Electron Spin Resonance (Weight 2)

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1 Abstract

In this experiment we generate an external magnetic field using two coils and measure the absorption for three different coils containing a small case of diphenylpicryl hydrazyl (DPPH). We find that the peak current and corresponding frequency have a roughly linear relationship, although the linear regression is a poor fit to the data. Additionally, the average gyromagnetic ratio is computed to be: 1.240 x 10^{11} ± 1.6 x 10^{9} (double coil), 1.195 x 10^{11} ± 1.6 x 10^{9} (long coil) and 1.220 x 10^{11} ± 1.6 x 10^{9} (short coil). From this the average Landé g factor values are found to be: 1.411 ± 0.019 (double coil), 1.361 ± 0.018 (long coil), 1.389 ± 0.018 (short coil). The theoretical Landé g factor for a free electron is 2.002 which does not match the results of this experiment but it is reasoned that this discrepancy is due to the idea that this experiment was non-ideal.

2 Introduction

Electron intrinsic spin is the very useful tool for studying electron's behaviors in the presence of external electromagnetic fields. This behaviour is described by:

$$E = -\vec{\mu} \cdot \vec{B} = -\gamma \vec{S} \cdot \vec{B} = \gamma S_z B_z$$

$$for: \ \ v = \frac{1}{2\pi}\gamma B_z \ \ and \ \ B_Z = \left(\frac{4}{5}\right)^{\frac{3}{2}} \frac{\mu_o n I}{R}$$

Where $\vec{\mu}$ is the magnetic moment, $\vec{B}$ is the external magnetic field, γ is the gyromagnetic ratio, $\vec{S}$ is the spin angular momentum, v is the frequency, $\mu_o = 4\pi \, x \, 10^{-7} \ (N/A^2)$, R is the radius of the coils, n is the number of turns in the coil and I is the measured current going into the coils.. Additionally, applying the idea that $S_z = \ \pm\hbar/2$, this gives rise to a difference in energy of:

$$\nabla E = \gamma \hbar B_z$$

It is expected under the assumption that an electron is a uniform sphere with homogeneous charge distribution that the gyromagnetic ration is $e/2m_e$ (energy of an electron over twice its mass). However, experimentally there is a discrepancy which is written in terms of the Landé g factor:

$$g = \frac{\gamma}{e/2m_e}$$

In this lab, we aim to demonstrate the fact that electron spin gives rise to a potential energy difference in the presence of an external magnetic field, due to the different quantum spin numbers of electron. The appropriate electromagnetic frequency will cause absorption to occur and generate a current spike. We aim to show that as the incident frequency increases, a stronger magnetic field will be required to allow absorption, and in particular, it is a linear relation.

3 Methods and Materials

The materials utilized for this experiment were: an ESR basic unit, an adaptor, a frequency counter, a triple output power supply, a standard power supply, a multimeter, a 1 Ohm resistor, an oscilloscope, a pair of Helmholtz coils (320 turns each), a small case of diphenylpicryl hydrazyl (DPPH), three different small copper coils (referred throughout the text as double coil, long coil and short coil with most to least number of turns in that order) and enough wires to connect the system. The data is analyzed using the Jupyter Notebooks software.

4 Experimental Procedure

First, the experimental system was setup according to Figure 1. In particular the distance between the coils was set to equal their radius to create a uniform magnetic field. The Helmholtz coils will produce a magnetic field, B_z, as the AC current flows through the system. The current going into the coils is measured with the multimeter and by looking at the voltage drop across the resistor. The current and frequency of the system is varied and measured at the points where the two peaks on the oscilloscope merge into one (corresponds to peak current at resonance). This was repeated for all three small copper coils.

5 Results

The acquired data is of peak current (A) and the corresponding frequency at resonance (MHz) so by plotting these two parameters for all three copper coils we can determine the nature of the relationship between them. A linear regression (similar to previous labs) is also applied to each data set and from visual analysis it seems that indeed there exists a linear relationship between these two parameters. However for the double coil (most turns) there are quite a few outlier points and the linear fit does not seem to approximate the data as well as it does for the other two coils. It also seems that for less turns in the coil, the linear fit becomes a progressively better approximation.

