ENGINEERING MATHEMATICS

ALL BRANCHES

Probability and Statistics

DPP 01 Discussion Notes
(Part-04)

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TOPICS TO BE COVERED

01 Question

02 Discussion

Q. Questions

Let X be a continuous random variable with probability function

$$\boxed{f(x) = \frac{1}{2}\,e^{-|x-1|}}, -\infty < x < \infty$$

Find the value of $P(1 < |X| < 2)$

$f(x) = \frac{1}{2} e^{-|x-1|}$

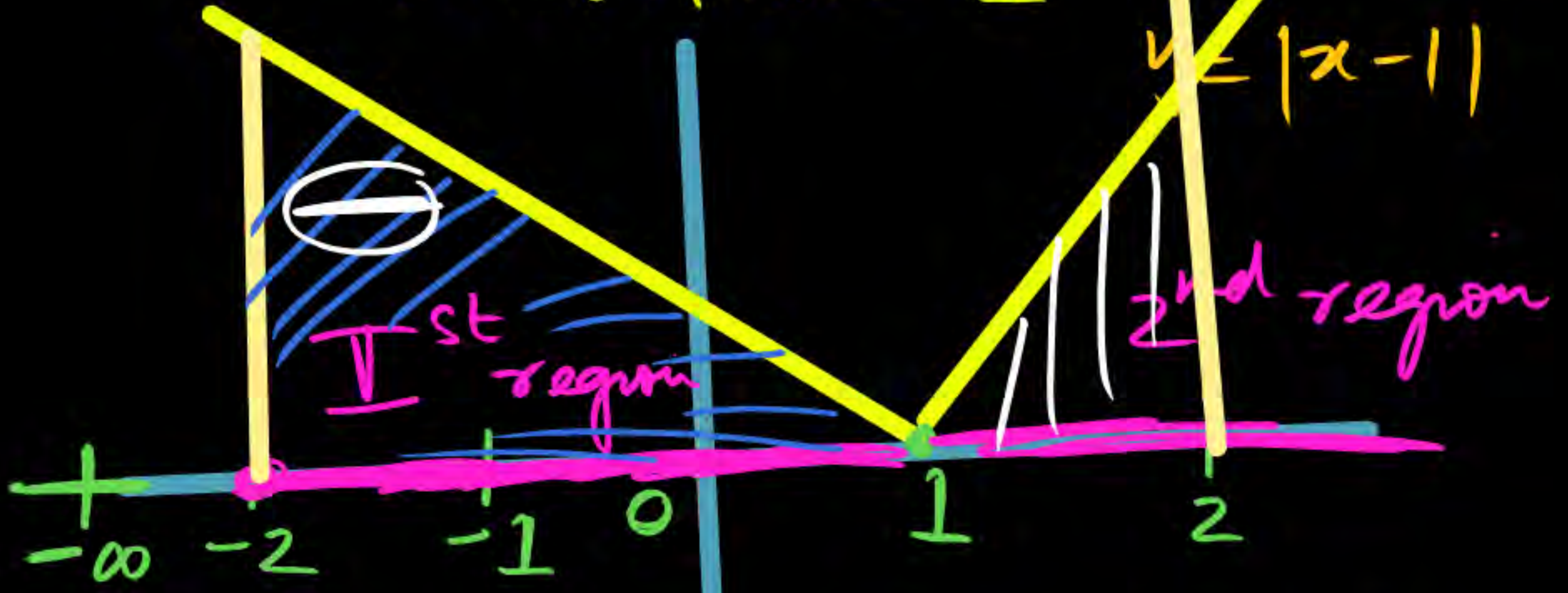
Find the value.

