GATE

ALL BRANCHES

ENGINEERING MATHEMATICS

Single Variable Calculus

Lecture No. 1

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01 Single Variable calculus

Limit ✓ Existence of limit

✓ How to evaluate the limit

Existence of Limit:

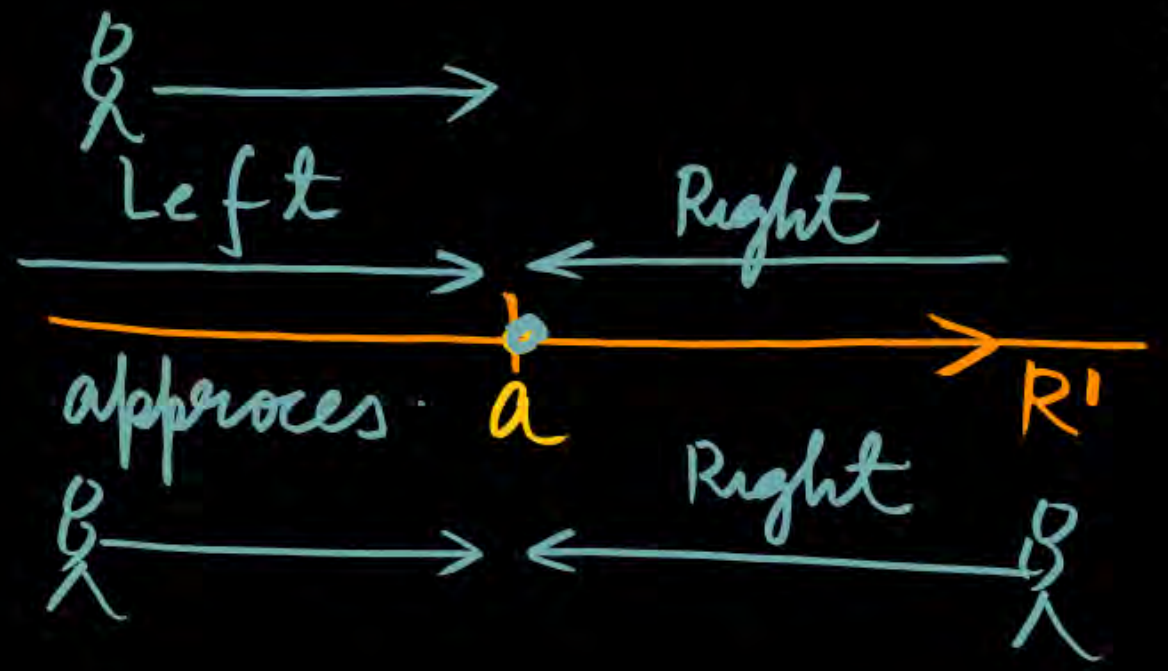
$$y = f(x)$$

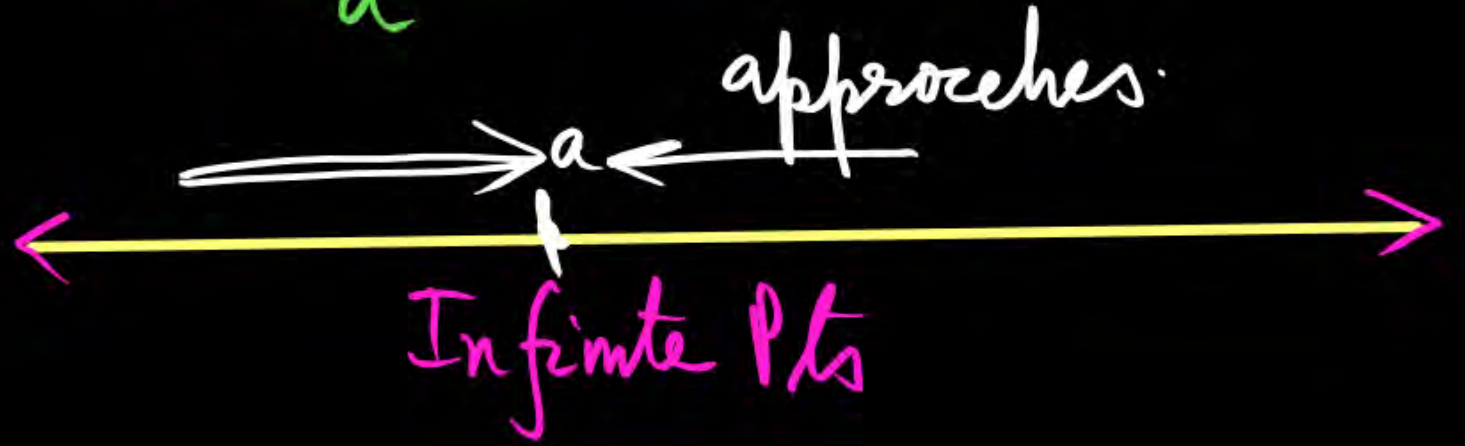
$$\underset{x \to a}{\mathrm{Lt}}\ f(x)$$