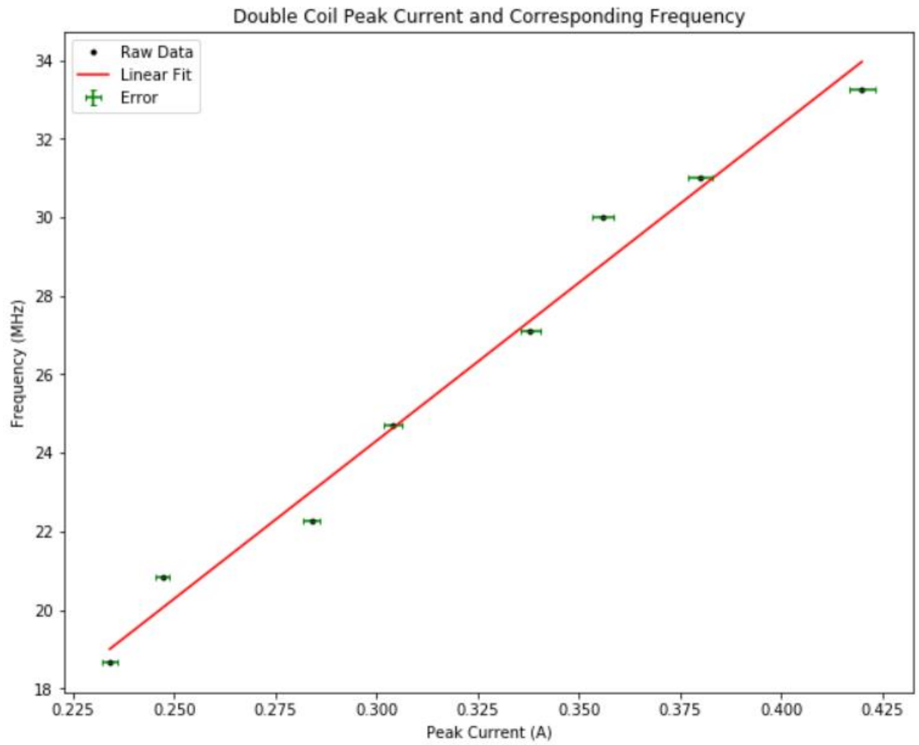

Figure 2: *Double coil peak current plotted against corresponding resonance frequency with linear fit approximation.*

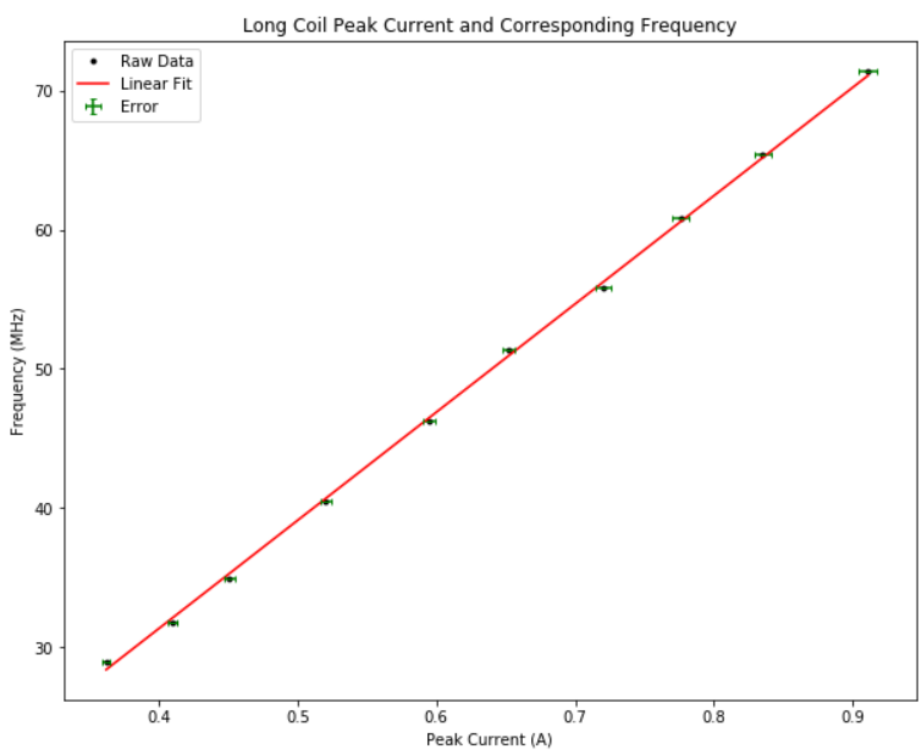

Figure 3: *Long coil peak current plotted against corresponding resonance frequency with linear fit approximation.*

