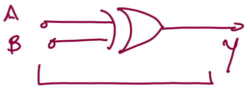
# Chapter 2 : Boolean Algebra & Logic Gates

→ **Ex-OR :-**

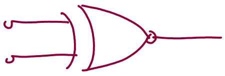


| A | B | $y = \underline{A \oplus B}$ |  |
|---|---|---|---|
| 0 | 0 | 0 | 1 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 |

$\Rightarrow [A\bar{B} + \bar{A}B]$

↑ Ex-NOR

**Ex-NOR :-**



* **Postulates & Theorems :-** [ Boolean ]
↳ 0/1

Postulate 2 (a) $x + 0 = x$ (b) $x . \textcircled{1} = x$

" 5 (a) $x + x' = 1$ (b) $x . x' = 0$
$(x + \bar{x})$

Theorem 1 (a) $x + x = x$ (b) $x . x = x$
[x OR x]

Theorem 2 (a) $x + 1 = 1$ (b) $x . 0 = 0$

Theorem 3 → $(x')' = x$

Postulate 3 (Commutative) (a) $x + y = y + x$ (b) $xy = yx$

Theorem 4 associative (a) $x + (y+z) = (x+y) + z$ (b) $x(yz) = (xy)z$

Distributive (a) $x(y+z) = xy + xz$ (b) $x + yz = (x+y)(x+z)$

Theorem 5 De Morgan (a) $(x+y)' = x' . y'$ (b) $(xy)' = x' + y'$

Theorem 6 Absorption (a) $x + xy = x$ (b) $x(x+y) = x$

**Proof :-**

LHS = $x + yz$ & RHS = $(x+y)(x+z)$

$= x.x + x.z + x.y + y.z$

$= [x + x.y + x.z] + y.z$

$= x[1 + y + z] + yz$

① Logical Equations ⇒

$= x\cdot 1 + yz$

$= x + yz = LHS$

② Truth Table method:

| $x$ | $y$ | $z$ | $f_1 = y.z$ | LHS ✓<br>$\boxed{f_2 = x+yz}$ | $f_3 = x+y$ | $f_4 = x+z$ | RHS ✓<br>$\boxed{f_5 = f_3 \cdot f_4}$ |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 1 | 1 | 1 | 1 |
| 1 | 0 | 1 | 0 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 0 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

Absorption.

$LHS = x + xy =$

$= x(1+y)$

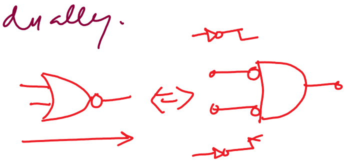
$= x = RHS$

## * De-Morgan's Theorem :-

Statement 1 : " The complement of a sum is equal to the product of complements of each variable individually."

i.e. $\left[\overline{(A+B)} = \overline{A}\cdot\overline{B}\right]$

NOR Bubbled AND



✓ $\left[\overline{(A+B+C)} = \overline{A}\cdot\overline{B}\cdot\overline{C}\right]$

$\overline{(A+B+C+ \cdots +N)} = \overline{A}\cdot\overline{B}\cdot\overline{C}\cdots\overline{N}$

| $A$ | $B$ | $A+B$ | LHS<br>$\boxed{\overline{A+B}}$ | $\overline{A}$ | $\overline{B}$ | RHS<br>$\boxed{\overline{A}\cdot\overline{B}}$ |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | 0 | 0 | 1 | 0 |
| 1 | 1 | 1 | 0 | 0 | 0 | 0 |

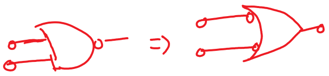
Statement 2 :- " Complement of a product is equal to sum of complements of individual variables."

–

sum of complements of individual variables.

