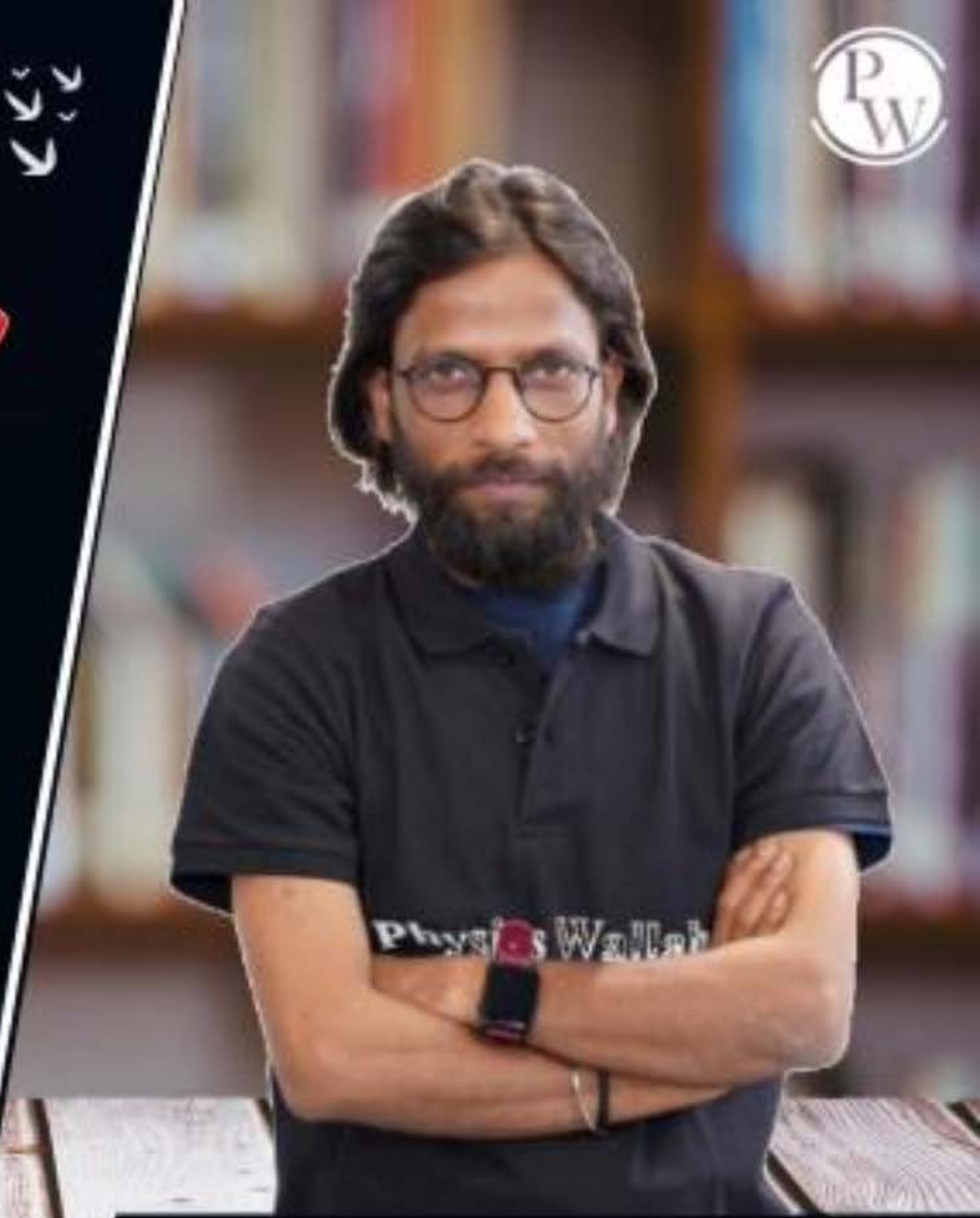
GATE-ALL BRANCHES

ENGINEERING MATHEMATICS

Linear Algebra

Lecture No.- 03



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Recap of Previous Lecture

Topic Concepts of a matrix

Topics to be Covered

Topic Problems on Determinant of a matrix

Topic Matrix multiplication

Topic Adjoint and inverse of a matrix

Topic : Determinant of a matrix

gate

#Q. The determinant of the matrix given below

$$\begin{array}{c|l} \begin{bmatrix} 0 & 1 & 0 & 2 \\ -1 & 1 & 1 & 3 \\ 0 & 0 & 0 & 1 \\ 1 & -2 & 0 & 1 \end{bmatrix} & \begin{array}{l} - R_1 \\ - R_2 \\ - R_3 \\ - R_4 \end{array} \end{array}$$

