GATE
ALL BRANCHES
ENGINEERING MATHEMATICS

Probability and Statistics

Lecture No. 01

BY– RAHUL SIR

13-15 marks

Eng. Point of view

✓ EE/EC/ME/CE,/PI/CH — 13-15 marks (EM)

01 Fundamentals of Probability

02 Question Based on Probability

✓ Class - regular - attend
Continuity - maintain
✓ DPP - regular Solve
✓ Weekly TEST ✓

3500 questions

XE ESE

8:00 to 10:30
11:00 to 1:30
Date sum 12 NOV
Timing

ESE
- ✓ Probability and statistics
- ✓ Single variable Calculus
- ✓ Linear Algebra
- ✓ multivariable Calculus
- ✓ Vector Calculus
- ✓ Differential Eqn^n + P.D.E + Transforms (Laplace)
- ✓ fourier Series
- ✓ Numerical methods
- ✓ Complex Analysis

CSE

CE

Mechanical Eng.

EE/EC

(Numerical methods)

Probability: study of Uncertainity → Results ARE Unknown

✓ deterministic

✓ Random exp

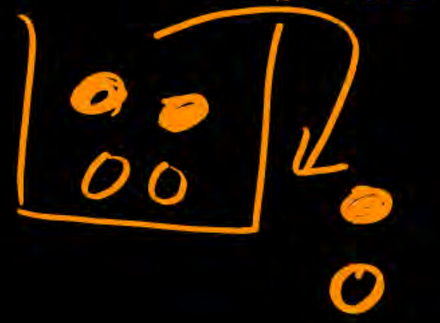
(A) Random experiment

→ Tossing A coin, Throwing A Die

→ Results ARE Different

→ Playing cards.

→ Pick A Ball



≠ Tossing A Fair coin

T (H) coin (Mass Distribution SAME) → Fair coin

weight more (Biased)

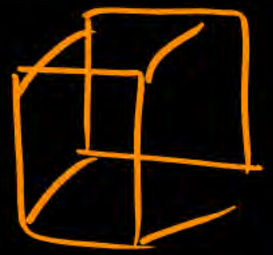
Every time Toss ↓ Result HEAD

If Mass Distribution Are Different Then coin is Biased

≠ Balanced Die

weight (Unfair Die) more Than (Biased Die) other surface

Exp- deterministic (Results-SAME)



Die

Coin

Fairness

experiment-random

Results- Different

✓ Random Experiment $\sqrt{2}$

✓ Results / SAMPLE Point / SAMPLE space / SET S

✓ Events E

] Probability

✓ Illustration 01 Tossing A coin (Fair coin) [$\sqrt{2}$]

$S = \{H, T\}$

Events → HEAD appears (A)

Tail appears (B)

Throwing A Die

$S = \{1, 2, 3, 4, 5, 6\}$

all possible outcomes

$S = \{1, 2, 3, 4, 5, 6\}$

Dots Events

1 E_1
2 E_2
3 E_3
4 E_4
5 E_5
6 E_6

Prime No E_7

Composite No E_8

greater Than 4 E_9

Even/odd

3)

Box

Total events

(Yellow, Red, blue)

event E_1 Event E_2 Event E_3

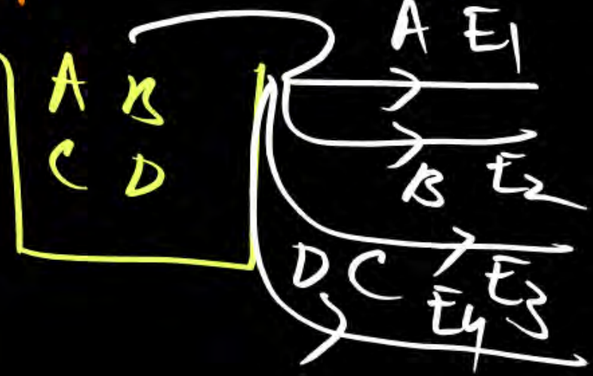
(Draw A Ball)

4) (Playing cards)

face card
52 cards

King (E_1)
Queen (E_2)
Jack (E_3)

(5)



Probability – Relative Frequency $\Rightarrow$ $\frac{\text{No. of Favourable outcomes}}{\text{Total No. of Possible outcomes}}$

$\Rightarrow$ [most likely NO]

$$P(E) = \frac{\text{No. of desired outcomes}}{\text{Total No. of Possible outcomes}} = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{\text{favourable region}}{\text{Total region}}$$

$$= \frac{\bigcirc \text{ fav}}{\bigcirc + \square}$$

Totalgadha coin

$P(\text{HEAD}) = \frac{n(H)}{n(S)} = \frac{(H)}{(H,T)} = \frac{1}{2}$

$= 50\%$ (HEAD)

