Computer Science & IT

Discrete Mathematics

Set Theory & Algebra

Lecture No. 05

By- Vishal Sir

Recap of Previous Lecture

Topic: Principle of Inclusion and Exclusion

Topic: Multi-set

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Topics to be Covered

Topic Questions based on Multi-set

Topic Cartesian Product

Topic Relation

Topic Types of Relation

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Topic : Multi-set

$$P = \{m_1 \cdot a_1, m_2 \cdot a_2, m_3 \cdot a_3, \ldots, m_k \cdot a_k\}$$

$$Q = \{n_1 \cdot a_1, n_2 \cdot a_2, n_3 \cdot a_3, \ldots, n_k \cdot a_k\}$$

- m_i is multiplicity of a_i in P
- n_i is multiplicity of a_i in Q

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Topic : Multi-set

(i) Multiplicity of a_i in $P \cup Q$ ✓

multiplicity of $a_i = \max(m_i, n_i)$

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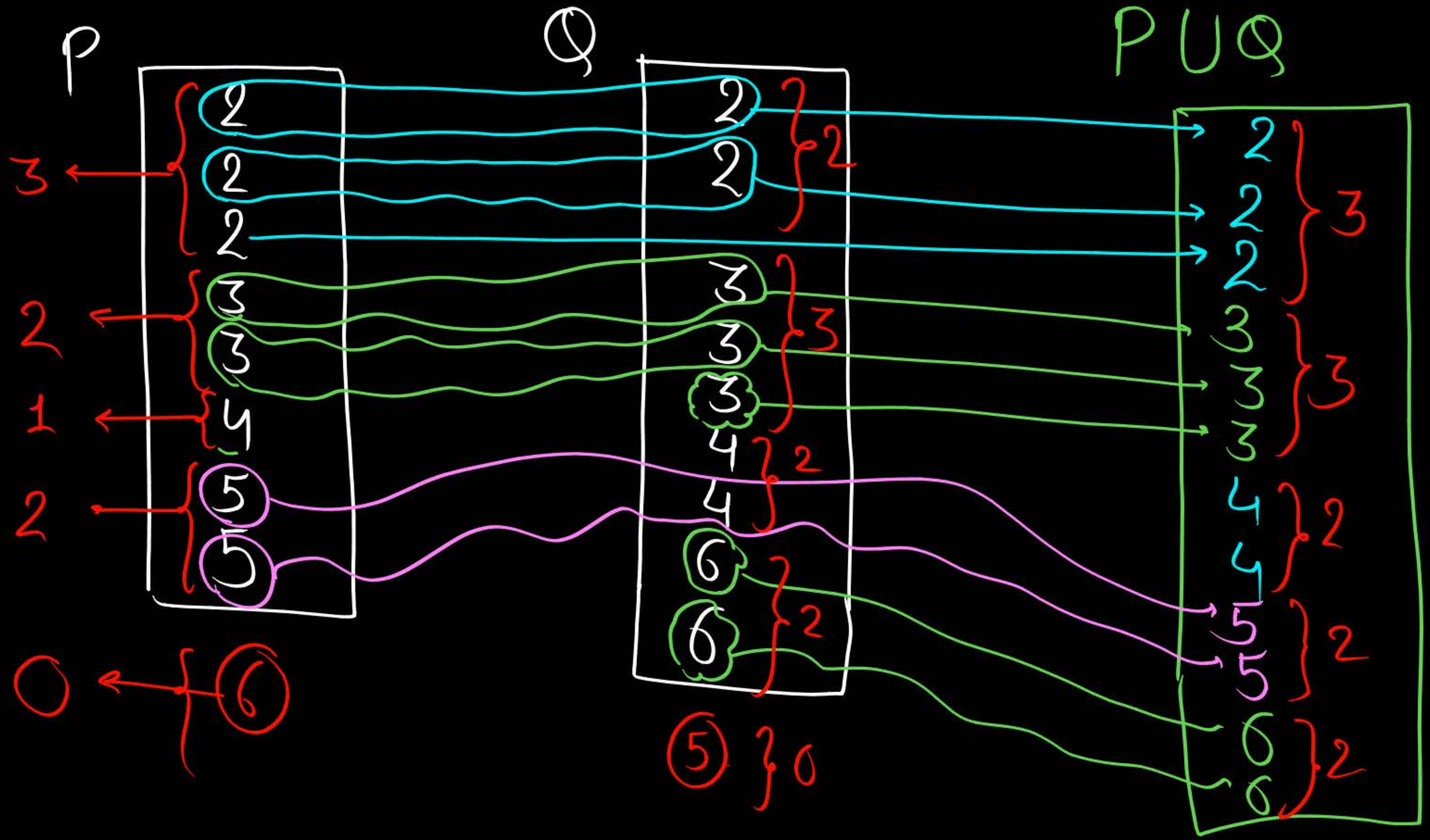