Limiting value of $f(x)$

$x \to a$

x Tends Towards to a

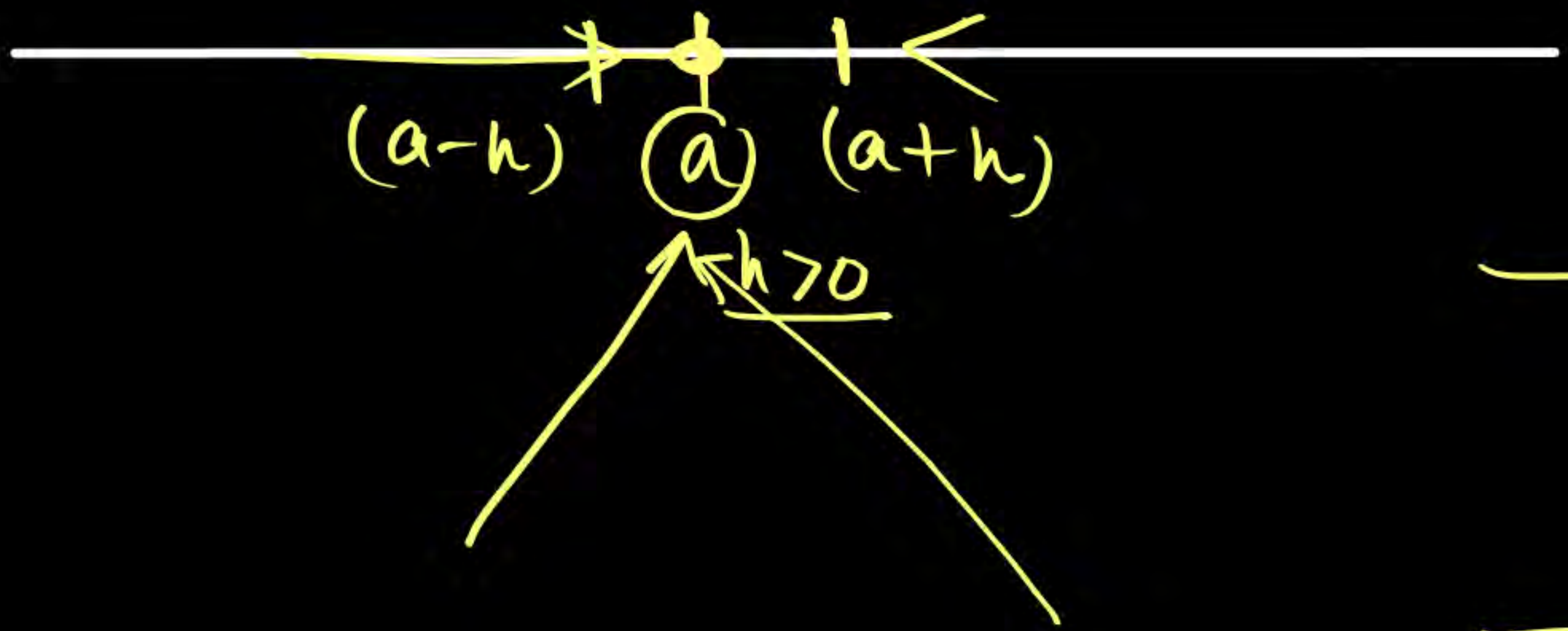


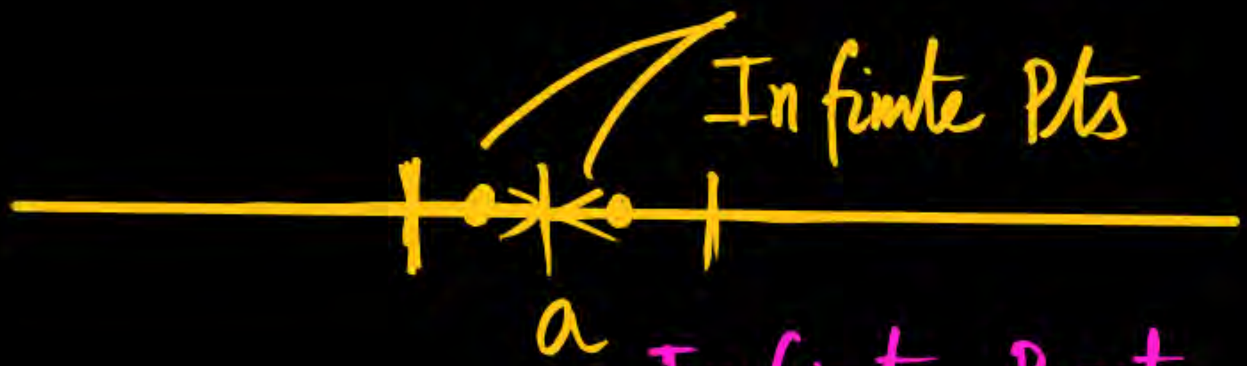


Right ward approaches $= (2+h)$