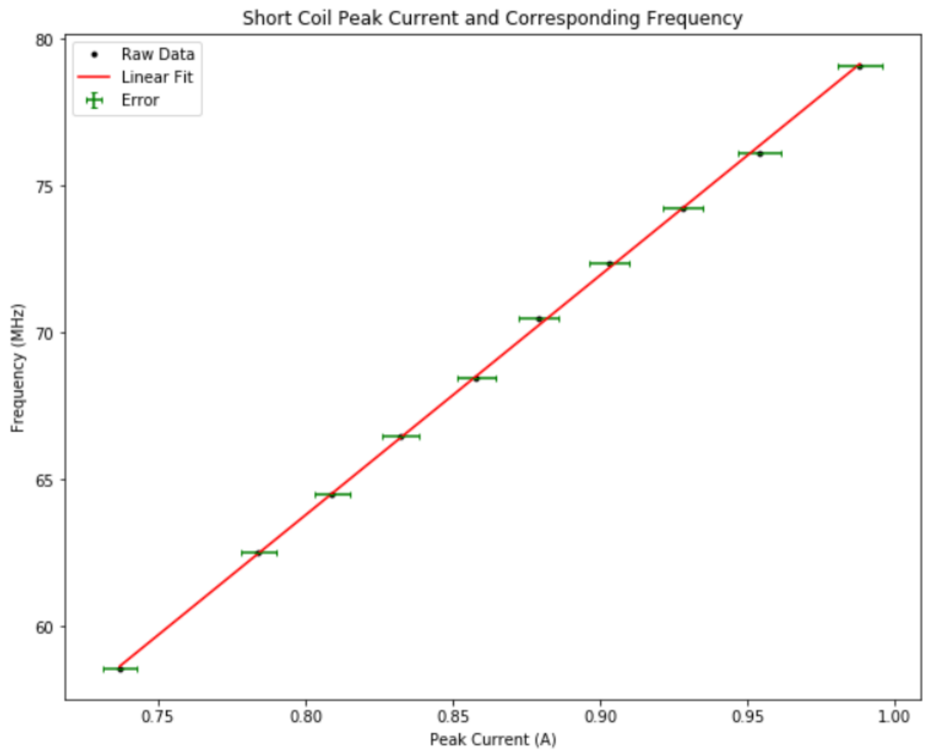

Figure 4: *Short coil peak current plotted against corresponding resonance frequency with linear fit approximation.*

Looking at the χ^2 values it is computed that $\chi^2_{doublecoil}$ = 8.5 x 10^6, $\chi^2_{longcoil}$ = 9.8 x 10^5, and $\chi^2_{shortcoil}$ = 7.5 x 10^4. This suggests that the linear model is a poor fit for the data sets. This can be seen in the plots as there are multiple outliers for which the fit does not lie within the margin of error (especially in Figure 2 and 3).

Since the linear curve fit is a poor approximation of the data, the gyromagnetic ratio and Landé g factor were computed directly from the data points using the equations discussed in sections 2 and 4. The full results for these parameters can be found as tables in the appendix section, however the average gyromagnetic ratio for each coil is: 1.240 x 10^{11} ± 1.6 x 10^9 (double coil), 1.195 x 10^{11} ± 1.6 x 10^9 (long coil) and 1.220 x 10^{11} ± 1.6 x 10^9 (short coil). Furthermore, the average Landé g factors are: 1.411 ± 0.019 (double coil), 1.361 ± 0.018 (long coil), 1.389 ± 0.018 (short coil).

