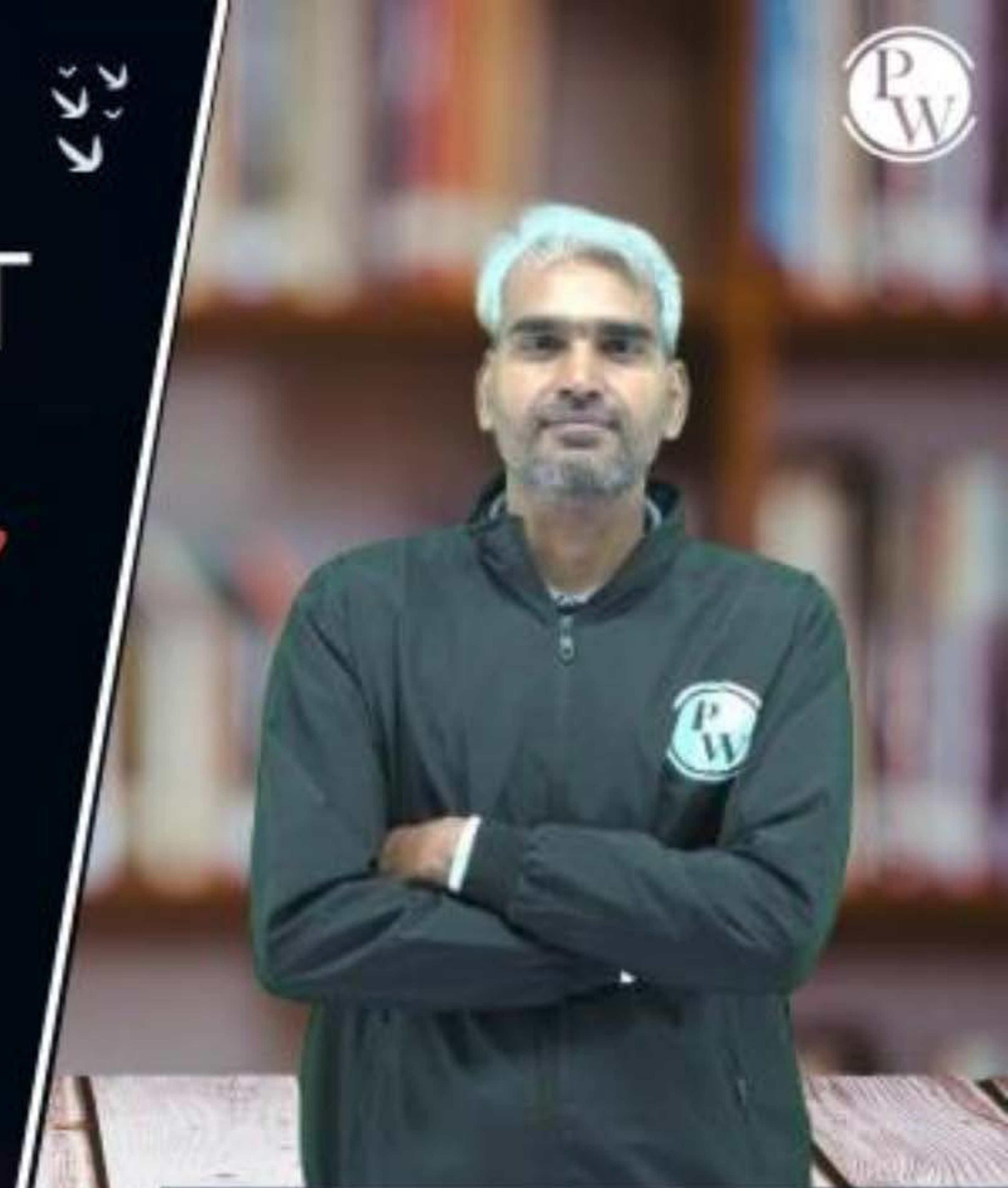
# Computer Science & IT

## Discrete Mathematics

**Set Theory & Algebra**

**Lecture No. 08**



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# Recap of Previous Lecture

**Topic** Types of Relations

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# Topics to be Covered

- **Topic** Reflexive Closure
- **Topic** Symmetric Closure
- **Topic** Transitive Closure
- **Topic** Equivalence Relation

Slide

# Topic : Reflexive Closure

Let $R$ be a relation on set $A$, then reflexive closure of relation $R$ is the smallest reflexive relation on set $A$ containing relation $R$

eg: Let $A = \{1, 2, 3, 4\}$.

