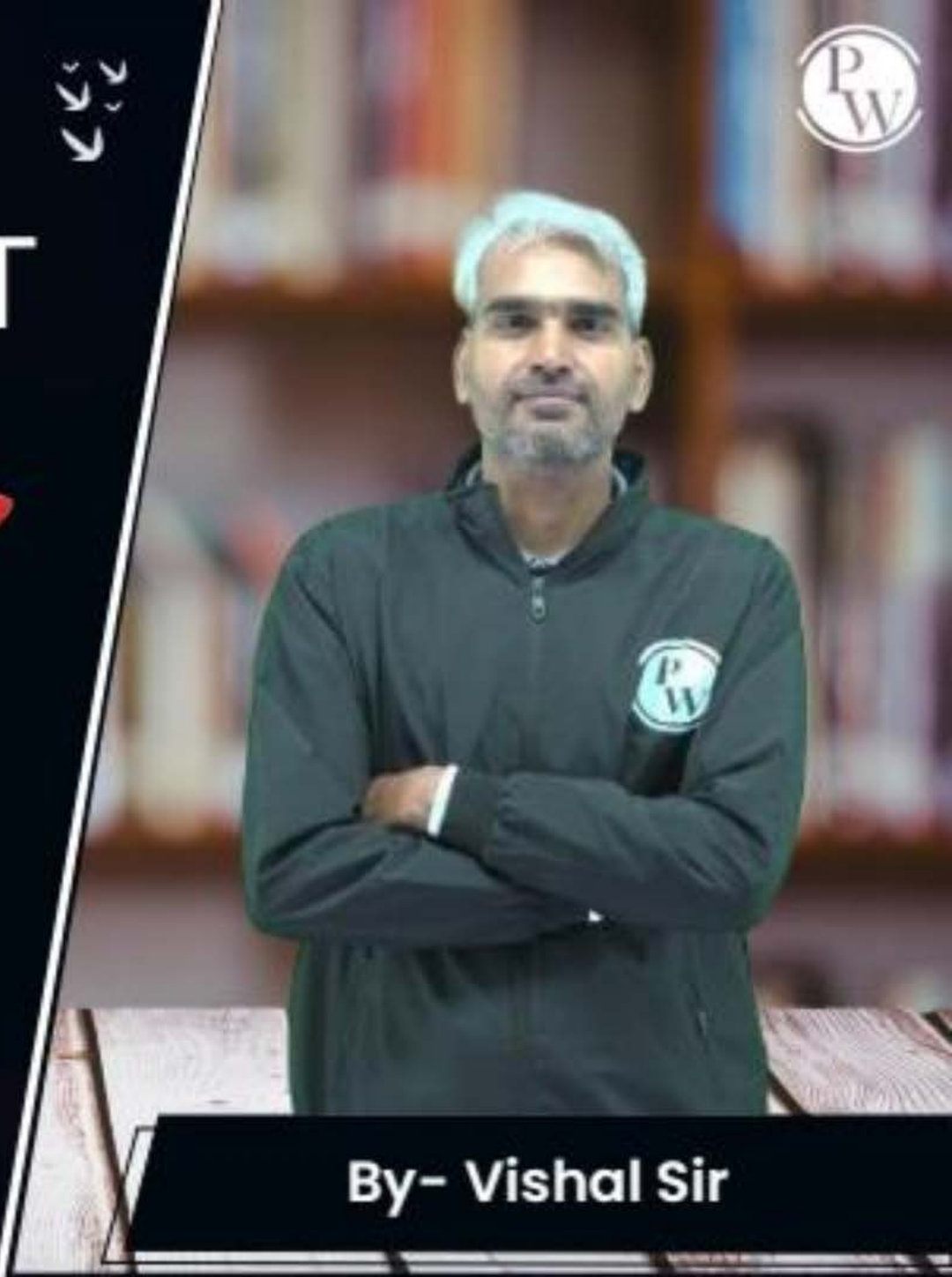
Computer Science & IT

Discrete Mathematics

Set Theory & Algebra

Lecture No. 04



By- Vishal Sir

Recap of Previous Lecture

- Topic: Power Set
- Topic: Venn Diagram
- Topic: Set Operations
- Topic: Properties of set operations

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Topics to be Covered

Topic Principle of Inclusion and Exclusion

Topic Multi-set

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Topic : Properties of Set Operations

