ALL BRANCHES **Hinglish**

Engineering Mathematics
Probability and Statistics
Topic : Fundamentals of Probability

DPP-01

1. If there are 6 girls and 5 boys who sit in a row. then the probability that no two boys sit together is

(a) $\frac{6!6!}{2!11!}$ (b) $\frac{7!5!}{2!11!}$

(c) $\frac{6!7!}{2!11!}$ (d) None of these

2. Twelve balls are distributed among three axes. The probability that the first box contains 3 balls is

(a) $\frac{110}{9}\left(\frac{2}{3}\right)^9$ (b) $\frac{9}{110}\left(\frac{2}{3}\right)^{10}$

(c) $\frac{{}^{12}C_3}{12^3}\cdot 2^9$ (d) $\frac{{}^{12}C_3}{3^{12}}$

3. A cricket club has 15 members of whom only 5 can bowl. If the names of 15 members are put into a box and 11 are drawn at random. Then the probability of obtaining an eleven containing at least 3 bowlers Is:

(a) 7/13 (b) 6/13

(c) 11/15 (d) 12/13

4. Three integers are chosen at random from the first 20 integers. The probability that their product is even

(a) 2/19 (b) 3/29

(c) 17/19 (d) 4/29

5. One hundred cards are numbered from 1 to 100. The probability that a randomly chosen card has a digit 5 is

(a) 1/100 (b) 9/100

(c) 19/100 (d) None of these

6. Three six faced dice are tossed together, then the probability that exactly two of the three numbers are equal is:

(a) 165/216 (b) 177/216

(c) 51/216 (d) 90/216

7. If the letters of word 'REGULATIONS' be arranged at random, the probability that there will be exactly 4 letters between R and E is:

(a) 6/55 (b) 3/55

(c) 49/55 (d) None of these

8. $2n$ boys are randomly divided into two subgroups containing n boys each. The probability that the two tallest boys are in different groups is:

(a) $\frac{n}{2n-1}$ (b) $\frac{n-1}{2n-1}$

(c) $\frac{2n-1}{4n^2}$ (d) None of these

9. In a bag there are three tickets numbered 1, 2, 3. A ticket is drawn at random and put back, and this is done four times the probability of that the sum of the numbers is even is:

(a) 41/81 (b) 39/81

(c) 40/81 (d) None of these

10. A pack of cards consists of 15 cards numbered 1 to 15. Three cards are drawn at random with replacement. Then, the probability of getting 2 odd and one even numbered card is:

(a) 348/1125 (b) 398/1125

(c) 448/1125 (d) 498/1125

11. Three persons A, B and C are to speak at a function along with five others. If they all speak in random order, the probability that A speaks before B and B speaks before C is:

(a) 3/8 (b) 1/6

(c) 3/5 (d) None of these

12. An elevator starts with m passengers and stops at n floors $(m \le n)$ the probability that no two passengers alight at same floor is:

(a) $\frac{{}^nP_m}{m^n}$ (b) $\frac{{}^nP_m}{n^m}$

(c) $\frac{{}^nC_m}{m^n}$ (d) $\frac{{}^nC_m}{n^m}$

13. There are n persons sitting in a row. Two of them are selected at random. The probability that two selected persons are not together is:

(a) $\frac{2}{n}$ (b) $1-2/n$

(c) $\frac{n(n-1)}{(n+1)(n+2)}$ (d) None of these

14. A and B play a game where each is asked to select a number from 1 to 25. If the two numbers match both of them win a prize. The probability that they will not win a prize in a single trial is:

(a) 1/25 (b) 24/25

(c) 2/25 (d) None of these

15. Fifteen persons among whom are A and B, sit down randomly at round table. The probability that there are 4 persons between A and B is:

(a) $\frac{9!}{14!}$ (b) $\frac{10!}{14!}$

(c) $\frac{9!}{15!}$ (d) None of these

16. The probability that the 13th day of a randomly chosen month is a second Saturday is:

(a) 1/7 (b) 1/12

(c) 1/84 (d) 19/84

17. Three of the six vertices of a regular hexagon are chosen at random. The probability that the triangle with three vertices is equilateral, is:

(a) 1/2 (b) 1/5

(c) 1/10 (d) 1/20

18. The probability that out of 10 persons, all born in April, at least two have the same birthday is:

(a) $\frac{{}^{30}C_{10}}{(30)^{10}}$ (b) $1-\frac{{}^{30}C_{10}}{30!}$

(c) $\frac{(30)^{10}-{}^{30}C_{10}}{(30)^{10}}$ (d) None of these

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Answer Key

1. (c)
2. (a)
3. (d)
4. (c)
5. (c)
6. (d)
7. (a)
8. (a)
9. (a)
10. (c)
11. (b)
12. (a)
13. (b)
14. (b)
15. (d)
16. (c)
17. (c)
18. (c)

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Topic : Classification of Events

1. If the probability that A and B will die within a year are p and q respectively. Then the probability that only of one of them will be alive at the end of the year is:
(a) $p + q$ (b) $p + q - 2pq$
(c) $p + q - pq$ (d) $p + q + pq$

2. If A and B each toss three coins. The probability that both get the same number of heads is:
(a) 1/9 (b) 3/16
(c) 5/16 (d) 3/8

3. If A and B are two independent events such that $P(\bar{A} \cap B) = 2/15$ and $P(A \cap \bar{B}) = 1/6$, then $P(B)$ is:
(a) 1/5 (b) 1/6
(c) 4/5 (d) 5/6

4. If A and B are two events, the probability that exactly one of them occurs is given by:
(a) $P(A) + P(B) - 2P(A \cap B)$
(b) $P(A \cap \bar{B}) + P(\bar{A} \cap B)$
(c) $P(A \cup B) - P(A \cap B)$
(d) $P(\bar{A}) + P(\bar{B}) - 2P(\bar{A} \cap \bar{B})$

5. If $\bar{E}$ and $\bar{F}$ are the complementary events of events E and F respectively and if 0 < P (F) < 1, then:
(a) $P(E/F) + P(\bar{E}/F) = 1$
(b) $P(E/F) + P(E/\bar{F}) = 1$
(c) $P(\bar{E}/F) + P(E/\bar{F}) = 1$
(d) $P(E/\bar{F}) + P(\bar{E}/\bar{F}) = 1$

6. If A and B are two events. The probability that at most one of A, B occurs is:
(a) $1 - P(A \cap B)$
(b) $P(\bar{A}) + P(\bar{B}) - P(\bar{A} \cap \bar{B})$
(c) $P(\bar{A}) + P(\bar{B}) + P(A \cup B) - 1$
(d) $P(A \cap \bar{B}) + P(\bar{A} \cap B) + P(\bar{A} \cap \bar{B})$

7. The probability of the simultaneous occurrence of two events A and B is p. If the probability that exactly one of A, B occurs is q then:
(a) $P(\bar{A}) + P(\bar{B}) = 2 + 2q - p$
(b) $P(\bar{A}) + P(\bar{B}) = 2 - 2p - q$
(c) $P(A \cap B / A \cup B) = \frac{p}{p+q}$
(d) $P(\bar{A} \cap \bar{B}) = 1 - p - q$

8. For two events A and B it is given that $P(A) = P(A/B) = \frac{1}{4}$ and $P(B/A) = \frac{1}{2}$. Then:
(a) A and B are mutually exclusive events
(b) A and B are independent events
(c) $P(\bar{A}/B) = \frac{3}{4}$
(d) $P(\bar{A}/B) = \frac{1}{2}$

9. If A and B are two independent events such that $P(A) = \frac{1}{4}$ and $P(B) = \frac{1}{2}$. Then:
(a) $P(A \cup B) = \frac{3}{5}$
(b) $P(A/B) = \frac{1}{2}$
(c) $P(A/A \cup B) = \frac{2}{5}$

