GATE
ALL BRANCHES
ENGINEERING MATHEMATICS

Single Variable Calculus

Lecture No. 02

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01 Single variable calculus

01 How to calculate limits

Indeterminate forms

NOTE:

Indeterminate form

Following forms are NOT indeterminate forms,

$(1)^{\to\infty} = 1, \frac{\to 0}{0} =$ not defined, $\frac{0}{0} =$ not defined,

$\frac{0}{\to 0} = 0, \frac{\to 0}{\to \infty} = \to 0,\ 0 \times \to \infty = 0, (\to 0)^{\to\infty} = \to 0, 0^0 =$ not defined,

7 Indeterminate forms

$1^{\to\infty}$, $(\to 1)^{\to\infty}$, $\frac{\to 0}{\to 0}$, $\frac{\to\infty}{\to\infty}$, $\to 0 \times \to\infty$, $(\to 0)^{\to\infty}$, 0^0 = Not defined

↙ approaches

Q. Questions

Evaluate: $\lim_{x\to 1}\left(4x^3-3x^2+6\right)$.

$\frac{\to 0}{\to 0}$, $\frac{\to \infty}{\to \infty}$, $(\to \infty)^{\to \infty}$, $\to \infty - \infty$, $(\to 1)^{\to \infty}$ Indeterminate form

→ free

↓ Remove The Indeterminate form

$\frac{\to 0}{\to 0}$ OR $\frac{\to \infty}{\to \infty}$ OR ← consult The limit (Plug-in)

$\lim_{x\to 1}\left(4x^3-3x^2+6\right)$

Plug-In method

$\lim_{x\to 1}\left(4x^3-3x^2+6\right)$

Direct Plug-in Limits

$= 4(1)^3-3\times 1+6$

$= 4+6-3$

$= 7$ Ans

Direct Plug-in ← Evaluation of lt not

Plug-in

$\lim_{x\to 0} \frac{\sin x}{x} = \left(\frac{\to 0}{\to 0}\right)$ Form $= $ Don't apply Plug in Rule

$\lim_{x\to 0} \frac{\tan x}{x} = \left(\frac{\to 0}{\to 0}\right)$ form $=$ " " " " "

$\lim_{x\to \infty} \frac{\sin x}{x} = \left(\frac{\to \infty}{\to \infty}\right)$ form $=$ " " " " "

$\lim_{x\to \infty} \sqrt{x^2+1} - \sqrt{x} = \to\infty - \to\infty =$ Direct Plug in Not allowed

