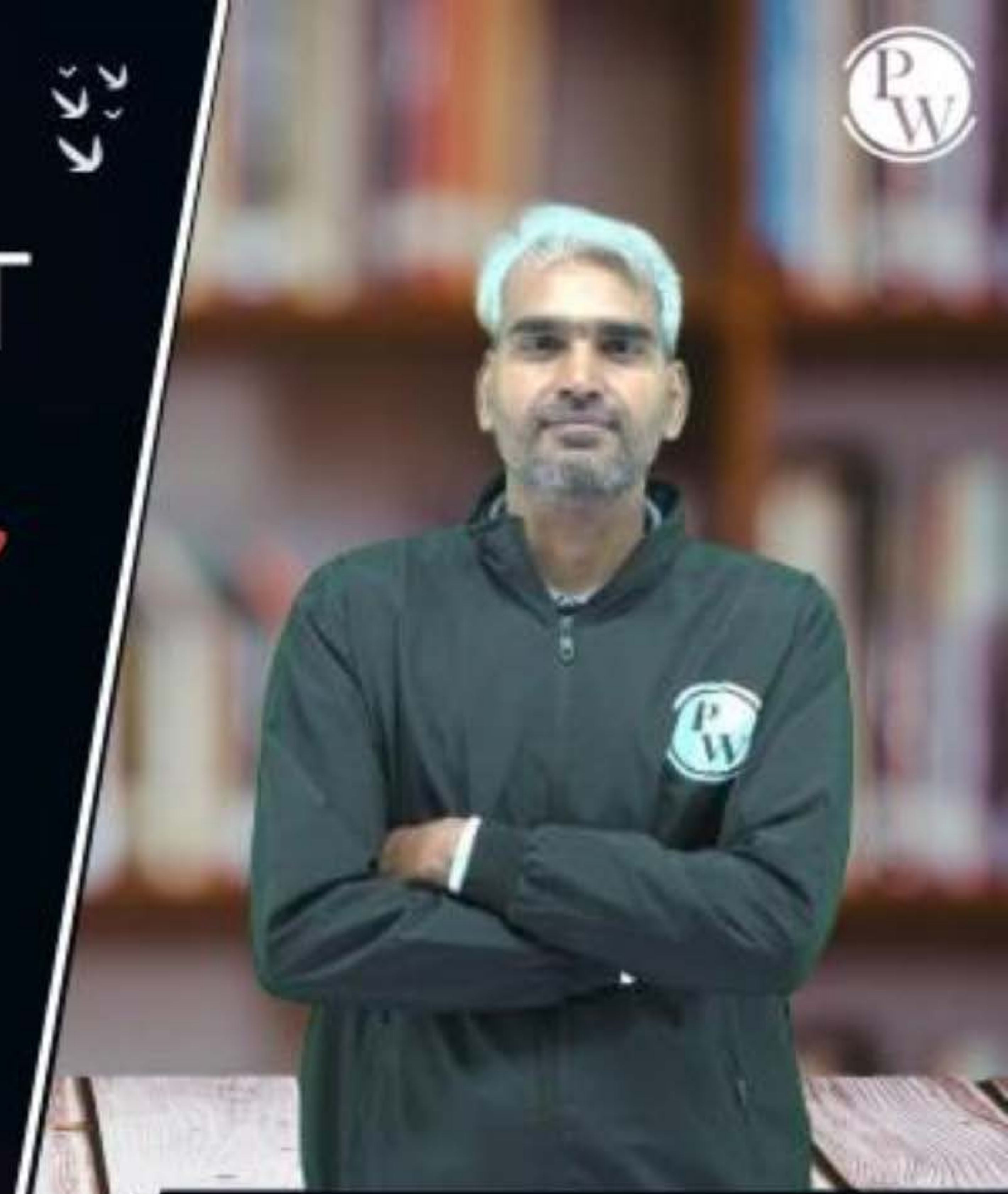
Computer Science & IT

Discrete Mathematics

Set Theory & Algebra

Lecture No. 03



By- Vishal Sir

Recap of Previous Lecture

- **Topic** Set
- **Topic** Representation of Set
- **Topic** Types of sets
- **Topic** Terminologies related to sets

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Topics to be Covered

Topic Power Set

Topic Venn Diagram

Topic Set Operations

Topic Properties of set operations

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9 integers → 1, 2, 3, ~~4~~, 5, 6, 7, 8, 9

How many 5-digit integer can be formulated using digits 1, 2, 3, ...9, such that there is no repeatation

$$= \underset{1^{st}}{\underline{9}} * \underset{2^{nd}}{\underline{8}} * \underset{3^{rd}}{\underline{7}} * \underset{4^{th}}{\underline{6}} * \underset{5^{th}}{\underline{5}}$$

$$= 9*8*7*6*5*\frac{4*3*2*1}{4*3*2*1}$$

$$= \frac{9!}{4!} = \frac{9!}{(9-5)!}$$

$(9-5)! \rightarrow 4!$

$\boxed{9^5 \text{ : when repeatation is allowed}}$

$\boxed{5! = \text{Permutation (Arrangements) of 5 distinct elements}}$

$\boxed{{}^9C_5 = \text{No. of ways to Choos 5 distinct elements out of 9-distinct elements.}}$

$$\boxed{{}^{n}P_{r} = \frac{n!}{(n-r)!}}$$

Permulation of n-elements taken 'r' at a time without repeatation

$${}^{n}P_{r} = {}^{n}C_{r} * r!$$

$$\boxed{{}^{n}C_{r} = \frac{{}^{n}P_{r}}{r!} = \frac{n!}{(n-r)!\,r!}}$$

Topic : Power Set

Power Set: Let A is any finite set, Power set of set A is a set Containing all subsets of set A.