(d) $P\left(A \cap B / \bar{A} \cup \bar{B}\right) = 0$

10. If the independent events A and B are such that $0 < P(A) < 1$ and $0 < P(B) < 1$. Then:

(a) A and B are mutually exclusive

(b) A and $\bar{B}$ are independent

(c) $\bar{A}$ and $\bar{B}$ are independent

(d) $P(A/B) + P\left(\bar{A}/B\right) = 1$

11. If A and B are events at the same experiments with P(A) = 0.2, P(B) = 0.5, then maximum value of $P\left(A' \cap B\right)$ is

(a) 1/4 (b) 1/2

(c) 1/8 (d) 1/16

12. The probabilities that a student passes in mathematics, physics and chemistry are m. p and c respectively. Of these subjects, a student has a 75% chance of passing in at least one, a 50% chance of passing in at least one, 50% chance of passing in at least two and a 40% chance of passing in exactly two subjects. Which of the following relations are true?

(a) $p + m + c = \frac{19}{20}$

(b) $p + m + c = \frac{27}{20}$

(c) $pmc = \frac{1}{10}$

(d) $pmc = \frac{1}{4}$

13. A coin is tossed n times. The probability of getting at least one head is greater than that of getting at least two tails by 5/32. Then n is:

(a) 5 (b) 10

(c) 15 (d) None of these

14. A pair of fair dice is rolled together till a sum of either 5 or 7 is obtained, the probability that 5 comes before 7 is

(a) 0.2 (b) 0.3

(c) 0.4 (d) 0.5

15. 'A' can hit the target 3 times out of 5 times, 'B' can hit 2 times out of 5 and C can hit 3 times out of 4. They aim at each other simultaneously. What is the probability that 2 out of 'A', 'B' and 'C' will hit the target?

16. A, B and C in order toss a coin. First one to get a head wins. What are their respective chances of winning?

17. An urn contains 6 white and 4 black balls. A fair die is rolled and that number of balls are chosen from the urn. The probability that the balls selected are white is:

(a) 1/5 (b) 1/6

(c) 1/7 (d) 1/8

18. There are four machines and it is known that exactly two of them are faulty. They are tested, one by one, in a random order till both the faulty machines are identified. Then the probability that only two tests are needed is:

(a) 1/3 (b) 1/6

(c) 1/2 (d) 1/4

19. A biased coin with probability p, $0 < p < 1$ of heads is tossed untill a head appears for the first time. If the probability that the number of tosses required is even is 2/5, then p equals:

(a) 1/3 (b) 2/3

(c) 2/5 (d) 3/5

20. Let $0 < P(A) < 1$, $0 < P(B) < 1$ and $P(A \cup B) = P(A) + P(B) - P(A)P(B)$. Then :

(a) $P(B/A) = P(B) - P(A)$

(b) $P(A^c \cup B^c) = P(A^c) + P(B^c)$

(c) $P(A \cup B)^c = P(A^c) + P(B^c)$

(d) $P(A/B) = P(A)$

Answer Key

1. **(b)**
2. **(c)**
3. **(b,c)**
4. **(a,b,c,d)**
5. (a)
6. **(a,b,c,d)**
7. **(b,c,d)**
8. **(a,c,d)**
9. **(None)**
10. **(b,c,d)**
11. **(b)**
12. **(b,c)**
13. **(a)**
14. **(c)**
15. **(0.45)**
16. **(P(A)=4/7,P(B)=2/7,P(C)=1/7)**
17. **(a)**
18. **(a)**
19. **(a)**
20. **(c,d)**

Topic : Random Variable

1. A fair coin is tossed 3 times. Let the random variable X denote the number of heads in 3 tosses of the coin. Find the probability density function of X.