$\lim_{x\to 0} \left(1+\frac{1}{x}\right)^{x}$ $=$ direct Plug - Not allowed

$(\to 1)^{\to \infty}$

Some manipulations → Rationalize, Substitute, Template, L-Hospital

Remove → Direct Plug-in

Remove The Indeterminate form

Q. Questions

Evaluate: $\lim_{x \to 0} \frac{\cos^3 x - 3\cos x + 7}{3x^2 + 5x - 14}$

Using Direct Plug-in method

$= \lim_{x \to 0} \frac{\cos^3 x - 3\cos x + 7}{3x^2 + 5x - 14}$

$\Rightarrow \frac{1 - 3 + 7}{0 + 5 \times 0 - 14}$

$\Rightarrow \frac{1 - 3 + 7}{0 + 0 - 14} = \frac{-5}{14}$

Q. Questions

(i) Evaluate: $\lim_{x \to 2} \frac{x^3 - 2x - 4}{x^2 - 3x + 2}$

$= \lim_{x \to 2} \frac{x^3 - 2x - 4}{x^2 - 3x + 2}$

$= \frac{8 - 4 - 4}{4 - 6 + 2} = \frac{\to 0}{\to 0}$ form

$x^2 - 3x + 2 = 0$

$= x^2 - 2x - x + 2 = 0$

$\Rightarrow x(x-2) - 1(x-2) = 0$

Plug in → solve

Using Factorization — Ist strategy

$= \lim_{x \to 2} \frac{x^3 - 2x - 4}{(x^2 - 3x + 2)}$

$= \lim_{x \to 2} \frac{(x-2)(x^2 + 2x + 2)}{(x-1)(x-2)}$

$= \lim_{x \to 2} \frac{(x^2 + 2x + 2)}{(x-1)}$ Plug-In Rule

$\Rightarrow \frac{(2)^2 + 2 \times 2 + 2}{2 - 1} = 10$

Q. Questions

(ii) Evaluate: $\lim_{x \to a} \frac{x^3 - a^3}{x^2 - ax}$

$\lim_{x \to a} \frac{x^3 - a^3}{(x-a)x} \left(\frac{\to 0}{\to 0}\right)$ Form

Using Factorization

$= \lim_{x \to a} \frac{(x-a)(x^2+a^2+ax)}{(x-a)x}$

$= \lim_{x \to a} \frac{(x^2+a^2+ax)}{x}$

$= \frac{a^2+a^2+a^2}{a}$

$= \frac{3a^2}{a} = 3a$

Q. Questions

(iii) Evaluate: $\lim_{x\to 2} \dfrac{x^3-6x^2+11x-6}{x^2-6x+8}$

$$\lim_{x\to 2} \frac{(x^3-6x^2+11x-6)}{(x^2-6x+8)} \quad \left(\frac{\to 0}{\to 0}\right)$$

$x^3-6x^2+11x-6=0$

$\downarrow$

$(x-1)(x-2)(x-3)$

$x^2-6x+8=0$

$x^2-4x-2x+8=0$

$\Rightarrow x(x-4)-2(x-4)=0$

$\Rightarrow (x-2)(x-4)=0$

$(x-2)(x-4)$

$$= \lim_{x\to 2} \frac{(x-1)(x-2)(x-3)}{(x-2)(x-4)}$$

$$= \frac{(2-1)(2-3)}{(2-4)} = \frac{1\times -1}{-2}$$

$$= \frac{1}{2} \text{ Ans}$$

Q. Questions

Do It own NEST

Evaluate: $\lim_{x \to 3} \frac{x^3 - 7x^2 + 15x - 9}{x^4 - 5x^3 + 27x - 27}$

(a) 2/9

(b) 1/9

(c) -2/9

(d) None of these

Polynomial Divide Rule

Q. Questions

Evaluate: $\lim_{x \to 5} \frac{x^4 - 625}{x^3 - 125}$

$$\lim_{x \to 5} \left\{ \frac{x^4 - 625}{x^3 - 125} \right\}$$

$$= \lim_{x \to 5} \frac{\frac{x^4 - 625}{x - 5}}{\frac{x^3 - 125}{x - 5}}$$

$$\lim_{x \to a} \frac{x^n - a^n}{x - a} = n a^{n-1}$$

$$\lim_{x \to a} \frac{x^n - a^n}{x^m - a^m} =$$

$$= \lim_{x \to 5} \frac{\frac{x^4 - 5^4}{(x - 5)}}{\frac{x^3 - (5)^3}{x - 5}}$$

$$= \frac{4(5)^{4-1}}{3(5)^{3-1}}$$

$$= \frac{4 \times 5^3}{3 \times 52} = \frac{20}{3}$$ Ans

Q. Questions

Evaluate: $\lim_{x\to 3} \dfrac{\sqrt{3x+7}-4}{\sqrt{x+1}-2}$

$$\lim_{x\to 3} \frac{\sqrt{3x+7}-4}{\sqrt{x+1}-2} = \frac{\sqrt{9+7}-4}{\sqrt{3+1}-2} = \left(\frac{\to 0}{\to 0}\right) \text{Form}$$

