Computer Science & IT

Discrete Mathematics

Set Theory & Algebra

Lecture No. 07

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Recap of Previous Lecture

Topic Types of Relations

- Diagonal
- Reflexive
- Irreflexive
- Symmetric
- Anti-symmetric

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Topics to be Covered

Topic Types of Relations

Asymmetric Rel^n

Transitive Rel^n

Complement of a Rel^n

Inverse of a Rel^n

Composite of two Rel^n

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$$x \neq y$$

	$\{(x,y),(y,x)\}$	$\{(x,y), \cancel{(y,x)}\}$	$\{\cancel{(x,y)}, (y,x)\}$	$\{\cancel{(x,y)}, \cancel{(y,x)}\}$
Symmetric	Allowed	Not allowed	Not allowed	Allowed
Anti-symm.	Not allowed	Allowed	Allowed	Allowed
Asymmetric	Not allowed	Allowed	Allowed	Allowed

Topic : Asymmetric Relation

A relation R on set A is said to be asymmetric only if, $\underbrace{\text{if } (x,y) \in R \text{ then } (y,x) \notin R \ \forall x,y \in A}$

i.e. if $x R y$ then $y \not R x \ \forall x, y \in A$

If $(x,y) \in R$ then $(y,x) \notin R$ even if $x = y$

↓

i.e. diagonal order pairs are not allowed in Asymmetric Relⁿ

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eg: let $A = \{1, 2, 3\}$

$R_1 = \{ \}$ smallest asymmetric relⁿ on set A

$R_2 = \{(1,2), (3,1), (3,3)\}$

diagonal order pairs are not allowed

$\therefore$ Not Asymmetric Relⁿ

$R_3 = \{(1,2), (2,3), (3,1)\}$ it is an asymmetric Relⁿ

Note:- Empty relation is the only relation which is symmetric, anti-symmetric as well as asymmetric

Note: Every asymmetric Rel^n is anti-symmetric Rel^n, but every anti-symmetric relation need not be an asymmetric Rel^n.

Note: All subsets of an anti-symmetric relation are also anti-symmetric

Similarly,

All subsets of an asymmetric relation are asymmetric

Q: let A is a set with n-elements,

How many asymmetric relations are possible

on set A.

Asymmetric Relation = None of the diagonal order Pair should be present and Possible Choices w.r.t. non-diagonal order pairs

i.e. order pairs w.r.t. distinct elements

Number of Asymmetric Rel^n $= {}^{n}C_{0} * 3^{\left({}^{n}C_{2}\right)}$

number of pairs of two distinct elements

for each pair of two distinct elements we have 3 choices, s.t. it is asymmetric

$= 1 * 3^{\frac{n(n-1)}{2}}$

$= 3^{\left(\frac{n^2-n}{2}\right)}$

Topic : Transitive Relation

A relation R on set A is said to be transitive only if,

if $(x,y) \in R$ and $(y,z) \in R$, then $(x,z) \in R$ $\forall x,y,z \in A$

i.e if $x R y$ and $y R z$, then $x R z$ $\forall x,y,z \in A$

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eg: Let $A = \{1, 2, 3\}$

* $R_1 = \{\ \}$: smallest transitive relation on set A.

* $R_2 = \{(1,2), (2,1)\}$

$(1,2) \in R_2$ and $(2,1) \in R_2$ $\therefore$ $(1,1)$ should be present ✓
$\therefore$ Not transitive

$R_3 = \{(1,2), (2,1), (1,1)\} \equiv \{(2,1), (1,2), (1,1)\}$

$(2,1) \in R_3$ & $(1,2) \in R_3$ $\therefore$ $(2,2)$ should be present in R_3

$(2,2) \notin R_3$ $\therefore$ Not transitive

$R_4 = \{(1,2), (2,1), (1,1), (2,2)\}$ ✓

$R_4 = \{(1,1), (1,2), (2,1), (2,2)\}$ ✓ ✓ ✓ ✓

Note: Presence of diagonal order pair will never create any Problem w.r.t. transitivity

Order pairs in which 1^{st} element is '1'

2nd element is '1'

$(1,1)$ & $(1,1)$ $\therefore$ $(1,1)$ already belong to R_4

same

$(1,1)$ & $(1,2)$ $\therefore$ $(1,2)$ already belongs to R_4

same

all order pairs are okay w.r.t. definition of transitivity, $\therefore$ R_4 is transitive relation

order pairs
with '2' as their 1^{st} element

$(2,1) \longrightarrow \therefore (1,1)$ { already belongs to R_4 }

$(1,2)$ & $(2,1)$

$(1,2)$ & $(2,2)$

ends with '2'