$P(\text{tail}) = \frac{n(T)}{n(S)} = \frac{\{T\}}{(T,H)}$

$= \frac{1}{2}$ (50% Tail)

$P(\text{Die on } 1) = \frac{n(1)}{n(S)} = \frac{(1)}{\{1,2,3,4,5,6\}}$

$P(1) = \frac{1}{6}$ $\quad P(2) = P(3) = P(4) = P(5) = P(6) = \frac{1}{6}$

#

$P(\text{yellow ball}) \Rightarrow \frac{\text{●●}}{\{\text{●●}, \text{○○}\}} = \frac{2}{4}$

#

Prob(Ace) $= \{A_1, A_2, A_3, A_4\}$ / 52

$P(\text{Ace}) = \frac{4}{52} = \frac{1}{13}$

1) $\Rightarrow P(H) = P(T) = \frac{1}{2}$ (Fair coin) (single experiment) you are guaranteed

$P(H) = 50\%$

$P(T) = 50\%$

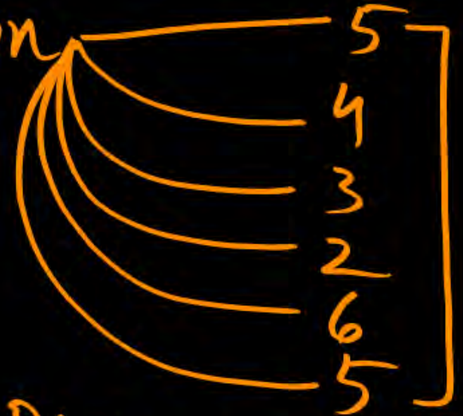
Ten times Coin Toss

$\Rightarrow$ H H H H H H H H T T

1 2 3 4 5 6 7 8 9 10

2) Die Thrown

Balanced Die



$P(1) = 0$

Die Thrown

$P(1) = \frac{1}{6}$ ✓

$P(1) = \frac{1}{6}$ ✓

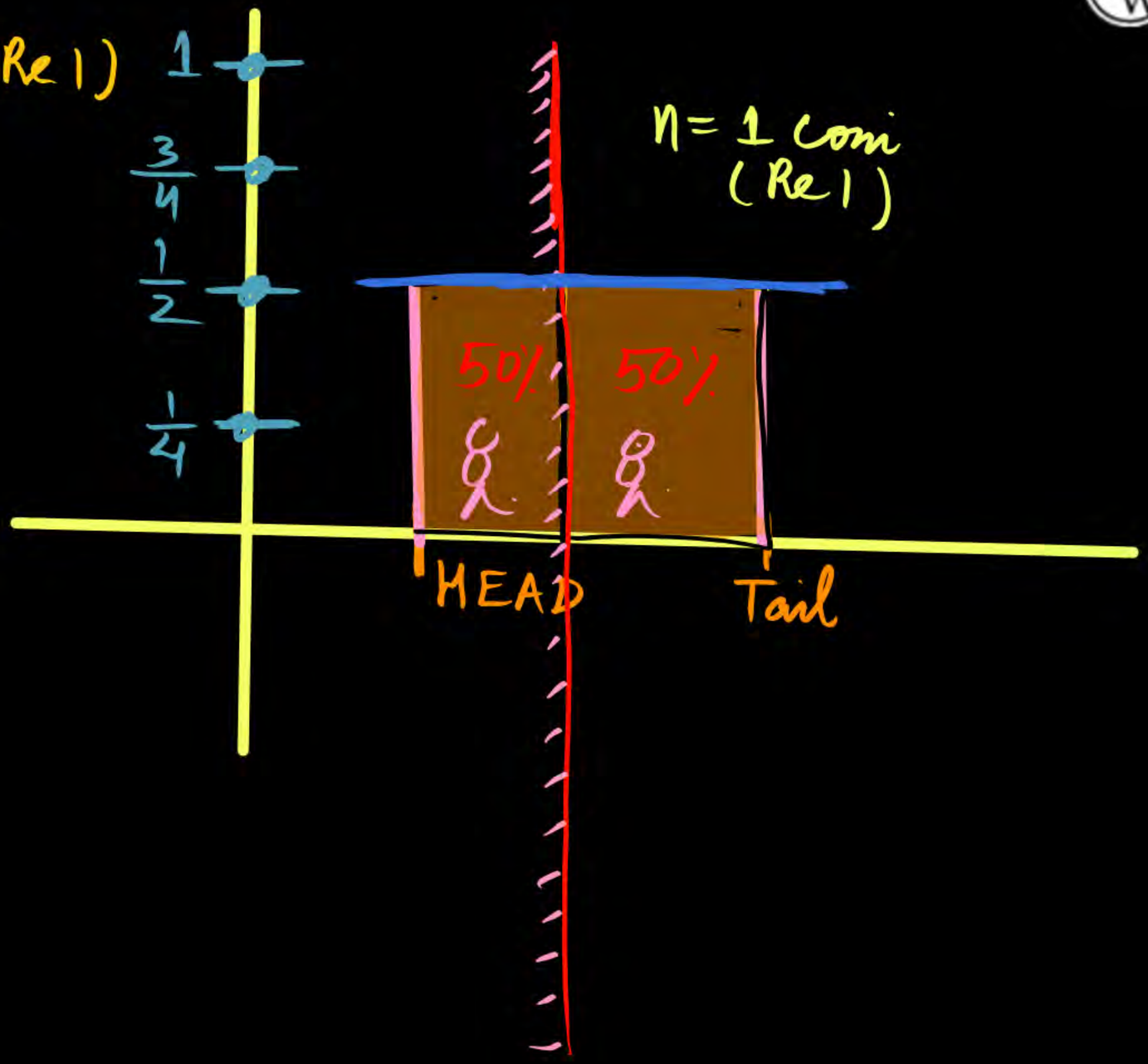
Tossing A coin $n = 1$ (Re 1)