P
Q
P U Q
2 2 2 3 3 4 5 5
2 2 3 3 3 4 4 6 6
2 2 2 3 3 3 4 4 5 5 6 6
3
2
1
2
0
6
5
0

Topic : Multi-set

* Multiplicity of a_i in $P \cap Q$,

multiplicity of $a_i = \text{Min}(m_i, n_i)$

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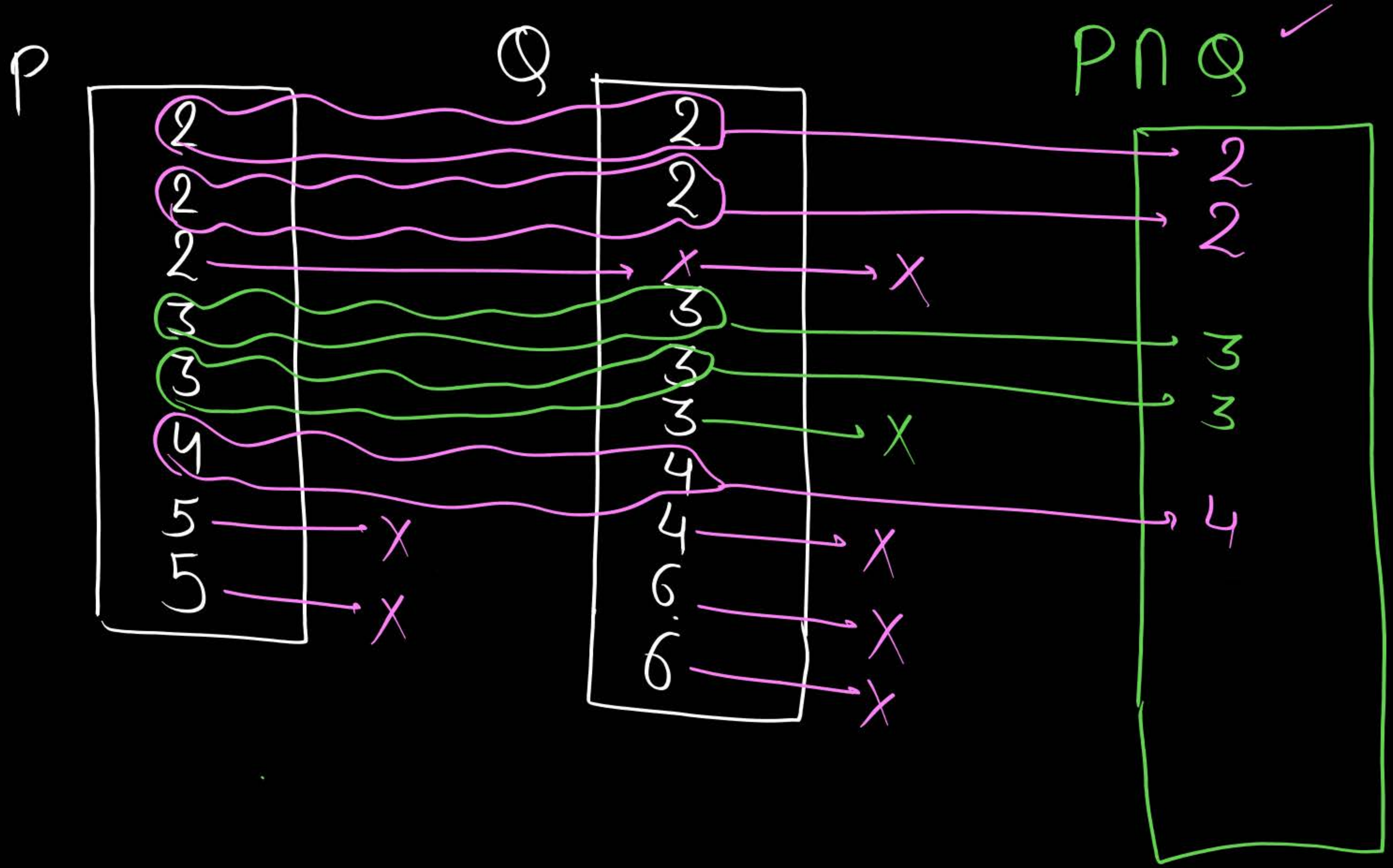
P
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Q
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6
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P ∩ Q
2
2
3
3
4