4×4

$(-1)^{i+j}$

A. -1 ✓

B. 0

C. 1

D. 2

$(-1)^7 \begin{vmatrix} 0 & 1 & 0 \\ -1 & 1 & 1 \\ 1 & -2 & 0 \end{vmatrix}$

$= (-1)(-1) \begin{vmatrix} -1 & 1 \\ 1 & 0 \end{vmatrix}$

$= 1 \begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix} = -1$

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Topic : Determinant of a matrix

#Q. The determinant of the matrix given below $\begin{bmatrix} 6 & -8 & 1 & 1 \\ 0 & 2 & 4 & 6 \\ 0 & 0 & 4 & 8 \\ 0 & 0 & 0 & -1 \end{bmatrix}$

4×4 matrix

→ upper Triangular matrix

$\det A = |A| = 6 \times 2 \times 4 \times -1$

$= \underline{-48}$

A 11

B -48

C 0

D -24

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Topic : Determinant of a matrix

#Q. The determinant of the matrix given below $\begin{bmatrix} 2 & 0 & 0 & 0 \\ 8 & 1 & 7 & 2 \\ 2 & 0 & 2 & 0 \\ 9 & 0 & 6 & 1 \end{bmatrix}$

A 4

B 0

C 15

D 20

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Topic : Determinant of a matrix

H.W

#Q. If $\Delta = \begin{bmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{bmatrix}$ then which of the following is a factor of Δ.

(a) a + b

(b) a - b

(c) abc

(d) a + b + c

$\boxed{\text{Ans} = (a-b)}$

Slide

Properties of Inverse matrix:

(A) If A, B, C Are Non Singular matrix

$(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$

Reversal Law

$(ABCD)^{-1} = D^{-1}C^{-1}B^{-1}A^{-1}$

$A^{-1} = \frac{\text{adj } A}{|A|}$

If $|A| \neq 0$

Non Singular matrix

(B) $(KA)^{-1} = \frac{1}{K}A^{-1}$

K = constant

$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $(8A)^{-1} = \frac{1}{8}A^{-1}$ ✓

(C) $(A^T)^{-1} = (A^{-1})^T$

$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $(A^T)^{-1} = (A^{-1})^T$

D) $(A^K)^{-1} = (A^{-1})^K$

$(A^2)^{-1} = (A^{-1})^2$

(E) $|A^{-1}| = \frac{1}{|A|}$

$|A^{-1}| = \frac{1}{|A|}$

Trace of matrices:

Trace(A) = SUM of diagonal elements

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{3\times 3}$$

$Tr(A) = \text{spur} = a_{11} + a_{22} + a_{33}$

for $n \times n$ matrix (square matrices)

$$\boxed{Tr(A) = \sum_{i=j}^{n} a_{ij}}$$

1) $Trace(A \pm B) = Trace(A) \pm Trace\ B$ A and B Square matrices

2) $Trace(KA) = K\,Trace(A)$ where A is Square matrices

Some special matrices:

Symmetric Matrix:

In Symmetric matrix
Diagonal element → any thing

$$A = \begin{bmatrix} 1 & 4 & 5 \\ 4 & 2 & 6 \\ 5 & 6 & 3 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & 4 & 5 \\ 4 & 2 & 6 \\ 5 & 6 & 3 \end{bmatrix}_{3\times3}$$

Principal Diagonal

(mirror/Reflection)
SAME Elements

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{\substack{n\times n \\ (3\times3)}} = \begin{bmatrix} a_{11} & \boxed{\odot} & \boxed{\odot} \\ \boxed{a_{21}} & a_{22} & \boxed{\odot} \\ \boxed{a_{31}} & \boxed{a_{32}} & a_{33} \end{bmatrix}$$

mirror

Symmetric matrix $\boxed{A^T = A}$

Skew Symmetric matrix :

