GATE-All BRANCHES
Engineering Mathematics

Linear Algebra

Lecture No.- 05

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Topics covered in previous lecture

Topic Properties of adjoint of a matrix

Topic Question based on matrices

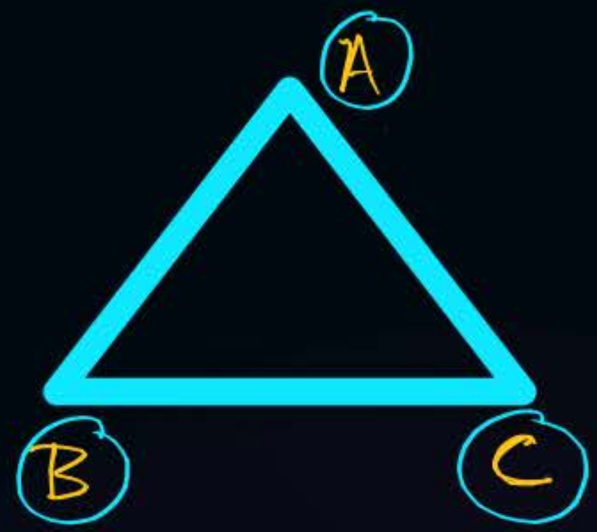
Topics to be Covered

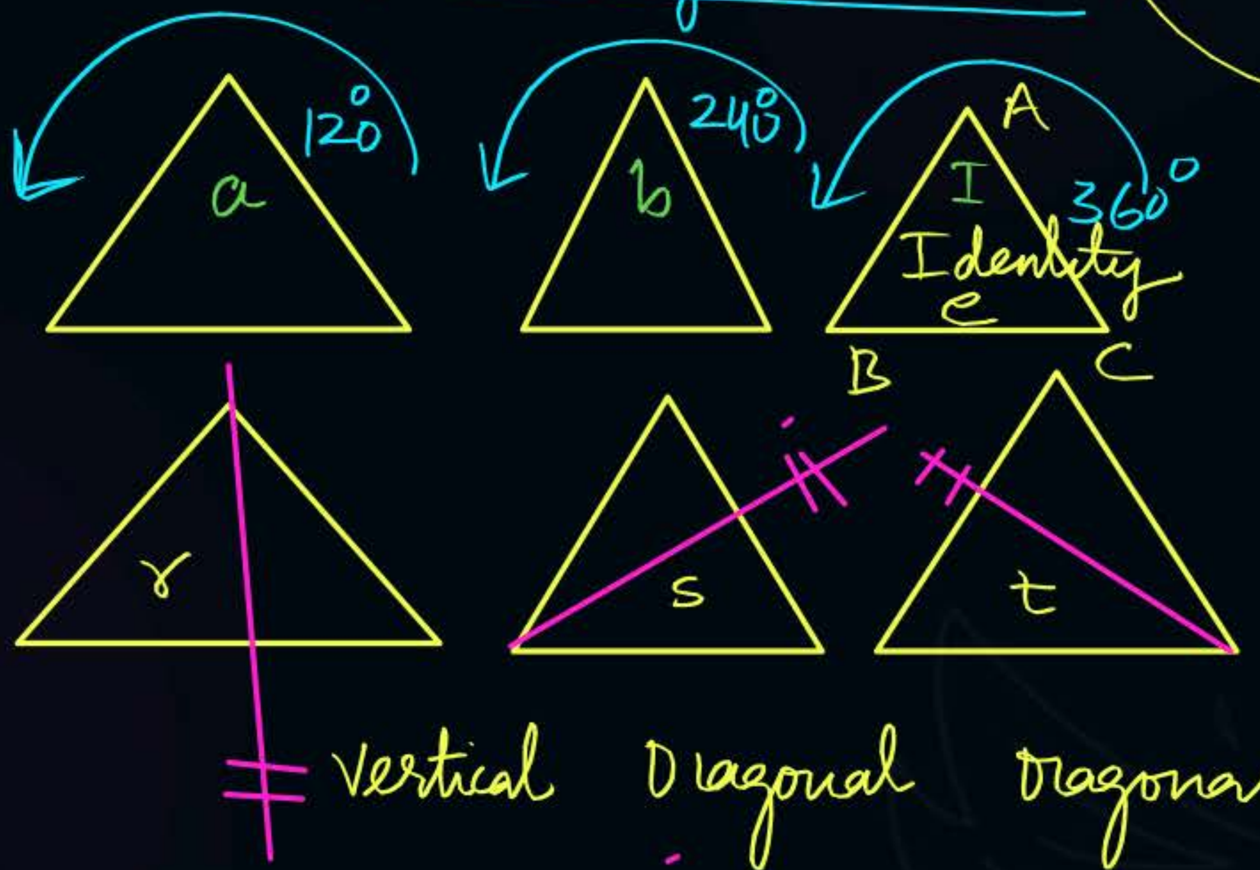
- **Topic** Vector space
- **Topic** Linear combination
- **Topic** Spanning set
- **Topic** Linear dependence and independence
- **Topic** Basis

