GATE
ALL BRANCHES
ENGINEERING MATHEMATICS

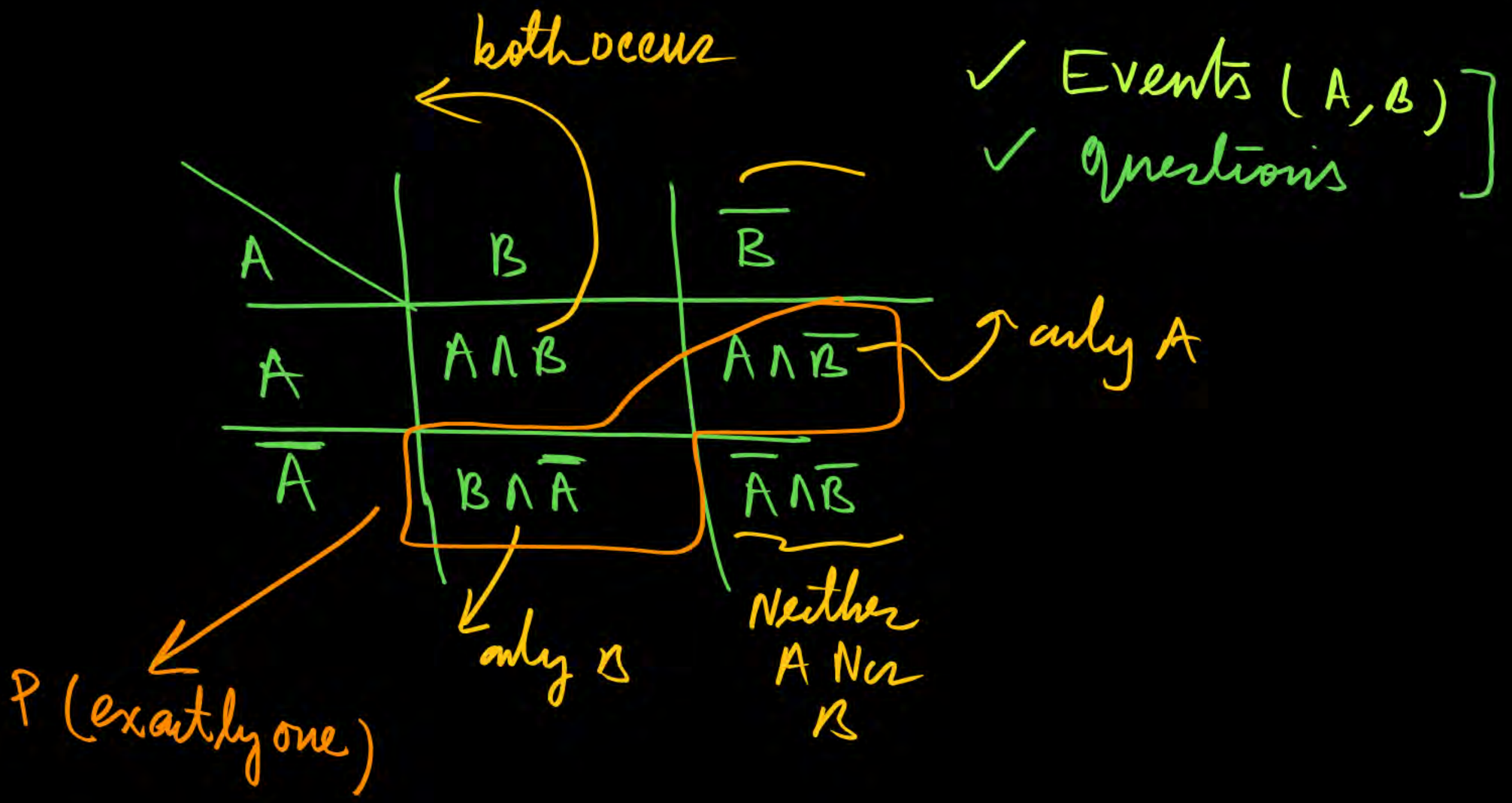
Probability and Statistics

Lecture No. 05

BY– RAHUL SIR

01 Conditional Probability and Bayes' Theorem

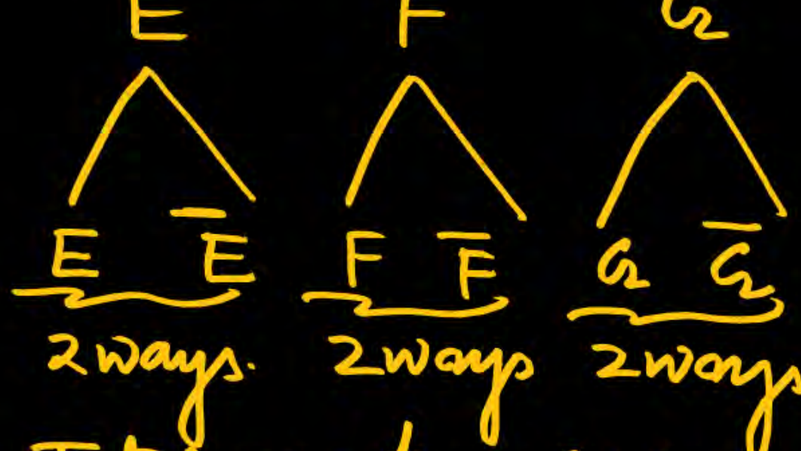
02 Questions Based on Conditional Probability and Bayes' Theorem


both occur
A
B
$\overline{B}$
A
$A \wedge B$
$A \wedge \overline{B}$
only A
$\overline{A}$
$B \wedge \overline{A}$
$\overline{A} \wedge \overline{B}$
P (exactly one)
only B
Neither A Nor B
✓ Events (A, B)
✓ Questions

THREE EVENTS :

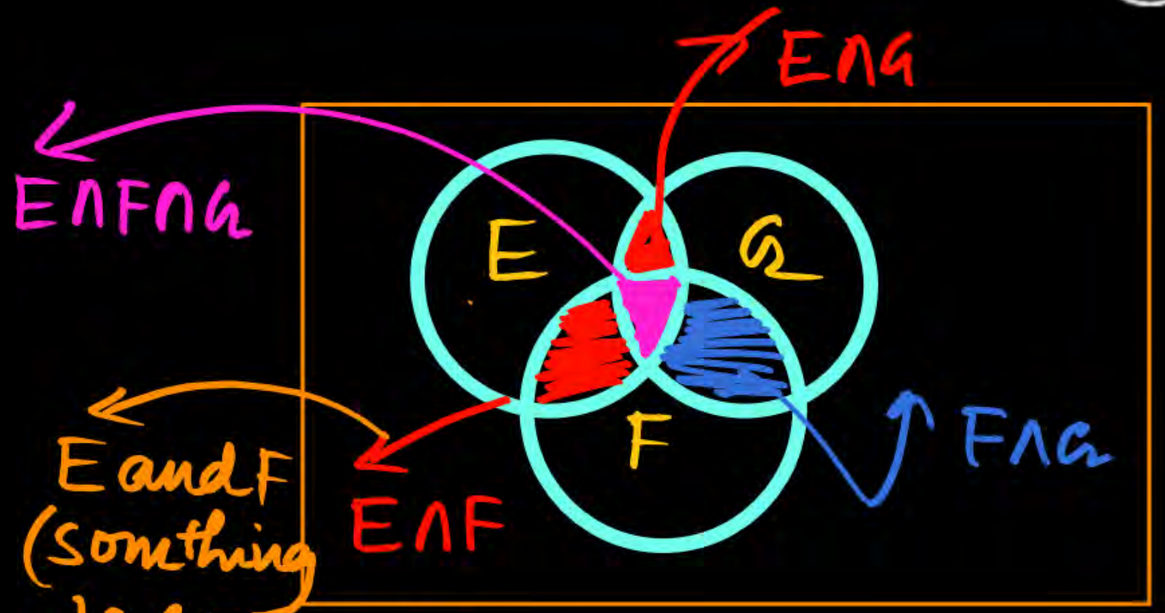
E, F, G

" something is common"



