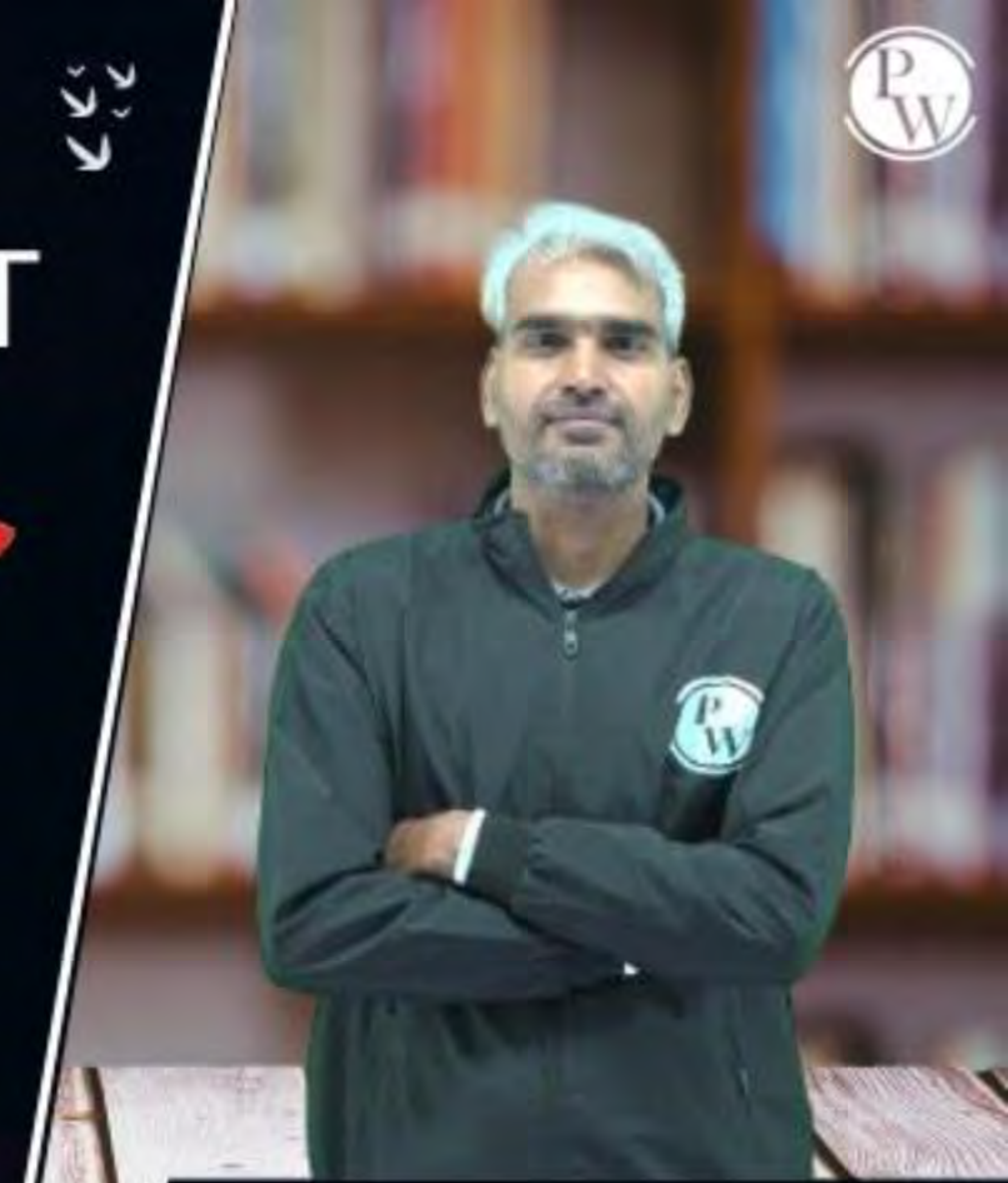
Computer Science & IT

Discrete Mathematics

Graph Theory

Lecture No. 13



By- Vishal Sir

Recap of Previous Lecture

Topic Spanning tree

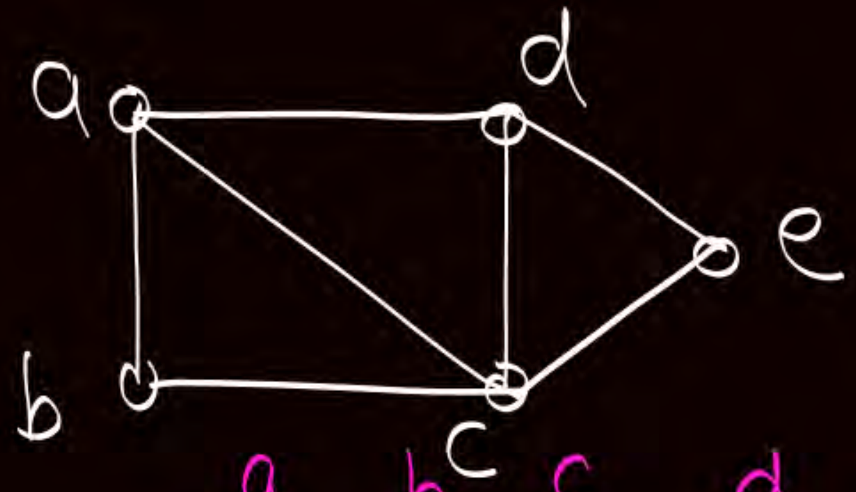
Slide

Topics to be Covered

Topic Connectivity

Topic Distance, Eccentricity, Diameter, Radius and Girth

Slide



