GATE
ALL BRANCHES
ENGINEERING MATHEMATICS

Probability and Statistics

Lecture No. 06

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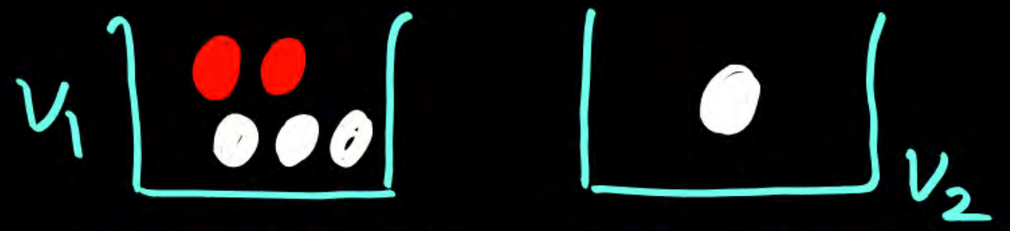
01 Conditional Probability and Bayes' Theorem

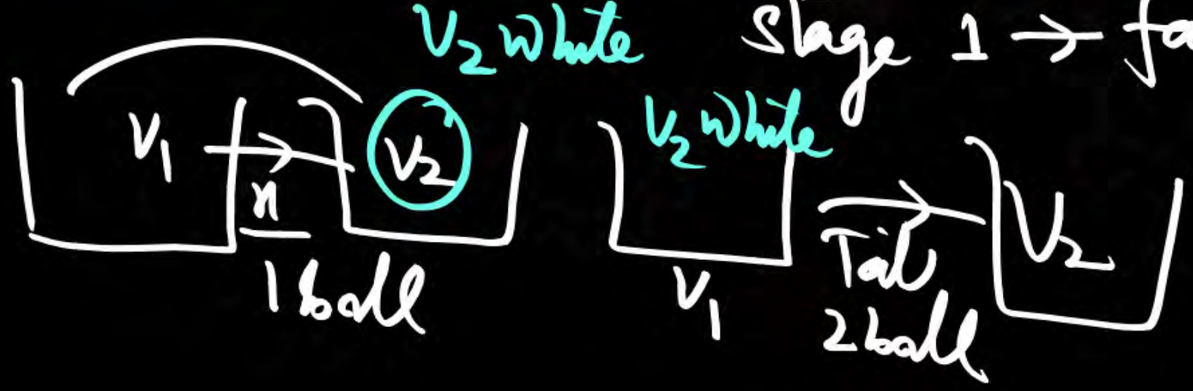
02 Questions Based on Conditional Probability and Bayes' Theorem

