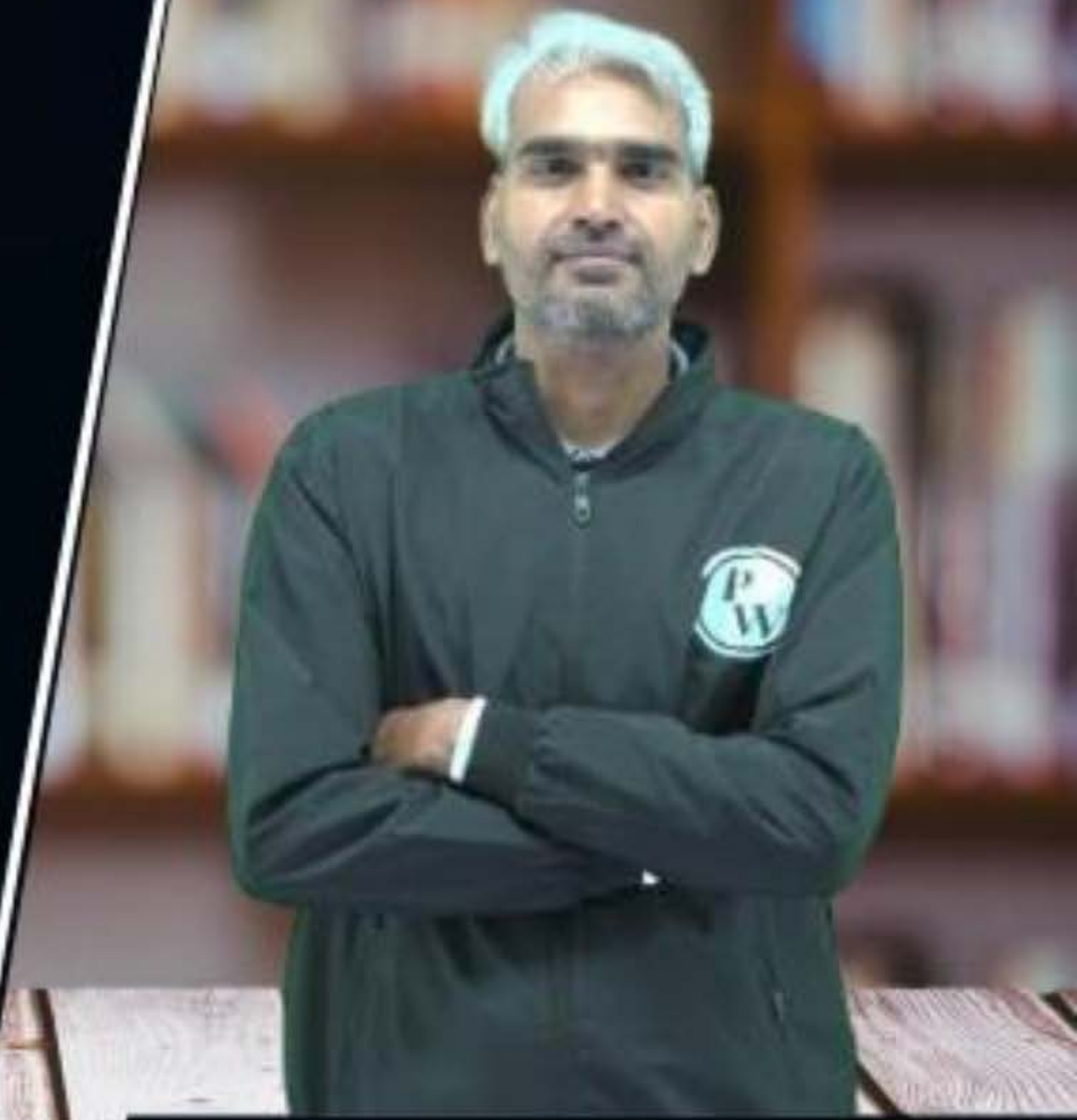
Computer Science & IT

Discrete Mathematics

Set Theory & Algebra

Lecture No. 10



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Recap of Previous Lecture

Topic Equivalence Relation

Topic Partition of a set

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Topics to be Covered

Topic Equivalence Class

Topic Number of equivalence relation on a set

Topic Bell Number

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Topic : Partition of a set

H.W. Q: Let $A = \{1, 2, 3, 4, 5, 6\}$

How many partitions are possible of set A

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6

(1) $1+1+1+1+1+1 = \dfrac{6\times5\times4\times3\times2\times1}{6!} = 1$

(2) $1+1+1+1+2 = \dfrac{{}^6C_1 * {}^5C_1 * {}^4C_1 * {}^3C_1 * {}^2C_2}{4!} = 15$

(3) $1+1+1+3 = \dfrac{{}^6C_1 * {}^5C_1 * {}^4C_1 * {}^3C_3}{3!} = 20$

(4) $1+1+2+2 = \dfrac{{}^6C_1 * {}^5C_1}{2!} * \dfrac{{}^4C_2 * {}^2C_2}{2!} = 15 * 3 = 45$

(5) $1+1+4 = \dfrac{{}^6C_1 * {}^5C_1 * {}^4C_4}{2!} = 15$

(6) $1+2+3 = {}^6C_1 * {}^5C_2 * {}^3C_3 = 60$

(7) $1+5 = {}^6C_1 * {}^5C_5 = 6$

(8) $2+4 = {}^6C_2 * {}^4C_4 = 15$

(9) $2+2+2 = \dfrac{{}^6C_2 * {}^4C_2 * {}^2C_2}{3!} = 15$

(10) $3+3 = \dfrac{{}^6C_3 * {}^3C_3}{2!} = 10$

(11) $6 = 1$

Ans = $\underline{\underline{203}}$

Q: let $A = \{1,2,3,4,5\}$

How many partitions of set A are possible

5 — $1+1+1+1+1 = 1$ $\quad 5$

7 — $1+1+1+2 = \frac{{}^5C_1 * {}^4C_1 * {}^3C_1 * {}^2C_2}{3!} = 10$

11 — $1+1+3 = \frac{{}^5C_1 * {}^4C_1 * {}^3C_3}{2!} = 10$

9 — $1+2+2 = \frac{{}^5C_1 * {}^4C_2 * {}^2C_2}{2!} = 15$

