GATE-All BRANCHES
Engineering Mathematics

Linear Algebra

Lecture No.- 07

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Topics covered in previous lecture

Topic Properties of matrices

Topic Question based on properties of the matrix

Topic Question based on properties of the matrix, Orthogonal basis

Topic Linear dependence and independence, Rank space and null space

Topic System of equations

Topic Question based on system of equations,

Topics to be Covered

Topic Question based on properties of the matrix, Orthogonal basis

Topic Linear dependence and independence, Rank space and null space

Topic System of equations

Topic Question based on system of equations, Span of vectors

System of Equations $\boxed{AX = 0}$ Homogeneous solution

Dot Product $= 0$

$$\left\{\begin{aligned} a_{11}x + a_{12}y + a_{13}z &= 0 \\ a_{21}x + a_{22}y + a_{23}z &= 0 \\ a_{31}x + a_{32}y + a_{33}z &= 0 \end{aligned}\right]$$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{3\times 3}$$

Always gives a solution

System of Eqn $AX = 0$ (always consistent)

Trivial soln

Infinite solution Non Trivial

$\boxed{\text{If } |A| \neq 0,\ \rho(A) = n \ \text{(unique solution)}}$

$\left.\begin{aligned} x &= 0 \\ y &= 0 \\ z &= 0 \end{aligned}\right]$ (Trivial solution)

$\boxed{\text{If } |A| = 0\ \ \rho(A) < n \ \text{(Infinite solution)}}$

Topic: Vector space

$$C = \left[\begin{array}{ccc|c} -1 & 5 & 0 & -1 \\ 1 & -1 & 0 & 2 \\ 1 & 3 & 0 & 3 \end{array}\right]$$

#Q. How many solutions does the following system of linear equations have

$$\left.\begin{aligned} -x + 5y &= -1 \\ x - y &= 2 \\ x + 3y &= 3 \end{aligned}\right]$$

$\underline{z = 0}$

$\underline{3 \text{ equations}}$

$\left.\begin{aligned} \rho(A) &= 2 \\ \rho(C) &= 2 \\ n &= 3 \end{aligned}\right]$ Infinite solutions

Using elimination

A. Infinitely many ✓

B. Two distinct solutions

C. Unique

D. None

Slide

Topic: Vector space

#Q. Consider the following system of equations in three real variable x_1, x_2 and x_3:

$2x_1 - x_2 + 3x_3 = 1, 3x_1 + 2x_2 + 5x_3 = 2, -x_1 + 4x_2 + x_3 = 3$

This system of equations has

Using elimination

$$C = \left[\begin{array}{ccc|c} 2 & -1 & 3 & 1 \\ 3 & 2 & 5 & 2 \\ -1 & 4 & 1 & 3 \end{array}\right]$$

A. No solution

B. A unique solution

C. More than one but a finite number of solutions

D. An infinite number of solutions

$$\left\{\begin{array}{l} \rho(A) = 3 \\ \rho(C) = 3 \\ n = 3 \end{array}\right\}$$ Unique solution

Slide

Topic: Vector space

Read

$r(A) = 2$, $r(C) = 2$, $n = 3$ — Infinite

$\alpha = 2$, $\beta = 7$

$AX = B$ (Non Homogenous)

$C =$ augmented matrix

#Q. For what values of α and β the following simultaneous equations have an $\boxed{\text{infinite number of solutions}}$

CE

$x + y + z = 5,$

$x + 3y + 3z = 9,$

$x + 2y + \alpha z = \beta$

α, β Infinite sol^n

$\boxed{\rho(A) = \rho(C) < n}$

$$C = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 1 & 3 & 3 & 9 \\ 1 & 2 & \alpha & \beta \end{array}\right] \begin{array}{l} R_1 \\ R_2 \\ R_3 \end{array} \rightarrow 0\text{'s}$$

using elimination

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 0 & 2 & 2 & 4 \\ 0 & 1 & \alpha-1 & \beta-5 \end{array}\right]$$

$2 \times$

$R_3 \to R_3 - R_1$

A. 2, 7

B. 3, 8

C. 8, 3

D. 7, 2

$R_3 \to R_3 - R_1$

$R_2 \to R_2 - R_1$

$R_3 \to 2R_3 - R_2$

$$= \left[\begin{array}{cccc} 1 & 1 & 1 & 5 \\ 0 & 2 & 2 & 4 \\ 0 & 0 & 2\alpha-4 & 2\beta-14 \end{array}\right]$$