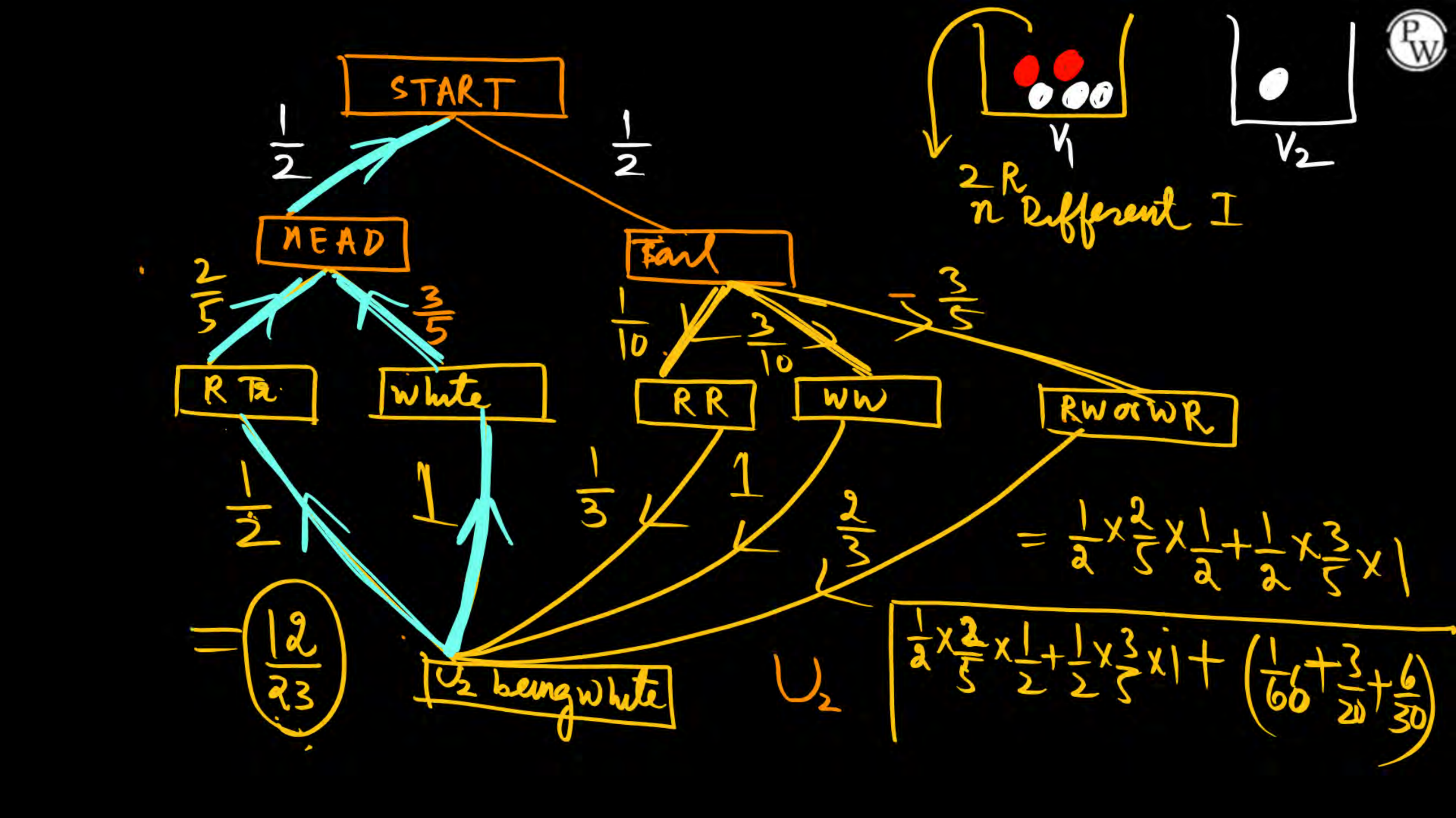
Q. Questions

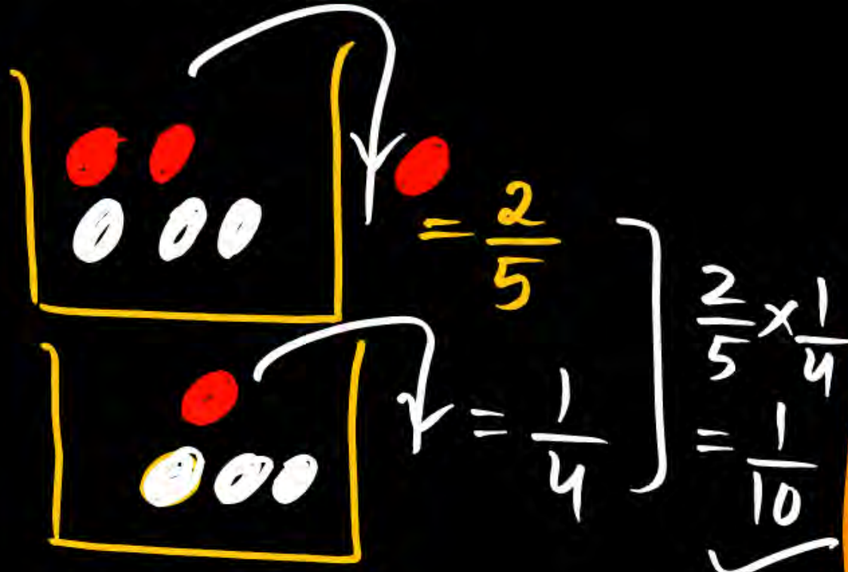


Let v_1 and v_2 be two urns box such that v_1 contains 3 White and 2 Red balls and v_2 contains only 1 White ball. A fair coin is tossed. If head appears then 1 ball is drawn at random from v_1 and put Into v_2. However If tail appears then 2 balls are drawn at random from v_1 and put into v_2. Now one ball is drawn at random from v_2 given that the drawn ball from v_2 is white then the probability that head appeared on the coin.

Bayes THEOREM STAGE 2



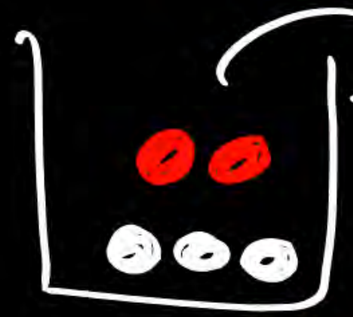

START
1/2
1/2
HEAD
TAIL
2/5
3/5
1/10
3/10
3/5
RT
white
RR
WW
RW or WR
1/2
1
1/3
1
2/3
U2 being white
U2
2R
n Different I
V1
V2
= 12/23
= 1/2 x 2/5 x 1/2 + 1/2 x 3/5 x 1
1/2 x 2/5 x 1/2 + 1/2 x 3/5 x 1 + (1/66 + 3/20 + 6/30)



$\frac{2}{5}$

$\frac{1}{4}$

$\frac{2}{5} \times \frac{1}{4} = \frac{1}{10}$



$\frac{3}{5} \times \frac{2}{4} = \frac{3}{10}$

$\begin{matrix} RW \\ WR \end{matrix} + \frac{2}{5} \times \frac{3}{4} \times 2 = \frac{3}{5}$

n Different Items
Taken 2 at a time

RR $\rightarrow \frac{{}^2C_2}{{}^5C_2}$

WW $\rightarrow \frac{{}^3C_2}{{}^5C_2}$

$\begin{bmatrix} RW \\ WR \end{bmatrix} \rightarrow \frac{{}^3C_1 \, {}^2C_1}{{}^5C_2}$

Q. Questions

A hydraulic structure has four gates which operate independently. The probability of failure of each gate is 0.2. Given that gate 1 has failed, the probability that both gates 2 and 3 will fail is

(a) 0.240

(b) 0.200

(c) 0.040

(d) 0.008

Q. Questions

An examination consists of two papers, paper 1 and paper 2. The probability of failing in paper 1 is 0.3 and that in paper 2 is 0.2. Given that a student has failed in paper 2, the probability of failing in paper 1 is 0.6. The probability of a student failing in both the papers is

(a) 0.5

(b) 0.18

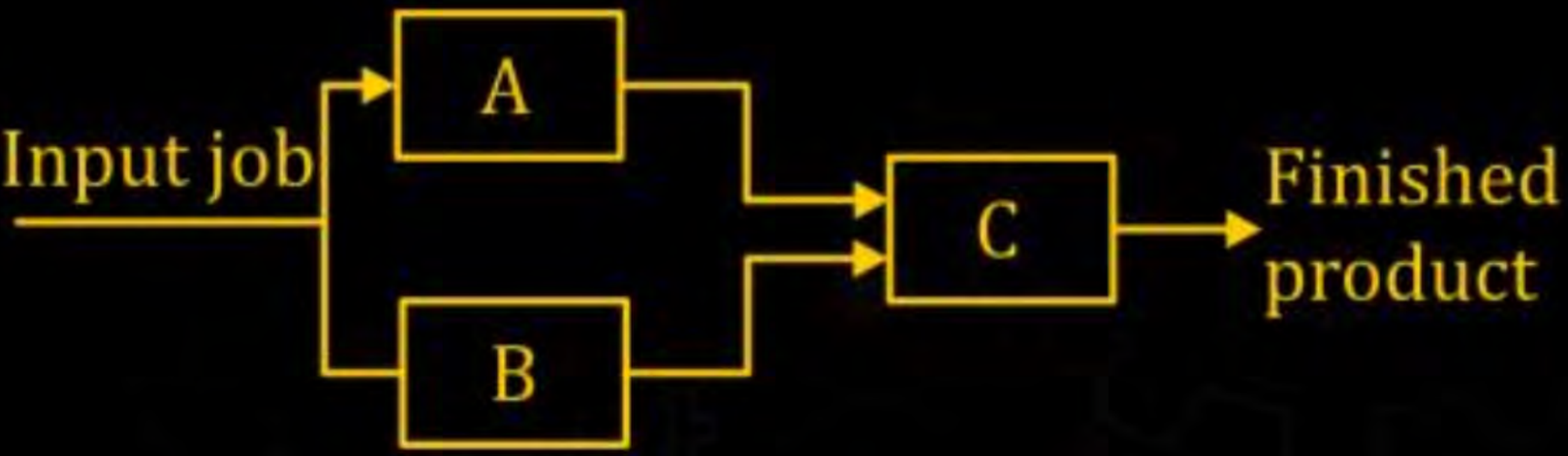
(c) 0.12

(d) 0.06

Q. Questions

The figure shown the schematic of a production process with machines A, B and C. An input job needs to be pre-processed either by A or by B before it is fed to C, from which the final finished product comes out. The probabilities of failure of the machines are given as:

$P_A = 0.15$, $P_B = 0.05$ & $P_C = 0.1$



Assuming independence of failures of the machines, the probability that a given job is successfully processed (up to the third decimal place) is

Q. Questions

In a given day in the rainy season, it may rain 70% of the time. If it rains, chance that a village fair will make a loss on that day is 80%. However, if it does not rain, chance that the fair will make a loss on that day is only 10 %. If the fair has not made a loss on a given day in the rainy season, what is the probability that it has not rained on that day?