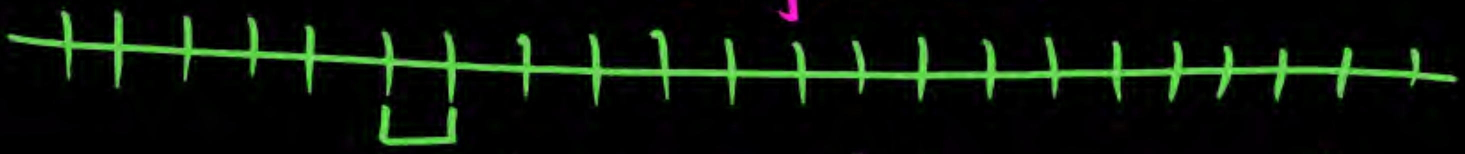
Left ——— $= (2-h)$

step·size $h > 0$



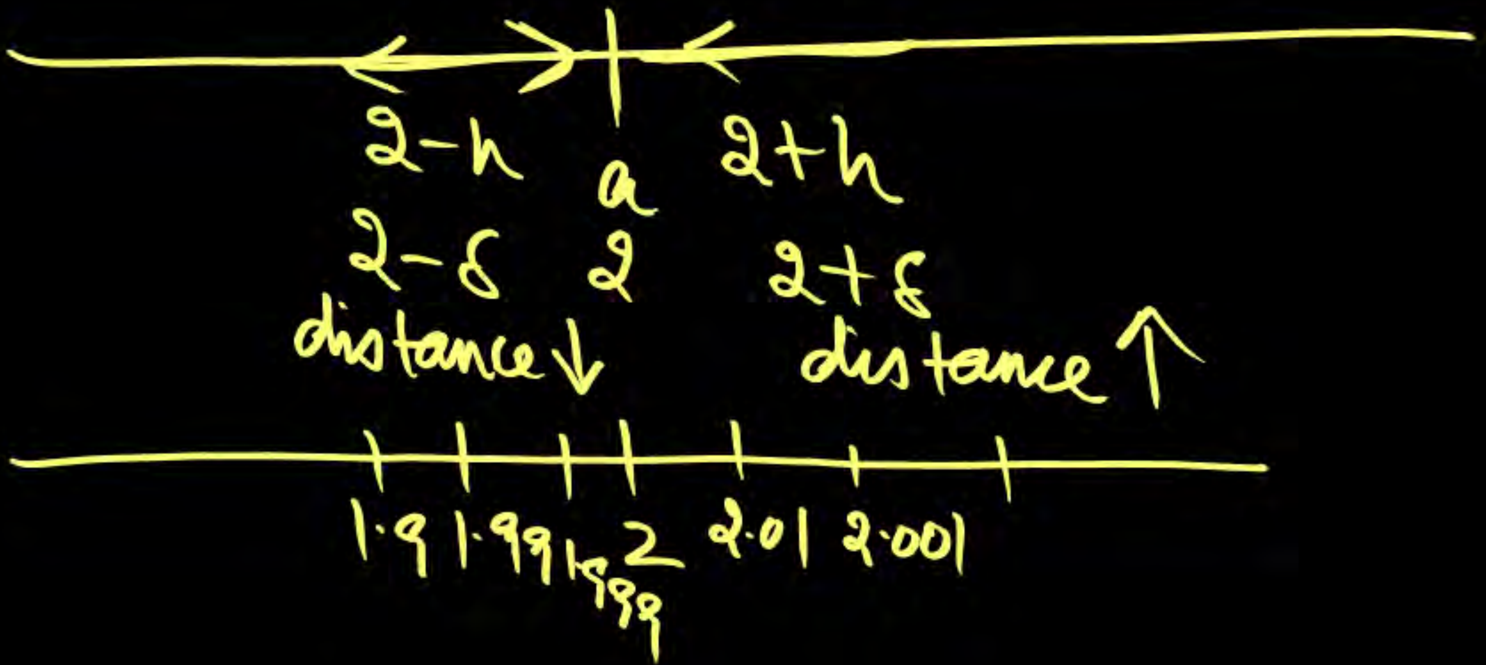


Infinite Point

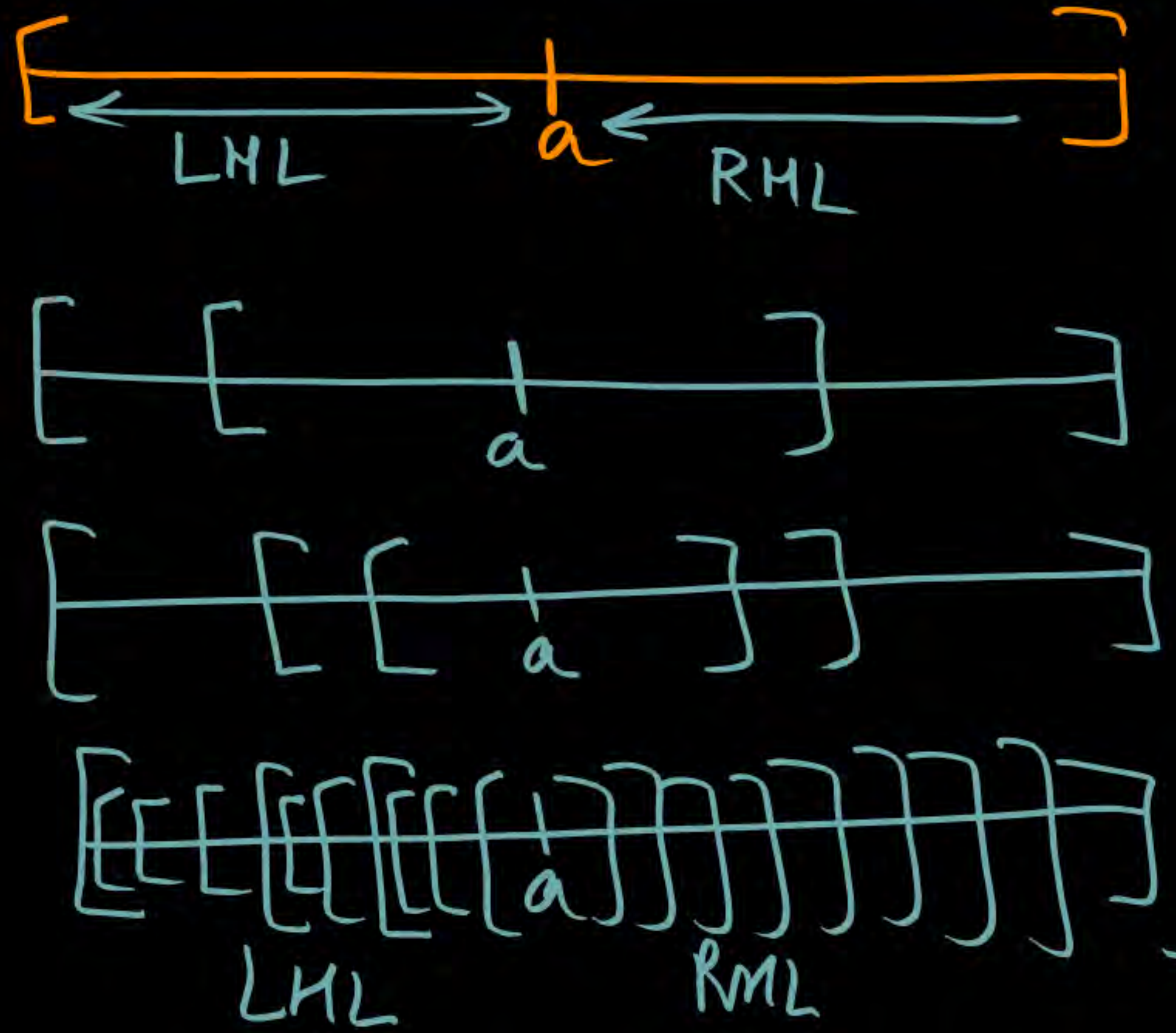