$\alpha = 2$ $\quad \beta = 7$

$2\alpha - 4 = 0$

$\alpha = 2$

$2\beta - 14 = 0$

$\beta = 7$

Slide

Topic: Vector space

H.W

#Q. For the set of equations

$$x_1 + 2x_2 + x_3 + 4x_4 = 2,$$
$$3x_1 + 6x_2 + 3x_3 + 12x_4 = 6.$$

The following statement is true

A. Only the trivial solution $x_1 = x_2 = x_3 = x_4 = 0$ exist

B. There are no solutions

C. A unique non-trivial solution exits

D. Multiple non-trivial solution exist

Slide

Topic: Vector space

#Q. The system of linear equations $\begin{pmatrix} 2 & 1 & 3 \\ 3 & 0 & 1 \\ 1 & 2 & 5 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 5 \\ -4 \\ 14 \end{pmatrix}$ has

A. A unique solution

B. Infinite many solutions

C. No solutions

D. Exactly two solutions

$$C = \left[\begin{array}{ccc|c} 2 & 1 & 3 & 5 \\ 3 & 0 & 1 & -4 \\ 1 & 2 & 5 & 14 \end{array}\right]$$

$$\det A = 2\begin{vmatrix} 0 & 1 \\ 2 & 5 \end{vmatrix} - 1\begin{vmatrix} 3 & 1 \\ 1 & 5 \end{vmatrix} + 3\begin{vmatrix} 3 & 0 \\ 1 & 2 \end{vmatrix}$$

$$= 2(-2) - 1(15-1) + 3(6-0)$$

$$= -4 - 14 + 18$$

$$= \det A = 0$$

$\rho(A) = 2$

$\rho(C) = 2$

$n = 3$

Infinite solutions

$2R_1 - R_3 \to R_1$

$R_2 - R_1 \to R_2$

Slide

Topic: Vector space

#Q. Given a system of equations

$$x + 2y + 2z = b_1$$
$$5x + y + 3z = b_2$$

Which of the following is true about its solutions

A) The system has a unique solution for any given b_1 and b_2

B) Whether or not a solution exists depends on the given b_1 and b_2.

C) The system will have infinitely many solutions for any given b_1 and b_2

D) The system would have no solution for any values of b_1 and b_2

$A = \begin{bmatrix} 1 & 2 & 2 \\ 5 & 1 & 3 \end{bmatrix}$ $B = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$

$C = \begin{bmatrix} 1 & 2 & 2 & b_1 \\ 5 & 1 & 3 & b_2 \end{bmatrix} \times 5$ $R_2 - 5R_1$

$= \begin{bmatrix} 1 & 2 & 2 & b_1 \\ 0 & -9 & -7 & b_2 - 5b_1 \end{bmatrix}$

$\rho(A) = 2$
$\rho(C) = 2$
$n = 3$

Infinite solution given b_1 b_2

Slide

Topic: Vector space

#Q. Consider the system of linear equations:

$x - 2y + 3z = -1$

$x - 3y + 4z = 1$ and

$-2x + 4y - 6z = k,$

The value of 'k' for which the system has infinitely many solutions is ____.

$$C = \left[\begin{array}{ccc|c} 1 & -2 & 3 & -1 \\ 1 & -3 & 4 & 1 \\ -2 & 4 & -6 & K \end{array}\right]$$

$R_3 \to R_3 + 2R_1$

$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & -1 \\ 1 & -3 & 4 & 1 \\ 0 & 0 & 0 & k-2 \end{array}\right]$$

$\rho(A) = 2$

$\rho(C) = 2$

$n = 3$

infinite solution

$K - 2 = 0$

$\boxed{K = 2}$

$K = 2$

Slide

Topic : Vector space

$AX = 0$ — Non Trivial

$\rho(A) < n$

$|A| = 0$

$(p-q)^2+(q-r)^2+(r-p)^2 \Rightarrow p=q=r$

#Q. If the following system has non – trivial solution

$px + qy + rz = 0$

$qx + ry + pz = 0$

$rx + py + qz = 0$

$$\begin{vmatrix} p & q & r \\ q & r & p \\ r & p & q \end{vmatrix} = 0$$

$C_1 \quad C_2 \quad C_3$

$$\begin{vmatrix} p+q+r & q & r \\ p+q+r & r & p \\ p+q+r & p & q \end{vmatrix} = 0$$

