GATE
ALL BRANCHES
ENGINEERING MATHEMATICS

Probability and Statistics

Lecture No. 12

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01 Problems based on Probability Distribution

only Single · Bernoulli

$E[X] = p$

$V(X) = (1-p)p$

$S.D = \sqrt{p(1-p)}$

Independent trials

only one Parameter

Binomial n Large

$P(X=x) = {}^{n}C_{r}\, p^{r} q^{n-r}$

$E[X] = np$

$V(X) = npq$

$S.D = \sqrt{npq}$

$B(n, p)$

Q. Questions

An unbiased coin is tossed an infinite number of times. The probability that the fourth head appears at the tenth toss is

(a) 0.067

(b) 0.073

(c) 0.082

(d) 0.091

HHH T T T T T T

H H T T H T T T T T

T T T H H H T T T

H T T T T T T T H H

Many Sequence Are Possible.

Tenth Toss

HEAD

$\frac{1}{2}$ (4^{th} HEAD)

10^{th} Toss

(HEAD)

Bernoulli Trials $\xrightarrow[n=9]{}$ binomal distribution

4th Head 10th Toss

$\frac{1}{2}$

1

$S = \{H, T\}$

$P(H) = \frac{1}{2}$

$P(T) = \frac{1}{2}$

$P(H) = \frac{1}{2} \quad P(T) = \frac{1}{2} \quad n = 9 \quad r = 3$

$P(X = 3\,\text{Head}) = {}^{9}C_{3} \left(\frac{1}{2}\right)^{3} \left(\frac{1}{2}\right)^{9-3}$

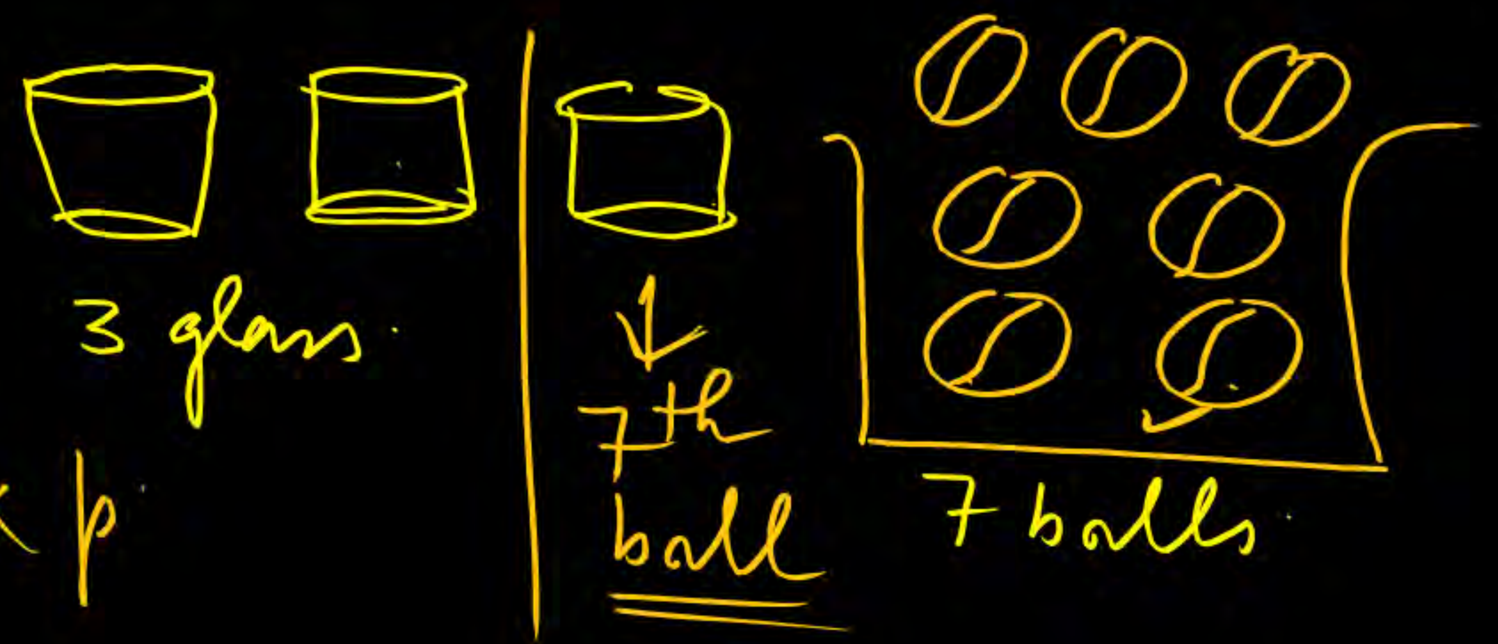
$$\boxed{P(X = 3\,\text{HEAD}) = \frac{{}^{9}C_{3}}{2^{9}}}$$

