Computer Science & DA

Probability and Statistics

Probability

Lecture No. 03

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Recap of previous lecture

Topic Types of Events & Various theorem(Fundamental Question)

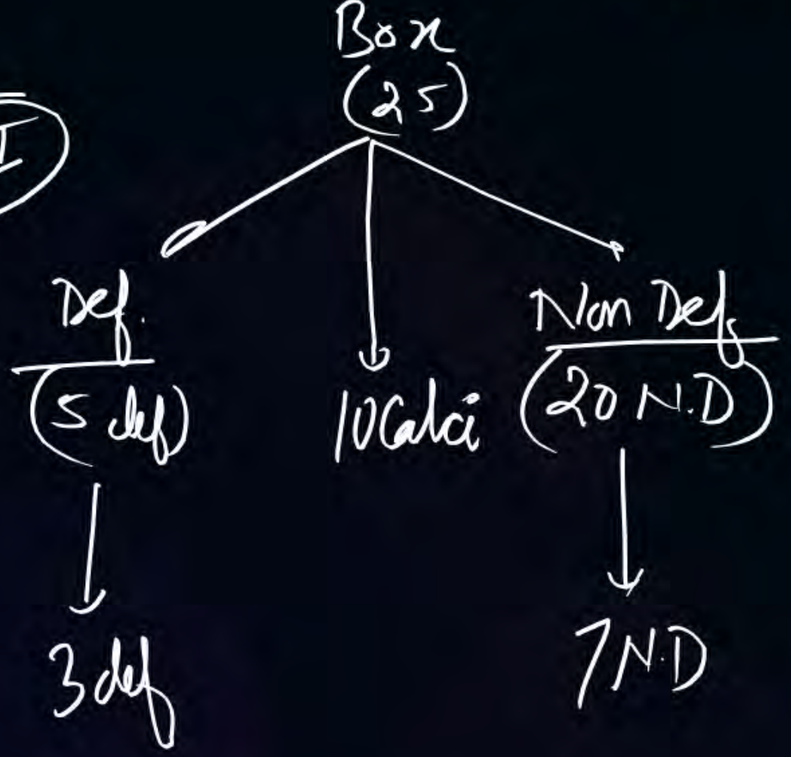
Topics to be Covered

Topic Conditional probability

Qe A Box contains 25 Calci in which 5 are defective. If we are choosing 10 Calci at a time then P(there will be exactly 3 def. Calci) = ?

Sol:

App II



$$\text{Req Prob} = \frac{f}{T} = \frac{{}^{5}C_3 \times {}^{20}C_7}{{}^{25}C_{10}}$$

GATE

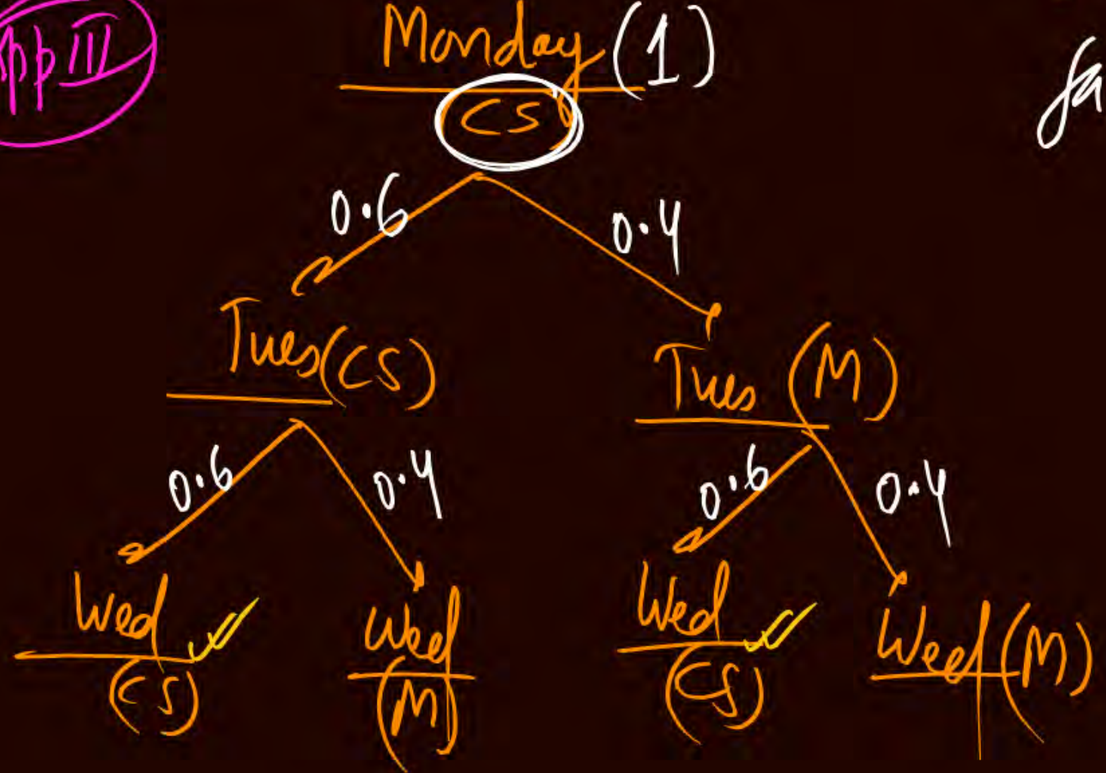
Q. Aishwarya studies either CS or M on each day.