Total events = 2×2×2

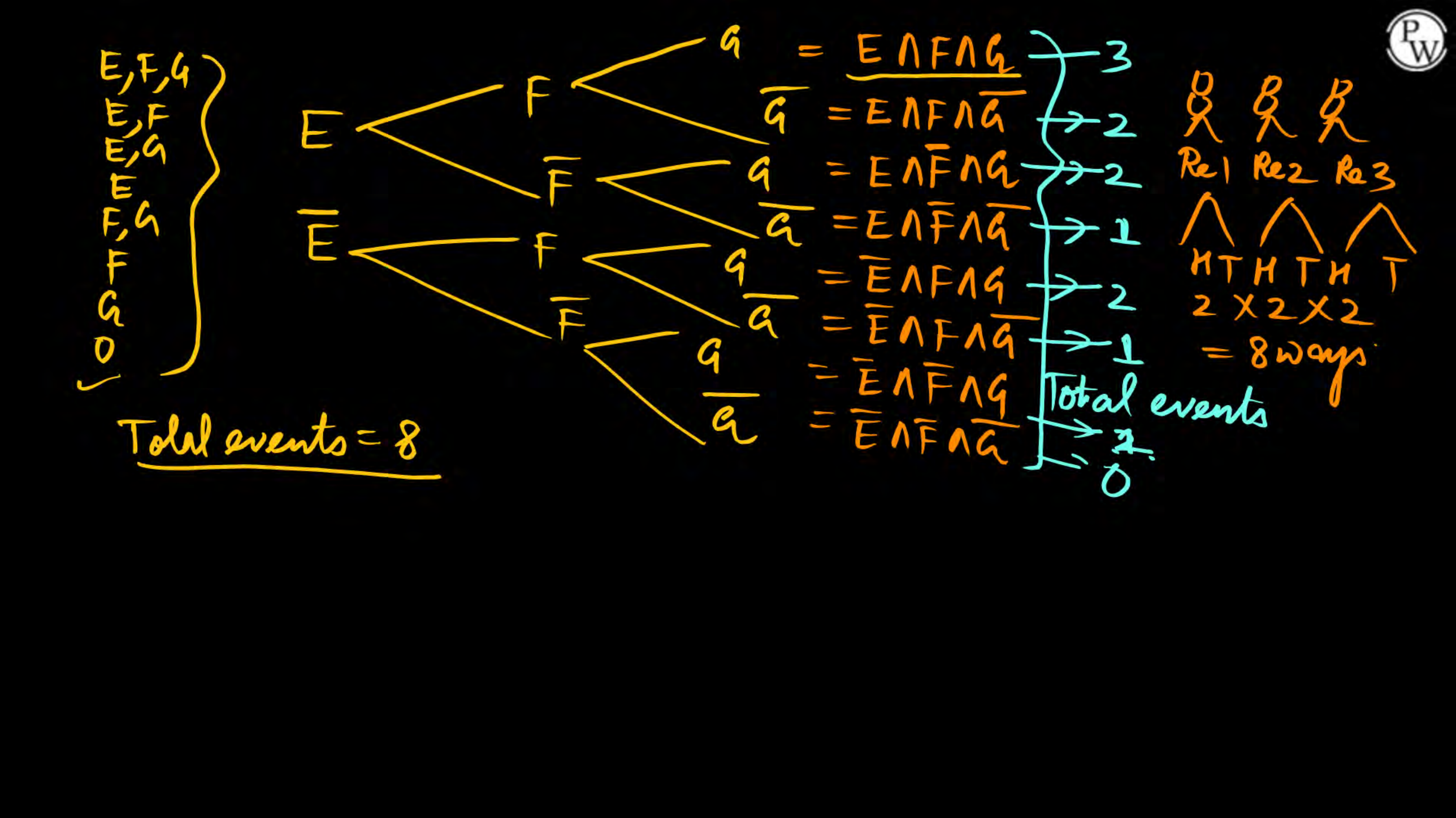
E, F, G = 8 ways.



Remember

$$P(E \cup F \cup G) = \underbrace{P(E)}_{\text{only } E} + \underbrace{P(F)}_{\text{only } F} + \underbrace{P(G)}_{\text{only } G} - \underbrace{P(E \cap F)}_{E,F} - \underbrace{P(F \cap G)}_{F,G} - \underbrace{P(G \cap E)}_{\text{only } G,E} + \underbrace{P(E \cap F \cap G)}_{E,F,G}$$

8 events


E, F, G
E, F
E, G
E
F, G
F
G
0
E
$\bar{E}$
F
$\bar{F}$
F
$\bar{F}$
G = E ∧ F ∧ G
$\bar{G}$ = E ∧ F ∧ $\bar{G}$
G = E ∧ $\bar{F}$ ∧ G
$\bar{G}$ = E ∧ $\bar{F}$ ∧ $\bar{G}$
G = $\bar{E}$ ∧ F ∧ G
$\bar{G}$ = $\bar{E}$ ∧ F ∧ $\bar{G}$
G = $\bar{E}$ ∧ $\bar{F}$ ∧ G
$\bar{G}$ = $\bar{E}$ ∧ $\bar{F}$ ∧ $\bar{G}$
3
2
2
1
2
1
Total events
2
0
Total events = 8
Re1 Re2 Re3
H T H T H T
2 X 2 X 2
= 8 ways

$\frac{100}{2} = 50$

$\frac{100}{3} = 33$

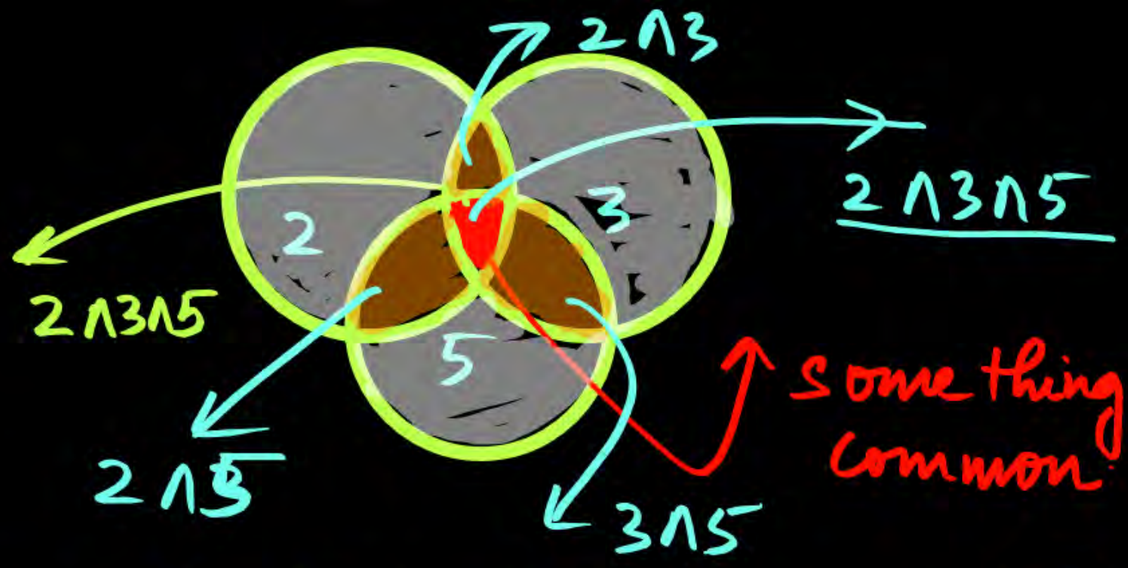
$\frac{100}{5} = 20$

$\frac{100}{2\times3} = 16$

$\frac{100}{5\times2} = 10$

$\frac{100}{3\times5} = 6$

$\frac{100}{2\times3\times5} = 3$



$$P(2 \vee 3 \vee 5) = P(2) + P(3) + P(5) - P(2 \wedge 3) - P(2 \wedge 5) - P(3 \wedge 5) + P(2 \wedge 3 \wedge 5)$$

$$= \frac{50}{100} + \frac{33}{100} + \frac{20}{100} - \frac{16}{100} - \frac{10}{100} - \frac{6}{100} + \frac{3}{100}$$

$$= \frac{74}{100} = \frac{37}{50} \text{ Ans}$$

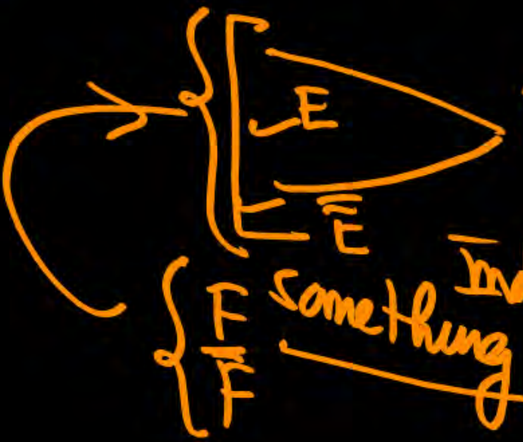
$\frac{100}{15} = 6$

1-100 digits

Div by 2 | div by 5 | Div by 3

What is the Prob.

P(either div by 2 or 3 or 5)



bernoulli Trials.