17 — $1+4 = {}^5C_1 * {}^4C_4 = 5$

25 — $5 = 1$

13 — $2+3 = {}^5C_2 * {}^3C_3 = 10$

$1 + 10 + 10 + 15 + 5 + 1 + 10 = 52$

Topic : Equivalence Class

* Let R be an equivalence relation on set A. For any element $x \in A$. Equivalence class of element 'x' w.r.t. equivalence relation R is denoted by $[x]$ and it is defined as

$$[x] = \{ y \mid (x, y) \in R \}$$

eg: let $A = \{1, 2, 3, 4, 5\}$

1 relates with 2

Equivalence Rel^n $R = \{(1,1), (2,2), (3,3), (4,4), (5,5), (1,2), (2,1), (3,4), (4,3)\}$

2 relates with '1'

* Equivalence Class: for elements of set A. w.r.t. rel^n R

* Equivalence class of '1' $= [1] = \{1, 2\}$
* Equivalence Class of '2' $= [2] = \{2, 1\} = \{1, 2\}$
* Equivalence Class of 3 $= [3] = \{3, 4\}$
* Equivalence class of 4 $= [4] = \{4, 3\} = \{3, 4\}$
* Equivalence class of 5 $= [5] = \{5\}$

same

same

Three distinct equivalence classes for the elements of set A, w.r.t. R

Note: (1) Equivalence class of element x may be equal to the equivalence class of element y, even when $\underline{x \neq y}$.

(2) Set of all distinct equivalence classes of elements of set A will define partition of set A w.r.t. given Equivalence Relⁿ

in the above eg: $A = \{1, 2, 3, 4, 5\}$

Set of all distinct equivalence class $= \{\{1, 2\}, \{3, 4\}, \{5\}\}$ {it is partition of A w.r.t. given equivalence Relⁿ R}

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Note ③: Given the equivalence relation on set A. We can determine the partition of set A w.r.t. given equivalence relation, using concept of equivalence class.

