GATE
ALL BRANCHES
ENGINEERING MATHEMATICS

Probability and Statistics

Lecture No. 02

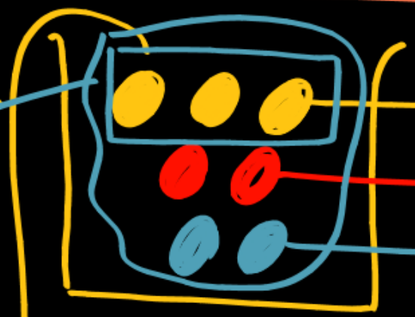
BY– RAHUL SIR

01 Fundamentals of Probability

02 Question Based on Probability

Problem Solving Strategies :- Without Replacement

✓ CASE(A)



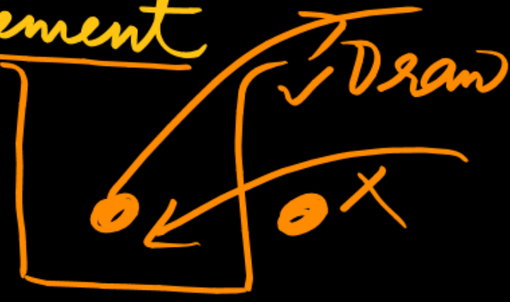
3 yellow Ball
2 Red ball
2 blue balls

for Choice

1st yellow ball

II nd yellow

What is The Prob: 3 yellow ball are Drawn Randomly Taken one at time



✓ Draw

$$P(\text{yellow}_1) = \frac{3}{7} \longrightarrow W_1 \text{ Ob.}$$

$$P\left(\frac{\text{yellow 2}}{\text{yellow 1}}\right) = \frac{2}{6} \longrightarrow W_2 \text{ Ob.}$$

yellow 1 / yellow 2 → yellow 2 is already occured
yellow 1 is occuring

$P\left(\frac{\text{yellow 2}}{\text{yellow 1}}\right)$ = yellow 1 is already occured
yellow 2 occuring

3rd yellow ball

$P\left(\frac{Y_3}{Y_1, Y_2}\right) = \frac{1}{5}$ → W_3 Job

Y_3 — occuring

$\frac{Y_3}{Y_1\, Y_2}$ → occured

$P(3 \text{ yellow balls Are Drawn}) = \underbrace{W_1 \times W_2 \times W_3}_{\text{Working Together}}$

$= \frac{3}{7} \times \frac{2}{6} \times \frac{1}{5}$

(A) sample Space Are Reduced / Prob / chances Are change

(B) Next prob effected via Previous Prob.

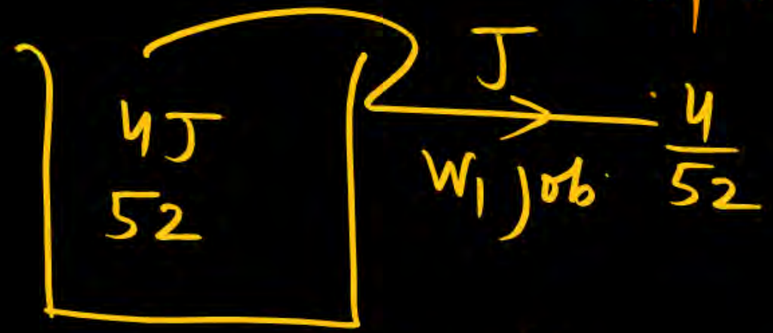
(C) Dependent events / without replacement

✓ without Replacement

52 cards
4 Jack
4 King
4 Queen
3 cards draw

Ans-① $P(JKQ) = \frac{4}{52} \times \frac{4}{51} \times \frac{4}{50}$

$W_1 \times W_2 \times W_3$

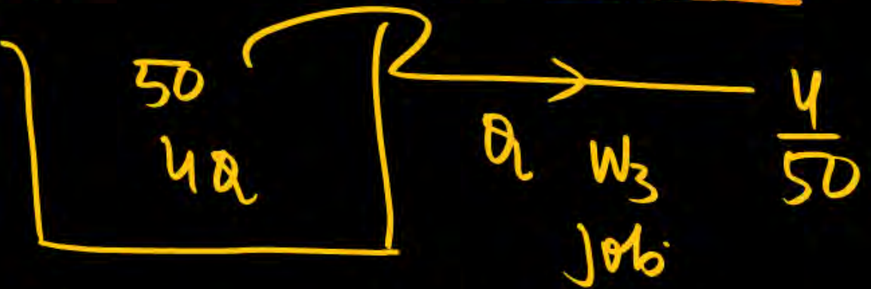


What is the prob. THREE (order is specified) cards.

J K Q
① ② ③

✓ Imp # What is the Prob if THREE cards Drawn (ONE of THOSE)

CARDS — J, Q, K



$P(JKQ) = \frac{4}{52} \times \frac{4}{51} \times \frac{4}{50}$

Dependency.

