GATE

ALL BRANCHES

ENGINEERING MATHEMATICS

Probability and Statistics

Lecture No. 07

BY- RAHUL SIR

Random Variable:

01 Types of Random Variables

02 CDF (Cumulative Distribution)

03 PDF (Probability Density Function)

04 Statistical Averages (Mean, Median, Mode)

05 Expectation, Variance, Standard Deviation

Random Variable:

Random variable is a mathematical Function

x (Input) → $\boxed{X}$ → output

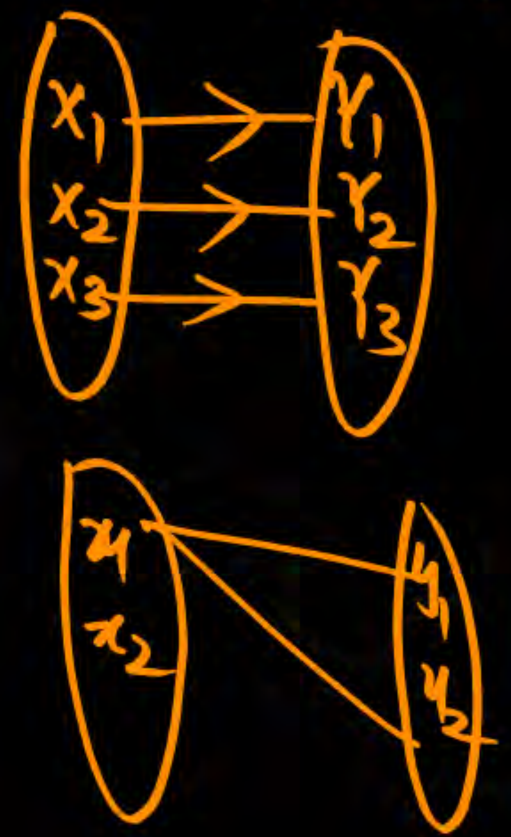
$f ; x \to y$

$x \in R$

$y \in R$

prob — Experiment, Event, sample space

Experiment, Event, sample space] Ω_x, E, S



Input {(SAMPLE Points, SAMPLE space)} → $\boxed{f}$ → output (prob)

$f ; X_S \to \frac{P}{y}$

Tossing A Two coin: Re 1, Re 2 $\underbrace{\text{Re 1, Re 2}}_{\text{simultaneously Draw}}$

→ Input (SAMPLE Points)

↓ output (Prob)

Re 1 — HEAD / Tail

Re 2 — H / T

	H	T
H	HH	HT
T	TH	TT

$S = \{HH, HT, TH, TT\}$

Input → Function → Prob

(HH, HT, TH, TT}

(Unique Value)

$S = \{\underbrace{HH, HT, TH, TT}\}$

We define The Random variable 'X'

\# variable — No of HEADS / No. of Tails

X = No. of HEADS / No. of tails

SAMPLE Point	HH	HT	TH	TT
X	2	1	1	0

If X = No. of HEADS

$X = 0, 1, 2$

Y = No. of Tails. $Y = 0, 1, 2$

SAMPLE Points (NO)

S	HH	HT	TH	TT
Y	0	1	1	2

$$\xrightarrow{\text{Input}} \boxed{\text{Function}} \xrightarrow{} \text{Unique value (Prob. of Event)}$$

X = No. of HEADS.
X = 0, 1, 2

$\{HH, HT, TH, TT\}$

P(0 HEAD) = $\frac{1}{4}$

P(1 HEAD) = $\frac{2}{4}$

P(2 H) = $\frac{1}{4}$

$P[X = x]$ = Prob. of An Event

$P[Y = y]$ = Random var.

Random variable.

$P[X=0] = \frac{1}{4}$

$P[X=1] = \frac{2}{4}$

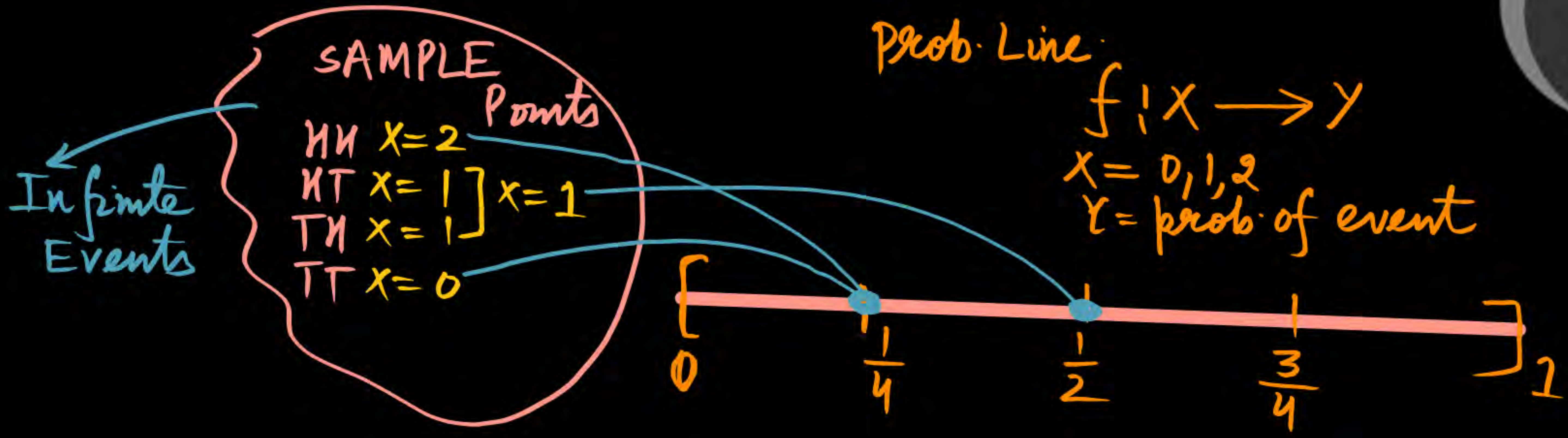
$P[X=2] = \frac{1}{4}$

$P[Y=0] = \frac{1}{4}$

$P[Y=1] = \frac{2}{4}$

$P[Y=2] = \frac{1}{4}$

X = 0, 1, 2
X = No. of HEADS.



