# GATE-All BRANCHES Engineering Mathematics

# Linear Algebra



By- Rahul sir

#### **Topics covered in previous lecture**









Topic

Properties of matrices

Topic

Question based on properties of the matrix

Topic

Question based on properties of the matrix,

Orthogonal basis

Topic

Linear dependence and independence, Rank space and null space

Topic

System of equations

Topic

Question based on system of equations,

### **Topics to be Covered**









Topic

Question based on properties of the matrix, Orthogonal basis

Topic

Linear dependence and independence, Rank space and null space

Topic

System of equations

Topic

Question based on system of equations, Span of vectors



# System of Equations AX=0

$$A X = D$$

Honogenons solution

$$\begin{array}{l} \text{bot} \\ \text{Product} \\ = 0 \\ \begin{array}{l} a_{21}x + a_{12}y + a_{33}z = 0 \\ a_{31}x + a_{32}y + a_{33}z = 0 \end{array} \end{array}$$

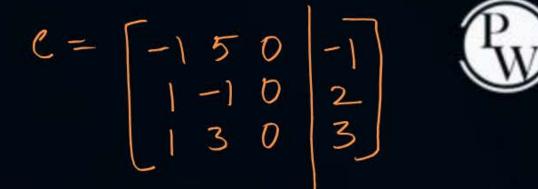
Always gives a solution System of Egin AX=0 (always consistent)

Trivial sol

Infinite solution Won Trivial

If 
$$|A|=0$$
  $f(A)  
(Infinite solution)$ 







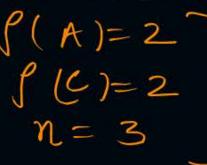
#Q. How many solutions does the following system of linear equations have

$$-x + 5y = -1$$

$$x - y = 2$$

$$x + 3y = 3$$

x - y = 2 x + 3y = 3 3 equations



S(A)=2 Infinite S(C)=2 Solutions N=3 Vome elimentum

Infinitely many

Two distinct solutions

Unique

None





#Q. Consider the following system of equations in three real variable  $x_1$ ,  $x_2$  and  $x_3$ :  $2x_1 - x_2 + 3x_3 = 1$ ,  $3x_1 + 2x_2 + 5x_3 = 2$ ,  $-x_1 + 4x_2 + x_3 = 3$ 

This system of equations has

A No solution

A unique solution

More than one but a finite number of solutions

An infinite number of solutions

Vering 
$$C = \begin{bmatrix} 2 & -1 & 3 & 1 \\ 3 & 2 & 5 & 2 \\ -1 & 4 & 1 & 3 \end{bmatrix}$$
 elimination

$$S(A) = 3$$

$$S(e) = 3$$

$$N = 3$$
solution



Rend

AX=B

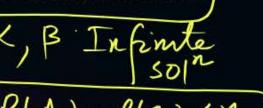
For what values of  $\alpha$  and  $\beta$  the following simultaneous equations have an

#### infinite number of solutions

$$x + y + z = 5,$$

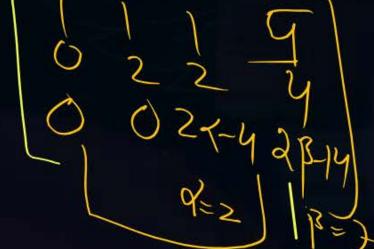
$$x + 3y + 3z = 9,$$

$$x + 2y + \alpha z = \beta$$



2,7 
$$R_3 \rightarrow R_2 - R_1$$





Slide





#Q. For the set of equations

$$x_1 + 2x_2 + x_3 + 4x_4 = 2$$
,  
 $3x_1 + 6x_2 + 3x_3 + 12x_4 = 6$ .

The following statement is true



- B There are no solutions
- A unique non-trivial solution exits
- Multiple non-trivial solution exist

Slide





C No solutions

Exactly two solutions

$$\begin{pmatrix} 2 & 1 & 3 \\ 3 & 0 & 1 \\ 1 & 2 & 5 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 5 \\ -4 \\ 14 \end{pmatrix} \text{ has}$$

$$= \frac{2|0|}{|2|} - \frac{3|}{|5|} + \frac{3|30|}{|5|} + \frac{3|30|}{|5|} = \frac{3(-2) - 1(|5-1|) + 3(6-0)}{|5-1|} + \frac{3|30|}{|5-1|} = \frac{3|5|}{|5-1|} + \frac{3|5|}{|5-1|} + \frac{3|5|}{|5-1|} + \frac{3|5|}{|5-1|} = \frac{3|5|}{|5-1|} + \frac{3|5$$

$$= -4 - 14 + 18$$

$$= \det A = 0$$

$$\int (C) = 2$$

$$n = 3$$

$$2R_1-R_3\rightarrow R_1$$

$$R_2-R_1\rightarrow R_2$$





$$A = \begin{bmatrix} 1 & 2 & 2 \\ 5 & 1 & 3 \end{bmatrix} \quad B = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$$x + 2y + 2z = b_1$$

$$5x + y + 3z = b_2$$

Which of the following is true about its solutions

 $C = \begin{bmatrix} 1 & 2 & 2 & b_1 \\ 5 & 1 & 3 & b_2 \end{bmatrix} \times 5^{-1} R_2 - 5R_1$ 

The system has a unique solution for any given  $b_1$  and  $b_2$ 

 $\frac{-9-7}{P(A)=3}$ 

- **B** Whether or not a solution exists depends on the given  $b_1$  and  $b_2$ .
- The system will have infinitely many solutions for any given  $b_1$  and  $b_2$
- The system would have no solution for any values of b<sub>1</sub> and b<sub>2</sub>