$\overline{(A \cdot B)} = \bar{A} + \bar{B}$ ⇒

$\underbrace{\overline{(A \cdot B)}}_{\text{NAND}}$ $\underbrace{\bar{A} + \bar{B}}_{\text{Bubbled OR}}$

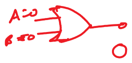


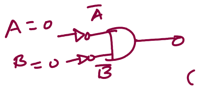
$$\overline{(A \cdot B \cdot C)} = \bar{A} + \bar{B} + \bar{C}$$

$$\overline{(A \cdot B \cdot C \cdot \cdots N)} = \bar{A} + \bar{B} + \bar{C} + \cdots + \bar{N}$$

## * Canonical & standard Forms :–

↳ Minterms & Maxterms





$\frac{1}{(\bar{A}\bar{B})}$

| Dec. | A | B | | (AND) minterms = (1) | (OR) Maxterm = 0 |
|---|---|---|---|---|---|
| 0 | 0 | 0 | = ( 1 ) | $m_0 = \bar{A}\bar{B}$ | $M_0 = A + B$ |
| 1 | 0 | 1 | | $m_1 = \bar{A}B$ | $M_1 = A + \bar{B}$ |
| 2 | 1 | 0 | | $m_2 = A\bar{B}$ | $M_2 = \bar{A} + B$ |
| 3 | 1 | 1 | | $m_3 = AB$ | $M_3 = \bar{A} + \bar{B}$ |

$m_3 = AB \rightarrow$ 1 → 0

| A | B | C | minterms | Maxterms |
|---|---|---|---|---|
| 0 | 0 | 0 | $m_0 = \bar{A} \cdot \bar{B} \cdot \bar{C}$ | $M_0 : A + B + C$ |
| 0 | 0 | 1 | $m_1 = \bar{A} \cdot \bar{B} \cdot C$ | $M_1 = A + B + \bar{C}$ |
| ⋮ | | | ⋮ | |
| 1 | 1 | 1 | $m_7 = \underline{A \cdot B \cdot C}$ <br> 1 | $M_7 = \bar{A} + \bar{B} + \bar{C}$ |

→ **Canonical Forms:** –

$F_2 = (AB) + (\bar{A}B\bar{C})$ ⇒ standard Form

$F_1 \underbrace{(A,B,C)}$ = $\underbrace{A} + \underbrace{BC}$ ⇒ Standard Form

↳ (1) Sum of minterm =

(2) Product of Maxterms

**Ex. 1** Derive the canonical form in terms of sum of minterms.

$$F_1 \underbrace{(A, B, C)} = A + BC$$

$$= A \cdot \underbrace{1} + BC \cdot ①$$

$$= A(B + \bar{B}) + BC(A + \bar{A})$$

$$AB\cdot(1)$$
$$AB\cdot(C+\bar{C})$$

$$= AB + A\bar{B} + A\overset{\checkmark}{B}C + \bar{A}\overset{\checkmark}{B}C$$

$$= AB(C+\bar{C}) + A\bar{B}(C+\bar{C}) + ABC + \bar{A}BC$$

$$= \underset{1\,1\,1}{ABC} + \underset{1\,1\,0}{AB\bar{C}} + \underset{1\,0\,1}{A\bar{B}C} + \underset{1\,0\,0}{A\bar{B}\bar{C}} + \overset{1\,1\,1}{ABC} + \underset{0\,1\,1}{\bar{A}BC}$$

$$= m_7 + m_6 + m_5 + m_4 + \boxed{m_7} + m_3$$

$$= m_3 + m_4 + m_5 + m_6 + m_7$$

$$A+BC \rightarrow$$
$$F(A,B,C) = \Sigma(3,4,5,6,7)$$

(2) $F_2(A,B,C) = A\bar{B} + \bar{C} = A\bar{B}C + A\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + A\bar{B}\bar{C} + AB\bar{C}$

$$= \Sigma(0,2,4,5,6) \checkmark$$

(3) $F(A,B,C,D) = A\bar{B} + \bar{C}D = \Sigma(1,5,8,9,10,11,13)$

- - - - - - - - - - - - - - - - - - - - - - - -

## [B] Product of Maxterms:-

Maxterms are complement of minterms

$$\begin{tikzcd} \text{minterm} \arrow[r] & \text{Maxterm} \arrow[l, bend left] \end{tikzcd}$$

