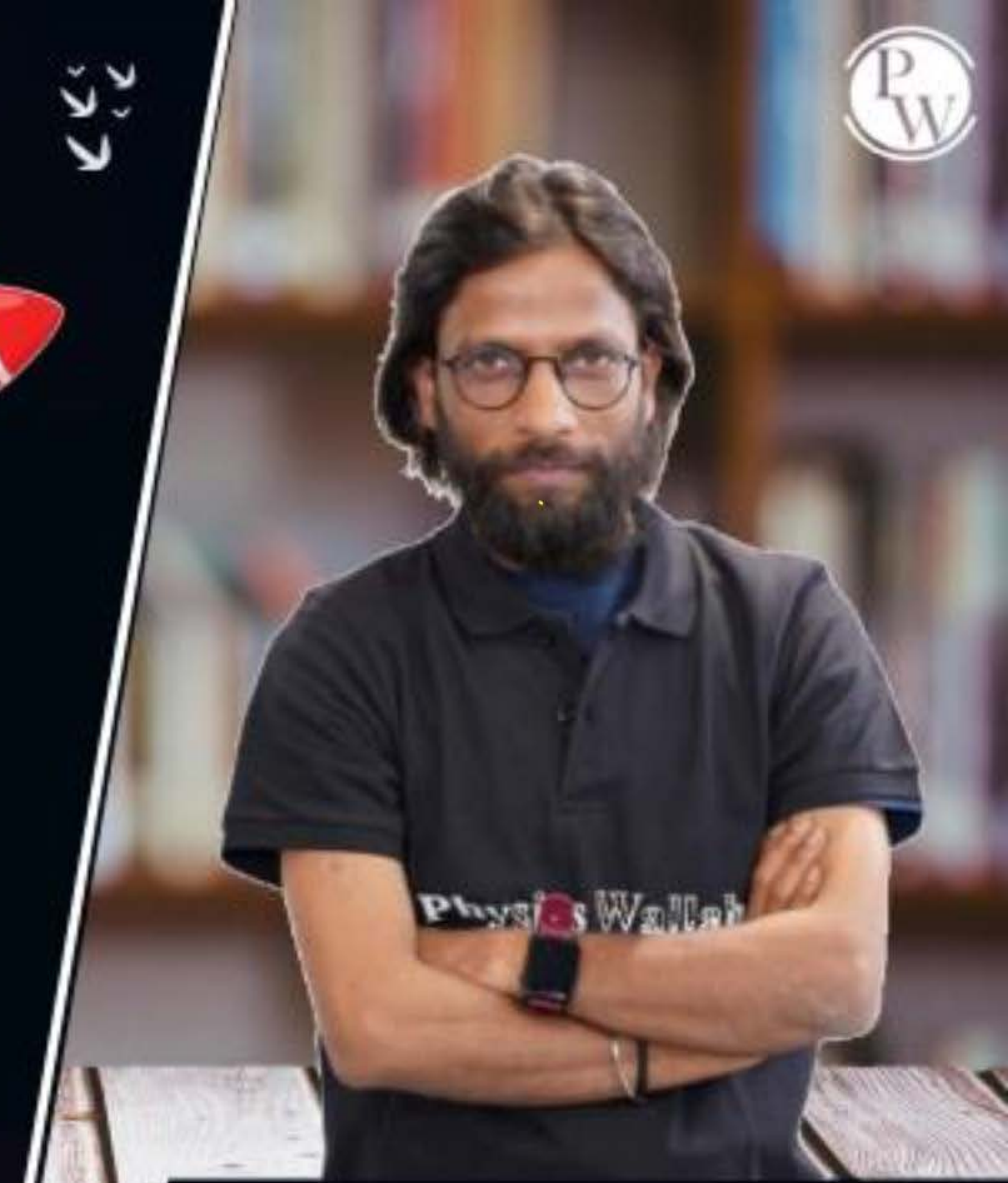
GATE-ALL BRANCHES

ENGINEERING MATHEMATICS

Linear Algebra

Lecture No.- 1



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Recap of previous lecture

Topic Concept of taylor's series

Topic Problems based on taylors series

Topic Partial derivatives

Topic Problems based on partial derivatives

Topics to be Covered

Topic Determinant of matrix

Topic Matrix properties

Questions

#Q. Let $f = y^x$. What is $\frac{\partial^2 f}{\partial x \partial y}$ at $x = 2, y = 1$?