vector space (V)

6th class.

Symmetry counts
→ Rotational symmetry
→ Reflectional "





Rotational symmetry

Reflection

a
b
e (Identity)
r
s
t

Total symmetry

set of symmetries

SET of symmetries

	a	b	c	r	s	t
a	b	c	a	s	t	r
b						
c						
r						
s						
t						

✓ Closure $\rightarrow$ $v_1 \in V$

$v_2 \in V$

$$\left\{ \begin{array}{l} v_1 + v_2 \in V \\ v_1 \cdot v_2 \in V \end{array} \right]$$

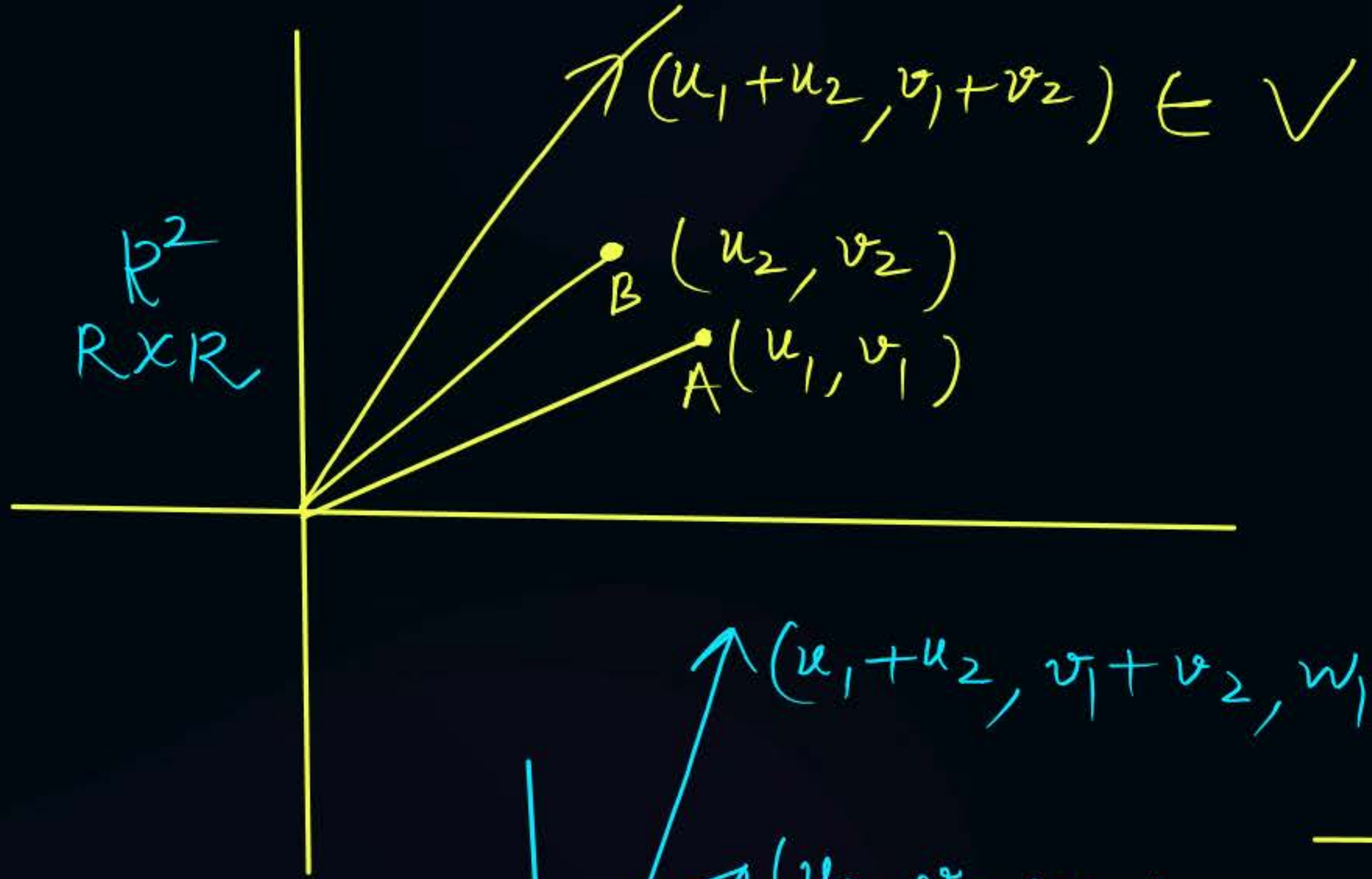
$v_1 \in$ Rational R

$v_2 \in$ Rational R

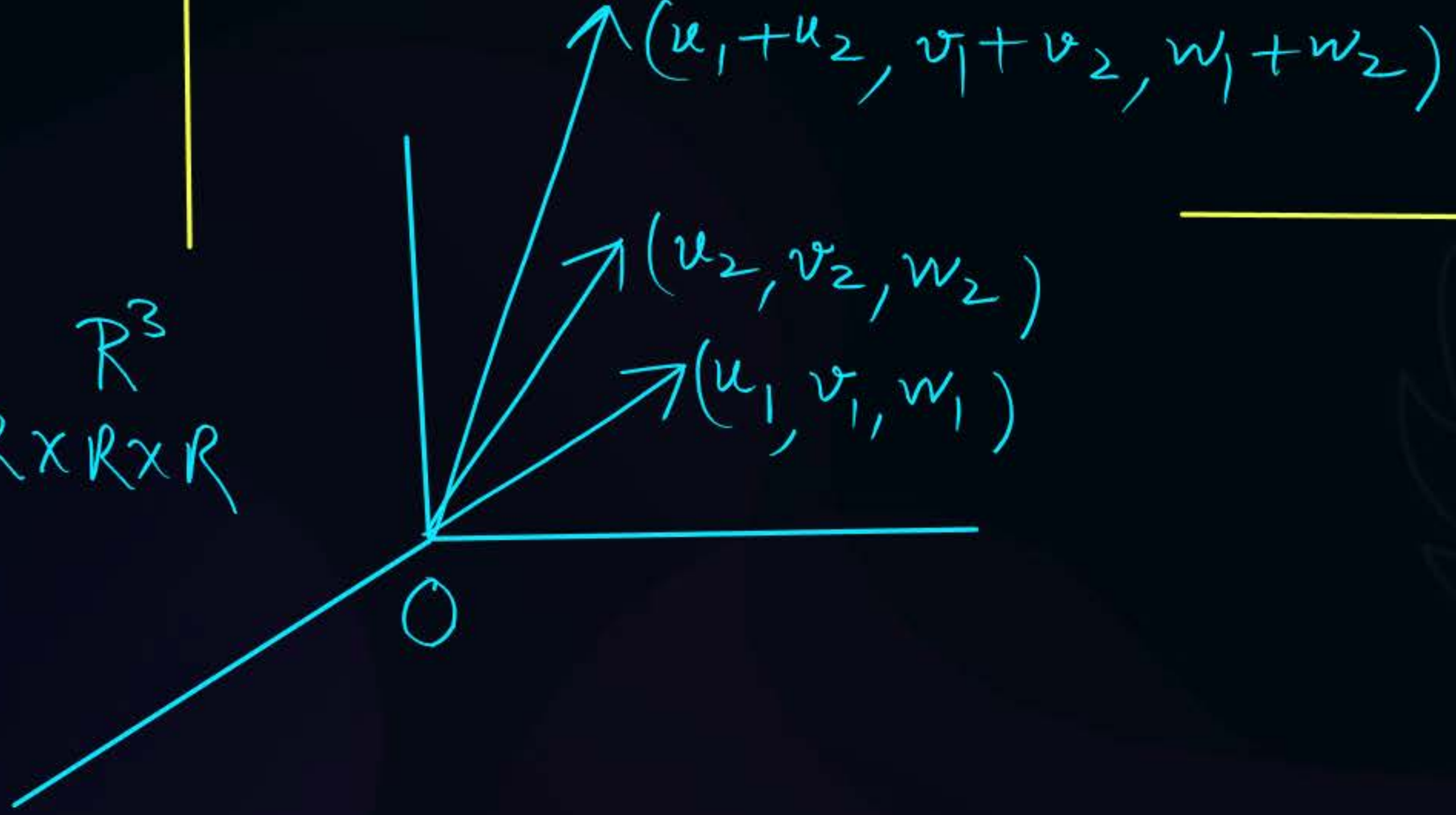