if she studies CS on a day then the Prob of studying M on next day is 0.4

if " " M " " " " " " CS " " " " is 0.6

Given that Aish studies CS on Monday then P(she will also study CS on Wed) = ?

App II



fav Path = (M T W: CS CS CS) + (M T W: CS M CS)

$= (1 \times 0.6 \times 0.6) + (1 \times 0.4 \times 0.6)$

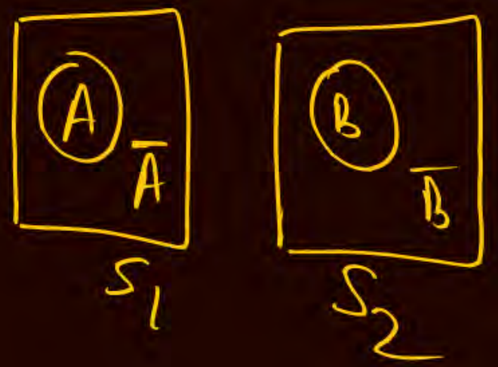
$= 0.36 + 0.24 = 0.6$

Concept of ME and Independency in a Single Question →

Eg There are two Gangster Munna Mobile (A) & Pappu Pazer (B). They both fire at the target once with prob of their hitting is $\frac{4}{5}$ and $\frac{3}{4}$ resp. then write it's S-Space ?

$P(A) = \frac{4}{5}$, $P(\bar{A}) = \frac{1}{5}$

$P(B) = \frac{3}{4}$, $P(\bar{B}) = \frac{1}{4}$

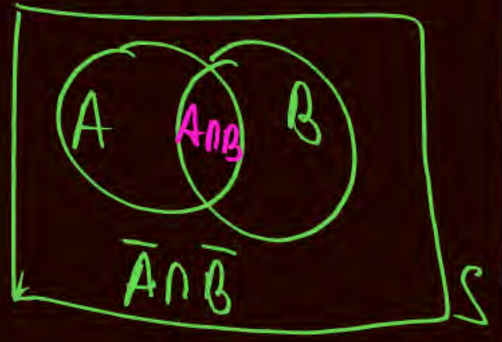


Here A & B are Ind

and A & $\bar{B}$ are Ind

$\bar{A}$ & B " Ind

$\bar{A}$ & $\bar{B}$ " "



Exactly one will hit

(None will hit) or (A hit & B missed) or (A Missed & B hit) or (Both will hit) = Total

At least one will hit

$(\bar{A} \cap \bar{B}) \cup (A \cap \bar{B}) \cup (\bar{A} \cap B) \cup (A \cap B) = S$

$E_1 \cup E_2 \cup E_3 \cup E_4 = S$

where E_1, E_2, E_3, E_4 are M.E events bcoz at a time More than one can't occur simultaneously while Individual events are Ind Events

Here $P(E_1) + P(E_2) + P(E_3) + P(E_4) = 1$

Q. A & B fire at the target once with prob of their hitting is $\frac{4}{5}$ & $\frac{3}{4}$ resp then find ?

$P(A)=\frac{4}{5}$, $P(\bar{A})=\frac{1}{5}$, $P(B)=\frac{3}{4}$, $P(\bar{B})=\frac{1}{4}$, A & B are (Ind.)

(1) $P[\text{Both will hit}]=? = P(A \cap B)=\frac{4}{5}\cdot\frac{3}{4}=\frac{3}{5}$

(2) $P(\text{None will hit})=? = P(\bar{A}\cap\bar{B})=\frac{1}{5}\cdot\frac{1}{4}=\frac{1}{20}$

(3) $P(\text{at least one will hit})=? = 1-P(\text{None will hit})=1-\frac{1}{20}=\frac{19}{20}$

(4) $P(\text{target will be hit})=? = \text{Same as part (3)} = \frac{19}{20}$

(5) $P(\text{either A or B or Both will hit})=? = \text{Same as above}$

$$P(A\cup B)=P(A)+P(B)-P(A\cap B)=\frac{4}{5}+\frac{3}{4}-\frac{3}{5}=\frac{19}{20}$$