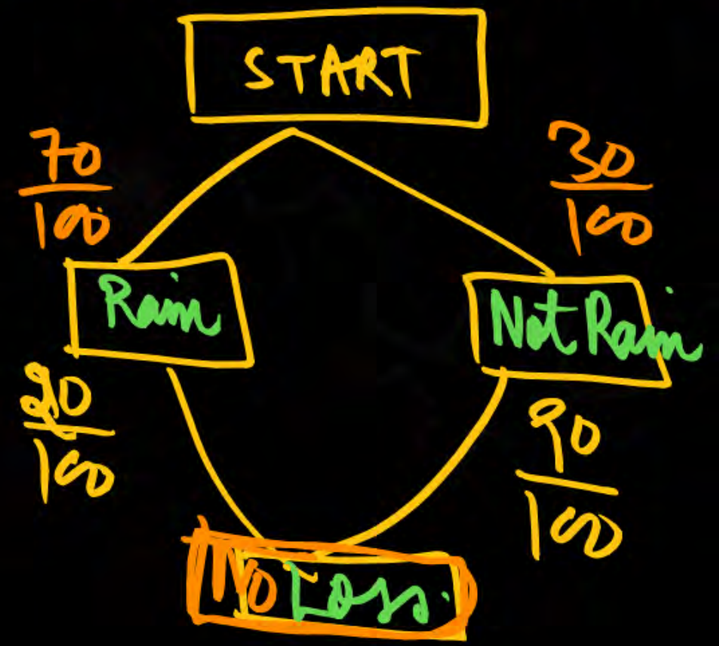
(a) 3/10

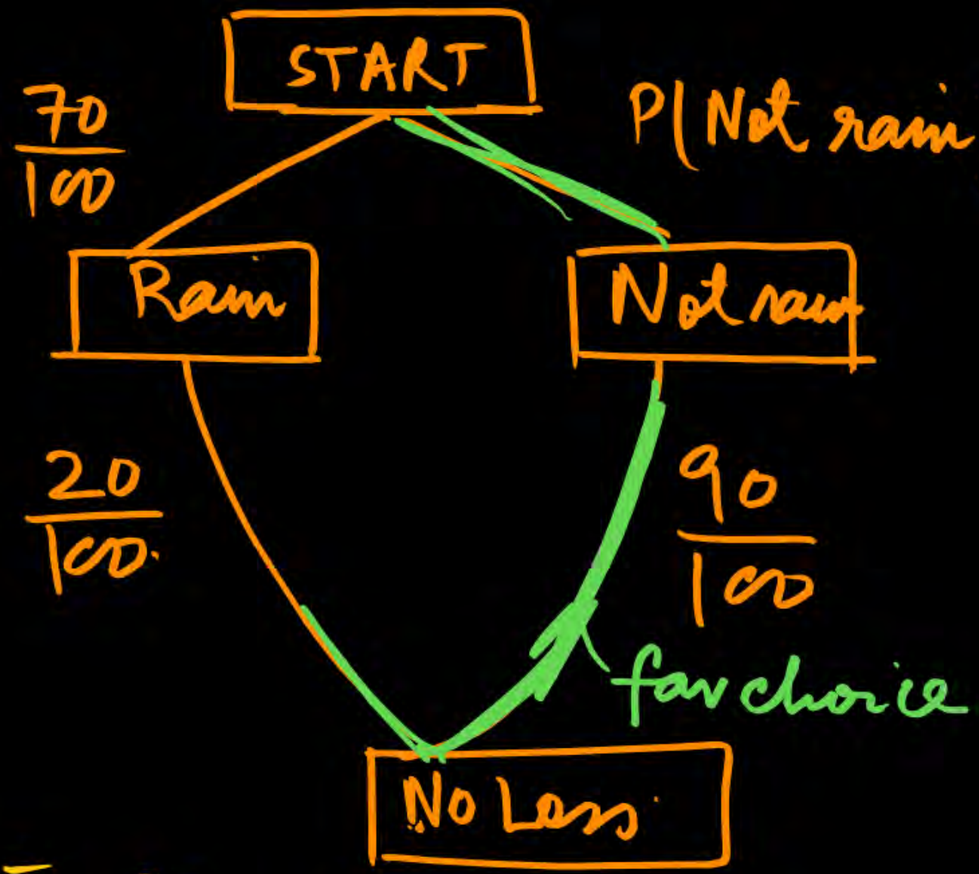
(b) 9/11

(c) 14/17

(d) 27/41

$$\Rightarrow \frac{\frac{90}{100}\times\frac{30}{100}}{\frac{90}{100}\times\frac{30}{100}+\frac{70}{100}\times\frac{20}{100}} = \frac{27}{41}$$





$P(\text{Not rain}) = 1 - \frac{70}{100} = \frac{30}{100}$

$$= \frac{\frac{30}{100} \times \frac{90}{100}}{\frac{30}{100} \times \frac{90}{100} + \frac{70}{100} \times \frac{20}{100}} = \frac{27}{41}$$

Q. Questions

$P(\text{GREEN}) = \frac{4}{5}$ $P(\text{red}) = \frac{1}{5}$

A signal which can be green or red with probability 4/5 and 1/5 respectively, is received by station A and then transmitted to station B. The probability of each station receiving the signal correctly is 3/4. If the signal received at station B is green, then the probability that the original signal was green is