$P(1 < |x| < 2)$

$f(x) = \frac{1}{2} e^{-|x-1|}$

It's a valid pdf

Total Area = 1



$P(1 < |x| < 2)$

$|x| > 1$, $|x| < 2$ And

OR

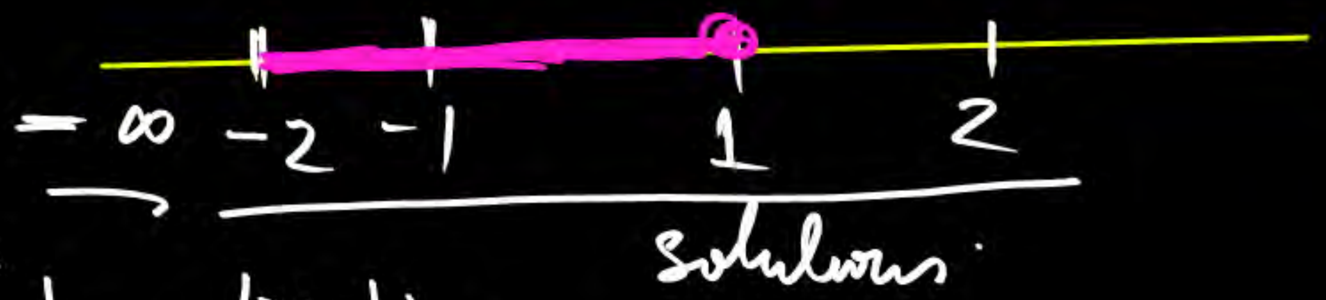
Two Inequalities

$(-\infty, -1) \cup (1, \infty)$

OR

$-2 < x < 2$

Inequality



$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$= \int_{-2}^{-1} \frac{1}{2} e^{+(x-1)}\,dx + \int_{1}^{2} \frac{1}{2} e^{-(x-1)}\,dx$$

$$\underbrace{\int_{-2}^{-1} \frac{1}{2} e^{+(x-1)}\,dx}_{\text{I}^{st}\text{ region}} + \underbrace{\int_{1}^{2} \frac{1}{2} e^{-(x-1)}\,dx}_{\text{II}^{nd}\text{ region}}$$

Ans

$$P(\underbrace{1<|x|<2}) = \boxed{\frac{1}{2}(e^{-1}+e)(e^{-1}+e^{-2})}$$

$$\left[\begin{array}{ll} |x|>1 & (-\infty,-1)\cup(1,\infty) \\ |x|<2 & (-2<x<2 \end{array}\right]$$

Q. **Questions**

$f(x) \Rightarrow \dfrac{k|x|}{[1+|x|]^4} \quad -\infty < x < \infty$

Let $f(x) = \dfrac{k|x|}{(1+|x|)^4}, -\infty < x < \infty$

Then the value of k for which $f(x)$ is a probability density function is

If $f(x)$ is a valid pdf means:

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

(a) $\dfrac{1}{6}$

(b) $\dfrac{1}{2}$

(c) 3 ✓

(d) 6

$$\int_{-\infty}^{\infty} \frac{K|x|}{[1+|x|]^4}\,dx = 1$$

Even FUNCTION
$f(x) = |x|$
$f(-x) = |-x| = |x|$
$\boxed{f(-x) = f(x)}$ Even FUNCTION