Power set of set A is denoted by $P(A)$ or 2^A

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eg: $A = \{a, b\}$ $\boxed{|A| = 2}$

$$P(A) = \{\{\ \}, \{a\}, \{b\}, \{a, b\}\}$$ $\boxed{|P(A)| = 4}$

eg: $A = \{1, 2, \{1,2\}\}$ $\boxed{|A| = 3}$

$$P(A) = \{\underbrace{\{\ \}}, \underbrace{\{1\}, \{2\}, \{\{1,2\}\}}, \underbrace{\{1,2\}, \{1, \{1,2\}\}, \{2, \{1,2\}\}}, \underbrace{\{1, 2, \{1,2\}\}}\}$$

$\boxed{|P(A)| = 8}$

Topic : Cardinality of Power Set

* Let A is any finite set, then Cardinality of $P(A)$ = No. of elements in $P(A)$

= No. of subsets of set A

$= 2^{|A|}$

$\therefore$ for any finite set A,

$$\boxed{|P(A)| = 2^{|A|}}$$

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Q: Let ϕ be an empty set,

Find the Cardinality of

$$P(P(\phi))$$

$\{\ \} \not\equiv \{\{\ \}\}$

$|\{\ \}| = 0$

$|\{\{\ \}\}| = 1$

$\phi \equiv \{\ \}$ {The only subset of empty set is empty set itself}

$P(\phi) = \{\text{Subsets of } \phi\}$

$= \{\ \{\ \}\ \}$ ✓

$P(P(\phi)) = \{\text{subsets of } P(\phi)\}$

$= \{\ \{\ \}, \{\{\ \}\}\ \}$

$|P(P(\phi))| = 2$

$|\underbrace{P(P(\phi))}_{A}| = 2^{|P(\phi)|}$ $\quad |A|$

$$\boxed{|\underbrace{P(\phi)}_{A}| = 2^{|\phi|} = 2^0 = 1}$$

$|\phi| = 0$

$\therefore |P(P(\phi))| = 2^{|P(\phi)|} = 2^1 = 2$

$S = \{1, 2\}$

$P(S) = \{\{1\}, \{2\}, \{1,2\}, \{\}\}$

$P(S) \cap S = \phi$

Q1. Let $P(S)$ denote the power set of a set S. Which of the following is always true?

A. $\boxed{P(P(S)) = P(S)}$ → Never Possible

B. $\boxed{\underbrace{P(S)}_{\text{Set of Subsets of } S} \cap \underbrace{P(P(S))}_{\text{Set of Subsets of } P(S)} = \{\emptyset\}}$ {Always True}

C. $P(S) \cap S = P(S)$ {Always False} →

D. $S \notin P(S)$ {S always belong to P(S)}

$\{1, 2, \{1,2\}\}$

$P(S) = \{\{\}, \{1\}, \{2\}, \{\{1,2\}\}, \{1,2\}, \{1,\{1,2\}\}, \{2,\{1,2\}\}, \{1, 2, \{1,2\}\}\}$

$P(P(S)) = \{\{1,2\}\}$

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Note: For any finite set A

$\emptyset \in P(A)$ and $\emptyset \subseteq P(A)$

Both are always true

$A = \{1, 2, 3\}$

$1 \in A \quad 1 \nsubseteq A$

Q2. For a set A, the power set of A is denoted by 2^A. If $A = \{5, \{6\}, \{7\}\}$ which of the following options are true?

1. $\emptyset \in 2^A$
2. $\emptyset \subseteq 2^A$
3. $\{5, \{6\}\} \in 2^A$
4. $\{5, \{6\}\} \subseteq 2^A$

$P(A) = \{ \{\},$
$\{5\}, \{\{6\}\}, \{\{7\}\},$
$\{5, \{6\}\}, \{5, \{7\}\},$
$\{\{6\}, \{7\}\}$
$\{5, \{6\}, \{7\}\}$
$\}$

$\{\{5, \{6\}\}\} \subseteq P(A)$

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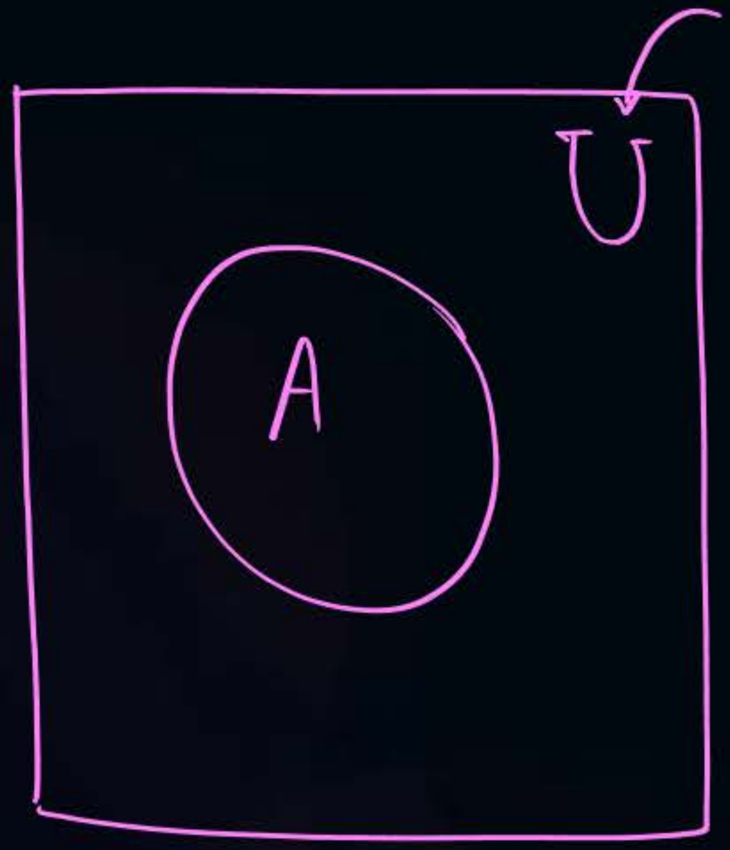
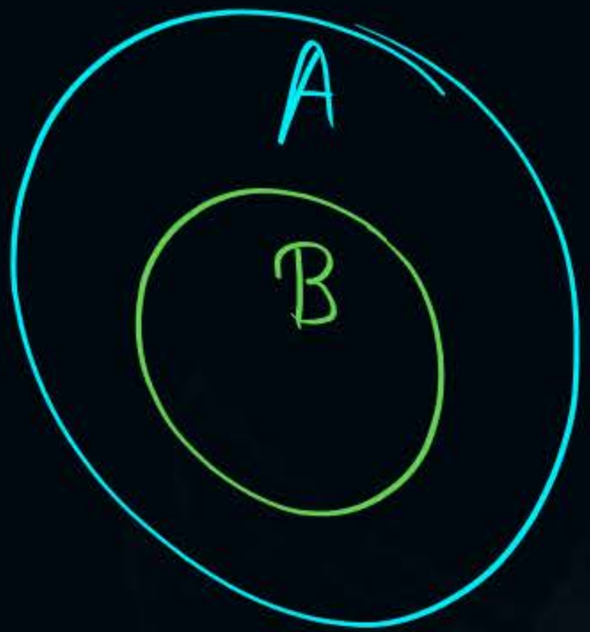
Topic : Venn Diagram