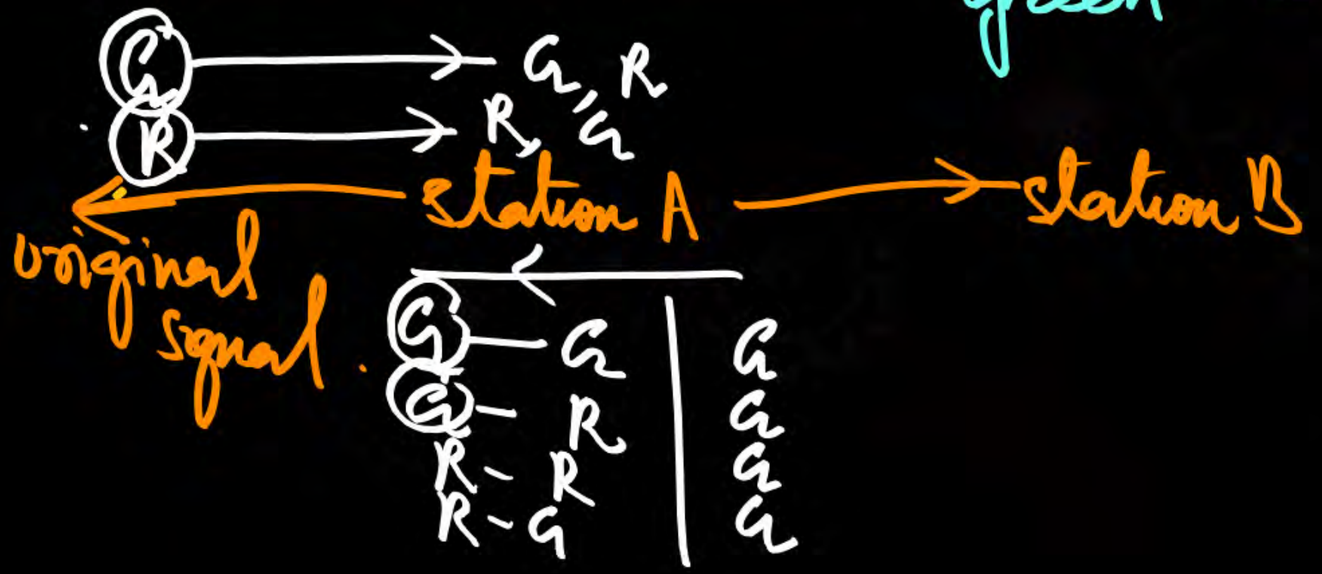
GREEN

(a) 3/5

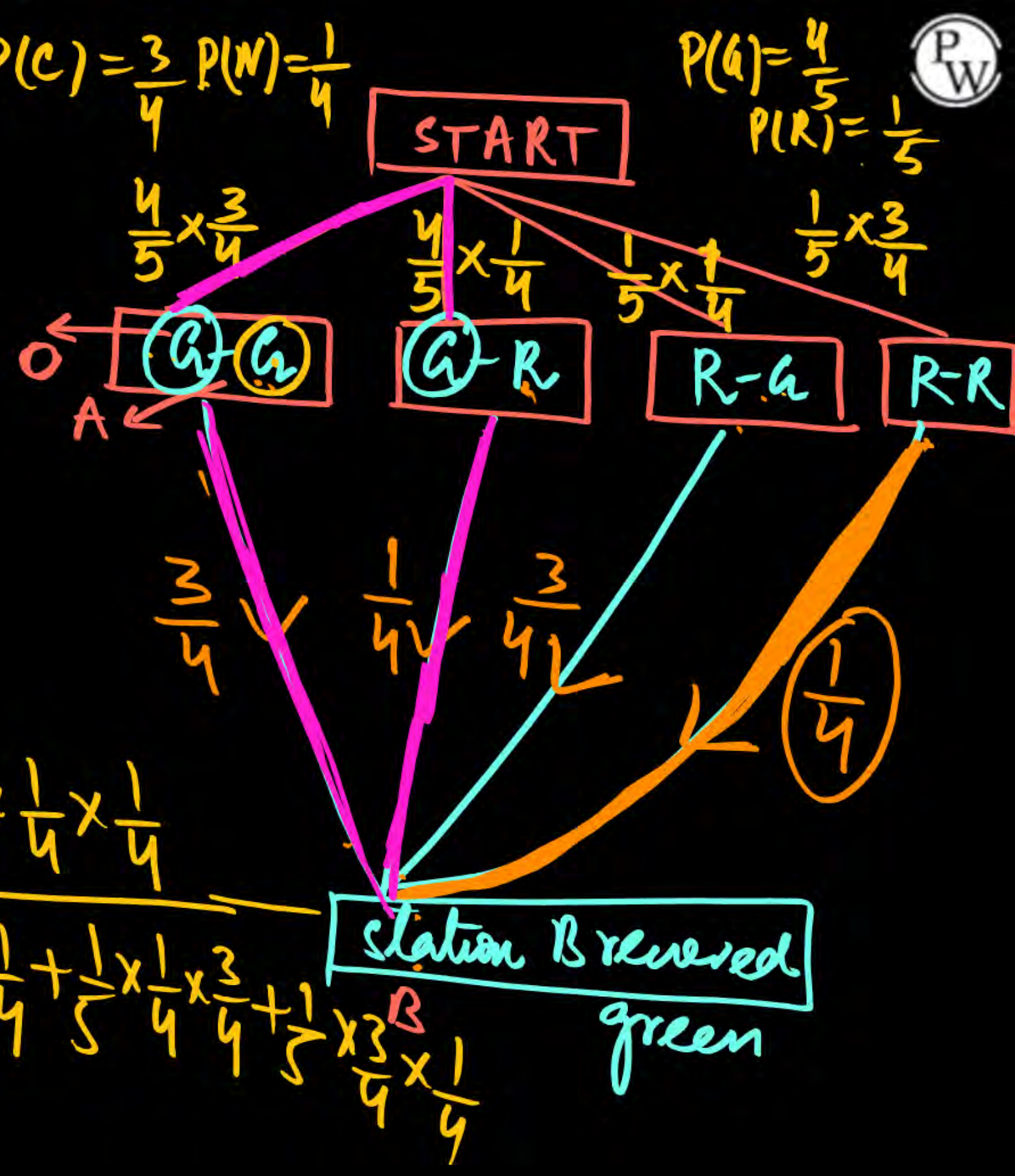
(b) 6/7

(c) 20/23

(d) 9/20



Original	A station	B station
Original for: G → (C) G	G	✓ Green
G → (I) R	R	✓ Green
R → (I) G	G	Green
R → (C) R	R	Green