Independent

$P(\text{div by } 2 \text{ or } 3 \text{ or } 5) = P(2 \vee 3 \vee 5)$

$2 \wedge \bar{3} \wedge \bar{5} = P(2) = P(5) = (\bar{2} \wedge \bar{3} \wedge 5)$

$3 \wedge \bar{2} \wedge \bar{5} = P(3)$

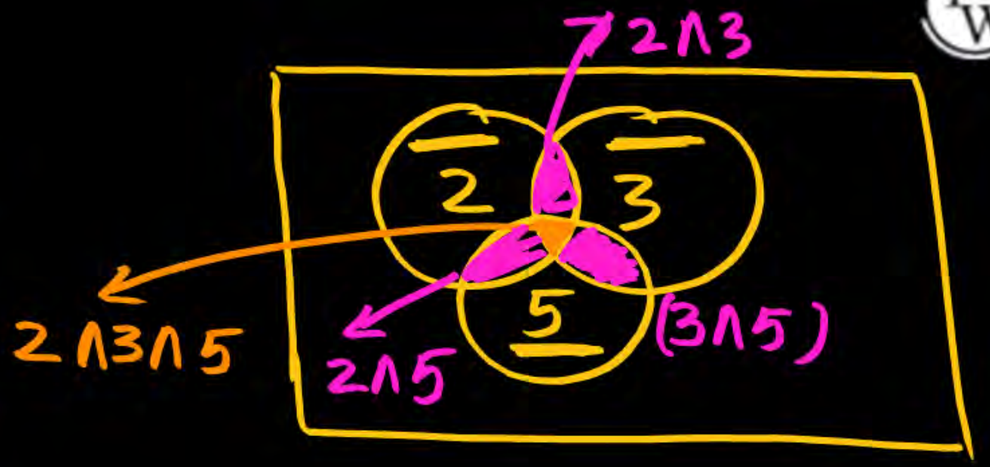
$2 \wedge 3 \wedge \bar{5} = P(2 \wedge 3)$

$3 \wedge 5 \wedge \bar{2} = P(3 \wedge 5)$

$5 \wedge 2 \wedge \bar{3} = P(5 \wedge 2)$

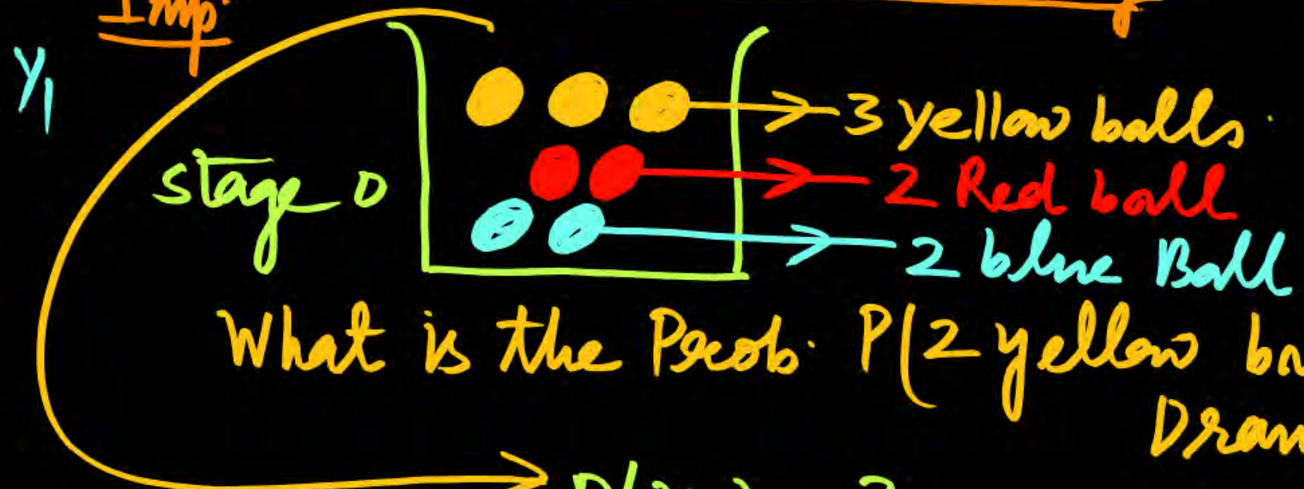
$2 \wedge 3 \wedge 5 = P(2 \wedge 3 \wedge 5)$

Add Them $= \frac{74}{100}$



Conditional Probability :

Imp ✓



What is the Prob. P(2 yellow balls Are Drawn at random)

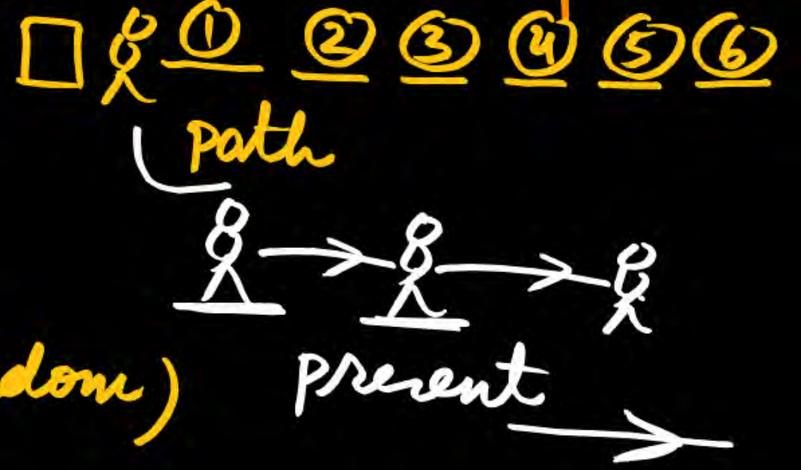
$P(Y_1) = \frac{3}{7}$

Ist yellow

Y_1 and Y_2 ARE working Together:

$P(Y_1 \text{ and } Y_2 \text{ both occur}) = P(Y_1)\, P\left(\frac{Y_2}{Y_1}\right)$

Relative Prob : (without Replacement)



$P(Y_1 \text{ and } Y_2 \text{ both occurs}) = P(Y_1)\,P\left(\frac{Y_2}{Y_1}\right)$

$\Rightarrow P(Y_1 \wedge Y_2) = \boxed{P\left(\frac{Y_2}{Y_1}\right)}\,P(Y_1)$

$\Rightarrow (Y_1 \wedge Y_2) \Rightarrow$

Y_1 Y_2

both occurs

Conditional Prob. given condition Y_1

✓ $\boxed{P\left(\frac{Y_2}{Y_1}\right) = \frac{P(Y_1 \wedge Y_2)}{P(Y_1)}}$ Conditional Prob. $P(Y_1) \neq 0$

Yellow y_1 = A Yellow 2 → B.

✓ $\boxed{P\left(\frac{B}{A}\right) = \frac{P(A \wedge B)}{P(A)}}$ $P(A) \neq 0$

A is Already Happen: B → Happening.

$P\left(\frac{A}{B}\right) = \frac{P(A \wedge B)}{P(B)}$ $P(B) \neq 0$

Conditional Prob.

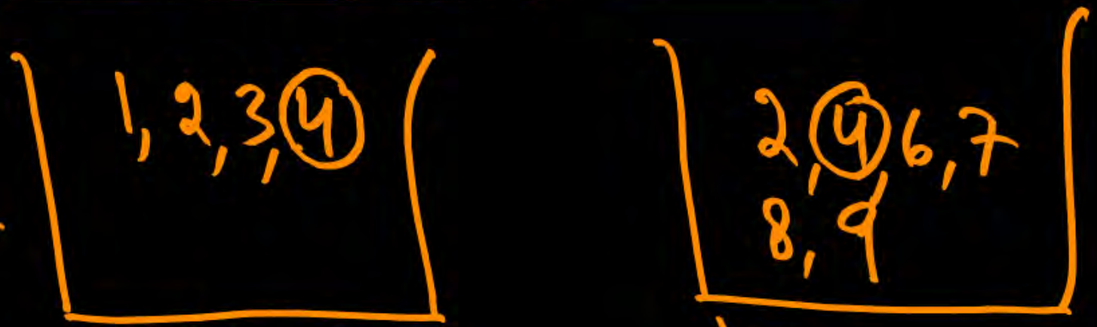
00_ _ _99

Q. Questions

M.Imp.

