* r's & (r-1)'s complements:-

↳ Used in the subtraction process in the machines like computers.

2, 8, 10, 16, 'x'

[1] Finding r's complement of a Number:-

- Binary → r = 2 ⇒ 2's complement
- Octal → r = 8 ⇒ 8's "
- Decimal → r = 10 ⇒ 10's "
- Hexa → r = 16 ⇒ 16's "
- Base → r = 7 ⇒ 7's "

A Number is given in base r ⇒ $(\text{Number})_r$

r's complement ⇒ $r^n - N$

(n = 3)

$(339.45)_x$

r = Base of the system
n = No. of digits in the integer part
N = Given number

⇒ $(102)_{10}$ ⇒ Find out r's complement.

$r = 10$
$n = 3$
$N = (102)_{10}$

$$(r^n) - N = (10)^3 - (102)_{10}$$

$$= \begin{array}{r} 1000 \\ -\ 102 \\ \hline (898)_{10} \end{array}$$

(2) $(1001)_2$ ⇒ r's complement ?

$r = 2$ $\quad r^n - N$

(1) $(1001)_2 \Rightarrow$ r's complement

$r = 2$

$n = 4$

$N = (1001)_2$

$r^n - N$

$(2)^4 - (1001)_2$

$\Rightarrow (10000)_2 - (1001)_2$

$$\begin{array}{r} 10000 \\ -\ 1001 \\ \hline (0111)_2 \end{array}$$

$(10000)_2 = 1 \times 2^4 + 0 + 0 + 0 + 0 = (2^4)_{10}$

Decimal $(10)^4_{10} \Rightarrow (10000)_{10}$

$(r)^n_r \Rightarrow (1 \rightarrow n \text{ zeros})_r$

$(2)^4_2 \Rightarrow (10000)_2$

$(16)^3_{16} \Rightarrow (1000)_{16}$

$(8)^3_8 \Rightarrow (1000)_8$

r's complement

(2) $(100111)_2 \Rightarrow (011001)_2$

(3) $(742)_8 \Rightarrow$

$$\begin{array}{r} 1000 \\ -\ 742 \\ \hline (036)_8 \end{array}$$

(4) $(91AB)_{16} \Rightarrow$

$r^n - N$

$(16)^4 - (91AB)_{16}$

$$\begin{array}{r} 10000 \\ -\ 91AB \\ \hline (06E55)_{16} \end{array}$$

(5) $(3125)_7 \Rightarrow$

$$\begin{array}{r} 10000 \\ 3125 \\ \hline (3542)_7 \end{array}$$

(312.)

$(339.45)_{10} \Rightarrow$ r's complement

$n = 3$

$(10)^3 - (389.45)_{10} = (660.55)_{10}$

$(11011.1011)_2 \Rightarrow (00100.0101)_2$

$(356.421)_8 \Rightarrow (421.357)_8$ ✓

---

* $(r-1)$'s complement:-

$r^n - r^{-m} - N$

n = no. of digits in the integer part

m = no. of digits in the fractional part

N = Given number

$(100.53)_{10}$ => $(r-1)$'s complement

$n = 3$ | $N = 100.53$

$m = 2$ |

$(10)^3 - (10)^{-2} - (100.53)$

1000.00

$- \quad 0.01$

$- \quad 100.53$

$(899.46)_{10}$

(2) $(11101.0101)_2 \Rightarrow (00010.1010)_2$ $\quad (8)^{-2} \Rightarrow (0.01)_8$

(3) $(732.45)_8 \Rightarrow (045.32)_8$ $\quad r^n - r^{-m} - N$

(4) $(AB02.DE)_{16} \Rightarrow (54FD.21)_{16}$

$n = 4$

$m = 2$

$(8)^{-2} \Rightarrow (0.01)_8$ (8^{-1}, 8^{-2})

$(16)^{-3} \Rightarrow (0.001)_{16}$ (16^{-1}, 16^{-2}, 16^{-3})

$(16)^4 - (16)^{-2} - (AB02.DE)_{16}$

$0FFFF(16)$

$r^n - 1$'s comp.

$r = 9 \Rightarrow$ 9's comp.

$(16) - (16) - (AB02.DE)_{16}$

$(r-1)$'s comp.

$r = 9 \Rightarrow$ 8's comp

0 F F F F F (16)
1 0 0 0 0 . 0 0
− 0 . 0 1
− A B 0 2 . D E

$(54FD.21)_{16}$

$(r-1)$'s ← r's

$(AB02.DE)_{16} \Rightarrow (54FD.21)_{16}$ $\quad (54FD.22)_{16}$

$(732.45)_8 \Rightarrow (045.32)_8$ $\quad (045.33)_8$

$(154)_{10} \Rightarrow (845)_{10}$ $\quad (846)_{10}$

$(11010)_2 \Rightarrow (00101)_2$ $\quad (00110)_2$

$(\quad)_r \Rightarrow$ → Largest single digit value of that base. $(r-1)$.

$(543.2)_{10} \Rightarrow$

$$\begin{array}{r} 999.9 \\ -\ 543.2 \\ \hline 456.7 \end{array}$$ → $(r-1)$'s comp.

$$\begin{array}{r} .1 \\ + \\ \hline 456.8 \end{array}$$ → r's complement

$(543)_{10} \Rightarrow$ ($n = 3$)

$$\begin{array}{r} 999 \\ 543 \\ \hline 456 \\ +\ 1 \\ \hline 457 \end{array}$$ → r's comp.

$\Rightarrow (10)^3 - N$

$(1000)_{10} - N$

$(999+1)_{10} - N$

$(999 - N) + 1$

↑ $(r-1)$'s comp.

