ENGINEERING MATHEMATICS

ALL BRANCHES

Probability and Statistics

DPP 01 Discussion Notes
(Part-02)

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TOPICS TO BE COVERED

01 Question

02 Discussion

DPP02 Answer key update

Multiple Select → 10

Q. Questions

If the probability that A and B will die within a year are p and q respectively. Then the probability that only of one of them will be alive at the end of the year is:

A and B will Die with in a YEAR

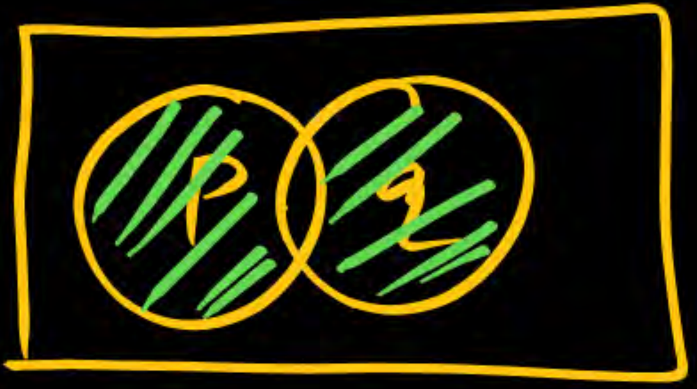
(a) $p + q$

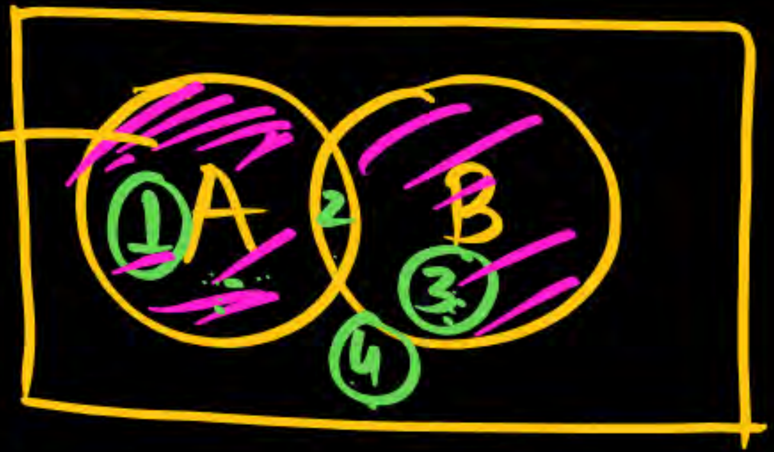
(b) $p + q - 2pq$

(c) $p + q - pq$

(d) $p + q + pq$

$$P(\text{exactly one}) = P(\bar{A} \cap B) + P(\bar{B} \cap A)$$
$$= \text{only } B + \text{only } A$$
$$= P(A) + P(B) - 2P(A \cap B)$$
$$= P(A) + P(B) - 2P(A)P(B)$$
$$= p + q - 2pq$$





region(1+3) exactly one

$P(\text{Exactly one})$

$= P(A \cap \bar{B}) + P(\bar{A} \cap B)$

$P(A) + P(B) - 2P(A \cap B)$

$(3+4) + (1+4) - 2(2)$

only A + only B

$= (1+3)$

exactly one

A, B } Independent events

$= P(\text{exactly one}) = P(A \cap \bar{B}) + P(B \cap \bar{A})$

$= P(A)\,P(\bar{B}) + P(B)\,P(\bar{A})$

$= P(A)[1 - P(B)] + P(B)[1 - P(A)]$

$= P(A) + P(B) - 2P(A)P(B)$

Q. Questions

2 min

If A and B each toss three coins. The probability that both get the same number of heads is:

(a) 1/9

(b) 3/16

(c) $\boxed{5/16}$

(d) 3/8

(A) 3 coins
Re 1, Re 2, Re 3

(B) 3 coins
Re 1, Re 2, Re 3

Three coins: 0 H, 1 H, 2 H, 3 H

A	B
SAME HEAD	SAME (A)
0	0
1	1
2	2
3	3

CASE 01: A = 0 HEAD   B = 0 HEAD = SAME NO. of HEADS

$n = 3$

1 — ① $= {}^3C_0\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^6$

3 — ② ${}^3C_1\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^3$

3 — ③ ${}^3C_2\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^3$

1 — ④ ${}^3C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^3$

$= \frac{5}{16}$

TTT	TTT
0H	0H

${}^3C_0\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^3$

$= \left(\frac{1}{2}\right)^6$

HTT ×	THT
HHT ×	THH
HHH ×	TTT

Q. Questions

Two Independent events $P(A \cap B) = P(A)P(B)$

If A and B are two independent events such that $P(\bar{A} \cap B) = 2/15$ and $P(A \cap \bar{B}) = 1/6$, then P(B) is:

(a) 1/5

(b) 1/6

(c) 4/5

(d) 5/6

[B, C]

$P(\bar{A} \cap B) = \frac{2}{15}$

$P(\bar{A})P(B) = \frac{2}{15}$

$[1 - P(A)]P(B) = \frac{2}{15}$

$P(A \cap \bar{B}) = [1 - P(B)]P(A) = \frac{1}{6}$

$P(\bar{A} \cap B) = \frac{2}{15}$

$P(A \cap \bar{B}) = \frac{1}{6}$

$P(B) =$

[(B) (C)] ✓ ✓

$[1-P(A)]P(B)=\frac{2}{15}$ — (1)

$[1-P(B)]P(A)=\frac{1}{6}$ — (2)

$\Rightarrow P(B)-P(A)P(B)=\frac{2}{15}$ — (1)

$\Rightarrow P(A)-P(A)P(B)=\frac{1}{6}$ — (2)

$$P(A)-P(A)P(B)=\frac{1}{6}$$

$$P(B)-P(A)P(B)=\frac{2}{15}$$

$$- \quad + \qquad\qquad - $$

$$P(A)-P(B)=\frac{1}{6}-\frac{2}{15}$$

$$=\frac{15-12}{90}=\frac{1}{30}$$

$$\boxed{P(A)=\frac{1}{30}+P(B)}$$

$P(A)[1-P(B)]=\frac{1}{6}$

$\left[\frac{1}{30}+P(B)\right][1-P(B)]=\frac{1}{6}$

→ quadratic Equ^n

$P(B)=\frac{1}{6},\frac{4}{5}$ → (B), (C)

Q. Questions

MSQ

$\overline{A \cap B}$

$P(\text{Exactly one}) = (1+3)$

If A and B are two events, the probability that exactly one of them occurs is given by:

✓ A B C D

(a) $P(A) + P(B) - 2P(A \cap B)$

(b) $P(A \cap \bar{B}) + P(\bar{A} \cap B)$

(c) $P(A \cup B) - P(A \cap B)$

(d) $P(\bar{A}) + P(\bar{B}) - 2P(\bar{A} \cap \bar{B})$

$(3+4) + (1+4) - 2 \times (4)$

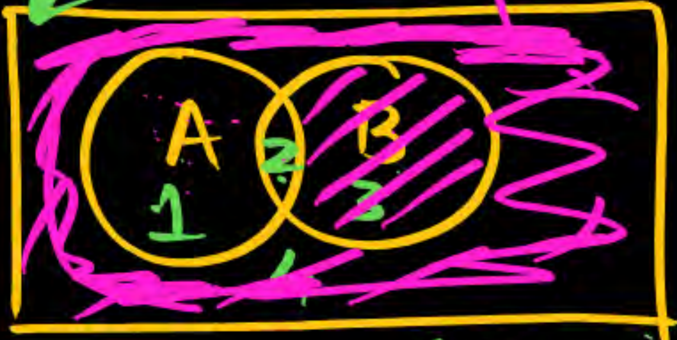
$= (1+3)$ region

(A) $P(A) + P(B) - 2P(A \cap B)$

$(1+2) + (2+3) - 2(2)$

region $= (1+3)$

exactly one



{region mark

(B) $P(A \cap \bar{B}) + P(\bar{A} \cap B) = (1+3)$ region

(C) $P(A \cup B) - P(A \cap B)$

$(1+2+3) - 2$

$= (1+3) \rightarrow$ region

exactly one

Q. Questions

$\Rightarrow$ B C D HW $\Rightarrow$

The probability of the simultaneous occurrence of two events A and B is p. If the probability that exactly one of A, B occurs is q then:

(a) $\boxed{P(\bar{A})+P(\bar{B})=2+2q-p}$

(b) $P(\bar{A})+P(\bar{B})=2-2p-q$

(c) $P(A\cap B/A\cup B)=\frac{p}{p+q}$

(d) $P(\bar{A}\cap\bar{B})=1-p-q$

$P(A\cap B)=p$

$P[(\bar{A}\cap B)\cup(\bar{B}\cap A)]=q$

Q. Questions

2 minutes

0 ✓ $\downarrow$ maxm

atmost 0 1

$\uparrow$ 0, 1

(D)

If A and B are two events. The probability that at most one of A, B occurs is:

$(1+3+4)$

(a) $\boxed{1 - P(A \cap B)}$

(b) $\boxed{P(\bar{A}) + P(\bar{B}) - P(\bar{A} \cap \bar{B})}$

(c) $\boxed{P(\bar{A}) + P(\bar{B}) + P(A \cup B) - 1}$

(d) $\boxed{P(A \cap \bar{B}) + P(\bar{A} \cap B) + P(\bar{A} \cap \bar{B})}$

(A) $1 - P(A \cap B)$

$= (\underbrace{1+2+3+4}_{\text{region}}) - (2)$

$= 1+3+4$ ✓

$\Rightarrow (3+4) + (1+4) - 4$

$= 1+3+4$ ✓

$1 = (1+2+3+4)$

(C) $(3+4) + (1+4) + (1+2+3)$

$- (1+2+3+4)$

$= 1+3+4$ ✓

$$\underline{1+3+4} = (3+4)+(1+4)+[(1+2+3+4)-(1+2+3)]$$

$$\Rightarrow P(\bar{A}\cap B)+P(\bar{B}\cap A)+P(\bar{A}\cap \bar{B})$$

(A)

(B)

(C)

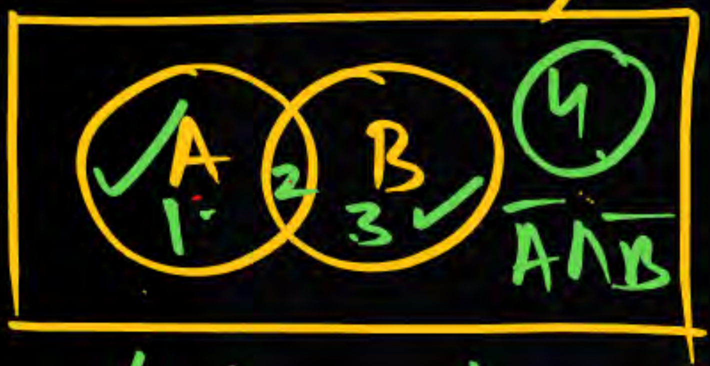
(D)

$$(3+4)+(1+4)+[(1+2+3+4)-(1+2+3)]$$

$$=(1+3+4)$$

$$1-P(A\cup B)$$

P(at most one of them)

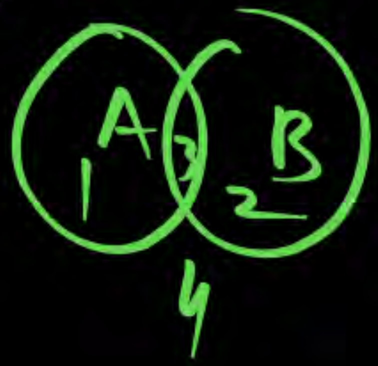


$(1+3+4)$

atmost one

$$(3+4)+(1+4)+[(1+2+3+4)-(1+2+3)]$$

$$=1+3+4$$



$(A \cap \bar{B}) + P(\bar{A} \cap B)$

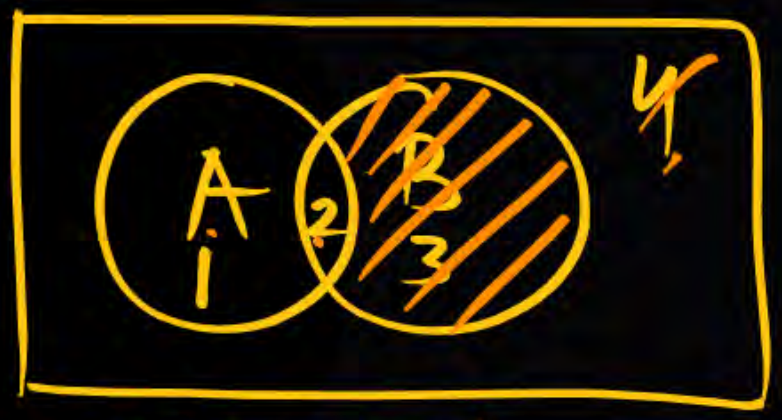
$+ P(\bar{A} \cap \bar{B})$

A

B

C

D



$(3+4) + (1+4)$

$+ \{(1+2+3+4) - (1+2+3)$

$P(\bar{A} \cap \bar{B})$

$= 1 - P(A \cup B)$

$= (1+2+3+4)$

$- (1+2+3)$

$= 1+3+4$

Q. Questions

A, B] A and B Are Disjoint Events

$P(\bar{A} \cap B) = 0.5$

If A and B are events at the same experiments with P(A) = 0.2, P(B) = 0.5, then maximum value of $P(A' \cap B)$ is: $\boxed{P(\text{only } B)}$

maximum value

$P(A) = 0.2$
$P(B) = 0.5$

(a) 1/4

(b) 1/2 ✓

(c) 1/8

(d) 1/16

$P(\bar{A} \cap B)$

$P(A \cap B) = 0$

max value $= 0.5$

Q. Questions

A is correct

An urn contains $\boxed{\text{6 white and 4 black balls}}$. A fair die is rolled and that number of balls are chosen from the urn. The probability that the balls selected are white is:

6 white
4 black

→ Die Rolled
1, 2, 3, 4, 5, 6

1 2 3 4 5 6
W W W W W W

$\boxed{\text{(a) } 1/5}$

(b) 1/6

(c) 1/7

(d) 1/8

6W
4R

Die 1 − 1W
Die 2 → 2W
Die 3 → 3W
Die 4 → 4W
Die 5 → 5W
Die 6 → 6W

$$\underbrace{\frac{1}{6}\times\frac{6}{10}}_{1} + \underbrace{\frac{1}{6}\times\frac{6}{10}\times\frac{5}{9}}_{2} + \underbrace{\frac{1}{6}\times\frac{6}{10}\times\frac{5}{9}\times\frac{4}{8}}_{3}$$

Die A White Die 2 2W

1

$$+\underbrace{\frac{1}{6}\times\frac{6}{10}\times\frac{5}{9}\times\frac{4}{8}\times\frac{3}{7}}_{4}$$

$$+\underbrace{\frac{1}{6}\times\frac{6}{10}\times\frac{5}{9}\times\frac{4}{8}\times\frac{3}{7}\times\frac{2}{6}}_{5}$$

$$+\frac{1}{6}\times\frac{6}{10}\times\frac{5}{9}\times\frac{4}{8}\times\frac{3}{7}\times\frac{2}{6}\times\frac{1}{5}$$

Without replacement

$$= \Rightarrow \frac{1}{5}$$

Q. Questions

$\frac{1}{5}$

A biased coin with probability p, 0 < p < 1 of heads is tossed until a head appears for the first time. If the probability that the number of tosses required is even is 2/5, then p equals:

(a) 1/3

(b) 2/3

(c) 2/5

(d) 3/5

A

Biased coin p $0<p<1$

$P(\text{HEAD appears}) = P(2\text{ times}) + P(4\text{ times}) + P(6\text{ times}) + \cdots$

EC/EE/CS

2 ✓
4 ✓
6 ✓
8 ✓
10 ✓

$\sum$ Add $= \frac{2}{5}$

2 $\longrightarrow (1-p)p$

4 $\longrightarrow (1-p)(1-p)(1-p)\cdot p$

6 $\longrightarrow (1-p)(1-p)(1-p)(1-p)p$

6 $\longrightarrow (1-p)^5 \cdot p$

8 $\longrightarrow (1-p)^7 \cdot p$

Coin Toss

HEAD = p

Tail = $p(1-p)$

$\longrightarrow$ even No.

✓ 2 $\longrightarrow p(1-p)$ ✓

✓ 4 $\longrightarrow (1-p)^3 \cdot p$ ✓

$P(H) = p$

$P(T) = (1-p)$

$$\frac{2}{5} = S = \underbrace{\boxed{p(1-p)}}_{2\text{ times}} + \underbrace{p(1-p)^3}_{4\text{ times}} + \underbrace{(1-p)^5 p}_{6\text{ times}} + \underbrace{(1-p)^7 \cdot p}_{8\text{ times}} + \dots \infty$$

$$S_\infty = \frac{a}{(1-r)}$$

$$\frac{2}{5} = \frac{p(1-p)}{1-(1-p)^2}$$

T_1, T_2, T_3, T_4

$\frac{T_2}{T_1} \quad \frac{T_3}{T_2} = r \longrightarrow$ G.P

$(1-p)^2 \quad (1-p)^2$

$T_2 = p(1-p)^3$

$T_1 = (1-p)p$

$\frac{T_2}{T_1} = (1-p)^2$

$$p = \frac{1}{3}$$

$$\frac{2}{5} = \frac{p(1-p)}{1-(1-p)^2}$$

$$= 2\left[1-(1+p^2-2p)\right] = 5(p-p^2)$$

$$= 2\left[\cancel{1}-\cancel{1}-p^2+2p\right] = 5p-5p^2$$

$$= -2p^2+4p-5p+5p^2 = 0$$

$$= 3p^2-p = 0$$

$$\Rightarrow p(3p-1) = 0$$

$$\Rightarrow p = 0 \quad \times$$

$$3p-1 = 0$$

$$p = 1/3$$

Thank you

GW
Soldiers !