GATE
ALL BRANCHES

ENGINEERING MATHEMATICS

Probability & Statistics

Lecture-14

BY- RAHUL SIR

ME/CE

TOPICS TO BE COVERED

01 Poisson's distribution

02 Problems based on Poisson's distribution.

Poisson Distribution: Poisson Distribution – models.

Per

Poisson → Discrete Distribution (Arrival)

Average No. of success in a given time Interval.

1 success

Δt

0 (failure

1 success

$\boxed{1}$ success

$\boxed{0}$ -

$\boxed{1}$

$\boxed{0}$

$\boxed{1}$

event की Prob. Rare.

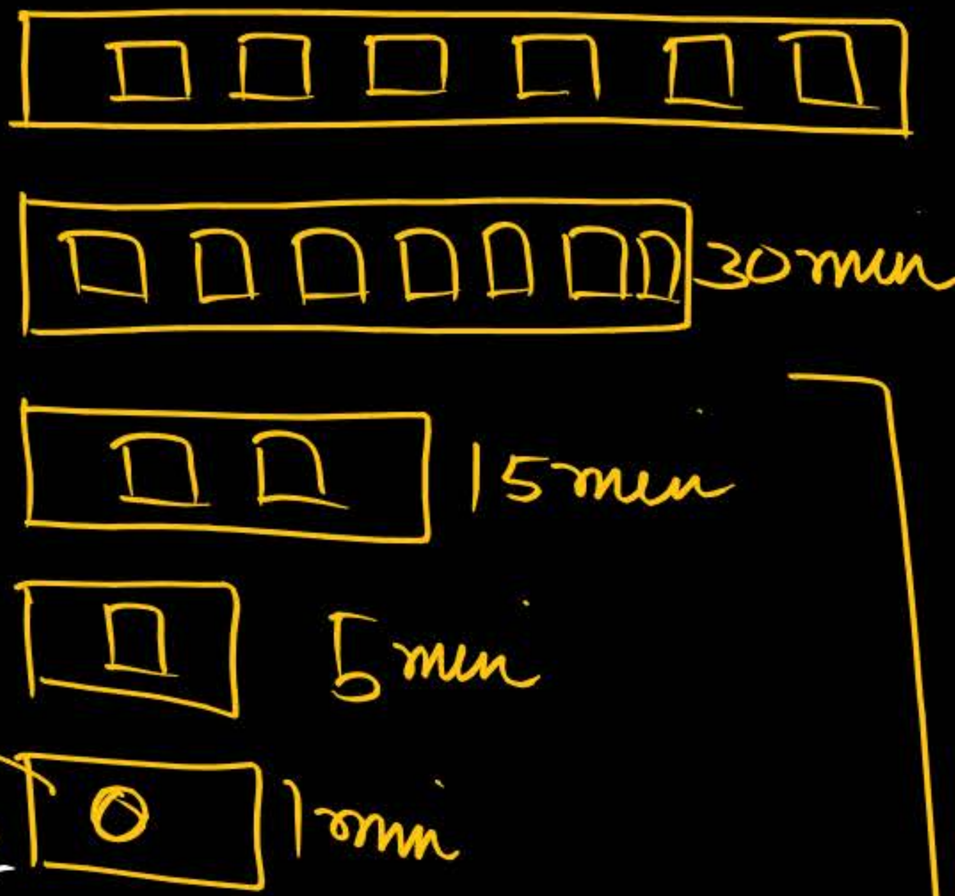
Average No. of claims.
Average No. of customers
Average No. of HEADS.
Average No. of vehicle coming Traffic lights
Average No. of Telephone calls.