$v_1 + v_2 \in v$

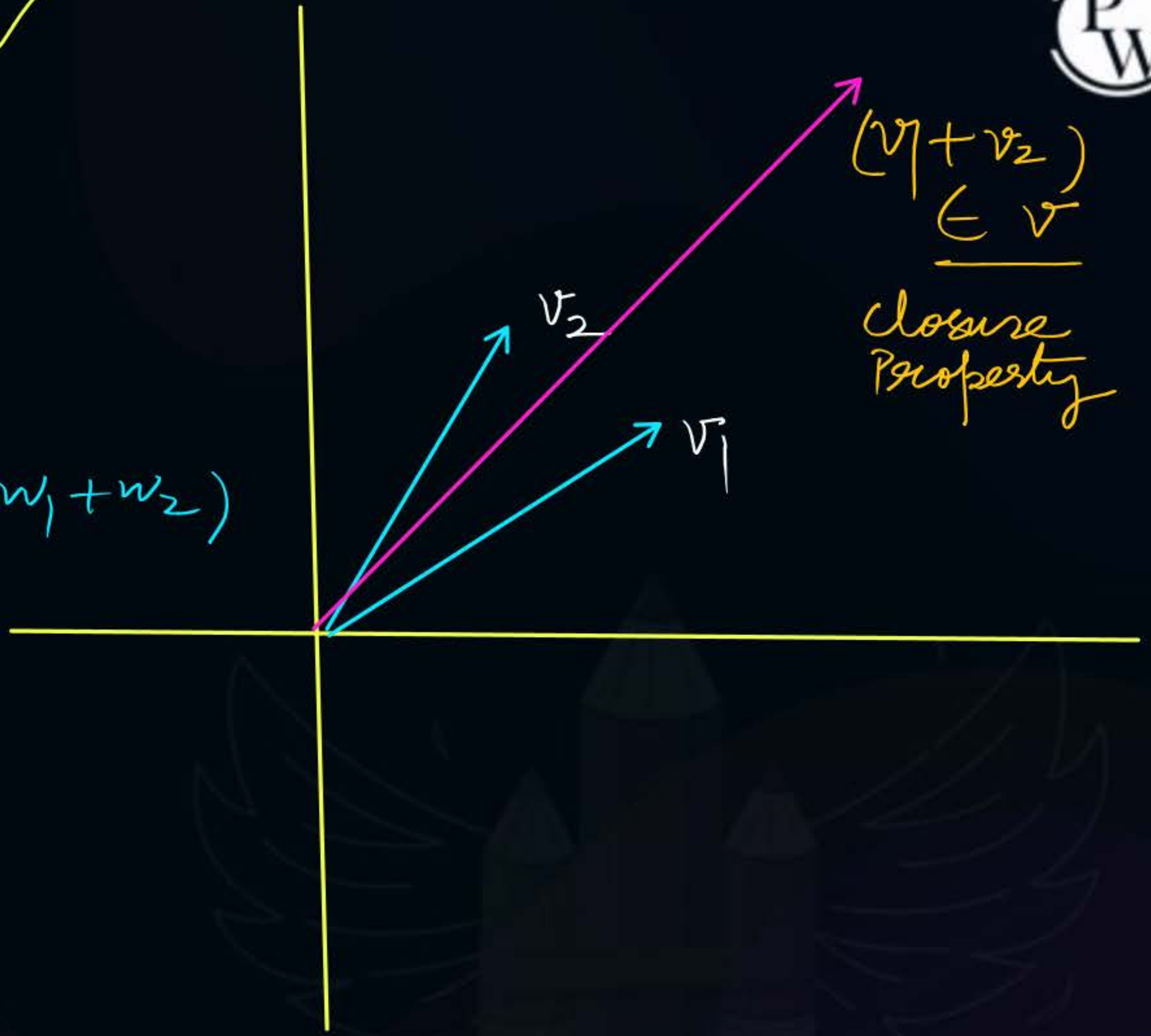
$v_1 \cdot v_2 \in v$

R^2
$R \times R$



R^3
$R \times R \times R$



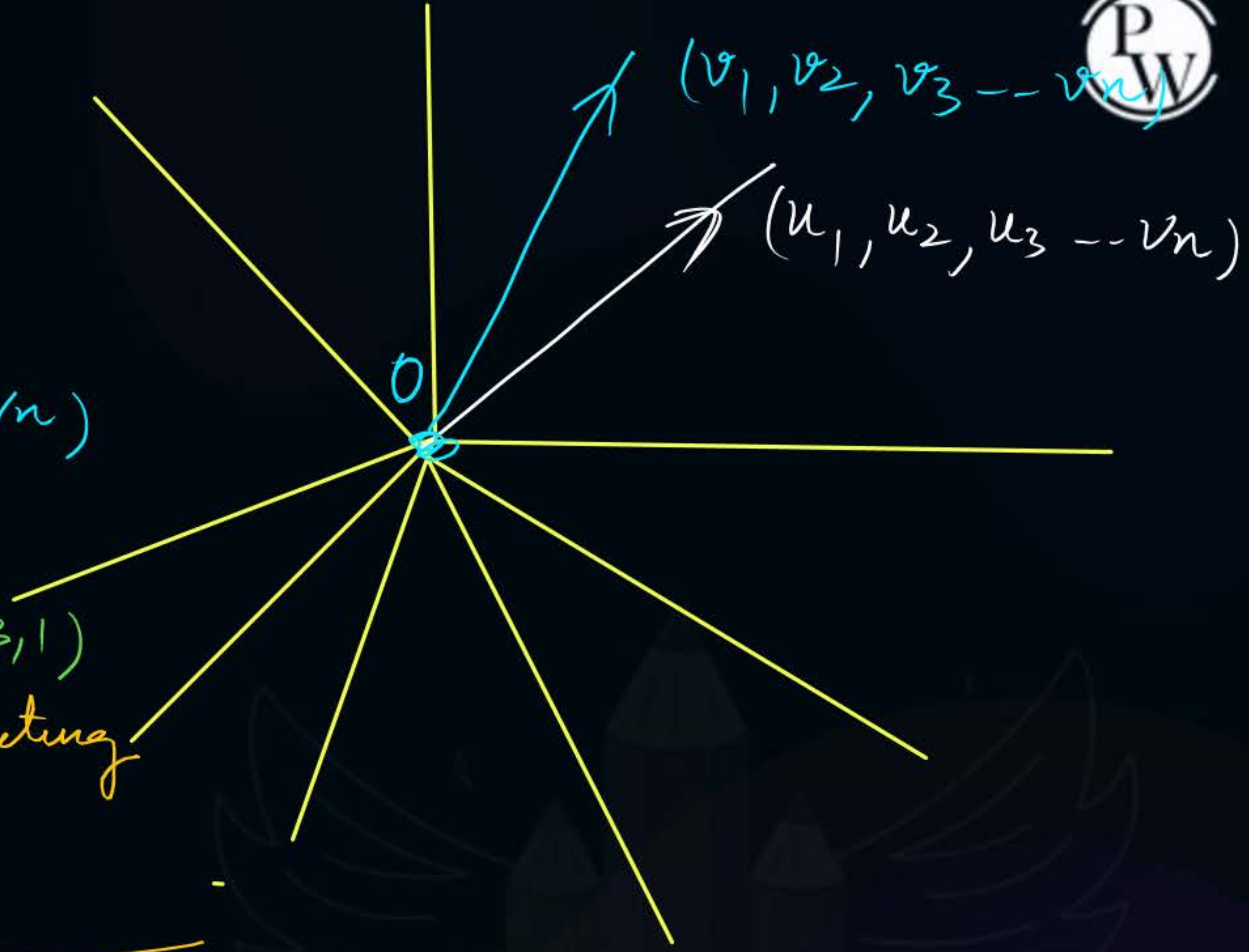


closure Property

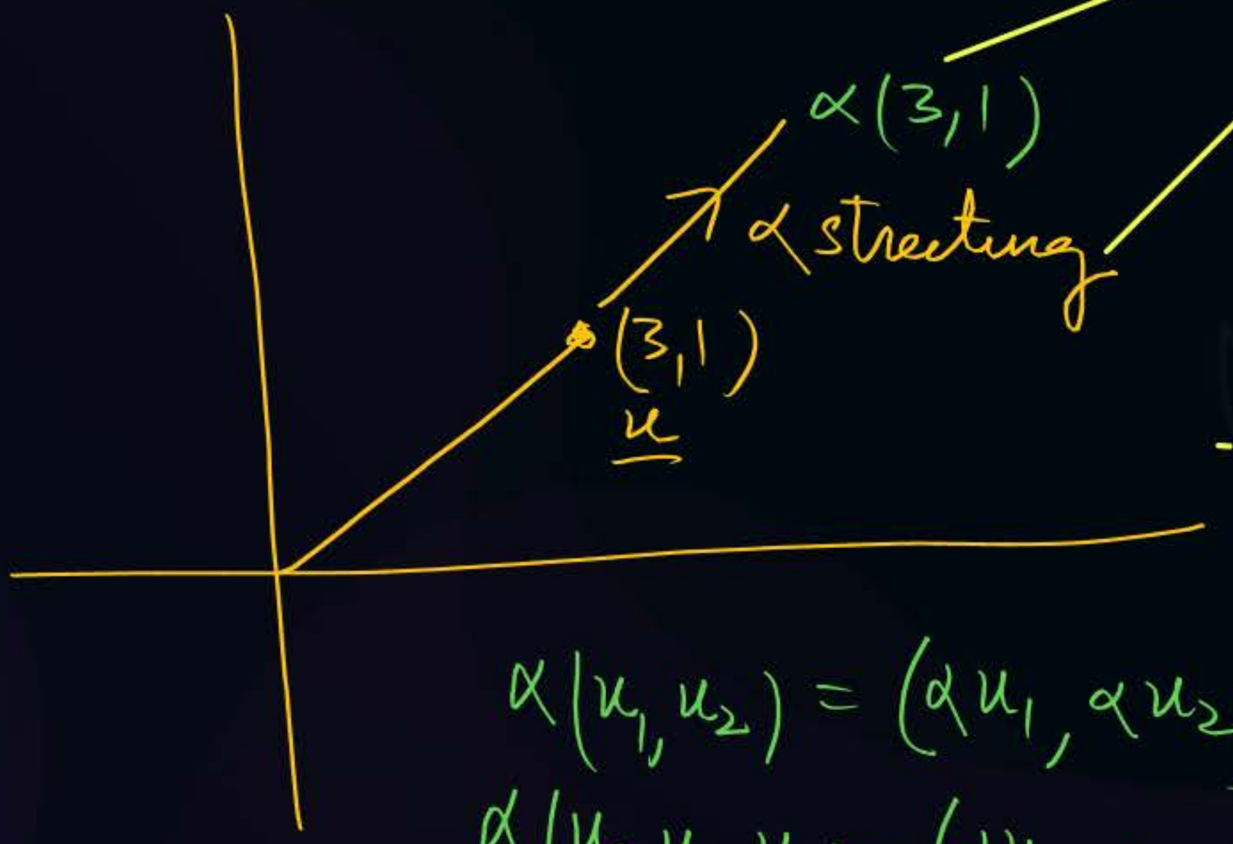
Add $(u_1, u_2, u_3 \ldots v_n)$

$$+ (v_1, v_2, v_3 \ldots v_n)$$

$$= (u_1 + v_1, u_2 + v_2, u_3 + v_3 \ldots u_n + v_n)$$



Scaling



$$\alpha(u_1, u_2) = (\alpha u_1, \alpha u_2)$$

$$\alpha(u_1, u_2, u_3) = (\alpha u_1, \alpha u_2, \alpha u_3)$$

Properties (Add)

1) Closure: for all $u, v \in R^2$

$u + v \in R^2$ (Closure Property)

2) Associative: for all $u, v, w \in R^3$

$$u + (v + w) = v + (u + w) = w + (u + v)$$

3) Additive Identity $\Rightarrow$ for $v \in R^n$

$$v + 0 = v \in R^n$$

Additive Identity '0'