Prob. Line:

$f: X \longrightarrow Y$

$X = 0, 1, 2$

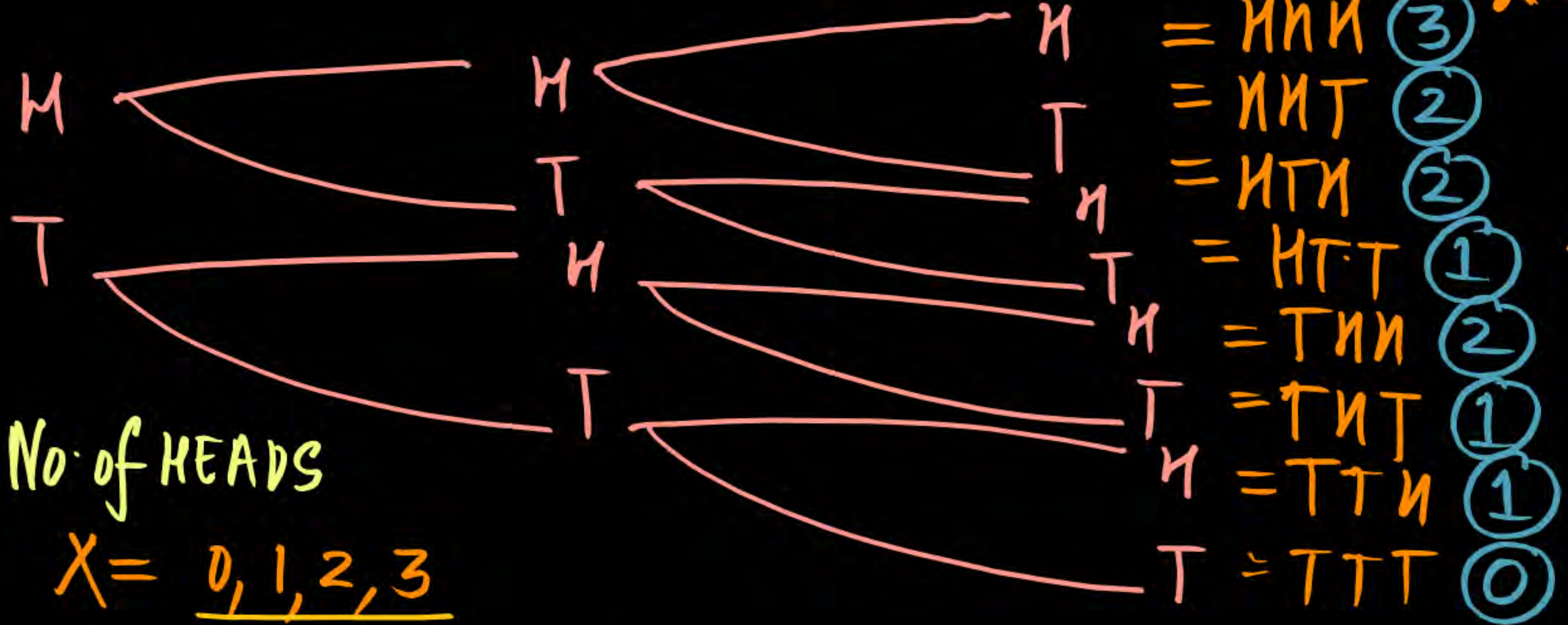
Y = prob. of event

Tossing A THREE coins

Re 1, Re 2, Re 3

X = No. of HEADS

X = 0, 1, 2, 3



X = No. of HEADS

X = 0, 1, 2, 3

SAMPLE	HHH	HHT	HTH	HTT	THH	THT	TTH	TTT
$P(X=x)$	$1/8$	$1/8$	$1/8$	$1/8$	$1/8$	$1/8$	$1/8$	$1/8$

$P(X = 0 \text{ HEAD}) = \frac{1}{8}$ $\quad P(X = 1 \text{ HEAD}) = \frac{3}{8}$ $\quad P(X = 2) = \frac{3}{8}$