1 1 0 1 0 1 0 1

$\Rightarrow$ S S F S F S F S

Average No. of success

6 vehicle



1 Hrs.

30 min

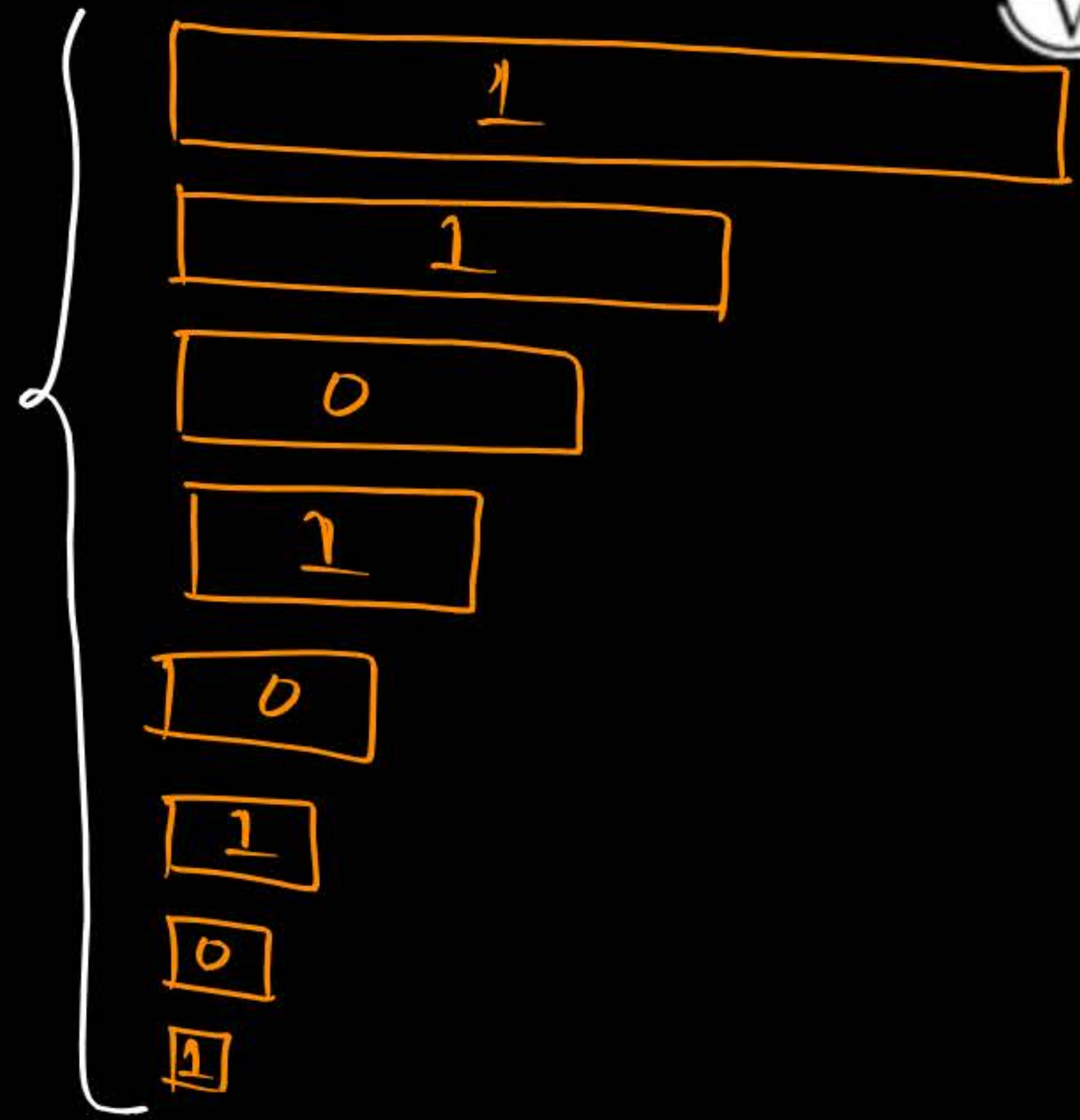
15 min

5 min

0 vehicle

1 min

Poisson distribution



Rare chances

If Number of Trials Are Increased and Prob. of event is lesser

$$\boxed{P[X = r \text{ success}] = \frac{e^{-\mu}\mu^{r}}{r!}}$$

Where μ = Average = mean

r = No. of success

$r = 0, 1, 2 \ldots$ (Arrival)