8. Absorption

8. a. $A \cup (A \cap B) = A$

8. b. $A \cap (A \cup B) = A$

All elements of A will be present + Some extra elements

9. De Morgan's

a. $(A \cup B)^c = A^c \cap B^c$

b. $(A \cap B)^c = A^c \cup B^c$

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Topic : Properties of Set Operations

10. Distributive

a. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

b. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Topic : Principle of Inclusion & Exclusion

- Let A and B are two sets derived from universal set U, then

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$|A \cup B| = |A| + |B| - |A \cap B|$$

$$|A^c \cap B^c| = |U| - |A \cup B|$$

$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$A = \{2, 3, 5, 7\}$

$B = \{2, 4, 7, 8, 9\}$

$(2, 3, 4, 5, 7, 8, 9)$

$|A| = 4, \quad |B| = 5$

$|A \cap B| = 2$

$|A \cup B| = |A| + |B| - |A \cap B|$

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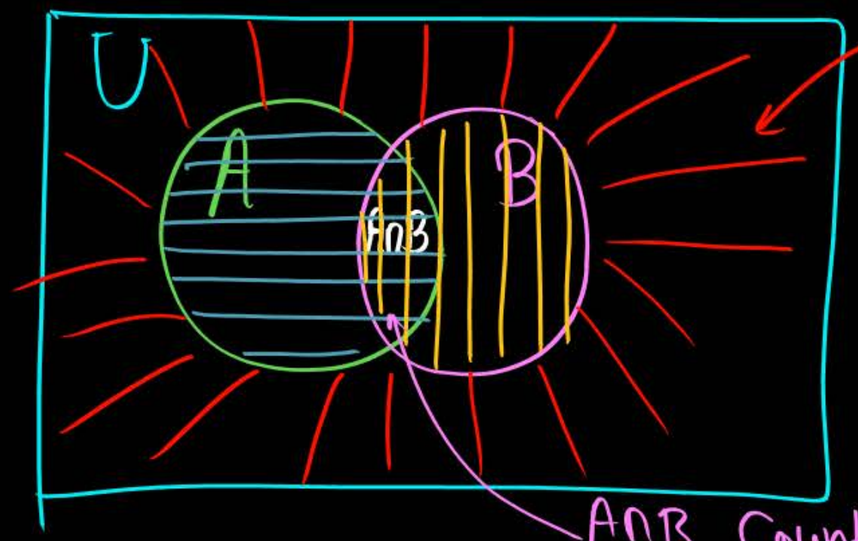