A bag contains 4 ticket numbers (1, 2, 3, 4) and another bag contains 6 ticket numbers (2, 4, 6, 7, 8, 9). One bag is chosen and ticket is drawn. The probability that the ticket bears the number 4 is _____.

box1
box2



$\frac{1}{2} \times \frac{1}{4} + \frac{1}{2} \times \frac{1}{6} = \frac{5}{24}$

Stage 1
bag choose
→ stage 02
Tkt drawn

'What is The Prob. one bag is chosen. And green ball is drawn"

$$= \frac{1}{3} \times \frac{2}{4} + \frac{1}{3} \times \frac{3}{4} + \frac{1}{3} \times \frac{4}{6}$$

bag 1 bag 2 bag 3

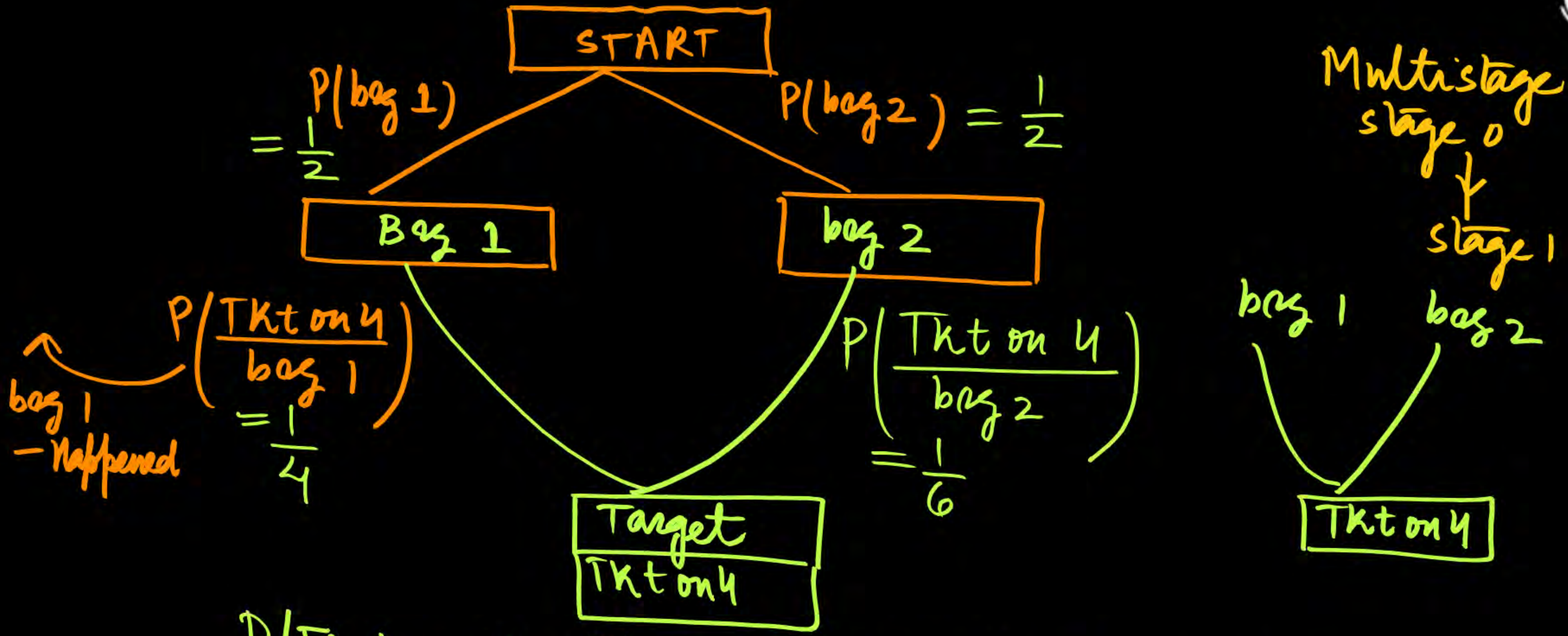
= Ans

$\frac{1}{3}$ ✓Bag 1 | bag 2 | bag 3

$\times \frac{2}{4}$ ↓ | ↓ | ↓

green ball | g | green

+ | + | 1



Multistage

stage 0

↓

stage 1

bag 1 bag 2

Tkt on 4

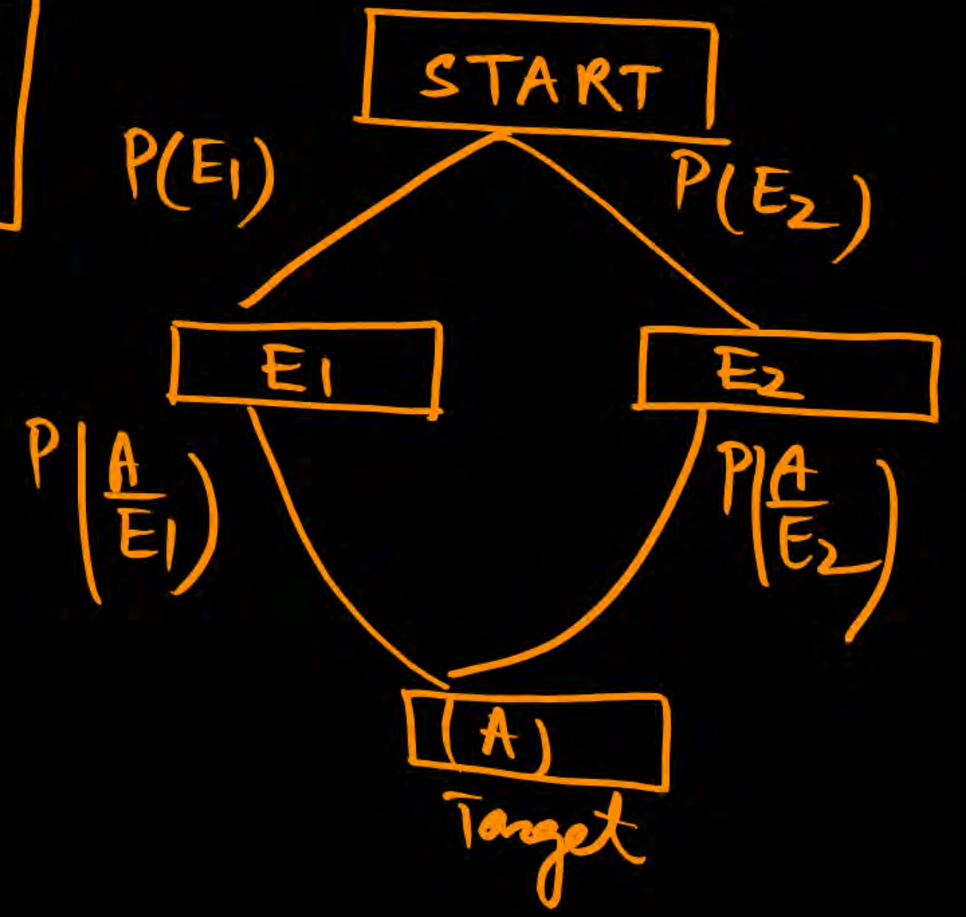
$$P(\text{Tkt on 4}) = P(\text{bag 1})\,P\left(\frac{\text{Tkt on 4}}{\text{bag 1}}\right) + P(\text{bag 2})\,P\left(\frac{\text{Tkt on 4}}{\text{bag 2}}\right)$$