Then which one of the following options is True

$$(p+q+r)\begin{vmatrix} 1 & q & r \\ 1 & r & p \\ 1 & p & q \end{vmatrix} = 0$$

$p+q+r=0$

$p=q=r$

A $p - q + r = 0$ or $p = q = -r$

B $p + q - r = 0$ or $p = -q = r$

C $p + q + r = 0$ or $p = q = r$ ✓

D $p - q + r = 0$ or $p = -q = -r$

Slide

$$C = \left[\begin{array}{ccc|c} 1 & 2 & -3 & a \\ 2 & 3 & 3 & b \\ 5 & 9 & -6 & c \end{array}\right]$$

$R_2 \to R_2 - 2R_1$
$R_3 \to R_3 - 5R_1$
$R_3 \to R_3 - R_2$

$\boxed{3a + b - c = 0}$

#Q. Consider the following linear system

$$x + 2y - 3z = a$$
$$2x + 3y + 3z = b$$
$$5x + 9y - 6z = c$$

Always give a sol^n

This system is consistent if a, b and c satisfy the equation

unique Infinite

A. $7a - b - c = 0$

B. $\boxed{3a + b - c = 0}$

C. $3a - b + c = 0$

D. $7a - b + c = 0$

Slide

Topic : Vector space

No. of Non ZERO Rows = rank

#Q. Consider the following system of equations

Infinite soln

$$x + y + z = 3$$
$$x + 2y + 3z = 4$$
$$x + 4y + kz = 6$$

will not have a unique solution for k equal

$$\begin{bmatrix} 1 & 1 & 1 & 5 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\rho(A) = 2$

$\rho(C) = 2$

$n = 3$

$k = 7$

Infinite soln

A. 0

B. 5

C. 6

D. 7

$$C = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 1 & 2 & 3 & 4 \\ 1 & 4 & k & 6 \end{array}\right] \begin{array}{l} R_1 \\ R_2 \\ R_3 \end{array}$$

$R_3 \to R_3 - R_1$

$R_2 \to R_2 - R_1$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 1 & 2 & 3 & 4 \\ 0 & 3 & k-1 & 3 \end{array}\right] \begin{array}{l} R_1 \\ R_2 \\ R_3 \end{array}$$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & 1 & 2 & 1 \\ 0 & 3 & k-1 & 3 \end{array}\right] \; 3\times$$

$R_3 \to R_3 - 3R_2$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & k-7 & 0 \end{array}\right]$$

$k - 7 = 0$

$k = 7$

Slide

Topic : Vector space

#Q. The system of equations

$x + y + z = 6$

$x + 4y + 6z = 20$

$x + 4y + \lambda z = \mu$

has No solution of values of λ and μ given by

A $\lambda = 6, \mu = 20$

B $\lambda = 6, \mu \neq 20$

C $\lambda \neq 6\ \mu = 20$

D $\lambda \neq 6, \mu \neq 20$

$$C = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 1 & 4 & 6 & 20 \\ 1 & 4 & \lambda & \mu \end{array}\right]$$

$R_3 \to R_3 - R_2$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 1 & 4 & 6 & 20 \\ 0 & 0 & \lambda-6 & \mu-20 \end{array}\right]$$

$\rho(A) \neq \rho(C)$

No solution

(A) $\lambda = 6\ \mu = 20$ $\rho(A) = 2$ $n = 3$

$\rho(C) = 2$ Infinite

B) $\lambda = 6\ \mu \neq 20$ $\rho(A) = 2$ $\rho(A) \neq \rho(C)$

$\rho(C) = 3$ No sol^n

(C) $\lambda \neq 6\ \mu = 20$ $\left.\begin{array}{l}\rho(A) = 3 \\ \rho(C) = 3\end{array}\right]$ no of variables $= 3$ Unique sol^n

$r(A) = r(C) = n$

THANK - YOU