Computer Science & DA

Probability and Statistics

Probability

Lecture No. 04

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Recap of previous lecture

Topic Conditional probability

Topics to be Covered

Topic Bayes' Theorem

Q: Parcels are sending from Sender S to Receiver R Sequentially through two post offices. The Prob of loosing an incoming parcel by each post office is $\frac{1}{5}$ independently by each Post office. Given that Parcel is Lost then Find the Prob that it was lost by 2^{nd} P.O ?

Cond^n

Sol: Original Prob $= P(S) = 1$

Reduced Prob $= P(\text{Cond}^n)$

$= P(\text{parcel has been lost})$

$= P[\text{either (lost by } 1^{st}) \text{ or (Lost by } 2^{nd})]$

$= P(\text{Lost by } 1^{st}) + P(\text{lost by } 2^{nd})$

$= P(\text{lost by } 1^{st}) + P[\text{NL by } 1^{st} \ \& \text{ L by } 2^{nd}]$

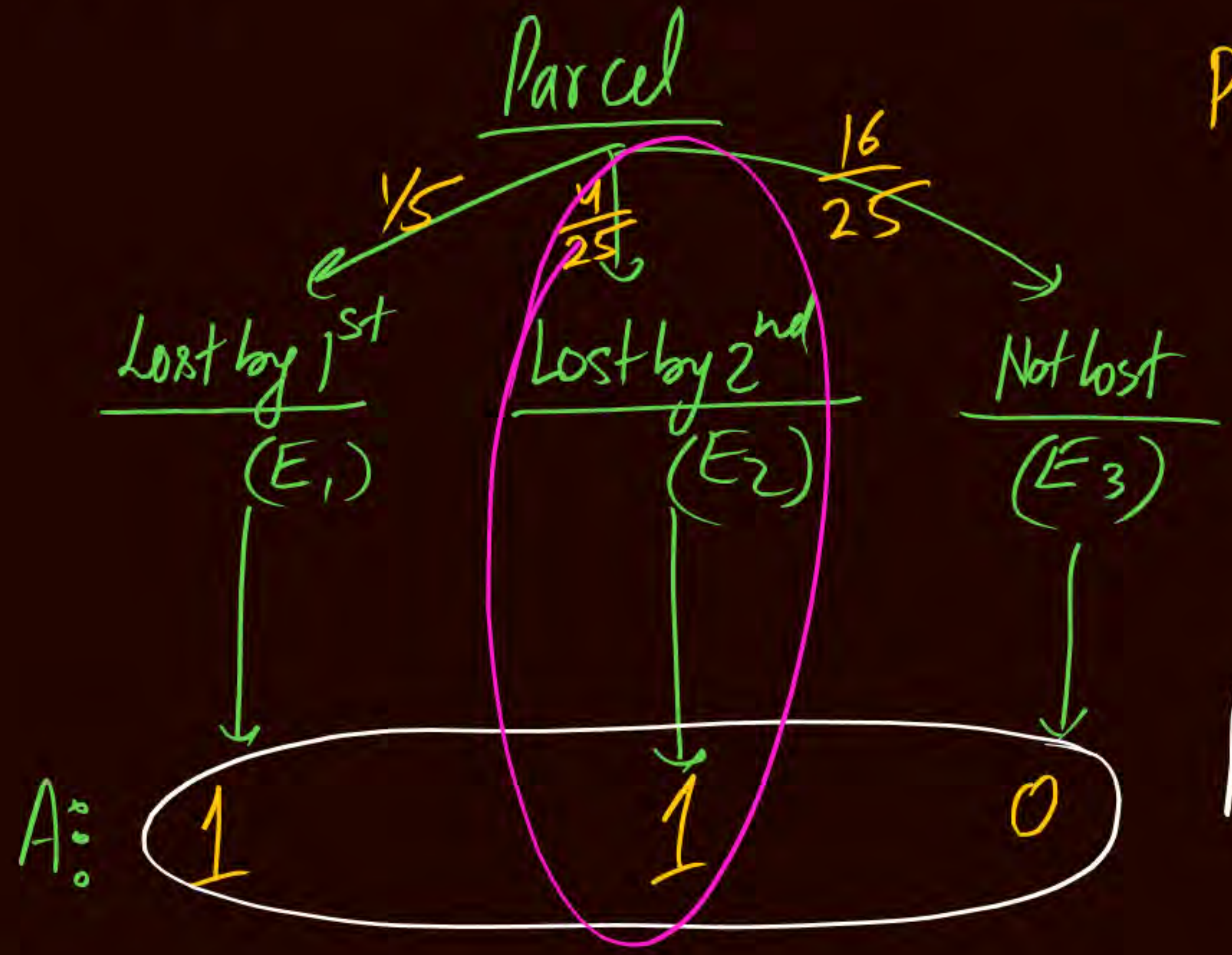
$= \frac{1}{5} + \frac{4}{5} \times \frac{1}{5} = \frac{9}{25}$