$S = \{H, T\}$

H ⟶ HEAD occurs.

T ⟶ Tail event

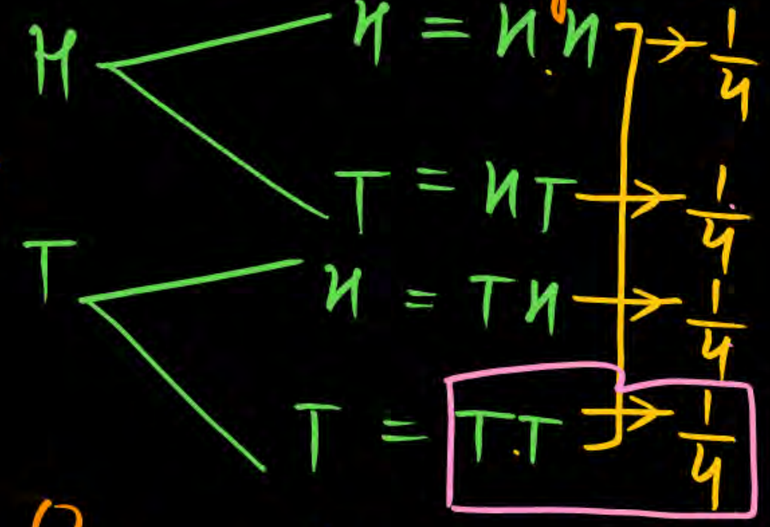
$P(H) = \frac{1}{2}$ $P(T) = \frac{1}{2}$



$n=2$ coins

Re 1, Re 2 simultaneously Draw

Re 1 Tree Diagram:



0 → 0	0 → 0	0 HEAD
H	H	1 HEAD
H	T	2 HEAD
T	H	0 Tail
T	T	1 Tail
		2 Tail

Re 1 < H (W1 ob) / T

Re 2 < H (W2 ob) / T

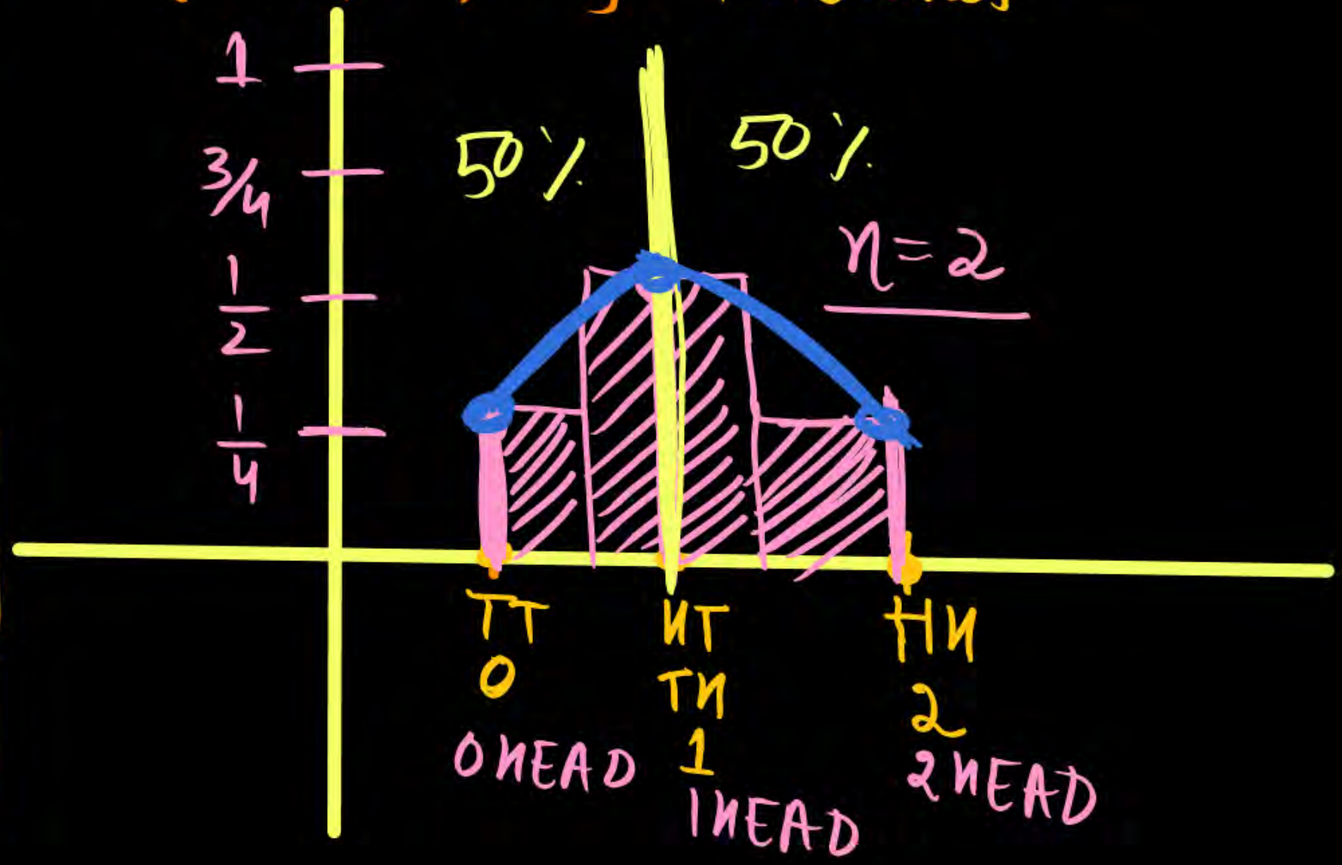
Work Together

$W = W_1 \times W_2$

$= 2 \times 2$

$= 4$

$S = \{HH, HT, TH, TT\}$ = 4 outcomes



For $n=3$ coins (Simultaneously Draw)

(Re 1, Re 2, Re 5)