$(1011.110)_2 \Rightarrow (r-1)$' comp.

$$\begin{array}{r} 1111.111 \\ -\ 1011.110 \\ \hline (0100.001)_2 \end{array}$$

$(342.15)_8 \Rightarrow$

$$\begin{array}{r} 777.77 \\ -\ 342.15 \\ \hline (435.62)_8 \end{array}$$

↑ $(r-1)$'s ↓ +) r's $(435.63)_8$

$\Rightarrow$ $(231.58)_9 \Rightarrow$ r's & $(r-1)$'s comp.

$$\begin{array}{r} 888.88 \\ -\ 231.58 \\ \hline (657.30)_9 \end{array}$$ → $(r-1)$'s comp.

$$\begin{array}{r} +\ .01 \\ \hline (657.31)_9 \end{array}$$ → r's comp.

* Subtraction using Complements :-

[1] r's comp. :-

[2] $(r-1)$'s comp. :-

← B

× ⊖ 1

· complements

\+

[1] r's complements :-

$(A)_r - (B)_r$

Step 1 :- Find r's complement of $(B)_r$ i.e. number to be subtracted.

Step 2 :- Add it to $(A)_r$ [A + (r's comp. of B)] ⇓

Step 3 :- Decision based on carry :-

(a) If there is a carry, discard it & the answer is positive & its actual magnitude form.

(b) If there is no carry, ... & ...

(Ans)
↓
Actual magnitude [− (r's comp.)]

- the answer is negative its in r's complement form.
- To get the actual negative magnitude, one can take r's complement again of the answer.

[1] $(35)_{10} - (32)_{10} \Rightarrow (03)_{10}$

(A) (B)

step 1:- Find out r's complement of B.

$(32)_{10} \Rightarrow n = 2 \Rightarrow r^n - N$

$= 100 - 32$

r's complement of $B = (68)_{10}$

step 2:- Add it to (A).:-

```
  1 1
  3 5
+ 6 8
-------
1 0 3
```

carry = 1

step 3:- Ans. $\pm (03)_{10}$

(2) $(1101)_2 - (1001)_2 = (0100)_2$

$(13)_{10} - (9)_{10} = (4)_{10}$

$= (0100)_2$

(3) $(32)_{10} - (35)_{10} = -(03)_{10}$

step 1:- r's comp of $B \Rightarrow (35)_{10} \Rightarrow$

$r^n - N \Rightarrow (10)^2 - (35)$

$= 100 - 35$

$(65)_{10}$

step 2:- Add it with (A)

$$\begin{array}{r} 32 \\ +\ 65 \\ \hline 0\,|\,97 \end{array}$$

No carry ⇒ 97 ⇒ [r's comp. of $(3)_{10}$]

step 3:- Ans. is negative & its in r's complement form.

Actual magnitude = r's comp. of Ans. in step 2

$$r^n - (\text{Ans}) = 10^2 - 97$$
$$= -(3)_{10}$$

(4) $(342)_8 - (430)_8 =$

step 1:- r's complement of $(430)_8$ ⇒

$$(8)^3 - (430)_8$$
$$= (1000)_8 - 430$$

$$\begin{array}{r} {}^{0}7{}^{(8)}\\ (1000)_8 \\ -\ 430 \\ \hline (350)_8 \end{array}$$

step 2:- Add it to (A) =

$$\begin{array}{r} {}^{1} \\ (342)_8 \\ +(350)_8 \\ \hline 0\,|\,712 \end{array}$$

No carry ⇒ 0|712

step 3:- Ans is negative & its in r's complement form: take r's complement again.

$$\begin{array}{r} 0\,7\,1\,8 \\ 1\,0\,0\,0 \\ -\ \ 7\,1\,2 \\ \hline -(0066) \end{array}$$

→ Actual magnitude of Ans.

(5) $(1602)_{16} - (0602)_{16} = (1000)_{16}$

step 1 — r's comp ⟹ $(F9FE)_{16}$

step 2

$$\begin{array}{r} 1602 \\ +\ F9FE \\ \hline \not{1}\,(1001)_{16} \end{array}$$

step 3: Since there is a carry in step 2, hence discard it & the ans. is in actual magnitude form.

(b) $(1011.01)_2 - (0101.001)_2$

→ $r^n - N$ — n is no. digits in int part.

$(r^4 - N)$

$$\begin{array}{r} 1011.010 \\ -\ 0101.001 \\ \hline 0110.001 \end{array}$$

⟹

[B] Subtraction using (r-1)'s complement.

Rules: $(A)_r - (B)_r$

step 1: Find (r-1)'s complement of $(B)_r$

step 2: Add it to $(A)_r$

step 3: Decision Based on Carry:

(a) If there is a carry in step 2, (it will be known as "end around carry): add the carry bit to

result - & now, the result is positive & in actual magnitude form.

(b) If there is no carry in step 2) the answer is negative & in (r-1)'s complement form.

Ex 1. $(35)_{10} - (32)_{10} \Rightarrow (03)_{10}$

↓

Step 1: Find (r-1)'s comp. of $(B)_{10} = (32)_{10}$

$= r^n - r^{-m} - 1$; n = no. of digits in integer part

n = 2; m = 0 ; m = no. of digits in fractional part

$= (10)^2 - (10)^0 - (32)_{10}$

$= 100$
$- \quad 1$
$= 32$

$(67)_{10}$

Step 2: Add it to (A)

$$\begin{array}{r} 1 \\ 35 \\ +\ 67 \\ \hline 1\ 02 \\ \rightarrow\ 1 \\ \hline (03)_{10} \end{array}$$

End around carry

→ Actual Answer

(2) $(32)_{10} - (35)_{10} \Rightarrow -(3)_{10}$