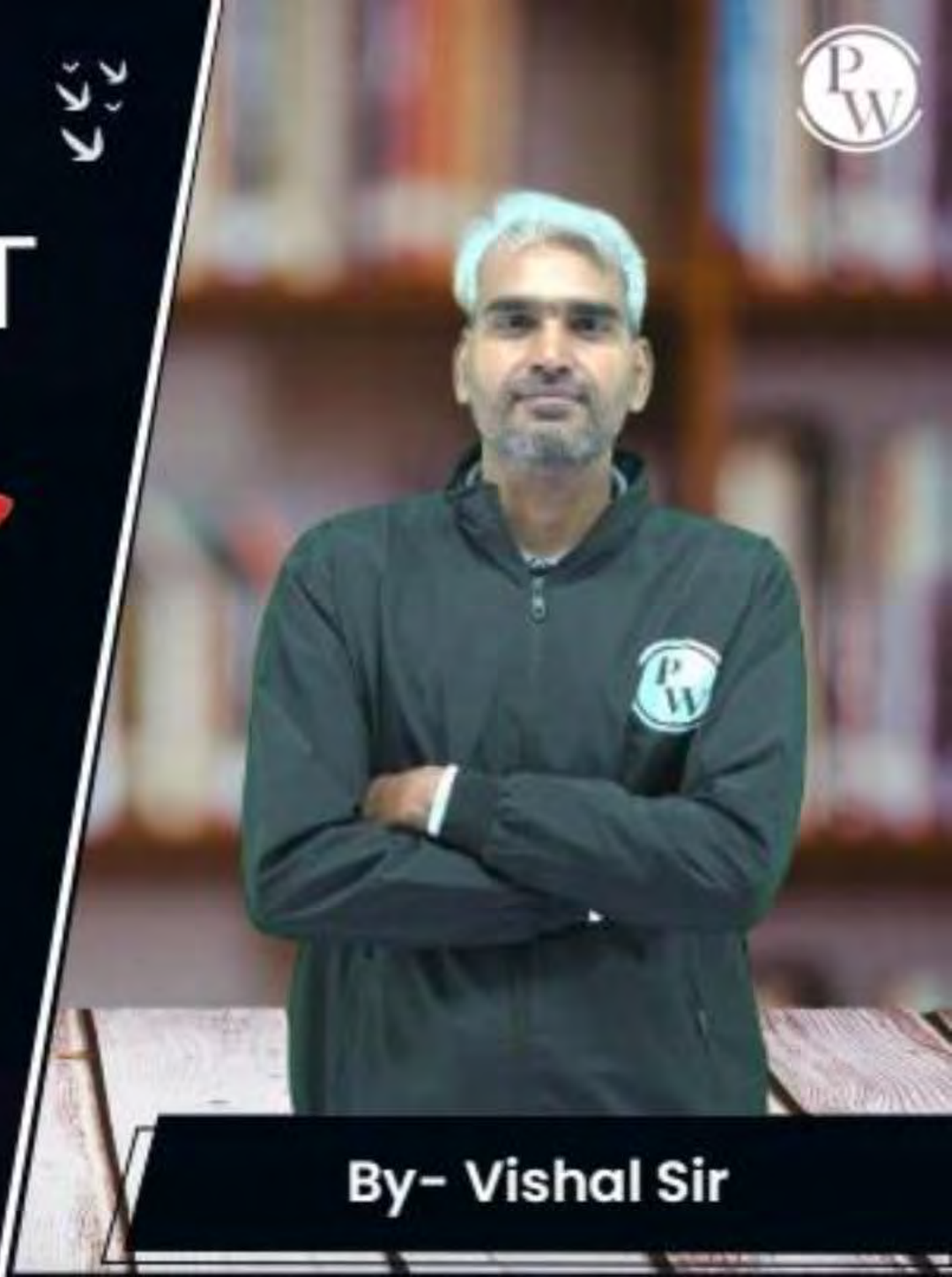
Computer Science & IT

Discrete Mathematics

Mathematical Logic

Lecture No. 02



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Recap of Previous Lecture

Topic Propositions and their types

Topic Connectives

Topic Tautology, Contradiction

Topic Contingency and Satisfiable propositional functions

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Topics to be Covered

- Topic: Logical implications and logical equivalences
- Topic: Important equivalences
- Topic: Important statements
- Topic: Argument / Inference
- Topic: Rules of inference

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Topic : Logical Implication / Implication

Let P and Q are any two propositional functions.