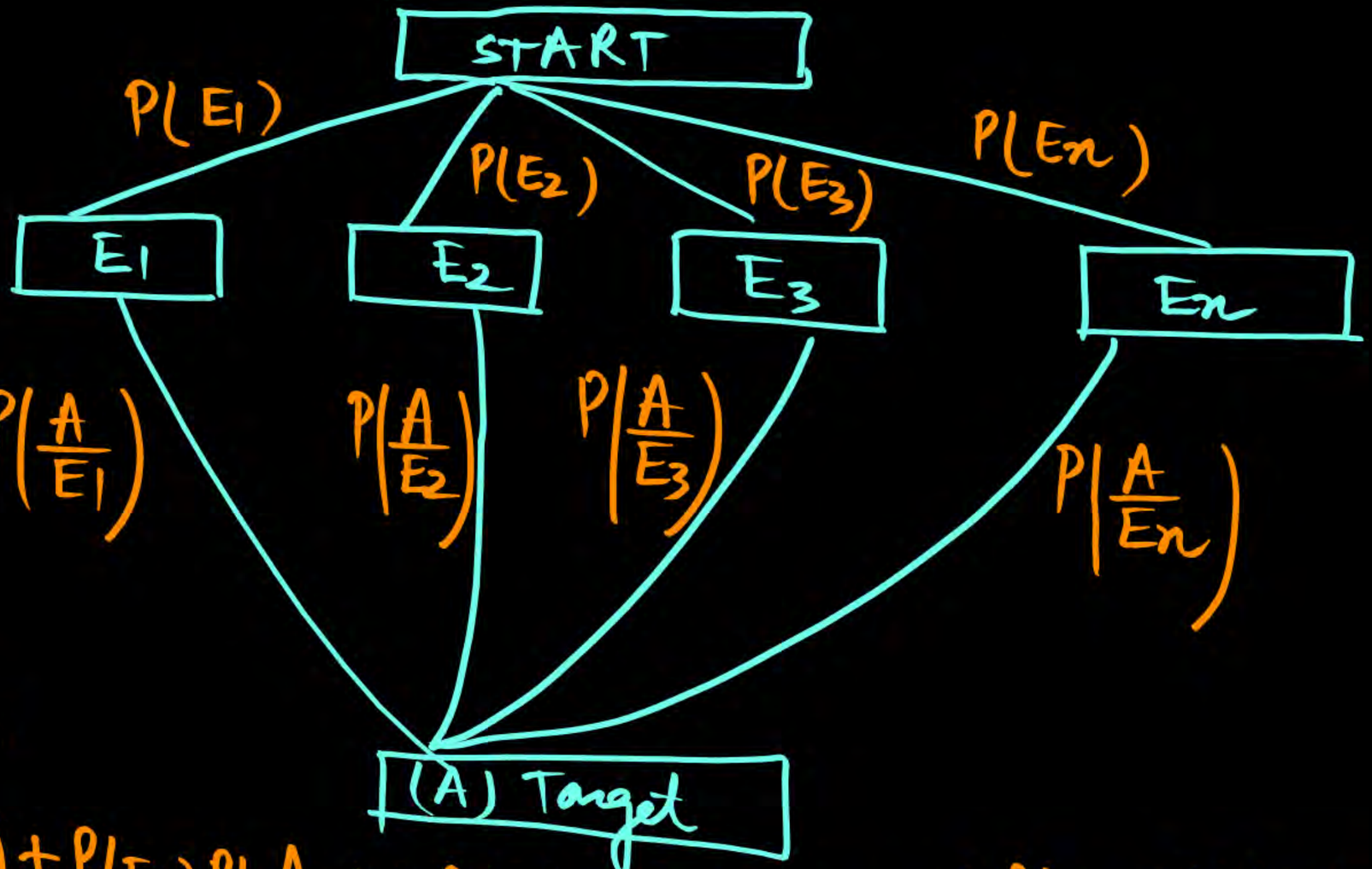
$$P(\text{Tkt on 4}) = P(\text{bag 1})\,P\left(\frac{\text{Tkt on 4}}{\text{bag 1}}\right) + P(\text{bag 2})\,P\left(\frac{\text{Tkt on 4}}{\text{bag 2}}\right)$$

$$P(\text{Tkt on 4}) = \frac{1}{2}\times\frac{1}{4} + \frac{1}{2}\times\frac{1}{6} = \frac{5}{24}\ \checkmark$$

$$\boxed{P(A) = P(E_1)\,P\left(\frac{A}{E_1}\right) + P(E_2)\,P\left(\frac{A}{E_2}\right)}$$

→ Total prob. Rule for dependent events (multistage)





Total prob.

$$P(A) = P(E_1)\,P\left(\frac{A}{E_1}\right) + P(E_2)\,P\left(\frac{A}{E_2}\right) + P(E_3)\,P\left(\frac{A}{E_3}\right) + \cdots + P(E_n)\,P\left(\frac{A}{E_n}\right)$$

What is the Prob.

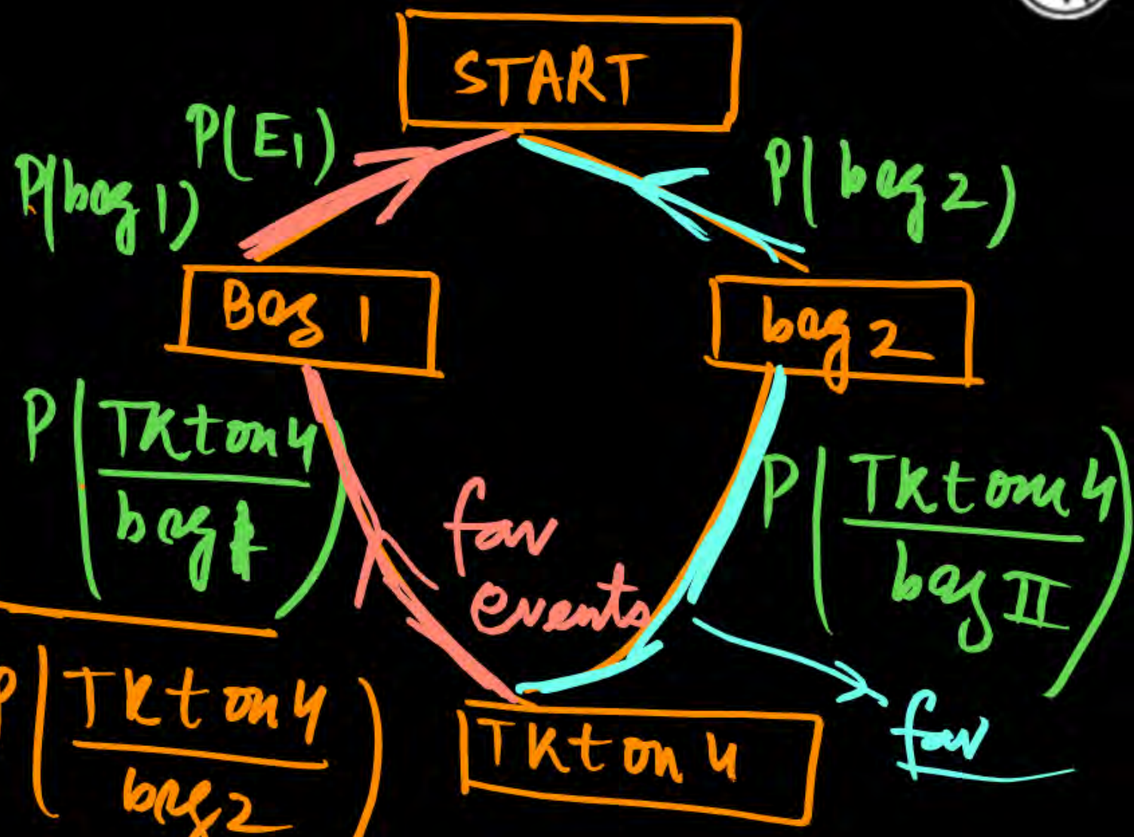
$$P\left(\begin{matrix}\text{Tkt on 4}\\ \text{bent bag 1}\end{matrix}\right) = P\left(\frac{\text{bag 1}}{\text{Tkt on 4}}\right)$$

$$\Rightarrow P(\text{bag 1})\, P\left(\frac{\text{Tkt on 4}}{\text{bag 1}}\right)$$

$$\frac{P(\text{bag 1})\, P\left(\frac{\text{Tkt on 4}}{\text{bag 1}}\right) + P(\text{bag 2})\, P\left(\frac{\text{Tkt on 4}}{\text{bag 2}}\right)}$$

$$= \frac{\frac{1}{2}\times\frac{1}{4}}{\frac{5}{24}} = \frac{3}{5}$$

$$P\left(\frac{\text{bag 1}}{\text{Tkt on 4}}\right) = \frac{3}{5}$$



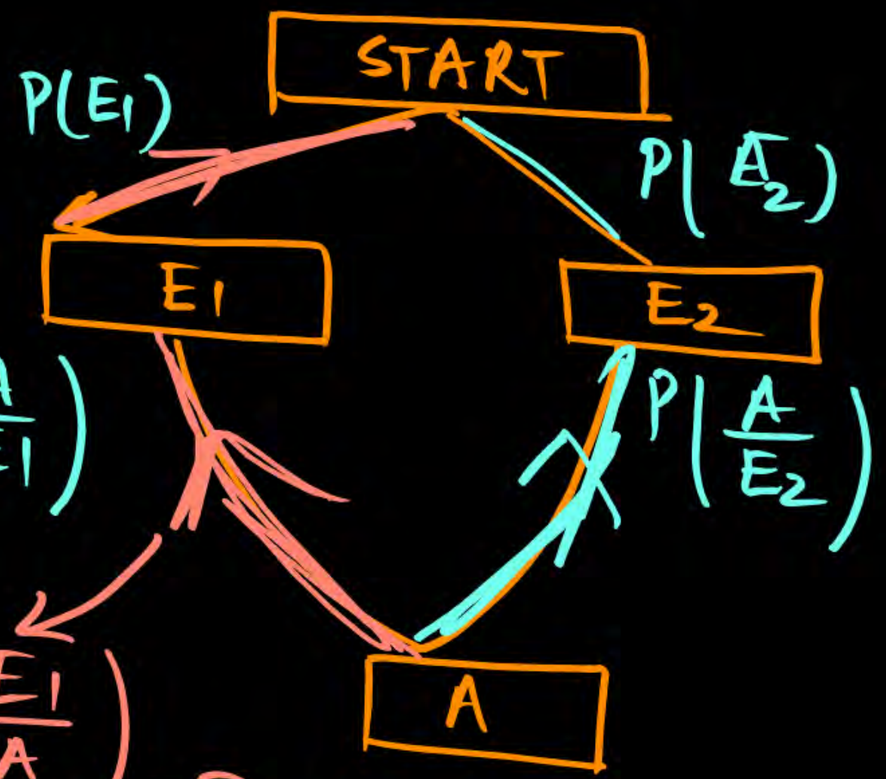
$$P\left(\frac{E_1}{A}\right) \Rightarrow \frac{P(E_1)\,P\left(\frac{A}{E_1}\right)}{P(E_1)\,P\left(\frac{A}{E_1}\right)+P(E_2)\,P\left(\frac{A}{E_2}\right)}$$

or

$$P\left(\frac{E_2}{A}\right) = \frac{P(E_2)\,P\left(\frac{A}{E_2}\right)}{P(E_1)\,P\left(\frac{A}{E_1}\right)+P(E_2)\,P\left(\frac{A}{E_2}\right)}$$