Infinte solution green b, b,





#Q. Consider the system of linear equations:

$$x - 2y + 3z = -1$$
$$x - 3y + 4z = 1 \text{ and}$$

$$-2x + 4y - 6z = k,$$

$$R_3 \rightarrow R_3 + 2R_1$$

$$R_3 \rightarrow R_3 + 2R_1$$

The value of 'k' for which the system has infinitely many solutions is

$$S(A)=2$$

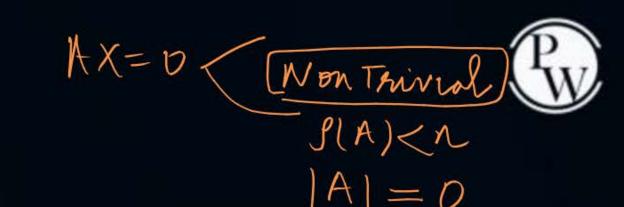
$$S(e)=2$$

$$N=3$$
Monte

\*\*solution

$$\frac{1}{1} = 0$$





#Q. If the following system has non - trivial solution

$$px + qy + rz = 0$$

$$qx + ry + pz = 0$$

$$rx + py + qz = 0$$

$$p = 2$$

$$r = 0$$

$$p = 2$$

$$r = 0$$

$$p =$$

$$p-q+r=0 \text{ or } p=q=-$$

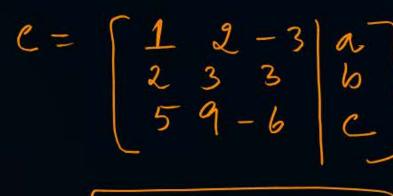
$$p + q - r = 0 \text{ or } p = -q = r$$

$$p + q + r = 0 \text{ or } p = q = r$$

$$p-q+i$$

$$p - q + r = 0$$
 or  $p = -q = -r$ 





#Q. Consider the following linear system

$$x + 2y - 3z = a$$
$$2x + 3y + 3z = b$$

2x + 3y + 3z = b 5x + 9y - 6z = c Always give a solu

Vingne Infinite

This system is consistent if a, b and c satisfy the equation

$$7a - b - c = 0$$

$$3a+b-c=0$$

$$3a - b + c = 0$$

$$7a - b + c = 0$$



#Q. Consider the following system of equations

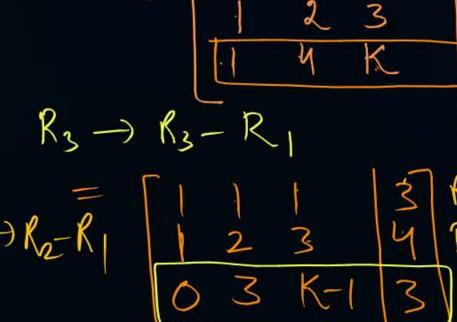
Twinte 
$$x+y+z=3$$

$$x+2y+3z=4$$

$$x+4y+kz=6$$

will not have a unique solution for k equal





$$R_{3} + R_{3} - 3R_{2} = 0.1 - 1.3$$

$$R_{3} + R_{3} - 3R_{2} = 0.3 - 1.3$$

Slide



#Q.

#### **Topic: Vector space**

The system of equations



$$R_{3} \rightarrow R_{5} - R_{2}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

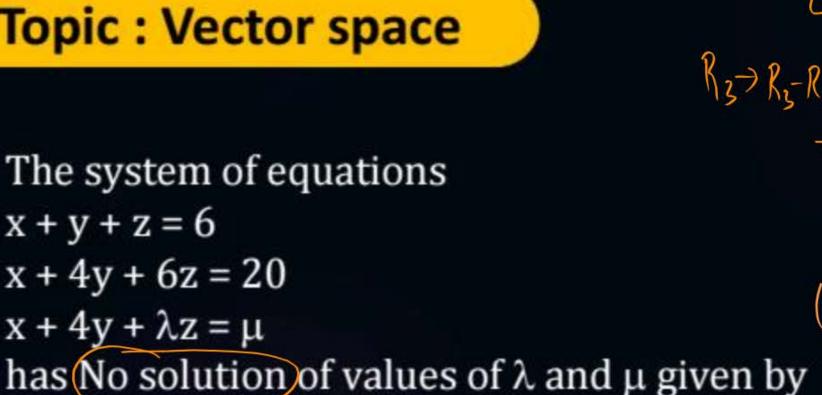
$$= \begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & 1 & 1 & 6 \\ 1 & 1 & 1 & 6 \end{bmatrix}$$

P(A) + SC) No so her

$$(A) = 6 M = 20 P(A) = 2 M = 3$$
  
 $f(C) = 2 Infinite$ 

B) 
$$\lambda = 6 \mu \neq 20$$
  $f(A) = 2 f(A) \neq f(C)$   
 $\lambda = 6, \mu \neq 20$   $f(C) = 3$  Nosol<sup>n</sup>



B) 
$$\lambda = 6 \mu \neq 20$$
  $f(A) = 2 f(A) \neq f(C)$   
 $\lambda = 6, \mu \neq 20$   $f(C) = 3$  NOSOLY

$$\lambda = 6$$
,  $\mu \neq 20$ 

$$\lambda \neq 6 \mu = 20$$

 $\lambda = 6$  ,  $\mu = 20$ 

x + y + z = 6

x + 4y + 6z = 20

 $x + 4y + \lambda z = \mu$ 

$$\lambda \neq 6$$
,  $\mu \neq 20$ 



## THANK - YOU