No. of sample space reduced

4K
4Q 52
4J

$$\frac{{}^4C_1 \times {}^4C_1 \times {}^4C_1}{{}^{52}C_3}$$

Too much confusion

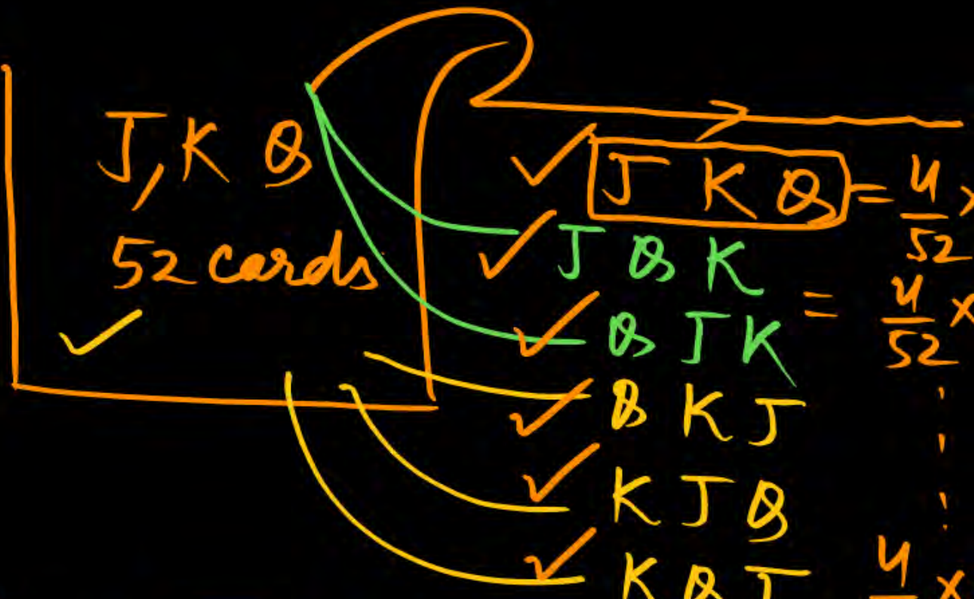
$$\frac{4}{52} \times \frac{4}{50} \times \frac{4}{51}$$

Fundamental Principal of Counting

1) 6 options

J, K, Q

52 cards



$= \frac{4}{52} \times \frac{4}{51} \times \frac{4}{50}$

$= \frac{4}{52} \times \frac{4}{51} \times \frac{4}{50}$

$\frac{4}{52} \times \frac{4}{51} \times \frac{4}{50}$

THREE cards drawn

One of THOSE cards

✓ J, K, Q [ordering]

$= 6 \times \frac{4}{52} \times \frac{4}{51} \times \frac{4}{50}$

$= 3! \times \frac{4}{52} \times \frac{4}{51} \times \frac{4}{50}$

→ 3 balls Are drawn

One of THOSE balls Are: Orange, White, blue

(NO Two balls ARE SAME colour)

$6 \times \frac{3}{7} \times \frac{2}{6} \times \frac{2}{5}$

$= 3! \times \frac{3}{7} \times \frac{2}{6} \times \frac{2}{5}$ Ans

#

With Replacement

What is The Prob:
3 balls Are Drawn one of THOSE
(with replacement)
Yellow, Green, blue