Fav Prob $= P(\text{Lost by } 2^{nd}) = \frac{4}{5} \times \frac{1}{5} = \frac{4/25}{1}$

Hence Cond^n Prob $= \frac{\text{fav Prob}}{\text{R. Prob}} = \frac{4/25}{9/25}$

$= \frac{4}{9}$

M-II Using Baye's Th → $A = \{\text{Parcel in Lost}\}$



$\because P(E_1)+P(E_2)+P(E_3)=1$

$$P(\text{lost by }2^{nd}) = P(NL \text{ by } 1^{st} \,\&\, L \text{ by } 2^{nd}) = \frac{4}{5}\times\frac{1}{5} = \frac{4}{25}$$

$$P(NL \text{ by any P.O}) = P(NL \text{ by } 1^{st} \,\&\, NL \text{ by } 2^{nd}) = \frac{4}{5}\times\frac{4}{5} = \frac{16}{25}$$

$$P(A) = \left(\frac{1}{5}\times 1\right) + \left(\frac{4}{25}\times 1\right) + \left(\frac{16}{25}\times 0\right) = \frac{9}{25}$$

$$P(E_2/A) = \frac{\frac{4}{25}\times 1}{\left(\frac{9}{25}\right)} = \frac{4}{9}$$

BAYE'S Theorem

Exhaustive Events → if $(E_1 \cup E_2 \cup E_3 = S) \Longleftrightarrow$ then (E_1, E_2, E_3 are Called Exhaustive events)

$S = \{1, 2, 3, 4, 5, 6\}$

(i) $E_1 = \{1, 3, 5\}$, $E_2 = \{2, 4, 6\}$

$\because E_1 \cup E_2 = S$ So these are Exhaustive Events.

(ii) $E_1 = \{1 \le No \le 3\}$, $E_2 = \{3 < No < 6\}$, $E_3 = \{6\}$

$E_1 = \{1, 2, 3\}$, $E_2 = \{4, 5\}$, $E_3 = \{6\}$

Again $E_1 \cup E_2 \cup E_3 = S \Rightarrow E_1, E_2, E_3$ are Exh. events

Mutually Exclusive Events →

if $E_i \cap E_j = \phi \Longleftrightarrow$ then Events are ME

ME & Exhaustive Events →

E_1, E_2, E_3 are ME & Exh. events

if
- $E_i \cap E_j = \phi \quad \forall\, i \ne j$
- $E_1 \cup E_2 \cup E_3 = S$

Note: if (E_1, E_2, E_3 are ME and Exh) then $\Leftrightarrow$ $\boxed{P(E_1)+P(E_2)+P(E_3)=1}$