$A^c \cap B^c$

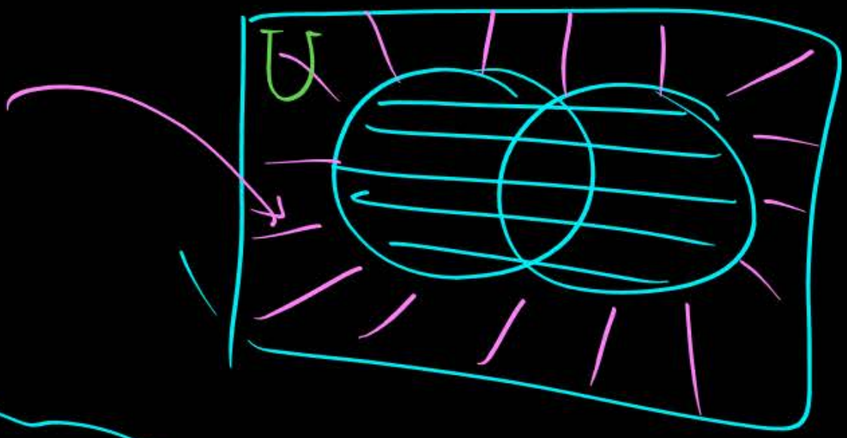
$|A^c \cap B^c| = |U| - |A \cup B|$

A∩B Counted twice {ie Subtract once}

$|A \cup B| = |A| + |B| - |A \cap B|$

How many elements are there which are neither in A nor in B

$|U| - |A \cup B|$



elements which are not in $(A \cup B)$

not in $= (A \cup B)^c$

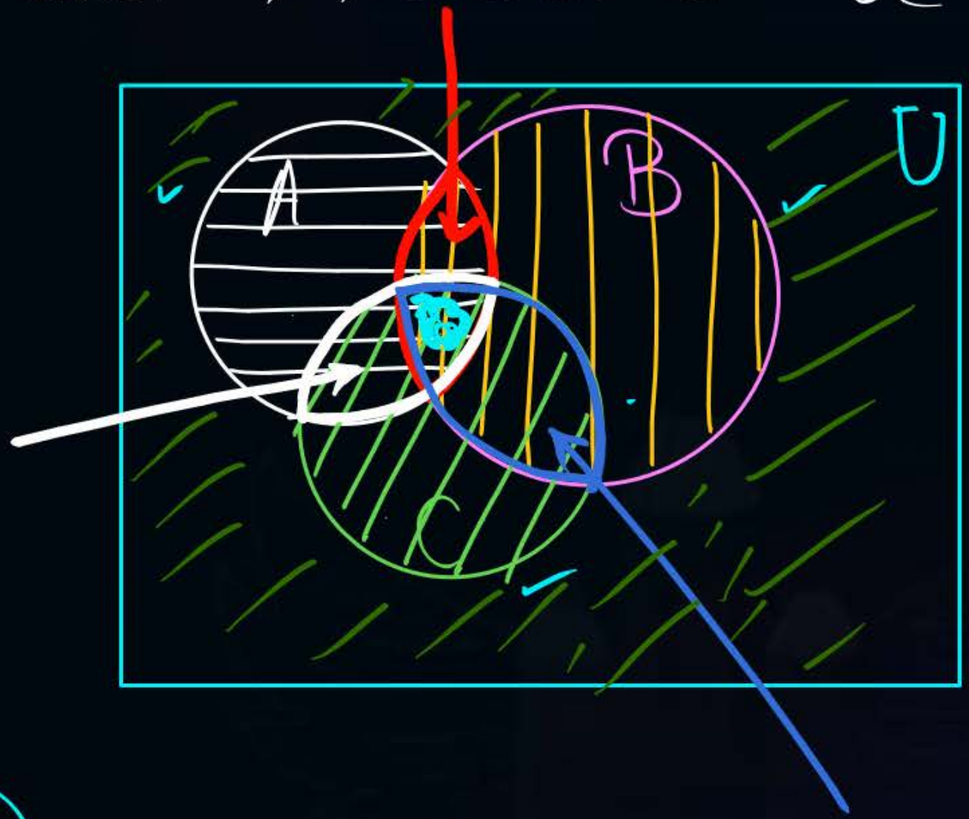
not in A and not in B

$A^c \cap B^c$

Topic : Principle of Inclusion & Exclusion

* let U is the universal set and A, B and C be some Sets derived from U

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$



$$|A^c \cap B^c \cap C^c| = |U| - |A \cup B \cup C|$$

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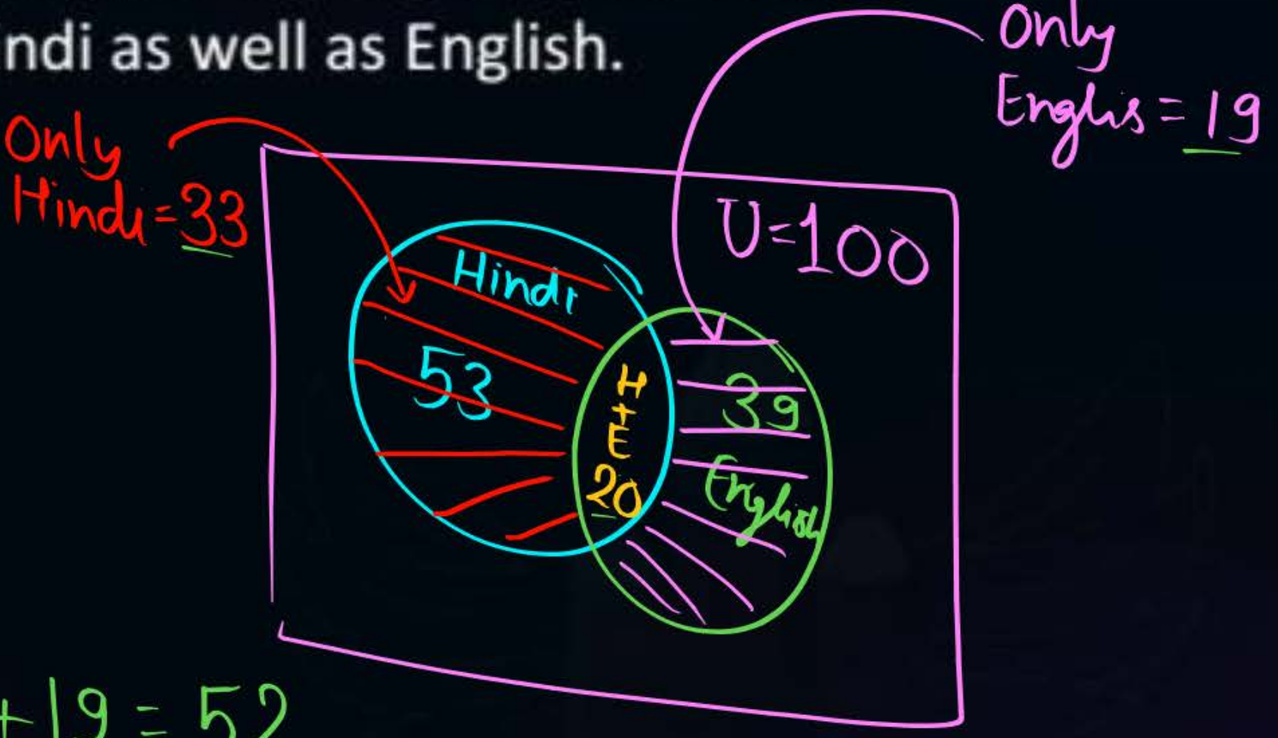
Topic : Principle of Inclusion & Exclusion

In a class of 100 students 53 students can speak Hindi , 39 students can speak English, 20 students can speak Hindi as well as English.
How many students can speak

a) At least one out of two languages.
b) Only one of the two languages.
c) None of the two languages.



(a) $|H \cup E| = |H| + |E| - |H \cap E| = 72$

(b) Only Hindi + Only English $= 33 + 19 = 52$

(c) $|H^c \cap E^c| = |U| - |H \cup E| = 100 - 72 = 28$

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$$n(A \cup B \cup C \cup D) = n(A) + n(B) + n(C) + n(D)$$
$$- n(A \cap B) - n(A \cap C) - n(A \cap D) - n(B \cap C) - n(B \cap D) - n(C \cap D)$$
$$+ n(A \cap B \cap C) + n(A \cap B \cap D) + n(A \cap C \cap D) + n(B \cap C \cap D)$$
$$- n(A \cap B \cap C \cap D)$$

Topic : Principle of Inclusion & Exclusion

In a class of 100 students 40 students failed in Maths , 30 students failed in Physics, 25 students failed in Chemistry, 20 students failed in Maths and Physics, 15 students failed in Physics and Chemistry, 10 students failed in Maths and Chemistry, and 5 students failed in all three subjects.