$R \to R$ Correct

$R \to G$ Incorrect

$G \xrightarrow{I} R$

$G \xrightarrow{C} G$

$$= \frac{\frac{4}{5}\times\frac{3}{4}\times\frac{3}{4} + \frac{4}{5}\times\frac{1}{4}\times\frac{1}{4}}{\frac{4}{5}\times\frac{3}{4}\times\frac{3}{4} + \frac{4}{5}\times\frac{1}{4}\times\frac{1}{4} + \frac{1}{5}\times\frac{1}{4}\times\frac{3}{4} + \frac{1}{5}\times\frac{3}{4}\times\frac{1}{4}} = \frac{20}{23}$$

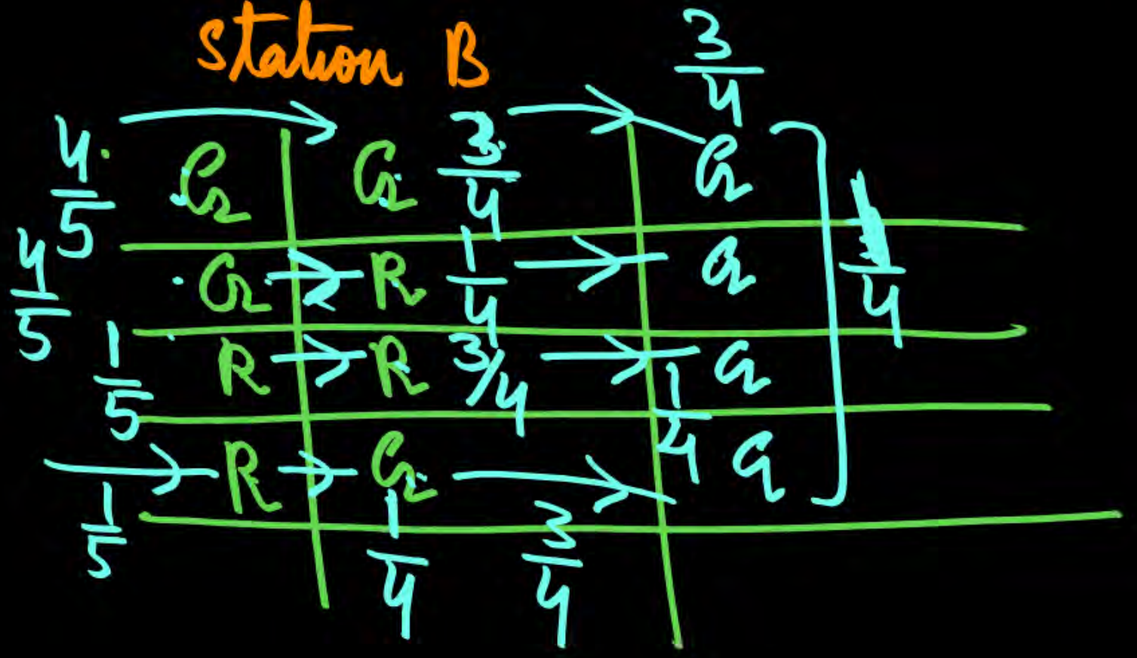
Original

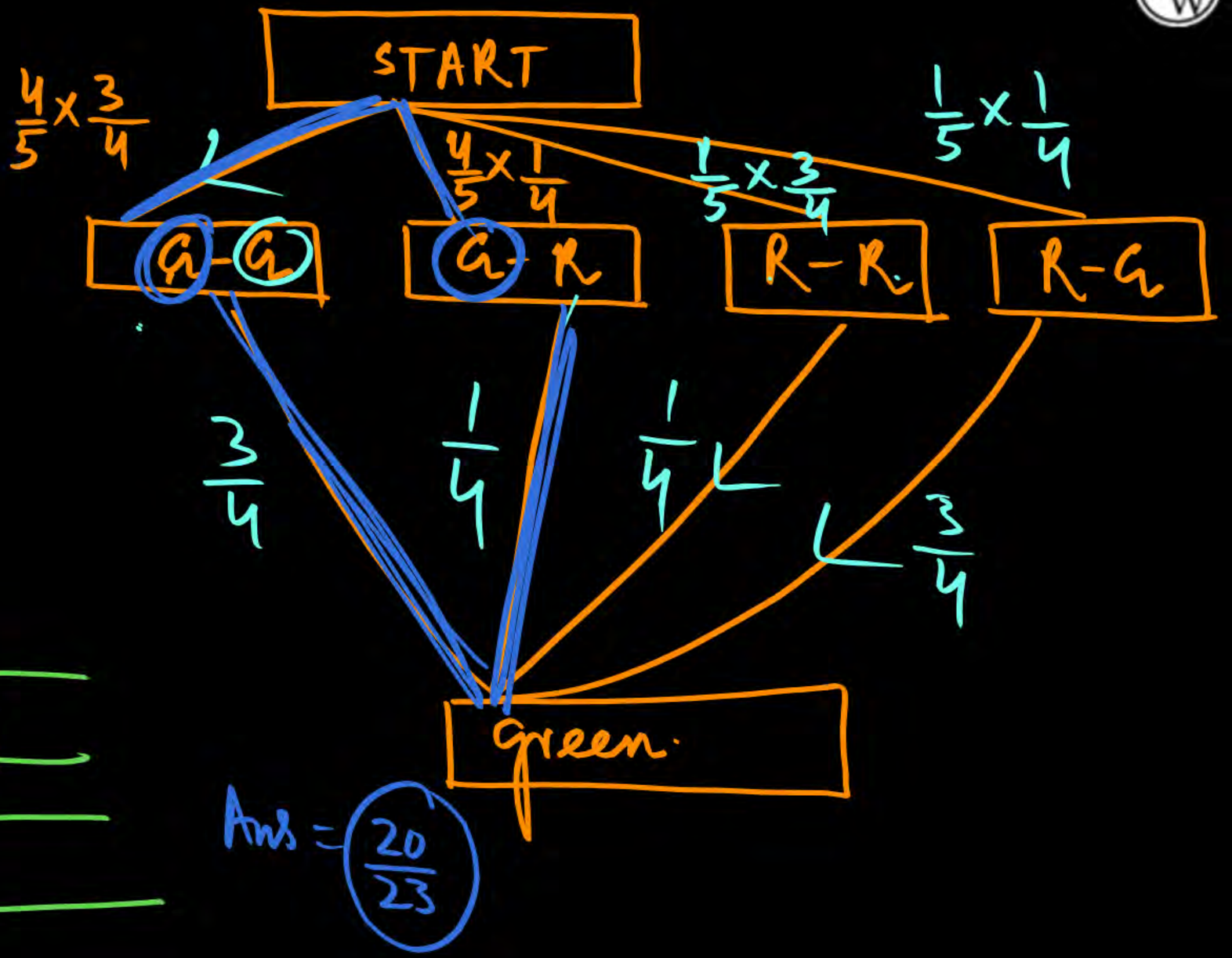
↓

station A