(a) 0

(b) ln 2

(c) 1

(d) $\frac{1}{\ln 2}$

$f = y^x \quad \frac{\partial^2 f}{\partial x \partial y}$ at $x=2$, $y=1$

$\left[\frac{\partial f}{\partial x}\right]_{\text{w.r.t } x}$ y const $\Rightarrow y^x \ln y$

$a^x = a^x \ln a$

$= y^x \ln y$

$\left[\frac{\partial f}{\partial y}\right]$ y variable x const $\Rightarrow$

$y^x = x^n$

$x y^{x-1} \cdot \ln y + \frac{1}{y} \cdot y^x$

$\frac{\partial f}{\partial x} = y^x \ln y$

$\frac{\partial}{\partial y}\left[\frac{\partial f}{\partial x}\right] = \left[\frac{\partial^2 f}{\partial y \partial x}\right]_{y \text{ variable}} = x y^{x-1} \ln y + \frac{1}{y} y^x = (2)(1)^{2-1} \ln 1 + \frac{1}{1}(1)^2$

$0 + 1$

$= 1$

Slide

Questions

#Q. If $z = xy\ln(xy)$, then

(a) $x\boxed{\frac{\partial z}{\partial x}} + y\boxed{\frac{\partial z}{\partial y}} = 0$

(b) $y\frac{\partial z}{\partial x} = x\frac{\partial z}{\partial y}$

(c) $x\frac{\partial z}{\partial x} = y\frac{\partial z}{\partial y}$

(d) $y\frac{\partial z}{\partial x} + x\frac{\partial z}{\partial y} = 0$

$z = \underset{\text{I}}{xy} \times \underset{\text{II}}{\ln(xy)}$

$x\left(\frac{\partial z}{\partial x}\right)_{\substack{x \text{ vari} \\ y \text{ const}}} \Rightarrow \cancel{x}y \cdot \frac{1}{\cancel{xy}} y + \ln xy \cdot y$

$= y(1+\ln xy)$

$\left(\frac{\partial z}{\partial y}\right)_{\substack{y \text{ vari} \\ x \text{ cons}}} = \cancel{xy} \cdot \frac{1}{\cancel{xy}} \cdot x + \ln xy \cdot x$

$y\boxed{\frac{\partial z}{\partial y} = x[1+\ln xy]}$

$$\left[\begin{aligned} x\frac{\partial z}{\partial x} &= xy(1+\ln xy) \\ y\frac{\partial z}{\partial y} &= xy(1+\ln xy) \end{aligned}\right]$$

Slide

Questions

curve - simple closed curve

2 marks.

#Q. The $\boxed{\text{contour}}$ on the x-y plane, where the partial derivative of $x^2 + y^2$ with respect to *y* is equal to the partial derivative of $6y + 4x$ with respect to 'x', is

(a) $y = 2$

(b) $x = 2$

(c) $x + y = 4$

(d) $x - y = 0$

$$\frac{\partial}{\partial y}(x^2+y^2) = \frac{\partial}{\partial x}(6y+4x)$$

$$2y = 4$$

$$\boxed{y=2} \text{ Ans}$$

Slide

Questions

#Q. Let f (x) = e^{x+x^2} for real x. From among the following, choose the Taylor series approximation of f(x) around x = 0, which includes all powers of x less than or equal to 3.

[DoIt Home]

$f(x) = e^{x+x^2}$ $\quad x = 0 \quad f'(0) = \quad f''(0) \quad 3 \quad f'''(0)$

Using Taylor SERIES / Using maclurins SERIES $x=0$

(a) $1 + x + x^2 + x^3$

(b) $1 + x + \frac{3}{2}x^2 + x^3$

(c) $1 + x + \frac{3}{2}x^2 + \frac{7}{6}x^3$

(d) $1 + x + 3x^2 + 7x^3$

$$f(x) = f(a) + (x-a)\frac{f'(a)}{1!} + (x-a)^2\frac{f''(a)}{2!} + \cdots$$

$$= f(0) + (x-0)\frac{f'(0)}{1!} + (x-0)^2\frac{f''(0)}{2!} + (x-0)^3 f'''(0) + \cdots$$

$$= f(0) + x\frac{f'(0)}{1!} + x^2\frac{f''(0)}{2!} + x^3\frac{f'''(0)}{3!} + \cdots$$

Slide

Questions

#Q. Consider a function $f(x, y, z)$ given by $f(x, y, z) = (x^2 + y^2 - 2z^2)(y^2 + z^2)$. The partial derivative of this function with respect to x at the point $x = 2$, $y = 1$ and $z = 3$ is ____.