Topic : Multi-set

* Multiplicity of a_i in $P-Q$

$$\text{multiplicity of } a_i = \begin{cases} m_i - n_i & \text{if } m_i > n_i \\ 0 & \text{if } m_i \leq n_i \end{cases}$$

$m_i \geq n_i$

$m_i < n_i$

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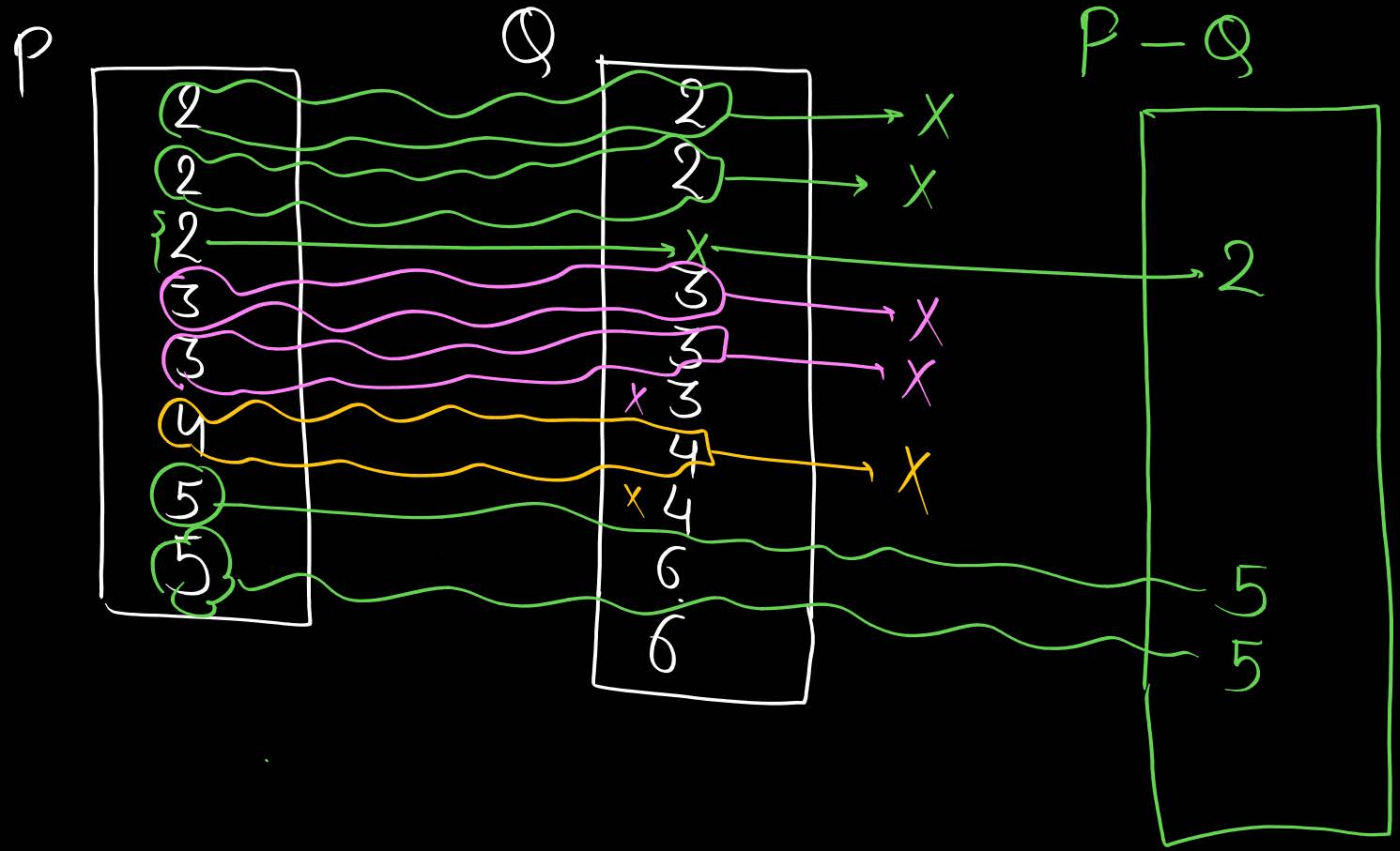
P
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P − Q
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* How many multisets are possible with an element 'a'

$\{0.a\} \leftarrow$ multiset of size = 0

$\{a\} \leftarrow$ Multiset of size = 1

$\{a, a\} \leftarrow$ Multiset of size = 2

$\{a, a, a\} \leftarrow$ Multiset of size = 3

$\vdots$

We can use 'a' infinite times, $\therefore$ infinite multisets are possible

Q: How many multisets are possible with the elements of set $A = \{1, 2, 3, \ldots n\}$

(a) 2^n (b) n^2 (3) n^n (d) None

With a single element infinite multi-sets are possible if we do not restrict the size of multiset

Q:- Let $A = \{1, 2, 3, 4, \ldots n\}$

How many Multisets of size = 4 are possible using the elements of set A, such that at least one element appears exactly twice in the multiset.