* In general universal set is denoted by U

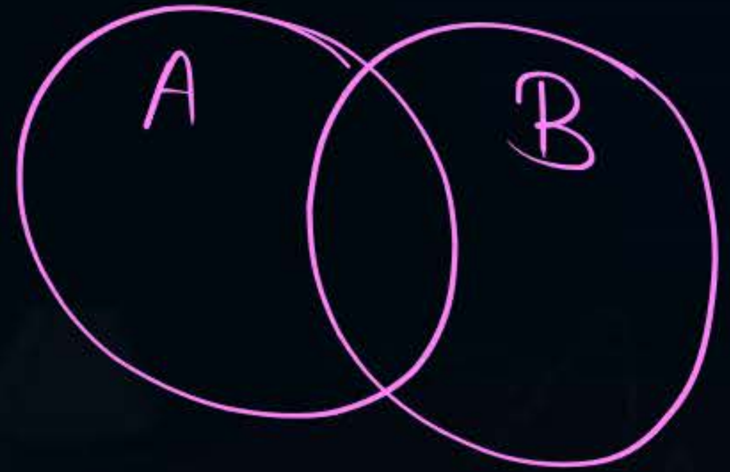
Venn diagram is used to represent the relationship among the sets pictorially.



$A \subseteq U$



$B \subseteq A$



Neither $A \subseteq B$ nor $B \subseteq A$

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Topic : Set Operations

- ❑ Complement of a set
- ❑ Union of two sets
- ❑ Intersection of two sets
- ❑ Set difference
- ❑ Symmetric difference of two sets

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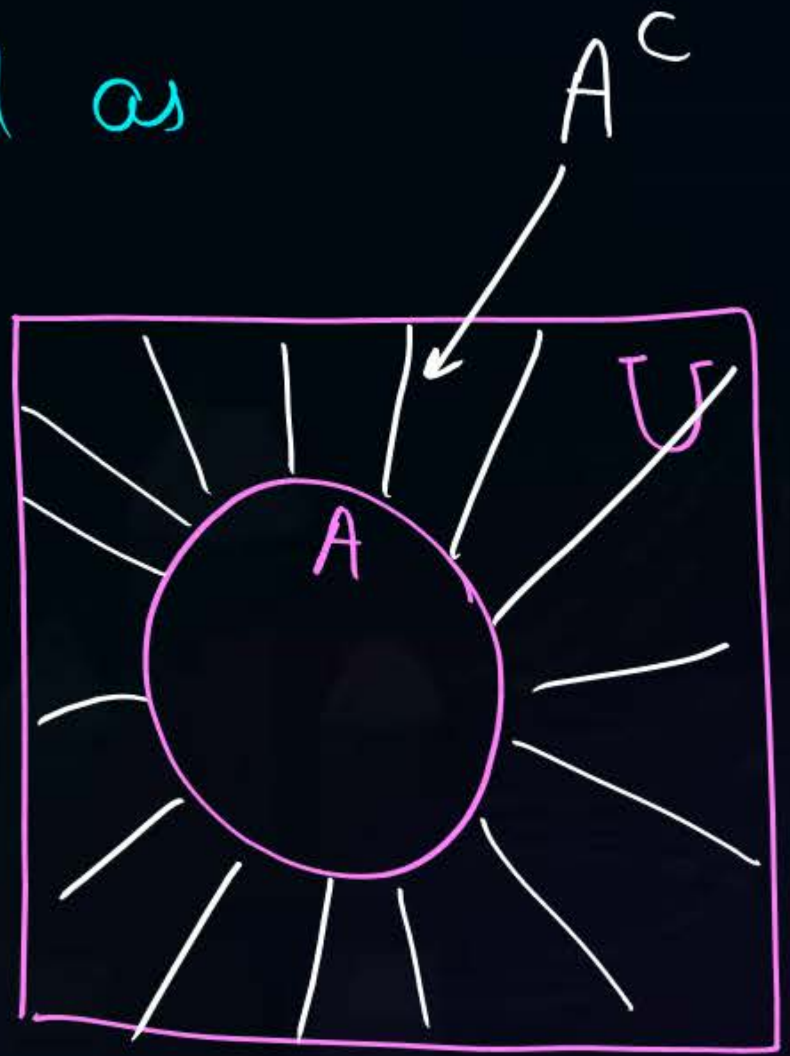
Topic : Complement of a set