* Whenever P is true if Q is also true then P logically implies Q. is true.

If there exist any case for which P is true but Q is false, then P logically implies Q is invalid

{i.e. P does not logically implies Q}

P logically implies Q if and only if P → Q is a tautology.

Topic : Logical Equivalence / Equivalence

- If P and Q are any two propositional functions, then P equivalent to Q is written as $P \equiv Q$. or $P \cong Q$
- P and Q are said to be equivalent if and only if they have same truth table. let $P : \sim a \vee b$ & $Q : a \to b$ then $P \equiv Q$
- $P \equiv Q$ if and only if $P \leftrightarrow Q$ is a tautology.
- $P \equiv Q$ if and only if P logically implies Q and Q logically implies P.
- $P \leftrightarrow Q$ is a tautology if and only if $P \to Q$ is a tautology as well as $Q \to P$ is a tautology.

Topic : Some important equivalences

① $\sim(\sim p) \equiv p$ {Double negation}

② $p \vee q \equiv q \vee p$
$p \wedge q \equiv q \wedge p$ } {Commutative}

③ $(p \vee q) \vee r \equiv p \vee (q \vee r)$
$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$ } {Associative}

Topic : Some important equivalences

(4) $p \wedge (q \vee R) \equiv (p \wedge q) \vee (p \wedge R)$

$p \vee (q \wedge R) \equiv (p \vee q) \wedge (p \vee R)$

} Distributive.

(5) $\sim(p \wedge q) \equiv \sim p \vee \sim q$

$\sim(p \vee q) \equiv \sim p \wedge \sim q$

} De' Morgan's

Topic : Some important equivalences

*(6) $p \wedge (p \vee q) \equiv p$
$p \vee (p \wedge q) \equiv p$
} Absorption law

(7) $p \wedge p \equiv p$
$p \vee p \equiv p$

(8) $p \vee T \equiv T$
$p \wedge f \equiv f$

(9) $p \wedge T \equiv p$
$p \vee f \equiv p$

(10) $T \wedge T \equiv T$
$T \vee T \equiv T$
$f \wedge f \equiv f$
$f \vee f \equiv f$
$T \wedge f \equiv f$
$T \vee f \equiv T$

Some important Equivalences :-

① $P \rightarrow Q \equiv \sim P \vee Q$

② $P \rightarrow Q \equiv \sim Q \rightarrow \sim P$

③ $P \leftrightarrow Q \equiv (P \rightarrow Q) \wedge (Q \rightarrow P)$

Topic : Some important statements

1. P implies Q $\equiv p \rightarrow q$
2. If P then Q $\equiv p \rightarrow q$
3. P only if Q $\equiv p \rightarrow q$
4. P is sufficient condition for Q $\equiv p \rightarrow q$
5. Q is necessary condition for P $\equiv p \rightarrow q$

Continuity is necessary for differentiability $\equiv$ Differentiability $\rightarrow$ Continuity