Bayes Theorem

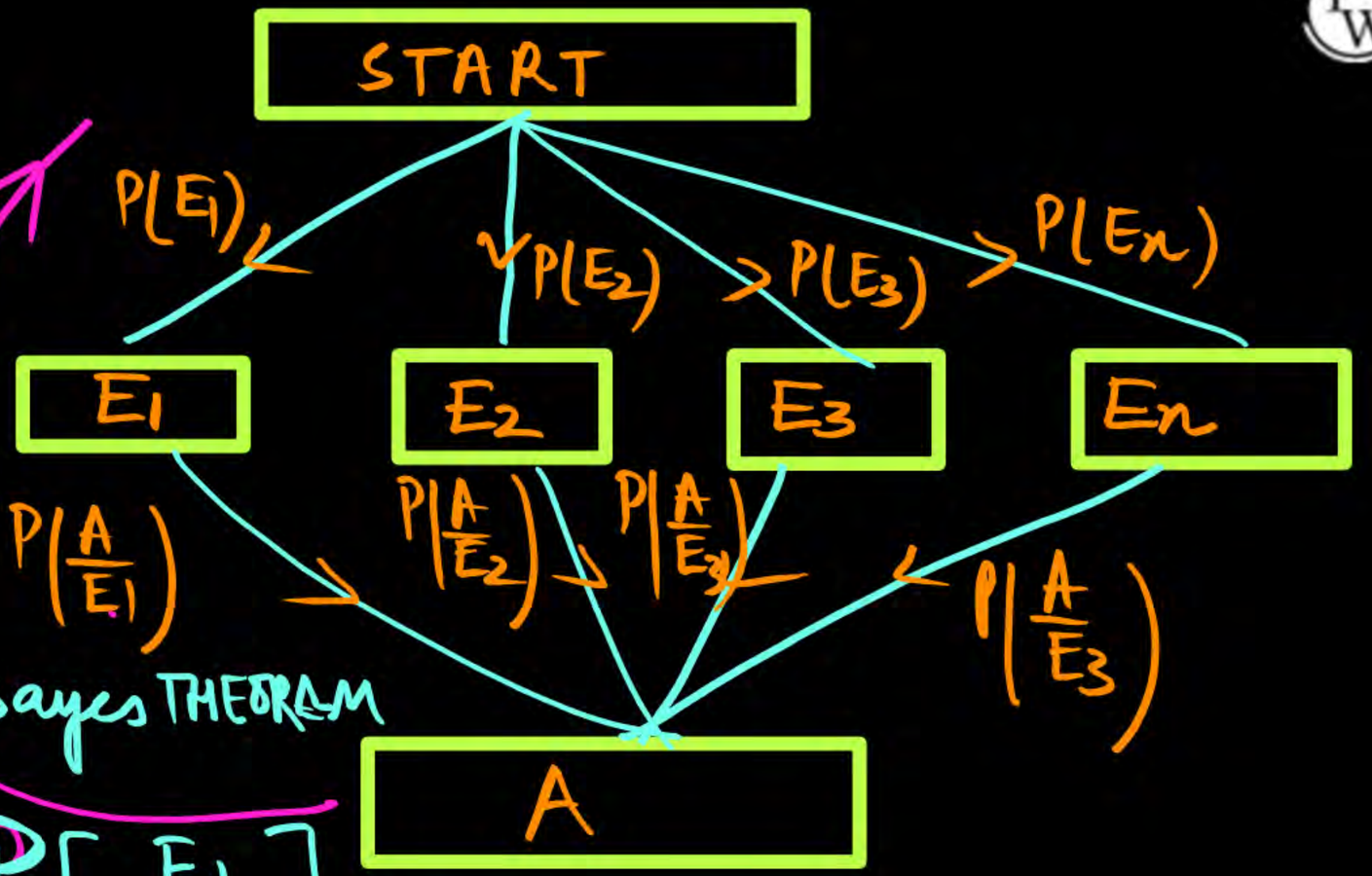
(Rule for Inverse Prob)



Total Prob.

$$\Rightarrow \sum_{i=1}^{n} P(E_i)\, P\left(\frac{A}{E_i}\right)$$

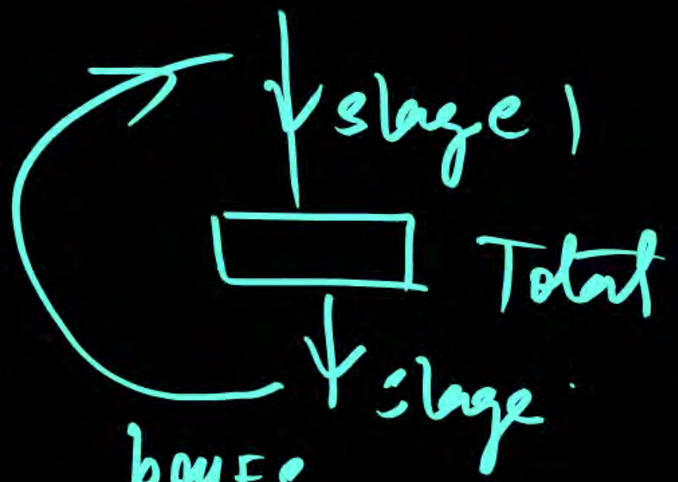
$$P\left[\frac{E_i}{A}\right] = \frac{P(E_i)\, P\left(\frac{A}{E_i}\right)}{\sum_{i=1}^{n} P(E_i)\, P\left(\frac{A}{E_i}\right)}$$