↓

station B





Ans = 20/23

Q. Questions

MSQ

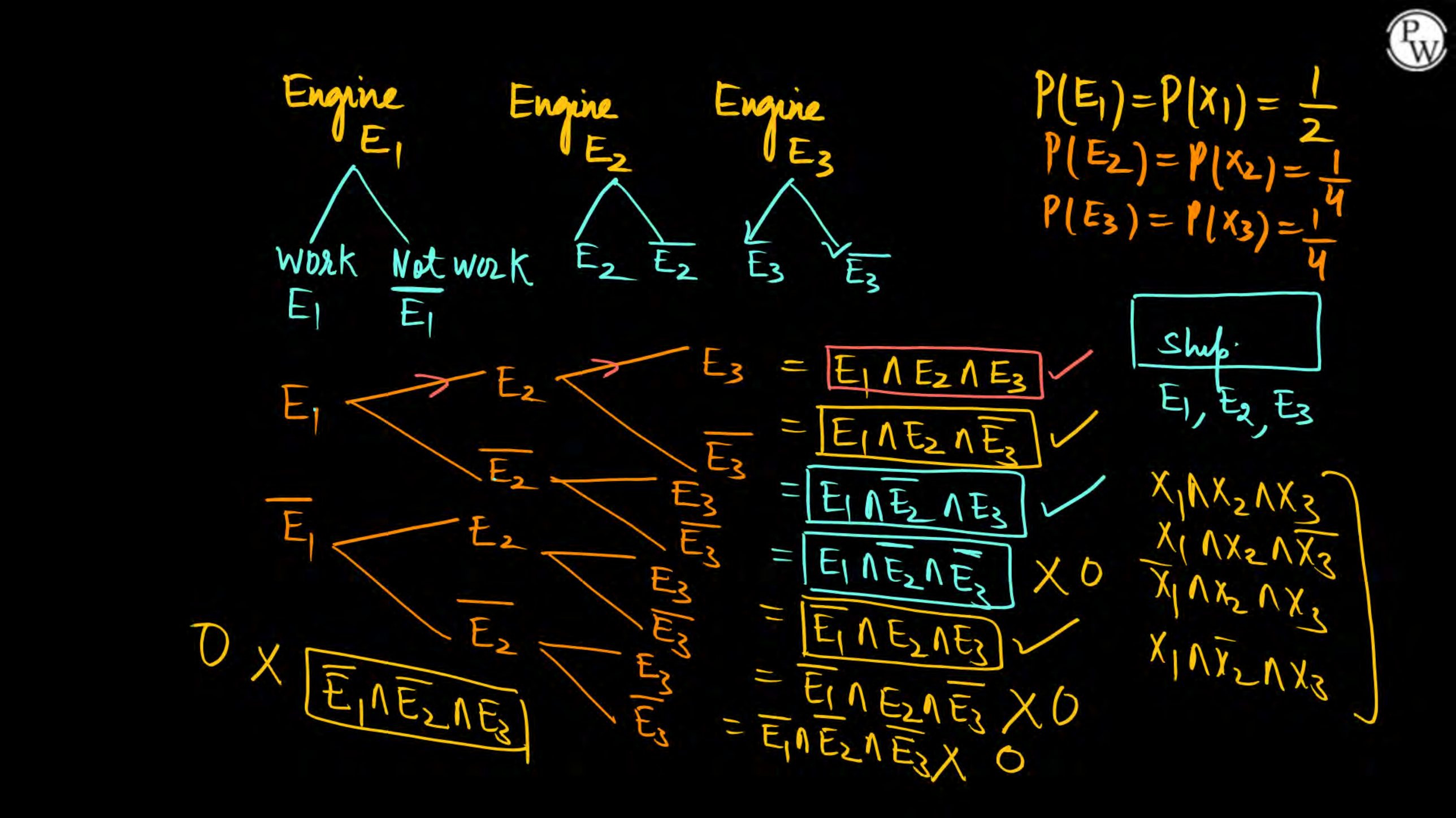
A ship is fitted with three engines E_1, E_2 and E_3. The engines function independently of each other with respective probabilities 1/2, 1/4, 1/4. For the ship to be operational at least two of its engines must function. Let X denote the event that the ship is operational and let X_1, X_2 and X_3 denote respectively the events that the engines E_1, E_2 and E_3 are functioning. Which of the following is/are true?