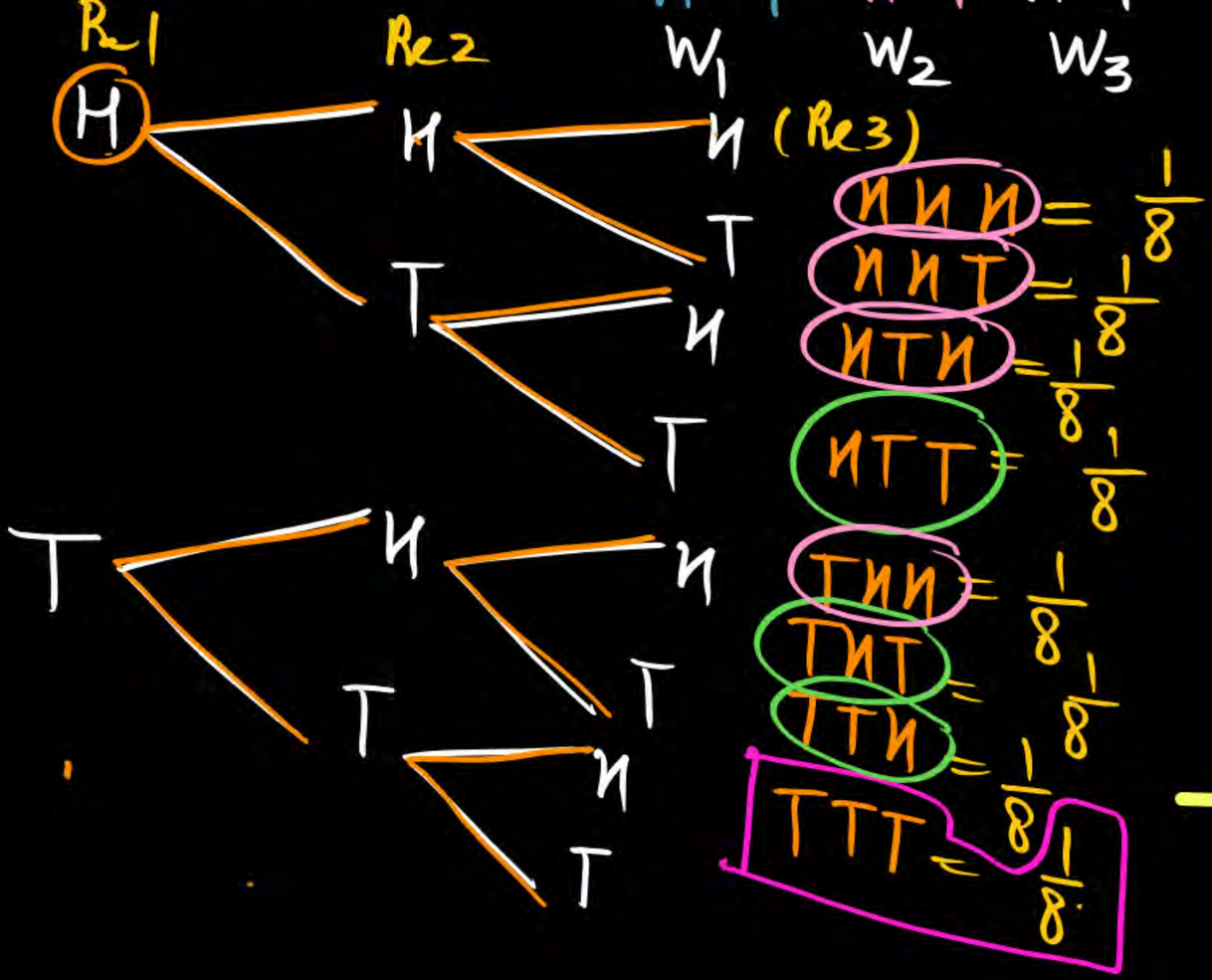
H T H T H T

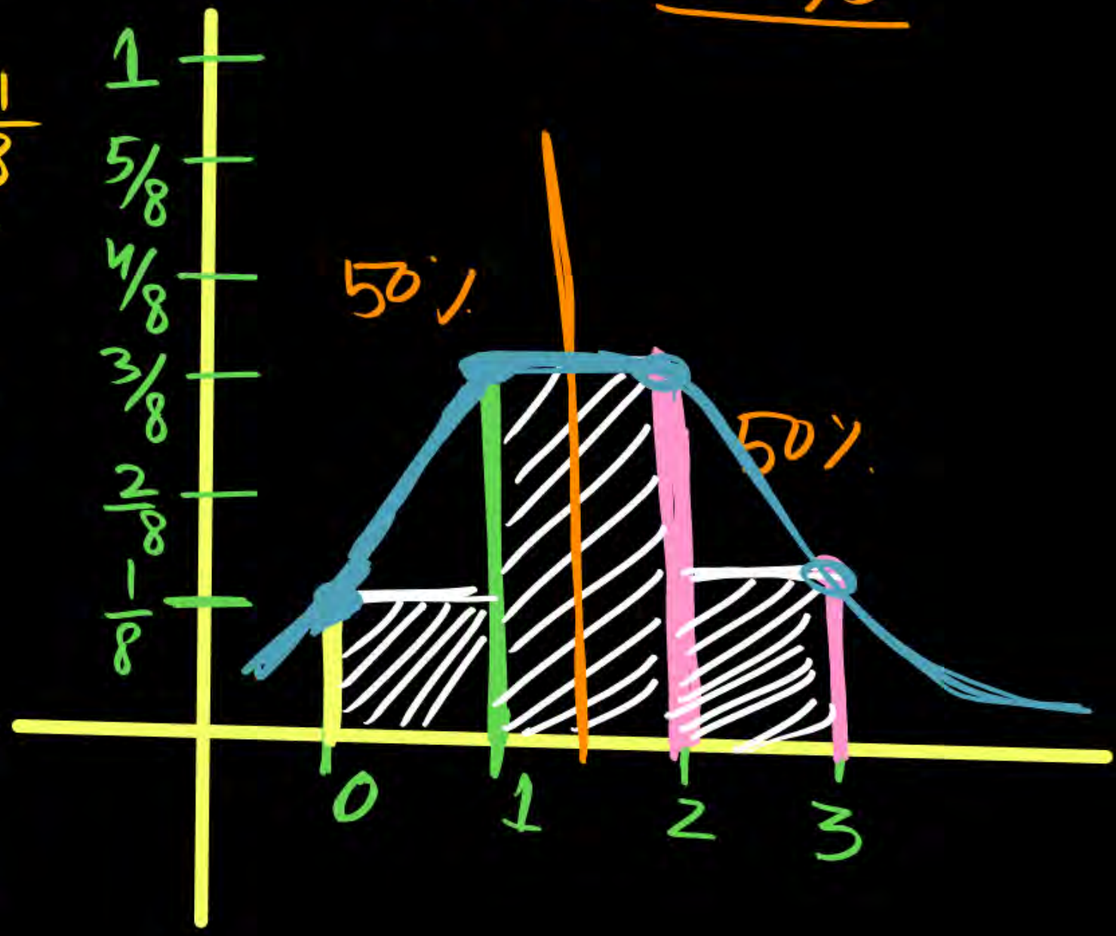
W_1 W_2 W_3

All are working Together.

$W = W_1 \times W_2 \times W_3$

$= 2 \times 2 \times 2 =$ 8 ways





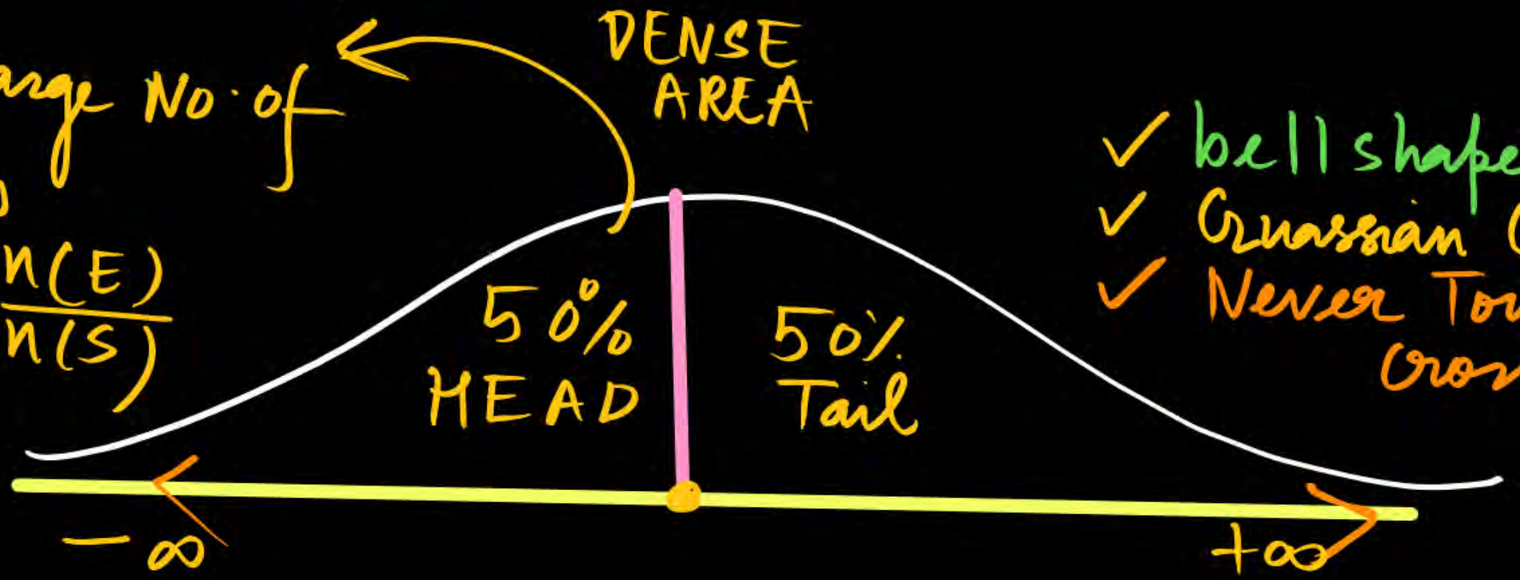
If n is Large No. of trials Then Curve is bell shaped

✓ If n is large No. of trials

Then $P(E) = \frac{n(E)}{n(S)}$

$P(H) = \frac{1}{2}$

$P(T) = \frac{1}{2}$



- ✓ bell shaped
- ✓ Gaussian Curve
- ✓ Never Touches or Crosses The Horizontal axis

$$P(E) = \frac{n(E)}{n(S)}$$

Large No. of trials

Throwing A Die (Two Die)