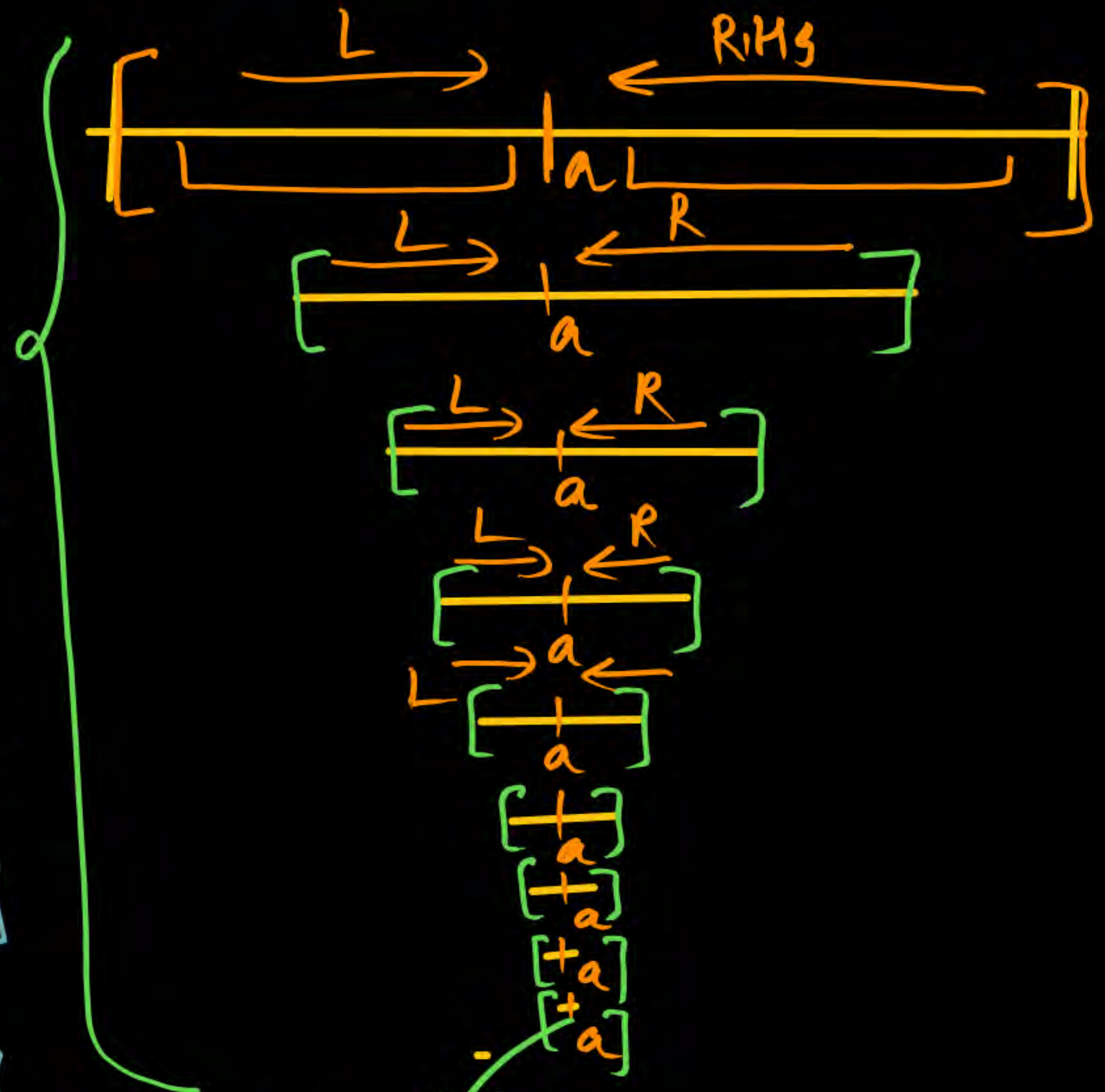
distance step·size $= h$
$= \delta$

$\delta > 0,\ h > 0$



If Interval Are Reduced Half-Half Approaches The Point =