⑥ $P(\text{A hit \& B missed}) = ? = P(A \cap \bar{B}) = \frac{4}{5} \times \frac{1}{4} = \frac{1}{5}$

⑦ $P(\text{only one person will hit}) = ? = P[(A \cap \bar{B}) \text{ or } (\bar{A} \cap B)]$

$= P[E_2 \cup E_3]$

$= P(E_2) + P(E_3)$ $(\because E_2 \& E_3 \text{ are ME})$

$= P(A \cap \bar{B}) + P(\bar{A} \cap B)$

$= \frac{4}{5} \times \frac{1}{4} + \frac{1}{5} \times \frac{3}{4} = \frac{7}{20}$

⑧ if exactly one person hit (Condition) then find the prob that A hit & B missed = ? 4/7

(6) $P(A \cap \bar{B}) = \frac{1}{5}$ (7) $P(\text{only one will hit}) = \frac{7}{20}$

(8) original Prob is Prob of S. Space $= 1$

Reduced Prob $= P(\text{condition}) = \frac{7}{20}$

fav Prob $= P(A \cap \bar{B}) = \frac{1}{5} = \frac{1/5}{1}$

So conditional Prob $= \frac{\text{fav Prob}}{\text{R. Prob}} = \frac{1/5}{7/20} = \frac{4}{7}$

Conceptual Questions of Conditional Prob → (just Reduce S-Space according to Condition)

Qe A Couple has 3 Children then ① Find the prob that there are exactly 2 B ?

Sol: $S = \{(BBB), (BBG), (BGB), (BGG), (GBB), (GBG), (GGB), (GGG)\}$

$n(S) = 8$

$fav = \{(BBG), (BGB), (GBB)\}$

$= 3 \Rightarrow$ Req Prob $= \frac{f}{T} = \frac{3}{8}$

M-II of Q2-e

App III

Req Prob $= P[(B)BG \text{ or } B(G)B]$ ← ME

$= \left(1 \times \frac{1}{2} \times \frac{1}{2}\right) + \left(1 \times \frac{1}{2} \times \frac{1}{2}\right) = \frac{1}{2}$

② A Couple has 3 children. If 1st child is Boy given then find the Prob that there are exactly 2 B ?

M-I

App I

Reduced S.Sp $= \{1^{st} \text{ child is Boy}\}$

$= \{(BBB), (BBG), (BGB), (BGG)\}$ ← 4

fav Cases $= \{(BBG), (BGB)\} = 2$

Conditional Prob $= \frac{fav}{R.Cases} = \frac{2}{4} = \frac{1}{2} \neq \frac{4}{8}$

Q. A Coin is tossed twice and 1st time Head is occurring then Find the Prob that H will again come?

Condition

App III

Req Prob = $P[(H)H]$

$= 1 \times \frac{1}{2} = \frac{1}{2}$

App I

origin ssp = $\{(HH),(HT),(TH),(TT)\} = 4$

Reduced ssp = $\{(HH),(HT)\} = 2$

fav Cases = $\{(HH)\} = 1$

Req Cond^n Prob = $\frac{\text{fav}}{\text{R Cases}} = \frac{1}{2}$

Q. Two integers are to be selected from integers 1, 2, 3, 4, ..., 10, 11. If their sum is Even (Condition), then find the prob that both the selected integers are odd?

Sol: $1, 2, 3, \ldots, 10, 11$ ($^{11}C_2$)

$\rightarrow 2, 4, 6, 8, 10 \rightarrow {}^5C_2 = 10$

$\rightarrow 1, 3, 5, 7, 9, 11 \rightarrow {}^6C_2 = 15$

Total ways of selecting two integers $= {}^{11}C_2 = 55$

Reduced " " " " (according to Cond")

$= \{$Sum is Even$\}$

= either (Both are odd) or (Both are Even)

$= {}^6C_2 + {}^5C_2 = 25$

Fav Cases $= \{$Both should be odd$\} = {}^6C_2 = 15$

Conditional Prob $= \frac{\text{Fav Cases}}{\text{R Cases}} = \frac{{}^6C_2}{{}^6C_2 + {}^5C_2} = \frac{15}{25}$

Note - Had the Cond" were not there then answer would have been = ?