$= \text{Type 1 Multisets} + \text{Type 2 Multisets}$

$$= \frac{n(n-1)(n-2)}{2} + \frac{n(n-1)}{2}$$

$$= \frac{n(n-1)^2}{2}$$

$A = \{1, 2, 3, \ldots, n\}$

order is not important

Possible Cases

Type 1 multiset

Type 2 multiset

$\{a, b, c, a\} \equiv \{a, a, b, c\}$

$\{b, b, a, a\} \equiv \{a, b, a, b\} \equiv \{a, a, b, b\}$

$= {}^{n}C_1 * 1 * {}^{(n-1)}C_2 * 1$

$= \boxed{n * \frac{(n-1)(n-2)}{2}}$

Or

$+$

$\left[\frac{{}^{n}C_1 * 1 * {}^{(n-1)}C_1 * 1}{2}\right]$

$= \frac{n(n-1)}{2}$

a, b, c

a, a, b, c

b, b, a, c

c, c, a, b

$= {}^{n}C_3 * {}^{3}C_1 * 1 * 1$

$= \frac{n(n-1)(n-2)}{3 * 2} * 3 * 1 * 1$

$= \frac{n(n-1)(n-2)}{2}$

${}^{n}C_2 * 1 = \frac{n(n-1)}{2}$

H.W. Let $A = \{1, 2, 3, \ldots\ldots n\}$

How many multisets of size $= 4$ are possible from the elements of set A.

$\longrightarrow$

Topic : Cartesian Product

- let A and B are any two finite sets, then Cartesian product $A \times B$ is defined as set of all ordered pair (x, y) such that $x \in A$ and $y \in B$

$$A \times B = \{ (x, y) \mid \underset{\substack{\uparrow \\ \text{first element} \\ \text{from set A}}}{x} \in A \text{ and } \underset{\substack{\uparrow \\ \text{second element} \\ \text{from set B}}}{y} \in B \}$$

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Order pair (x, y)

$\downarrow$

Order in which the elements appear is important

in general, $(x, y) \neq (y, x)$

$\therefore$ In general

$$A \times B \neq B \times A$$

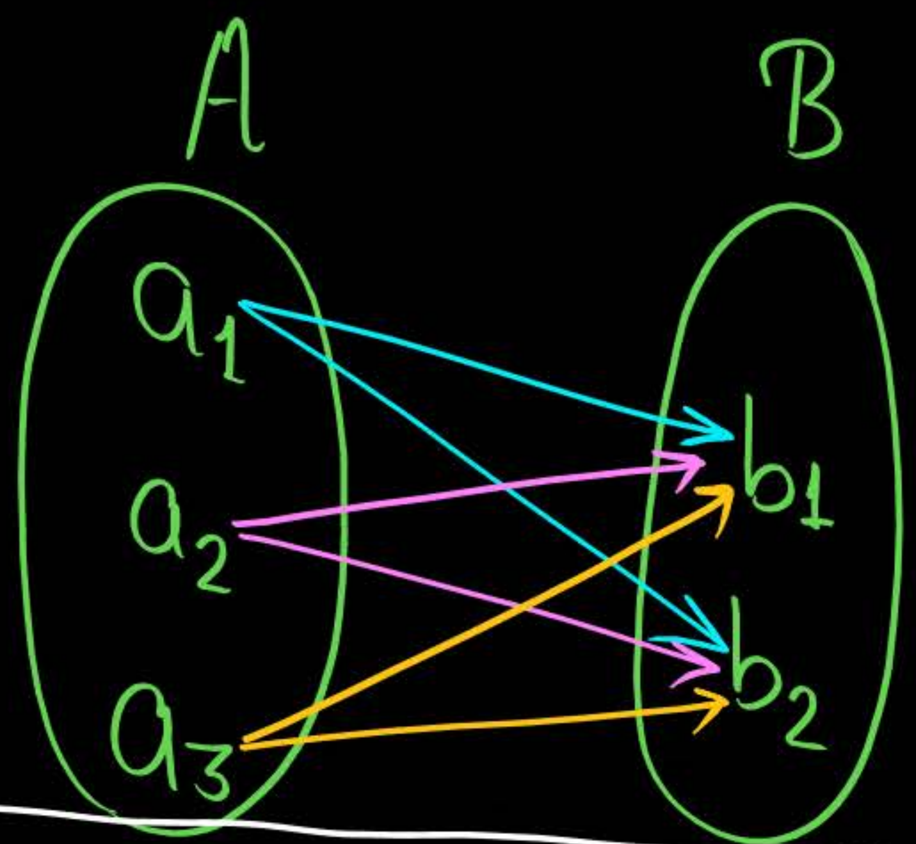
Note: If $A \times B = B \times A$, then either $A = B$

