Mathematical Logic

DPP: 1

Q1 Which of the following is tautology?

(A) $(\sim p \wedge (p \to q)) \to \sim q$

(B) $\sim (p \to q)) \to \sim q$

(C) $[(\sim p \wedge q) \wedge (q \to (p \to q))] \to \sim r$

(D) None of these

Q2 The statement $[P \vee (P \longleftrightarrow Q) \vee Q]$ is equivalent to

(A) P (B) Q

(C) A tautology (D) $(P \wedge Q)$

Q3 Consider the following statement

S_1: $[(p \Rightarrow q) \wedge (q \Rightarrow r)] \Rightarrow (r \Rightarrow p)$

S_2: $[(p \Rightarrow q) \wedge (q \Rightarrow r)] \Rightarrow (p \Rightarrow r)$

Which of the following is/are correct?

(A) S_1 is contingency

(B) S_2 is tautology

(C) S_1 and S_2 both contingency

(D) S_1 and S_2 both Tautology

Q4 Which of the following is valid?

S_1: $p \Rightarrow (q \vee r) \equiv (p \Rightarrow q) \vee (p \Rightarrow r)]$

S_2: $p \Rightarrow (q \wedge r) \equiv (p \Rightarrow q) \wedge (p \Rightarrow r)]$

(A) S_1 is valid and S_2 is not valid

(B) S_1 is not valid and S_2 is valid

(C) Both S_1 and S_2 are valid

(D) Neither S_1 and S_2 is valid

Q5 Which of the following is/are true.

(A) $\{R \to S, P \to Q, R \vee P\} \Rightarrow S \vee Q$

(B) $\{\sim R \to (S \to \sim T), \sim R \vee W, \sim P \to S, \sim W\} \Rightarrow T \to P$

(C) $\{\sim P \wedge Q, Q \to (P \to R)\} \Rightarrow \sim R$

(D) $\{P \to R, P, Q \vee \sim R\} \Rightarrow Q$

Q6 Consider the following logical inferences.

I_1 : "I will study discrete math or I will study English literature"

"I will not study discrete math"

Inference : I will not study English literature

I_2 : "If A works hard then B or C will enjoy themselves"

"If B enjoys himself then A will not work hard"

"If C enjoys himself then D will not enjoy himself"

Inference: - If A work hard then D will not enjoy himself

(A) Both I_1 and I_2 are correct inferences

(B) I_1 is correct but I_2 is not a correct inference

(C) I_1 is not correct but I_2 is a correct inference

(D) Both I_1 and I_2 are not correct inferences

Q7 Which of the following is/are logical equivalence?

I. $\sim (p \to q)$

II. $\sim (p \to q) \wedge (q \to r)$

III. $p \wedge \sim q$

IV. $(p \vee q) \to r$

(A) I and II (B) I and III

(C) II and IV (D) II and III

Q8 Consider the following statement

$S_1 : \sim (p \longleftrightarrow q)$

$S_2 : p \longleftrightarrow \sim q$

Which of the following is correct?

(A) S_1 is tautology

(B) S_2 is contradiction

(C) S_1 is equivalent to S_2

(D) None of these

Q9 Consider the following statement

$S_1 : \sim (p \vee (\sim p \wedge q))$

$S_2 : \sim p \wedge \sim q$

Which of the following is correct?

(A) S_1 is tautology

(B) S_2 is contradiction

(C) S_1 is equivalence to S_2

(D) S_1 is not equivalence to S_2

Q10 $(p \rightarrow (q \rightarrow r)) \leftrightarrow ((p \rightarrow q) \rightarrow r)$ is

(A) Tautology

(B) Contingency

(C) Contradiction

(D) Can't be determined.

Answer Key

Q1	(B)
Q2	(C)
Q3	(A, B)
Q4	(C)
Q5	(A, B, D)
Q6	(C)
Q7	(A, B, D)
Q8	(C)
Q9	(C)
Q10	(B)

Hints & Solutions

Q1 Text Solution:

(a) $(\sim p \wedge (p \to q)) \to q$

$\equiv (\sim p \wedge (\sim p \vee q)) \to \sim q$

$\equiv ((\sim p \wedge \sim p) \vee (\sim p \wedge q)) \to \sim q$

$\equiv \sim p \to \sim q$

$\equiv p \vee \sim q$ (Not a tautology)

(b) $(\sim(p \to q)) \to \sim q$

$\equiv (\sim(\sim p \vee q)) \to \sim q$

$\equiv (p \wedge \sim q) \to \sim q$

$\equiv \sim(p \wedge \sim q) \vee \sim q$

$\equiv \sim p \vee q \vee \sim q$

$\equiv$ T (tautology)

(c) $[(\sim p \wedge q) \wedge (q \to (p \to q))] \to \sim r$

$\equiv [(\sim p \wedge q) \wedge (\sim q \vee \sim p \vee q)] \to \sim r$

$\equiv (\sim p \wedge q) \to \sim r$

$\equiv p \vee \sim q \vee \sim r$ (Not tautology)

Q2 Text Solution:

$p \vee (p \leftrightarrow q) \vee Q$

$\equiv p \vee (p \wedge Q) \vee (\sim p \wedge \sim Q) \vee Q$

$\equiv p \vee (\sim p \wedge \sim q) \vee Q$

$\equiv T$

Q3 Text Solution:

$S_1 : [(p \to q) \wedge (q \to r)] \to (r \to p)$

$\equiv [(\sim p \vee q) \wedge (\sim q \vee r)] \to (\sim r \vee p)$

$\equiv [(\sim p \wedge \sim q) \vee (\sim p \wedge r) \vee (q \wedge r)] \to (\sim r \vee p)$

$\equiv [(p \vee q) \wedge (p \vee \sim r) \wedge (\sim q \vee \sim r)] \vee (\sim r \vee p)$

$\equiv p \vee \sim r$ (Contigency)

S_2: $[(p \to q) \wedge (q \to r)] \to (p \to r)$

It is the valid transitivity property of implication.