Note ④ :- If we know the partition of set A, then we can determine the equivalence relation on set A corresponding to the partition, by performing the self cross product of the subsets in the partition, and by taking the union of all those cross products eg. let $A = \{1, 2, 3, 4, 5\}$

let Partition of A w.r.t. some equivalence Rel^n R is $= \{\{1,2\}, \{3,4\}, \{5\}\}$

$\therefore$ Equivalence Rel^n R $= \{1,2\} \times \{1,2\} \cup \{3,4\} \times \{3,4\} \cup \{5\} \times \{5\}$

$= \{(1,1),(1,2),(2,1),(2,2)\} \cup \{(3,3),(3,4),(4,3),(4,4)\} \cup \{(5,5)\}$

$= \{(1,1), (2,2), (3,3), (4,4), (5,5), (1,2), (2,1), (3,4), (4,3)\}$

Note ⑤ :- These is a one-to-one correspondance between the partitions of set A and Equivalence relations on set A.

i.e.

Number of Equivalence Relation on set A	=	No. of Partitions of set A.

Topic : Number of equivalence relation on a set

Q: Let $A = \{1\}$

How many equivalence rel^n on set A.

Ans = 1 $\left[R_1 = \{(1,1)\} \right.$ [It is w.r.t. partition $\{\{1\}\}$]

Q: Let $A = \{1, 2\}$

How many equivalence rel^n are possible on set A

Ans = 2 $\left[R_1 = \Delta_A = \{(1,1), (2,2)\} \right.$ [It is w.r.t. partition $\{\{1\}, \{2\}\}$

$R_2 = A \times A = \{(1,1), (1,2), (2,1), (2,2)\}$ [It is w.r.t. partition $\{\{1, 2\}\}$

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Q: Let $A = \{1,2,3\}$

How many equivalence Rel^n are possible on set A.

$R_1 = \Delta_A = \{(1,1), (2,2), (3,3)\}$ [it is w.r.t. partition $\{\{1\}, \{2\}, \{3\}\}$

$R_2 = \{(1,1),(2,2),(3,3), (1,2),(2,1)\}$ [it is w.r.t. partition $\{\{1,2\}, \{3\}\}$

$R_3 = \{(1,1), (2,2), (3,3), (1,3), (3,1)\}$ $\{\{1,3\}, \{2\}\}$

$R_4 = \{(1,1), (2,2), (3,3), (2,3), (3,2)\}$ $\{\{2,3\}, \{1\}\}$

$R_5 = \{(1,1),(2,2),(3,3), (1,2),(2,1), (1,3),(3,1)\}$ $\cancel{(2,3)}$ $\therefore$ Not transitive

$R_5 = A \times A =$ $\longrightarrow$ [It is w.r.t Partition $\{\{1,2,3\}\}$

Q:- Let $A = \{1, 2, 3, 4, 5\}$

How many equivalence relⁿ are possible on set A, such that number of order pairs in the relation are exactly '9'

Solⁿ: Equivalence Relⁿ on set A with exactly 9 order pairs will be of type

$$\{(1,1), (2,2), (3,3), (4,4), (5,5), \overset{1\,2}{(x, y)}, \overset{2\,1}{(y, x)}, \overset{3\,4}{(p, q)}, \overset{4\,3}{(q, p)}\}$$

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Topic : Bell Number

Bell number B_n gives the number of partitions of set A, where $|A| = n$

i.e. the number of equivalence relations

Cardinality of set

number of partitions

Bell triangle

Last number of previous row

We need to perform this addition as long as there is a number above it in the previous row

B_0	1					
B_1	1	2				
B_2	2	3	5			
B_3	5	7	10	15		
B_4	15	20	27	37	52	
B_5	52	67	87	114	151	203
B_6	203					

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2 mins Summary

Topic Equivalence Class

Topic Number of equivalence relation on a set

Topic Bell Number

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THANK - YOU

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