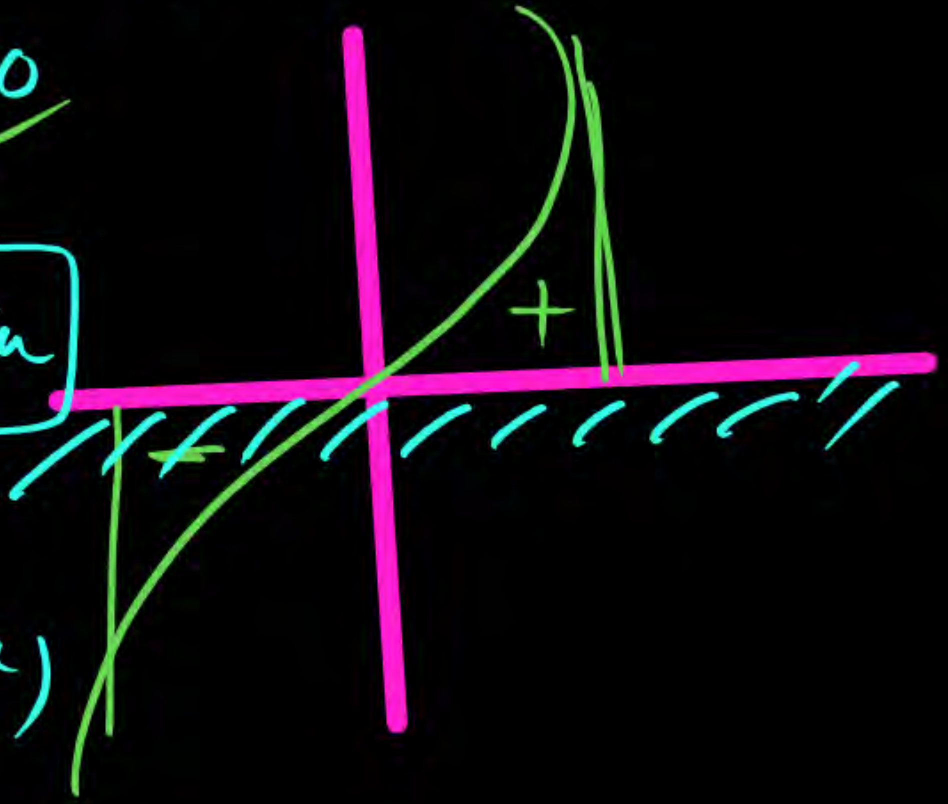
odd function $f(-x) = -f(x)$

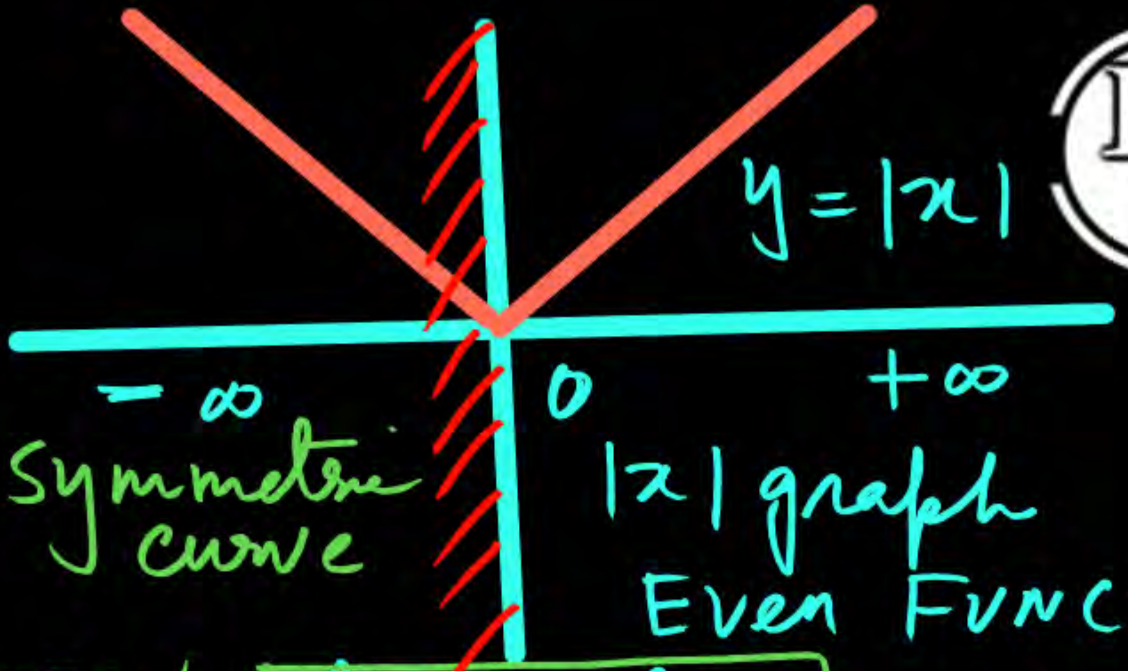
Area = 0 ✓

$y = x^3$ odd function

$\boxed{\text{odd Function}}$

$f(-x) = (-x)^3$
$= -x^3 = -f(x)$

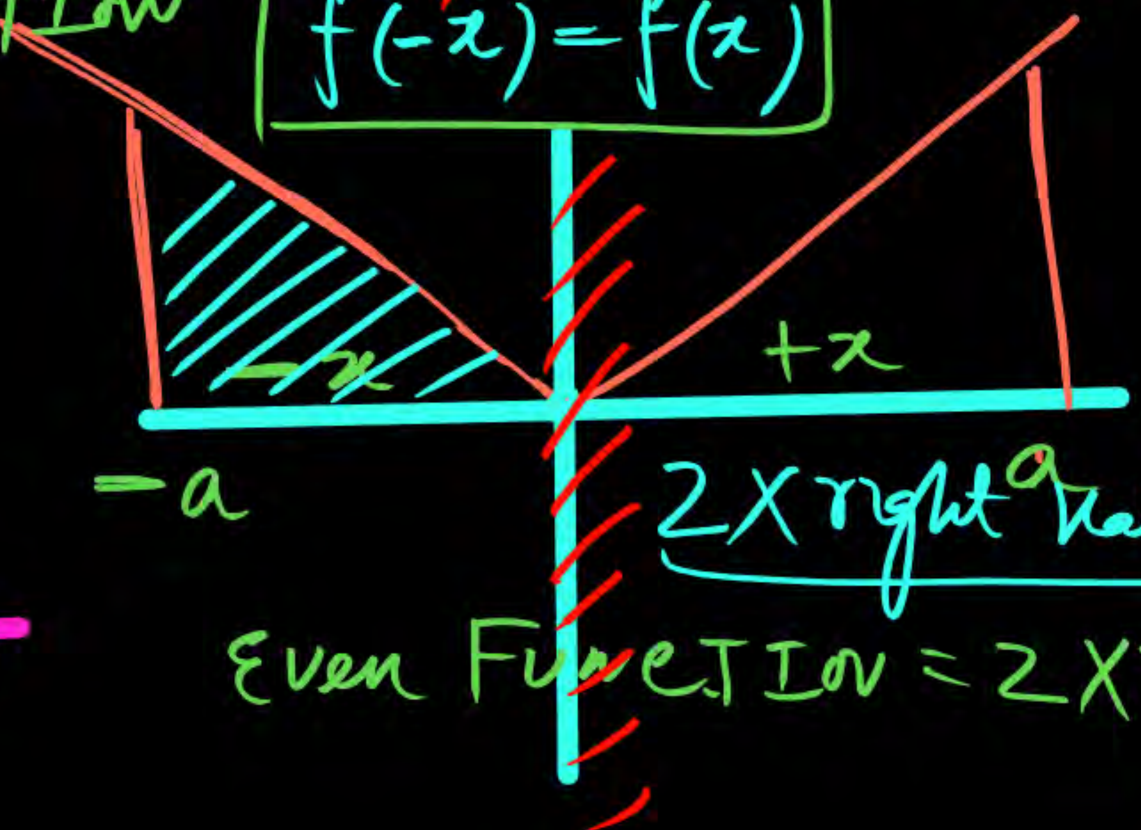




Symmetric curve

$|x|$ graph
Even FUNCTION

$\boxed{f(-x) = f(x)}$



2 X right half

Even FUNCTION = 2 X right

$$\int_{-\infty}^{\infty} \frac{K|x|}{[1+|x|]^4}\,dx = 1$$

Even Function

$-\infty \longrightarrow \infty$

$-\infty$ to 0 $+$ 0 to ∞

$$\Rightarrow 2\int_0^{\infty} \frac{Kx}{(1+x)^4}\,dx = 1$$

$$\int_{-\infty}^{\infty} f(x)\,dx \longrightarrow 2\int_0^{\infty} f(x)\,dx$$

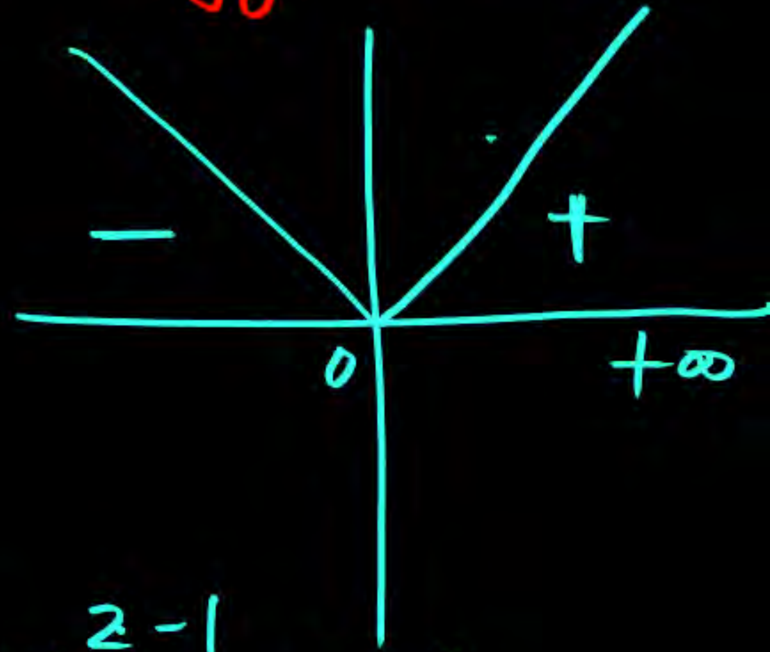
Beta FUNCTION