{ Complement is defined w.r.t. universal set }

For any set A, Complement of set A is denoted by A^c/A' and it is defined as

$$A^c = \{x \mid x \notin A \text{ and } x \in U\}$$



Q: let $U = \{1, 2, 3, 4, 5\}$

$A = \{2, 3, 4\}$

$A^c = \{1, 5\}$

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Topic : Union of two sets

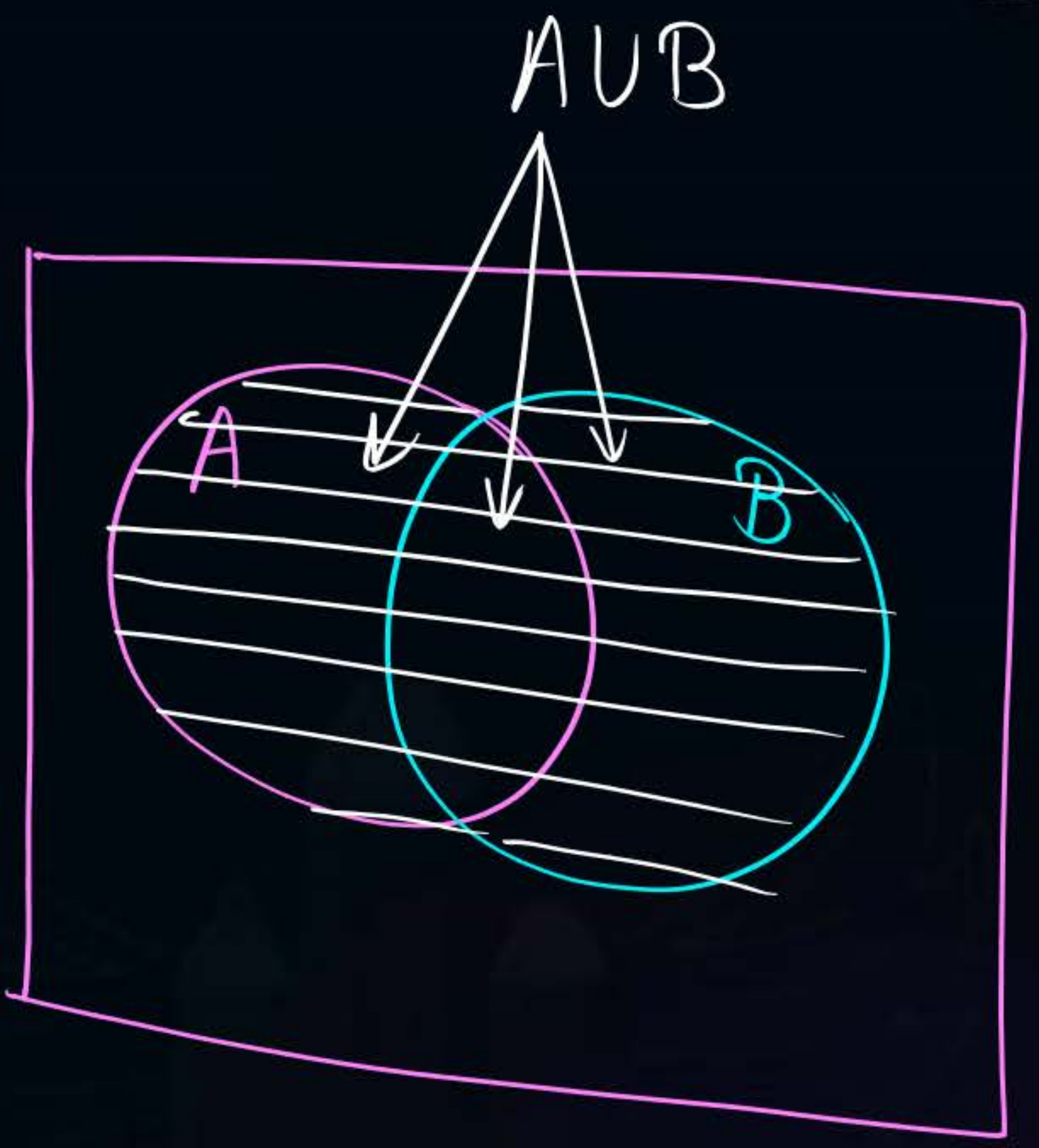
Let A & B are any two sets.