$$= 3! \times \frac{3}{8} \times \frac{2}{8} \times \frac{1}{8}$$

green

3 Different arrange: YGB

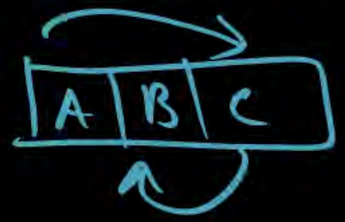
YGB
YBG
BGY
BYG
GYB
GBY

for choice

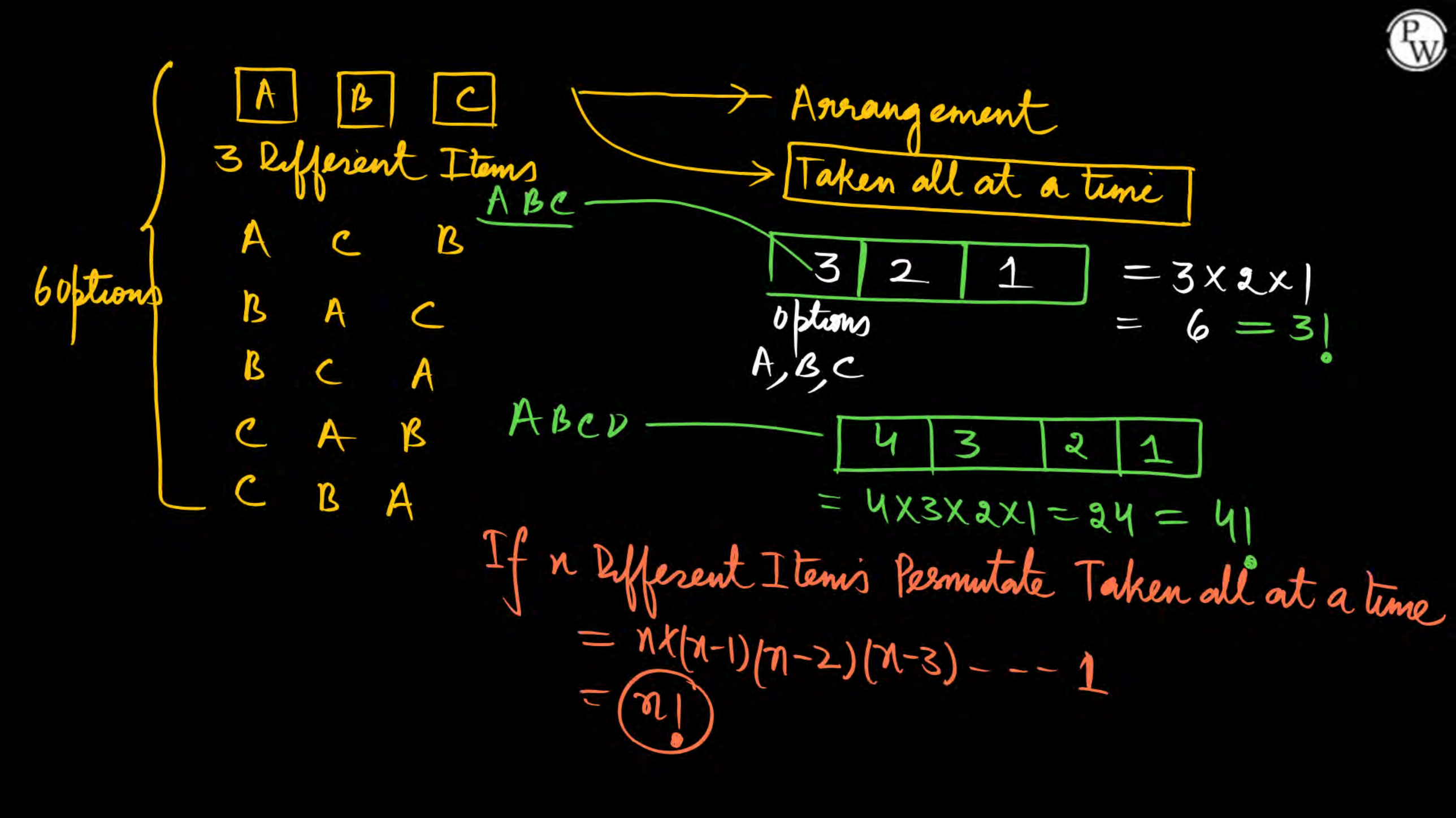
A) SAMPLE space Not reduced

2) Next prob don't effect previous one

3) Independent events → with replacement



Permutation – arrangement order matters

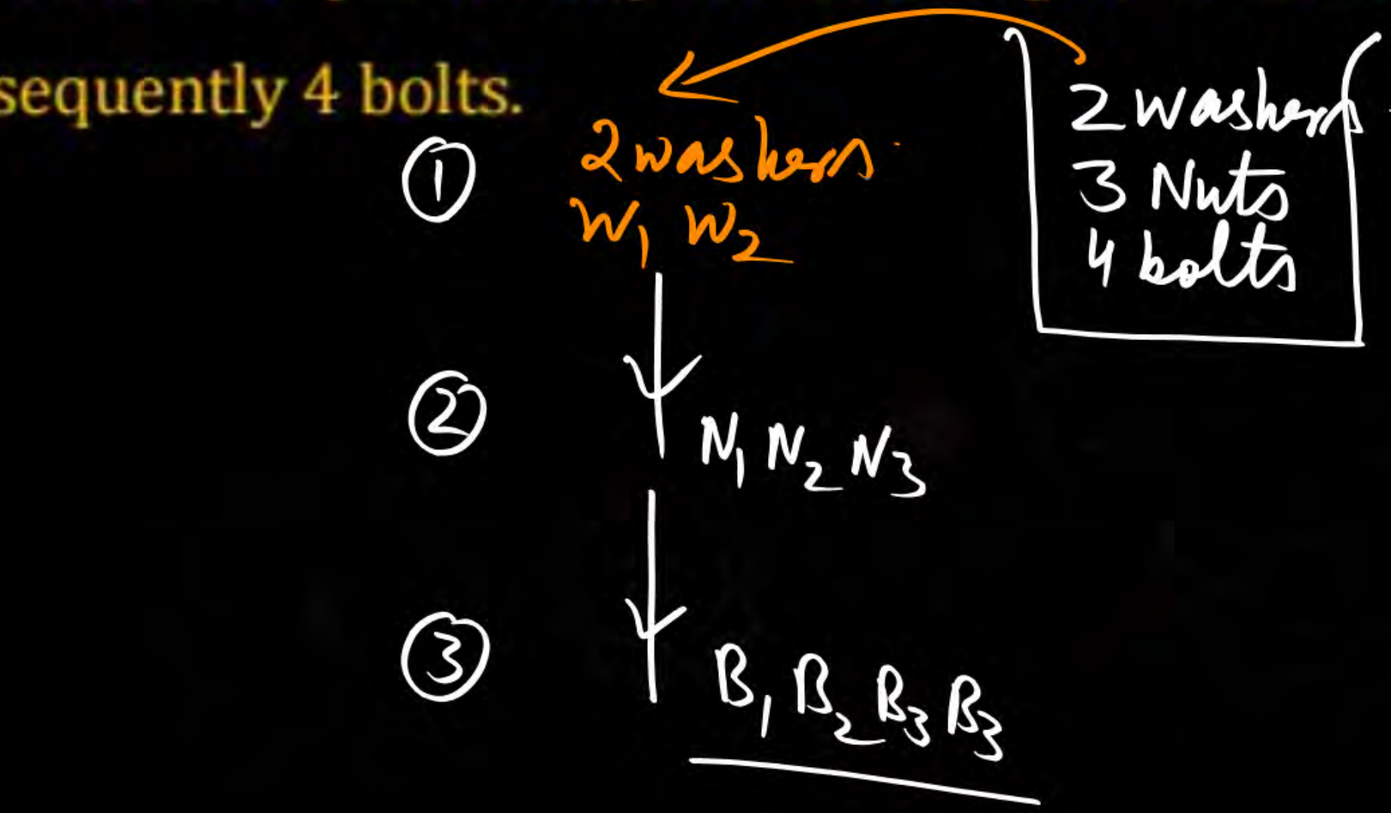

A B C
3 Different Items
Arrangement
Taken all at a time
6 Options
A C B
B A C
B C A
C A B
C B A
A B C
3 2 1
= 3 x 2 x 1
= 6 = 3!
Options
A, B, C
A B C D
4 3 2 1
= 4 x 3 x 2 x 1 = 24 = 4!
If n Different Items Permutate Taken all at a time
= n x (n-1)(n-2)(n-3) - - - 1
= n!