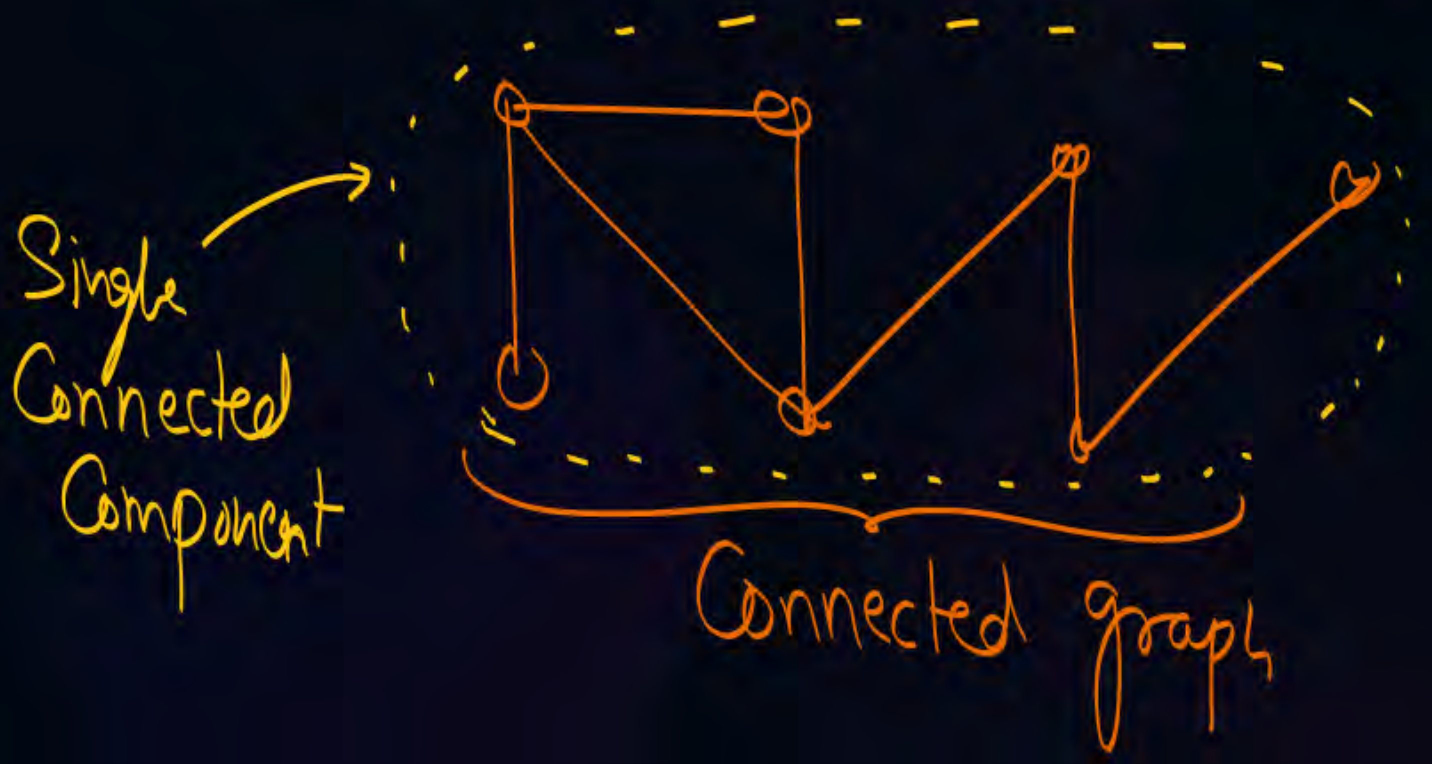
$$M = \begin{array}{c|c} & \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{array}{c} a \\ b \\ c \\ d \\ e \end{array} & \begin{bmatrix} 3 & -1 & -1 & -1 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ -1 & -1 & 4 & -1 & -1 \\ -1 & 0 & -1 & 3 & -1 \\ 0 & 0 & -1 & -1 & 2 \end{bmatrix} \end{array}$$

