab can be found below as well as on the author's Github as "Photoelectric Effect.py":
https://github.com/AyushPandhi/Pandhi_Ayush_PHY224/tree/master/Photoelectric%20Effect.

```
#Photoelectric Effect
#Author: Ayush Pandhi (1003227457)
#Date: November 22, 2018

#Importing required modules
import numpy as np
import matplotlib.pyplot as plt
from scipy.optimize import curve_fit

#Defining the linear model function
def f(x, a, b):
    return a*x + b

#Loading Data Sets for Exercise 1
wavelength = (10**(-9))*(np.loadtxt('PE Effect Ex1 1.txt', skiprows=2, usecols=(0,)))
vstop = (np.loadtxt('PE Effect Ex1 1.txt', skiprows=2, usecols=(1,)))

#Loading Data Sets for Exercise 2
intensitylvl = np.loadtxt('PE Effect Ex2 1.txt', skiprows=2, usecols=(0,))
photocurrent = np.loadtxt('PE Effect Ex2 1.txt', skiprows=2, usecols=(1,))
vstop2 = np.loadtxt('PE Effect Ex2 1.txt', skiprows=2, usecols=(2,))
```

```
#Computing frequency from wavelength
frequency = (3*10**8)/wavelength

#Estimated errors from equipment
v_error = np.empty(len(vstop))
for i in range(len(vstop)):
   v_error[i] = max(vstop[i]*0.0010, 0.01)

#Linear regression
p_opt_1, p_cov_1 = curve_fit(f, frequency, vstop, (0, 0), v_error, True)
lin_output = f(frequency, p_opt_1[0], p_opt_1[1])

#Calculating chi squared
chi_sq = (1/5)*(np.sum(((vstop - lin_output)/v_error)**2))
print('Chi squared for linear regression: ', chi_sq)

#Plot of Vstop vs Frequency
plt.figure(figsize=(10,6))
plt.scatter(frequency, vstop, label = 'linear raw data', marker='.', color='k')
plt.plot(frequency, lin_output, 'r-', label = 'linear fit')
plt.title('Vstop vs. Frequency with Linear Fit')
plt.xlabel('Frequency (Hz)')
plt.ylabel('Vstop (V)')
plt.errorbar(frequency, vstop, xerr=0, yerr=v_error, linestyle='none', ecolor='g', label='Error', capsize=2)
plt.legend(loc='lower right')
plt.show()

#Outputting Planck's Constant h
h = p_opt_1[0]*(1.6*10**(-19))
h_error = p_cov_1[0,0]*(1.6*10**(-19))
print('Estimated Plancks Constant: ', h, '+/-', h_error)

#Outputting the Work Function
wf = -p_opt_1[1]*(1.6*10**(-19))
wf_error = p_cov_1[1,1]*(1.6*10**(-19))
print('Estimated Work Function: ', wf, '+/-', wf_error)

#Outputting the cut-off frequency
f0 = -(1.6*10**(-19))*p_opt_1[1]/h
f0_error = p_cov_1[1,1]*(1.6*10**(-19))/h
print('Estimated Cut-off Frequency: ', f0, '+/-', f0_error)

#Estimated errors from equipment
```

```
v_error2 = np.empty(len(vstop2))
for i in range(len(vstop2)):
    v_error2[i] = max(vstop2[i]*0.0010, 0.01)

i_error = np.empty(len(photocurrent))
for i in range(len(photocurrent)):
    i_error[i] = max(photocurrent[i]*0.0030, 0.01)

#Linear regression
p_opt_2, p_cov_2 = curve_fit(f, intensitylvl, vstop2, (0, 0), v_error2, True)
lin_output2 = f(intensitylvl, p_opt_2[0], p_opt_2[1])

#Linear regression
p_opt_3, p_cov_3 = curve_fit(f, intensitylvl, photocurrent, (0, 0), i_error, True)
lin_output3 = f(intensitylvl, p_opt_3[0], p_opt_3[1])

#Calculating chi squared
chi_sq = (1/2)*(np.sum(((vstop2 - lin_output2)/v_error2)**2))
print('Chi squared for linear regression: ', chi_sq)

#Plot of Vstop vs Intensity Level
plt.figure(figsize=(10,6))
plt.scatter(intensitylvl, vstop2, label = 'linear raw data', marker='.', color='k')
plt.plot(intensitylvl, lin_output2, 'r-', label = 'linear fit')
plt.title('Vstop vs. Intensity Level with Linear Fit')
plt.xlabel('Intensity Level')
plt.ylabel('Vstop (V)')
plt.errorbar(intensitylvl, vstop2, xerr=0, yerr=v_error2, linestyle='none', ecolor='g', label='Error',
capsize=2)
plt.ylim(0.8, 1.2)
plt.legend(loc='lower right')
plt.show()

#Calculating chi squared
chi_sq = (1/2)*(np.sum(((photocurrent - lin_output3)/i_error)**2))
print('Chi squared for linear regression: ', chi_sq)

#Plots of Photocurrent vs Intensity Level
plt.figure(figsize=(10,6))
plt.scatter(intensitylvl, photocurrent, label = 'linear raw data', marker='.', color='k')
plt.plot(intensitylvl, lin_output3, 'r-', label = 'linear fit')
plt.title('Photocurrent vs. Intensity Level with Linear Fit')
plt.xlabel('Intensity Level')
plt.ylabel('Photocurrent')
```

```
plt.errorbar(intensitylvl, photocurrent, xerr=0, yerr=i_error, linestyle='none', ecolor='g', label='Error',
capsize=2)
plt.legend(loc='lower right')
plt.show()
```