Computer Science & Information Technology

Discrete Mathematics

DPP : 4

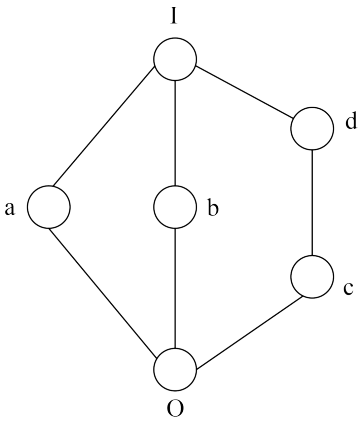
Set Theory and Algebra

Q1 The POSET
({2,3,5,30,60,120,180,360}; |) is_____.
(A) Join semi lattice but not a meet semi lattice
(B) Not join semi lattice but a meet semi lattice
(C) A lattice
(D) Not a semi lattice

Q2 The POSET ({2,3, 4,6,12,18}; |)is
(A) Join semi lattice but not a meet semi lattice.
(B) Not join semi lattice but a meet semi lattice.
(C) A lattice.
(D) Not a semi lattice.

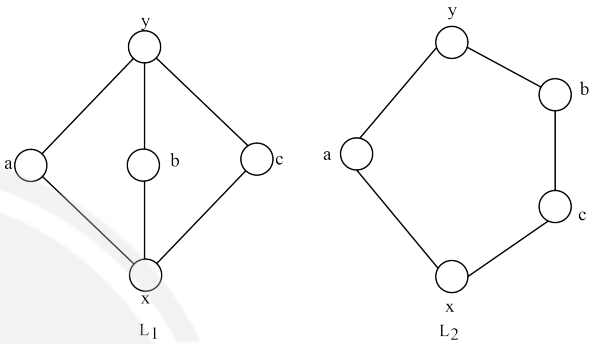
Q3 Consider the following statements
S_1: Every lattice is a totally ordered set.
S_2: Every totally ordered set is a lattice.
(A) S_1 is true and S_2 is false
(B) S_1 is false but S_2 is true.
(C) Both S_1 and S_2 are true.
(D) Neither S_1 nor S_2 are true.

Q4



Which of the following is/are true for above hasse diagram?
(A) Above hasse diagram represent a complemented lattice.
(B) Above hasse diagram represent a distributive lattice
(C) Elements a, b, c, and d have equal number of complements
(D) Every element of the above lattice has at most one comeplement.

Q5



Which of the following is/are true for the lattice 'L' with respect to POSET ({1, 2, 5, 3, 9, 90} , |)
(A) L has a sub-lattice which is isomorphic to L1.
(B) L has a sub-lattice which is isomorphic to L2.
(C) L has no sub-lattice which is isomorphic to either L1 or L2.
(D) L is not a distributive lattice.

Q6 In a Boolean algebra with respect to set A where |A| = n consider the following statements.
S1: Number of vertices in the hasse diagram are 2^n.
S2: Number of edges in the hasse diagram are $n \cdot 2^{n-1}$
Which of the following is true?
(A) S1 is true and S2 is false
(B) S1 is false but S2 is true.
(C) Both S1 and S2 are true.
(D) Both S1 and S2 are false.

Q7 Let P be the partial order defined on the set {1,2,3,4} as follows
$P = \{(x,x) \mid x \in \{1, 2, 3, 4\}\} \cup \{(1, 2), (3, 2), (3, 4)\}$
The number of total orders on {1, 2, 3, 4} that contains P is_____.

Q8 Which of the following is/are always true for any lattice?

(A) There exists exactly one minimum and exactly one maximum element.

(B) There exists at most one minimal and at most one maximal element.

(C) Least upper bound and greatest lower bound exists for every pair of elements.

(D) Every element has a unique complement.

Answer Key

Q1	(A)	Q5	(A, B, D)
Q2	(D)	Q6	(C)
Q3	(B)	Q7	5~5
Q4	(A, D)	Q8	(B, C)

Hints & Solutions

Q1 Text Solution:
Join semi lattice but not a meet semi lattice.

Q2 Text Solution:
Not a semi lattice.

Q3 Text Solution:
S_1: Every lattice is a totally ordered set: FALSE
S_2: Every totally ordered set is a lattice: TRUE

Q4 Text Solution:
Above hasse diagram represent a complemented lattice.
Every element of the above lattice has at most one comeplement.

Q5 Text Solution:
L has a sub-lattice which is isomorphic to L1.
L has a sub-lattice which is isomorphic to L2.
L is not a distributive lattice.

Q6 Text Solution:
S1: Number of vertices in the hasse diagram are 2^n. (TRUE)
S2: Number of edges in the hasse diagram are $n \cdot 2^{n-1}$ (TRUE)

Q7 Text Solution:
The number of total orders on {1, 2, 3, 4} that contains P is 5.

Q8 Text Solution:
There exists at most one minimal and at most one maximal element.
Least upper bound and greatest lower bound exists for every pair of elements.