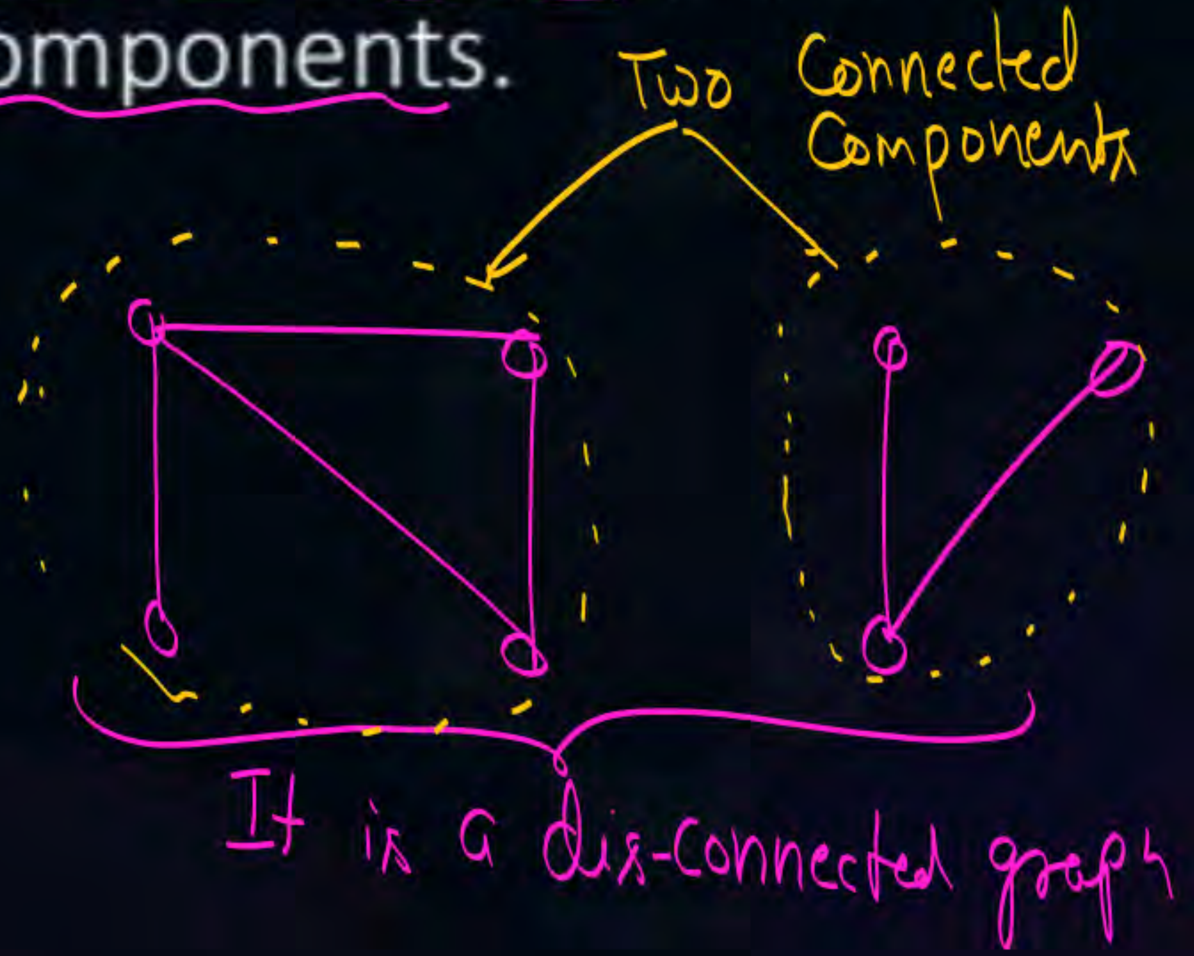
$$\text{Cofactor of } a_{11} = (-1)^{1+1} \cdot \begin{vmatrix} 2 & -1 & 0 & 0 \\ -1 & 4 & -1 & -1 \\ 0 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{vmatrix}$$

Topic : Connectivity

* A graph is said to be connected if there exists a path between every pair of vertices.

