Introduction to Angles

An angle is formed when two rays (or line segments) share a common endpoint, called the **vertex**. Angles are fundamental in geometry, trigonometry, and various fields of engineering and physics.

Units of measurement for angles include **degrees** ($^\circ$) and **radians** (rad).

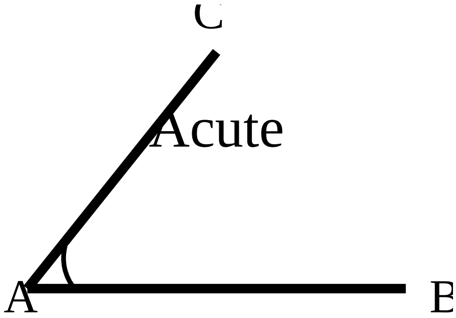
Basic Angle Definitions

- **Ray:** A part of a line that has one endpoint and extends infinitely in one direction.
- **Vertex:** The common endpoint where two rays meet to form an angle.
- **Arms/Sides:** The two rays that form the angle.

Types of Angles by Measurement

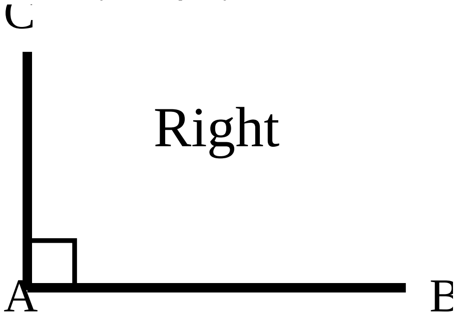
1. Acute Angle

An angle whose measure is **greater than** 0° **but less than** 90°.
Example: 30°, 45°, 89°.



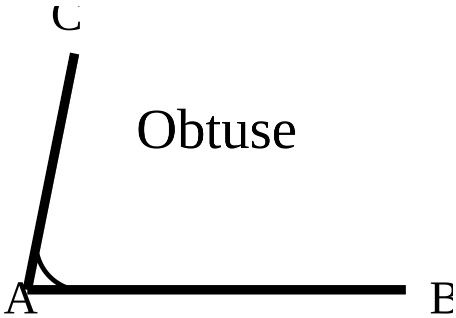
2. Right Angle

An angle whose measure is **exactly** 90°.
Often indicated by a small square symbol at the vertex.



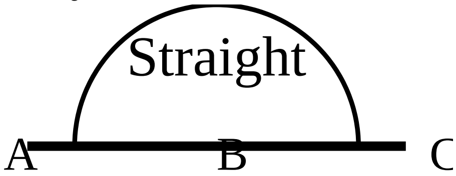
3. Obtuse Angle

An angle whose measure is **greater than** 90° **but less than** 180°.
Example: 100°, 135°, 170°.



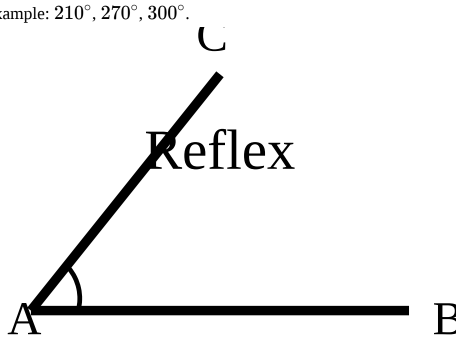
4. Straight Angle

An angle whose measure is **exactly** 180°.
Forms a straight line.



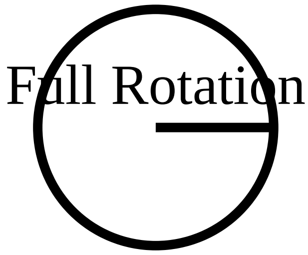
5. Reflex Angle

An angle whose measure is **greater than** 180° **but less than** 360°.
Example: 210°, 270°, 300°.



6. Full Rotation (or Full Angle)

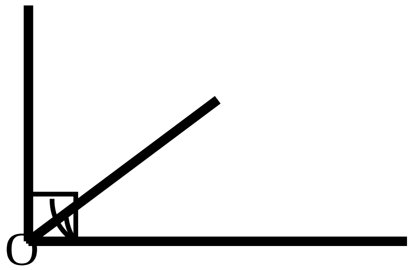
An angle whose measure is **exactly** 360°.
Represents a complete circle.



Angle Pairs

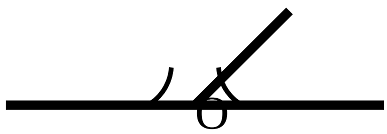
1. Complementary Angles

Two angles whose sum is 90°.
If $\angle A + \angle B = 90^\circ$, then $\angle A$ and $\angle B$ are complementary.
Example: 30° and 60° are complementary.

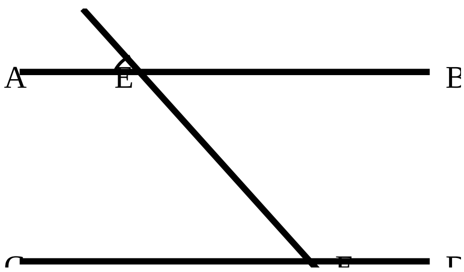


2. Supplementary Angles

Two angles whose sum is 180°.
If $\angle A + \angle B = 180^\circ$, then $\angle A$ and $\angle B$ are supplementary.
Example: 120° and 60° are supplementary.