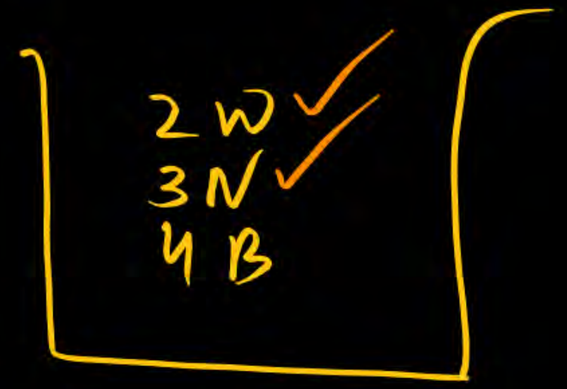
Q. Questions

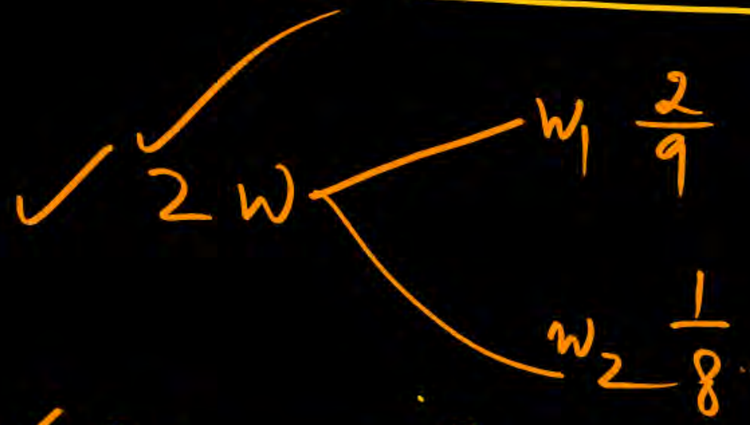
ME-GATE

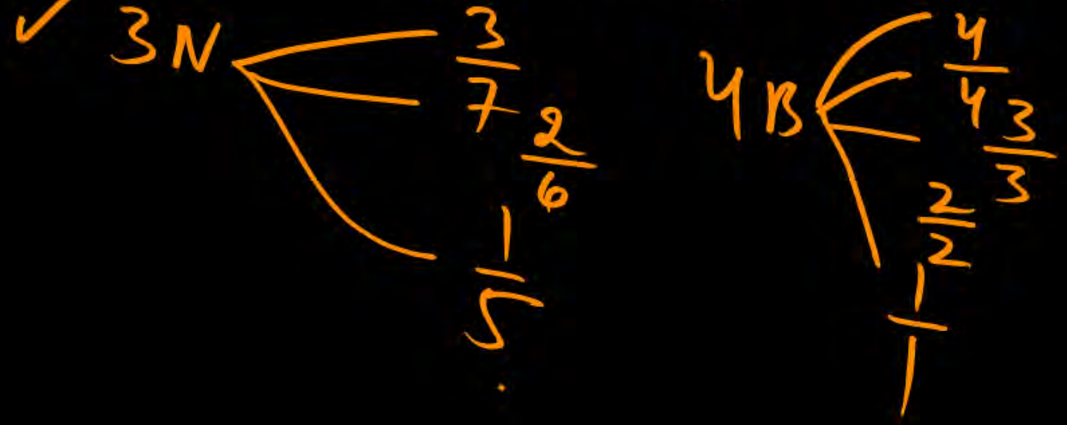
(without replacement)

A box contains 2 washers, 3 nuts and 4 bolts. are drawn from the box randomly one at a time. Find the probability of drawing 2 washers first followed by 3 nuts and subsequently 4 bolts.









$\times 9!$

$W_1\ W_2\ N_1\ N_2\ N_3\ B_1\ B_2\ B_3\ B_4$

$= \underbrace{\frac{2}{9} \times \frac{1}{8}}_{\text{2 washers}} \times \underbrace{\frac{3}{7} \times \frac{2}{6} \times \frac{1}{5}}_{\text{3 Nuts}} \times \underbrace{\frac{4}{4} \times \frac{3}{3} \times \frac{2}{2} \times \frac{1}{1}}_{\text{4 bolts}}$