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$

$$\boxed{\underline{\text{Note}}: \quad A \cup B = B \cup A}$$

eg $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6\}$

$$A \cup B = \{1, 2, 3, 4, 5, 6\}$$



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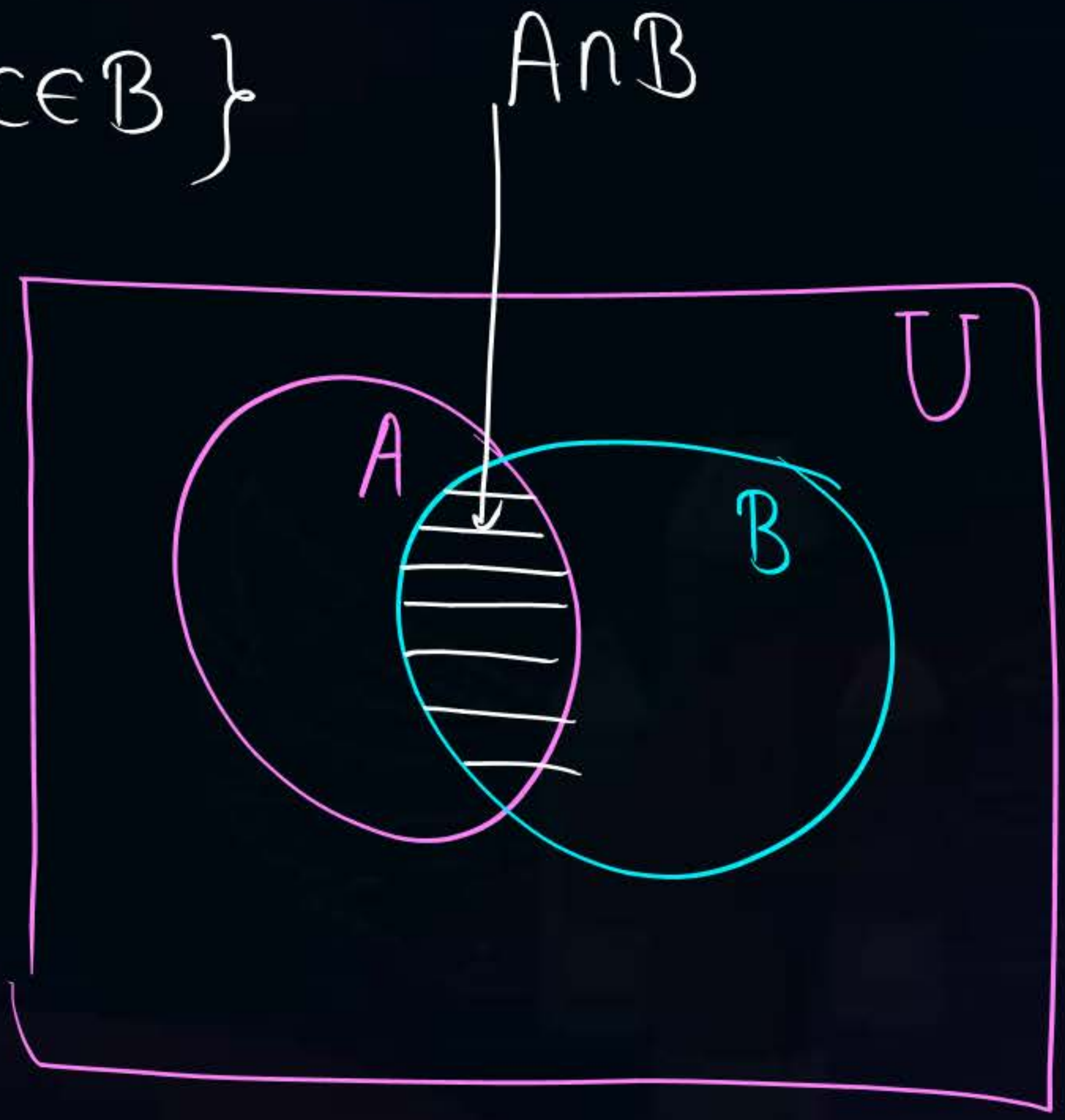
Topic : Intersection of two sets

For any two sets A & B

$$A \cap B = \{ x \mid x \in A \text{ and } x \in B \}$$

Note: $\boxed{A \cap B = B \cap A}$



eg let $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6\}$

$A \cap B = \{3, 4\}$

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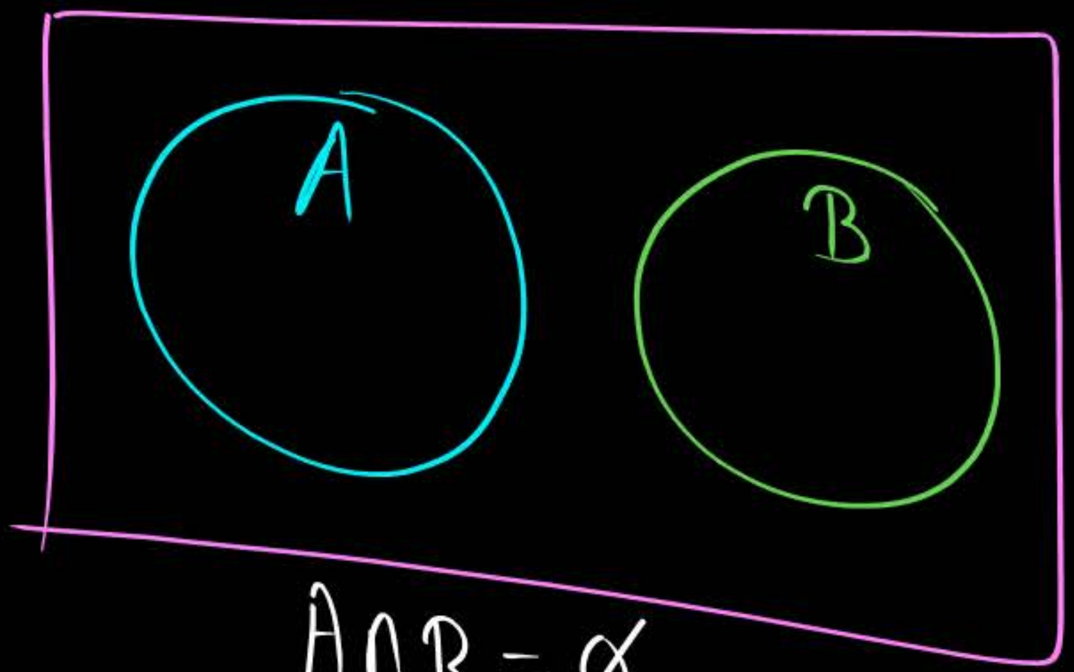
Disjoint Sets :- Two sets A and B are said to be disjoint sets only if $A \cap B = \phi$



$A \cap B = \phi$

$\therefore$ A & B are disjoint Sets.