$P(X = 3) = \frac{1}{8}$

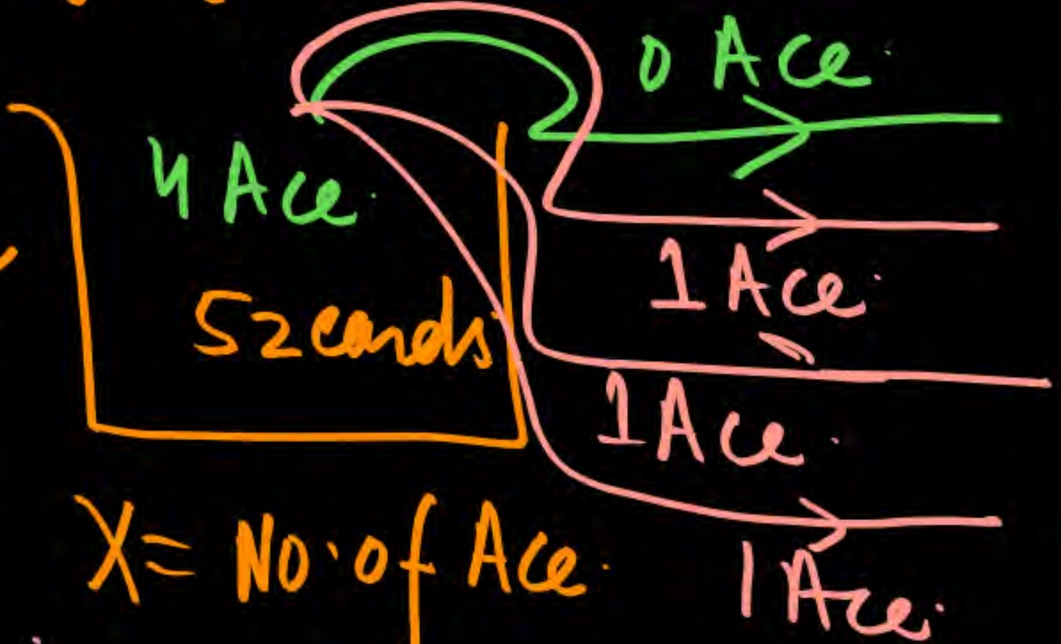
1) SUM of all prob = 1

$P[X=0H) + P(X=1H) + P(X=2H) + P(X=3H) = 1$

$$\boxed{\sum_{i=0}^{n} P[X=x_i] = 1}$$

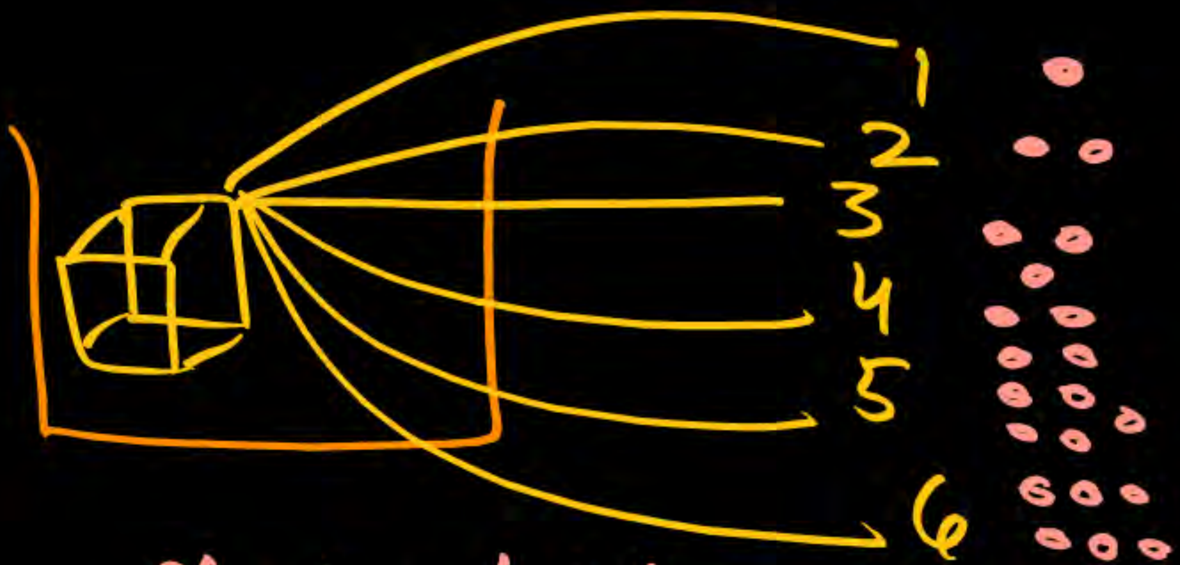
Total Prob = 1

Playing cards :



$X = 0, 1$

X = No. of Ace



X = No. of dots

X = 1, 2, 3, 4, 5, 6

4 Aces
52 cards

$X \begin{cases} 0 & \text{No Ace} \\ 1 & \text{Ace represent} \end{cases}$

Ace – club, spade, HEART, Diamond → $X = 0$

= 48 cards

X	0	1
$P(X=x)$	$\frac{48}{52}$	$\frac{4}{52}$

X = No Ace, 1 Ace

#

2 balls Are Random selected

X	0	1	2	3
P				

X = No. of green balls

0 green → 2 red → $\frac{2}{5} \times \frac{1}{4}$ ——

1 green → 1 red, 1 green
1 green, 1 red +

2 green → 2 green

Different Kind of Random Var:

Throwing A Die X = No. of dots

X = 1, 2, 3, 4, 5, 6

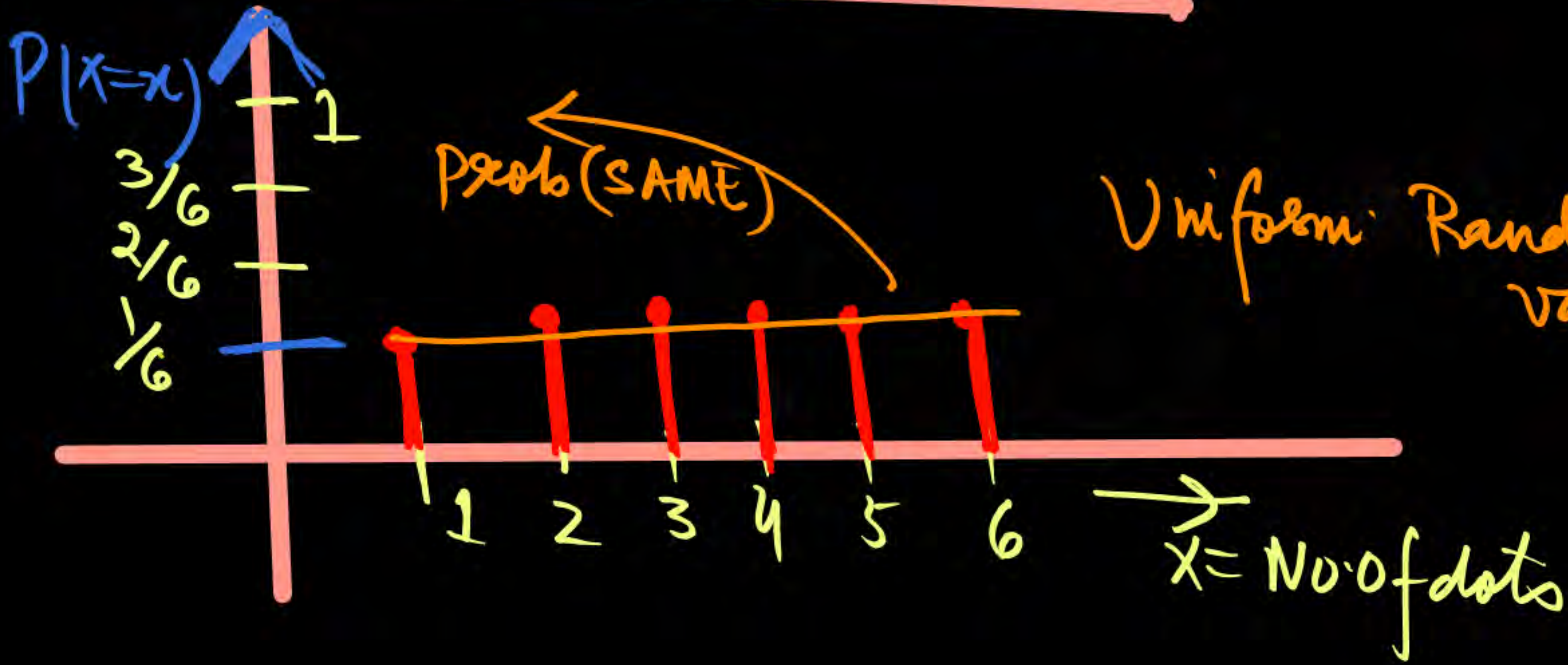
X value

y value

X	1	2	3	4	5	6
$P(X=x_i)$	1/6	1/6	1/6	1/6	1/6	1/6

$P[X=x]$ → Random variable

$\sum_{x=0}^{n} P[X=x] = 1$

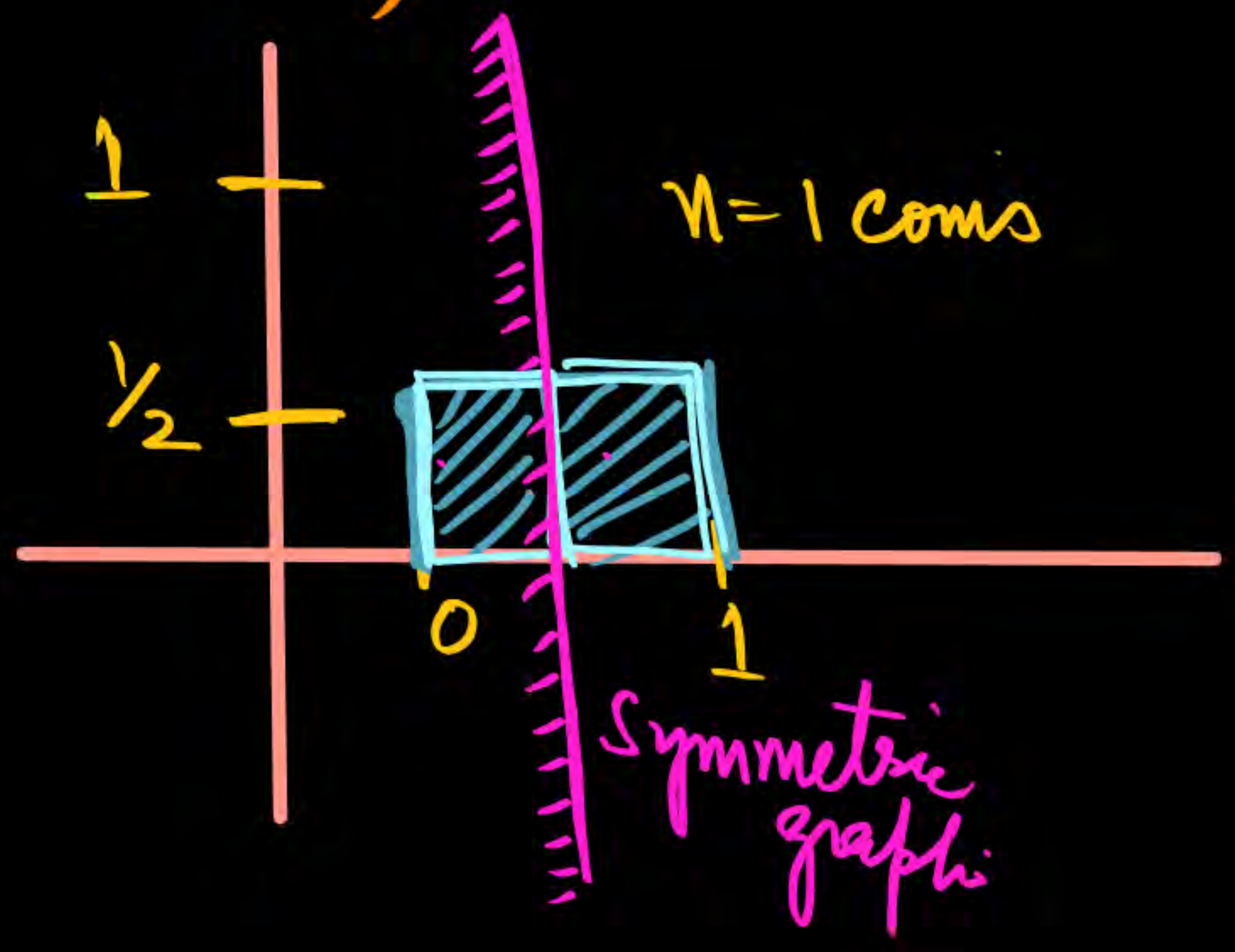


Uniform Random variable

Tossing A single coin

X	$H(1)$	$T(0)$
$P(X=x)$	$1/2$	$1/2$

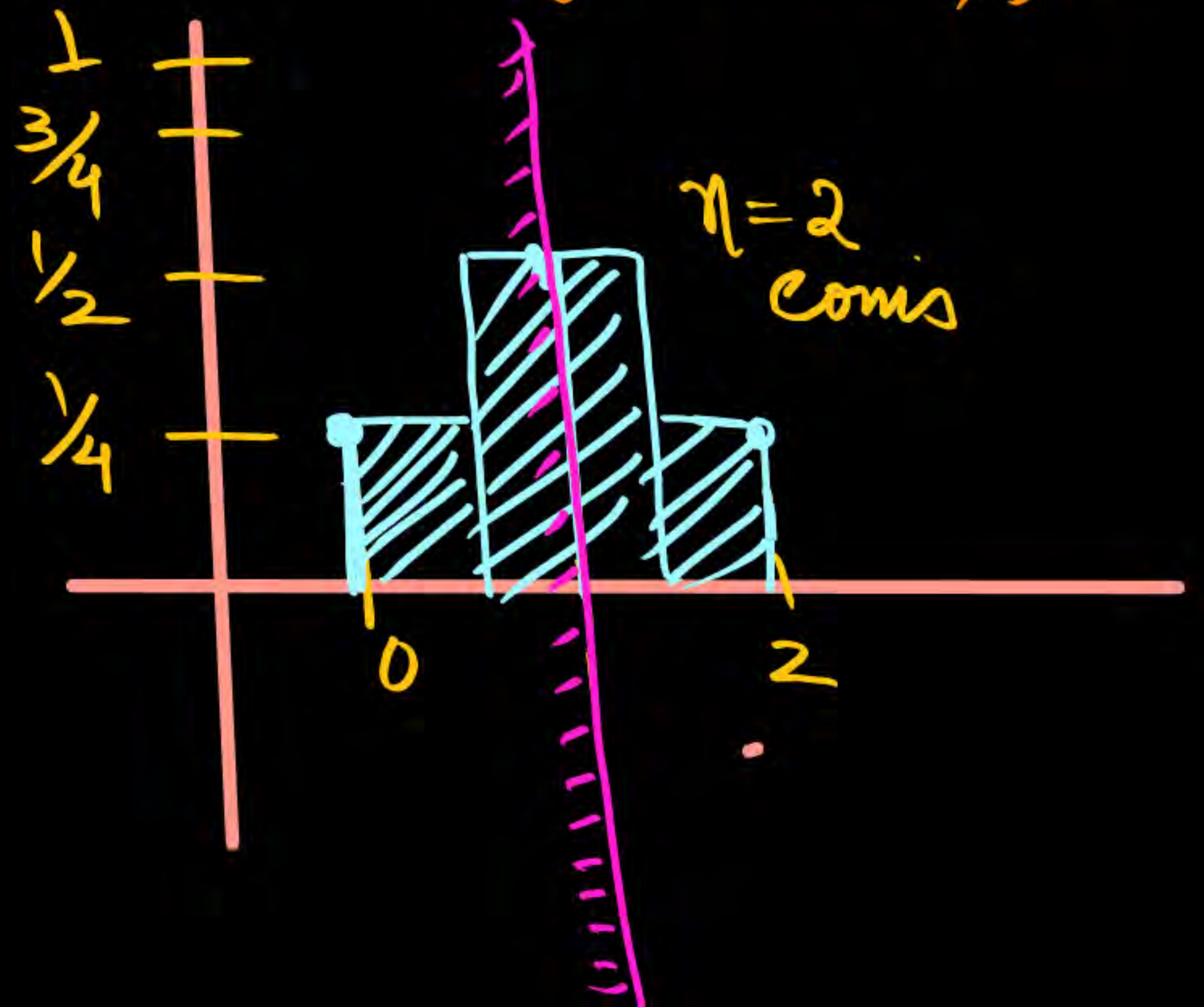