If a graph (with 2 or more vertices) is not connected, then it will have two or more connected components.

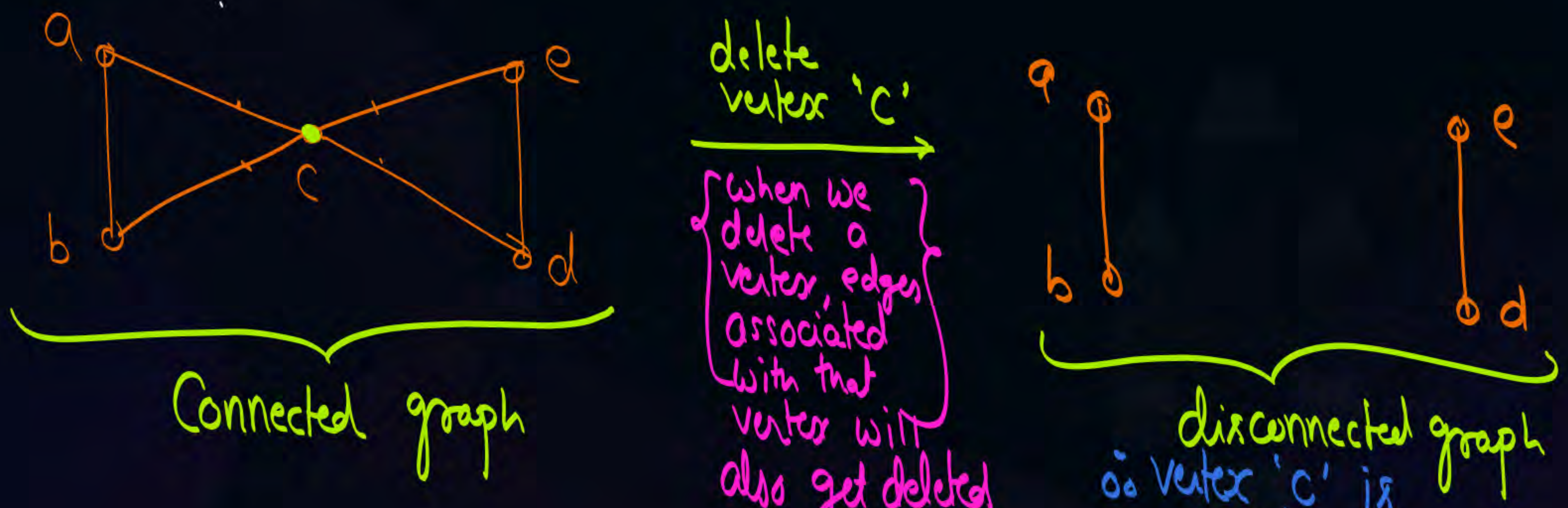




Topic : Cut-vertex

Articulation point

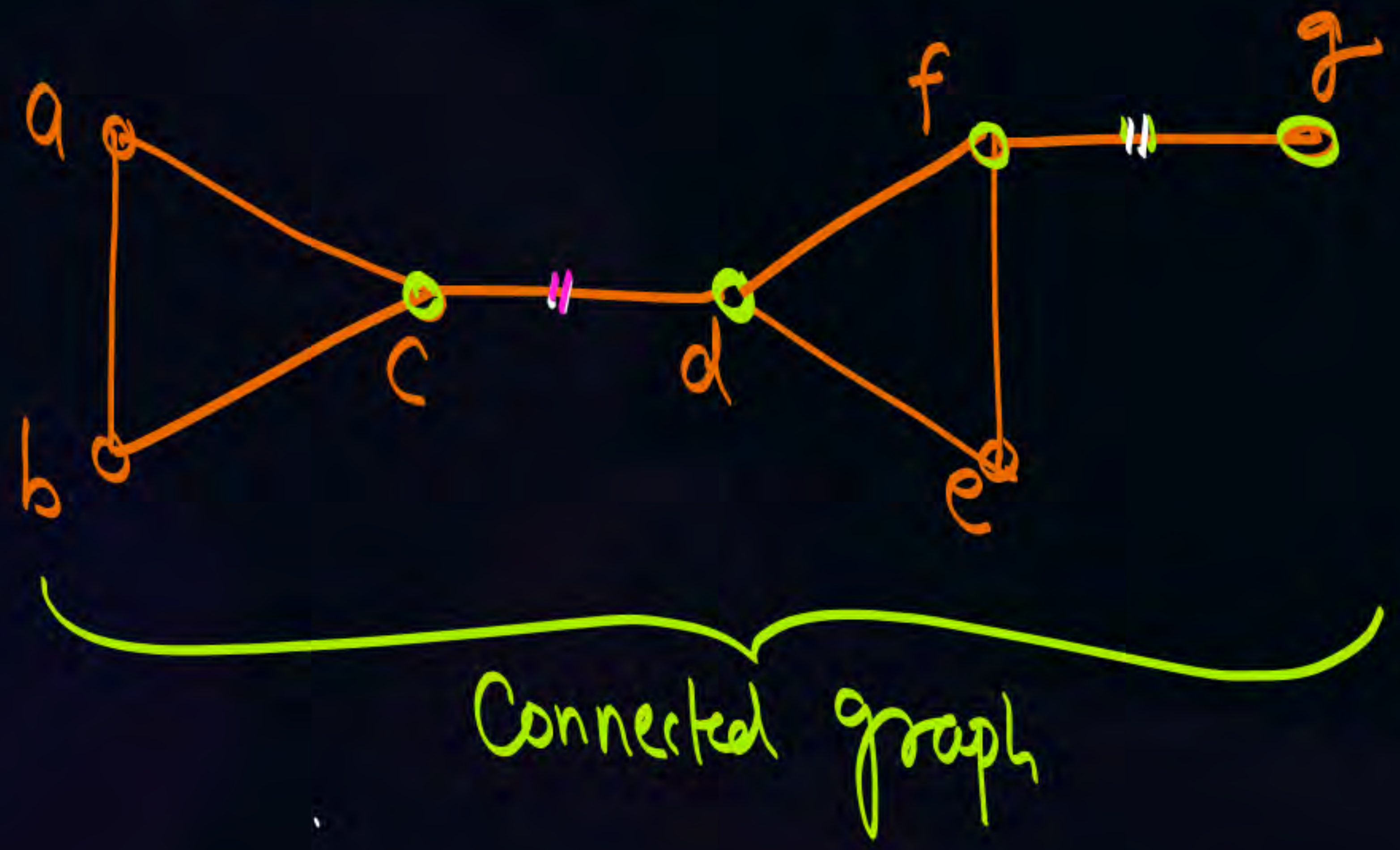
Let G be a connected graph, a vertex $v \in G$ is called a cut vertex if deletion of vertex "v" from graph G makes the graph disconnected.



∴ vertex 'c' is a cut-vertex

Topic : Cut-edge / Bridge

Let G be a connected graph, an edge $e \in G$ is called a cut edge or bridge if deletion of edge "e" from graph G makes the graph disconnected.



- {c,d} is a cut-edge as well as {f,g} is a cut-edge

Topic : NOTE

① An edge of Connected graph G is a cut-edge if and only if that edge is not part of any cycle in graph G

② At least one vertex associated with a cut-edge is also a cut-vertex

eg. In the above eg. w.r.t. cut-edge {c, d} both 'c' & 'd' are cut vertices.