(a) $\binom{3}{x}\left(\frac{1}{2}\right)^{2x}\left(\frac{1}{2}\right)^{2-x}$

(b) $\binom{3}{2x}\left(\frac{1}{2}\right)^{x}\left(\frac{1}{2}\right)^{1-x}$

(c) $\binom{3}{x}\left(\frac{1}{2}\right)^{x}\left(\frac{1}{2}\right)^{3-x}$

(d) $\binom{3}{x}\left(\frac{1}{2}\right)^{x}\left(\frac{1}{2}\right)^{4-x}$

2. If the probability of a random variable X is given by $f(x) = k\,(2x - 1), x = 1, 2, 3…., 12$. Find k.

3. The density function for the continuous random variable X is

$$f_X(x) = \begin{cases} e^{-x} \text{for } x > 0 \\ 0 \text{ for } x \le 0 \end{cases}$$

Find the Probability P [X ≤ 2 | X > 1].

4. A continuous random variable X has density function

$$f(x) = \begin{cases} 2x & 0 < x < \frac{1}{2} \\ \frac{4-2x}{3} & \frac{1}{2} \le x < 2 \\ 0 & \textit{elsewhere} \end{cases}$$

Find P [0.25 < x ≤ 1.25]

5. Let X be a continuous random variable with probability density function

$$f(x) = \frac{1}{2} e^{-|x-1|}, -\infty < x < \infty$$

Find the value of P(1 < | X | < 2)

6. The probability function of a random variable X is given by

$$f(x) = \begin{cases} \frac{1}{4} & |x| \le 1 \\ \frac{1}{4x^2} & \textit{otherwise} \end{cases}$$

Then $P\left(-\frac{1}{2} \le X \le 2\right) =$ __________.

7. Let X be a continuous random variable with the probability density function

$$f(x) = \begin{cases} \frac{x}{8} & 0 < x < 2 \\ \frac{k}{8} & 2 \le x \le 4 \\ \frac{6-x}{8} & 4 < x < 6 \\ 0 & \text{otherwise} \end{cases}$$

where k is a real constant. Then P (1 < X < 5) equals __________.

8. Suppose the random variable X has a probability density function

$$f(x) = \begin{cases} \frac{|x|}{4}, & -c \le x \le c \\ 0 & \text{otherwise} \end{cases}$$

The value of c is

(a) 0.5 (b) 1

(c) 2 (d) 4

9. A random variable X has probability density function

$$f(x) = \begin{cases} kx(1-x), & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

The value of k is

(a) 2 (b) 6

(c) 5 (d) 4

10. The probability distribution of a discrete random variable X is given in the table below.

x	0	1	2	3	4	5
P (X = x)	0.1	0.3	0.15	0.25	0.15	0.05

The P $(1 < X \le 4)$ is

(a) 0.55 (b) 0.85
(c) 0.70 (d) 0.40

11. Suppose the random variable X has a probability density function

$$f(x) = \begin{cases} kx^3 e^{-x/2}, & x > 0 \\ 0 & \text{otherwise} \end{cases}$$

The vale of k is
(a) 1/96 (b) 96
(c) 8/3 (d) 1/4

12. Let X be a continuous random variable with pdf

$$f_X(x) = \begin{cases} cx^2, & \text{for } 0 < x \le 1, \\ 0 & \text{otherwise} \end{cases}$$

For some positive constant c. The value of P $\left(X \le \frac{2}{3} | X > \frac{1}{3}\right)$ is
(a) 3/26 (b) 5/26
(c) 7/26 (d) 11/26

13. Suppose the random variable X has the probability density function

$$f(x) = \begin{cases} ce^{x/3}, & x \le 0, \\ ce^{-x/3}, & x > 0, \end{cases}$$

For some positive constant c. The value of P $[X > 6/X > 0]$ is

(a) e^{-2} (b) ce^{-2}

(c) 0 (d) $1 - e^{-2}$

14. Let X be a discrete random variable with probability function $P(X = x) = \frac{2}{3^x}$, for $x = 1, 2, 3, \ldots\ldots$ What is the probability that X is even?