$\Rightarrow \boxed{\frac{1}{1260}}$ Ans

Q. Questions

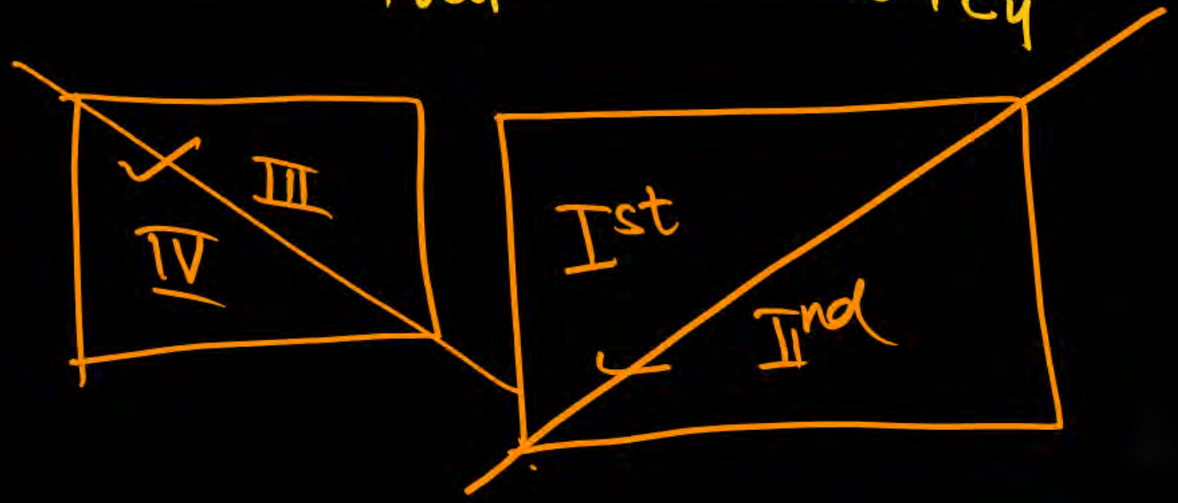
64 squares → 4 squares

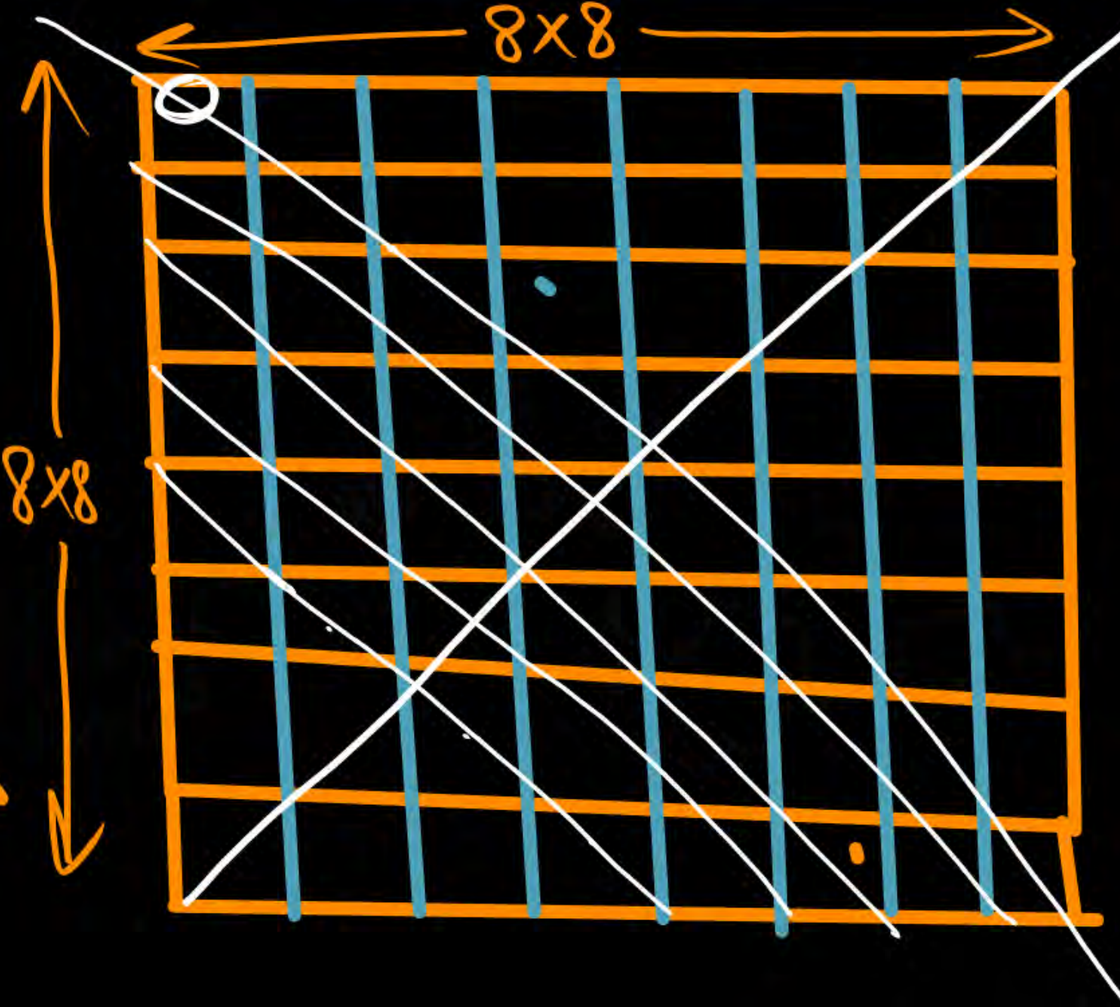
$^{64}C_4$

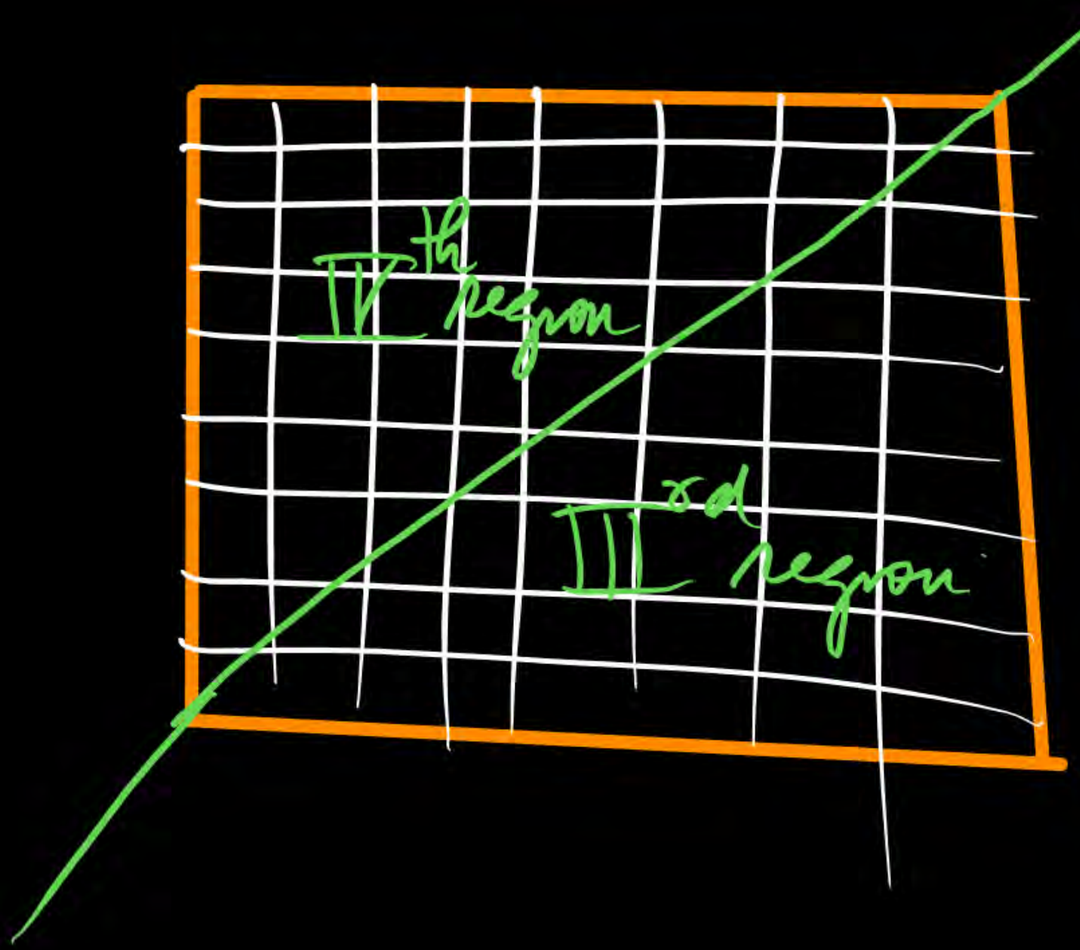
(X) square CHOOSE CHOICE / committee

4 squares are chosen at random on the chessboard. Find the probability that they lie along the same diagonal line.

