ENGINEERING MATHEMATICS

ALL BRANCHES

Probability and Statistics

DPP 01 Discussion Notes
(Part-01)

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TOPICS TO BE COVERED

01 Question

02 Discussion

Question 1

If there are 6 girls and 5 boys who sit in a row. then the probability that no two boys sit together is

6 girls + 5 boys. (No Two Boys sit together)

A $\frac{6!6!}{2!11!}$

B $\frac{7!5!}{2!11!}$

C $\frac{6!7!}{2!11!}$

D None of these

(C)

3 objects: A B C

B C A
B A C
A C B
C A B
C B A

6 possible arrangement

Permutation

Relative Position Change.

n objects

n	$n-1$	$n-2$		1
I	II	III	IV	

Total No. of choices:

$n(n-1)(n-2)\cdots 1$

$= \underline{|n}$

$= n!$

$\left.\begin{matrix} A \\ B \\ C \end{matrix}\right\} 3! = 3\times2\times1 = 6$

$\begin{matrix} A \\ B \\ C \\ D \end{matrix} = 4!\ \ 4\times3\times2\times1 = 24$

n objects $= n!$ or $\lfloor n$

n objects $\longrightarrow$ r objects $\Rightarrow {}^nP_r$ Total P_r

arrange

3 objects $\boxed{A}$ $\boxed{B}$ $\boxed{C}$ $\longrightarrow$ Two objects $\longrightarrow {}^3P_2 = \dfrac{\lfloor 3}{\lfloor 3-2} = \lfloor 3 = 6$ ✓

$${}^nP_r = \frac{\lfloor n}{\lfloor n-r}$$

$\left.\begin{matrix} A, B \\ B, C \\ C, A \\ B, A \\ C, B \\ A, C \end{matrix}\right\}$ Permutate Arrangement $= 6$ ✓

6 girls 5 boys   No Two boys sit together

No Two Boys sit together ✓ $\boxed{B}$ G $\boxed{B}$ G $\boxed{B}$ G $\boxed{B}$ G $\boxed{B}$ G $\boxed{B}$ G $\boxed{B}$ → Fav outcomes

$$P(\text{No Two Boys sit together}) = \frac{n(\text{No Two Boys sit together})}{n(\text{Total})}$$

B $\boxed{G}$ B $\boxed{G}$ B $\boxed{G}$ B $\boxed{G}$ B $\boxed{G}$ B G B

n objects = $n!$

Total arrangement $= 11!$

B G B G B G B G B G B G B

$7 \to 5$

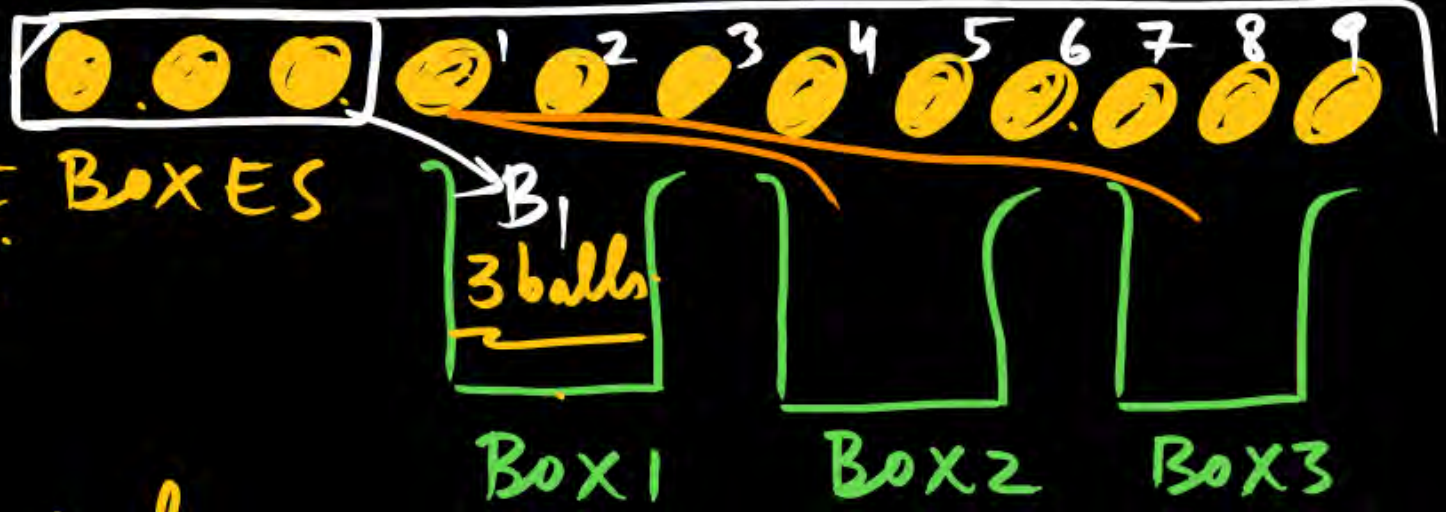
Favourable outcomes $= 6!\ {}^7P_5$

$$P(E) = \frac{6! \cdot {}^7P_5}{11!} = \frac{6! \cdot 7!}{2! \cdot 11!} = \frac{6! \cdot 7!}{11! \cdot 2!}$$

