Computer Science & DA

Probability and Statistics

Probability

Lecture No. 02

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Recap of previous lecture

Topic Probability-Basic definition

Topics to be Covered

Topic Types of Events & Various theorem(Fundamental Question)

(1) if A & B are ME then $A \cap B = \phi$
- $P(A \cap B) = 0$
- $P(A \cup B) = P(A) + P(B) - 0$

(2) if A & B are Ind then
- $P(A \cap B) = P(A) \cdot P(B)$
- $P(A \cup B) = P(A) + P(B) - P(A) \cdot P(B)$

Concept of one by one with Replacement & W/O Replacement ⟶

eg (3) cards are drawn from a well shuffled pack of (52) cards then find the Total number of ways in which cards are drawn;

(1) At Random = $^{52}C_3$

(2) one by one <u>with</u> Replacement = $^{52}C_1 \times {}^{52}C_1 \times {}^{52}C_1$

(3) one by one (w/o) " = $^{52}C_1 \times {}^{51}C_1 \times {}^{50}C_1$

Qe SEVEN Surgical Strikes occurred in a week from INDIA on PAKISTAN then find the chance that all will occur on a same day?

Prob

Sol → Appli

$$\frac{\text{Total ways of occurring 5 Strikes (No Restriction) /R.A}}{} = \frac{7}{S_1}\times\frac{7}{S_2}\times\frac{7}{S_3}\times\frac{7}{S_4}\times\frac{7}{S_5}\times\frac{7}{S_6}\times\frac{7}{S_7} = 7^7 \text{ ways}$$

fav ways of occurring 5 Strikes = ?

$$\frac{M}{S_1}\ \frac{M}{S_2}\ \frac{M}{S_3}\ \frac{M}{S_4}\ \frac{M}{S_5}\ \frac{M}{S_6}\ \frac{M}{S_7}$$

or T T T T T T T

or W W W W W W W

⋮

or S S S S S S S

} 7 ways

So Req Prob $= \frac{^7C_1}{7^7} = \frac{1}{7^6}$ ways.

Qe If the letters of the word PROBABILITY are arranged in all possible ways then find the chance that two B's are together & two I's are also together?

Sol:→ P, R, O, A, L, I, T, B B, I, I

1 2 3 4 5 6 7 8 9 10 11

Total arrangements = $\frac{11!}{2!\,2!}$

P R O A L I T (B B) (I I)

1 2 3 4 5 6 7 B 9

fav arrangements = $9! \times \frac{2!}{2!} \times \frac{2!}{2!}$

So Req Prob = $\frac{f}{T} = \frac{9!}{\frac{11!}{2!2!}} = \frac{9! \times 2! \times 2!}{11!} = \frac{4}{11 \times 10}$

Fundamental Question → A coin is tossed 6 times then write it's S.Space.

$S = \{(HHHHHH), (HHHHHT), (HHHHTT), (HHHTTT), (HHTTTT), (HTTTTT), (TTTTTT)\}$

$^6C_6 = 1 \quad ^6C_5 = 6 \quad ^6C_4 = 15 \quad ^6C_3 = 20 \quad ^6C_2 = 15 \quad ^6C_1 = 6 \quad ^6C_0 = 1$

$n(S) = {}^2C_1 \times {}^2C_2 \times {}^2C_3 \times {}^2C_4 \times {}^2C_5 \times {}^2C_6 = 2^6 = 64$, Ordered 6 tuples And All the tosses are Ind

(1) find the Prob that all the outcomes are identical = ?

App I

fav outcomes $= \{(HHHHHH), (TTTTTT)\}$

$= 2$

So Req Prob $= \frac{f}{T} = \frac{2}{2^6} = \frac{2}{64} = \frac{1}{32}$

App II

Req Prob $= P[(HHHHHH) \text{ or } (TTTTTT)]$ (ME)

$= \left(\frac{1}{2}\right)^6 + \left(\frac{1}{2}\right)^6$

$= \left(\frac{1}{2}\right)^5 = \frac{1}{32}$

(2) find the prob that H and Tail appears alternately ?

App I) fav Cases = {(HTHTHT), (THTHTH)} ⇒ 2 ⇒ Prob = $\frac{f}{T} = \frac{2}{64} = \frac{1}{32}$

App III) Req Prob = P[(HTHTHT) or (THTHTH)] = $\left(\frac{1}{2}\right)^6 + \left(\frac{1}{2}\right)^6 = \frac{1}{32}$ (ME)

(3) find the Prob that Both H and T appears at least once ?