$$P(E) = \frac{\text{No. of fav}}{\text{Total}} = \frac{\square}{^{64}C_4}$$







8C4
7C4
6C4
5C4
4C4

II nd part

I st part

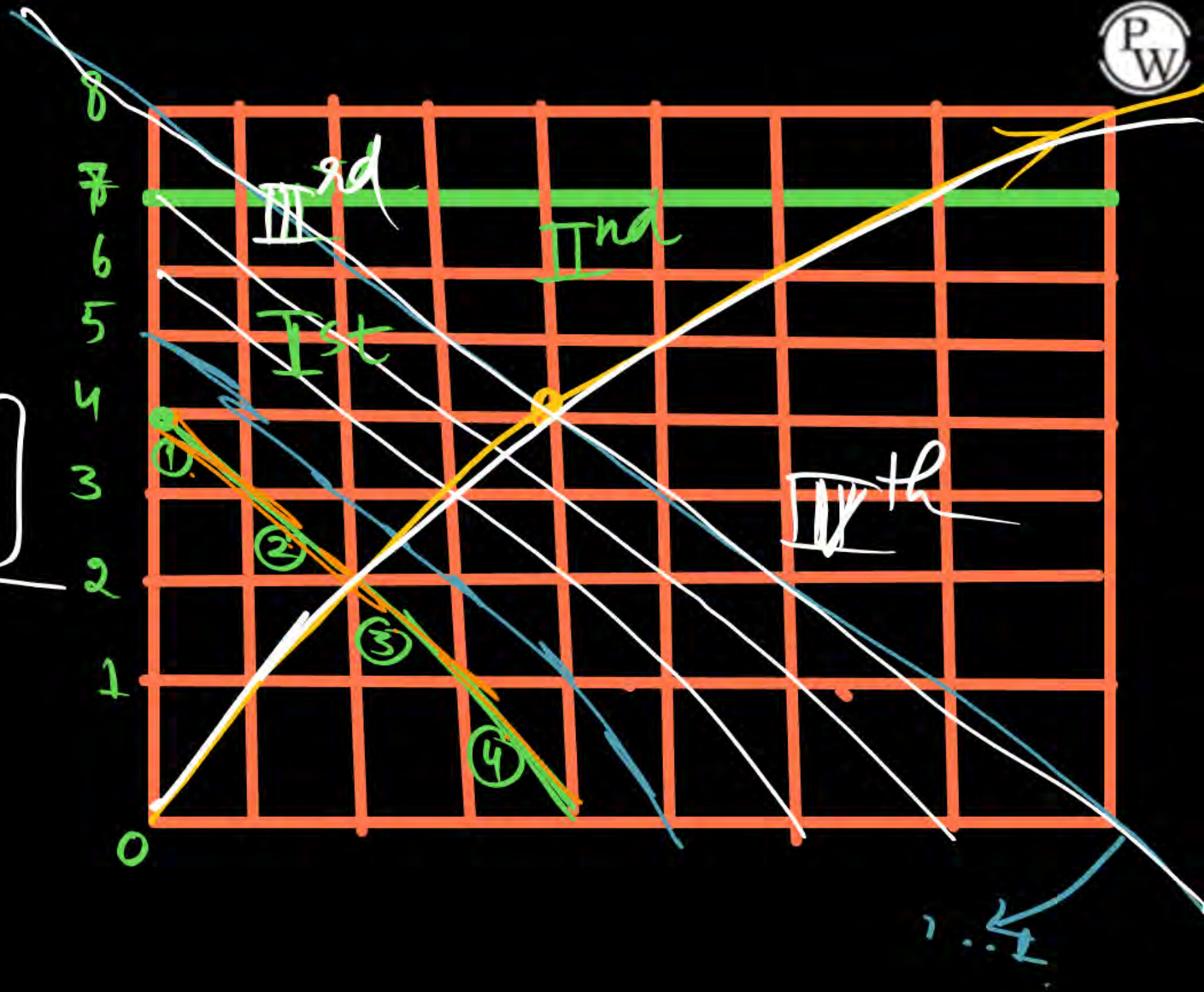
$$\left\{2\left[2\left[{}^{4}C_{4}+{}^{5}C_{4}+{}^{6}C_{4}+{}^{7}C_{4}\right]+{}^{8}C_{4}\right]\right.$$