Proof: $\because$ Events are Exhaustive so $E_1 \cup E_2 \cup E_3 = S$

$$P(E_1 \cup E_2 \cup E_3) = P(S)$$

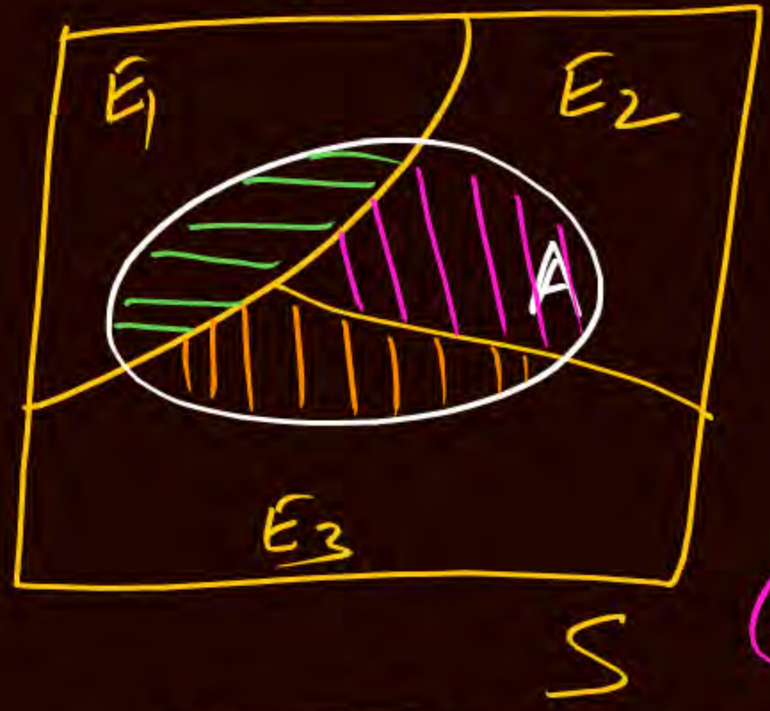
$$P(E_1)+P(E_2)+P(E_3)-\cancel{P(E_1 \cap E_2)}^{0}-\cancel{P(E_2 \cap E_3)}^{0}-\cancel{P(E_2 \cap E_3)}^{0}+\cancel{P(E_1 \cap E_2 \cap E_3)}^{0}=1$$

i.e. $P(E_1)+P(E_2)+P(E_3)=1$ //

eg: $D=\{1,2,3,4,5,6\}$ $\to$ $E_1=\{1,3,5\}$, $E_2=\{2,4,6,1\}$

$\because E_1 \cup E_2 = S$ & these are Exhaustive But Not ME $\because E_1 \cap E_2 \neq \phi$

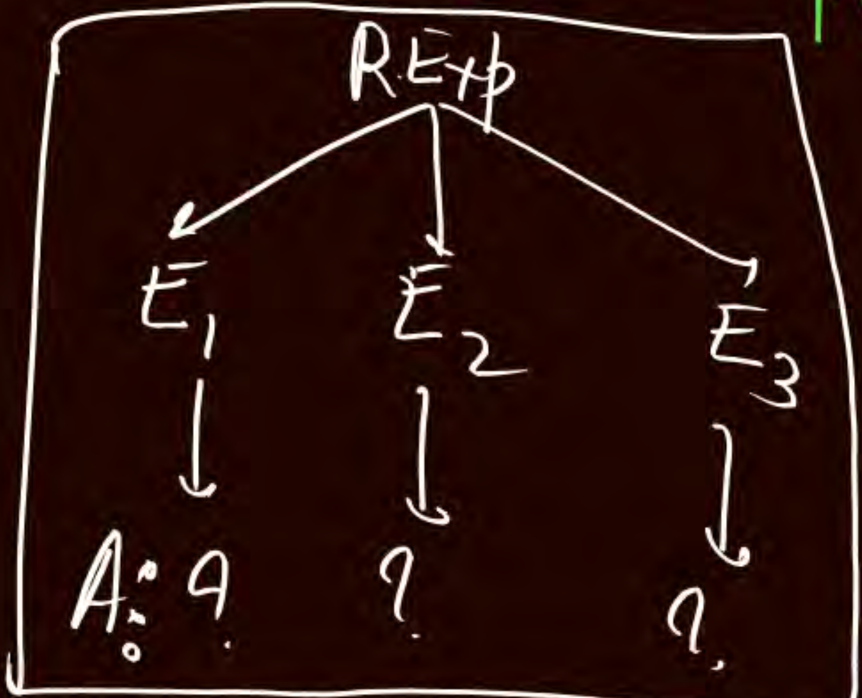
Law of Total Prob:- Let E_1, E_2, E_3 are ME & Exhaustive events and A is an Event that can occur with all E_1, E_2, E_3 then



$$\boxed{P(A) = P(E_1)P(A/E_1) + P(E_2)P(A/E_2) + P(E_3)P(A/E_3)} \quad \text{—— (1)}$$