Nested Intervals $h \to 0$

Existence of limit

$y = f(x)$

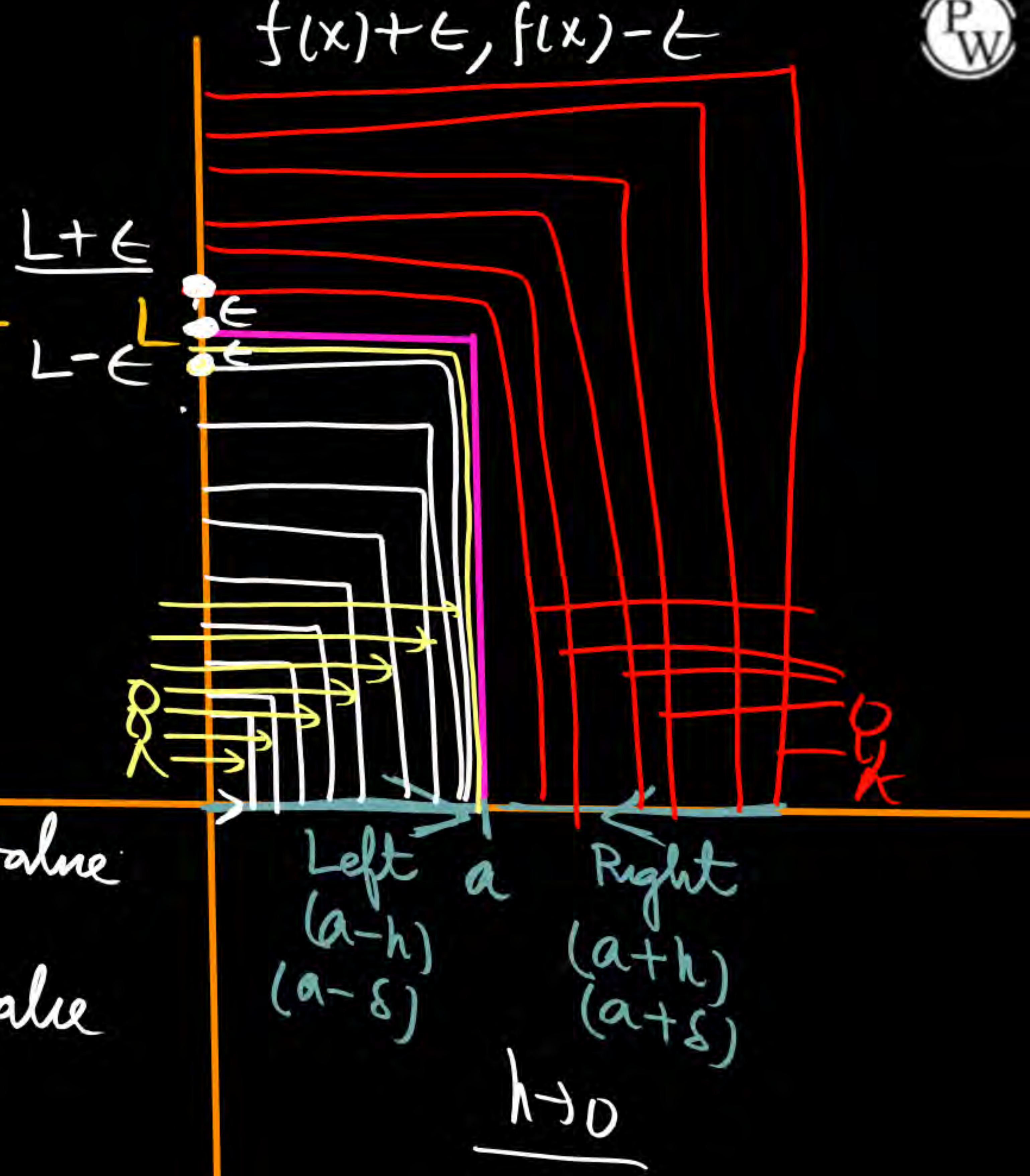
$\underset{x \to a}{Lt}\, f(x)$ = limiting value of $f(x)$