6 Discussion

The graph from the oscilloscope tells us that there is indeed only one specific magnetic field strength that will match the incident electromagnetic wave with a certain frequency because as we tune the current intensity up and down, there is only one current intensity that will cause absorption peaks to merge together. Also, there is a chance that we may have tuned the current to a lower strength, which would still show some absorption, but not the maximum absorption. Hence we avoided this by starting off with a very high large current and turning it down until we see peaks merge together the first time, so that the magnetic field strength we get is actually the correct one. In terms of the plots, we find that in general there is a roughly linear relationship between the peak current and corresponding resonance frequency, however the linear regression proved to be a poor fit to the data as there were multiple outliers and the χ^2 values were very large. Additionally, our computed g factors are slightly lower than the expected result for free electrons which could be due to our experiment being non-ideal since we had a lot of materials interacting with free electrons.

The uncertainty in the experiment first arises from error in the experimental values of R, v and I. The uncertainty for radius is simply the reading error from the measurement device (ruler). For current we take the error to be the maximum between the error of accuracy and error of precision: ± (0.75% of reading) as the error of accuracy and one digit (with position on the last digit and value 1) to be the error of precision. The error in frequency is taken to be ± 10 parts per million (or equivalently 0.001%) as per the frequency counter's instrumental guidelines. Furthermore, when computing B_z, γ and the g factor, the error is propagated as:

$$\Delta B = B \times \sqrt{(\frac{\Delta I}{I})^2 + (\frac{\Delta R}{R})^2}, \Delta R = radius\ uncertainty$$

$$\Delta \gamma = \gamma \times \sqrt{(\frac{\Delta B}{B})^2 + (\frac{\Delta v}{v})^2}, \Delta v = frequency\ uncertainty$$

$$\Delta g = \frac{\Delta \gamma}{e/2m_e}$$

The design of this experiment can be improved in the following ways. First, using a larger coil that will produce a much stronger magnetic field should be able to decrease the fluctuation of the steadiness of the field strength hence we would observe a much leaner graph for absorption on the oscilloscope. Secondly, is that our g factor has a value less than the theoretically predicted 2.0023, we think that since we are eyeballing the current measurement based on when is the time that we see peaks merge together, so we may have neglected the fluctuation in current readings, this can be done by measuring the current value several times after the first measurement was taken, i.e. turning the current away from the actual reading and go back to it again, and this way we can see the different reading that the ammeter gives, which will give us a bigger error range so that the theoretical value may fall into our error range. The same idea also applies to the frequency readings on the frequency counter, we can fix the magnetic field strength and vary the frequency back and forth to see if the same frequency does cause the merge to happen again.

7 Numbered Questions

Question 1:

The longer the coils are, or the more turns that the coils have, the longer it is for signals to travel and hence we predict that highest frequency will be generated from the short coil, a lower frequency by the long coil and the lowest frequency by the double coil.

Question 2:

The asymmetry about the peak current may be due to the fact that our magnetic field produced by the two coils is actually not strong enough to maintain on the exact B-field needed for absorption to happen at all time, and caused photon absorption to have a asymmetric behavior around peak current, although it is still constant enough so that most photons are indeed absorbed.

Question 3:

The relation is indeed linear. Because as the incident magnetic frequency increases to a high level, a much stronger magnetic field would be required to provide the corresponding potential difference of the electrons. In our experiments, or even in daily life, the magnetic field strength created by circuits or small solenoids like the ones we used is actually very tiny, but there do exist strong magnetic field and we can use that to provide the environment for the need of such high frequencies.

Question 4:

The width of the peak corresponds to the range of magnetic field that will cause photon absorption. But since theoretically only specific field strength will match the frequency, the width of the peak is a standard deviation of absorption occurrence, or the spread of absorption occurrence over a small range of different field strength.

Question 5:

The basic unit work as a time-signal emitter and receiver. It sends out electromagnetic waves at a certain frequency through the coils attached to it, and detects the amount of photons returned to the unit, if the absorption happens, then the unit sends out a signal to the oscilloscope for absorption, and if no absorption happens, it sends out a signal for transmission. The knob is connected to the adaptor. Since the photon absorption increases the electron's potential energy, it will cause a current spike when absorption happens, so the current intensity will be the parameter that is fed out of the Y socket.