Die A, Die B
(simultaneously Draw)

$P(\text{Die A} = \text{Die B}) = \frac{6}{36}$

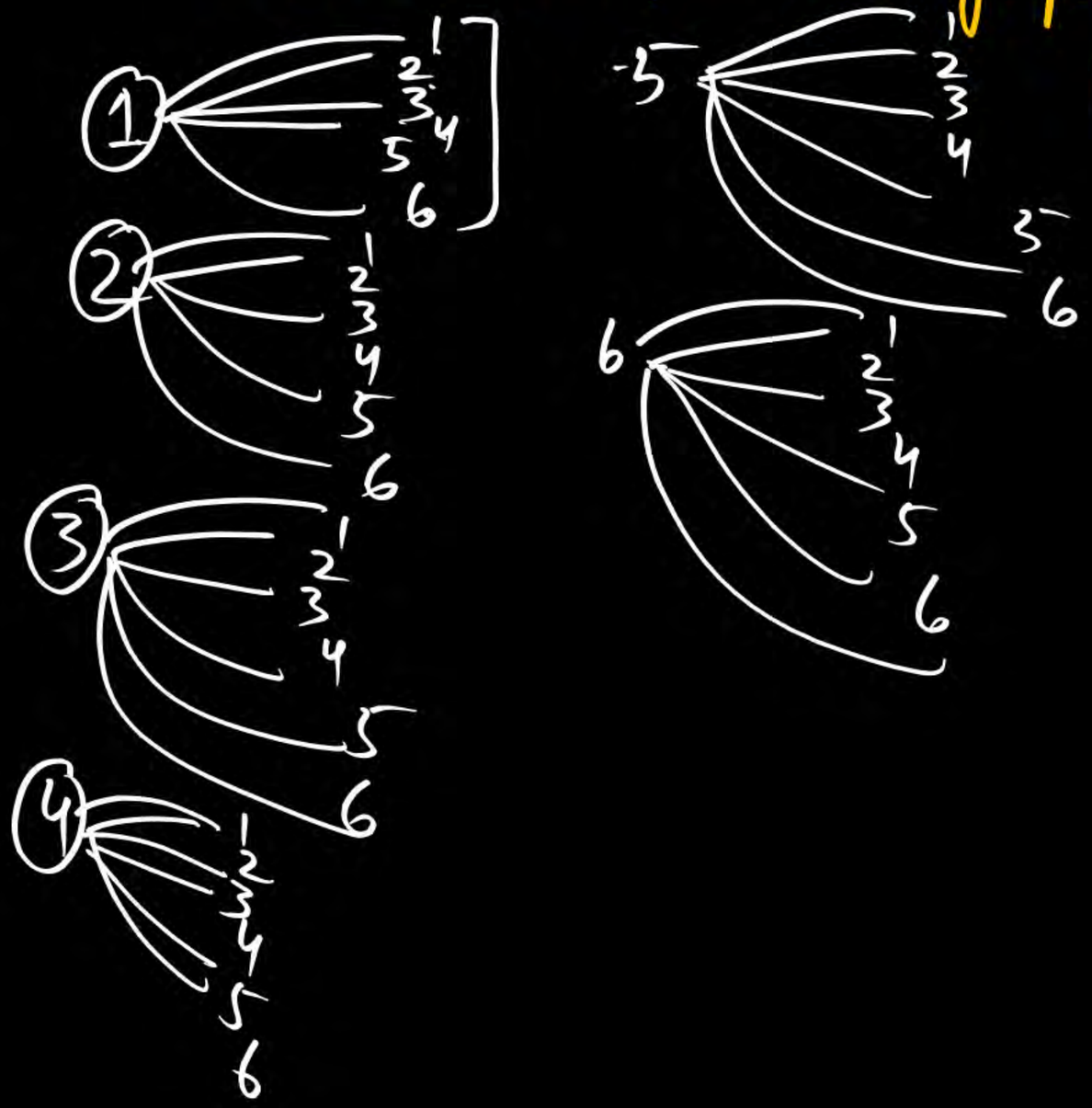
$P(\text{Die A} + \text{Die B} \geq 8) = \frac{15}{36}$ ✓

Trick

Rahul Sir PW Teleg

Die B \ Die A	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

P (Die A + Die B) = sum - graph



X \ Y	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

W_2 (X) · W_1 (Y)