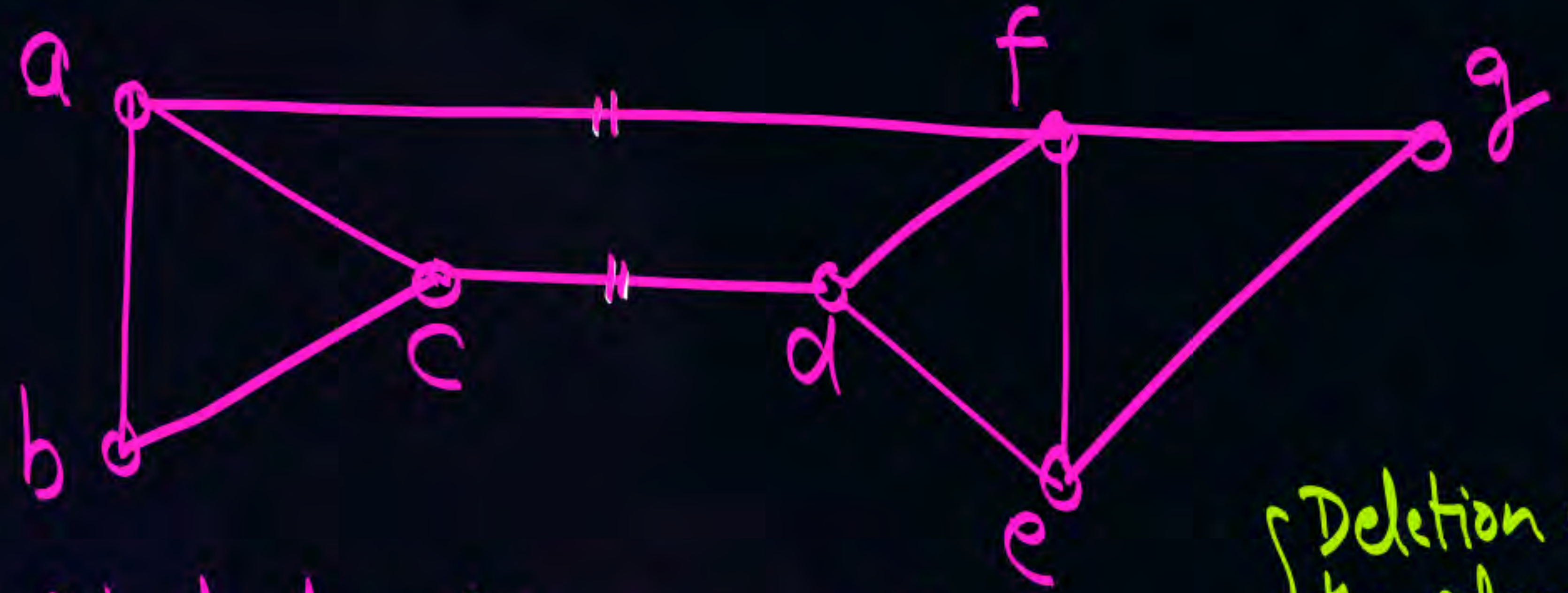
but w.r.t. cut-edge {f, g}, only 'f' is a cut-vertex because deletion of 'g' does not make the graph disconnected

③ Existance of cut-edge in a graph implies the existance of cut-vertex in that graph, but existance of cut-vertex does not imply existance of cut-edge

Topic : Cut-set

→ i.e., a set from which no element can be deleted without destroying its property

A minimal set of edges in connected graph G, such that removal of all the edges of that set from the graph G makes the graph disconnected is called cut-set of graph G.



Set of edges {{c,d},{a,f}} is a Cut-set of the above graph

{ Deletion of both the edges makes the graph disconnected }
if we remove any edge from that set and delete the remaining edges then graph never become disconnected.

{{d,e}, {e,g}, {f,g}}
→ Removal of all the edges of this set makes the graph disconnected but it is not a cut-set because it is not minimal. edge {d,e} can be removed from the set

Topic : Vertex connectivity

$K(G)$

Let G be a connected graph, the minimum number of vertices whose removal from graph G makes the graph disconnected is called vertex connectivity of graph G. Vertex connectivity of a graph G is denoted by $\kappa(G)$

Vertex connectivity of graph is 1 if and only if cut-vertex exists in the graph.

- If Cut-vertex exists in graph G, then $K(G) = 1$
- Vertex Connectivity of graph G is '0' {i.e. $K(G)=0$} if and only if graph is already a disconnected graph
- If graph G is a Connected graph with no Cut vertex then $K(G) \geq 2$

Topic : Edge connectivity

$\lambda(G)$

Let G be a connected graph, the minimum number of edges whose removal from graph G makes the graph disconnected is called edge connectivity of graph G. Edge connectivity of a graph G is denoted by $\lambda(G)$

- Edge connectivity of graph is 1 if and only if cut-edge exists in the graph.
- $\lambda(G) = 0$ iff G is already disconnected.