8 Conclusion

Concluding from our experiment, we see that there is absorption occurring at every different magnetic field strength with the corresponding frequency that it requires. We also found that there is in general a linear relationship between these two parameters but the linear curve fit proved to be poor based on the resulting χ^2 values. Thus the γ parameter was computed directly from the data points rather than as the slope of the fit, which would be very inaccurate. This leads to the resultant Landé g factor values which has a theoretical proven value of 2.0023. However, this theoretical value is only valid for free electrons, whereas in our experiment, we had materials consisting of matters that can interact with free electrons, so the magnetic field required to allow absorption may vary and hence the results will not perfectly reflect this factor.

9 References

[1] Pandhi, A. and Chen, X., University of Toronto, Toronto, ON. "PHY224 Laboratory Notes: Electron Spin Resonance", November 2018.

10 Appendices

10.1 Tables of computed parameters

Table 1: *Computed Magnetic Field, Gyromagnetic Ratio and Landé g Factor for the Double Coil*

Current (A)	Magnetic field (Teslas)	Gyromagnetic Ratio	Landé g Factor
0.420	$1.726 \times 10^{3} \pm 1.8 \times 10^{5}$	$1.210 \times 10^{11} \pm 1.5 \times 10^{9}$	1.378 ± 0.018
0.380	$1.562 \times 10^{3} \pm 1.6 \times 10^{5}$	$1.248 \times 10^{11} \pm 1.6 \times 10^{9}$	1.421 ± 0.018
0.356	$1.463 \times 10^{3} \pm 1.5 \times 10^{5}$	$1.288 \times 10^{11} \pm 1.6 \times 10^{9}$	1.466 ± 0.019
0.338	$1.389 \times 10^{3} \pm 1.4 \times 10^{5}$	$1.225 \times 10^{11} \pm 1.6 \times 10^{9}$	1.395 ± 0.018
0.304	$1.250 \times 10^{3} \pm 1.3 \times 10^{5}$	$1.242 \times 10^{11} \pm 1.6 \times 10^{9}$	1.414 ± 0.018
0.284	$1.167 \times 10^{3} \pm 1.2 \times 10^{5}$	$1.198 \times 10^{11} \pm 1.5 \times 10^{9}$	1.364 ± 0.017
0.247	$1.015 \times 10^{3} \pm 1.1 \times 10^{5}$	$1.289 \times 10^{11} \pm 1.6 \times 10^{9}$	1.468 ± 0.019
0.234	$9.62 \times 10^{4} \pm 1.0 \times 10^{5}$	$1.219 \times 10^{11} \pm 1.6 \times 10^{9}$	1.389 ± 0.018

Table 2: *Computed Magnetic Field, Gyromagnetic Ratio and Landé g Factor for the Long Coil*

Current (A)	Magnetic field (Teslas)	Gyromagnetic Ratio	Landé g Factor
0.911	$3.745 \times 10^{3} \pm 3.9 \times 10^{5}$	$1.198 \times 10^{11} \pm 1.5 \times 10^{9}$	1.364 ± 0.017
0.835	$3.432 \times 10^{3} \pm 3.5 \times 10^{5}$	$1.197 \times 10^{11} \pm 1.5 \times 10^{9}$	1.363 ± 0.017
0.776	$3.190 \times 10^{3} \pm 3.3 \times 10^{5}$	$1.198 \times 10^{11} \pm 1.5 \times 10^{9}$	1.364 ± 0.017
0.720	$2.960 \times 10^{3} \pm 3.1 \times 10^{5}$	$1.186 \times 10^{11} \pm 1.5 \times 10^{9}$	1.350 ± 0.017
0.652	$2.680 \times 10^{3} \pm 2.8 \times 10^{5}$	$1.203 \times 10^{11} \pm 1.5 \times 10^{9}$	1.370 ± 0.018
0.595	$2.446 \times 10^{3} \pm 2.5 \times 10^{5}$	$1.188 \times 10^{11} \pm 1.5 \times 10^{9}$	1.353 ± 0.017
0.520	$2.137 \times 10^{3} \pm 2.2 \times 10^{5}$	$1.190 \times 10^{11} \pm 1.5 \times 10^{9}$	1.355 ± 0.017
0.451	$1.854 \times 10^{3} \pm 1.9 \times 10^{5}$	$1.185 \times 10^{11} \pm 1.5 \times 10^{9}$	1.349 ± 0.017
0.410	$1.685 \times 10^{3} \pm 1.7 \times 10^{5}$	$1.184 \times 10^{11} \pm 1.5 \times 10^{9}$	1.349 ± 0.017
0.362	$1.489 \times 10^{3} \pm 1.5 \times 10^{5}$	$1.222 \times 10^{11} \pm 1.6 \times 10^{9}$	1.391 ± 0.018