Find out the maxterms for the function:

$$F_1 = A + BC$$

Apply distributive law

$$A + B + \underline{0}$$

$$= (A+B).(A+C)$$

$$= (A+B+0)\ (A+C+0)$$

$$= (\underbrace{A+B}_{x} + C.\bar{C})\ (A+C+B.\bar{B})$$

$$= \underset{0\ 0\ 0}{(A+B+C)}\ \underset{0\ 0\ 1}{(A+B+\bar{C})}\ \underset{0\ 0\ 0}{(A+B+C)}\ \underset{0\ 1\ 0}{(A+\bar{B}+C)}$$

$$= M_0 . M_1 . M_0 . M_2$$

$$= M_0 . M_1 . M_2$$

$$F_1 = A + BC = \Pi(0,1,2) = \Sigma(3,4,5,6,7)$$

| A | B | C | Maxterms ✓ / Minterms | BC | A + BC ✓ |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 | 0 |

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 1 | 0 | 1 |
| 1 | 1 | 0 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |

**Ex. 2** Find out the maxterms for following function:-

$$F_1 = A\bar{B} + C\bar{D}$$

$$= (A\bar{B} + C)(A\bar{B} + \bar{D})$$

$$= (A+C)(\bar{B}+C)(A+\bar{D})(\bar{B}+\bar{D})$$

$$= (A+C+B\bar{B})(A.\bar{A}+\bar{B}+C)(A+\bar{D}+B\bar{B})(A\bar{A}+\bar{B}+\bar{D})$$

$$= (A+B+C)(A+\bar{B}+C)(A+\bar{B}+C)(\bar{A}+\bar{B}+C)$$
$$(A+B+\bar{D})(A+\bar{B}+\bar{D})(A+\bar{B}+\bar{D})(\bar{A}+\bar{B}+\bar{D})$$

$$= (A+B+C+D\bar{D})(A+\bar{B}+C+D\bar{D})(\bar{A}+\bar{B}+C+D\bar{D})$$
$$(A+B+\bar{D}+C\bar{C})(A+\bar{B}+\bar{D}+C\bar{C})(\bar{A}+\bar{B}+\bar{D}+C\bar{C})$$

$$= \overset{M_0}{(\overset{0}{A}+\overset{0}{B}+\overset{0}{C}+\overset{0}{D})}\ \overset{M_1}{(\overset{0}{A}+\overset{0}{B}+\overset{0}{C}+\overset{1}{\bar{D}})}\ \overset{M_4}{(\overset{0}{A}+\overset{1}{\bar{B}}+\overset{0}{C}+\overset{0}{D})}$$

$$M_5\ (\overset{0}{A}+\overset{1}{\bar{B}}+\overset{0}{C}+\overset{1}{\bar{D}})\ (\overset{1}{\bar{A}}+\overset{1}{\bar{B}}+\overset{0}{C}+\overset{0}{D})^{M_{12}}\ \underset{1\ 1\ 0\ 1\ \ M_{13}}{(\bar{A}+\bar{B}+C+\bar{D})}$$

$$\boxed{M_1}\ (\overset{0}{A}+\overset{0}{B}+\overset{0}{C}+\overset{1}{\bar{D}})\ (\overset{0}{A}+\overset{0}{B}+\overset{1}{\bar{C}}+\overset{1}{\bar{D}}) \rightarrow M_3$$

$$\boxed{M_5}\ (\overset{0}{A}+\overset{1}{\bar{B}}+\overset{0}{C}+\overset{1}{\bar{D}})\ (\overset{0}{A}+\overset{1}{\bar{B}}+\overset{1}{\bar{C}}+\overset{1}{\bar{D}}) \rightarrow M_7$$