$P(10^{th}\ \text{Toss}\ 4\ \text{HEAD}) = \frac{{}^{9}C_{3}}{2^{9}} \times \frac{1}{2}$ Ans

$$= {}^6C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^{6-2}\times \frac{1}{2}$$

$\times p$

$B(n, p) \times p$



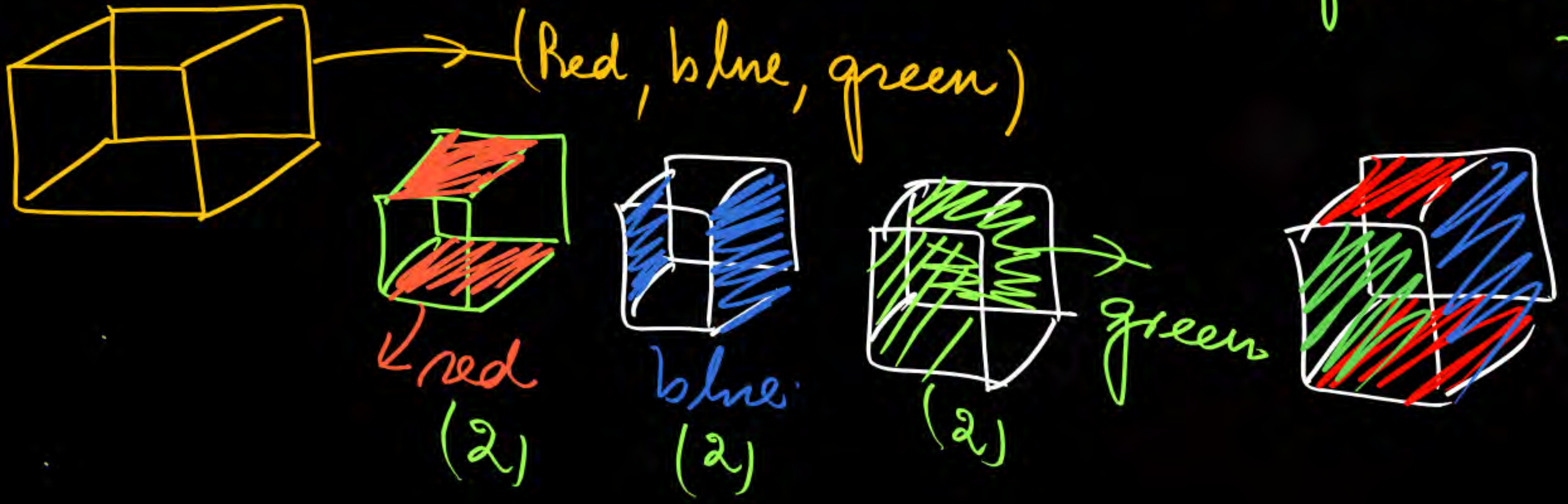
Q. Questions

(balanced Die)

Consider an unbiased cubic die with opposite faces coloured identically and each face coloured red, blue or green such that each colour appears only two times on the die. $\boxed{\text{If the die is thrown thrice,}}$ the probability of obtaining red colour on top face of the die at least twice is

No. of trials are fixed

$n = 3$



$n = 3$

Red colour = SUCCESS
Not red = failure

Bernoulli Trials (Independent)