BAYE'S Theorem:- Theory same as above
(Inverse Prob Th)

$$P(E_1/A) = \frac{P(E_1)P(A/E_1)}{P(A)}, \quad P(E_2/A) = \frac{P(E_2)\,P(A/E_2)}{P(A)}, \quad P(E_3/A) = \frac{P(E_3)P(A/E_3)}{P(A)}$$



Note (1) whenever there is a feeling of cross check the given condⁿ, we use B.Th
(2) N.Condⁿ for B.Th are Associated Events must be ME & Exhaustive
(3) A = {Assume A that event which is given as condition}
(4) B.Th is useful to solve complex Problems of Conditional Prob.

Q. Computers are supplied to an organisation as per the following chart;

Company	% of computers supplied	Probability of being defective
X	60%	0.01
Y	30%	0.02
Z	10%	0.03

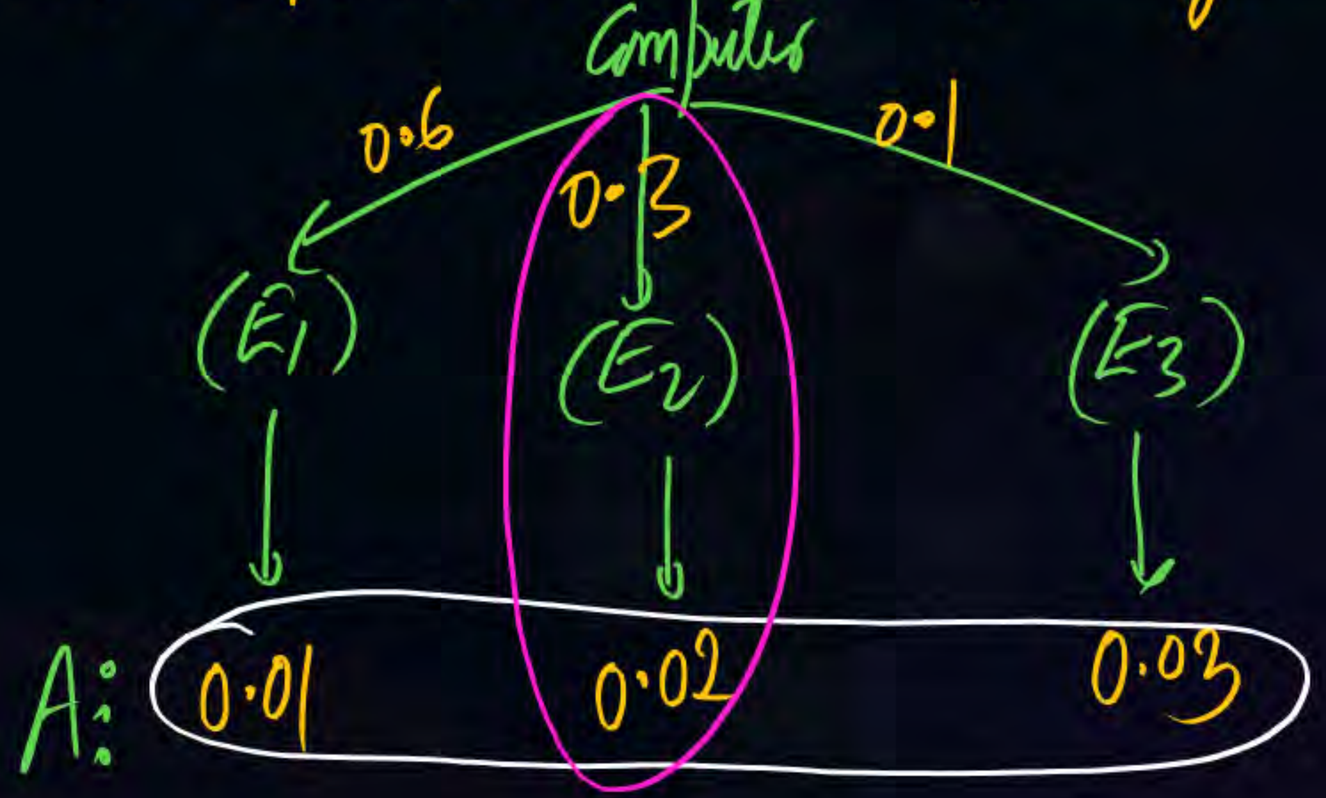
Given that, supplied computer is defective (A), then find the Prob that it was supplied by Company Y