X = No. of HEADS
$X = 0, 1$



Tossing A Two coin

X	0	1	2	
$P(X=x_i)$	$1/4$	$2/4$	$1/4$	

X = No. of HEADS 0, 1, 2

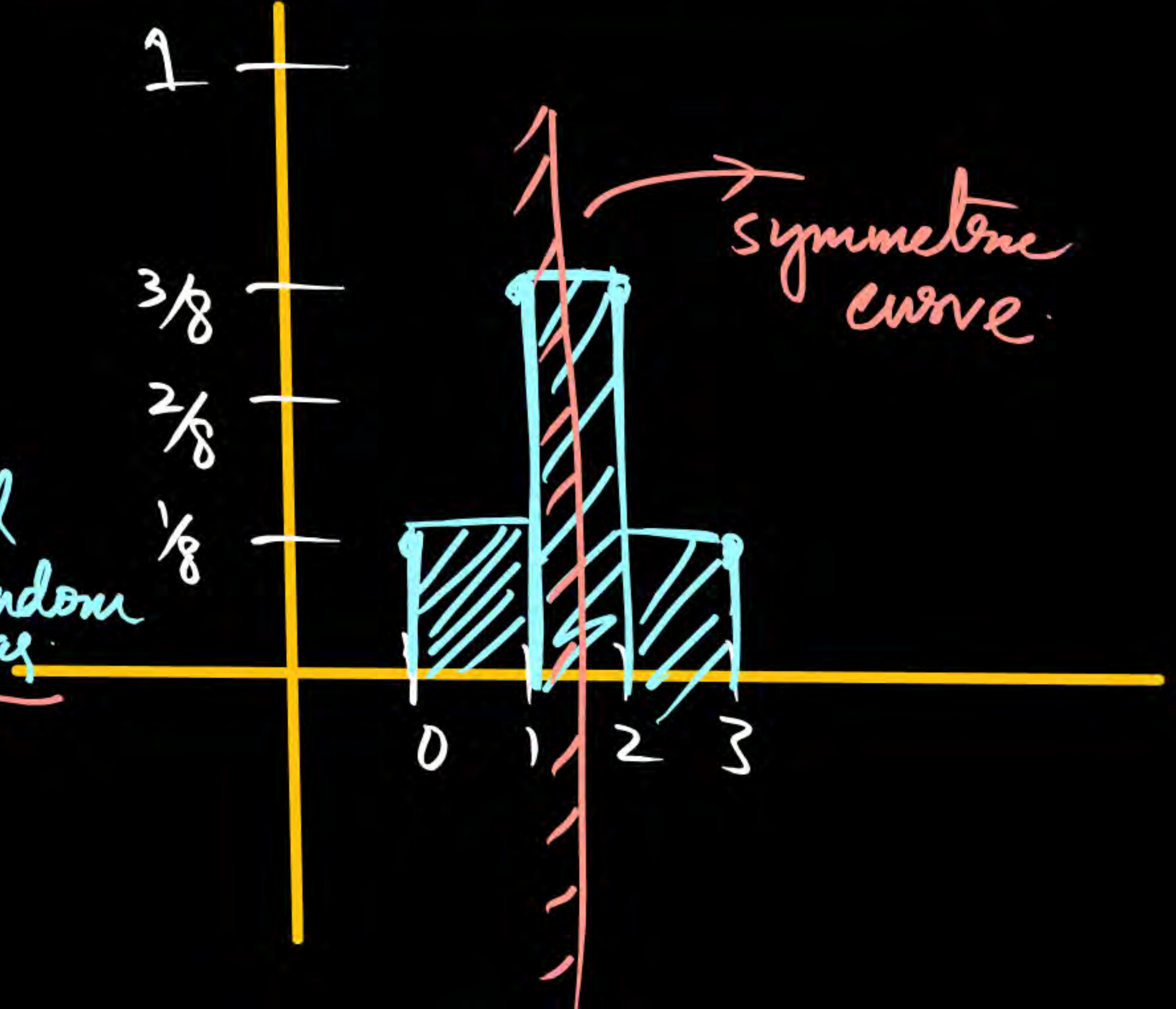


$X = 0, 1, 2, 3$

No. of HEADS.

$\underbrace{Re_1,\ Re_2,\ Re_3}_{3 \text{ coins}}$

x	0	1	2	3
$P(X=x_i)$	$1/8$	$3/8$	$3/8$	$1/8$.

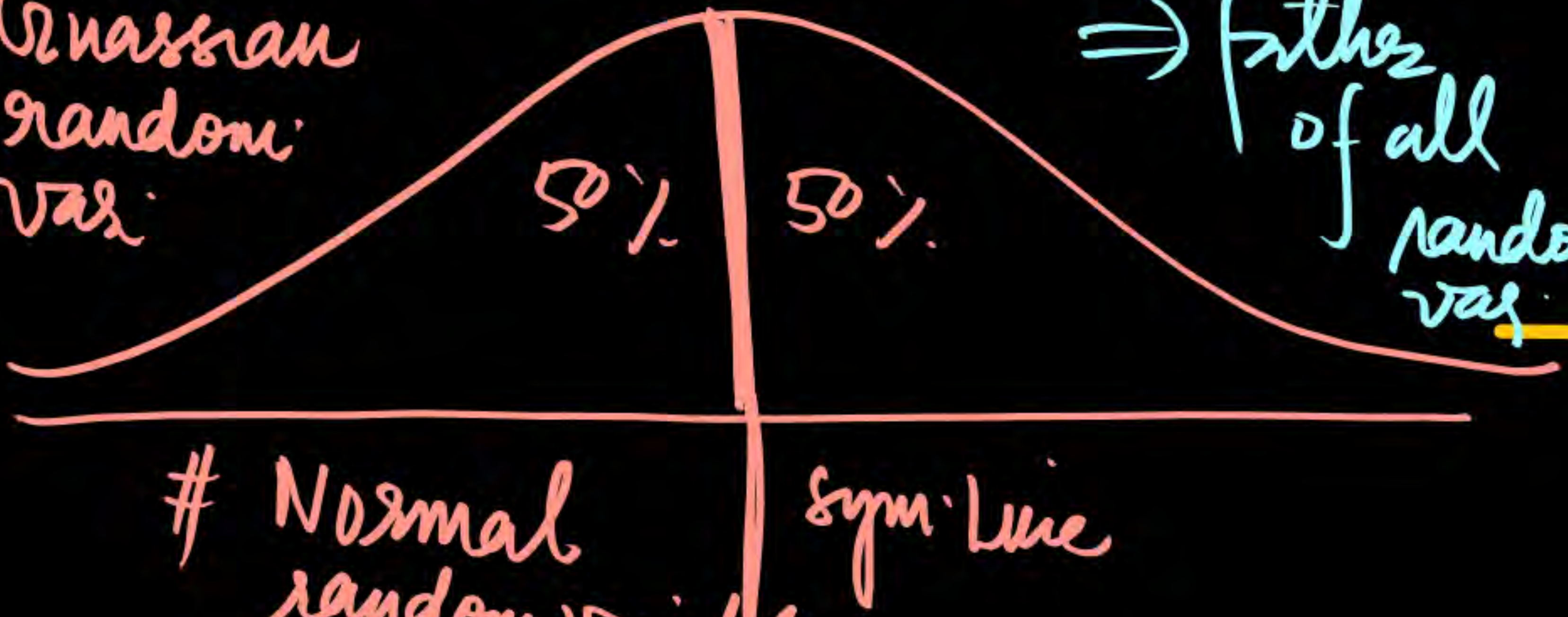


If n is large of trials

Guassian Distribution

Guassian random var.

⇒ father of all random var.



Normal random variable.

If any Random variable $\xrightarrow[\text{n is Large No. trials}]{}$ Central Limit THEOREM [Large No. of trials

↓

Guassian

- ✓ Binomial
- ✓ Poisson
- ✓ Normal
- ✓ St. Normal
- ✓ Geometric
- ✓ Uniform

$\xRightarrow[\text{n Large}]{}$ Guassian random var.