$${}^{6}C_{4}$$

Arrang. Rff.

$$\begin{bmatrix} 1,2,3,4 \\ 4,3,2,1 \\ 3,4,2,1 \\ 4,2,1,3 \end{bmatrix}$$ SAME = Combi

$$\frac{2\left[2\left[{}^{4}C_4+{}^{5}C_4+{}^{6}C_4+{}^{7}C_4\right]+{}^{8}C_4\right]}{{}^{64}C_4}$$



5 coupons: A, B, C, D, E → 2 select / group / committee

AB, BA → ${}^nP_r$ (order)

AB, BA → ${}^nC_r$ SAME ✗ order

Total No. of arrangement $= 5^2$

(Repetition allowed) $= 25$

$= N^2$

$= N^2 - N = N(N-1)$

$= 25 - 5 = 20$

No. of combination $= \dfrac{{}^nP_r}{r!} = {}^nC_r$

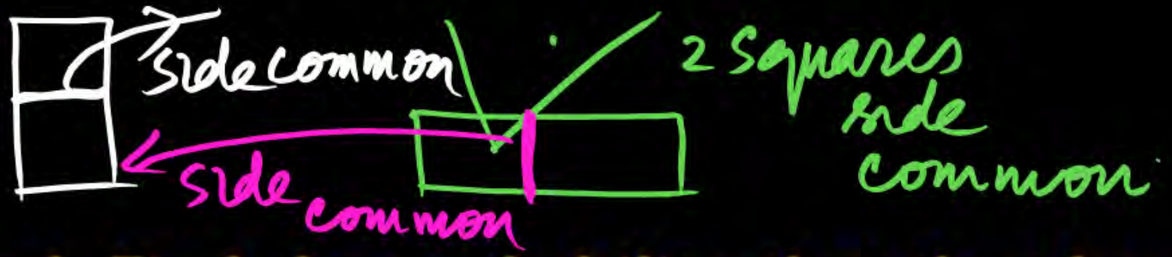
No. of combination $= \dfrac{N(N-1)}{2} = \dfrac{20}{2}$