H.W

Home work

Do yourself

Slide

Questions

#Q. Let $f(x, y) = \frac{ax^2 + by^2}{xy}$, where a and b are constants. If $\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y}$ at $x = 1$ and $y = 2$, then relation between a and b is

H.W

Do yourself

(a) $a = \frac{b}{4}$

(b) $a = \frac{b}{2}$

(c) $a = 2b$

(d) $a = 4b$

Slide

Questions

#Q. Taylor series expansion of $f(x)=\int_0^x e^{-\left(\frac{t^2}{2}\right)}dt$ around x = 0 has the form

$f(x) = a_0 + a_1x + a_2x^2 + \ldots..$

The coefficient a_2 (correct to two decimal places) is equal to ____.

$$f(x) = \int_0^x e^{-t^2/2}\,dt \text{ around } x=0$$

Newton Leibnitz Principle

$$f(x) = \int_{\phi(x)}^{\psi(x)} f(t)\,dt$$

Slide

$$f(x)=\int_{\phi(x)}^{\psi(x)} f(t)\,dt$$

Using Newton – Leibnitz Rule:

both sides Differentiate It w.r.t x

$$\frac{d}{dx}f(x)=\frac{d}{dx}\int_{\phi(x)}^{\psi(x)} f(t)\,dt$$

$$\Rightarrow f'(x)=f[\psi(x)]\frac{d}{dx}\psi(x)-f(\phi(x))\cdot\frac{d}{dx}[\phi(x)]$$

$$f(x)=\int_{x^2}^{x^3}\sin t\,dt$$

$$f'(x)=\sin x^3\frac{d}{dx}x^3-\sin x^2\frac{d}{dx}(x^2)=\underline{\sin x^3\cdot 3x^2-\sin x^2\cdot 2x}$$

$x = 0$

$$f(x) = \int_0^x e^{-t^2/2}\,dt$$

$$f'(x) = e^{-x^2/2} \frac{d}{dx}(x)$$

$$f'(x) = e^{-x^2/2}$$

$$f''(x) = e^{-\frac{x^2}{2}} \cdot \left(-\frac{\cancel{2}x}{\cancel{2}}\right)$$

$$= -x e^{-x^2/2}$$

$$\begin{cases} f(0) = \int_0^0 e^{-t^2/2}\,dt = 0 \\ f'(0) = 1 \\ f''(0) = 0 \end{cases}$$

coefficient of a_2 around $x = 0$

<u>Using maclurins SERIES</u>

$$= f(0) + a_1 f'(0) + a_2 \frac{f''(0)}{2!} + \cdots$$

$$= a_2 \cdot \frac{(0)}{2!}$$

<u>a_2 colfficient $= 0$</u>

Determinant of a matrix
matrix Properties
System of Equations
eigenvalues
eigenvectors
Diagonalization / Power of matrices.

Linear Algebra

Linear Algebra: Matrix: A SET of m Rows, n columns in a rectangular array [ ]

[A] = matrix

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \end{bmatrix}_{2\times 4}$$

$$\begin{array}{cl} A = \begin{bmatrix} \text{Elements} \\ \text{arrange} \end{bmatrix} & \begin{array}{l} R_1 \\ R_2 \\ R_3 \\ R_4 \\ R_n \end{array} \\ \quad C_1 \; C_2 \; C_3 \; C_4 \; C_n & \end{array}$$

Row Horizontal

Column Vertical

= OPERATOR

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{3\times 3}$$

i = Rows.

j = columns.

matrix Size | order | Total of elements = Row x columns.