Question 2

CHOOSE → ${}^nC_r$ Arrangement = ${}^nP_r$

Twelve balls are distributed among three ~~axes.~~ boxes. The probability that the first box contains 3 balls is



A $\frac{110}{9}\left(\frac{2}{3}\right)^9$

B $\frac{9}{110}\left(\frac{2}{3}\right)^{10}$

C $\frac{{}^{12}C_3}{12^3}\cdot 2^9$

D $\frac{{}^{12}C_3}{3^{12}}$

$P(E) = \frac{\text{Favourable outcomes}}{\text{Total outcomes}}$

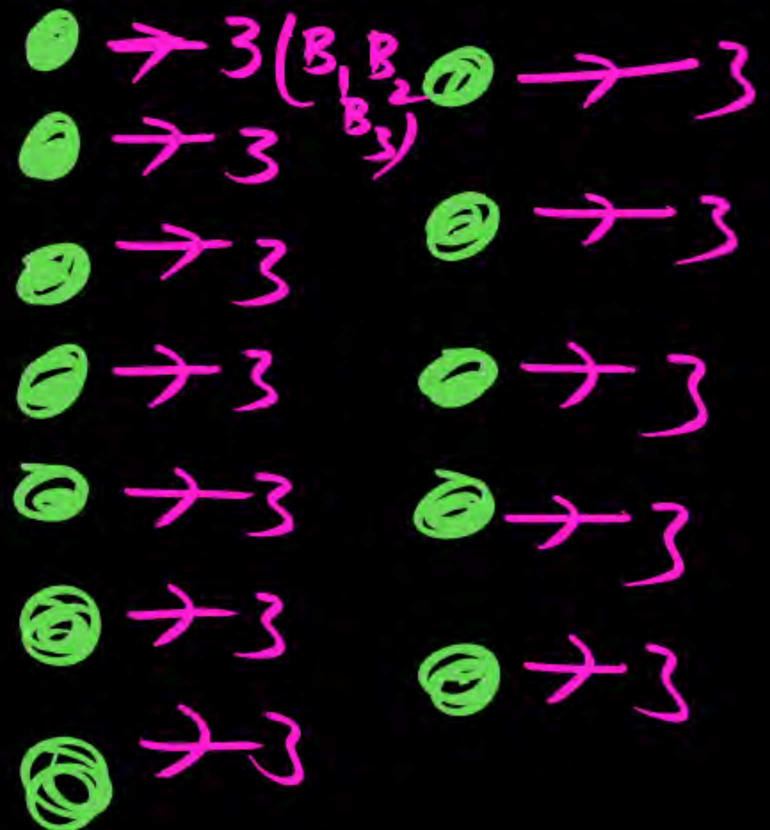
Total outcomes

$= 3\times3\times3\times3\times3\times3\times3\times3 \cdots$ 12 times

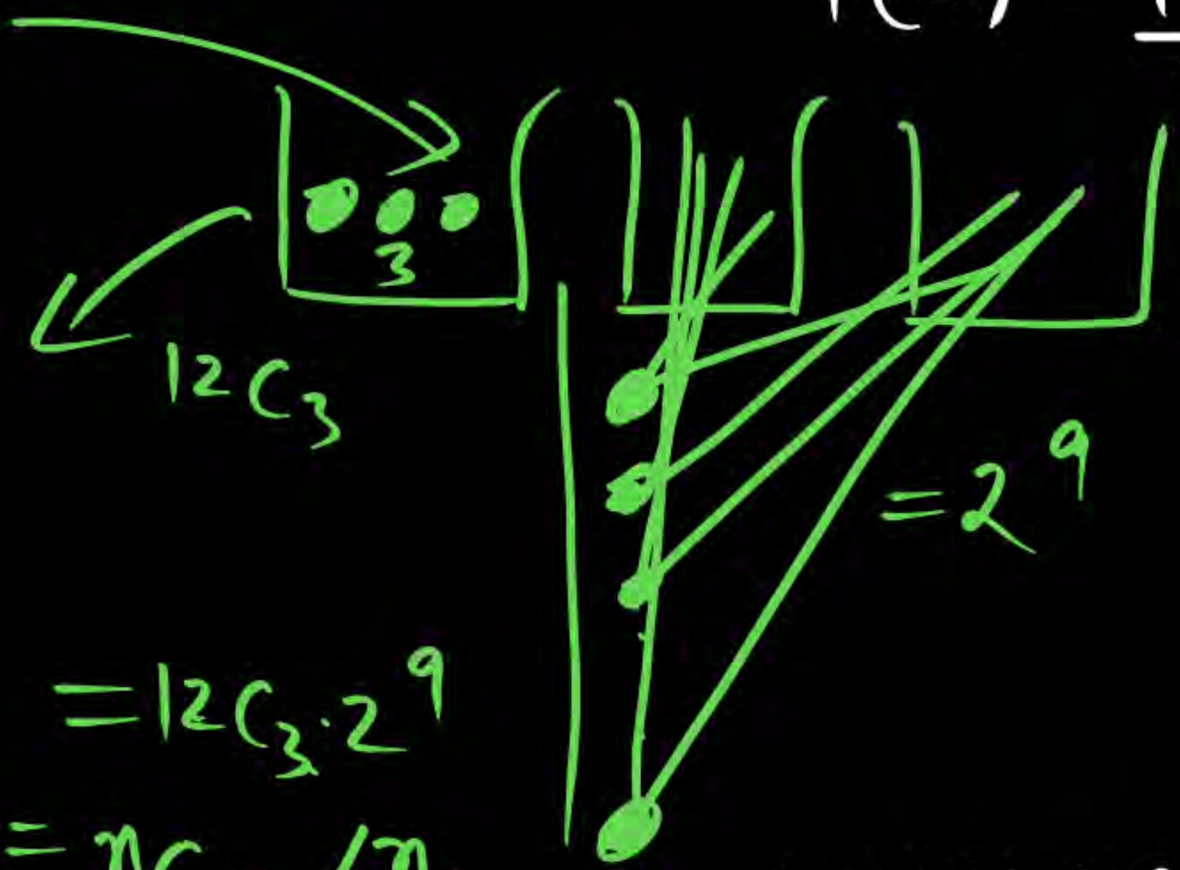
$= 3^{12}$

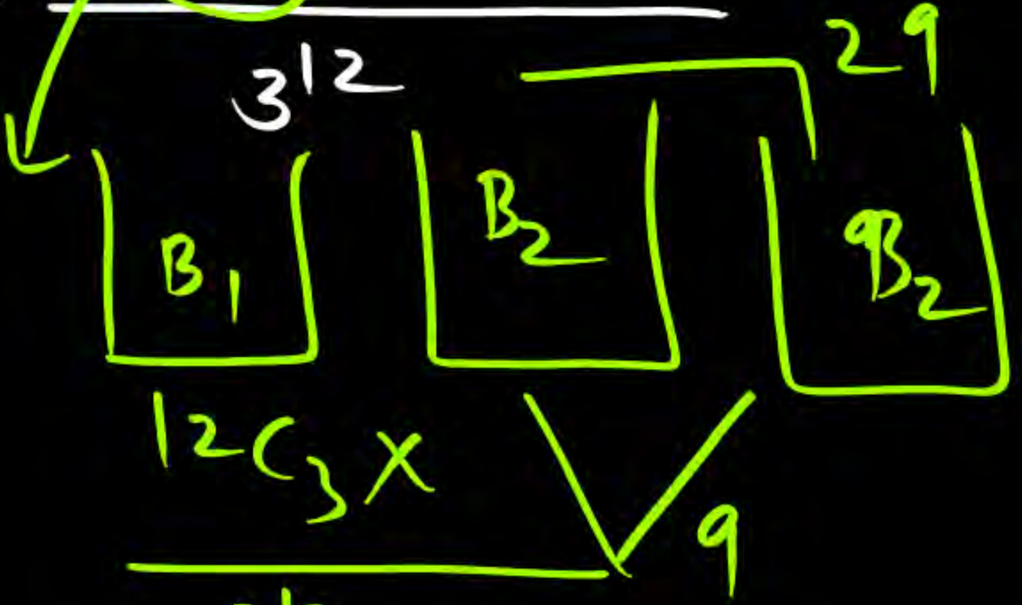
Fav outcomes $= {}^{12}C_3 \cdot 2^9$

$2\times2\times2\times2 \cdots$ 9 times $= 2^9$



$$P(E) = \frac{\text{Fav outcomes}}{\text{Total outcomes}} = \frac{{}^{12}C_3 \, 2^9}{3^{12}}$$





$$\frac{{}^{12}C_3 \times}{3^{12}}$$

$= {}^{12}C_3 \cdot 2^9$

$= {}^nC_r = \frac{\lfloor n}{\lfloor n-r \, \lfloor r}$

$= \frac{12!}{9! \cdot 3!} \cdot 2^9$

$= \frac{12\times 11\times 10\times \cancel{9!}\times 2^9}{\cancel{9!}\times 3!}$

$= \frac{\overset{4}{\cancel{12}}\times 11\times \overset{5}{\cancel{10}}}{\cancel{3}\times \cancel{2}\times 1}\times 2^9$

$= 20\times 11\times 2^9 = 210\times 2^9$

Question 3

$\underline{15} \rightarrow 11 \quad (3) \quad \dfrac{{}^5C_3\,{}^{10}C_8 + {}^5C_4\,{}^{10}C_7 + {}^5C_5\,{}^{10}C_8}{{}^{15}C_{11}}$

A cricket club has 15 members of whom only 5 can bowl. If the names of 15 members are put into a box and 11 are drawn at random. Then the probability of obtaining an eleven containing at least 3 bowlers Is:

A 7/13

B 6/13

C 11/15

D 12/13

P(at least Three Bowl)

15 → 5 can, 11

max 5 Bowl.