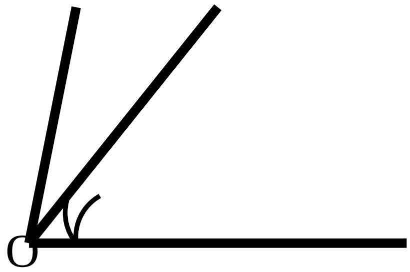


Problem: In the diagram below, lines AB and CD are parallel. If $\angle AEF = 65^\circ$, find the measure of $\angle EFD$ and $\angle CFE$.



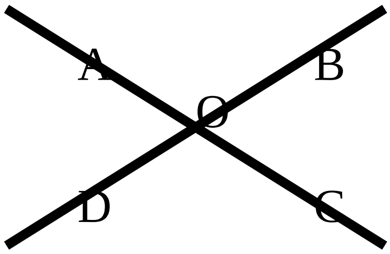
3. Adjacent Angles

Two angles that share a common vertex and a common arm, but do not overlap.



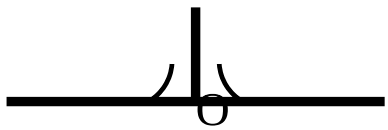
4. Vertically Opposite Angles

Angles formed when two straight lines intersect. They are opposite to each other at the vertex and are always equal.
If two lines intersect, $\angle A = \angle C$ and $\angle B = \angle D$.



5. Linear Pair

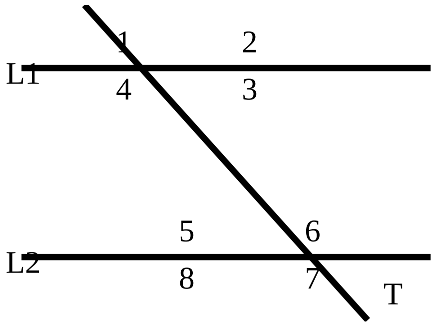
A pair of adjacent angles whose non-common arms form a straight line. They are always supplementary.
$\angle A + \angle B = 180^\circ$.



Angles Formed by a Transversal

When a transversal line intersects two other lines (often parallel lines), several angle pairs are formed.

Parallel Lines L_1 and L_2 intersected by Transversal T



- **Corresponding Angles:** Angles in the same relative position at each intersection. They are equal if lines are parallel.
  - $\angle 1 = \angle 5$
  - $\angle 2 = \angle 6$
  - $\angle 3 = \angle 7$
  - $\angle 4 = \angle 8$
- **Alternate Interior Angles:** Angles on opposite sides of the transversal and between the two lines. They are equal if lines are parallel.
  - $\angle 3 = \angle 6$
  - $\angle 4 = \angle 5$
- **Alternate Exterior Angles:** Angles on opposite sides of the transversal and outside the two lines. They are equal if lines are parallel.
  - $\angle 1 = \angle 8$
  - $\angle 2 = \angle 7$
- **Consecutive Interior Angles (or Same-Side Interior Angles):** Angles on the same side of the transversal and between the two lines. They are supplementary if lines are parallel.
  - $\angle 3 + \angle 5 = 180^\circ$
  - $\angle 4 + \angle 6 = 180^\circ$

Applications of Angles

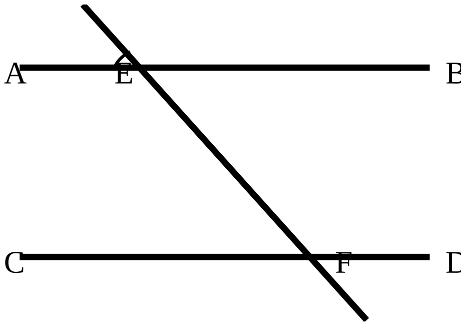
- **Construction & Architecture:** Ensuring right angles for stability, calculating roof pitches (acute/obtuse angles), designing structures.
- **Navigation:** Using compass bearings (angles relative to North) for direction. E.g., a bearing of 90° is East.
- **Physics:** Analyzing forces (vectors at angles), reflection and refraction of light (angles of incidence and reflection).
- **Engineering:** Design of gears, robotic arms, vehicle turning radii, stress analysis in materials.
- **Sports:** Calculating trajectories in basketball, golf, football; optimal angles for throwing or hitting.
- **Art & Design:** Creating perspective, balance, and visual interest using lines and angles.
- **Time:** The hands of a clock form angles. At 3:00, the angle is 90°.

Key Formulas & Relationships

- Sum of angles in a triangle: 180°.
- Sum of angles in a quadrilateral: 360°.
- Sum of interior angles of an n-sided polygon: $(n-2) \times 180^\circ$.
- Conversion: $1 \text{ radian} = \frac{180^\circ}{\pi}$ and $1^\circ = \frac{\pi}{180} \text{ radians}$.
- In a circle, the angle subtended by an arc at the center is twice the angle subtended by it at any point on the remaining part of the circle.

Example Problem

Problem: In the diagram below, lines AB and CD are parallel. If $\angle AEF = 65^\circ$, find the measure of $\angle EFD$ and $\angle CFE$.



Solution:

1. $\angle AEF$ and $\angle EFD$ are **alternate interior angles**. Since AB || CD, alternate interior angles are equal.
   So, $\angle EFD = \angle AEF = 65^\circ$.
2. $\angle AEF$ and $\angle CFE$ are **consecutive interior angles**. Since AB || CD, consecutive interior angles are supplementary.
   So, $\angle AEF + \angle CFE = 180^\circ$.
   $65^\circ + \angle CFE = 180^\circ$.
   $\angle CFE = 180^\circ - 65^\circ = 115^\circ$.