Sol: $A = \{$supplied computer is defective$\}$

$E_1 = \{$Supplied by X$\}$

$E_2 = \{$ " " Y$\}$

$E_3 = \{$ " " Z$\}$



$$P(A) = (0.6 \times 0.01) + (0.3 \times 0.02) + (0.1 \times 0.03) = \frac{0.015}{1} = \frac{15}{1000}$$

$P(\text{Cond}^n) = P(\text{supplied Comp is defective}) = 0.015$

out of 1000 supplied computers only 15 are defective

fav Prob $= 0.3 \times 0.02 = 0.006$

$$\text{Cond}^n \text{ Prob} = \frac{\text{fav. Prob}}{\text{R. Prob}} = \frac{0.006}{0.015} = \frac{6}{15}$$

ie $P(E_2/A) = \dfrac{P(E_2).P(A/E_2)}{P(A)} = \dfrac{0.006}{0.015} = \dfrac{6}{15}$

Similarly $P(E_1/A) = \dfrac{6}{15}$

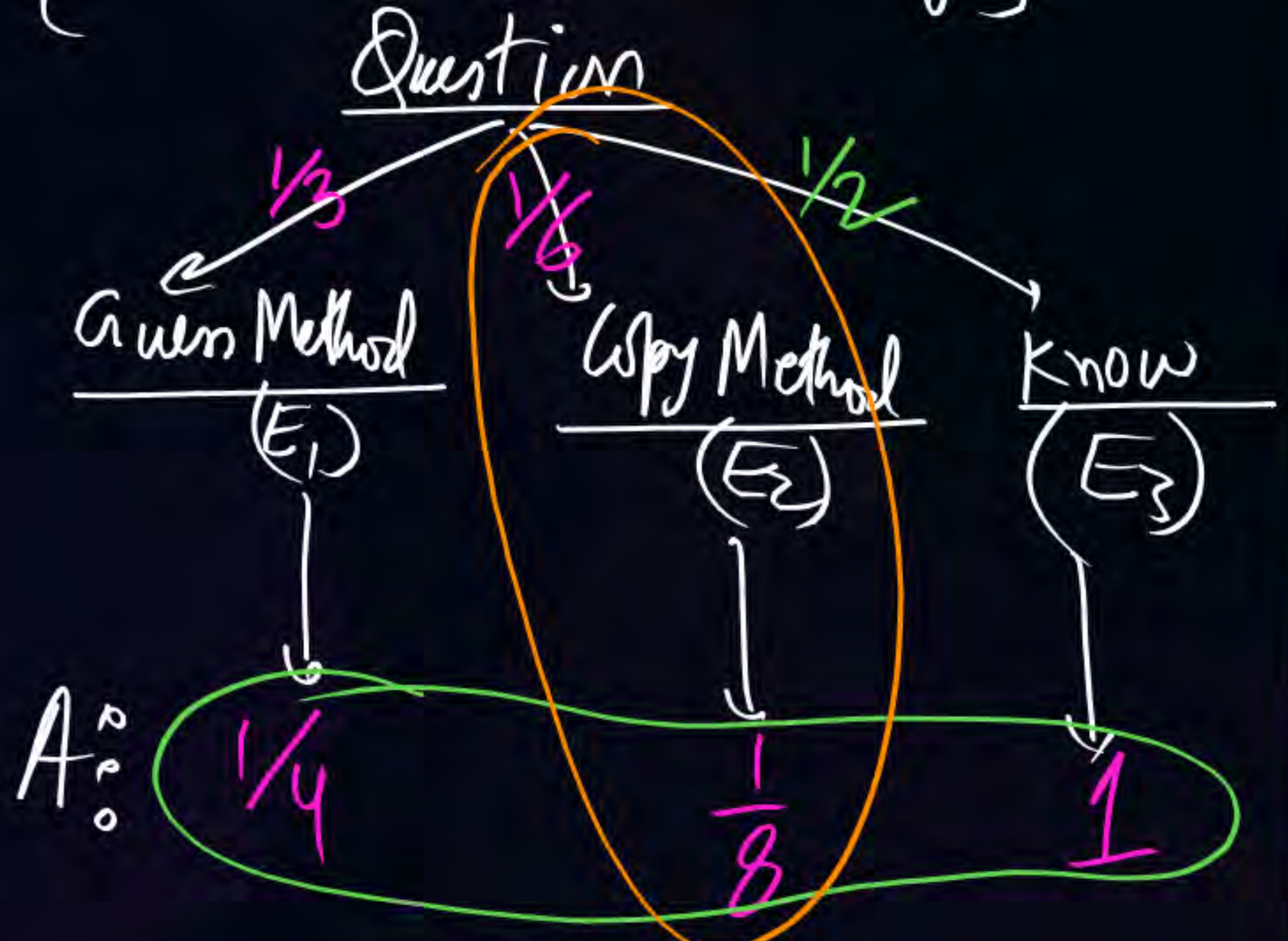
$P(E_3/A) = \dfrac{3}{15}$

out of 15 defective computers 6 are supplied from X, 6 from Y & 3 from Z.

Q: In a test an examinee either Guess or Copy or Know the answer of an objective type Q, having 4 choices each. The prob of making Guess is $\frac{1}{3}$ and Copying is $\frac{1}{6}$. The prob. that his answer is correct given that he copied it is $\frac{1}{8}$ then find the prob that he copy the answer if he answered correctly?

Condition.