Table 3: *Computed Magnetic Field, Gyromagnetic Ratio and Landé g Factor for the Short Coil*

Current (A)	Magnetic field (Teslas)	Gyromagnetic Ratio	Landé g Factor
0.988	$4.061 \times 10^{3} \pm 4.2 \times 10^{5}$	$1.224 \times 10^{11} \pm 1.6 \times 10^{9}$	1.393 ± 0.018
0.954	$3.921 \times 10^{3} \pm 4.1 \times 10^{5}$	$1.220 \times 10^{11} \pm 1.6 \times 10^{9}$	1.389 ± 0.018
0.928	$3.815 \times 10^{3} \pm 4.0 \times 10^{5}$	$1.223 \times 10^{11} \pm 1.6 \times 10^{9}$	1.393 ± 0.018
0.903	$3.712 \times 10^{3} \pm 3.8 \times 10^{5}$	$1.225 \times 10^{11} \pm 1.6 \times 10^{9}$	1.395 ± 0.018
0.879	$3.613 \times 10^{3} \pm 3.7 \times 10^{5}$	$1.226 \times 10^{11} \pm 1.6 \times 10^{9}$	1.396 ± 0.018
0.858	$3.527 \times 10^{3} \pm 3.7 \times 10^{5}$	$1.220 \times 10^{11} \pm 1.6 \times 10^{9}$	1.389 ± 0.018
0.832	$3.420 \times 10^{3} \pm 3.5 \times 10^{5}$	$1.221 \times 10^{11} \pm 1.6 \times 10^{9}$	1.390 ± 0.018
0.809	$3.325 \times 10^{3} \pm 3.4 \times 10^{5}$	$1.219 \times 10^{11} \pm 1.6 \times 10^{9}$	1.388 ± 0.018
0.784	$3.223 \times 10^{3} \pm 3.3 \times 10^{5}$	$1.219 \times 10^{11} \pm 1.6 \times 10^{9}$	1.388 ± 0.018
0.737	$3.029 \times 10^{3} \pm 3.1 \times 10^{5}$	$1.215 \times 10^{11} \pm 1.6 \times 10^{9}$	1.383 ± 0.018