15 MEMBERS.
5 Bowl
11 TEAM

Ans

$\dfrac{{}^5C_3\,{}^{10}C_8 + {}^5C_4\,{}^{10}C_7 + {}^5C_5\,{}^{10}C_6}{{}^{15}C_{11}}$

$= \dfrac{12}{13}$ Solved It

Question 4

Three integers are chosen at random from the first 20 integers. The probability that their $\boxed{\text{product is even}}$

- **A** 2/19
- **B** 3/29
- **C** 17/19
- **D** 4/29

THREE integers choose

$$\text{I} \times \text{II} \times \text{III}$$

$$O \times O \times O = \text{odd}$$

$$P(\text{even}) + P(\text{odd}) = 1$$

$$P(\text{product is even}) = 1 - P(\text{odd})$$

10 E

10 O

20

3

$$P(\text{odd}) = \frac{{}^{10}C_3}{{}^{20}C_3}$$

$$= 1 - \frac{{}^{10}C_3}{{}^{20}C_3}$$

Even odd

$A \quad \bar{A}$

$$P(A) + P(\bar{A}) = 1$$

$$P(\text{Happen}) + P(\text{Does Not Happen}) = 1$$

O O O

$A \times B \times C$

odd

$= 1 - P(\text{odd})$

$$P(\text{product is Even}) = 1 - \frac{{}^{10}C_3}{{}^{20}C_3}$$

$$n_{C_r} = \frac{\angle n}{\angle n-r \cdot \angle r}$$

$$\begin{cases} {}^{10}C_3 = \dfrac{\angle 10}{\angle 7\ \angle 3} \\ {}^{20}C_3 = \dfrac{\angle 20}{\angle 17 \cdot \angle 3} \end{cases}$$

$${}^{10}C_3 = \frac{10 \times 9 \times 8 \times \cancel{\angle 7}}{\cancel{\angle 7} \cdot \angle 3} = \frac{\overset{5}{\cancel{10}} \times \overset{3}{\cancel{9}} \times 8}{3 \times \cancel{2} \times 1} = 120$$

$${}^{20}C_3 = \frac{\overset{10}{\cancel{20}} \times 19 \times \overset{6}{\cancel{18}} \times \cancel{\angle 17}}{\cancel{\angle 17} \cdot \cancel{3} \times \cancel{2} \times 1} = 60 \times 19$$

$$P(\text{product is even}) = 1 - P(\text{product of odd}) = 1 - \frac{\overset{2}{\cancel{120}}}{\cancel{60} \times 19} = 1 - \frac{2}{19} = \frac{17}{19}$$

Question 5

One hundred cards are numbered from 1 to 100. The probability that a randomly chosen card has a digit 5 is

1-100 Cards.

Prob. a Digit 5

$$P(E) = \frac{\text{No. of Fav outcomes}}{\text{Total outcomes}}$$

$$P(E) = \frac{19}{100}$$

A 1/100

B 9/100

C 19/100

D None of these

5, 15, 25, 35, 45, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 65, 75, 85, 95

Question 6

a b c

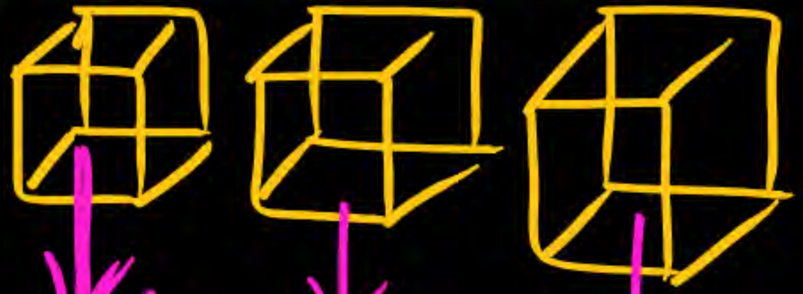
Three six faced dice are tossed together, then the probability that exactly two of the three numbers are equal is:

$P(E) = \dfrac{n(E)}{n(S)}$

Die A, Die B, Die C

SAME NUMBER

Total outcomes $= 6 \times 6 \times 6 = 216$



6 options, 6 options, 6 options

D1	D2	D3
$\boxed{a}$	$\boxed{a}$	b
$\boxed{a}$	b	$\boxed{a}$
$\boxed{a}$	$\boxed{a}$	b

A 165/216

B 177/216

C 51/216

D 90/216

✓ 1) CASE A

D_1 a D_2 a D_3 b

$6 \times 1 \times 5 = 30$ options

✓ 2) CASE B.

D_1 a D_2 b D_3 a

$6 \times 5 \times 1 = 30$ options

3) CASE 3

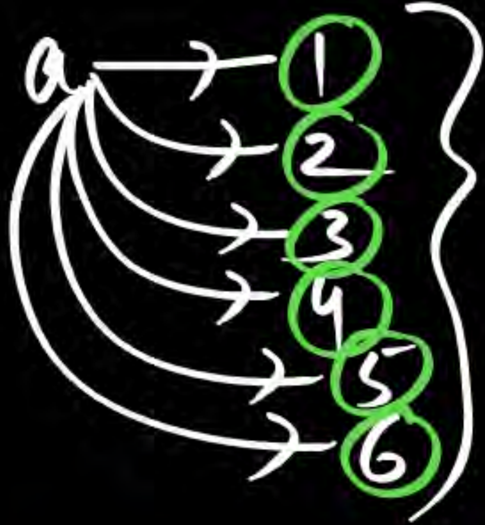
D_1 b D_2 a D_3 a

$6 \times 5 \times 1 = 30$ options

4 4 3   3, 4 4

↓ ↓ ↓   4 3 4

$\boxed{6 \; 1 \; 5}$



Total Fav outcomes $= 30 + 30 + 30 = 90$

$$P(E) = \frac{\text{Fav outcomes}}{\text{Possible outcomes}} = \frac{90}{216}$$

Question 7

If the letters of word 'REGULATIONS' be arranged at random, the probability that there will be exactly 4 letters between R and E is:

CHOOSE

Permutation

- **A** 6/55
- **B** 3/55
- **C** 49/55
- **D** None of these

A

'REGULATIONS'

1 2 3 4 5 6 7 8 9 10 11

$P(E) = \frac{n(E)}{n(S)}$

Total No. of arrangment = $11!$

No. of fav choice = ${}^9C_4 \cdot 4!\, 2!\, 6!$

$P(E) = \frac{{}^9C_4\, 4!\, 2!\, 6!}{11!}$

4 letters

R E → $2!$

4 letters Arrang → $6!$

1 2 R 1 2 3 4 E 4 5 6 Fix

1 2 3 4 arrangment $4!$

4 3 2 1

R E $2!$

$$P(E) = \frac{{}^9C_4 \cdot 4!\,2!\,6!}{11!}$$

$${}^9C_4 = \frac{9!}{5!\,4!}$$

$$= \frac{\frac{9!}{5!\,\cancel{4!}}\,\cancel{4!}\,2!\,6!}{11!} = \frac{9!\,2!\,6 \times \cancel{5!}}{\cancel{5!} \times 11!}$$

$$= \frac{\cancel{9!} \times \cancel{2!} \times 6}{11 \times \cancel{10}^{5} \times \cancel{9!}} = \frac{6}{55}$$

Question 8

$2n$ boys are randomly divided into two subgroups containing n boys each. The probability that the two tallest boys are in different groups is:

A $\frac{n}{2n-1}$

B $\frac{n-1}{2n-1}$

C $\frac{2n-1}{4n^2}$

D None of these

Question 9

In a bag there are three tickets numbered 1, 2, 3. A ticket is drawn at random and put back, and this is done four times the probability of that the sum of the numbers is even is:

P(SUM of No is Even)

A 41/81 — A

B 39/81

C 40/81

D None of these

1, 2, 3 — Four times

→ All odd — case A

$\underline{a+b+c+d}$

odd → Even

four times odd No

$\left\{\begin{matrix} 3+3+3+3 \\ 3+1+1+3 \end{matrix}\right]$ Even

Case-2 Two even + Two odd

$\underline{3+3+2+2}$ — even

Case-3 → No odd No

1, 2, 3 $\rightarrow$ Even: 4 odd No, SUM Even + Even: 2 odd No. or 2 even (SUM) Even + Even: No odd No

$P(\text{odd}) = \frac{2}{3}$

$P(\text{Even}) = \frac{1}{3}$

1 $\rightarrow$ case A: $4C_4\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)\left(\frac{2}{3}\right) = \left(\frac{2}{3}\right)^4 = \frac{2^4}{3^4}$

$\underline{a+b+c+d}$

4 Number

4 odd $4C_4$

$\rightarrow$ case 2: 2 odd No. and 2 even No. $= 4C_2\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^2 = 6 \times \frac{2^2}{3^4}$

a b c d

$\rightarrow$ case 3 $\longrightarrow$ 0 odd No. $4C_4\left(\frac{1}{3}\right)^4 = \left(\frac{1}{3}\right)^4$

a+b+c+d

Even

$P(\text{sum of } a+b+c+d = \text{even}) = {}^4C_4\left(\frac{2}{3}\right)^4 + {}^4C_2\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^2 + {}^4C_4\left(\frac{1}{3}\right)^4$

A+B+C+D

SUM - even

→ A, B, C, D odd

→ $\underline{A, B} \mid C, D$

2 odd

→ $\underline{A, B, C, D}$

No odd

$$= \frac{2^4}{3^4} + 6\cdot\frac{2^2}{3^4} + \frac{1}{3^4}$$

$$= \frac{2^4 + 6\times 4 + 1}{3^4}$$

$$= \frac{24+16+1}{3^4} = \frac{41}{81}$$

Question 10

A pack of cards consists of 15 cards numbered 1 to 15. Three cards are drawn at random with replacement. Then, the probability of getting 2 odd and one even numbered cards is:

A 348/1125

B 398/1125

C 448/1125

D 498/1125

OOE
EOO
OEO

1-15
15 cards.

THREE cards.
with replacement

2 odd, one even

OOE ✓
EOO ✓
OEO

15 cards.
8 odd
7 even

1, 2, 3, 15

$$P\left(\begin{matrix}\text{THREE}\\ \text{cards.}\end{matrix}\right) = P(OOE) + P(EOO) + P(OEO)$$

2 odd
1 even

$$\Rightarrow \frac{8}{15}\times\frac{8}{15}\times\frac{7}{15} + \frac{7}{15}\times\frac{8}{15}\times\frac{8}{15} + \frac{8}{15}\times\frac{7}{15}\times\frac{8}{15}$$

$$\Rightarrow \frac{64\times7\times\cancel{3}}{5\cancel{15}\times15\times15} = \frac{448}{1125}$$

Question 11

ABC n objects $= n!$
8 objects $= 8!$

A B C

Three persons A, B and C are to speak at a function along with five others. If they all speak in random order, the probability that A speaks before B and B speaks before C is:

A B C → communication

1 2 3 4 5 They speak random

A speaks before B and B speaks before C

8 objects A B C 1 2 3 4 5

Total No. of Possible outcomes $= 8!$

Fav outcomes $= {}^8C_3 \cdot 5!$

$$P(E) = \frac{{}^8C_3 \cdot 5!}{8!} = \frac{1}{6}$$

A. 3/8

B. 1/6

C. 3/5

D. None of these

Question 12

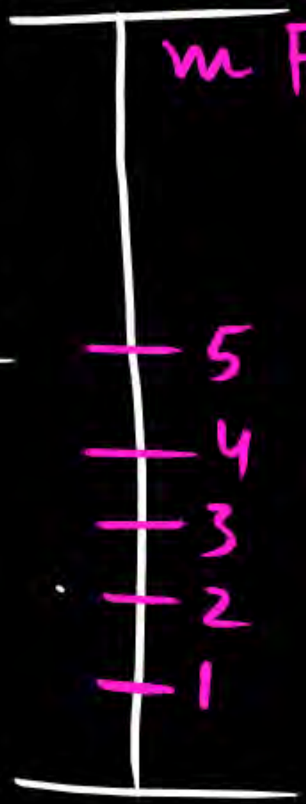
An elevator starts with m passengers and stops at n floors $(m \le n)$ the probability that no two passengers alight at same floor is:

A $\dfrac{{}^{n}P_{m}}{m^{n}}$ Ans

B $\dfrac{{}^{n}P_{m}}{n^{m}}$

C $\dfrac{{}^{n}C_{m}}{m^{n}}$

D $\dfrac{{}^{n}C_{m}}{n^{m}}$



m Floor n Person $m \le n$

$P_1, P_2, P_3, P_4, P_5, \ldots, P_n$ Permulate n Person

$P_1\ P_2\ P_3\ --\ P_n$

Arrangment

$= {}^{n}P_{m}$

Person floor

$P_1 \to 3$, $P_2 \to 2$, $P_3 \to 1$

$P(E) = \dfrac{{}^{n}P_{m}}{m^{n}}$

Person $\to$ Enter $\to m$

m Floor

$P_1 \to m$ choice

$P_2 \to m$ choice

$P_3 \to m$ "

$P_4 \to m$ "

$P_5 \to m$ "

$P_n \to m$ "

Total choice $= m \times m \times m \ldots$ n times

$= m^{n}$

Question 13

There are n persons sitting in a row. Two of them are selected at random. The probability that two selected persons are not together is:

A $\frac{2}{n}$

B $1-2/n$

C $\frac{n(n-1)}{(n+1)(n+2)}$

D None of these

Question 14

with replacement

A and B play a game where each is asked to select a number from 1 to 25. If the two numbers match both of them win a prize. The probability that they will not win a prize in a single trial is:

25 options | 1 to 25

- **A** 1/25
- **B** 24/25
- **C** 2/25
- **D** None of these

B

A

25 options

1 2 3 4 5 . . . 25

1 2 3 4 5 . . . 25

Total SAMPLE POINTS = 25X25

(1,1) (2,2) (3,3) (4,4) (5,5) (6,6) (7,7) (8,8) (9,9) (10,10) (11,11) (12,12) (13,13) (14,14) (15,15) (16,16) (17,17) (18,18) (19,19) (20,20) (21,21) (22,22) (23,23) (24,24) (25,25)