$P(\text{sum}=2) = \frac{1}{36}$

$P(\text{sum}=3) = \frac{2}{36}$

$P(\text{sum}=4) = \frac{3}{36}$

$P(\text{sum}=5) = \frac{4}{36}$

$P(6) = \frac{5}{36}$

$P(7) = \frac{6}{36}$

$P(8) = \frac{5}{36}$

$P(9) = \frac{4}{36}$

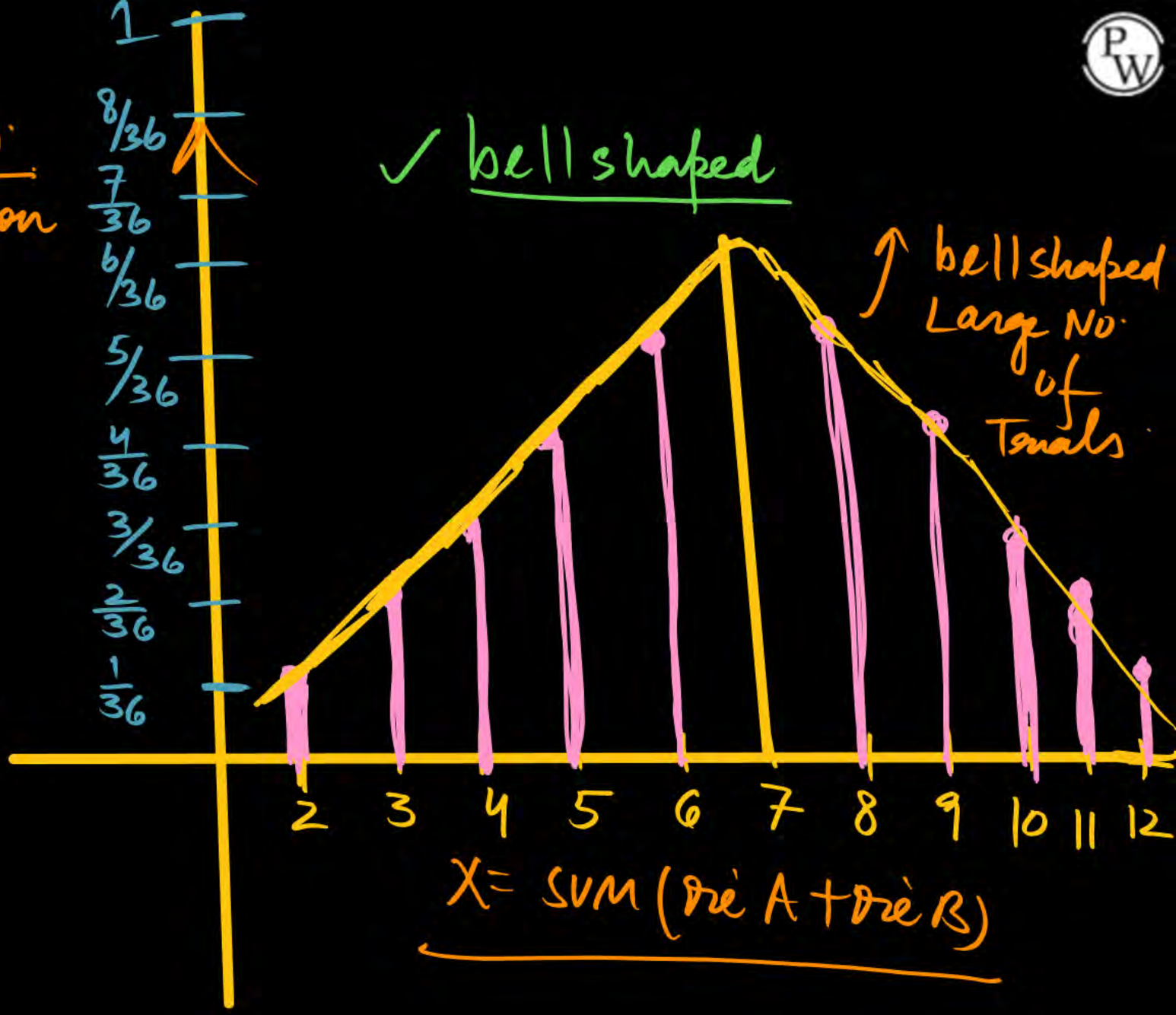
$P(10) = \frac{3}{36}$

$P(11) = \frac{2}{36}$

$P(12) = \frac{1}{36}$

Prob. Distribution

X = sum $(x+y)$



Thank You!

GW Soldiers