statistical Average

mean $E[X] = \mu$

variance $\sigma_X^2 = V(X) = \mu$

Standard deviation $= \sqrt{\mu}$

P($\boxed{\text{Rare event}}$) $\rightarrow$ common

Sell The ~~rare~~ event $\rightarrow$ Insurance company

Earn company

only Single Parameter Are Involved.

Poisson Dist = μ

Networks
computer

$$\boxed{P[X=r] = \frac{e^{-\mu}\mu^{r}}{r!}}$$ This Function is valid PMF $\frac{e^{-\lambda}\lambda^{r}}{r!}$ $\lambda \to \mu$

$$\sum_{r=0}^{n} \frac{e^{-\mu}\mu^{r}}{r!} = e^{-\mu}\Big[\underbrace{1+\frac{\mu}{1!}+\frac{\mu^2}{2!}+\frac{\mu^3}{3!}+ \cdots}_{e^{\mu}}\Big]$$

$$\boxed{\sum_{r=0}^{n} P[X=r] = 1}$$

Single coin → bernoulli

n coins
S F
Binomial
$\boxed{\mu = np}$ $\sigma_x^2 = npq$

Average No. of success

Poisson
λ mean
mean $= np$
variance $= \lambda = np$
$S.D = \sqrt{np}$

Q. Questions

If a random variable X satisfies the poission's distribution with a mean value of 2, then the probability that $X > 2$ is

$\boxed{\mu = 2}$ $\boxed{\text{mean} = 2}$

(a) $2e^{-2}$

$P(X>2) = P(X=3) + P(X=4) + P(X=5) + P(X=6) + \ldots$

(b) $1 - 2e^{-2}$

$\boxed{P(X>2) = 1 - P(X=0) - P(X=1)}$

(c) $3e^{-2}$

(d) $1 - 3e^{-2}$ ✓

$\checkmark 0 \quad \checkmark$

$$P(X \geq 2) = 1 - P(X=0) - P(X=1)$$

$$P(X=r) = \frac{e^{-\mu}\mu^{r}}{r!}$$

$\mu = 2$

$$P(X=0) = \frac{e^{-2}(2)^0}{0!} = e^{-2}$$

$$P(X=1) = \frac{e^{-2}(2)^1}{1!} = 2e^{-2}$$

$$P(X \geq 2) = 1 - e^{-2} - 2e^{-2}$$

$$= \underline{1 - 3e^{-2}}$$

Q. Questions

Suppose p is the number of cars per minute passing through a certain road junction between 5 PM and 6 PM, and p has a Poisson distribution with the mean 3. What is the probability of observing fewer than 3 cars during any given minute in this interval?

$\mu = 3$

$P(X<3)$

$\Rightarrow P(X=0) + P(X=1) + P(X=2)$

$P(X<3) = P(X=0) + P(X=1) + P(X=2)$

(a) $\frac{8}{(2e^3)}$

(b) $\frac{9}{(2e^3)}$

(c) $\frac{17}{(2e^3)}$

(d) $\frac{26}{(2e^3)}$

$\mu = 3$

$$P(X=r) = \frac{e^{-\mu}\mu^r}{r!}$$

$$P(X=0) = \frac{e^{-3}(3)^0}{0!} = e^{-3}$$

$$P(X=1) = \frac{e^{-3}(3)^1}{1!} = 3e^{-3}$$

$$P(X=2) = \frac{e^{-3}(3)^2}{2!} = \frac{9e^{-3}}{2}$$

$$P(X<3) = e^{-3} + 3e^{-3} + \frac{9e^{-3}}{2}$$

$$= \frac{2e^{-3} + 6e^{-3} + 9e^{-3}}{2}$$

$$= \frac{17e^{-3}}{2}$$

Q. Questions

$\mu = 5.2$

The number of accidents occurring in a plant in a month follows Poisson distribution with mean as 5.2. The probability of occurrence of less than 2 accidents in the plant during a randomly selected month is

