# CS & IT

## Discrete Mathematics

## Graph Theory

**DPP: 3**

**Q1** Which of the following options is/are correct?

(A) The chromatic number of a graph with at least 1 edge is at least 2.

(B) A graph is null graph if and only if its chromatic number is 1.

(C) For any graph G, chromatic number of G ≤ 1+Δ(G).

(D) For any graph G, chromatic number of G ≤ |V(G)|

**Q2** Which of the following options is/are correct?

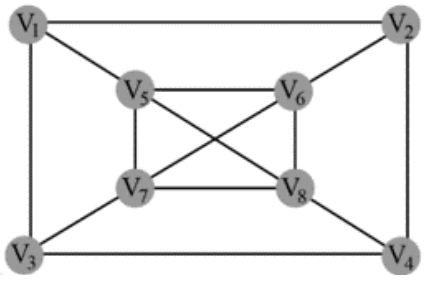
(A) If chromatic number of graph G is 'm', then graph G is a m-partite graph, but it can never be (less than m)-partite.

(B) If graph G is not a null graph and all the cycles of graph G are of even length, then chromatic number of G is 2.

(C) If graph G is not a null graph and all the cycles of graph G are of even length, then G is a bipartite graph.

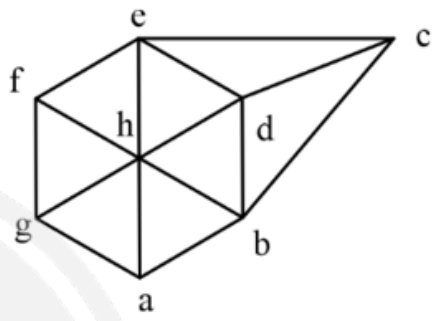
(D) A tree with at least one edge is always a bipartite graph.

**Q3** Consider the following graph,




Minimum number of edges that must be deleted from the graph such that the graph becomes a bipartite graph is_____

**Q4** For the graph G shown below

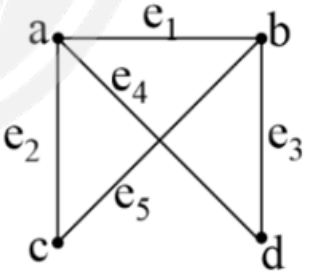



Chromatic number of G + matching number of G = _____.

(A) 6 (B) 7

(C) 5 (D) 8

**Q5** Number of maximal matching in the graph shown below is _____?




**Q6** Which of the following is not True?

(A) Number of perfect matching in $k_{2n} = \frac{(2n)!}{2^n n!}$

(B) Number of perfect matching in $K_{n,n} = n!$

(C) Number of perfect matching in $C_n$ (n is even) = 2

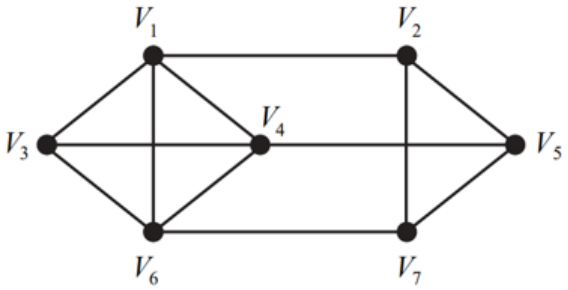
(D) Number of perfect matching in $w_{2n} = 2n$

**Q7** Number of perfect matching in a tree with n-vertices (n ≥ 2)

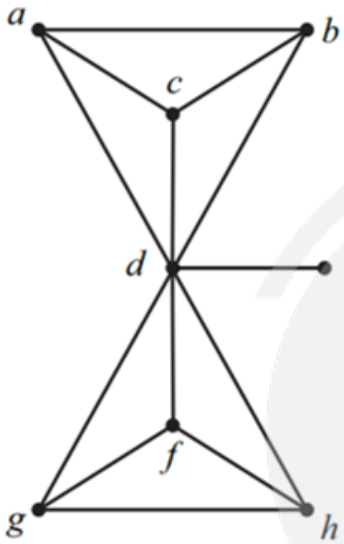
(A) >1 (B) ≥ 1

(C) ≤ 1 (D) = 1

**Q8** For the graph shown below, the chromatic number is _____.




**Q9** Find the matching number of the following graph




**Q10** Let G be a simple graph with 20 Vertices and 100 edges. The size of the minimum vertex cover of G is 8. Then, the size of the maximum independent set of G is_____

**Q11** What is the size of the smallest MIS (Maximal Independent Set) of a chain of nine nodes?

(A) 5 (B) 4

(C) 3 (D) 2

# Answer Key

| | |
|---|---|
| Q1 | (A, B, C, D) |
| Q2 | (A, B, C, D) |
| Q3 | 2~2 |
| Q4 | (B) |
| Q5 | 3~3 |
| Q6 | (D) |
| Q7 | (C) |
| Q8 | 4~4 |
| Q9 | 3 |
| Q10 | 12~12 |
| Q11 | (C) |

Android App | iOS App | PW Website

# Hints & Solutions

**Q1 Text Solution:**

(A) An edge connects two vertices and those two vertices have to be colored using two different colors (because they are adjacent).

(B) A null graph has no edge, so no vertices are adjacent. Hence, each vertex can be colored with a single color.

(C) It is a property of any graph.

(D) In worst case, we might have to color each vertex of a graph with a different color which makes the maximum chromatic number of a graph = |V(G)|. Hence in general, chromatic number of a graph < = |V(G)|.

Hence, all options are correct.

**Q2 Text Solution:**

(A) If chromatic number of a graph G is 'm', then it signifies that there is a vertex in G that is adjecent to m-1 vertices. So, these m vertices will make m partitions and the graph will be a m-partite graph.

(B) A cycle of even length can be colored alternately using two colors.

(C) This statement is derived using option A and option B.

(D) The chromatic number of a tree with atleast one edge is always 2.

Hence, all options are correct.

**Q3 Text Solution:**

$V_5$ $V_8$ and $V_6$ $V_7$ are the two edges to be deleted.

**Q4 Text Solution:**

The three partitions can be {a, d, f}, {b, e, g}, and {c, h}. Hence, chromatic number is 3. One possible matching set can be {ec, db, ga, fh}. Hence, matching number is 4.

**Q5 Text Solution:**

There are three possible maximal matching $\{e_1\}$, $\{e_2, e_3\}$, and $\{e_4, e_5\}$.

**Q6 Text Solution:**

The number of perfect matchings in $W_{2n}$ is 0 because a perfect matching is not possible in a wheel graph with an even number of vertices.

**Q7 Text Solution:**

A tree with an odd number of vertices cannot have a perfect matching because there will always be one vertex left unmatched.

If $n$ is even, the number of perfect matchings can be 1 or more depending on the tree's structure.

**Q8 Text Solution:**

The four partitions can be $\{V_1, V_5\}$, $\{V_2, V_3\}$, $\{V_6\}$ and $\{V_4, V_7\}$.

Hence, chromatic number is 4.

**Q9 Text Solution:**

One possible matching set can be {ad, bc, gf}. Hence, matching number is 3.

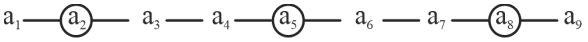
**Q10 Text Solution:**

Total no. of vertices = VC(G) + IS(G)

=> 20 = 8 + IS(G)

=> IS(G) = 20-8 = 12

**Q11 Text Solution:**




Android App | iOS App | PW Website