$$\# \quad I = \int_0^{\infty} \frac{x^{l-1}}{(1+x)^{l+m}} \Rightarrow \frac{\Gamma l \; \Gamma m}{\Gamma l+m}$$

$\Gamma l = (l-1)!$

$\Gamma m = (m-1)!$

l and m Are Integers



$$\Rightarrow 2\int_0^{\infty} \frac{k \cdot x^{\overset{l}{2}-1}}{(1+x)^{\underset{l+m}{2+2}}}$$

$$2k\int_0^{\infty} \frac{x^{2-1}}{(1+x)^{\underset{l+m}{2+2}}} \Rightarrow \frac{x}{(1+x)^4}$$

$$= 2k\int_0^{\infty} \frac{x^{2-1}}{(1+x)^{2+2}} = 1 \qquad l=2,\; m=2$$

$$\Rightarrow 2k \times \frac{\Gamma 2\,\Gamma 2}{\Gamma (2+2)} = 1$$

$$\left.\begin{aligned} \Gamma 2 &= (2-1)! = 1! = 1 \\ \Gamma 4 &= (4-1)! = 3! = 6 \end{aligned}\right\}$$

$$\Rightarrow \cancel{2}k \cdot \frac{1 \times 1}{\cancel{6}\,3} = 1$$

$$\Rightarrow \boxed{k=3}$$

Done

Q. Questions

The distribution function of a random variable X is given by

