Computer Science & Information Technology

Discrete Mathematics

Set Theory and Algebra

DPP: 2

Q1 Let A be {a, b, c}. Let the relation R on A and let R = {(b, a), (b, c), (c, a), (c, b), (b, b)}.
Which of the following statements about R is/are true?
(A) R is neither reflexive nor irreflexive.
(B) R is not symmetric.
(C) R is transitive.
(D) R is not anti-symmetric.

Q2 Let A and B be two sets such that A × B = {(1, 1), (2, 2), (3, 1), (3, 2), (1, 2), (1, 4), (2, 1), (2, 4), (3, 4)}.
What is the value of |P(A)| + |P(B)|? ___

Q3 The relation R is defined on Z = {0, ±1, ±2, ±3 . . .} by xRy if and only if$(x^2 + y^2)$ mod 4 = 0. Then R is:
(A) Reflexive, Symmetric
(B) Transitive, Symmetric
(C) Symmetric
(D) Reflexive, Symmetric and Transitive

Q4 Let R be the relation on the real numbers given by xRy iff (x – y)2 < 0. Then which of the following statement is/are true?

(A) R is an equivalence relation.
(B) R is symmetric.
(C) R is asymmetric and anti-symmetric both.
(D) R is transitive.

Q5 Let R be the relation on R (Where R is set of Natural Number) given by xRy if and only if x <2y + 1, then R is______
(A) Reflexive, but not symmetric and not transitive
(B) Reflexive, Symmetric, and not transitive
(C) Not Reflexive, not symmetric and Transitive
(D) Reflexive, but not symmetric and Transitive

Q6 Let R be the relation on A = {1, 2, 3, 4} given by R = {(1, 2), (2, 1), (2, 3), (4, 4), (3, 4)}. What is the cardinality of the reflexive closure of R? __________

Q7 Let R be the relation on A = {1, 2, 3, 4, 5} given by R = {(1, 2), (2, 2), (2, 4), (2,3), (1, 1), (3, 3), (3, 2)}. What is the cardinality of the symmetric closure of R?__________

Q8 Let R be the relation on A = {1, 2, 3, 4} given by R = {(1, 2), (2, 3), (4, 1), (3, 4)}. What is the cardinality of the transitive closure of R? __________

Q9 Suppose that R is the relation on positive real numbers such that xRy if and only if xy = 1. Then R satisfies how many of the properties given below? ____________

(i) Reflexive
(ii) Irreflexive
(iii) Symmetric
(iv) Anti-symmetric
(v) Transitive
(vi) Asymmetric

Q10 Let R be the relation {(a, b) | a ≠ b} on the set of integers. What is the reflexive closure of R?

(A) {(a, b) | a = b}
(B) {(a, b) | a ≤ b}
(C) {(a, b) | a ≥ b}
(D) Z x Z

Answer Key

Q1	(A, B, D)
Q2	16~16
Q3	(B)
Q4	(B, C, D)
Q5	(A)
Q6	8~8
Q7	9~9
Q8	16~16
Q9	1~1
Q10	(D)

Hints & Solutions

Q1 Text Solution:

R = {(b, a), (b, c), (c, a), (c, b), (b, b)}

(a) **True;** as (a, a), (c, c) is not there,

⇒ Not reflexive

(b) **True;** as (a, b) is not there.

(c) **False;** as (c, b), (b, c) is there but (c, c) is not there.

(d) **True;** due to existence of pair (b, c) &(c, b)

Q2 Text Solution:

A = {1, 2, 3} and B = {1, 2, 4}

|P (A)| = 2

3 = 8, and |P (B)| = 2

3 = 8.

So, the value of |P(A)| + |P(B)| = 8 + 8 = 16.

Q3 Text Solution:

R is not reflexive because (1, 1) ∉ R.

R is symmetric because if $(x^2 + y^2)$ mod 4 = 0 then y

2 + x

2 mod 4 = 0.

R is also transitive because R contains all those pairs which consists of even numbers.

R = {(2, 0), (2, 4), (4, 8), (8, 12) ...}

So, if x

2 + y

2 mod 4 = 0 and y

2 + z

2 mod 4 = 0, then x

2 + z

2 mod 4 = 0.

So, R is transitive.

Q4 Text Solution:

Square of any number can't be less than zero, so, it will contain only ∅. and ∅ other than reflexivity satisfies all other properties. Since R is not reflexive hence option (a) is not correct.

Q5 Text Solution:

xRy if (x < 2y + 1)

Reflexive as

x < 2x + 1 {always true}

Symmetric:

example

(1, 5) 1< 2 (5) + 1 but (5, 1) ∉ R (5 < 2(1) + 1)

Therefore, Not symmetric,

For Transitivity:

Take example of (5, 3) ∉ R & (3, 2) ∉ R but (5, 2) ∉ R

Therefore, Not transitive.

Q6 Text Solution:

The reflexive closure of R = {(1, 2), (2, 1), (2, 3), (4, 4), (3, 4), (1, 1), (2, 2), (3, 3)}

So, its cardinality = 8.

Q7 Text Solution:

The symmetric closure of R = {(1, 2), (2, 1), (2, 2), (2, 4), (4, 2), (2, 3), (1, 1), (3,3), (3, 2)}.

So, its cardinality = 9.

Q8 Text Solution:

The transitive closure of R= {(1, 1), (2, 2), (3, 3), (4, 4), (1, 2), (2, 1), (3, 2), (2, 3), (2, 4), (4, 2), (1, 3), (3, 1), (1, 4),(4, 1), (3, 4), (4, 3)}.

So, its cardinality = 16.

Q9 Text Solution:

(1, 1) ∈ R, but 2, 2 ∉ R. So, it is neither reflexive nor irreflexive. Also, it is not asymmetric.

If xy = 1 then yx = 1. So, it is symmetric.

Since (2, 1/2) ∈R and (1/2, 2) ∈ R but (2, 2)∉ R.

So, it is neither transitive nor anti-symmetric.

Q10 Text Solution:

Given {(a, b) |a≠b} means all possible pairs except self pair basically {(a, b) |a ≠ b} ∪ {(a, b) |a = b} = Z x Z therefore, (d) is correct.