$= 10$

$${}^nC_r = \frac{n!}{(n-r)!\,r!}$$

${}^nC_r = {}^5C_2 = \dfrac{5!}{(5-2)!\,2!} = 10$

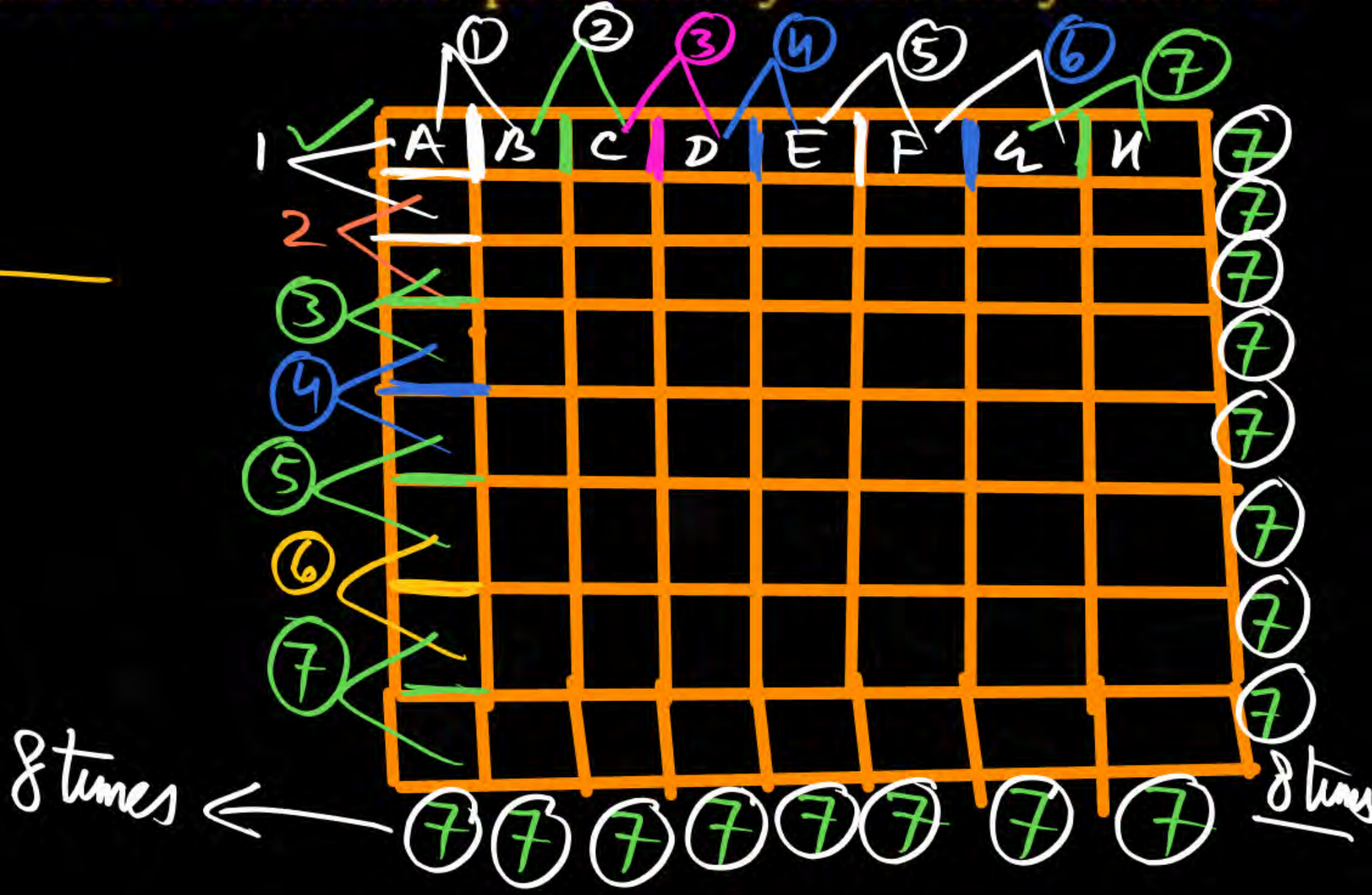
Q. Questions



2 squares are chosen on a chessboard. Find the probability that they have a side in common.

$$= \frac{7\times8+7\times8}{{}^{64}C_2}$$

$$= \frac{112}{\frac{64\times63}{2\times1}} \Rightarrow \frac{1}{18}$$



Q. Questions

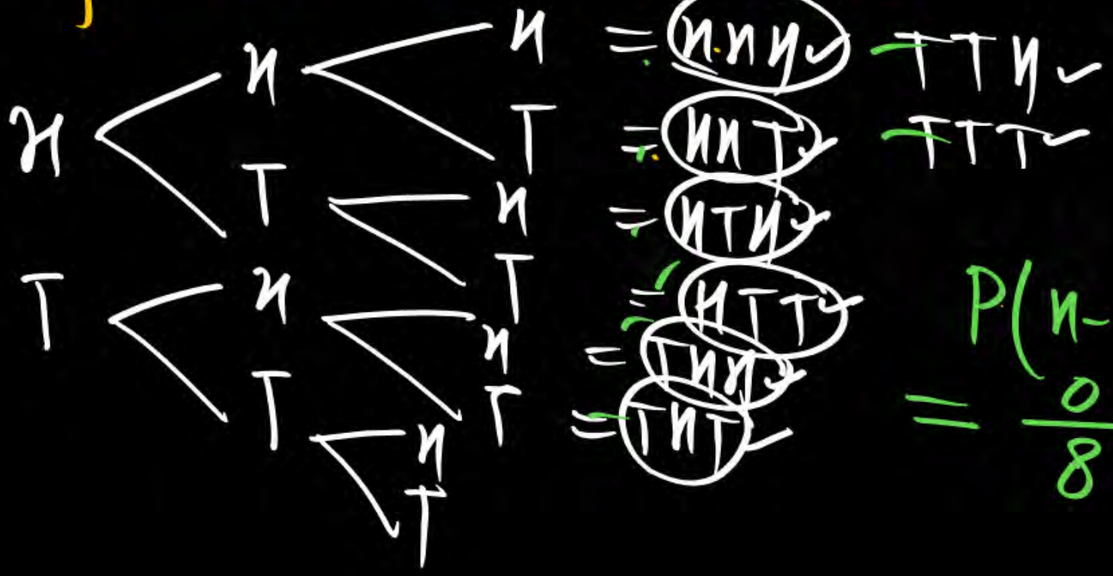
Rahul Sir PW

n=3 n=4

A fair coin is tossed n times, the probability that the difference between the number of heads and tail is (n-3) is _____.

n-3=1

→ A fair coin is Tossed 3 times The Prob That The Diff. between The No. of HEADS And tail is 0.



$P(H-T=0) = \frac{0}{8} = 0$

No. of H	No. of T	H-T
3	0	3-0=3
2	1	2-1=1
1	2	1-2=-1
0	3	0-3=-3

For n Trials:

Fav outcomes = 0

Total = $2 \times 2 \times 2 \ldots$ n times

$= 2^n$

$P(E) = \frac{0}{2^n} = 0$

No. of HEAD	No. of tail	n−T
n	0	n−0=n
n−1	1	n−1−1=n−2
n−2	2	n−2−2=n−4
n−3	3	n−6
⋮	⋮	n−8
		n−10
0	n	0−n=−n

Thank You!

GW Soldiers