Diagonal elements must (0,0,0)

Square matrix

$$A = \begin{bmatrix} 0 & -b & -c \\ b & 0 & -d \\ c & d & 0 \end{bmatrix}_{3\times 3}$$

reflected elements

= Square Matrix

Diagonal elements

skew sym. matrix $\boxed{A^T = -A}$

det odd order $= \underline{\underline{0}}$

Orthogonal matrix :

$AA^T = I$

Square matrix

$$A = \begin{bmatrix} \frac{3}{5} & \frac{4}{5} \\ -\frac{4}{5} & \frac{3}{5} \end{bmatrix}_{2\times 2} \longrightarrow \text{orthogonal matrix}$$

$$A^T = \begin{bmatrix} \frac{3}{5} & -\frac{4}{5} \\ \frac{4}{5} & \frac{3}{5} \end{bmatrix}$$

orthogonal matrix $= \underline{1 \text{ or } -1}$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2\times 2}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Conjugate of a matrices:

$$A = \begin{bmatrix} 1+i & 2 \\ 3 & 1-i \end{bmatrix} \qquad \bar{A} = A^c = \begin{bmatrix} 1-i & 2 \\ 3 & 1+i \end{bmatrix}$$

- $z = a+ib$
- $z^c = \bar{z} = a-ib$

(Diagonal - Anything)

Hermitian matrix $(\bar{A})^T = A$ or $A^\dagger = A$ or $A^* = A$

↑ Degan

Skew Hermitian matrix $(\bar{A})^T = -A$ or $A^\dagger = -A$ or $A^* = -A$

Diagonal elements → 0 or Purely Imaginary

Unitary matrix $AA^\dagger = I$

or

$A(\bar{A})^T = I$

or

$AA^* = I$

Nilpotent matrix $A^K = [0]$ = Null matrix

Idempotent matrix $\boxed{A^2 = A}$

Involutary matrix $\boxed{A^2 = I_{n\times n}}$

Periodic matrix $A^{K+1} = A$ $K=1$ $\boxed{A^2 = A}$

Idempotent

Singular matrix $|A| = 0$

Non Singular matrix $|A| \neq 0$

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}_{2\times 2}$ is a Nilpotent matrax What is The value of 'K'

$A^K = [0]$

$K = 2, 3, 4, \ldots$

Topic : Determinant of a matrix

#Q.Given that $A = \begin{bmatrix} -5 & -3 \\ 2 & 0 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, the value of A^3 is

Power of matrix

(a) 15 A + 12 I

(b) 19 A + 30 I

(c) 17 A + 15 I

(d) 17 A + 21 I

$A = \begin{bmatrix} -5 & -3 \\ 2 & 0 \end{bmatrix}_{2\times2}$ $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ my square matrix

$|A - I\lambda| = 0$ → STEP 1 (Polynomial) → quadratic eqn

$\begin{vmatrix} -5-\lambda & -3 \\ +2 & 0-\lambda \end{vmatrix} = 0$

$(-5-\lambda)(-\lambda) + 6 = 0$

$+5\lambda + \lambda^2 + 6 = 0$

Slide

$$\lambda^2+5\lambda+6=0 \longrightarrow \text{Polynomial}$$

STEP 02 Put $\lambda = A$

$$A^2+5A+6=0$$

$$\boxed{A^2=-5A-6}$$

$x^2=-5x-6$

$x^3=-5x^2-6$

$=-5(-5x-6)-6$

pre multiply A

$$\underline{AA^2}=-5A^2-6A$$

$$\underline{A^3}=-5(-5A-6)-6A$$

$$=+25A+30I-6A$$

$$\boxed{A^3=19A+30I}$$

$$A^3=19A+30I$$

multiply A

$$A^4=19A^2+30A$$

$$=\boxed{19(-5A-6I)+30A}$$

$$=$$

$$A^3=19\begin{bmatrix} & \\ & \end{bmatrix}+30\begin{bmatrix}1&0\\0&1\end{bmatrix}$$