If existance of Limit

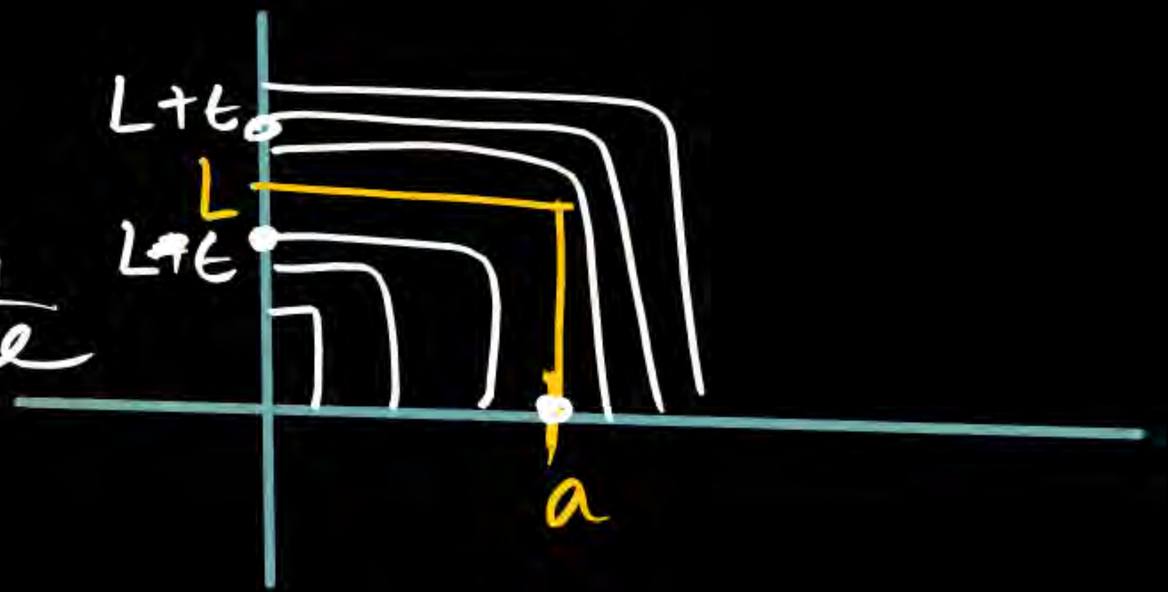
$LHL = RHL$

$\underset{h \to 0}{lt}\, f(a-h) = LHL$ = finite value

$\underset{h \to 0}{lt}\, f(a+h) = RHL$ = finite value



$$\left\{\begin{array}{l} LHL = RHL \\ \lim\limits_{h \to 0} f(a-h) = \lim\limits_{h \to 0} f(a+h) = \text{finite} \end{array}\right.$$



Limit existence :

If LHL ≠ RHL and Not finite

Limit does Not exists

left $f(x) = (x+2)$ LHL (left Hand)

✓ X	$(x+2) = f(x)$	$f(x) = x+2$
$x = 1.9$	$1.9 + 2 =$	3.9
$x = 1.99$	$1.99 + 2$	3.99
$x = 1.999$	$1.999 + 2$	3.9999
$x = 1.9999$	$1.99999 + 2$	3.999999

→ converge $= 4$

$f(x) = x+2$ RHL (Right Hand)

X	$x+2$	$f(x)$
2.01	$2.01 + 2$	4.01
2.001	$2.001 + 2$	4.001
2.0001	$2.0001 + 2$	4.0001