Die B = 6

Die A = 6

36

1–25

SAME

$\left.\begin{matrix} A = 5 \\ B = 5 \end{matrix}\right]$ win

1–25

W

LOOSE

$P(W) + P(L) = 1$

No. of Favourable outcomes = 25

Total No. of Possible outcomes = 25×25

$$P(\text{win}) = \frac{\cancel{25}}{25 \times \cancel{25}} = \frac{1}{25}$$

$$P(\text{wining}) = \frac{1}{25}$$

$$P(\text{Loose}) = 1 - \frac{1}{25} = \frac{24}{25}$$

$$= 1 - P(\text{win})$$

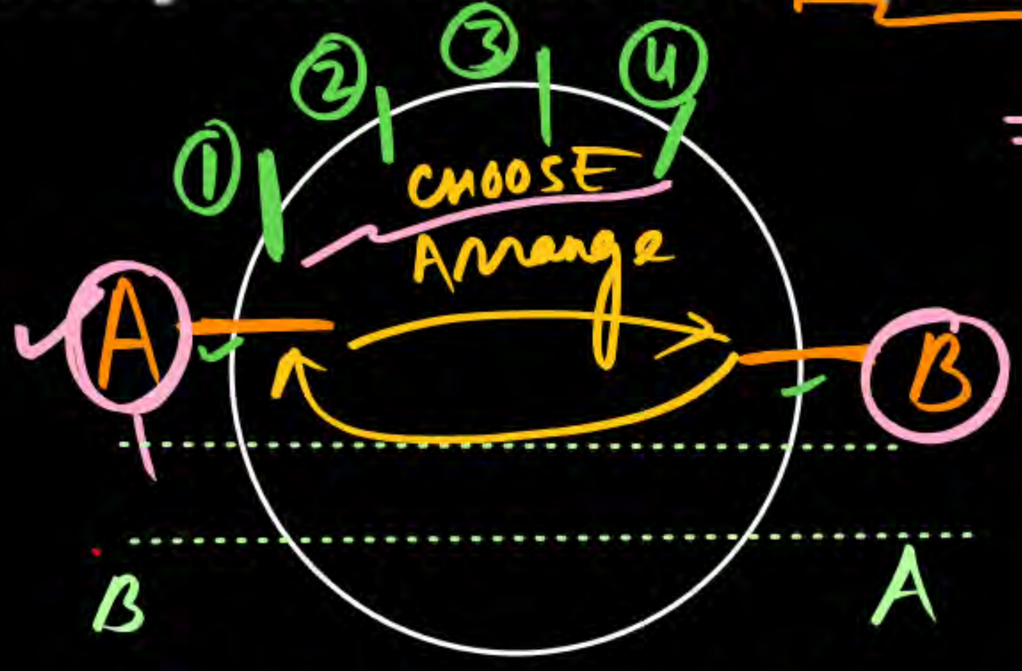
Question 15

$\lfloor 15-1 = \lfloor 14$

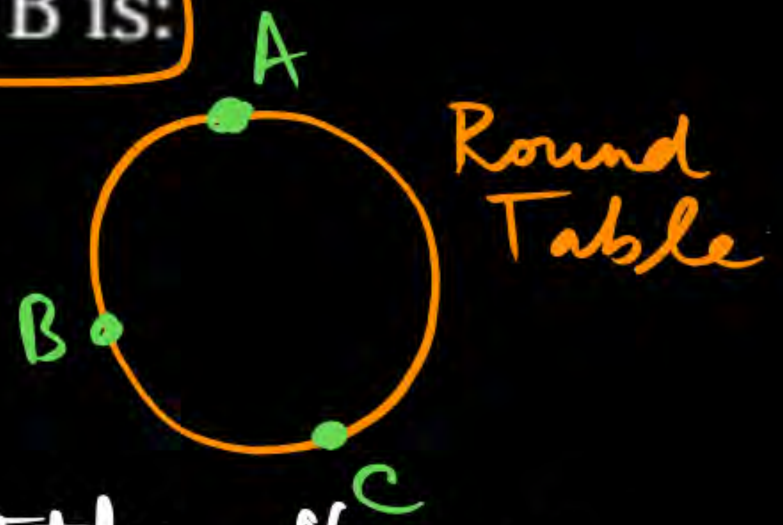
n objects

$\square\ \square\ \square \rightarrow n!$

Fifteen persons among whom are A and B, sit down at random at a round table. The probability that there are 4 persons between A and B is:



$= \frac{{}^{13}C_4 \cdot 4!\,2!\,9!}{\text{Fav choice}}$



Total No. of ways $= (n-1)!$

- **A** $\frac{9!}{14!}$
- **B** $\frac{10!}{14!}$
- **C** $\frac{9!}{15!}$
- **D** None of these $\frac{1}{7}$

$P(\text{Event}) = \frac{{}^{13}C_4 \cdot 4!\,2!\,9!}{14!} = \frac{1}{7}$

do yourself

Total No. of ways $= (15-1)! = 14!$

Question 16

The probability that the 13th day of a randomly chosen month is a second Saturday is:

Prob. of Month $= \frac{1}{12}$

Prob. of any Day in a week $= \frac{1}{7}$

Prob. of 13th Day second Saturday $= \frac{1}{12} \times \frac{1}{7}$

$= \frac{1}{84}$ ✓

A 1/7

B 1/12

C 1/84

D 19/84

Question 17

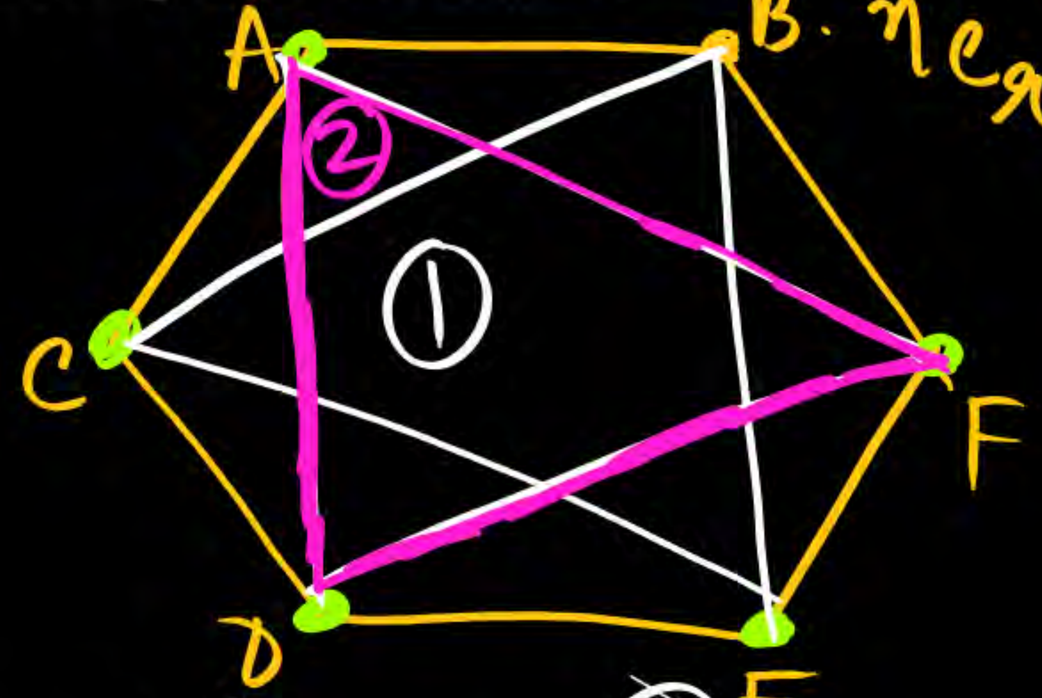
Three of the six vertices of a regular hexagon are chosen at random. The probability that the triangle with three vertices is equilateral, is:

CHOOSE ${}^{n}C_{r}$

Total No. of outcomes:

$\Rightarrow {}^{6}C_{3}$

Fav outcomes (equilateral Triangle) $= 2$



$$P(E) = \frac{\text{fav choice}}{\text{Total choice}} = \frac{2}{{}^{6}C_{3}} = \frac{\cancel{2}}{\frac{\cancel{2}\,6\times5\times4}{\cancel{3}\times\cancel{2}\times1}} = \frac{1}{10}$$

A	1/2
B	1/5
C	1/10
D	1/20

(C)

Question 18

The probability that out of 10 persons, all born in April, at least two have the same birthday is:

April month

P(at least Two have the same birthday)

= 1 − (None of Two are same birthday)

A $\dfrac{{}^{30}C_{10}}{(30)^{10}}$

B $1-\dfrac{{}^{30}C_{10}}{30!}$

C $\dfrac{(30)^{10}-{}^{30}C_{10}}{(30)^{10}}$

D None of these

→ 30 Days

→ 30 Days

30 Days

Total No. of ways

$=30\times30\times30\ldots$ 10 times

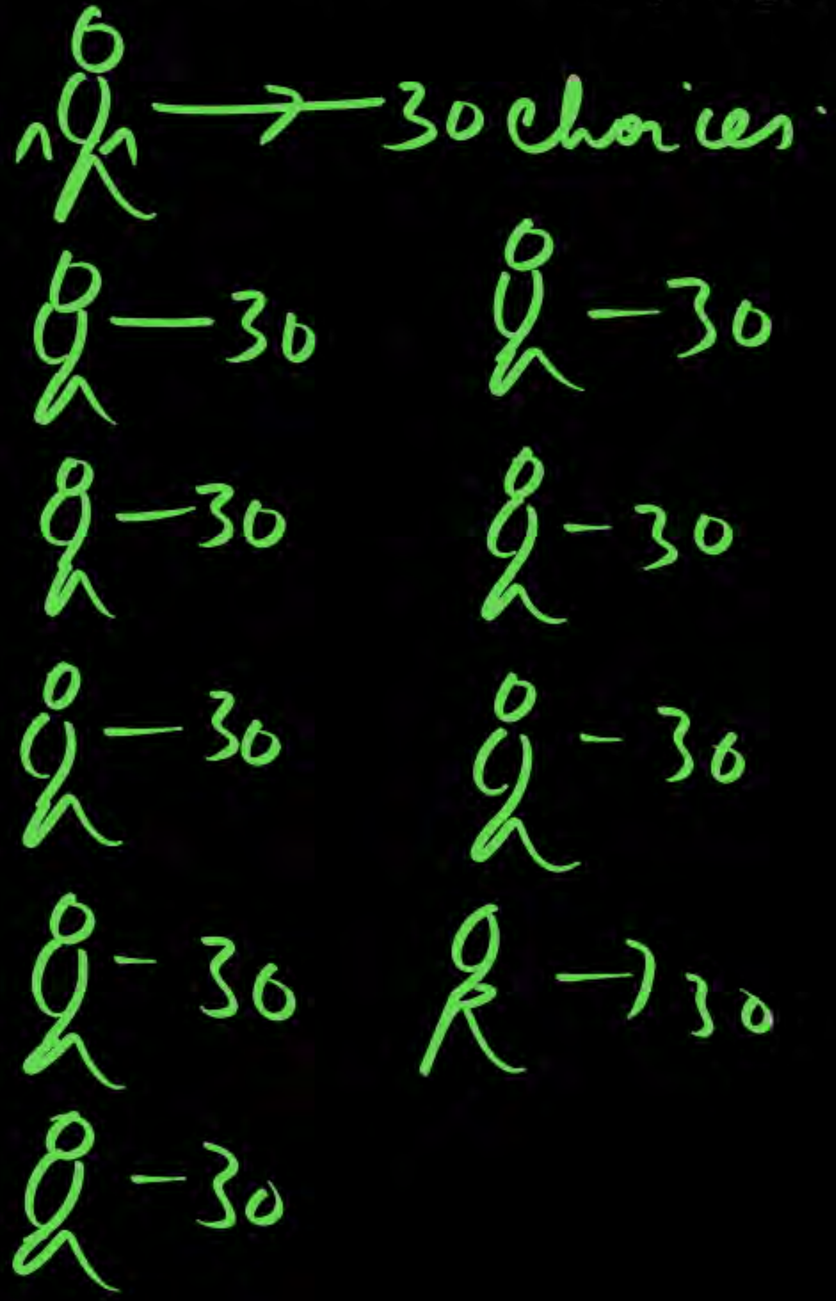
$=(30)^{10}$

A $\bar{A}$

30 Days

$\downarrow$

10 Persons



$$P(\text{atleast Two Persons SAME bday}) = 1 - P(\text{None of them Have SAME bday})$$

$$= 1 - \frac{{}^{30}C_{10}}{(30)^{10}}$$

$$= \frac{(30)^{10} - {}^{30}C_{10}}{(30)^{10}}$$

Thank you

GW Soldiers !