$$f(x)=\begin{cases}0 & x<0\\ \frac{1}{4} & 0\le x<\frac{1}{4}\\ \frac{1}{2} & \frac{1}{4}\le x<\frac{1}{2}\\ \frac{3}{4} & \frac{1}{2}\le x<\frac{3}{4}\\ \frac{x+3}{5} & \frac{3}{4}\le x<2\\ 1 & x\ge 2\end{cases}$$

using cdf

$P\left(\frac{1}{4}\le X\le 1\right)$

$P(X\le 1)-P\left(X\ge\frac{1}{4}\right)$

$=\left[\frac{x+3}{5}\right]_{x=1}-\frac{1}{4}$

$=\frac{4}{5}-\frac{1}{4}$

Then $P\left(\frac{1}{4}\le X\le 1\right)$ is $=\frac{11}{20}$

(a) $\frac{1}{20}$

(b) $\frac{11}{20}$

(c) $\frac{7}{20}$

(d) $\frac{13}{20}$

better understand using cdf

Q. Questions

$E[X]=2$ | $E[Y]=5$
$Var(X)=4$ | $V(Y)=1$

X has a mean of 2 and a variance of 4. $\boxed{Y = aX + b}$ has a mean of 5 and a variance of 1. What is ab assuming that $a > 0$?

