Graph Theory

Q1 Consider an undirected graph G, which is connected and have 8 vertices and 13 edges. Find the minimum number edges, whose deletion from graph G is always guarantee that it would be disconnected graph.

Q2 Consider a simple graph of 10 vertices. If the graph is disconnected, then the maximum number of edges, it can have is ________ ?

Q3 If G is a simple disconnected graph with 16 vertices and 3 components, then maximum number of edges possible in G is ______?

Q4 Consider the following statements

S1: A graph G(V, E) is called tree if there is an exactly one path between every 2 vertices.

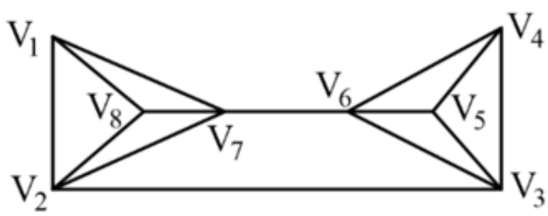
S2: A graph G(V, E) is tree iff it is connected, and it does not contain cycle.

Which of the following statements is true?

(A) S1 only (B) S2 only

(C) Both S1 and S2 (D) None of these

Q5 For the graph below, vertex.

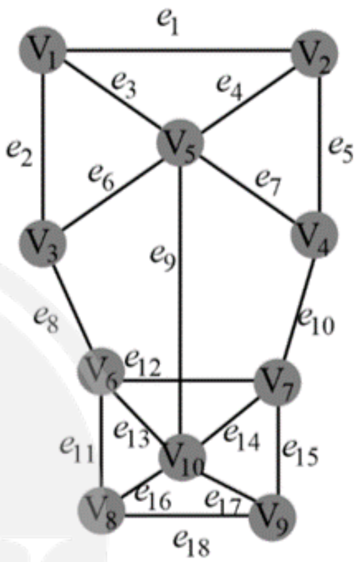


Let vertex connectivity is X and edge connectivity is Y

Value of X+Y is______

Q6 If G is a connected graph with 10 vertices and vertex connectivity is 3, then minimum number of edges necessary in G is_____.

Q7 Consider the given connected graph G



Which of the following is not the cut set?

(A) {e6, e7, e9}

(B) {e8, e9, e10, e12}

(C) {e8, e9, e10}

(D) {e1, e2, e3}

Q8 Which of the following is/are Euler Graph?

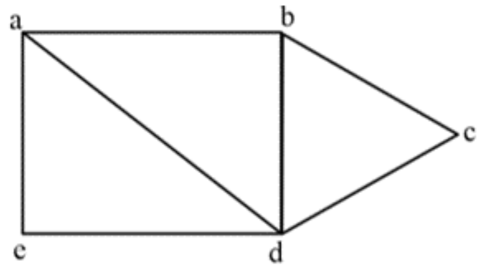
(A) K_{51}

(B) K_{50}

(C) $\bar{C}_{25}$ { Complement of C25}

(D) 10- regular graph

Q9 For the graph shown below



Which of the following statement is/are true?

(A) Euler path exists

(B) Euler circuit exists

(C) Hamiltonian cycle exists

(D) Hamiltonian path exists

Q10 If G is a bipartite graph with 9 vertices and maximum number of edges, then vertex connectivity of G = ____.

Answer Key

Q1	7~7	Q6	(15)
Q2	= 36	Q7	(A, B)
Q3	= 91	Q8	(A, C)
Q4	(C)	Q9	(A, C, D)
Q5	4~4	Q10	(4)

Hints & Solutions

Q1 Text Solution:

In worst case, a vertex may be present which is connected with all the remaining seven vertices. Hence, answer is 7.

Q2 Text Solution:

To achieve maximum number of edges, the components must have maximum number of vertices. Since, the graph is disconnected we can keep only 1 vertex in a component and rest 9 vertices in another component.

So, maximum number of edges possible in G = maximum number of edges possible in 1st component having 1 vertex + maximum number of edges possible in 2nd component having 9 vertices.