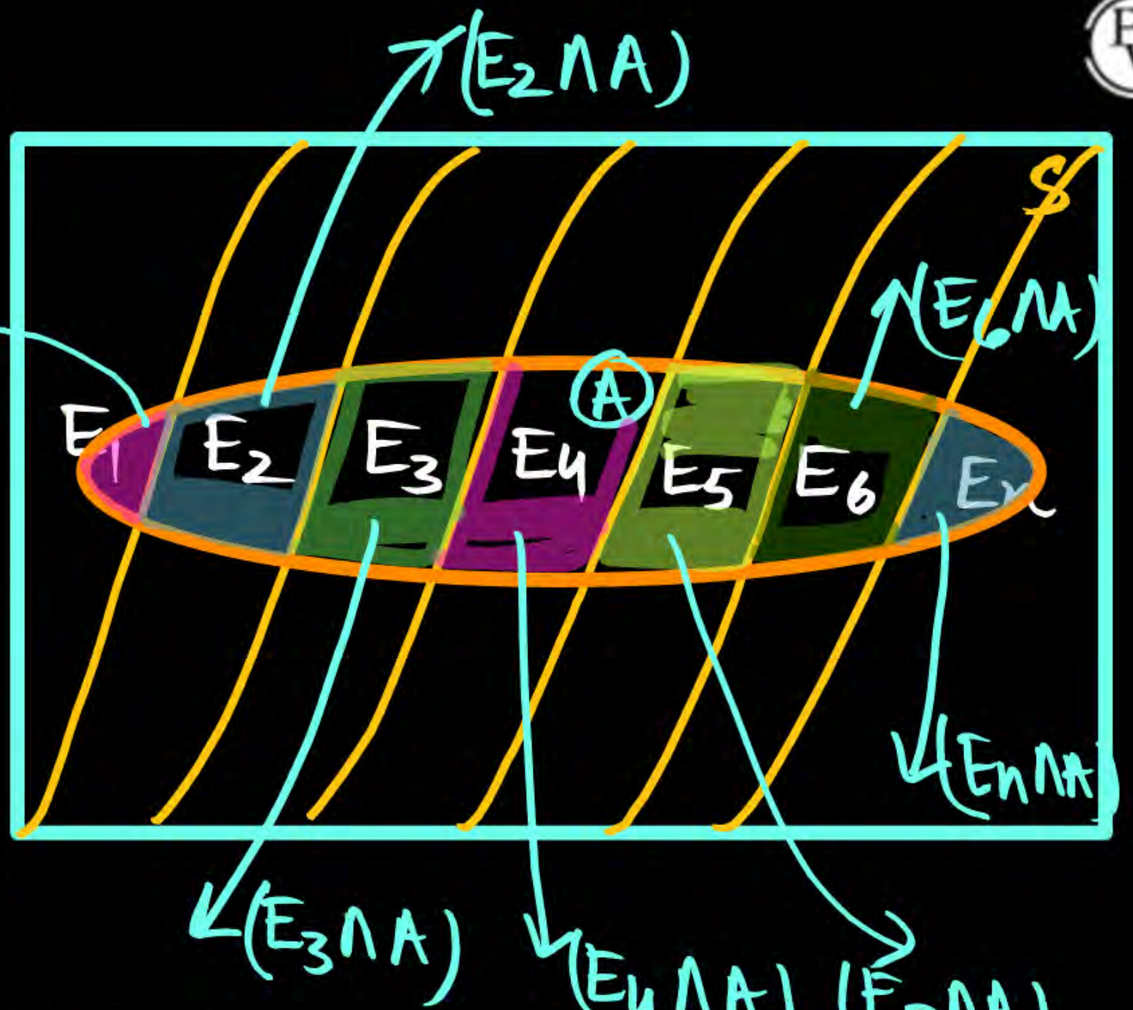


Bayes THEOREM

$$P\left[\frac{E_1}{A}\right] \Rightarrow \text{Inverse Probability}$$

↻ stage 1 ↑ stage 2





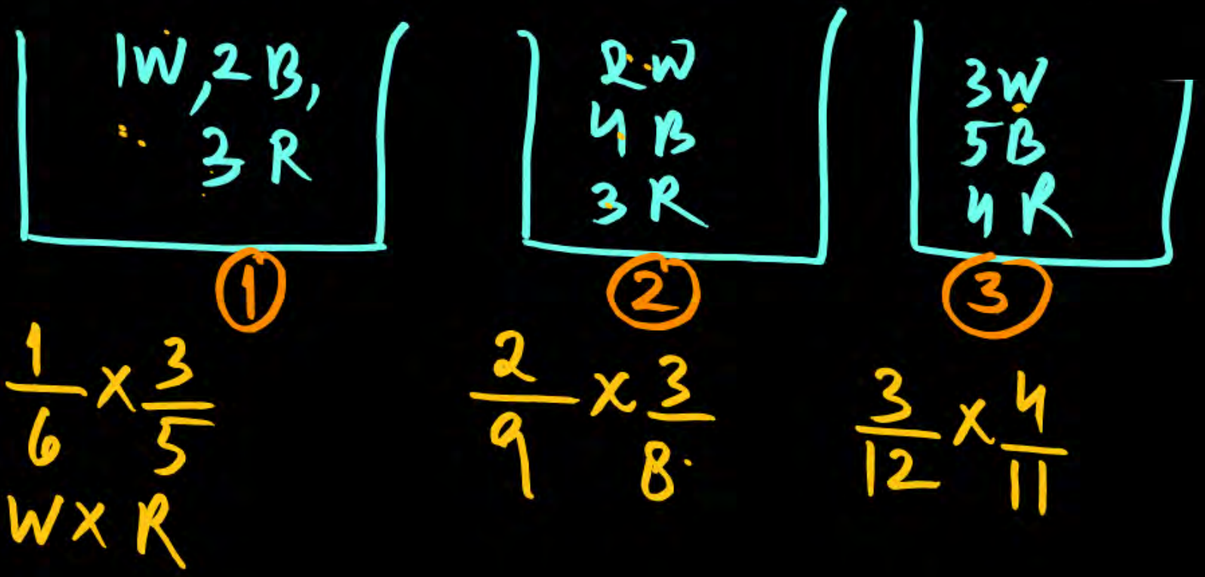
$$P(A) = P(A \cap E_1) + P(A \cap E_2) + P(A \cap E_3) + \text{---}$$

$$P(A) = P(E_1)\,P\left(\frac{A}{E_1}\right) + P(E_2)\,P\left(\frac{A}{E_2}\right) + \text{---}$$

Q. Questions

Three Boxes B_1, B_2, B_3 contains balls

$$\begin{cases} B_1 \rightarrow 1W, 2B, 3R \\ B_2 \rightarrow 2W, 4B, 3R \\ B_3 \rightarrow 3W, 5B, 4R \end{cases}$$



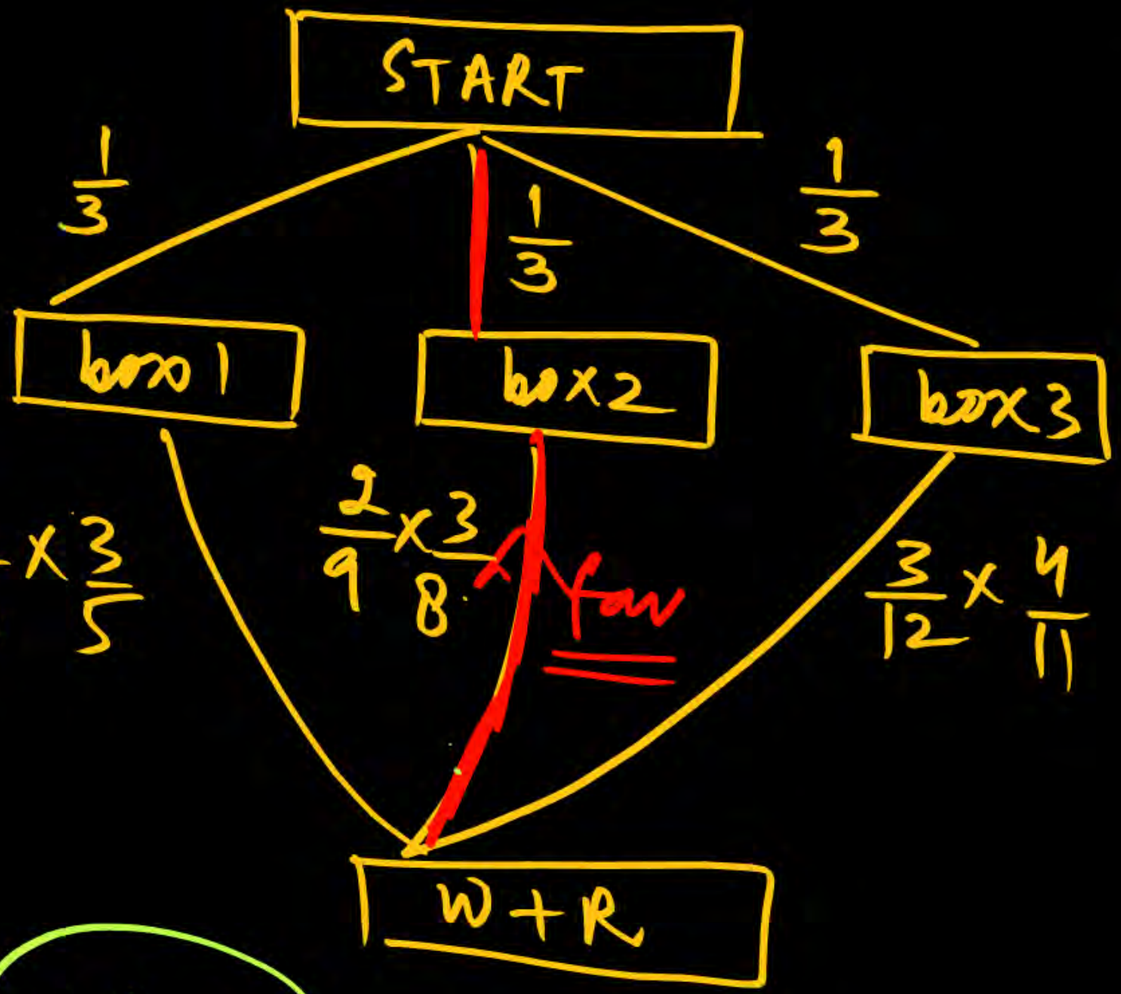
Without replacement, if 2 balls are drawn from randomly selected box. Find the probability one of the ball drawn is white and other ball is red from box 2 order is specified.