10.2 Python Code

The full code used for this lab can be found below as well as on the author's Github as "Electron Spin Resonance.py":

```
#Electron Spin Resonance
#Author: Ayush Pandhi (1003227457)
#Date: 11/01/2018

#Importing required modules
import numpy as np
import matplotlib.pyplot as plt
from scipy.optimize import curve_fit
```

```
#Defining a linear fit
def f(x, a, b):
    return a*x + b

#Loading the data for each of the three coils (freq from MHz -> Hz)
dbcoil_current = np.loadtxt('double coil data.txt', skiprows=2, usecols=(0,))
dbcoil_freq = 1000000*np.loadtxt('double coil data.txt', skiprows=2, usecols=(1,))
longcoil_current = np.loadtxt('long coil data.txt', skiprows=2, usecols=(0,))
longcoil_freq = 1000000*np.loadtxt('long coil data.txt', skiprows=2, usecols=(1,))
shortcoil_current = np.loadtxt('short coil data2.txt', skiprows=2, usecols=(0,))
shortcoil_freq = 1000000*np.loadtxt('short coil data2.txt', skiprows=2, usecols=(1,))

#Finding max of precision and accuracy error for dbcoil
dbcoil_ierror = np.empty(len(dbcoil_current))
for i in range(len(dbcoil_current)):
    dbcoil_ierror[i] = max(dbcoil_current[i]*0.0075, 0.1/1000)

dbcoil_freqerror = np.empty(len(dbcoil_freq))
for i in range(len(dbcoil_freq)):
    dbcoil_freqerror[i] = dbcoil_freq[i]*0.00001

#Finding max of precision and accuracy error for longcoil
longcoil_ierror = np.empty(len(longcoil_current))
for i in range(len(longcoil_current)):
    longcoil_ierror[i] = max(longcoil_current[i]*0.0075, 0.1/1000)

longcoil_freqerror = np.empty(len(longcoil_freq))
for i in range(len(longcoil_freq)):
    longcoil_freqerror[i] = longcoil_freq[i]*0.00001

#Finding max of precision and accuracy error for shortcoil
shortcoil_ierror = np.empty(len(shortcoil_current))
for i in range(len(shortcoil_current)):
    shortcoil_ierror[i] = max(shortcoil_current[i]*0.0075, 0.1/1000)

shortcoil_freqerror = np.empty(len(shortcoil_freq))
for i in range(len(shortcoil_freq)):
    shortcoil_freqerror[i] = shortcoil_freq[i]*0.00001

 #Field generated by the double coil
B_dbcoil = []
for i in dbcoil_current:
    B_dbcoil.append(((4/5)**(3/2))*(4*(np.pi)*10**(-7))*(320)*(1/0.07)*(i))
```

```
#Field generated by the long coil
B_longcoil = []
for i in longcoil_current:
    B_longcoil.append(((4/5)**(3/2))*(4*(np.pi)*10**(-7))*(320)*(1/0.07)*(i))

#Field generated by the short coil
B_shortcoil = []
for i in shortcoil_current:
    B_shortcoil.append(((4/5)**(3/2))*(4*(np.pi)*10**(-7))*(320)*(1/0.07)*(i))

#Gamma compted for each B
gamma_dbcoil = []
for i in range(len(B_dbcoil)):
    gamma_dbcoil.append(((2*np.pi)*(dbcoil_freq[i]))/(B_dbcoil[i]))

gamma_longcoil = []
for i in range(len(B_longcoil)):
    gamma_longcoil.append(((2*np.pi)*(longcoil_freq[i]))/(B_longcoil[i]))

gamma_shortcoil = []
for i in range(len(B_shortcoil)):
    gamma_shortcoil.append(((2*np.pi)*(shortcoil_freq[i]))/(B_shortcoil[i]))

#Delta E for each gamma
delE_dbcoil = []
for i in range(len(gamma_dbcoil)):
    delE_dbcoil.append((1.0545718*10**(-34))*(gamma_dbcoil[i])*(B_dbcoil[i]))

delE_longcoil = []
for i in range(len(gamma_longcoil)):
    delE_longcoil.append((1.0545718*10**(-34))*(gamma_longcoil[i])*(B_longcoil[i]))

delE_shortcoil = []
for i in range(len(gamma_shortcoil)):
    delE_shortcoil.append((1.0545718*10**(-34))*(gamma_shortcoil[i])*(B_shortcoil[i]))

#Lande g factor for each gamma
g_dbcoil = []
for i in range(len(gamma_dbcoil)):
    g_dbcoil.append((gamma_dbcoil[i])/((1.6*10**(-19))/(2*(9.10938356*10**(-31)))))

g_longcoil = []
for i in range(len(gamma_longcoil)):
```

```
    g_longcoil.append((gamma_longcoil[i])/((1.6*10**(-19))/(2*(9.10938356*10**(-31)))))

g_shortcoil = []