$P(\text{red color}) = \frac{2}{6} = \frac{1}{3}$ ✓

$P(\text{Not red}) = \frac{2G + 2B}{6} = \frac{4}{6} = \frac{2}{3}$ ✓

$P(X \geq 2) = P(\text{atleast Two}) = P(X=2) + P(X=3)$

Using Bionomial Distribution

$n = 3 \quad p = \frac{1}{3} \quad q = \frac{2}{3} \quad r \geq 2$

${}^nC_r\, p^r q^{n-r}$

$P(X \geq 2) = {}^3C_2 \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^{3-2} + {}^3C_3 \left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right)^{3-3} = \frac{3 \times 2}{2 \times 1} \times \frac{1}{3} \times \frac{1}{3} \times \frac{2}{3} + 1 \frac{1}{27} \left(\frac{2}{3}\right)^0$

✓ $\boxed{P(X \geq 2) = \frac{7}{27}}$

Q. Questions

$(r \geq 2)$ Indep. Bernoulli $\left[\begin{array}{l}\text{success} - 6 \text{ come} \\ \text{failure} - 6 \text{ Not come}\end{array}\right.$

The probability of obtaining at least two SIX' in throwing a fair dice 4 times is

No. of trials $n = 4$

Using Binomial Dis

$P(X=r) = {}^nC_r\, p^r q^{n-r}$

$P(S) = \frac{1}{6} = p$

$P(F) = \frac{5}{6} = q$

$r \geq 2$

(a) 425/432

(b) 19/144

(c) 13/144

(d) 125/432

$P(X \geq 2) = P(X=2) + P(X=3) + P(X=4)$

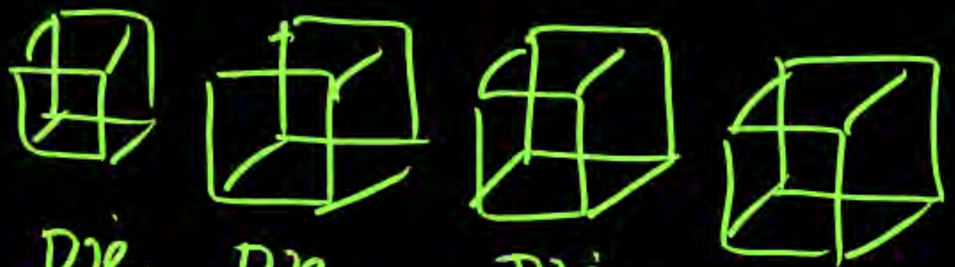
$= 1 - P(X=0) - P(X=1)$

$P(X \geq 2) = 1 - {}^4C_0 \left(\frac{1}{6}\right)^0 \left(\frac{5}{6}\right)^{4-0} - {}^4C_1 \left(\frac{1}{6}\right)^1 \left(\frac{5}{6}\right)^{4-1}$

$= \frac{19}{144}$ Ans

No 6		5	5	2	1
1	6	6	5	2	1
2	6	6	6	2	3
3	6	6	6	6	3
4	6	6	6	6	6

$$P(X \geq 2)$$
$$= P(X=2) + P(X=3)$$
$$= + P(X=4$$



Die 1   Die 2   Die 3   Die 4

$$P(X \geq 2) = 1 - P(X=0) - P(X=1)$$

Q. ## Questions

Difference between mean and variance of a binomial random variable is 1 and difference between their squares is 11. Find the probability of getting exactly three success.

$$\begin{cases} \text{MEAN} - \text{variance} = 1 \\ (\text{mean})^2 - (\text{var})^2 = 11 \end{cases}$$

$r = 3$

$p = P(S) = ?$

Number of trials $n = ?$

$q = P(F) = ?$

mean $E[X] = np$

variance $V(X) = npq$

$np - npq = 1$ — (1)

$n^2p^2 - n^2p^2q^2 = 11$ — (2)