4

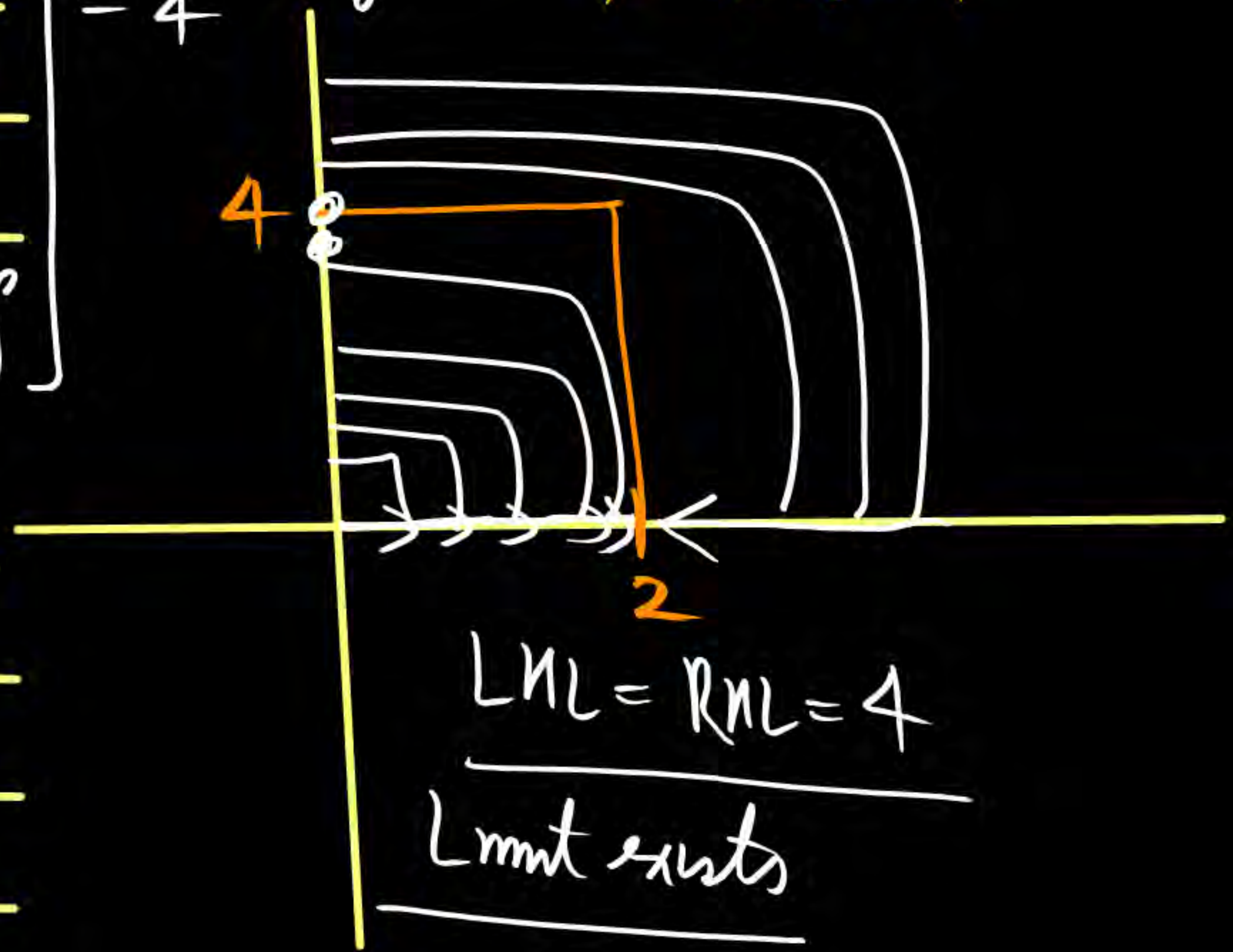
$y = (x+2)$

$\lim_{x \to 2} (x+2)$

Pnt $x = 2$

$y = x+2$

$y = 2+2 = 4$



$LHL = RHL = 4$

Limit exists

✓ **Method**:

$$\lim_{x\to 2}(x+2)$$

$f(x)=(x+2)$

$f(2-h)=(2-h)+2$

$=4-h$

$f(2+h)=(2+h)+2$

$=4+h$

Left Hand limit

$$=\lim_{h\to 0} f(a-h)$$

$$=\lim_{h\to 0} f(2-h)=\underbrace{\lim_{h\to 0}(4-h)}_{\text{Plug in Limits}}=4$$

Right Hand limit

$$=\lim_{h\to 0} f(a+h)$$

$$=\lim_{h\to 0} f(2+h)=\lim_{h\to 0}(4+h)=4$$

$\boxed{LHL=RHL}$

limit exists

Q. Questions

$f(x) = 2x-1$

$f(1-h) = 2(1-h)-1$

Show that the limit of: $f(x)=\begin{cases} 2x-1 \; ; & x \le 1 \\ x \; ; & x > 1 \end{cases}$ at x = 1 exists.

$f(x) = \begin{cases} 2x-1 & x \le 1 \\ x & x > 1 \end{cases}$ at $x = 1$

$(2x-1)$ | x

(LHL) 1 (RHL)

Left Hand limit $= \lim_{h\to 0} f(a-h) = \lim_{h\to 0} f(1-h)$

$= \lim_{h\to 0} 2(1-h)-1$

$= \lim_{h\to 0} 2-2h-1 = \lim_{h\to 0}(1-2h) = 1$

RHL

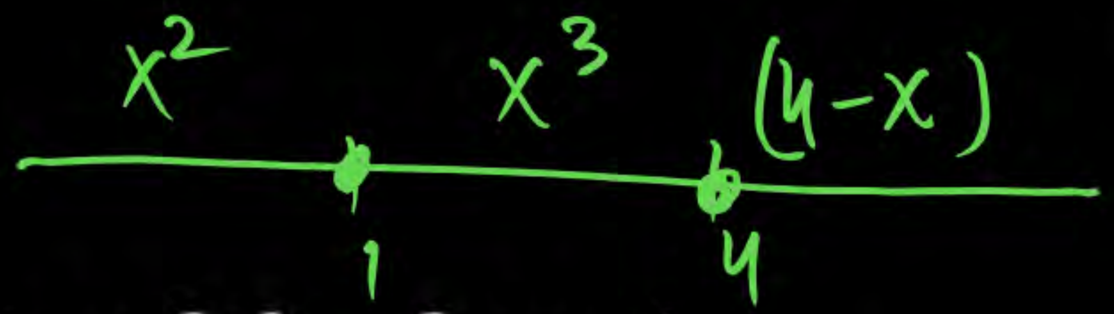
$= \lim_{h\to 0} f(1+h)$

$= \lim_{h\to 0}(1+h)$

$= 1$

LHL = RHL

Q. Questions



Evaluate the left hand and right hand limits of the function defined by

$$f(x)=\begin{cases} x^2 , & x<1 \\ x^3 , & 1<x<4 \\ 4-x , & x>4 \end{cases}$$

at $x = 1, 4$ and hence check existence of limit at $x = 1, 4$.