App I) unfav. outcomes = {(HHHHHH), (TTTTTT)} ⇒ 2 ⇒ fav outcomes = 64 − 2 = 62

Hence Req Prob = $\frac{f}{T} = \frac{62}{64}$

(4) find the Prob that Head appears at least once ?

App I) unfav = {(TTTTTT)} = 1 ⇒ fav = 63

⇒ Req Prob = $\frac{f}{T} = \frac{63}{64}$

App III) P(at least one H) = 1 − P(No Head)
= 1 − P(all T) = 1 − P(TTTTTT)
= $1 - \left(\frac{1}{2}\right)^6$

(5) App III if 1st three outcomes are all Heads (given) then Find the Prob of occurring T when Coin is tossed again?

Sol: App III Req Prob = P(occurring T in 4th toss) = $\frac{1}{2}$

Explanation: Req Prob = P[(HHH) T Something occurs] = $1^3 \times \frac{1}{2} \times 1^2 = \frac{1}{2}$ (HHH given)

(6) App III if 1st three outcomes are H, H, H (given) then Find the Prob of occurring T in remaining tosses.

Req Prob = P[(HHH) TTT] = $1^3 \times \left(\frac{1}{2}\right)^3 = \frac{1}{8}$ (HHH given)

(7) App II Find the Prob that Both H and T appears equal number of times.

Sol: P(getting equal number of H & T) = P(getting exactly 3H) = $\frac{f}{T} = \frac{20}{64} = \frac{{}^6C_3}{2^6}$

fav Cases = {eg (HTHHTT), ...} = $\frac{6!}{3!\,3!} = {}^6C_3 = 20$

App III → using Binomial Distribution → will be discussed Later.

(8) find the prob of getting exactly 2H = ? $\Rightarrow \frac{f}{T} = \frac{{}^6C_2}{2^6} = \frac{15}{64}$

(9) find the Prob that 1st two tosses produces H = ?

App III

Req Prob = P[HH something occurs] $= \left(\frac{1}{2}\right)^2 \times 1^4$

App II → fav. Cases = {eg (H H T/H T/H T/H T/H) ....}

$= 1 \times 1 \times 2 \times 2 \times 2 \times 2 = 16$

$\therefore$ Req Prob $= \frac{f}{T} = \frac{16}{64} = \frac{1}{4}$

(10) find the Prob that only 1st two tosses produces H ?

App I fav Cases = {(H H T T T T)} = 1.

Req Prob $= \frac{f}{T} = \frac{1}{64}$

App II Req Prob =

= P(only 1st tosses are H) = P[H H T T T T]

$= \left(\frac{1}{2}\right)^2 \cdot \left(\frac{1}{2}\right)^4 = \left(\frac{1}{2}\right)^6$

(11) if 1st two outcomes are H, H then find the prob of occurring T in Remaining tosses given.

Sol: Req Prob = $P[HHTTTT] = 1^2 \times \left(\frac{1}{2}\right)^4 = \frac{1}{16}$

(10) Find the Prob that only 1st two tosses produces H = ?

Req Prob = $P[HHTTTT] = \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^4 = \left(\frac{1}{2}\right)^6 = \frac{1}{64}$

Qs A Coin is tossed (10 times) then find the prob that ① Exactly 3H occurs = ? $= \frac{^{10}C_3}{2^{10}} = \frac{120}{1024}$

② 4^{th} Head occurs in 9^{th} toss = ?

$S = \{(HHHHHHHHHH), (HHHHHHHHHT), \ldots\ldots, (TTTTTTTTTT)\} \Rightarrow n(S) = 2^{10} = 1024$

App II

Fav Cases = {eg (exactly 3H ---------- (H) nT ... }

1st 8 tosses, 9^{th}, 10^{th}

$= {}^8C_3 \times 1 \times 2 = 112$

& Req Prob $= \frac{f}{T} = \frac{112}{1024} = \frac{^8C_3 \times 1 \times 2}{2^{10}} = \left(\frac{^8C_3}{2^8}\right) \times \left(\frac{1}{2}\right) \times \left(\frac{2}{2}\right)$

App III

Req Prob = P(4^{th} H in 9^{th} toss)