Q4 Text Solution:

S_1 $: p \to (q \vee r) \equiv [(p \to q) \vee (p \to r)]$

LHS : $p \to (q \vee r)$

$\equiv \sim p \vee q \vee r$

RHS: $(p \to q) \vee (p \to r)$

$\equiv \sim p \vee q \vee \sim p \vee r$

$\equiv \sim p \vee q \vee r$

$\because$ LHS = RHS, S_1 is valid.

S_2: $P \to (q \wedge r) \equiv [(p \to q) \wedge (p \to r)]$

LHS: $p \to (p \wedge r)$

$\equiv \sim p \vee (q \wedge r)$

RHS: $(p \to q) \wedge (p \to r)$

$\equiv (\sim p \vee q) \wedge (\sim p \vee r)$

$\equiv \sim p \vee (q \wedge r)$

$\because$ LHS $\equiv$ RHS, S_2 is valid.

Q5 Text Solution:

Option (A)

True by using 'constructive dilemma'

Option (B)

1. $\neg R \to (S \to \neg T)$
2. $\neg R \vee W$
3. $\neg P \to S$
4. $\neg W$

We need to determine if $T \to P$ follows.

- From premise 4 ($\neg W$), and premise 2 ($\neg R \vee W$), we can conclude $\neg R$.
- Given $\neg R$, from premise 1, we get $S \to \neg T$.
- From premise 3, $\neg P \to S$, or requivalently, $\neg S \to P$.

Combining $S \to \neg T$ with $\neg S \to P$:

- If T is true, then S must be false because $S \to \neg T$.
- If S is fale, then P must be true becasue $\neg S \to P$.

Thus, if T is true, P must be true, meaning $T \to P$.

option (B) is true.

Option (c)

Given premises:

1. $\neg P \wedge Q$
2. $Q \to (P \to R)$

we need to determine if $\neg R$ follows:

- From premise 1, $\neg P$ and Q are both true
- Given Q is true, we consider the implication $Q \to (P \to R)$
  - $P \to R$ must be true because Q is true
  - Since $P \to R$ is equivalent to $\neg P \vee R$, and $\neg P$ is true, $\neg P \vee R$ is true, satisfying $P \to R$.

There is no information that guarantees R to be false or true independently from P. Hence, $\neg R$ does not necessarily follow from the premises given. **Option (C) is false.**

Option (d)

Given premises:

1. $P \to R$
2. P
3. $Q \lor \neg R$

We need to determine if Q follows.

- From premise 2, P is true.
- Given $P \to R$ from premise 1, R must be true.
- Now, consider $Q \lor \neg R$. Since R is true, $\neg R$ is false.
- Hence, $Q \lor \neg R$ simplifies to $Q \lor$ false, which means Q must be true.

Thus, Q must be true. Option (D) is true.

Q6 Text Solution:

For I1,

Let p: "I will study discrete math"

q: "I will study English literature"

So premises will be,

1. $p \lor q$
2. $\neg p$

From premise 2, p is false. Hence, q has to be true from making premise 1 true.

So, I1 is incorrect.

For I2

Let p: "A works hard"

q: "B enjoys himself"

r: "C enjoys himself"

s: "D enjoys himself"

So premises will be,

1. $p \to (q \lor r)$
2. $q \to \neg p$
3. $r \to \neg s$

The inference is $p \to \neg s$

To proof by contradiction p and s has to true.

Premise 2 follows, if q is false,

Premise 1 follows, if r is false,

Now, in premise 3,

$r \to \neg s \equiv T \to F \equiv F$

So, antecedent becomes false and the conclusion follows.

Hence, I_2 is correct.

Q7 Text Solution:

I. $\sim(p \to q) \equiv p \land \sim q$

II. $\sim(p \to q) \land (q \to r) \equiv (p \land \sim q) \land (\sim q \lor r)$
$\equiv (p \land \sim q)$

III. $p \land \sim q$

IV. $(p \lor q) \to r \equiv (\sim p \land \sim q) \lor r$

So, I, II and III are equivalent to each other.

Q8 Text Solution:

S_1: $\sim(p \leftrightarrow q) \equiv p \oplus q$

S_2: $p \leftrightarrow (\sim q) \equiv (p \land \sim q) \lor (\sim p \land q) = p \oplus q$

Q9 Text Solution:

S_1 : $\sim(p \lor (\sim p \land q))$

$\equiv \sim p \land \sim(\sim p \land q)$

$\equiv \sim p \land (p \lor \sim q)$

$\equiv \sim p \land \sim q$

S_2 : $\sim p \land \sim q$

Q10 Text Solution:

Implication is not associative.

P	Q	R	$q \to r$	$p \to (q \to r)$	$p \to q$	$(p \to q) \to r$	$(p \to (q \to r)) \leftrightarrow ((p \to q) \to r)$
T	T	T	T	T	T	T	T
T	T	F	F	F	T	F	T
T	F	T	T	T	F	T	T
T	F	F	T	T	F	F	F
F	T	T	T	T	T	T	T
F	T	F	F	T	T	F	F
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	F