\# IInd strategy : Rationalize

$$= \lim_{x\to 3} \frac{\sqrt{3x+7}-4}{\sqrt{x+1}-2} \times \frac{\sqrt{x+1}+2}{\sqrt{x+1}+2} \times \frac{\sqrt{3x+7}+4}{\sqrt{3x+7}+4}$$

$$= \lim_{x\to 3} \frac{(3x+7)-16}{(x+1)-4} \times \frac{\sqrt{x+1}+2}{\sqrt{3x+7}+4}$$

$$= \lim_{x\to 3} \frac{3(x-3)}{(x-3)} \times \frac{\sqrt{x+1}+2}{\sqrt{3x+7}+4} \Rightarrow \frac{3}{2}$$

Remove Lt

$$= 3 \times \frac{\sqrt{4}+2}{\sqrt{16}+4} = \frac{3\times(2+2)}{(4+4)}$$

Q. Questions

Evaluate: $\lim_{x \to a} \frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}}$

H.W for NEST

$\frac{2}{\sqrt{3}}$

Q. Questions

Evaluate: $\lim_{x \to \infty} \left(x - \sqrt{x^2 + x} \right)$

(a) -1/2

(b) 1/2

(c) 1

(d) -1

$\lim_{x \to \infty} \left[x - \sqrt{x^2 + x} \right]$

$\to \infty - \to \infty$ Form

→ removed

⇓ Plug in limits

Imp: $\lim_{x\to\infty}\left[x-\sqrt{x^2+x}\right]$ → Rationalize It

$$=\lim_{x\to\infty}\frac{\left[x-\sqrt{(x^2+x)}\right]\left[x+\sqrt{(x^2+x)}\right]}{x+\sqrt{x^2+x}}$$

$$=\lim_{x\to\infty}\frac{x^2-(x^2+x)}{x+\sqrt{(x^2+x)}}$$

$$=\lim_{x\to\infty}\frac{-x}{x+\sqrt{x^2+x}}$$

$$=\lim_{x\to\infty}\frac{-\frac{x}{x}}{\frac{x}{x}+\sqrt{\frac{x^2}{x^2}+\frac{x}{x^2}}}$$

$$=\lim_{x\to\infty}\frac{-1}{1+\sqrt{1+\frac{1}{x}}}$$

If $x\to\infty$, $\frac{1}{x}\to 0$

$$=\lim_{x\to\infty}\frac{-1}{1+\sqrt{1+0}}$$

$$=\frac{-1}{2}=-\frac{1}{2}$$

Q. Questions

Evaluate: $\lim_{x \to \infty} \frac{x^3 - 2x^2 + 3x + 1}{5x^3 + 7x + 2}$

H.W for NEST

Q. Questions

Evaluate: $\lim_{n\to\infty} \dfrac{1^2+2^2+\ldots\ldots\ldots+n^2}{n^3}$

$$\lim_{n\to\infty} \frac{1^2+2^2+--+n^2}{n^3}$$

$$= \lim_{n\to\infty} \frac{n(n+1)(2n+1)}{6n^3}$$

$$= \frac{1}{6}\lim_{n\to\infty} \frac{(n^2+n)(2n+1)}{n^3} = \frac{1}{6}\lim_{n\to\infty} \frac{2n^3+3n^2+n}{n^3}$$

$$\lim_{n\to\infty} \frac{1^2+2^2+--+n^2}{n^3}$$

$$= \lim_{n\to\infty} \frac{1^2}{n^3}+\frac{2^2}{n^3}+\frac{3^2}{n^3}+--+\frac{n^2}{n^3}$$

$$0+0+0+0+0--$$

$$\frac{1^2+2^2+3^2+--+n^2}{\text{sum of } n^2}$$

$$\sum n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$= \frac{1}{6} \lim_{n \to \infty} \frac{2n^3 + 3n^2 + n}{n^3}$$

$$= \frac{1}{6} \lim_{n \to \infty} \frac{2n^3}{n^3} + \frac{3n^2}{n^3 n} + \frac{n}{n^3}$$

$$= \frac{1 \times 2}{6} + 0 + 0$$

$$= \frac{2}{6}$$

$$= \frac{1}{3}$$

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Calculus

Thank You!

GW Soldiers