for i in range(len(gamma_shortcoil)):
    g_shortcoil.append((gamma_shortcoil[i])/((1.6*10**(-19))/(2*(9.10938356*10**(-31)))))

#Propagating error for B
dbcoil_Berror = B_dbcoil*(((dbcoil_ierror/dbcoil_current)**2 + (0.0005/0.07)**2)**0.5)
longcoil_Berror = B_longcoil*(((longcoil_ierror/longcoil_current)**2 + (0.0005/0.07)**2)**0.5)
shortcoil_Berror = B_shortcoil*(((shortcoil_ierror/shortcoil_current)**2 + (0.0005/0.07)**2)**0.5)

#Propagating error for gamma
dbcoil_gammaerror = gamma_dbcoil*(((dbcoil_ierror/dbcoil_current)**2 + (dbcoil_Berror/B_dbcoil)**2)**0.5)
longcoil_gammaerror = gamma_longcoil*(((longcoil_ierror/longcoil_current)**2 + (longcoil_Berror/B_longcoil)**2)**0.5)
shortcoil_gammaerror = gamma_shortcoil*(((shortcoil_ierror/shortcoil_current)**2 + (shortcoil_Berror/B_shortcoil)**2)**0.5)

#Propagating error for g
dbcoil_gerror = dbcoil_gammaerror/((1.6*10**(-19))/(2*(9.10938356*10**(-31))))
longcoil_gerror = longcoil_gammaerror/((1.6*10**(-19))/(2*(9.10938356*10**(-31))))
shortcoil_gerror = shortcoil_gammaerror/((1.6*10**(-19))/(2*(9.10938356*10**(-31))))

#Linear regression
p_opt_1, p_cov_1 = curve_fit(f, dbcoil_current, dbcoil_freq, (0, 0), dbcoil_freqerror, True)
lin_output = f(dbcoil_current, p_opt_1[0], p_opt_1[1])

#Calculating chi squared
chi_sq_1 = (1/6)*(np.sum(((dbcoil_freq - lin_output) / dbcoil_freqerror)**2))
print('chi squared for linear regression is', chi_sq_1)

#Double coil plot
plt.figure(figsize=(10,8))
plt.plot(dbcoil_current, (1/1000000)*dbcoil_freq, 'k.', label='Raw Data')
plt.plot(dbcoil_current, (1/1000000)*lin_output, 'r-', label='Linear Fit')
plt.title('Double Coil Peak Current and Corresponding Frequency')
plt.xlabel('Peak Current (A)')
plt.ylabel('Frequency (MHz)')
plt.errorbar(dbcoil_current, (1/1000000)*dbcoil_freq, xerr=dbcoil_ierror, yerr=(1/1000000)*dbcoil_freqerror, linestyle='none', ecolor='g', label='Error', capsize=2)
plt.legend()
plt.show()
```

```
#Linear regression
p_opt_1, p_cov_1 = curve_fit(f, longcoil_current, longcoil_freq, (0, 0), longcoil_freqerror, True)
lin_output = f(longcoil_current, p_opt_1[0], p_opt_1[1])

#Calculating chi squared
chi_sq_1 = (1/8)*(np.sum(((longcoil_freq - lin_output) / longcoil_freqerror)**2))
print('chi squared for linear regression is', chi_sq_1)

#Long coil plot
plt.figure(figsize=(10,8))
plt.plot(longcoil_current, (1/1000000)*longcoil_freq, 'k.', label='Raw Data')
plt.plot(longcoil_current, (1/1000000)*lin_output, 'r-', label='Linear Fit')
plt.title('Long Coil Peak Current and Corresponding Frequency')
plt.xlabel('Peak Current (A)')
plt.ylabel('Frequency (MHz)')
plt.errorbar(longcoil_current, (1/1000000)*longcoil_freq, xerr=longcoil_ierror,
yerr=(1/1000000)*longcoil_freqerror, linestyle='none', ecolor='g', label='Error', capsize=2)
plt.legend()
plt.show()

#Linear regression
p_opt_1, p_cov_1 = curve_fit(f, shortcoil_current, shortcoil_freq, (0, 0), shortcoil_freqerror, True)
lin_output = f(shortcoil_current, p_opt_1[0], p_opt_1[1])

#Calculating chi squared
chi_sq_1 = (1/8)*(np.sum(((shortcoil_freq - lin_output) / shortcoil_freqerror)**2))
print('chi squared for linear regression is', chi_sq_1)

#Long coil plot
plt.figure(figsize=(10,8))
plt.plot(shortcoil_current, (1/1000000)*shortcoil_freq, 'k.', label='Raw Data')
plt.plot(shortcoil_current, (1/1000000)*lin_output, 'r-', label='Linear Fit')
plt.title('Short Coil Peak Current and Corresponding Frequency')
plt.xlabel('Peak Current (A)')
plt.ylabel('Frequency (MHz)')
plt.errorbar(shortcoil_current, (1/1000000)*shortcoil_freq, xerr=shortcoil_ierror,
yerr=(1/1000000)*shortcoil_freqerror, linestyle='none', ecolor='g', label='Error', capsize=2)
plt.legend()
plt.show()
```