$= \frac{1(1-1)}{2} + \frac{9(9-1)}{2}$

= 36.

Q3 Text Solution:

Maxiumu number of edges possible in G = maximum number of edges possible in 1st component having 1 vertex +
maximum number of edges possible in 2nd component having 1 vertex +
maximum number of edges possible in 3rd component having 14 vertices.

$= \frac{1(1-1)}{2} + \frac{1(1-1)}{2} + \frac{14(14-1)}{2}$

= 91.

Q4 Text Solution:

Both S1 and S2 are properties of a tree.

Q5 Text Solution:

One possible way to disconnected the graph with respect to vertices is by removing V_2 and V_7.

Hence, vertex connectivity (x) = 2

One possible way to disconnect the graph with respect to edges is by removing $V_2 V_3$ and V_7V_6,

Hence, edge connectivity (Y) = 2

value of x + y = 2 + 2 = 4

Q6 Text Solution:

n = 10

Vertex connectivity is 3, that means degree of each vertex is at least 3.

$\therefore$ Minimum number of edges in C_m = $\frac{10\times 3}{2} = 15$

Q7 Text Solution:

A cut set is the minimal set of edges whose removal can disconnect the graph.

(A) Removal of these three edges does not disconnects the graph.

(B) Although the graph gets disconnected but the set is not minimal, e_{12} is extra here. Option C is the minimal version of this set.

Q8 Text Solution:

(A) K_{51}

K_{51} is a complete graph with 51 vertices.

- In a complete graph K, each vertex has degree n-1.
- For K_{51}, each vertex has degree 50, which is even.
- K_{51} is connected.
- Therefore, K_{51} is an Eulerian graph because it satisfies the condition of having all vertices with even degree and being connected.

(B) K_{50}

K_{50} is a complete graph with 50 vertices.

- For K_{50}, each vertex has degree 49, which is odd.
- Since all vertices have an odd degree, K_{50} cannot have an Eulerian circuit or path.
- Therefore, K_{50} is not an Eulerian graph.

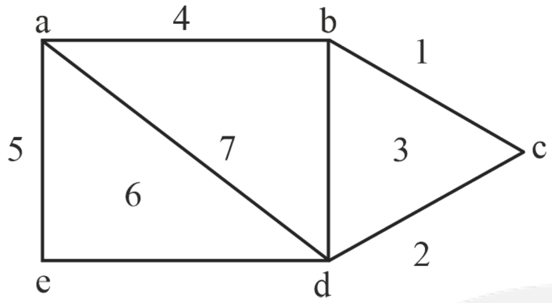
(C) $\overline{C_{25}}$ (component of C_{25})

- C_{25} is a cycle graph with 25 vertices.
- In a cycle graph C_n, each vertex has degree 2.
- The complement of C_{25} will have 25 vertices, and each vertex will be connected to all other vertices except the two vertices it was connected to in C_{25}.
- Thus each vertex = $\overline{C_{25}}$ wil have degree 25 – 3 = 22, which is even .
- The graph $\overline{C_{25}}$ is connected.
- Therefore $\overline{C_{25}}$ is an Eulerian graph because it satisfies the condition of having all vertices

with even degree and being connected.

(D) In a 10-regular graph, vertex of each degree is 10 (even) but the graph may be disconnected. Hence, it is not an Euler graph.

Q9 Text Solution:



The numbering of edges shows the euler path.
The Hamiltonian path/cycle can be a-b-c-d-e.
But, there is no euler circuit.

Q10 Text Solution:

For maximum number of edges in a bipartite graph of 9 vertices, the optimal bipartition is to have 4 vertices in one set and 5 in the other.

To disconnect the graph with respect to vertices, one of the set has to be completely removed.

Also, Vertex Connectivity is the minimum number of vertices that need to be removed to disconnect the graph.

Hence, Vertex connectivity = min(4,5) = 4