Types of Random Variable

Random variable

(Arrival Pattern) Discrete

Countable

(Infinite countable)

X = Discrete Random var.

X = Integer value

X = 0, 1, 2, 3, 4, 5 - - -

✓ Tossing A coin
✓ Throwing A Die
✓ Picking A ball
✓ Playing cards.

Time = continuous [Waiting Time]

0.1 + 0.11 + 0.111 + 0.1111 + - - -

continuous random

Infinite count

(MEASURE) → Infinite uncountable SET

⇒ always defined in Interval

$a \le x \le b$

✓ Height of a Person $1.2 \le x \le 1.3$
✓ Discharging A battery $1 \le x \le 2$
✓ Weight of Person

Discrete Random Var → Arrival Pattern

Continuous → Size / Waiting Time

Time

✓ Number Line

Infinite Pts

A B

$$\int_A^B \boxed{\text{Integrate}}$$

✓ Integration = Infinite

Tool

Uncountable

No (Add)

Cumulative distribution function : (cdf)

	I	I	I
	0	0	0

$$F_X(x_i) = P[X \le x_i] = \text{cdf}$$

X	x_1	x_2	x_3	x_i
$F_X(x)$	$F_X(x_1)$	$F_X(x_2)$	$F_X(x_3)$	$F_X(x_i)$

✓ $F_X(0) = P[X \le 0] = P_0$

✓ $F_X(1) = P[X \le 1] = P[X=0] + P[X=1]$
$= P_0 + P_1$

✓ $F_X(2) = P[X \le 2] = P[X=0] + P[X=1] + P[X=2] = P_0 + P_1 + P_2$

✓ $F_X(3) = P[X \le 3] = P_0 + P_1 + P_2 + P_3 \leftarrow P[X=0] + P[X=1] + P[X=2] + P[X=3]$

✓ $F_X(x_i) = P[X \le x_i] = P_0 + P_1 + P_2 + \cdots P_i$

CDF Throwing A Die : $F_X(x_i) = P[X \le x_i]$

$S = \{1, 2, 3, 4, 5, 6\}$

$F_X(1) = P[X \le 1] = P[X = 1] = \frac{1}{6}$

$F_X(2) = P[X \le 2] = P[X=1] + P[X=2]$

$= \frac{1}{6} + \frac{1}{6} = \frac{2}{6}$

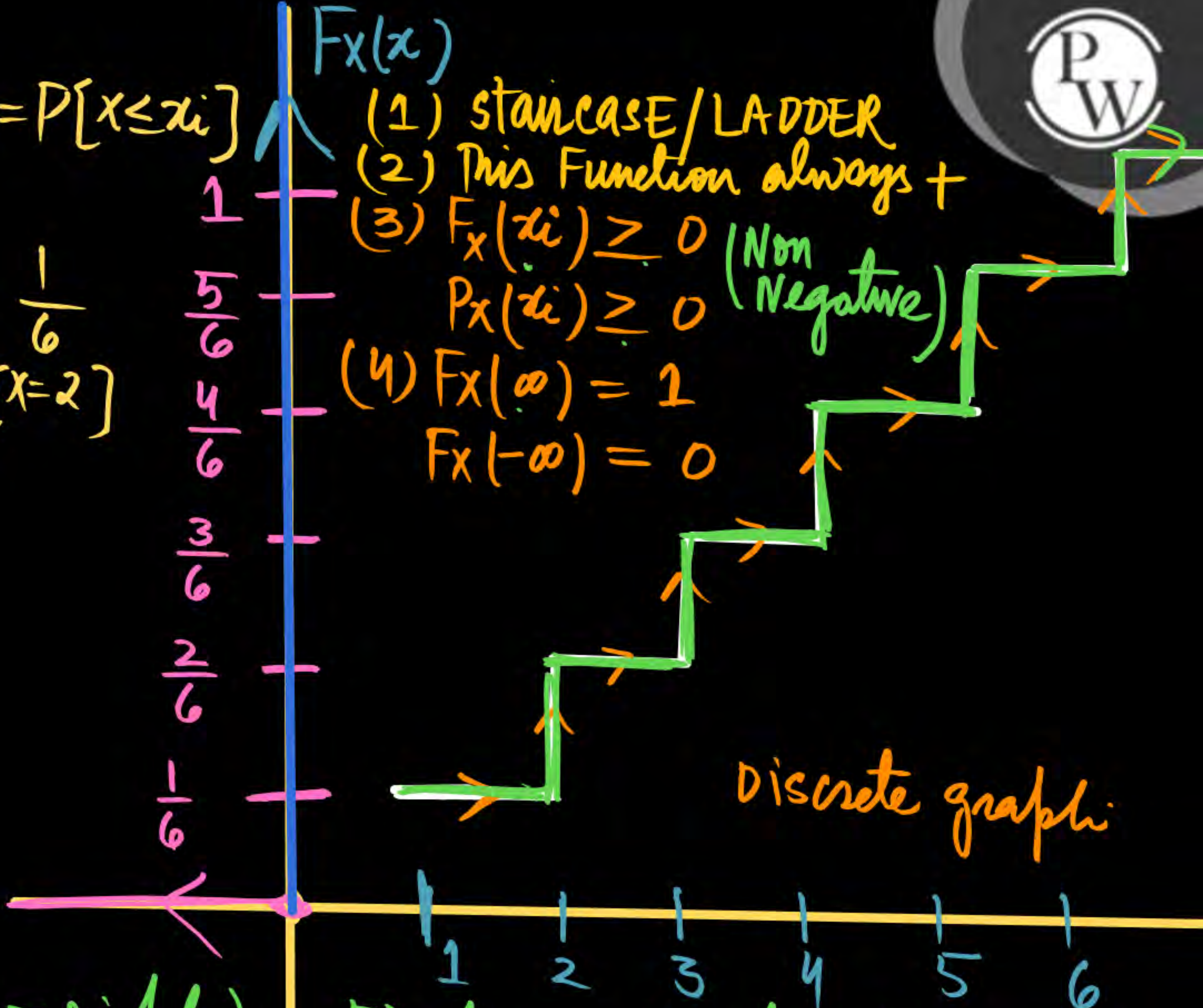
$F_X(3) = P[X \le 3] = \frac{3}{6}$

$F_X(4) = P[X \le 4] = \frac{4}{6}$

$F_X(5) = P[X \le 5] = \frac{5}{6}$

$F_X(6) = P[X \le 6] = 1$

(discrete random variable)