Total No. of element = 9 ($R \times C$)

$$A_{ij} = \begin{bmatrix} \text{square} \\ \text{matrix} \\ R = C \end{bmatrix}_{\substack{m\times n \\ n\times n}} \quad m = n$$

$$A_{ij} = \begin{bmatrix} \text{Non square} \\ \text{matrices} \\ R \neq C \end{bmatrix}_{m\times n} \quad m \neq n$$

Determinant of matrix : $|A|$ = determinant of matrix A

A_{ij} = square matrix

" determinant always Possible only square matrix "

$$[A_{ij}]_{2\times 2} = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}_{2\times 2} = \boxed{a_{11}a_{22} - a_{12}a_{21}}$$

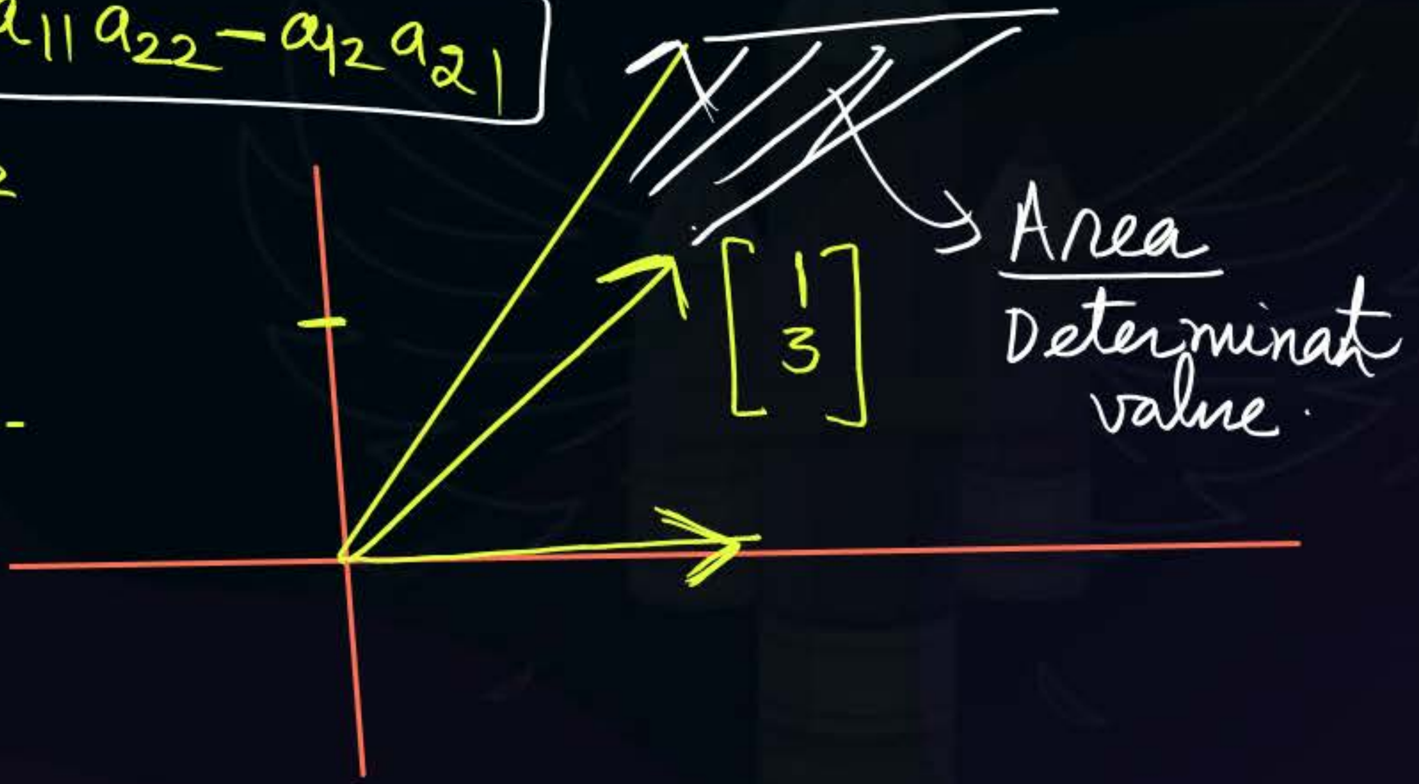
Area

Column

$$[A_{ij}] = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

Horizontal

$$\begin{aligned} x + 2y &= 0 \\ 3x + 4y &= 0 \end{aligned} \quad \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$



