Computer Science & Information Technology

Discrete Mathematics

DPP: 1

Set Theory and Algebra

Q1 Which of the following statements are true?

I. $\phi \in \phi$

II. $\phi \subset \phi$

III. $\phi \subseteq \phi$

IV. $\phi \in \{\phi\}$

V. $\phi \subset \{\phi\}$

Vi. $\phi \subseteq \{\phi\}$

Q2 If a set A has 63 proper subsets, then what is the cardinality of A?

Q3 If a set A has 64 subsets of odd cardinality, then what is |A|?

(A) 6
(B) 63
(C) 7
(D) 128

Q4 How many subset of {1, 2, 3, ..., 11} contain at-least one even integer?

Q5 Let A = {1, 2, 3, 4, 5, 7, 8, 10, 11, 14, 17, 18}
How many subsets of A contain six elements?

Q6 Let A = {2, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 15}
How many six-elements subsets of A contain four even integers and two odd integers?

Q7 Let A = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}
How many subsets of A contain only odd integers?

Q8 Suppose A, B, C and D are subsets of U (the universe) with A as a subset of B and C as subset of D
i.e A ⊆B and C ⊆ D, then consider the following statements

I. $A \cap C \subseteq B \cap D$

II. $A \cup C \subseteq B \cup D$

Which of the following is correct options?

(A) Only I is true
(B) Only II is true
(C) Neither I nor II is true
(D) Both I and II are true

Q9 Let A = {1, 2, 3, ... 15}. How many subsets of A contains all of the odd integers in A?

Q10 Let $A, B \subseteq$, where $A = \{x \mid x^2 - 7x = -12\}$ and $B = \{x \mid x^2 - x = 6\}$. Determine $A \cup B$ and $A \cap B$.

(A) $A \cup B$ = {5} and $A \cap B$ = {−2, 3, 4}
(B) $A \cup B$= {3} and $A \cap B$ = {−2, 3, 4}
(C) $A \cup B$ = {−2, 3, 4} and $A \cap B$ = {3}
(D) $A \cup B$ = {2, 3, 4} and $A \cap B$= {5}

Answer Key

Q1	4~4	Q6	225~225
Q2	6~6	Q7	63~63
Q3	(C)	Q8	(D)
Q4	1984~1984	Q9	128~128
Q5	924~924	Q10	(C)

Hints & Solutions

Q1 Text Solution:

I. $\phi \in \phi$ is false. The empty set has no members.

II. $\phi \subset \phi$ is false. The empty set is not a proper subset of itself.

III. $\phi \subseteq \phi$ is true. The empty set is a subset of every set

$\therefore$ subset of itself

IV. $\phi \in \{\phi\}$is true. phi is a member here.

V. $\phi \subset \{\phi\}$ is true. The empty set is a proper subset of itself.

VI. $\phi \subseteq \{\phi\}$ is true. The empty set is a subset of every set.

Q2 Text Solution:

If a set has n elements then the number of subsets will be 2^n and the number of proper subsets will be $2^n - 1$.

A has 63 proper subsets, so $2^n - 1 = 63$

$2^n = 63 + 1$

$2^n = 64$

$2^n = 2^6$

$\therefore n = 6$

The cardinality of A is 6

Q3 Text Solution:

The number of subsets for {1, 2, 3, ... n} with odd cardinality is 2^{n-1}.

Number of subsets with cardinality i = ${}^nC_i$

So, the number of subsets with odd cardinality

$\sum i = 1, 3 \ldots n - 1 \; {}^nC_i = 2^{n-1}$.

Now, given

$2^{n-1} = 64$

$2^{n-1} = 2^6$

$n - 1 = 6$

[Bases are same, so equating power]

$n = 7$

Q4 Text Solution:

2^{11} subset for {1, 2, 3, ...11}

2^6 subset for {1, 3, 5, 7, 9, 11} contains none of the even integers {2, 4, 6, 8, 10}.

Hence, there are $2^{11} - 2^6 = 1984$ subsets that contain at least one even integer.

Q5 Text Solution:

If we choose 6 elements from a set of 12 elements where order does not matter. Then we can do it in ${}^{12}C_6$ ways.

For example consider a set = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12} out of this set we have to choose a subset of 6 elements. This can be done in following ways:

1, 2, 3, 4, 5, 6}, {2, 3, 4, 5, 6, 7},{3, 4, 5, 6, 7, 8},

{4, 7, 8, 3, 11, 12}, {5, 7, 9, 10, 11, 12}..... and so on.

That means the arrangement or order of elements does not matter, therefore we can do it using combinations.

$$^{12}C_6 = \frac{12 \times 11 \times 10^{2} \times 9^{3} \times 8^{2} \times 7 \times 6!}{6! \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}$$

$= 11 \times 12 \times 7$

$= 77 \times 12$

$= 924$

Q6 Text Solution:

Out of 6 element subsets, we can choose 4 even integers in ${}^6C_4$ ways.

Similarly to find 2 odd integers out of 6 element subset, can be done in ${}^6C_2$ ways.

Therefore

$$^6C_4 * {}^6C_2 = \frac{6^{3} \times 5 \times 4!}{2 \times 4!} \times \frac{6^{3} \times 5 \times 4!}{4! \times 2}$$

$= 15 * 15 = 225$

Q7 Text Solution:

In the given set, there are 6 odd integers, we have two choices for each odd integer to be included or not included, therefore total possibilities = $2^6 - 1 = 63$.

Q8 Text Solution:

I. $A \cap C \subseteq B \cap D$, is True.

Let a be an arbitrary element of $A \cap C$, so $a \in A \cap C$ then $a \in A \subseteq B$, so $a \in B$ and $a \in C \subseteq D$, so $a \in D$. That concludes that $a \in B$ and $a \in D$, therefore by definition $a \in B \cap D$. If follows that every element of A ∩ C belongs to B ∩ D, which by definition means $A \cap C \subseteq B \cap D$.

II. $A \cup C \subseteq B \cup D$, is True.

If a is an arbitrary element that belongs to $A \cup C$ then it definitely belongs to $B \cup D$ as $A \subseteq B$ and $C \subseteq D$.

Q9 Text Solution:

In the given set A = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15}

There are 8 odd integers. For all odd integer we have choices whether to include it or not with the 7 even integers in the set.

Therefore possiblities = 2^7 = 128.

Q10 Text Solution:

$x^2 - 7x = -12 \Rightarrow x^2 - 7x + 12 = 0 \Rightarrow (x-4)(x-3) = 0 \Rightarrow x = 4, x = 3.$

$x^2 - x = 6 \Rightarrow x^2 - x - 6 = 0 \Rightarrow (x-3)(x+2) = 0 \Rightarrow x = 3, x = -2.$

Consequently, $A \cap B = \{3\}$ and $A \cup B = \{-2, 3, 4\}$.