(1) staircase/LADDER

(2) This Function always +

(3) $F_X(x_i) \ge 0$ (Non Negative)

$P_X(x_i) \ge 0$

(4) $F_X(\infty) = 1$

$F_X(-\infty) = 0$

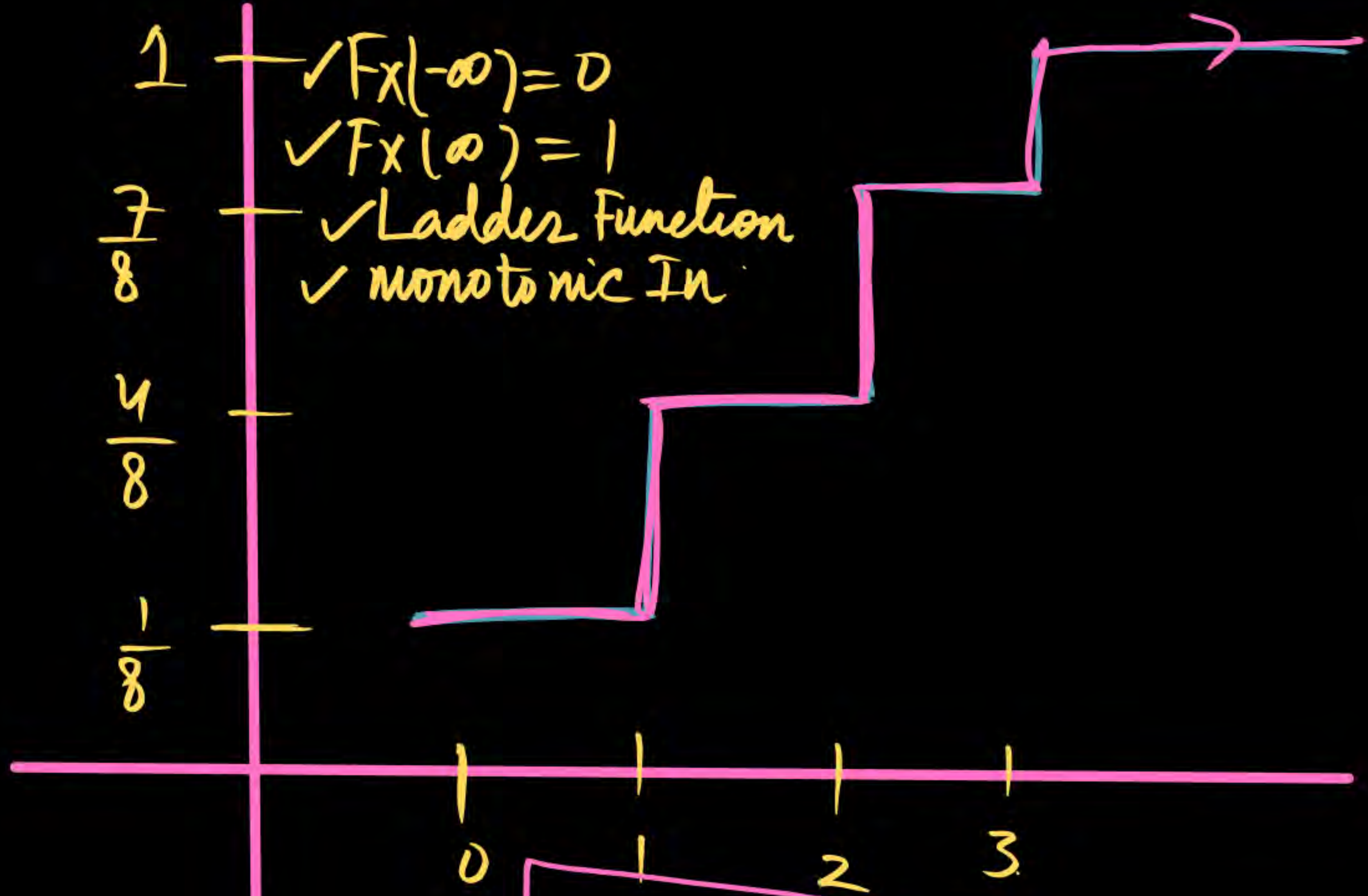
Discrete graph

5) always Monotonic Increasing.

2	
3	
4	
5	
6	
7	
8	
9	
10	

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Do yourself



cdf $\boxed{F_X(x_i)=P[X\leq x_i]}$

Tossing A Three coins

Re 1, Re 2, Re 3

X = No. of HEADS

$$X = 0,\ 1,\ 2,\ 3$$

$$P(X=x) = \frac{1}{8},\ \frac{3}{8},\ \frac{3}{8},\ \frac{1}{8}$$

$$F_X(0) = P(X\leq 0) = P_0 = \frac{1}{8}$$

$$F_X(1) = P[X\leq 1) = P_0+P_1 = \frac{1}{8}+\frac{3}{8} = \frac{4}{8}$$

$$F_X(2) = P[X\leq 2] = P_0+P_1+P_2 = \frac{7}{8}$$

$$F_X(3) = \frac{7}{8}+\frac{1}{8} = \frac{8}{8} = 1$$

Thank You!

GW Soldiers