or at least one of A or B is empty set.

* If any of the two sets A or B is an empty set, then $A \times B$ (or $B \times A$) will also be an empty set

Let $A = \{a_1, a_2, a_3\}$ $B = \{b_1, b_2\}$

$$A \times B = \{(a_1, b_1), (a_1, b_2), (a_2, b_1), (a_2, b_2), (a_3, b_1), (a_3, b_2)\}$$



$$B \times A = \{(b_1, a_1), (b_1, a_2), (b_1, a_3), (b_2, a_1), (b_2, a_2), (b_2, a_3)\}$$

* Cardinality of Cartesian product :-

let $|A| = m$ and $|B| = n$,

then $|A \times B| = |A| \cdot |B|$

$$\boxed{|A \times B| = m \cdot n}$$

Topic : Relation

* Let A and B are two sets.

A relation from A to B defines that how the elements of set A relates with elements of set B.

In castesion product $A \times B$, every element of set A relates with every element of set B.

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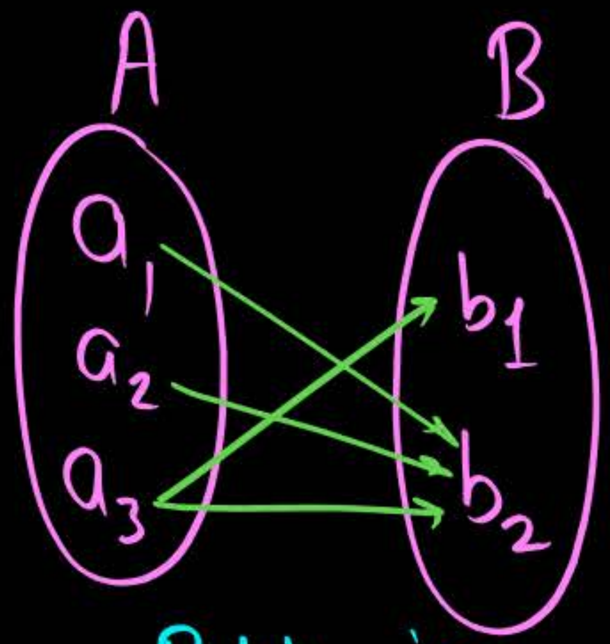
Note:- Every relation from A to B is a subset of $A \times B$.



Relation 'R'

$$\boxed{R = \{(a_1, b_2), (a_2, b_2), (a_3, b_1), (a_3, b_2)\}}$$

It is definitely a subset of $A \times B$

Cartesian product $A \times B$ is universal relation w.r.t. relations from set A to set B.

* Number of relations possible from set A to set B:

Let $|A| = m$ & $|B| = n$,

then number of relations possible from A to B $=$ Number of subsets of $A \times B$

Because each subset of $A \times B$ represent a relation from A to B

$$= 2^{|A \times B|}$$

$$= 2^{|A| \cdot |B|}$$

$$\boxed{\text{Number of relations possible from A to B} = 2^{m \cdot n}}$$

Note:- A relation from set A to set A itself is called a relation on set A.

* let $|A| = n$,

then Number of relations possible on set A $= 2^{|A \times A|}$

$= 2^{|A| \cdot |A|}$

$= 2^{n^2}$

Topic : Types of Relations

defined from A to A
i.e. Relations defined on same set

(1) Diagonal Relation (Identity Relation)

(2) Reflexive Relation

(3) Irreflexive Relation

(4) Symmetric Relation

(5) Anti-Symmetric Relation

(6) Asymmetric Relation

(7) Transitive Relation

(8) Complement of a Relation

(9) Inverse of a Relation

Relation can be defined from any set to Any set.

(10) Composite of two Relations

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2 mins Summary

Topic Cartesian Product & Relations

Topic Types of Relations

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THAK - YOU

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