Sign convention $(-1)^{i+j}$

$$A_{ij} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{3\times 3}$$

$$a_{11}x + a_{12}y + a_{13}z = 0$$
$$a_{21}x + a_{22}y + a_{23}z = 0$$
$$a_{31}x + a_{32}y + a_{33}z = 0$$

volume

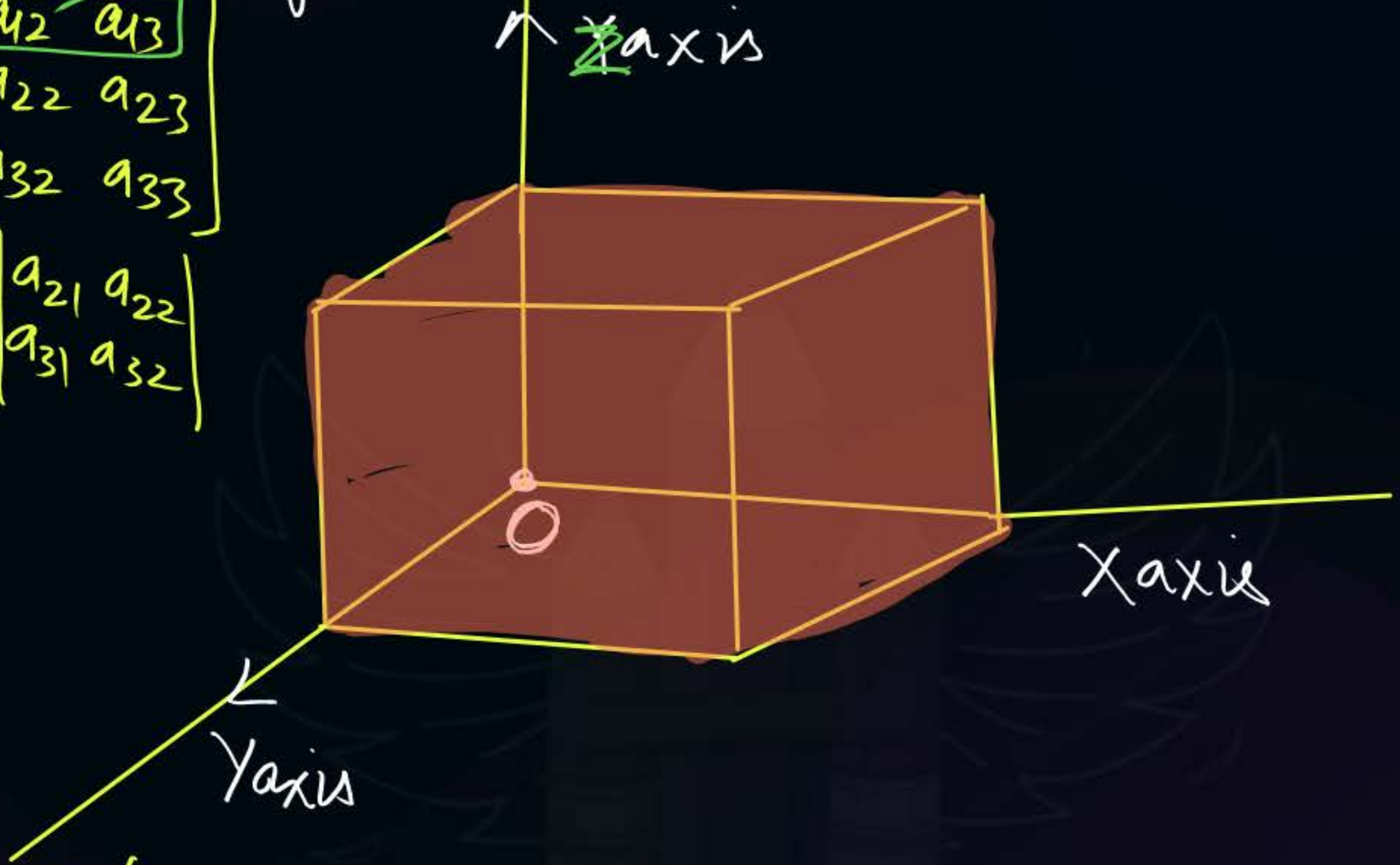
det = volume

det of matrix $A = |A|$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$\Rightarrow a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$$A_{3\times 3} = \begin{vmatrix} 1 & 0 & 2 \\ 1 & 3 & 5 \\ 5 & 2 & 1 \end{vmatrix}$$



