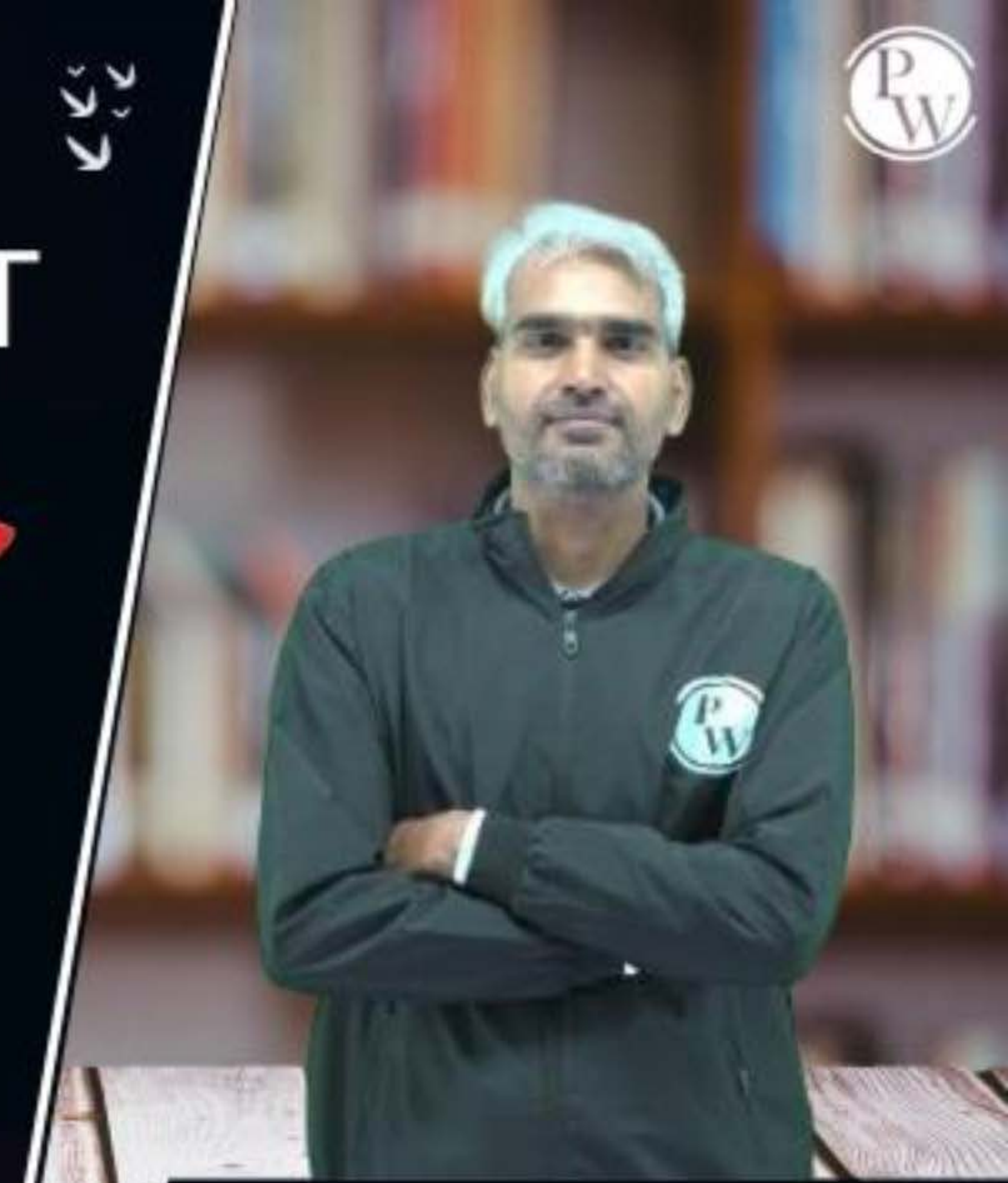
Computer Science & IT

Discrete Mathematics

Set Theory & Algebra

Lecture No. 02



By- Vishal Sir

Recap of Previous Lecture

Topic Introduction to Discrete Maths

- Set Theory & Algebra
- Graph Theory
- Mathematical Logic
- Recurrence Relⁿ + Generating Functions