How many students

a) Failed in at least one of the subjects. = 55

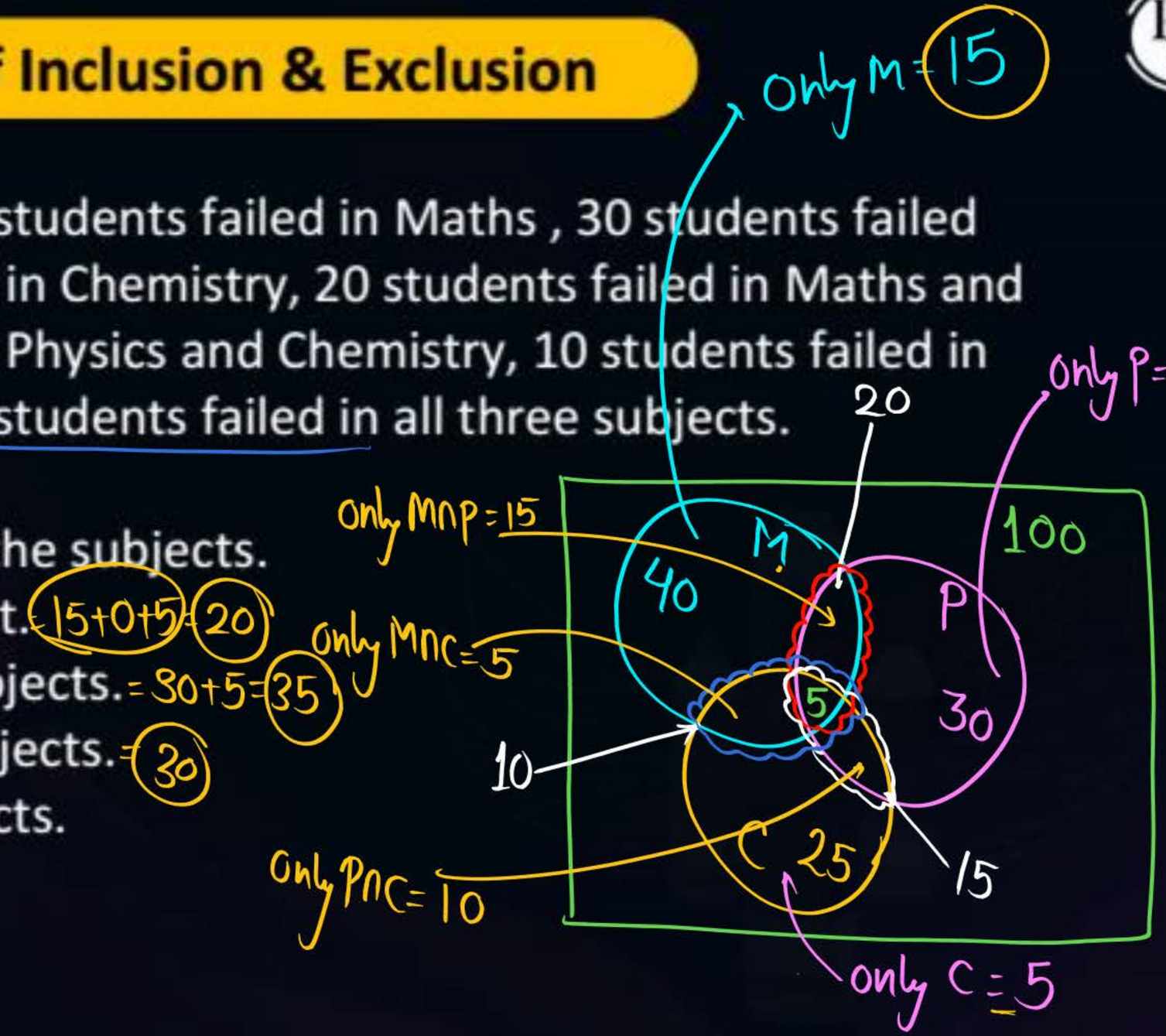
b) Failed in only one subject. = 15+0+5 = 20

c) Failed in at least two subjects. = 30+5 = 35

d) Failed in exactly two subjects. = 30

e) Passed in all three subjects.

100 − 55 = 45



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Topic : Multi-set

A well defined unordered Collection of elements in which element may appear more than once is called multiset (or mset).

eg $\{a, a, b, b, b, c, d, d\}$

$\equiv \{2.a, 3.b, 1.c, 2.d\}$

Multiplicities of respective elements.

Number of times an element appears in the multiset is called multiplicity of that element

Multiplicity of every element must be ≥ 0

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(or Size of a Multiset)

→ Cardinality of a multiset is defined as summation of the multiplicites of all the elements of that set

eg: let $A = \{a, a, b, b, b, c, d, d\}$

Cardinality of multiset $A = 2 + 3 + 1 + 2 = 8$

Topic : Multi-set

→ Let $P = \{m_1 \cdot a_1, m_2 \cdot a_2, m_3 \cdot a_3, \ldots\ldots\ldots m_k \cdot a_k\}$

P is a multiset where multiplicity of element a_i is m_i

and Let $Q = \{n_1 \cdot a_1, n_2 \cdot a_2, n_3 \cdot a_3, \ldots\ldots\ldots n_k \cdot a_k\}$

Q is also a multiset where multiplicity of element a_i is n_i

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Topic : Multi-set

H.W.

For above multi-sets P and Q

(i) What will be the multiplicity of a_i in P union Q

(ii) What will be the multiplicity of a_i in P intersection Q

(iii) What will be the multiplicity of a_i in $P - Q$

(difference)

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2 mins Summary

Topic Principle of Inclusion and Exclusion

Topic Multi-set

THANK - YOU