$\rightarrow n, p, q$

$np - npq = 1$, $n^2p^2 - n^2p^2q^2 = 11$ $n =$

$\Rightarrow np(1-q) = 1$ —① $n^2p^2(1-q^2) = 11$ —②

Equ^n ② ÷ ①

$$\Rightarrow \frac{\cancel{n^2p^2}(\cancel{1-q^2})}{\cancel{np}(\cancel{1-q})} = \frac{11}{1}$$

$$= np(1+q) = 11$$

Using Equ^n ①

$$np = \frac{1}{(1-q)}$$

$$\Rightarrow \frac{1}{(1-q)}(1+q) = \frac{11}{1}$$

$$\frac{1+q}{1-q} = \frac{11}{1}$$

$$1+q = 11 - 11q$$

$$q = \frac{5}{6} \quad p = 1-q = 1-\frac{5}{6} = \frac{1}{6}$$

Using Equ^n ① and Put The value P and q.

$$np(1-q) = 1$$

$$n \times \frac{1}{6}\left(1-\frac{5}{6}\right) = 1 \quad n = 36$$

$n = 36 \quad p = \frac{1}{6} \quad q = \frac{5}{6} \quad r = 3$

$$P(X=3) = {}^{36}C_3 \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right)^{36-3}$$

$$\boxed{P(X=3) = {}^{36}C_3 \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right)^{33}}$$

Ans

$$B(n, p)$$

$$= {}^nC_r\, p^r q^{n-r}$$

Q. Questions

In an Examination of 10 Multiple choice question (one or more can be correct) out of 4 options. A student decided to mark the answer at random. Find the probability he gets the exact two questions correct.

$n = 10$

Multiple choice question

A

B

C

D

IIT-Advanced

Question — 1 option

— 2 option

— 3 option

— 4 option

correct

Not correct

bernoulli Trials

What is the prob (for single question) correct

Not correct

1 option is correct = A B C D
= 4 options

2 options are correct: AB BC CD DA AC BD

3 options are correct = ABC BCD CDA DAB

4 options are correct = ABCD = 1

Total choices (question correct)

$$= {}^4C_1 + {}^4C_2 + {}^4C_3 + {}^4C_4 = 15$$

Prob. of single question $= \frac{1}{15}$ $P(\text{correct}) = \frac{1}{15}$

#

$\boxed{A}$
$\boxed{B}$
$\boxed{C}$
$\boxed{D}$
$= {}^4C_1 = 4$

A B, B C, C D, D A, A C, B D $= {}^4C_2 = 6$

A B C, B C D, C D A, D A B $= {}^4C_3 = 4$

$P(C) = \frac{1}{15}$ $P(\text{Not}) = \frac{14}{15}$ $n = 10$ $r = 2$

Using Binomial Distribution

$$P(X=r) = {}^nC_r\, p^r q^{n-r}$$

$$= {}^{10}C_2 \left(\frac{1}{15}\right)^2 \left(\frac{14}{15}\right)^{10-2}$$

$$= {}^{10}C_2 \left(\frac{1}{15}\right)^2 \left(\frac{14}{15}\right)^{8}$$ Ans

$${}^nC_0 + {}^nC_1 + {}^nC_2 + {}^nC_3 + \dots = 2^n - 1$$

No. of option = 4

$2 \times 2 \times 2 \times 2$

$= 2^4 = 16$ ways.

$= 2^4 - 1 = 15$ ways.

$2^5 - 1 =$

Q. Questions

ME

In a manufacturing plant, the probability of making a defective bolt is 0.1. The mean and standard deviation of defective bolts in a total of 900 bolts are respectively

$P(\text{def}) = 0.1$

$P(\text{Not}) = 0.9$

$n = 900$ bolts

(a) 90 and 9

(b) 9 and90

(c) 81 and 9

(d) 9 and 81

mean $\mu = E[X] = np = 900 \times 0.1 = 90$

variance $\sigma_x^2 = V(X) = npq = 900 \times 0.1 \times 0.9$

$= \sqrt{81}$

$S.D = \sqrt{81} = 9$

Thank You!

GW Soldiers