Slide

Topics to be Covered

Topic Set

Topic Representation of Set

Topic Types of sets

Topic Terminologies related to sets

Slide

Topic : Set

A well-defined unordered collection of distinct elements is called as set.

eg: $A = \{1, 2, 3\} = \{2, 1, 3\}$
$= \{3, 1, 2\}$

eg: $B = \{1, 2, a, b, \text{Jan}, \text{Sun}\}$

elements need not be of similar type.

eg: $C = \{1, 2, 2, 3, 4\}$

It is a multi-set

not a set

Set w.r.t. this collection of elements will be $\{1, 2, 3, 4\}$

eg: $D = \{1, 2, \{1, 2, 3\}, \{2, 3\}\}$

4 different elements

Slide

In general,

N : Set of all natural numbers

$$N = \{1, 2, 3, 4, \ldots\}$$

W : Set of all whole numbers

$$W = \{0, 1, 2, 3, 4, \ldots\}$$

R : Set of all real numbers

Z : Set of all integers

Z^+ : Set of all +ve integers

Topic : Representation of sets

There are three different ways in which set may be represented.

1. Roaster form or Tabular form:
2. Set-builder form:
3. Statement form:

Slide

Topic : Roaster form or Tabular form

In roaster notation all the elements of the set are listed within the curly braces '{'

eg: $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

{It may not be possible to represent all the sets in tabular/roaster form}

Slide

Topic : Set-builder form

In set-builer form property specifying the elements of the set is defined.

eg: $A = \{x \mid \underbrace{x \in N \text{ and } x \leq 10}\}$

A is a set | Containing all the elements 'x' | Such that | following property is satisfied

Slide

$B = \{0, 0.12, 0.25, 0.5, 0.72, 0.85\}$

it may not be possible to represent this set using set-builder form of representation

---

$C = \{x \mid x \in R \text{ and } 0 < x < 1\}$ ← It may not be possible to represent the elements of this set in roaster form.

$0 \le x < 17$

$0 \le x \le 10$

$0 < x \le 15$

Topic : Statement form

* The statement is used to define the elements of the set.

eg. A = Set of all natural numbers less than or equal to 10.

C = Set of all real numbers greater than '0' and less than '1'

Slide

Topic : Cardinality of a set

Cardinality is defined only for finite sets.

* Cardinality of a finite set A is defined as number of elements in set A, it is denoted by $|A|$.
* Cardinality of set A is also known as size of set A.

Slide

eg: Let $A = \{1, 2, 3\}$ then $|A| = 3$

eg: let $B = \{\underbrace{1}_{\text{integer '1'}}, \underbrace{\{1\}}_{\text{set containing integer '1'}}, \underline{\{2\}}, \underline{\{1, 2\}}\}$, then $|B| = 4$

Empty Set: A set containing no element in it is called an empty set, denoted by ϕ or $\{\ \}$

W.r.t. Empty Set $|\phi| = |\{\ \}| = 0$

Note: $\underbrace{\{\ \}}_{\text{it is an empty set}} \neq \{\{\ \}\}$

it is a set containing an empty set

$|\{\ \}| = 0$

$|\{\{\ \}\}| = 1$

Slide

Singleton Set: A set containing exactly one element is called singleton set

eg. $A = \{1\}$

$B = \{\{\ \}\}$

$C = \{\{1\}\}$

$D = \{\{1, 2, 3\}\}$

Slide

Topic : Types of sets

Finite Set: A set containing finite number of elements in it is called a finite set.

eg. $A = \{1, 2, 3\}$

$B = \{1, a, b, d, e\}$

Note:- Empty set is also a finite set.