White + red

$$P\left(\frac{E_2}{A}\right) =$$

$$= P\left(\frac{\text{box}2}{W+R}\right) = \frac{\frac{1}{3}\times\frac{2}{9}\times\frac{3}{8}}{\frac{1}{3}\times\frac{2}{9}\times\frac{3}{8}+\frac{1}{3}\times\frac{1}{6}\times\frac{3}{5}+\frac{1}{3}\times\frac{3}{12}\times\frac{4}{11}}$$

$$= \frac{55}{181}.$$



$^{W}_{R}\binom{}{}\times 2!$

Q. Questions

00, 01, 02, 03
04, 05, ... 99 → Complement

One ticket is selected at random from 100 ticket 00, 01, 02, ... 99. Suppose A and B are the sum and product of digits found on the ticket. Then $P\left[\frac{A=7}{B=0}\right]$ is given by

00, 01
02, 03
... 99

x y
0 0

$P\left(\frac{\text{sum}=7}{\text{product}=0}\right) = P\left(\frac{A=7}{B=0}\right) = P\left(\frac{x+y=7}{xy=0}\right)$

$P\left(\frac{A}{B}\right)$ → given condition

(a) 2/13

(b) 2/19

(c) 1/50

(d) None of these

$$P\left(\frac{x+y=7}{xy=0}\right) = \frac{P(x+y=7 \wedge xy=0)}{P(xy=0)}$$

$$P\left(\frac{A}{B}\right) = \frac{P(A \wedge B)}{P(B)} = \frac{\frac{2}{100}}{\frac{19}{100}}$$

$$= \frac{2}{19} \text{ Ans}$$

Q. Questions

Imp.

The probability that a student knows the correct answer to a multiple choice question is 2/3. If the student does not know the answer, then the student guesses the answer. The probability of the guessed answer being correct is 1/4. Given that the student has answered the question correctly, the conditional probability that the student knows the correct answer is

(a) 2/3

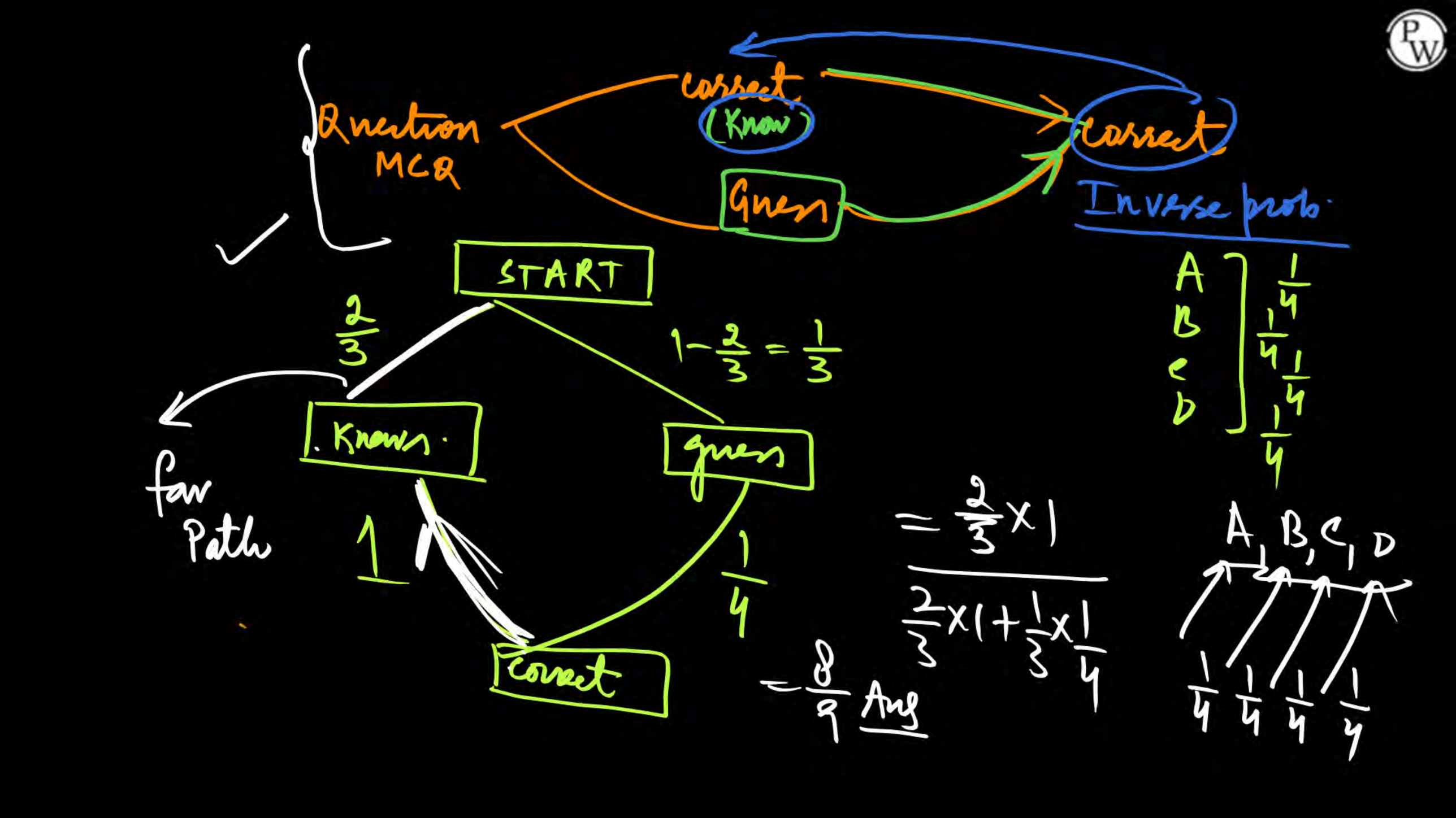
(b) 3/4

(c) 5/6

(d) 8/9

$P(\text{correct}) = \frac{2}{3}$

$P(\text{guess}) = \frac{1}{4}$


Question MCQ
correct
Know
Guess
correct
Inverse prob.
A
B
C
D
1/4
1/4
1/4
1/4
START
2/3
1 - 2/3 = 1/3
Known
guess
1
1/4
correct
far Path
= (2/3 x 1) / (2/3 x 1 + 1/3 x 1/4)
= 8/9 Ans
A, B, C, D
1/4
1/4
1/4
1/4

Q. Questions

Do yourself

The probability that a given positive integer lying between 1 and 100 (both inclusive) is NOT divisible by 2, 3 or 5 is

$$P(\text{div}) = \frac{74}{100}$$

$$P(\text{Not div}) = 1 - \frac{74}{100}$$

$$= \frac{26}{100} = \frac{13}{50}$$

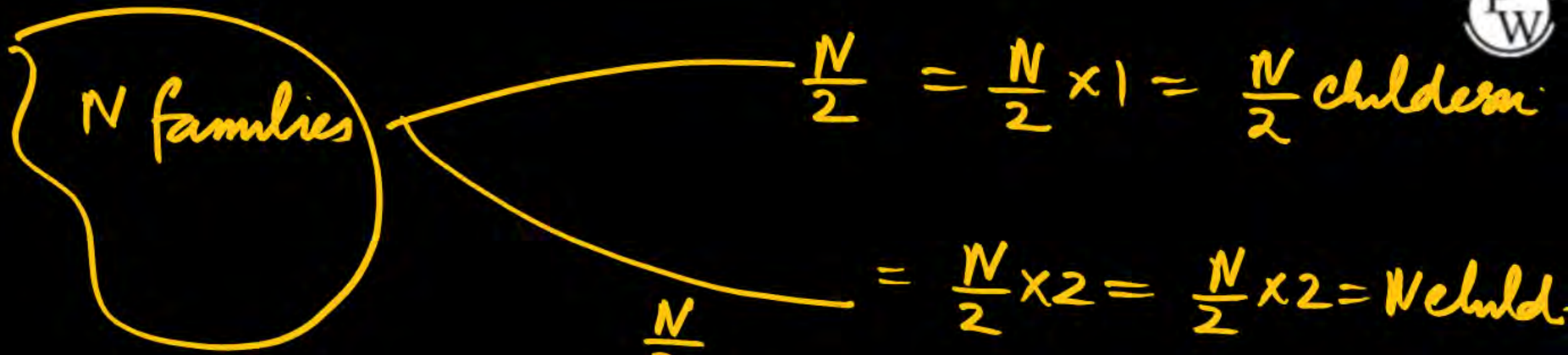
Q. Questions

In a housing society, half of the families have a single child per family, while the remaining half have two children per family. The probability that a child picked at random, has a sibling is _____.

50 — Total child = $50 \times 1 = 50$

50×2 — Total child = $50 \times 2 = 100$

$$= \frac{\text{fav choice}}{\text{Total}} = \frac{100}{100+50} = \frac{100}{150} = \frac{2}{3}$$



$$P(\text{sibling}) = \frac{\frac{N}{2}}{\frac{N}{2} + W} = \frac{\frac{N}{2}}{\frac{N}{2}} = \frac{2}{3}$$

Thank You!

GW Soldiers