Topic : Set difference

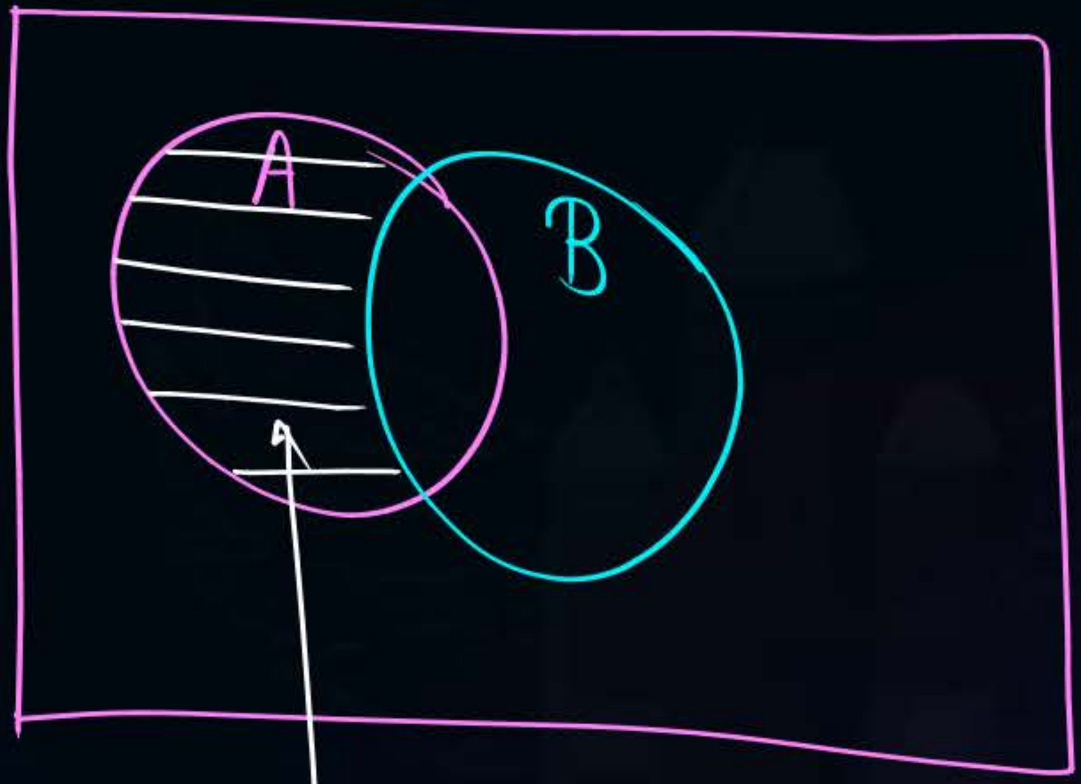
Let A & B are two sets,

$A-B$ = Set of all elements which are present in set A but not present in set B

$$A-B = \{x \mid x\in A \text{ and } x\notin B\}$$

Note: In general,
$$A-B \neq B-A$$

$A-B = B-A$ if and only if
$$A=B$$
in that case $A-B=B-A=\emptyset$



$$A-B = A\cap B^{c} = A-(A\cap B)$$

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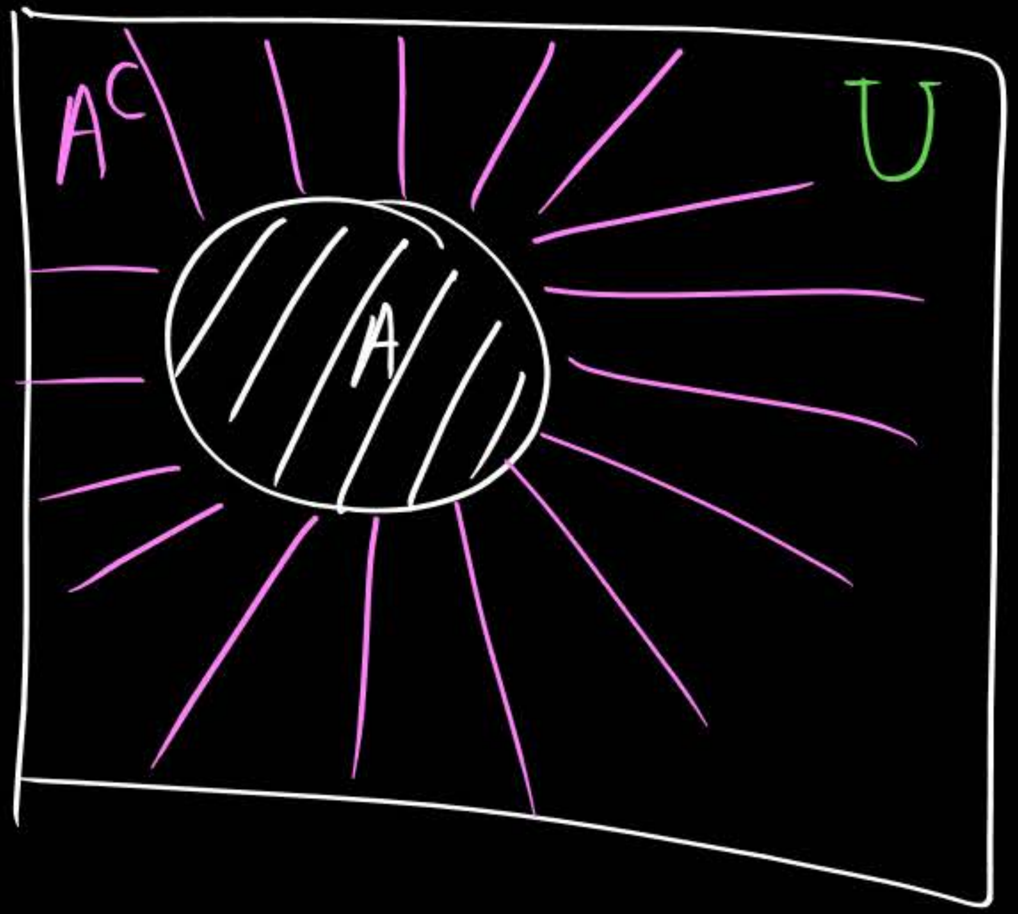
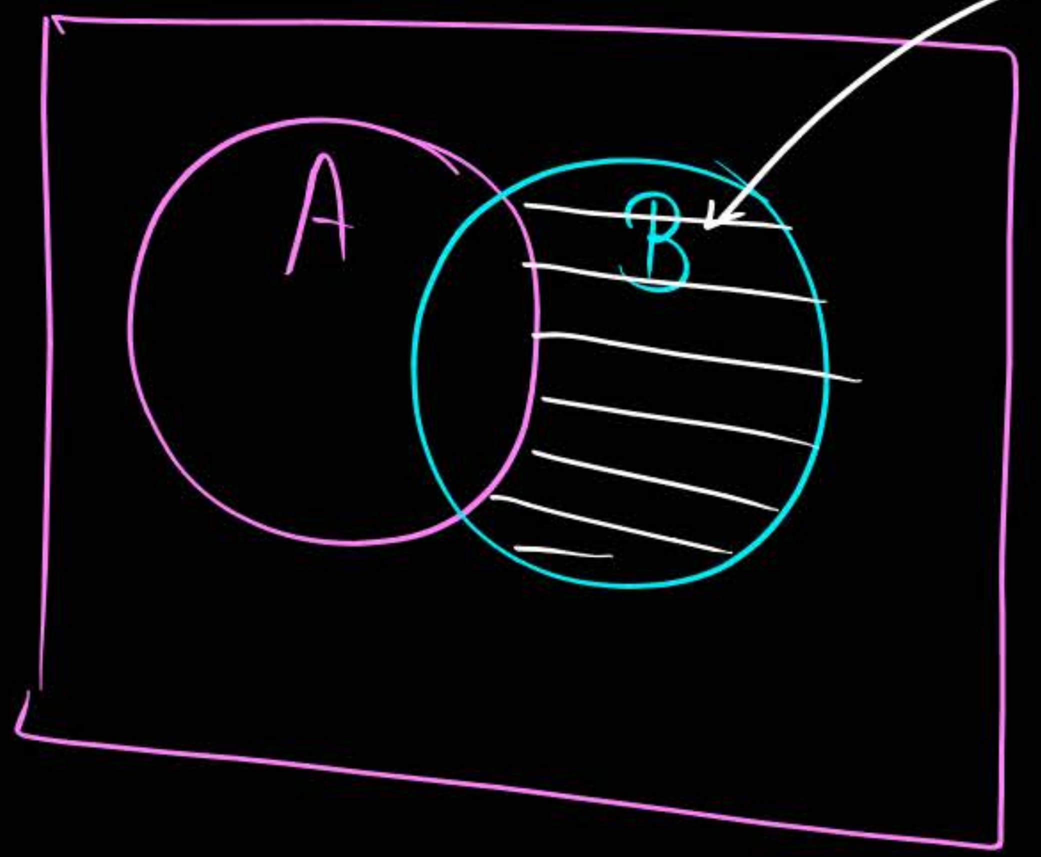