$$\boxed{M_{13}}\ (\overset{1}{\bar{A}}+\overset{1}{\bar{B}}+\overset{0}{C}+\overset{1}{\bar{D}})\ \underset{1\ 1\ 1\ 1}{(\bar{A}+\bar{B}+\bar{C}+\bar{D})} \rightarrow M_{15}$$

standard forms
↓
canonical forms

$$= M_0 . M_1 . M_4 . M_5 . M_{12} . M_{13} . M_3 .$$
$$M_7 . M_{15}$$

$$= \Pi(0,1,3,4,5,7,12,13,15)$$

* **Standard Forms:-**

↳ Product of sum ( POS ) $F_2 = (\underbrace{A+B}).(\bar{C}+\bar{D})$

↳ Product of Sum (POS) $F_2 = (\underbrace{A+B}).(C+D)$

↳ Sum of Product (SOP) $F_1 = (AB) + (\bar{C}\bar{D})$

→ Convert into standard form: (SOP) [Reduce the no. of terms/ variables]

$F_1 = \underbrace{(AB\bar{C}) + ABC + \bar{A}BC + \bar{A}B\bar{C}}$

$= AB\,(\underbrace{\bar{C}+C}_{1}) + \bar{A}B\,(\underbrace{C+\bar{C}}_{1})$

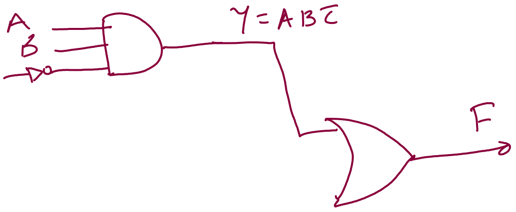
$= AB + \bar{A}B$

$= B\,(A+\bar{A})$

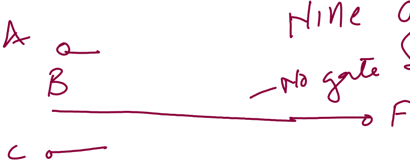
$= \boxed{B}$

Hardware will be more & hence cost of the system will be more.

$AB\bar{C} \Rightarrow$



Nine gates



—No gate required (Zero gate)

Exercise :-

(1) Reduce the following functions to minimum no. of literals:

(a) $AB + A(B+C) + \bar{B}(B+D)$ =

(b) $(X+Y+Z)(\bar{X}+\bar{Y}+\bar{Z})X$ = $X(Y'+Z')$ $x(Y'Z+Z')$

(c) $AB + \overline{AC} + A\bar{B}C\,(AB+C)$ = 1

[a] $AB + A(B+C) + \bar{B}(B+D)$

1 AND, 1 AND, 1 OR (2 i/p), AND, 1 OR (2 i/p), 1 OR (3 i/p)

AND - 3
OR - 3
6
NOT - 1
7

$= AB + AB + AC + B\bar{B} + \bar{B}D$

$= AB + AC + 0 + \bar{B}D$

$= A(B+C) + \bar{B}D \Rightarrow$

OR - 2
NOT - 1
AND - 2
5

[c] $AB + \overline{AC} + A\bar{B}C\,(AB+C)$

$$= AB + \overline{A} + \overline{C} + (A\overline{B}C \cdot AB + A\overline{B}C \cdot C)$$

$$= AB + \overline{A} + \overline{C} + A\overline{B}C$$

$$= A(B + \overline{B}C) + \overline{A} + \overline{C}$$

$$= A(B + \overline{B})(B + C) + \overline{A} + \overline{C}$$

$$(AC + \overline{AC})$$

$$= AB + \underset{\uparrow}{\underline{AC}} + \underline{\overline{AC}} \quad \Rightarrow (x + \overline{x}$$

$$= AB + 1$$

$$= 1$$

<u>Ex. 4</u> Prove that:

$$\overline{\overline{(A\overline{B} + ABC)} + A(B + A\overline{B})} = 0$$