Slide

Topic : Types of sets

Infinite Set: A set containing infinite number of elements in it.

eg: A = Set of all natural numbers

$B = \{x \mid x \in R, \text{ and } 0 < x \leq 1\}$

Slide

Topic : Types of sets

Equal Sets: Two sets A and B are said to be equal only if every element of set A is included in set B and every element of set B is included in set A.

eg $\left.\begin{array}{l} A = \{1, 2, a, b\} \\ B = \{2, a, b, 1\} \end{array}\right\} A = B$

eg: $\left.\begin{array}{l} A = \{1, 2, a, b\} \\ B = \{1, 2, 3, a, b\} \end{array}\right\} A \neq B$

Slide

Topic : Types of sets

Equivalent Sets: Two finite sets are said to be equivalent if they have same Cardinality

$\cong$ or $\sim$

eg. $A = \{1, 2, a, b\}$ $|A| = 4$

$B = \{2, 4, 7, 9\}$ $|B| = 4$

$\therefore A \cong B$ or $A \sim B$

but $A \neq B$

eg. $A = \{1, 2, 3, a\}$

$B = \{a, 3, 2, 1\}$

$\Rightarrow A = B \therefore A \cong B$

Slide

* If two sets A & B are equal, then they are also equivalent.
* But two equivalent sets may or may not be equal sets.

* Two infinite sets are said to be equivalent if there exists a $\underbrace{\text{bijective function}}_{\text{(one-one and onto)}}$ between them.

* Set of all +ve number is equivalent to set of all −ve numbers.

Topic : Types of sets

Universal Set: A set containing all the elements corresponding to the problem under discussion

Slide

$\subseteq$ $\supseteq$
Subset & Superset: Let A and B are any two arbitaray sets. set A is called subset of set B if every element of set A is included in set B, denoted by $A \subseteq B$

eg: $A = \{1, 2, 3\}$
$B = \{a, 1, b, 2, 3, d\}$

we can observe
$A \subseteq B$
but $B \not\subseteq A$

i.e, A is subset of B
or
B is superset of A

Slide

Note :- ① A is subset of B if and only if B is a superset of A.

Note :- ② If $A \subseteq B$ as well as $B \subseteq A$, then $A = B$

eg. $A = \{1, 2, a, 3\}$ $B = \{2, 1, 3, a\}$

$A \subseteq B$ ✓ and $B \subseteq A$ ✓

$\therefore A = B$

Note:- (3) $\emptyset$ is a subset of every set.

Note:- (4) For any set A, set A itself is always a subset of set A.

Q: Write all the subsets of set $A = \{1, 2, 3\}$
Where size of set $A = 3$

Subsets of set A are

$$\underbrace{\{\ \}}_{\text{subset of size}=0} \qquad \underbrace{\{1\}\ \{2\}\ \{3\}}_{\text{Subsets of size}=1} \qquad \underbrace{\{1,2\}\ \{1,3\}\ \{2,3\}}_{\text{Subsets of size}=2} \qquad \underbrace{\{1,2,3\}}_{\text{Subset of size}=3}$$

$A = \{1, 2, 3\}$

$\in$ ← belongs to

(a) $1 \in A$ ✓

(b) $\{1\} \in A$ X

(c) $1 \subseteq A$ X

(d) $\{1\} \subseteq A$ ✓

Q. Consider the following set $A = \{1, \underbrace{\{1\}}_{a}, 2, \underbrace{\{1,2\}}_{c}, 3\}$

Which of the following is/are true

(a) $\{1\} \in A$

(b) $\{1\} \subseteq A$

(c) $\{1,2\} \in A$

(d) $\{1,2\} \subseteq A$

Subsets of A

$\{\ \}, \underbrace{\{1\}}_{b}, \{\{1\}\}, \{2\}, \{\{1,2\}\}, \{3\}$

$\{1, \{1\}\}, \underbrace{\{1,2\}}_{d}, \{1, \{1,2\}\}, \{1,3\}, \ldots$

Topic : Types of sets

Proper Subset: "denoted by $\subset$"