(a) 1

(b) 2 ✓

(c) 3

(d) 4

$Y = aX+b$

$Var(Y) = a^2 Var(X)$

$1 = 4a^2 \Rightarrow a = \pm\frac{1}{2}$

$\boxed{a = \frac{1}{2}}$ $\quad a = -\frac{1}{2}$ (Neglect) $a>0$

$E[Y] = aE[X]+b$

$5 = a\times 2+b$

$2a+b=5$

$Var(aX+b) = a^2 Var(X)$

$E(aX+b) = aE[X]+b$

$2a+b=5, \quad a=\frac{1}{2}$

$2\times\frac{1}{2}+b=5 \quad b=4$

$ab \Rightarrow \frac{1}{2}\times 4 = 2$

Q. Questions

Variance = 0
$E[X] = X$

Let X be a random variable with E(X) = 5 and E(X^2) = 25. Then E(X + E(X))^3 is

(a) 0

(b) 125

(c) 1000

(d) 250

$V(X) = E[X^2] - [E[X]]^2$
$= 25 - (5)^2 = 25 - 25 = 0$

$\boxed{V(X) = 0 \quad E[X] = X}$

$E[X] = 5 \quad X = 5$

Then $E[X + E[X]]^3 = E[5+5]^3$
$= \underline{1000}$

Q. Questions

Let X be a continuous variable with the probability symmetric about 0. If $V(X) < \infty$. Then which of the following statement is true?

(a) $E(|X|) = E(X)$

(b) $V(|X|) = V(X)$

(c) $V(|X|) < V(X)$

(d) $V(|X|) > V(X)$

$$V(X) = E[X^2] - [E[X]]^2$$

MEAN $E[X] = 0$

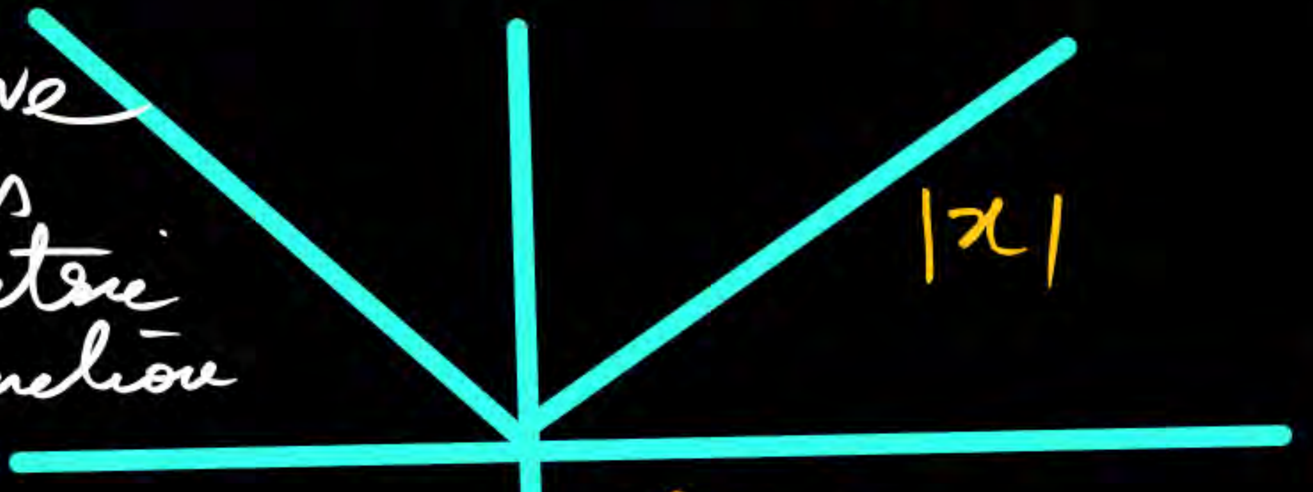
$$V(X) = E[X^2] - 0$$

$$\boxed{V(X) = E[X^2]}$$

$$V(|X|) = E[|X|^2] - [E[|X|]]^2$$

positive

This is symmetric function



$$V(|X|) = E[|X|^2] - [E[|X|]]^2$$

$$= E[X^2] - [E(X)]^2$$

$$\boxed{V(|X|) = V(X) - [E[|X|]]^2}$$

$$\boxed{V(|X|) < V(x)}$$

$$\int_{-\infty}^{\infty} x f(x)\,dx$$

$$|x| = 2\int_{0}^{\infty} x f(x)\,dx$$

DPP03
Part 02
fully Solved

Thank you