$$= 1\begin{vmatrix} 3 & 5 \\ 2 & 1 \end{vmatrix} - 0\begin{vmatrix} 1 & 5 \\ 5 & 1 \end{vmatrix} + 2\begin{vmatrix} 1 & 3 \\ 5 & 2 \end{vmatrix}$$

$$= 1 \times (-7) + 2(2-15) = -7 - 26 = (-33)$$

#

SHORTcut method "Sarrus (3X3) Rule"

$$A = \begin{array}{c} \\ \begin{bmatrix} 1 & 2 & -1 \\ 2 & 0 & 3 \\ 3 & 2 & 1 \end{bmatrix} \\ C_1 \quad C_2 \quad C_3 \end{array} \begin{array}{l} - R_1 \\ - R_2 \\ - R_3 \\ 3 \times 3 \end{array}$$

-5

$\downarrow$

Mag + Direction
$(-)$

starting Two columns Are repeated

$$\begin{matrix} 1 & 2 & -1 & 1 & 2 \\ 2 & 0 & 3 & 2 & 0 \\ 3 & 2 & 1 & 3 & 2 \end{matrix}$$

$\Rightarrow -4 - 6 + 0 = -10$

$$\begin{matrix} + & + & + & & \\ 1 & 2 & -1 & 1 & 2 \\ 2 & 0 & 3 & 2 & 0 \\ 3 & 2 & 1 & 3 & 2 \end{matrix}$$

$= -4 + 18 + 0$

$= \underline{14}$

SUM = White + yellow elements sum

$= -10 + 14$

$= 4$

$$A_{ij} = A_{3\times3} = \begin{bmatrix} 1 & 1 & 3 \\ -1 & 5 & 2 \\ 2 & -1 & 0 \end{bmatrix}$$

multiplication

$$\begin{array}{ccccc} 1 & 1 & 3 & 1 & 1 \\ -1 & 5 & 2 & -1 & 5 \\ 2 & -1 & 0 & 2 & -1 \end{array}$$

$\Rightarrow \underline{+2} - 30 + \underline{3 + 4} + 0$

$= \underline{-21}$

(A) If any Two Rows or Two columns

$\boxed{\det A = 0}$

Types of matrix :

Diagonal matrix

$$A_{3\times3} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{3\times3}$$

Diagonal Matrix always exists = Square matrices.

If $a_{ij} \Rightarrow \begin{cases} 0 & i \neq j \\ d_1, d_2, d_3 & i = j \end{cases}$

Principal Diagonal

$$A_{\text{diagonal matrix}} = \begin{bmatrix} d_1 & 0 & 0 \\ 0 & d_2 & 0 \\ 0 & 0 & d_3 \end{bmatrix} = \text{Diag}(d_1, d_2, d_3)$$

Identity matrix / scaler matrix / unit matrix

$d_1, d_2 d_3 \in 1$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$d_1, d_2 d_3 \in \text{Scaler}$

$$\begin{bmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Upper Triangular matrix

$$A = [A_{ij}] = \begin{bmatrix} a & d & e \\ 0 & b & f \\ 0 & 0 & c \end{bmatrix}$$

below The Diagonal elements Are ZERO (Lower elements)

Lower Triangular matrix:

$$A = [A_{ij}] = \begin{bmatrix} a & 0 & 0 \\ d & b & 0 \\ e & f & c \end{bmatrix}$$

Upper elements OR above The Diagonal elements Are ZERO determinant of matrix

$|A| = $ product of diagonal entries

Diagonal Matrix

$$\begin{bmatrix} d_1 & 0 & 0 \\ 0 & d_2 & 0 \\ 0 & 0 & d_3 \end{bmatrix} \quad |A| = d_1 d_2 d_3$$

Upper Triangular

$$\begin{bmatrix} a & d & e \\ 0 & b & f \\ 0 & 0 & c \end{bmatrix} \quad |A| = abc$$

Lower Triangular

$$\begin{bmatrix} a & 0 & 0 \\ d & b & 0 \\ e & f & c \end{bmatrix} \quad \det A = |A| = abc$$

scaler matrix

$$A = \begin{bmatrix} K & 0 & 0 \\ 0 & K & 0 \\ 0 & 0 & K \end{bmatrix} \quad |A| = K \cdot K \cdot K = \underline{K^3}$$

THANK - YOU