4) Additive Inverse

$$v + (-v) = 0$$

$-v$ = Additive Inverse

$2-2=0$

Axioms of vector space:

Add

1) closure $u, v \in V \quad u+v \in V$
2) Associative $u+(v+w) = v+(u+w) = w+(u+v)$
3) Additive Inverse $v+(-v) = 0$
4) Identity element $v+0 = v$
5) commutative Law $v_1+v_2 = v_2+v_1$

Multiply

6) closure $\alpha v \in V$
7) Associative $\alpha(\beta v) = (\alpha\beta)v$
8) Scaler multiplication $1 \cdot v = v$
9) $\alpha(v_1+v_2) = \alpha v_1 + \alpha v_2$
10) $(\alpha+\beta)v = \alpha v + \beta v$

$\boxed{\text{Symmetry}}$ ✓

Topic: Vector space

#Q. Let u = [1, 1, 1],v = [1, 2, n] be two vectors in R^n ,thus form vectors u + v, 2u.

$$u = [1, 1, 1]$$
$$v = [1, 2, n]$$
$$R^n$$

$$u + v = [1, 1, 1] + [1\ 2\ n]$$
$$= [1+1, 1+2, 1+n]$$
$$= [2, 3, n+1]$$

$$2u = 2[1\ 1\ 1]$$
$$= [2\ 2\ 2] \text{ Ans}$$

Topic: Vector space

#Q. Let u = (1, –1, 2, 0, –3), v = (0, 2, –1, 4, 0) be two vertices in R^5 ,thus form the vectors u + v and –3v.

$u = [1, -1, 2, 0, -3]$ R^5

$v = [0, 2, -1, 4, 0]$

$u + v = [1, -1\ 1, 4\ -3]$

$-3v = [0 - 6, +3, -12, 0]$

Topic: Vector space

#Q. Let u = (1, 0, 1, 0, 1 ....) and v = (1, –2, 3, –4, 5 ...) be two vectors in R^{∞} form u+ v and 5u.

$u = [1, 0, 1, 0, 1]$

$v = [1, -2, 3, -4, 5]$

$u + v = [2, -2, 4, -4, 6]$

$5u =$

Topic: Vector space

#Q. Prove that the following properties hold for vector addition in R^4

1) Cumulative property $u + v = v + u$

2) The additive property $v + o = v = o + v$

Prove

Topic: Vector space

#Q. If $u = \{1, 3, 5, 7\}$ and $v = (2, -1, -5, 6)$ in R^4 such that $\alpha = 3, \beta = 4$ Then find

(a) $\alpha(u + v)$ = Do yourself

(b) $(\alpha + \beta)\, v$ = Do yourself

Topic: Vector space

M·W

#Q. In R^3 calculate the linear combination

(a) $2V_1 + 3V_2$ where $V_1 = (1, 0, 3)$ and $V_2 = (0, 2, -1)$

(b) In R^4 calculate the linear combination $2V_1 + 3V_2 + 4V_3 - V_4$ when

$V_1 = (1, 0, 3, 1)$

$V_2 = (0, 2, 0, -1)$

$V_3 = (0, 1, -2, 0)$

$V_4 = (2, 10, -2, -1)$

H.W

(c) In R^2 let $V_1 = (0, 3)$ and $V_2 = (2, 1)$, thus calculate the linear combination $4V_1 - 2V_2$

(d) In R^4 let $V_1 = (1, 2, 1, 3)$ and $V_2 = (2, 1, 0, -1)$ thus calculate the linear combination $3V_1 + 2V_2$

Topic: Vector space

(e) Let V= M_{23} such that $V_1 = \begin{bmatrix} 1 & 0 & 2 \\ 0 & -1 & 3 \end{bmatrix}$, $V_2 = \begin{bmatrix} 2 & -1 & 0 \\ 0 & 3 & -4 \end{bmatrix}$ and $V_3 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 2 & 1 \end{bmatrix}$, thus find the linear combination $\underbrace{3V_1 - 2V_2 + V_3}$

Linear combination

M.W

Topic: Vector space

#Q. Determine whether (3, −1) can be expressed as a linear combination of each of the following.

(a) $V_1 = (2, 0)$ $V_2 = (1, 1)$

(b) $V_1 = (2, 2)$ $V_2 = (1, 1)$

(c) $V_1 = (9, -3)$ $V_2 = (-6, 2)$

H.W

THANK - YOU