Sol: $A = \{\text{Examinee answered correctly}\}$



Now by def^n; E_1, E_2, E_3 are ME & Exhaustive

So $P(E_1)+P(E_2)+P(E_3)=1 \Rightarrow P(E_3)=1-\frac{1}{3}-\frac{1}{6} = \frac{1}{2}$

$$P(A) = \left(\frac{1}{3}\times\frac{1}{4}\right)+\left(\frac{1}{6}\times\frac{1}{8}\right)+\left(\frac{1}{2}\times 1\right) = \frac{29}{48}$$

$$P(E_2/A) = \frac{\frac{1}{6}\times\frac{1}{8}}{29/48} = \frac{1}{29}$$

ANALYSIS :- (1) $P(E_2/A) = \frac{1}{29}$, (2) $P(E_1/A) = \frac{4}{29}$, (3) $P(E_3/A) = \frac{24}{29}$

(4) $P(A) = \frac{29}{48} \Rightarrow$ out of 48 attempted Questions only 29 are correct & out of 29 correct answers, he copied only 1

(5) Marks obtained by that student if each Q. is of 1 Marks (w/o -ve Marking)

$$= \frac{29}{48}$$

(6) Marks obtained by that Student (if Examiner is Puneet Sharma) = ? $= \frac{28}{48}$

Q. In a town 10% of the population is COVID (+ve). A New Diagonostic arrives in the market. This kit correctly identifies covid +ve individual 95% of time and covid −ve individual 89% of time. A person is tested by this kit and is found to be +ve then find the prob that person is actually +ve?