$R = \{(1,1), (1,2), (2,2), (2,3), (2,4), (3,1), (4,2)\}$

Reflexive Closure of $R$ $= \{(1,1), (1,2), (2,2), (2,3), (2,4), (3,1), (4,2), (3,3), (4,4)\}$

Slide

* If relation R is a reflexive relation on set A, then reflexive closure of R will be relation R itself.

We just need to add the missing diagonal order pairs (if any)

# Topic : Symmetric Closure

Let R be a relation on set A, then Symmetric Closure of R is the smallest symmetric relation on set A containing relation R.

eg. let $A = \{1, 2, 3, 4\}$

$R = \{(1,1), (1,2), (1,4), (2,1), (2,3), (3,3)\}$

Symmetric Closure of R $= \{(1,1), (1,2), (1,4), (2,1), (2,3), (3,3), (4,1), (3,2)\}$

Slide

* If R is a symmetric Relation on set A, then symmetric closure of R, will be relation R itself.

# Topic : Transitive Closure

Let R be a relation on set A, then transitive closure of relation R is the smallest transitive relation on set A containing relation R.

Slide

eg:- let $A = \{1, 2, 3\}$

$R = \{(1,1), (1,2), (2,1)\}$

Transitive Closure of $R$ $= \{(1,1), (1,2), (2,1), (2,2)\}$

eg: let $A = \{1, 2, 3\}$

$R = \{(1,1), (1,3), (2,2), (3,1), (3,2)\}$

Find Transitive Closure of R.

1st iteration $= \{(1,1), (1,3), (2,2), (3,1), (3,2), (1,2), (3,3)\}$

2nd iteration $= \{(1,1), (1,2), (1,3), (2,2), (3,1), (3,2), (3,3)\}$ {nothing new will be added in 2nd iteration}

When result of last iteration is same as previous iteration then the result of last iteration is the transitive closure of given Relation.

eg: $A = \{a, b, c, d\}$

$R = \{(a,d), (b,a), (b,c), (c,a), (c,d), (d,c)\}$

Find transitive closure of R.

1st iteration $= \{(a,d), (b,a), (b,c), (c,a), (c,d), (d,c), (a,c), (b,d), (c,c), (d,a), (d,d)\}$

2nd iteration $= \{(a,c), (a,d), (b,a), (b,c), (b,d), (c,a), (c,c), (c,d), (d,a), (d,c), (d,d), (a,a)\}$

3rd iteration $= \{(a,a), (a,c), (a,d), (b,a), (b,c), (b,d), (c,a), (c,c), (c,d), (d,a), (d,c), (d,d)\}$

No new order pair is added

Result of 3rd iteration = Result of 2nd iteration

$\therefore$ Transitive Closure of R.

Warshall's algorithm can be used to identify the transitive closure of a relation

We will discuss this during graph theory

Slide

Q: let $A = \{1, 2, 3, 4\}$

and $R = \{(1,1), (1,2), (2,2), (3,2), (2,3)\}$

find Reflexive symmetric closure of R

Smallest relation on set A that contains relation R, and it is reflexive as well as symmetric

$= \{(1,1), (1,2)\ (2,2)\ (3,2)\ (2,3), (3,3), (4,4), (2,1), \}$

Slide

# Topic : Equivalence Relation

Reflexive + Symmetric + Transitive

* A relation R on set A is called an equivalence relation if and only if relation R is ① Reflexive

and ② Symmetric

and ③ Transitive

Slide

eg: let $A = \{1, 2, 3\}$

$\Delta_A = R_1 = \{(1,1), (2,2), (3,3)\}$

→ Reflexive ✓

→ Symmetric ✓

→ Transitive ✓

$\therefore$ Equivalen Relation

* Diagonal relation on set A is the smallest equivalence relation on set A.

Slide

eg: let $A = \{1, 2, 3\}$

$A \times A = R_2 = \{(1,1), (1,2), (1,3),$

$(2,1), (2,2), (2,3),$

$(3,1), (3,2), (3,3)\}$

Reflexive ✓

Symmetric ✓

Transitive ✓

$A \times A$ is the largest Equivalence relation on set $A$.

H.W.

Q: Let $A = \{1, 2, 3\}$ ✓

Write all equivalence relations possible on set A.

Q: let $A = \{1, 2, 3, 4\}$ ✓

Write all equivalence relations possible on set A

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## 2-Min Summary :-

① Reflexive, Symmetric & Transitive Closure.

② Equivalence relation (Definition)

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# THANK - YOU

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