$$\boxed{A - A^c = A}$$

$B - A \neq A - B$



Topic : Symmetric difference of two sets

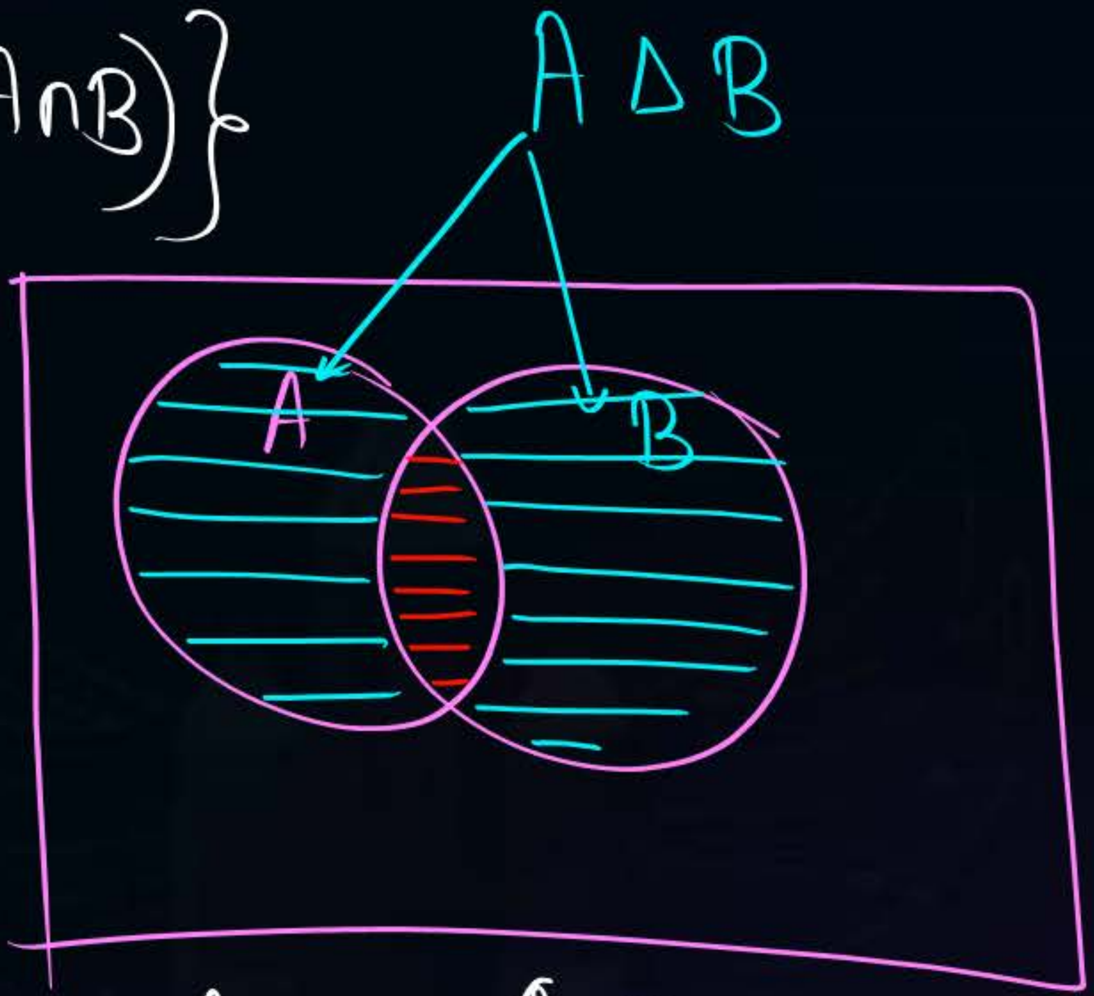
$\left\{ \begin{array}{l} \text{Denoted by} \\ \Delta \text{ or } \oplus \end{array} \right\}$