* Q follows from $P \equiv P \rightarrow Q$

Topic : Some important statements

6. P if Q $\equiv$ if q then $p \equiv q \to p$

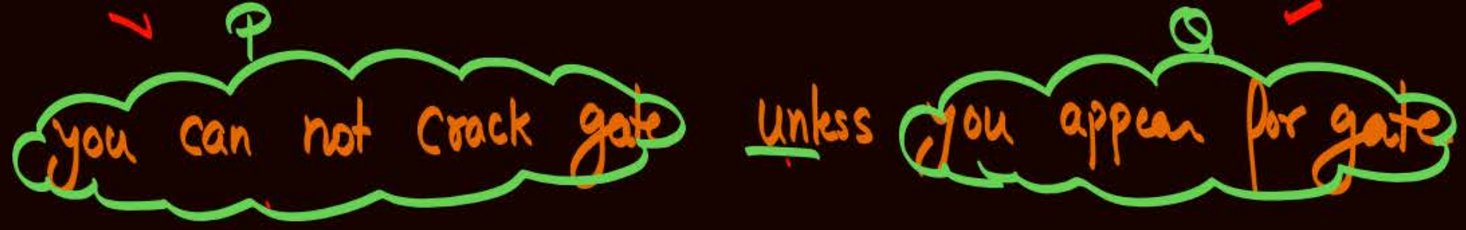
7. P when Q $\equiv q \to p$ (if)

8. P follows from Q $\equiv q \to p$

9. P unless $\sim Q \equiv \sim(\sim q) \to p \equiv q \to p$

Simply replace unless by $\vee \equiv p \vee \sim q \equiv \sim q \vee p \equiv q \to p$

10. P unless $q \equiv \sim q \to p \equiv \sim p \to q$



If you do not appear for gate $\equiv$ $\sim Q$ then you can not crack gate $\equiv$ P

$$\sim Q \Longrightarrow P$$

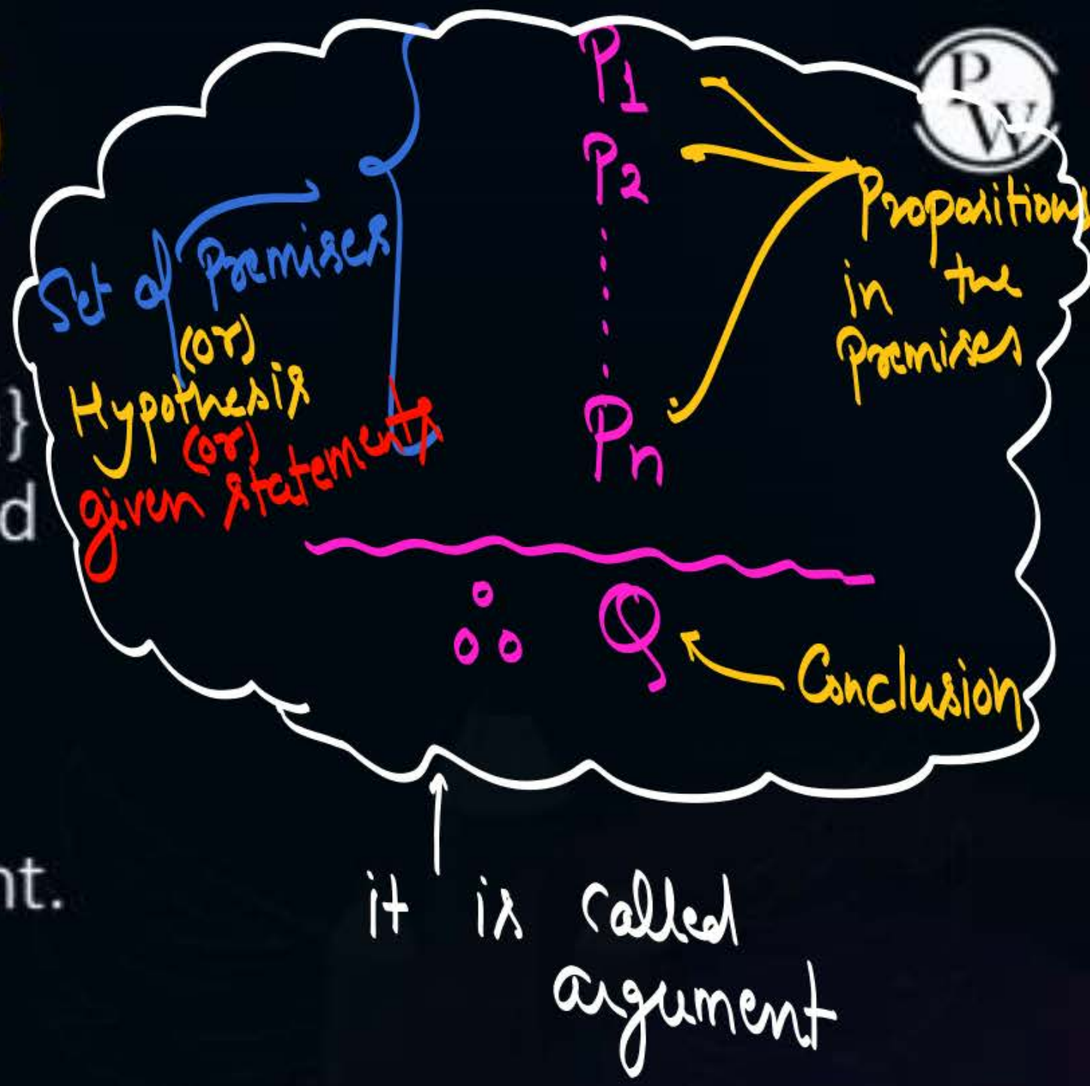
$$P \text{ unless } Q \equiv \sim Q \rightarrow P$$