(a) $P\left(\frac{X_1^C}{X}\right) = \frac{3}{16}$

(b) P (exactly two engines of the ship are functioning /X) = 7/8

(c) $P\left(\frac{X}{X_2}\right) = \frac{5}{16}$

(d) $P\left(\frac{X}{X_1}\right) = \frac{7}{16}$

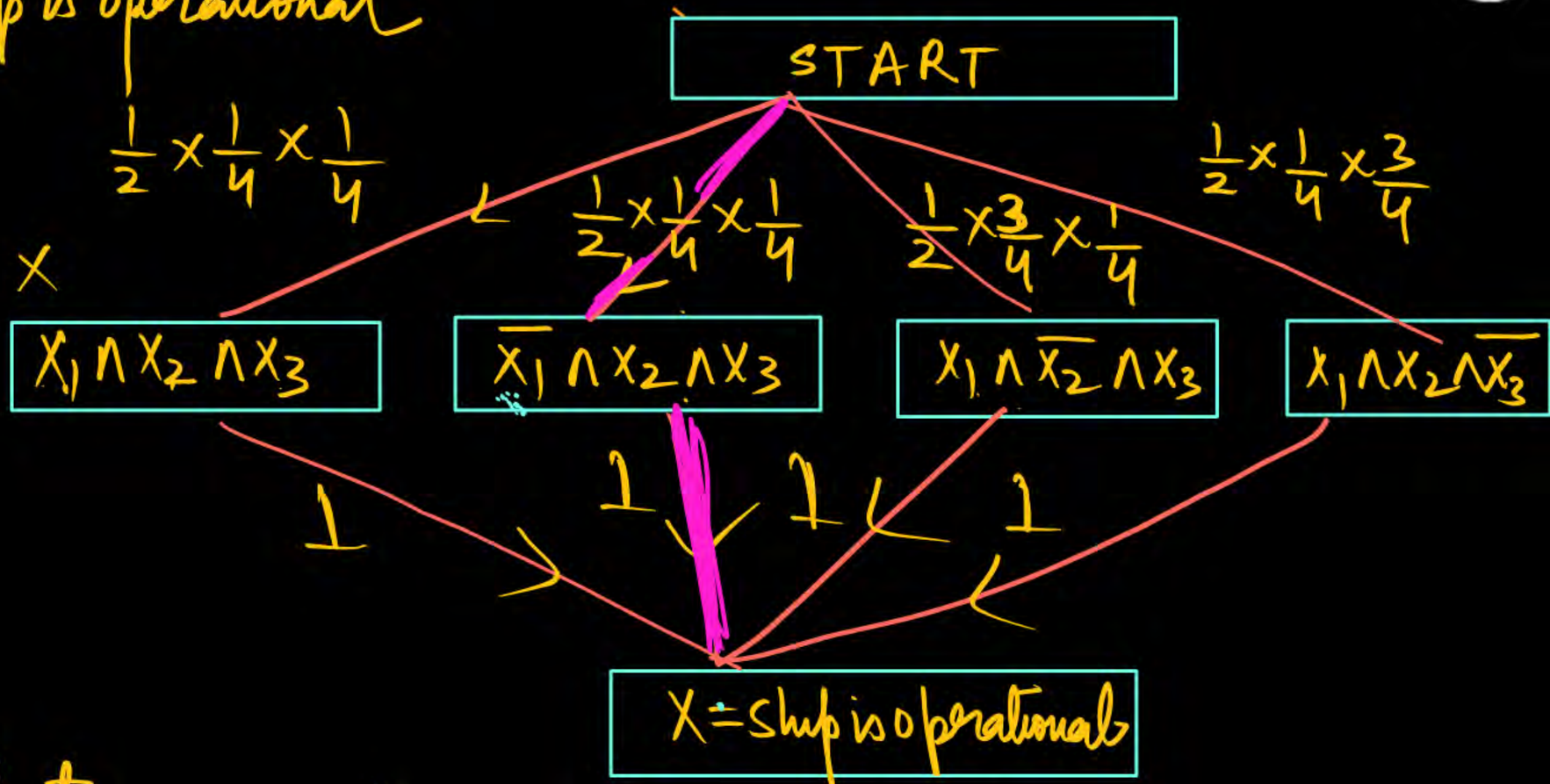

Engine E_1
Work E_1
Not work $\overline{E_1}$
Engine E_2
E_2 $\overline{E_2}$
Engine E_3
E_3 $\overline{E_3}$
$P(E_1)=P(X_1)=\frac{1}{2}$
$P(E_2)=P(X_2)=\frac{1}{4}$
$P(E_3)=P(X_3)=\frac{1}{4}$
E_1 E_2 E_3 $= E_1 \wedge E_2 \wedge E_3$
$\overline{E_3}$ $= E_1 \wedge E_2 \wedge \overline{E_3}$
$\overline{E_2}$ E_3 $= E_1 \wedge \overline{E_2} \wedge E_3$
$\overline{E_3}$ $= E_1 \wedge \overline{E_2} \wedge \overline{E_3}$ X 0
$\overline{E_1}$ E_2 E_3 $= \overline{E_1} \wedge E_2 \wedge E_3$
$\overline{E_3}$ $= \overline{E_1} \wedge E_2 \wedge \overline{E_3}$ X 0
$\overline{E_2}$ E_3 $= \overline{E_1} \wedge \overline{E_2} \wedge E_3$ X 0
$\overline{E_3}$
0 X $\overline{E_1} \wedge \overline{E_2} \wedge \overline{E_3}$
Shuffle
E_1, E_2, E_3
$X_1 \wedge X_2 \wedge X_3$
$X_1 \wedge X_2 \wedge \overline{X_3}$
$X_1 \wedge \overline{X_2} \wedge X_3$
$X_1 \wedge \overline{X_2} \wedge X_3$

X = ship is operational

$$P(X_1) = \frac{1}{2}$$
$$P(X_2) = \frac{1}{4}$$
$$P(X_3) = \frac{1}{4}$$
$$P(\overline{X_1}) = 1 - \frac{1}{2} = \frac{1}{2}$$
$$P(\overline{X_2}) = 1 - \frac{1}{4} = \frac{3}{4}$$
$$P(\overline{X_3}) = 1 - \frac{1}{4} = \frac{3}{4}$$

X_1, X_2, X_3 Are independent

$$P(X_1 \cap X_2 \cap X_3) = P(X_1)\,P(X_2)\,P(X_3)$$



$$P(\text{ship is operational}) = \frac{1}{32} + \frac{1}{32} + \frac{3}{32} + \frac{3}{32}$$
$$= \frac{8}{32}$$

$$P\left(\frac{X_1^c}{X}\right) = P\left[\frac{X_1 \text{ ship is Not working}}{\text{ship is operational}}\right] = \frac{\frac{1}{32}}{\frac{8}{32}} = \frac{1}{8}$$

only (2)

$$P\left[\frac{\text{Exactly Two engines Are working}}{\text{Ship is operational}}\right] = P\left[\frac{\text{Two engines.}}{X}\right] = \text{Case (2)} + \text{case (3)} + \text{case (4)}$$

$$\Rightarrow \frac{\frac{1}{32} + \frac{3}{32} + \frac{3}{32}}{\frac{8}{32}} = \frac{7}{8}$$

Ans

B Option is correct

(3) $P\left(\frac{X}{X_2}\right)$ = conditional prob.