(a) 0.029

(b) 0.034

(c) 0.039

(d) 0.044

$$P(X<2) = P(X=0) + P(X=1)$$

$$= \frac{e^{-5.2}(5.2)^0}{0!} + \frac{e^{-5.2}(5.2)^1}{1!}$$

$$= \underline{0.034}$$

Q. Questions

The second moment of a Poisson-distributed random variables is 2. The mean of the random variable _____ is

$$\begin{cases} E[X] = \text{first moment} \\ E[X^2] = \text{SECOND moment} \\ E[X^n] = n^{th} \text{ moment} \end{cases}$$

$E[X^2] = 2$ mean of The random variable.

$V(X) = E[X^2] - (E[X])^2$

$\mu = 2 - \mu^2$

$= \mu^2 + \mu - 2 = 0$ quadratic equⁿ

$\Rightarrow \mu^2 + 2\mu - \mu - 2 = 0$

$\Rightarrow \mu(\mu+2) - 1(\mu+2) = 0$

$\Rightarrow (\mu-1)(\mu+2) = 0$

$\Rightarrow$

$\mu = 1$

$\mu = -2$ ✗

$\mu = 1$

mean = 1

$E[X^2] = 2$

$E[X] = \text{mean} = \mu$

$\text{var}(X) = \mu$

Q. Questions

ME I

Consider a Poisson distribution for the tossing of a biased coin. The mean for this distribution is μ. The standard deviation for this distribution is given by :

(a) $\sqrt{\mu}$

(b) μ^2

(c) μ

(d) $1/\mu$

$\sigma = \sqrt{\mu}$

S.D

Q. Questions

If a random variable X has a Poisson distribution with mean 5, then the expectation $E[(X+2)^2]$ equals ______.

$E[aX+b]$
$= aE[X]+b$

$\mu = 5$

$V(aX+b)$
$= a^2 \operatorname{var}(X)$

$V(-X) = (-1)^2 \operatorname{var}(X)$

$E[(X+2)^2] = E[(X^2+4+4X]$

$\operatorname{var}(aX+bY) = a^2 V(X) + b^2 V(Y)$

$$E[(x+2)^2] = E[(x^2+4+4x)]$$

$$= E[x^2] + 4 + 4E[x]$$

$$= [V(x) + (E(x))^2] + 4 + 4E[x]$$

$$= [(5 + (5)^2] + 4 + 4 \times 5]$$

$$= 5 + 25 + 4 + 20$$

$$= 54$$

Ans

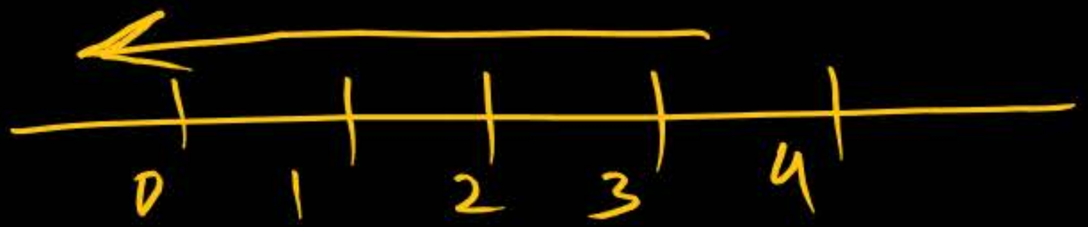
$$V(x) = E[x^2] - (E[x])^2$$

$$E[x^2] = V(x) + (E[x])^2$$

$$E[x] = 5$$

$$V(x) = 5$$

Q. **Questions**



The probability of a resistor being defective is 0.02. There are 50 such resistors is a circuit. The probability of two or more defective resistors in the circuit (round off to two decimal places) is __________.