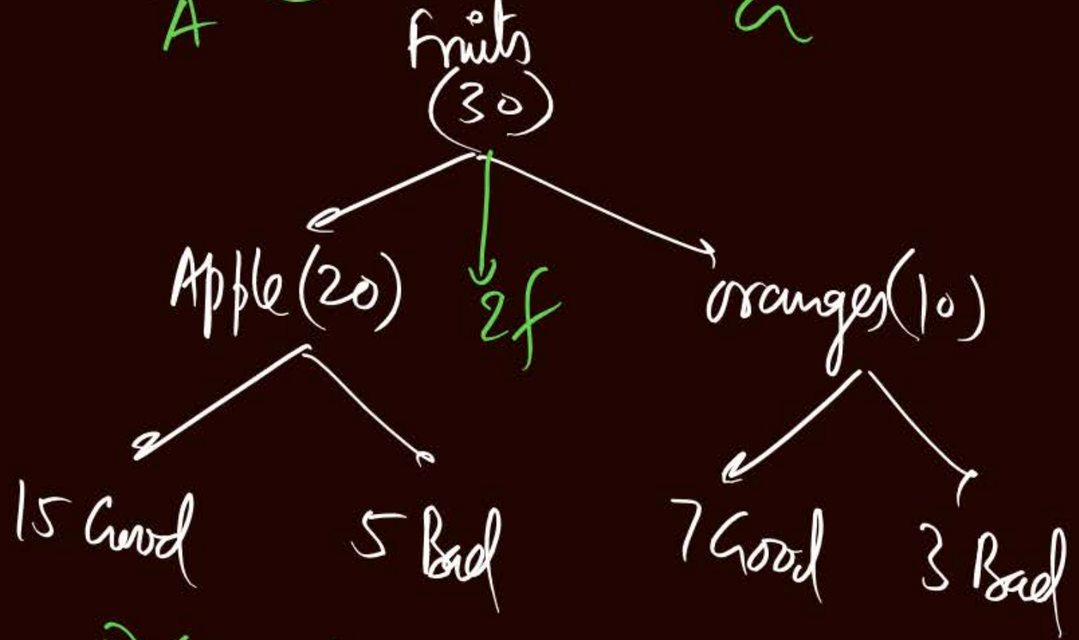
= P[exactly 3H in 1st 8 tosses] × P[H in 9^{th} toss]

× P(Something occurs in 10^{th} toss)

$= \left(\frac{^8C_3}{2^8}\right) \times \frac{1}{2} \times 1$

Q. A Basket Contains 20 Appl and 10 oranges in which 5 Appl and 3 oranges are Rotten if two fruits are choosen at Random then find the prob that either Both are apples (or) both are Good.

A G



$P(A \cup G) = P(A) + P(G) - P(A \cap G)$

$= \text{Ans.}$

Total ways of Selecting 2 f $= {}^{30}C_2$

$A = \{\text{Both fruits are Apples}\} \Rightarrow P(A) = \frac{{}^{20}C_2}{{}^{30}C_2}$

$G = \{\text{Both fruits are Good}\} \Rightarrow P(G) = \frac{{}^{22}C_2}{{}^{30}C_2}$

$A \cap G = \{\text{Both fruits are good Apples}\} \Rightarrow \frac{{}^{15}C_2}{{}^{30}C_2}$

Q: A Box contains 2 Nuts, 3 washers, and 4 Bolts. Items are drawn (selection) one by one w/o replacement then find the prob of drawing two Nuts 1st, followed by 3 Washers and finally 4 Bolts?

Sol: Req Prob $= \left(\frac{{}^{2}C_1}{{}^{9}C_1} \times \frac{{}^{1}C_1}{{}^{8}C_1}\right) \times \left(\frac{{}^{3}C_1}{{}^{7}C_1} \times \frac{{}^{2}C_1}{{}^{6}C_1} \times \frac{{}^{1}C_1}{{}^{5}C_1}\right) \times \left(\frac{{}^{4}C_1}{{}^{4}C_1} \times \frac{{}^{3}C_1}{{}^{3}C_1} \times \frac{{}^{2}C_1}{{}^{2}C_1} \times \frac{{}^{1}C_1}{{}^{1}C_1}\right)$

(Nuts) (Washers) (Bolts)

$= \left(\frac{2}{9} \times \frac{1}{8}\right) \times \left(\frac{3}{7} \times \frac{2}{6} \times \frac{1}{5}\right) \times 1$

THANK - YOU