$$P\left(\frac{X}{X_2}\right) = P\left[\frac{X \wedge X_2}{P(X_2)}\right]$$

$$P\left(\frac{A}{B}\right) = \frac{P(A \wedge B)}{P(B)}$$

$$\Rightarrow \frac{P(\text{ship is operational} \wedge X_2 \text{ is working})}{P(X_2)} = \text{Case } ① + ② + ④$$

$$= \frac{\frac{1}{32} + \frac{1}{32} + \frac{3}{32}}{\frac{1}{4}} = \frac{\frac{5}{32}}{\frac{1}{4}} = \frac{20}{32} = \frac{10}{16} = \frac{5}{8}$$ X

$$P\left(\frac{X}{X_1}\right) = \frac{P(X \cap X_1)}{P(X_1)} = \underline{\text{Case } ① + ③ + ④}$$

$$= \frac{\frac{1}{32} + \frac{3}{32} + \frac{3}{32}}{\frac{1}{2}}$$

$$= \frac{\frac{7}{32}}{\frac{1}{2}} = \frac{14}{32} = \frac{7}{16}$$

$P(X_1) = \frac{1}{2}$
$P(X_2) = \frac{1}{4}$
$P(X_3) = \frac{1}{4}$

(b) option is right

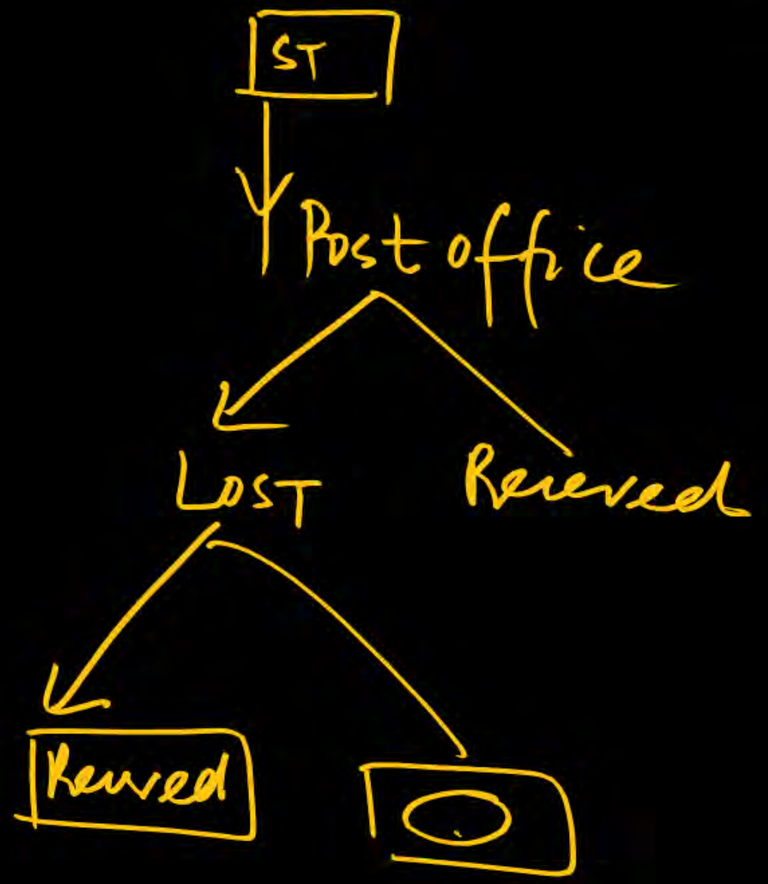
Q. Questions

Parcels from sender S to receiver R pass sequentially through two post - offices. Each post - office has a probability 1/5 of losing an incoming parcel, independently of all other parcels. Given that a parcel is lost, the probability that it was lost by the second post - office is _____.

SENDER — Reciever

Ans ⇒ $\frac{y}{9}$ Ans

✓ Do yourself



Q. Questions

If $P(X) = 1/4$, $P(Y) = 1/3$, and $P(X \cap Y) = 1/12$, then value of $P(Y/X)$ is

(a) 1/4

(b) 4/25

(c) 1/3

(d) 29/50

$$P\left(\frac{Y}{X}\right) = \frac{P(Y \cap X)}{P(X)}$$

$$= \frac{\frac{1}{12}}{\frac{1}{4}}$$

$$= \frac{4}{12} = \frac{1}{3}$$

$P(X) = \frac{1}{4}$

$P(Y) = \frac{1}{3}$

$P(X \cap Y) = \frac{1}{12}$

$P\left(\frac{Y}{X}\right) = ?$

Q. Questions

Ram and Ramesh appeared in an interview for two vacancies in the same department. The probability of Ram's selection is 1/6 and that of Ramesh is 1/8. What is the probability that only one of them will be selected?

(a) 47/48

(b) 1/4

(c) 13/48

(d) 35/48

$P(\text{only one Them selected}) \Rightarrow (\text{Ram} \wedge \overline{\text{Rameeh}}) + P(\text{rameeh} \wedge \overline{\text{Ram}})$

$\Rightarrow P(A \wedge \overline{B}) + P(B \wedge \overline{A})$

$$P(\text{only one selected}) = P(A \cap \bar{B}) + P(B \cap \bar{A})$$

$$\Rightarrow P(A)\,P(\bar{B}) + P(B)\cdot P(\bar{A})$$

$$= P(\text{Ram})\,P(\overline{\text{Ramesh}}) + P(\text{ramesh})\,P(\overline{\text{Ram}})$$

$$= \frac{1}{6}\left(1-\frac{1}{8}\right) + \frac{1}{8}\times\left(1-\frac{1}{6}\right)$$

$$= \frac{1}{6}\times\frac{7}{8} + \frac{1}{8}\times\frac{5}{6}$$

$$= \frac{12}{48} = \frac{1}{4}$$ Ans

Q. Questions

Do yourself

X and Y are two random independent events. It is known that $P(X) = 0.40$ and $P(X \cup Y^c) = 0.7$. Which one of the following is the value of $P(X \cup Y)$?

(a) 0.7

(b) 0.5

(c) 0.4

(d) 0.3

Q. Questions

DO yourself

P and Q are considering to apply for a job. The probability that P applies for the job is 1/4, the probability that P applies for the job given that Q applies for the job is 1/2, and the probability that Q applies for the job given that P applies for the job is 1/3. Then the probability that P does not apply for the job given that Q does not apply for the job is

(a) 4/5

(b) 5/6

(c) 7/8

(d) 11/12

Q. Questions

Do yourself

The probabilities of occurrence of events F and G are $P(F) = 0.3$ and $P(G) = 0.4$, respectively. The probability that both events occur simultaneously is $P(F \cap G) = 0.2$. The probability of occurrence of at least one event $P(F \cup G)$ is _____.

Q. Questions

Do yourself

The probability that a student passes in Mathematics, Physics and Chemistry are m, p and c respectively. Of these subjects, the students has a 75% chance of passing in atleast one, a 50% chance of passing in atleast two and a 40% chance of passing in exactly two. Which of the following relations is true?

(a) $p+m+c = 19/20$

(b) $p+m+c = 27/20$

(c) $pmc = 1/10$

(d) $Pmc = 1/4$

Thank You!

GW Soldiers