Topic : NOTE

① In a Connected graph G if $\delta(G)$ is the minimum of the degree of all the vertices of graph G.
then $\lambda(G) \leq \delta(G)$ — eqⁿ ①

② In a Connected graph G,
$K(G) \leq \lambda(G)$ — eqⁿ ②

By eqⁿ ① & ② $K(G) \leq \lambda(G) \leq \delta(G)$

* Find Vertex Connectivity & Edge Connectivity for the following graphs.

① $K_n \Rightarrow$ $\lambda(K_n) = (n-1)$

$\kappa(K_n) = (n-1)$

② $C_n \Rightarrow$ $\lambda(C_n) = 2$

$\kappa(C_n) = 2$

③ $W_n \Rightarrow$ $\lambda(W_n) = 3$

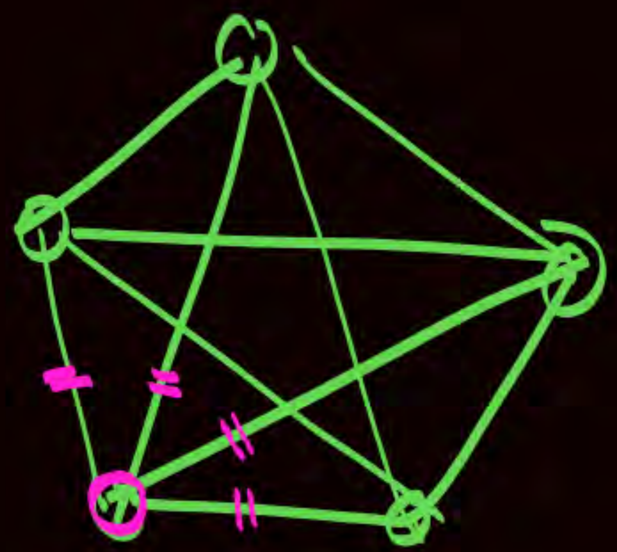
$\kappa(W_n) = \underset{\text{Hub}}{1} + \underset{\text{w.r.t. } C_{n-1}}{2} = 3$

④ $K_{m,n} \Rightarrow$ $\lambda(K_{m,n}) = \min(m,n)$

$\kappa(K_{m,n}) = \min(m,n)$

⑤ Star graph with n-vertices. $\Rightarrow$ $\lambda(G) = 1$

$\kappa(G) = 1$



$$\underbrace{K_4 \to K_3 \to K_2}_{\text{deletion of } (4-1) \text{ vertices}} \to K_1$$

Note:

* A single isolated vertex is actually a connected graph, but some-times it may considered as a disconnected graph { e.g. when we are trying to obatain a disconnected graph from a complete graph K_n, by deleting vertices from K_n }

#Q. If G is a forest with n vertices and k connected components, how many edges does G have?

Ans = $n - k$

[MCQ]

#Q. In a connected graph G if we delete an edge then no. of components are

A 0 or 1

B 1 or 2

C 2

D ≥2

(i) If the deleted edge is not a Cut-edge, then graph will remain Connected {∴ only one component}

(ii) If the deleted edge is a Cut-edge, then because of deletion of that edge the number of Components will increase by '1' {∴ Total No. of Components = 1+1 = 2}

on deleting an edge the no. of Components will either remain same or increase by exactly '1'

[MCQ]

with n-vertices $(n \geq 2)$

#Q. In a connected graph G ~~is~~ if we delete a vertex then number of components are

{Note: we are not considering a single isolated vertex (as $n \geq 2$)}

↳ if there is only one isolated vertex, then on deletion of a vertex there will be '0' components!

A. ≤ 2

B. 1 or 2

C. Lies between 1 and n -1

D. ≥2

If vertex is not a cut-vertex, then only one component after deletion

If vertex is a cut-vertex then there may be atmost (n-1) components after deletion of a vertex {eg: delete hub vertex from star graph with n-vertices}

H.W.

#Q. A simple graph with n vertices is necessarily connected if number of edges are more than E, then find the value of E.

H.W.

#Q. Maximum number of edges possible in an undirected graph with n-vertices and k components ?

Topic : Girth

In graph theory, the girth of an undirected graph is the length of a shortest cycle contained in the graph. If the graph does not contain any cycles (that is, it is a forest), its girth is defined to be infinity.

2 mins Summary

Topic Connectivity

Topic Distance, Eccentricity, Diameter, Radius and Girth

THANK - YOU

Slide