* Any subset of set A except set A itself is called proper subset of set A.

eg. let $A = \{a, b, c\}$

Proper Subsets of Set A $\{ \}$, $\{a\}$, $\{a, b\}$, $\{b\}$, $\{a, c\}$, $\{c\}$, $\{b, c\}$

$\{a, b, c\}$ is a subset of $\{a, b, c\}$ but not a proper subset of set $\{a, b, c\}$

Slide

Topic : Number of subsets of a set 'A' of cardinality 'n'

eg: $A = \{1, 2, 3\}$ $|A| = 3$

8 different subsets of set A

Subsets of A are

$\underbrace{\{\ \}}_{\text{Size}=0}$	$\{1\}$	$\{1, 2\}$	$\underbrace{\{1, 2, 3\}}_{\text{Size}=3}$
	$\{2\}$	$\{1, 3\}$	
	$\underbrace{\{3\}}_{\text{Size}=1}$	$\underbrace{\{2, 3\}}_{\text{Size}=2}$	

+ If $|A| = n$, then number of subsets of set $A = 2^{|A|} = 2^n$

Slide

* If there are 'n' distinct elements, then out of those n distinct elements we can Choose 'r' distinct elements in $^{n}C_{r}$ ways.

Selection {not arrangement}

$$^{n}C_{r} = \frac{n!}{(n-r)!\,r!}$$

$$^{5}C_{3} = \frac{5!}{(5-3)!\,3!} = \frac{5 * 4 * \cancel{3!}}{2! * \cancel{3!}} = \frac{20}{2} = 10$$

Let A is a set of size = n

{ Every subset of set A will be of size $\leq n$ where $|A| = n$

No. of subsets of size = 0 → ${}^{n}C_0$

No. of subsets of size = 1 → ${}^{n}C_1$

No. of subsets of size = 2 → ${}^{n}C_2$

. . . . .

No. of subsets of size = k $(k < n)$ → ${}^{n}C_k$

. . .

No. of subsets of size = n → ${}^{n}C_n$

Let A is a set of size = n

{ Every subset of set A will be of size $\leq n$ where $|A| = n$ }

No. of subsets of size = 0	No. of subsets of size = 1	No. of subsets of size = 2	....	No. of subsets of size = K ($K < n$)	....	No. of subsets of size = n
$\downarrow$	$\downarrow$	$\downarrow$		$\downarrow$		$\downarrow$
${}^{n}C_{0}$	${}^{n}C_{1}$	${}^{n}C_{2}$		${}^{n}C_{K}$		${}^{n}C_{n}$

Total number of subsets of set A $= {}^{n}C_{0} + {}^{n}C_{1} + {}^{n}C_{2} + \cdots + {}^{n}C_{K} + \cdots + {}^{n}C_{n} = 2^{n}$

Number of subsets of size $\leq K$

Number of subsets of size $= K$

No. of subsets of size $\geq K$

$$(1+x)^n = {}^nC_0 \cdot x^n + {}^nC_1 \cdot x^{n-1} + {}^nC_2 \cdot x^{n-2} + \cdots + {}^nC_n \cdot x^{n-n}$$

$x \downarrow 1$

$$(1+1)^n = {}^nC_0 (1)^n + {}^nC_1 \cdot (1)^{n-1} + \cdots \; {}^nC_n \cdot (1)^{n-n}$$

$$\boxed{(2)^n = {}^nC_0 + {}^nC_1 + {}^nC_2 + \cdots {}^nC_n}$$

* If A is a finite set of size = n {i.e $|A| = n$}, then number of proper subsets of set A $= \underbrace{2^n}_{\text{total number of subsets}} \underbrace{-}_{\text{Except}} \underbrace{1}_{\text{set A itself}}$

2 mins Summary

Topic Set

Topic Representation of Set

Topic Cardinality of a set

Topic Types of sets

Topic Number of subsets, Concept of power set, and Cardinality of power set of a set

Slide

THANK - YOU

Slide