$$f(x)=\begin{cases} x^2 & x<1 \\ x^3 & 1<x<4 \\ 4-x & x>4 \end{cases}$$

At a Point $x = 1$

$$LHL = \lim_{h\to 0} f(a-h)$$

$$= \lim_{h\to 0} f(1-h)$$

$$= \lim_{h\to 0} (1-h)^2$$

$$= \lim_{h\to 0} (1+h^2-2h)$$

$$\boxed{LHL = 1}$$

x^2 | 1 x^3 | 4 $(4-x)$

LHL | RHL

x^2 | 1 x^3

LHL | RHL

x^3 | 4 $(4-x)$

$$RHL = \lim_{h\to 0} f(a+h) = \lim_{h\to 0} f(1+h)$$

$$= \lim_{h\to 0} (1+h)^3$$

$$= \lim_{h\to 0} [1+h^3+3h^2+3h]$$

$$RHL = 1$$

RHL = LHL exists limits

At point $x=4$

$$LHL = \lim_{h\to 0} f(a-h)$$

$$\Rightarrow \lim_{h\to 0} f(4-h)$$

$$\Rightarrow \lim_{h\to 0} (4-h)^3$$

$$\boxed{LHL \Rightarrow 64}$$

$$\left.\begin{array}{l} LHL \neq RHL \\ 64 \neq 0 \end{array}\right]$$ does Not exists

x^3 | $(4-x)$

LHL 4 Right

$$RHL = \lim_{h\to 0} f(a+h)$$

$$= \lim_{h\to 0} f(4+h)$$

$$= \lim_{h\to 0} \left[\cancel{4}-\cancel{(4}+h)\right]$$

$$= 0$$

$f(x) = x-4$

$f(4+h) = \cancel{4}+h-\cancel{4}$

$= h$

Q. Questions

Evaluate the left hand and right-hand limits of the function

$$f(x)=\begin{cases}\dfrac{\sqrt{(x^2-6x+9)}}{(x-3)}, & x \neq 3 \\ 0, & x=3\end{cases}$$

✓ Square Root Function
✓ modulus function
$\Big]$ does Not exists

at $x = 3$ and hence comment on the existence of limit at x = 3.

$$f(x) = \begin{cases} \dfrac{\sqrt{x^2-6x+9}}{x-3} & x \neq 3 \\ 0 & x = 3 \end{cases}$$

$$f(x) = \begin{cases} \dfrac{\sqrt{(x-3)^2}}{x-3} & x \neq 3 \\ 0 & x = 3 \end{cases}$$

$$f(x) = \frac{|x-3|}{(x-3)} \quad x \neq 3$$

$$RHL = \lim_{h\to 0} f(a+h)$$

$$= \lim_{h\to 0} f(3+h)$$

$$x \longrightarrow (3+h)$$

$$= \lim_{h\to 0} \frac{|\cancel{3}+h-\cancel{3}|}{(\cancel{3}+h-\cancel{3})} = \lim_{h\to 0} \frac{|h|}{h}$$

$$RHL = \lim_{h\to 0} \frac{|h|}{h} = \lim_{h\to 0} \frac{h}{h} = 1$$

$$LHL = \lim_{h\to 0} f(a-h) = \lim_{h\to 0} f(3-h)$$

$$= \lim_{h\to 0} \frac{|\cancel{3}-h-\cancel{3}|}{(3-h-3)} = \lim_{h\to 0} \frac{|-h|}{-h} = \lim_{h\to 0} \frac{h}{-h} = -1$$

$$LHL \neq RHL$$

= does Not exists

Q. Questions

If $f(x)=\begin{cases}\dfrac{x-|x|}{x}, & x\neq 0\\ 2, & x=0\end{cases}$ the $\lim_{h\to 0} f(x)$ is

(a) 2

(b) 0

(c) 1

(d) Does not exist

$f(x)=\begin{cases}\dfrac{x-|x|}{x} & x\neq 0\\ 2 & x=0\end{cases}$

$f(-h)=\dfrac{-h-|-h|}{-h}$

LHL $=\lim_{h\to 0} f(a-h)$ $\quad a^- \quad a^+$

$=\lim_{h\to 0} f(0-h)$

$=\lim_{h\to 0} f(-h)$

LHL $=\lim_{h\to 0}\dfrac{-h-|-h|}{-h}$

$=\lim_{h\to 0}\dfrac{-h-h}{-h}$

$=\lim_{h\to 0}\dfrac{-2h}{-h}=2$

RHL $=\lim_{h\to 0}\dfrac{+h-|h|}{h}=\dfrac{h-h}{h}=0$

LHL $\neq$ RHL

Does Not exists

Limit evaluate

Thank You!

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