$= \frac{f}{T} = \frac{{}^6C_2}{{}^{11}C_2} = \frac{15}{55}$

(M-II) App.I ← 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 (RNA) = Plastic Toys

S.Space = { (1,1) (1,2) (1,3) ---- (1,10) (1,11)
(2,1) (2,2) (2,3) -- (2,10) (2,11)
(3,4) (3,5) ---- (3,11)

(10,11) } = 55 Pair

Reduced Cases = {Sum is Even}

= { (2,4), (2,6), (2,8), (2,10)
(4,6), (4,8) ..... (8,10)
(1,3), (1,5) ---- (1,11)
(3,5), (3,7) ---- (9,11) } = 25 Pair

fav Cases = { (1,3) (1,5) ---- (1,11)
(3,5) ---- (9,11) } = 15 ⟹ ∴ Prob = $\frac{f}{R} = \frac{15}{25}$

Standart Result of Conditional Prob →

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = P(A \text{ when } B \text{ has already occurred})$$

$$P(B/A) = \frac{P(B \cap A)}{P(A)} = P[B \text{ when } A \text{ " " " "}]$$

$$P(A \cap B/C) = \frac{P(A \cap B \cap C)}{P(C)} = P[\text{Simultaneous occurrence of } A \& B \text{ when } C \text{ has already occurred}]$$

Note → (1) In Case of Ind Events, Condition has No Significance. i.e

Let A & B are Ind Events then

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A) \cdot P(B)}{P(B)} = P(A)$$

Similarly $P(B/A) = P(B)$

eg $P\left(\frac{\text{odd Number on dice}}{\text{Head on Coin}}\right) = ? = P(\text{odd No. on Dice}) = \frac{3}{6} = \frac{1}{2}$

$\because$ Dice and Coin are Ind.

(2) To check Independency of Events, we have following three methods;

(M-I) By using Def^n.

(M-II) if $[P(A \cap B) = P(A) \cdot P(B)]$ $\Longleftrightarrow$ then [A & B are Ind]

(M-III) if $[P(A/B) = P(A)]$ $\Longleftrightarrow$ then [A & B are Ind]

Q. if $P(A)=\frac{1}{3}$, $P(B)=\frac{1}{4}$, $P(A/B)=\frac{1}{6}$ then find the prob of their Simultaneous occurrence?

Sol:- (a) $\frac{1}{12}$ (b) $\frac{1}{24}$ (c) $\frac{1}{6}$ (d) None

$\because P(A/B) \neq P(A) \Rightarrow$ A & B are Not Ind. (By M-II)

Hence using Multi Th,

$$P(A \cap B) = P(A/B) \cdot P(B)$$

$$= \frac{1}{6} \times \frac{1}{4} = \frac{1}{24}.$$

Q. if $P(A)=1$, $P(B)=\frac{1}{2}$ → $P(A/B)=?=1$, $P(B/A)=?=\frac{1}{2}$

Sol:- $P(A)=1$

⇓

A is sure Event

⇓

A will definitely occur

⇓

A is Ind from B (By M-I)

⇓

B is also Ind from A

Qe An Hydraulic structure has 4 Gates, which operates Independently. The Prob of failure of each Gate is 0.2. Given that Gate 1 has failed then Find the Prob that Gate 2 and Gate 3 will also fail?

Condition

Sol: $P(G_1) = P(G_2) = P(G_3) = P(G_4) = 0.2$

$$P\left(\frac{G_2 \cap G_3}{G_1}\right) = P(G_2 \cap G_3) = 0.2 \times 0.2 = 0.04$$

M-II $\text{Req. Prob} = P[G_1 \cap G_2 \cap G_3] = 1 \times 0.2 \times 0.2 = 0.04$

Q. p & q are considering to apply for a job. The prob that p applies for a job is 1/4.
the prob that p applies for job given that q applies for job is 1/2.
The " " q " " " " " " p " " " is 1/3. then find the prob
that p does not apply for a job given that q does not apply for job ? (Ans = 4/5)

$P(p) = \frac{1}{4}$, $P(q) = ?$

$P(p/q) = \frac{1}{2}$

$P(q/p) = \frac{1}{3}$

$P(\bar{p}/\bar{q}) = ?$

$$P(\bar{p}/\bar{q}) = \frac{P(\bar{p} \cap \bar{q})}{P(\bar{q})} = \frac{1 - P(p \cup q)}{1 - P(q)} = \frac{1 - \{P(p) + P(q) - P(p \cap q)\}}{1 - P(q)}$$

$\because P(q/p) = \frac{1}{3}$

$\frac{P(q \cap p)}{P(p)} = \frac{1}{3}$

$P(q \cap p) = \frac{1}{3} \times \frac{1}{4}$

$P(p/q) = \frac{1}{2}$

$\frac{P(p \cap q)}{P(q)} = \frac{1}{2}$

$\frac{1/12}{P(q)} = \frac{1}{2} \Rightarrow P(q) = \frac{1}{6}$

$$= \frac{1 - \frac{1}{4} - \frac{1}{6} + \frac{1}{12}}{1 - \frac{1}{6}} = \frac{4}{5}$$

(1)

THANK - YOU