Let A & B are two sets.

$$A \Delta B = \{ x \mid (x \in A \text{ or } x \in B) \text{ and } (x \notin A \cap B) \}$$

$\boxed{\text{Note: } A \Delta B = B \Delta A}$

$A \Delta B$ is set of all elements which are present either in set A or in set B, but not common for A & B.



$$A \Delta B = (A - B) \cup (B - A)$$

or $$A \Delta B = (A \cup B) - (A \cap B)$$

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Topic : Properties of Set Operations

1. Idempotent:

   a. $A \cap A = A$

   b. $A \cup A = A$

1- Clean

1- Not clean

2. Identity:

   a. $A \cup \emptyset = A$

   b. $A \cap U = A$

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Topic : Properties of Set Operations

3. Domination:

   a. $A \cap \emptyset = \emptyset$

   b. $A \cup U = U$

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Topic : Properties of Set Operations

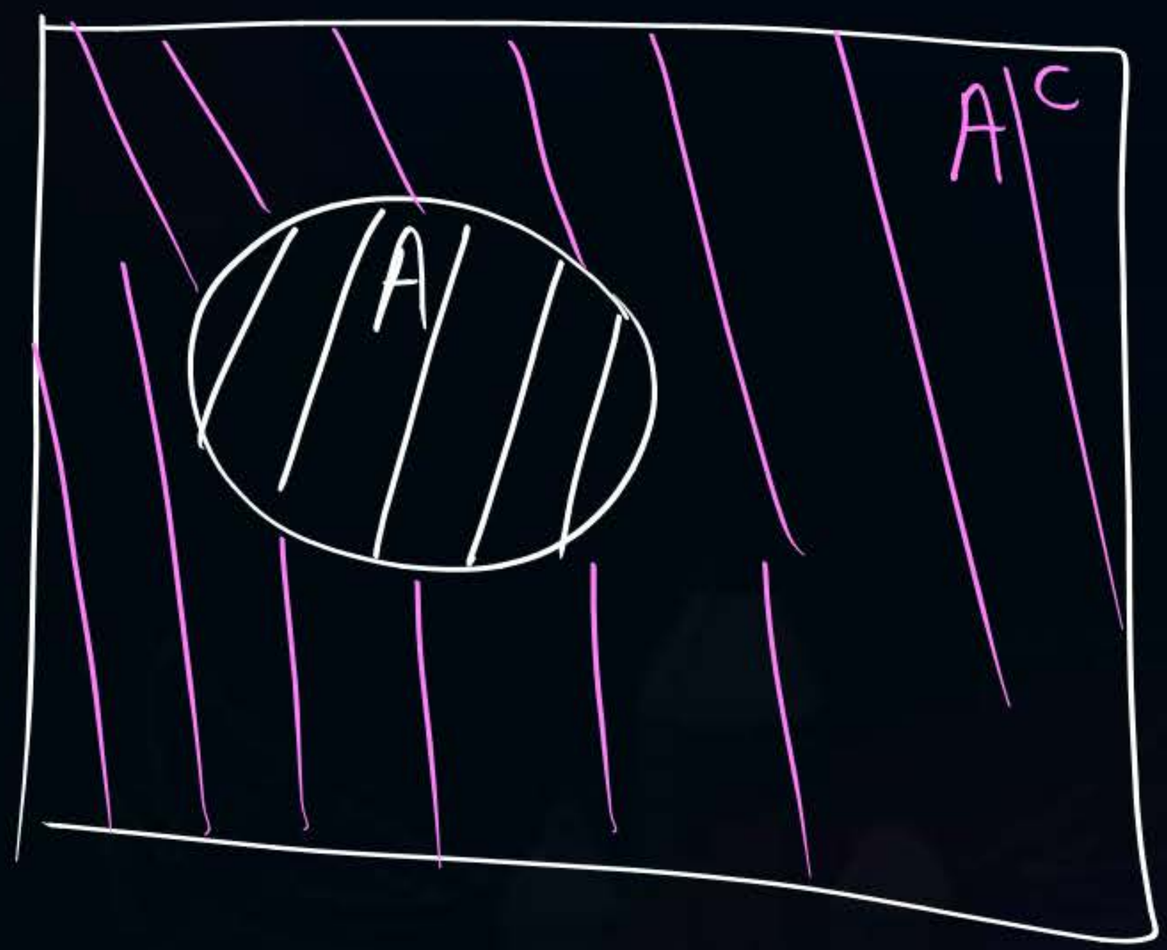
4. Complementation:

   a. $A \cup A^c = U$

   b. $A \cap A^c = \emptyset$

5. Double Complement:

   a. $(A^c)^c = A$



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Topic : Properties of Set Operations

6. Commutative

   a. $A \cup B = B \cup A$

   b. $A \cap B = B \cap A$

7. Associative

   a. $A \cup (B \cup C) = (A \cup B) \cup C$

   b. $A \cap (B \cap C) = (A \cap B) \cap C$

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Topic : Properties of Set Operations

$A \cap B \subseteq A$ | $A \cup \underbrace{\text{subset of } A} = A$

8. Absorption

   a. $A \cup (A \cap B) = A$

   b. $A \cap (A \cup B) = A$

   all elements of A & B

9. DeMorgan's

   a. $(A \cup B)^C = A^c \cap B^c$

   b. $(A \cap B)^C = A^c \cup B^c$

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Topic : Properties of Set Operations

10. Distributive

a. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

b. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

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2 mins Summary

Topic Power set

Topic Venn Diagram

Topic Set operations

Topic Properties of set operations

Topic Principle of Inclusion and Exclusion

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THANK - YOU

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