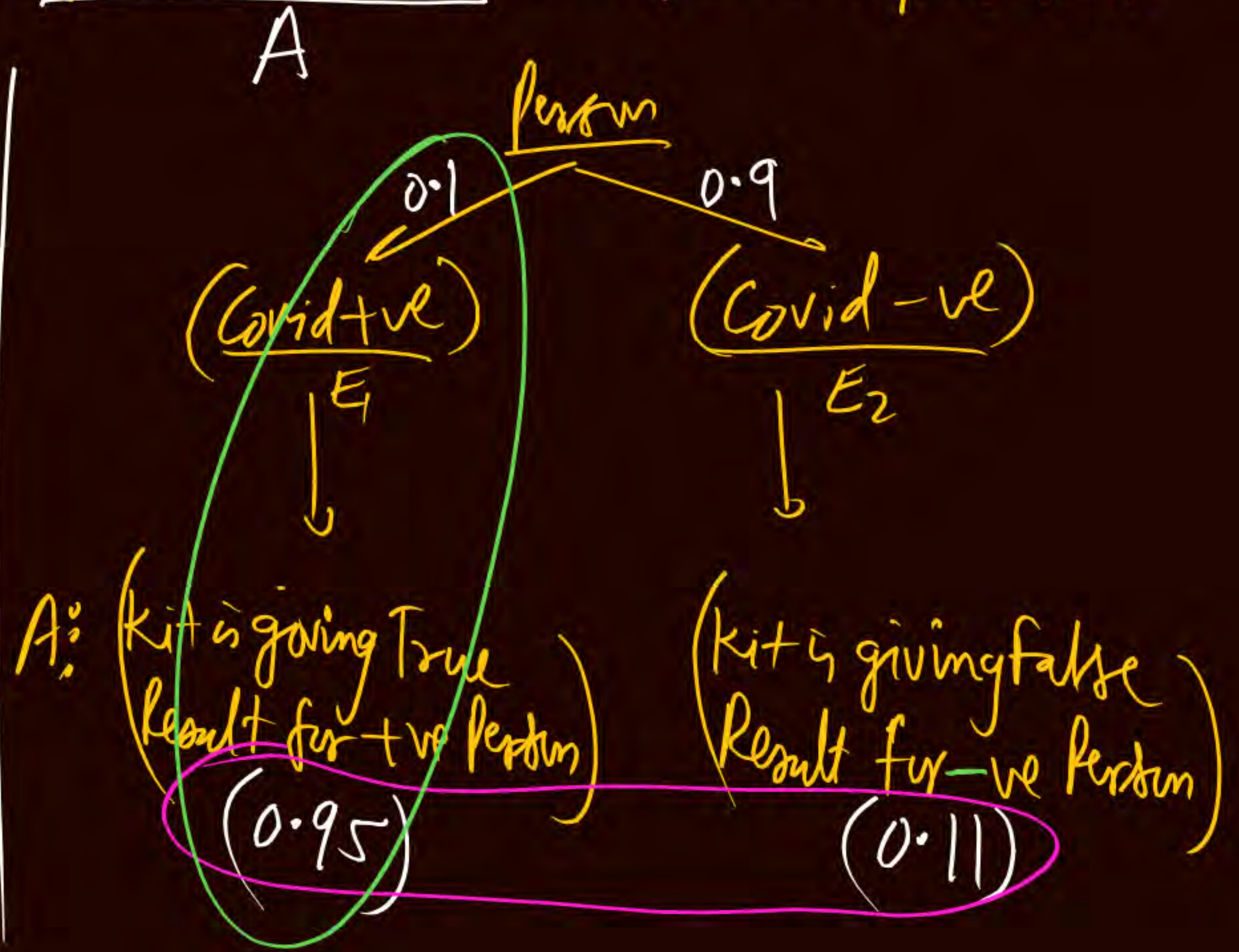
R.Exp.

Soln: $A = \{\text{Person declared +ve by Kit}\}$

$$P(A) = (0.1 \times 0.95) + (0.9 \times 0.11) = 0.194$$

$$P(\text{actually +ve}) = P(E_1/A) = \frac{0.1 \times 0.95}{0.194}$$

$$= \frac{95}{194}$$



① $P\left(\begin{array}{c}\text{person declared +ve} \\ \text{by kit}\end{array}\right) = \dfrac{0.194}{1} = \dfrac{194}{1000}$ & $P(E_2/A) = P(\text{actually +ve}) = \dfrac{95}{194}$

out of 1000 persons tested by kit, 194 are declared +ve by kit & out of 194 +ve declared persons, only 95 are actually +ve

②

1000 persons
- 100 person are actually +ve
  - $\xrightarrow{95\%}$ 95 are declared +ve by kit (Correct Result)
  - $\xrightarrow{5\%}$ 5 " " -ve by kit (wrong R.)
- 900 " " actually -ve
  - $\xrightarrow{89\%}$ 801 are declared -ve by kit (Correct R)
  - $\xrightarrow{11\%}$ 99 " " +ve by kit (wrong R)

3 1000 persons

- 194 are declared +ve by kit
  - 95 are actually +ve
  - 99 are actually −ve
- 806 are declared −ve by kit
  - 801 are actually −ve
  - 5 are actually +ve

THANK - YOU