Prob (Resistor) = 0·02 $\quad n = 50$ Resistor

$P(X \geq 2) = ?$

$p = 0.02 \quad n = 50$

Mean $\mu = np = 50 \times 0.02 = 1$

If No of trials ↑
P ↓

$P(X \geq 2) = 1 - P(X=0) - P(X=1) - P(X=2)$

$P(X \geq 2)$

$0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 50$

$P(X=r) = \dfrac{e^{-\mu} \mu^{r}}{r!}$

$= 1 - \dfrac{e^{-1}(1)^0}{0!} - \dfrac{e^{-1}(1)^1}{1!} - \dfrac{e^{-1}(1)^2}{2!}$

$= 1 - e^{-1} - e^{-1} - \dfrac{e^{-1}}{2}$

$= 1 - 2e^{-1} - \dfrac{e^{-1}}{2}$

$= 0.08$

$P(X<3)$

$P(X \geq 2)$ $P(X>2)$

$0 \quad 1 \quad 2 \quad 3 \quad 4$

Q. Questions

$\mu = 5$

A traffic office imposes on an $\boxed{\text{average}}$ 5 number of penalties daily on traffic violators. Assume that the number of penalties on different days is independent and follows a Poisson distribution. The probability that there will be less than 4 penalties in a day is ______.

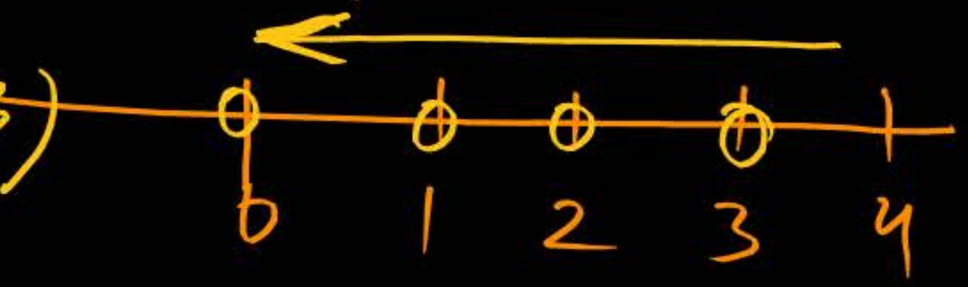
$$P(X<4) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$$

Less than
4 Penalties

$\boxed{\mu = 5}$ Using Poisson Distri: $P(X=r) = \dfrac{e^{-\mu}\mu^{r}}{r!}$

Average

$$P(X<4) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$$



$$= \frac{e^{-5}(5)^{0}}{0!} + \frac{e^{-5}(5)^{1}}{1!} + \frac{e^{-5}(5)^{2}}{2!} + \frac{e^{-5}(5)^{3}}{3!}$$

$$= e^{-5} + 5e^{-5} + \frac{25}{2}e^{-5} + \frac{125}{6}e^{-5}$$

$$= 0.26$$

Q. Questions

An $\boxed{\text{observer}}$ counts 240veh/h at a specific highway location. Assume that the vehicle arrival at the location is Poisson distributed, the probability of having one vehicle arriving over a 30-second time interval is

240 veh/hr

P(x=1 vehicle)

30 SEC

240 veh/hr $\longrightarrow \frac{240}{60}$ veh/min

$\boxed{1 \text{ hr} = 60 \text{ min}}$

$\longrightarrow$ 4 veh/min

mean $\mu = 2$ veh/30SEC

$r = 1$ vehicle. $P(X=r) = \dfrac{e^{-\mu}\mu^r}{r!}$

$$P(X=1) = \frac{e^{-2}(2)^1}{1!}$$

$$= 2e^{-2}$$ Ans

Thank You!

GW Soldiers