(a) $\frac{1}{4}$ (b) $\frac{2}{7}$

(c) $\frac{1}{3}$ (d) $\frac{2}{3}$

15. Let $f(x) = \frac{k|x|}{(1+|x|)^4}$, $-\infty < x < \infty$

Then the value of k for which $f(x)$ is a probability density function is

(a) $\frac{1}{6}$ (b) $\frac{1}{2}$

(c) 3 (d) 6

16. A random variable X has a probability mass of 0.2 at X = 0 and a probability mass of 0.1 at X = 1. For all other values, X has the following density function

$$f(x) = \begin{cases} 0 & x < 0 \\ x & 0 < x < 1 \\ 2x & 1 < x < c, \text{ where c is constant} \\ 0 & x \ge c \end{cases}$$

Find P $(X < 1/X > 0.5)$
(a) (0, 0.6) (b) (0.6, 0.7)
(c) (0.7, 0.8) (d) (0.8, 0.9)

17. The distribution function of a random variable X is given by

$$f(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{4} & 0 \le x < \frac{1}{4} \\ \frac{1}{2} & \frac{1}{4} \le x < \frac{1}{2} \\ \frac{3}{4} & \frac{1}{2} \le x < \frac{3}{4} \\ \frac{x+3}{5} & \frac{3}{4} \le x < 2 \\ 1 & x \ge 2 \end{cases}$$

Then P $\left(\frac{1}{4} \le X \le 1\right)$ is

(a) $\frac{1}{20}$ (b) $\frac{11}{20}$

(c) $\frac{7}{20}$ (d) $\frac{13}{20}$

18. Let X be a random variable with cumulative distribution function

$$F_x(x) = \begin{cases} 0 & for\ x \le 0 \\ 1-e^{-x} & for\ x > 0 \end{cases},$$

What is P ($0 \le e^x \le 4$) ?

(a) e^{-4} (b) $\frac{1}{4}$

(c) $\frac{1}{2}$ (d) $\frac{3}{4}$

19. Let X is a random variable with density

$$f(x) = \frac{1}{4}e^{-\frac{|x|}{2}}, -\infty < x < \infty$$

Then E(| X |) = ____________ .

20. If X is a random variable with density function

$$f(x) = \begin{cases} 1.4e^{-2x} + 0.9e^{-3x}, & x > 0, \\ 0 & \text{elsewhere} \end{cases}$$

Then E [X] =

(a) $\frac{9}{20}$ (b) $\frac{5}{6}$

(c) 1 (d) $\frac{230}{126}$

21. You are given a random variable X such that its density is

$$f_{x}(x) = \begin{cases} 3x^2 & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

A square with diagonal of length X is constructed. Find the expected value of the area of that square.

(a) 0.1 (b) 0.25

(c) $\frac{4}{7}$ (d) 0.3

22. X has a distribution which is partly continuous and partly discrete

$$f(x) = \begin{cases} \frac{1-p}{2}, & 0 < x < 1 \\ p & x = 1 \\ \frac{1-p}{2}, & 1 < x < 2 \\ 0 & \text{otherwise} \end{cases}$$

Find the variance of X in terms of p

(a) $\frac{1-p}{3}$ (b) $\frac{2-p}{3}$

(c) $\frac{1-p}{2}$ (d) $\frac{2-p}{2}$

23. X has a mean of 2 and a variance of 4. Y = aX + b has a mean of 5 and a variance of 1. What is ab assuming that a > 0?

(a) 1 (b) 2

(c) 3 (d) 4

24. Let X be a random variable with E(X) = 5 and $E(X^2)$= 25. Then $E(X + E(X))^3$ is

(a) 0 (b) 125

(c) 1000 (d) 250

25. Let X be a continuous variable with the probability density function symmetric about 0.

If V(X) < ∞. Then which of the following statement is true?

(a) E(| X |) = E(X)

(b) V(| X |) = V(X)

(c) V(|X|) < V(X)

(d) V(|X|) > V(X)

Answer Key

1. (c)
2. (0.0069)
3. (0.63)
4. (0.75)
5. (0.78)
6. (0.5)
7. (0.875)
8. (c)
9. (b)
10. (a)
11. (a)
12. (c)
13. (a)
14. (a)
15. (c)
16. (a)
17. (b)
18. (d)
19. (2)
20. (a)
21. (d)
22. (a)
23. (b)
24. (c)
25. (c)

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