If you cracked gate $(\sim P)$ $\Longrightarrow$ then you appeared for gate (Q) $\equiv \sim P \rightarrow Q$

Topic : Argument / Inference

The statement that,
"A set of premises { P1, P2, P3,.....,Pn} yields another proposition Q" is called an argument.

Q is called conclusion of the argument.



given statements

(i) If today is Sujay's B'day then today is 13th Aug.

(ii) Today is 13th Aug.

∴ Today is Sujay's B'day

Conclusion

Argument { it may be a valid argument (or) an invalid argument }

Topic : Argument / Inference

- Argument may be valid or invalid.
- The process of reasoning whether the argument is valid or invalid is called inference.

If conclusion Q can be inferred from the set of premises by applying some rules of inference and equivalences, then argument is said to be valid otherwise invalid.

i.e. whenever $P_1, P_2, P_3, \ldots P_n$ are true if Q is guaranteed to be true

Topic : Argument / Inference

Following statements are equivalent,

- Argument {P1, P2, P3,.....,Pn} $\vdash Q$ is valid.
- {P1, P2, P3,.....,Pn} Logically implies Q is true/valid.
- {P1 $\wedge$ P2 $\wedge$ P3 $\wedge$ ..... $\wedge$ Pn} $\rightarrow Q$ is a tautology.

Topic : Rules of inference

Any valid reasoning is rule of inference.

Topic : Rules of inference

1. Simplification

Topic : Rules of inference

2. Addition

Topic : Rules of inference

3. Conjunction

Topic : Rules of inference

4. Disjunctive Syllogism

Topic : Rules of inference

5. Conjunctive Syllogism

Topic : Rules of inference

6. Modus Ponens

7. Fallacy of affirming the consequent

Topic : Rules of inference

8. Modus Tollen's

9. Fallacy of denying the antecedent

Topic : Rules of inference

10. Transitivity

Topic : Rules of inference

11. Dilemma

Topic : Rules of inference

12. Constructive Dilemma

Topic : Rules of inference

13. Destructive Dilemma

Topic : Rules of inference

14. Resolution

$$p \vee q$$

$$\sim p \vee r$$

$$\therefore$$

Topic : Rules of inference

15. Non Sequitur

2 mins Summary

Topic Logical implications and logical equivalences

Topic Important equivalences

Topic Important statements

Topic Argument / Inference

Topic Rules of inference

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THANK - YOU

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