Topic : Determinant of a matrix

GATE (EE)

$P = I_6 + \alpha J_6$

#Q. Consider the matrix

$$I_6 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} + \alpha J_6 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$P = I_6 + \alpha J_6$

$$P = \begin{array}{c} R_1 \\ \\ \\ \\ \\ R_6 \end{array} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & \alpha \\ 0 & 1 & 0 & 0 & \alpha & 0 \\ 0 & 0 & 1 & \alpha & 0 & 0 \\ 0 & 0 & \alpha & 1 & 0 & 0 \\ 0 & \alpha & 0 & 0 & 1 & 0 \\ \alpha & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad \alpha = 1$$

both Are SAME

If Two Rows - SAME $\det A = 0$

Which is obtained by reversing the order of the columns of the identity matrix I_6. Let $\boxed{P = I_6 + \alpha J_6}$, where α is a non - negative real number. The value of α for which $\boxed{\det(P) = 0}$ is ______. $\det(P) = 0$ $\quad \alpha = 1$

$\alpha = ?$

$$\begin{matrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{matrix} \begin{matrix} - R_1 \\ - R_6 \end{matrix}$$

Slide

Topic : Determinant of a matrix

$\boxed{x = -1 \quad x = 2}$

#Q. The solution set of the equation $\begin{vmatrix} 1 & 4 & 20 \\ 1 & -2 & 5 \\ 1 & 2x & 5x^2 \end{vmatrix} = 0$ is

The Solution SET

$\det A = 0$

____

$x = 2$

$x = -1$

$x = 2$

$$\begin{vmatrix} 1 & 4 & 20 \\ 1 & -2 & 5 \\ 1 & 4 & 20 \end{vmatrix} \begin{matrix} - R_1 \\ - R_2 \\ - R_3 \end{matrix}$$

$\det A = 0$

$x = 2$

$x = -1$

$$\begin{vmatrix} 1 & 4 & 20 \\ 1 & -2 & 5 \\ 1 & -2 & 5 \end{vmatrix} \begin{matrix} R_1 \\ R_2 \\ R_3 \end{matrix}$$

$\det A = 0$

Slide

Topic : Determinant of a matrix

#Q. Let $P = [a_{ij}]$ be a 3×3 matrix and let $Q = [b_{ij}]$, where $b_{ij} = 2^{i+j} a_{ij}$ for $1 \le i, j \le 3$. If the determinant of P is 2, then the determinant of the matrix Q is

(a) 2^{10}

(b) 2^{11}

(c) 2^{12}

(d) 2^{13}

$b_{ij} = 2^{i+j} a_{ij}$

$b_{11} = 2^{1+1} a_{11}$

$b_{12} = 2^{1+2} a_{12}$

$b_{13} = 2^{1+3} a_{13}$

$$Q = \begin{bmatrix} 2^2 a_{11} & 2^3 a_{12} & 2^4 a_{13} \\ 2^3 a_{21} & 2^4 a_{22} & 2^5 a_{23} \\ 2^4 a_{31} & 2^5 a_{32} & 2^6 a_{33} \end{bmatrix}_{3\times 3}$$

$$Q = 2^2 \times 2^3 \times 2^4 \begin{vmatrix} a_{11} & 2a_{12} & 2^2 a_{13} \\ a_{21} & 2a_{22} & 2^2 a_{23} \\ a_{31} & 2a_{32} & 2^2 a_{33} \end{vmatrix} = 2^2 \times 2^3 \times 2^4 \times 2 \times 2^2 \left| a_{ij} \right| = 2^{12} \times 2 = 2^{13}$$

$|P| = 2$

Slide

Topic : Determinant of a matrix

$a+e+i=9$

sum of diagonal entries $= \underline{\underline{9}}$

$M = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 12 \end{bmatrix}$

#Q. Let M be a 3 × 3 matrix satisfying

$M \times \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$

$$M\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}, M\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \text{ and } M\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 12 \end{bmatrix}.$$

Then the sum of the diagonal entries of M is

Principal diagonal.

Matrix = sum of diagonal

$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ +1 \\ -1 \end{bmatrix},$

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THANK - YOU