$\underbrace{(2,2)} \longrightarrow \therefore (1,2)$ { already belongs to R_4 }

diagonal order pair
$\therefore$ it will not create any problem

Q: Let $A = \{1, 2, 3\}$

$R_5 = \{(1,2), (2,1), (1,1), (2,2), (1,3), (3,1), (3,3)\}$

$= \{(1,1), (1,2), (1,3), (2,1), (2,2), (3,1), (3,3)\}$

$(2,1) \in R_5$ & $(1,3) \in R_5$

$\therefore (2,3)$ should belong to R_5

but $(2,3) \notin R_5$

$\therefore$ Not transitive

Topic : Complement of a Relation

Let R be a relation from set A to set B $\{$i.e $R \subseteq A \times B\}$, Complement of relation R is also a relation from A to B and it is defined as $R^c = A \times B - R$

or

$\overline{R}$

or

R'

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$A \times B = \{(a,1), (a,2), (b,1), (b,2), (c,1), (c,2)\}$

eg let $A = \{a, b, c\}$ $B = \{1, 2\}$

<u>and</u> $R = \{(a,1), (b,2), (c,1), (c,2)\}$

$R^c = ? = A \times B - R$

$= \{(a,2), (b,1)\}$

Topic : Inverse of a Relation

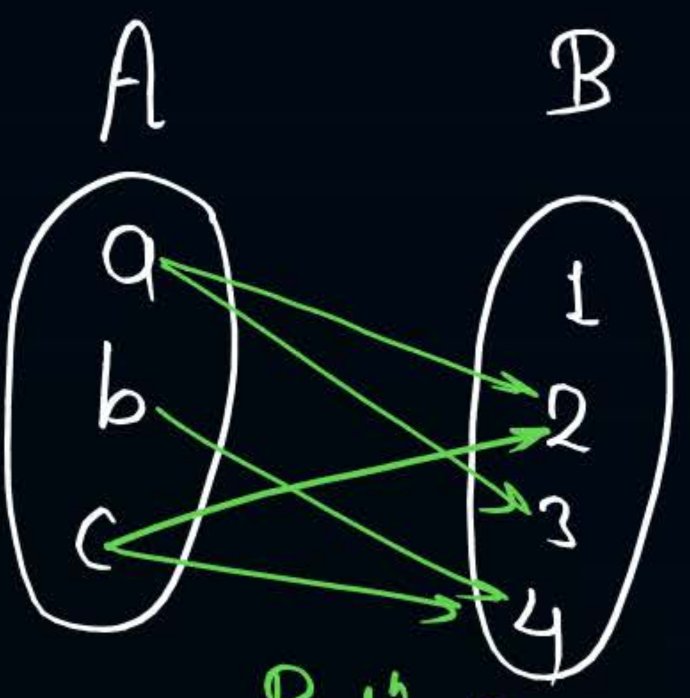
* Let R be a relation from set A to set B $\{$i.e $R \subseteq A \times B\}$,

Inverse of relation R is a relation from set B to set A, and it is defined as

✓ $R^{-1} = \{(b,a) \mid (a,b) \in R\}$

$R^{-1} \subseteq B \times A$

eg:



Rel^n R

$R = \{(a,2), (a,3), (b,4), (c,2), (c,4)\}$

$R^{-1} = \{(2,a), (3,a), (4,b), (2,c), (4,c)\}$

elements from B

elements from A

Topic : Composite of two Relations

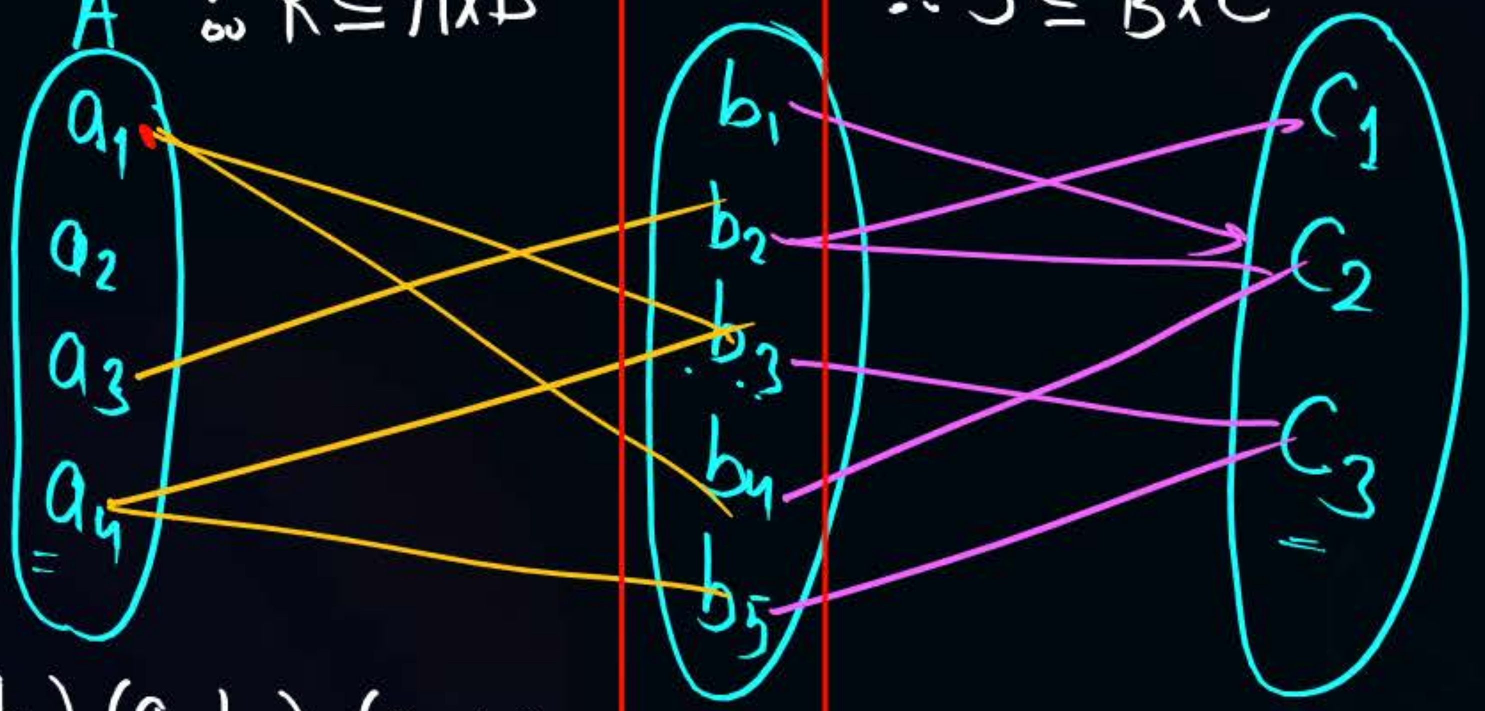
$R : A \to B$ $\therefore R \subseteq A \times B$

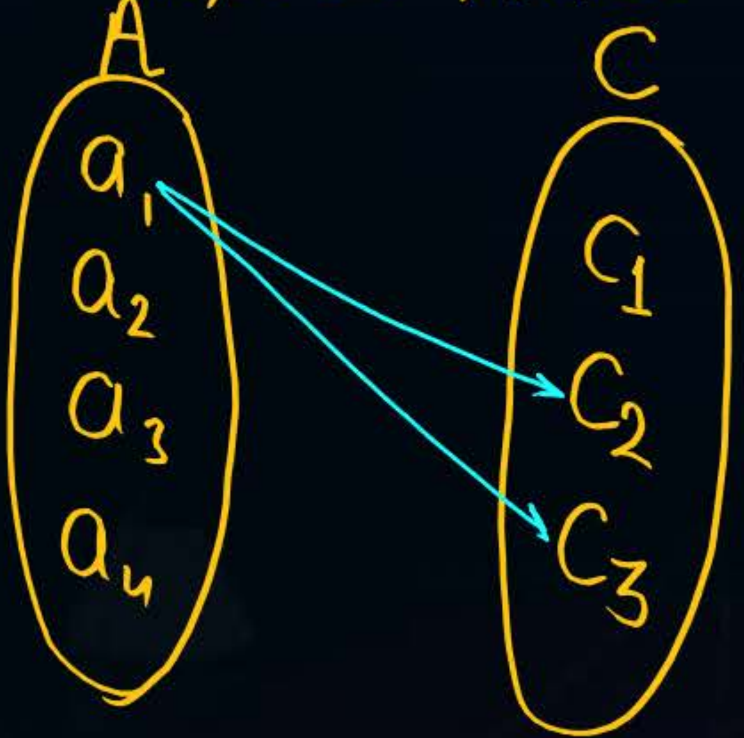
$S : B \to C$ $\therefore S \subseteq B \times C$

Composite Relation of R & S

$R;S : A \to C$

$\therefore R;S \subseteq A \times C$





$R = \{(a_1, b_3), (a_1, b_4), (a_3, b_2), (a_4, b_3), (a_4, b_5)\}$

$S = \{(b_1, c_2), (b_2, c_1), (b_2, c_2), (b_3, c_3), (b_4, c_2), (b_5, c_3)\}$

$R;S = \{(a_1, c_3), (a_1, c_2), (a_3, c_1), (a_3, c_2), (a_4, c_3)\}$

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Topic : Composite of two Relations

R is a relation from X to Y

S is a relⁿ from Y to Z

Let X,Y and Z be three sets, and

If $R \subseteq X \times Y$ and $S \subseteq Y \times Z$ are two binary relations, then their composition $R;S$ is the relation defined as,

$R;S = \{ (x,z) \mid (x,z) \in X \times Z$ and there exists $y \in Y$ such that $(x,y) \in R$ and $(y,z) \in S\}$

i.e., $R;S \subseteq X \times Z$ is defined by the rule that $(x,z) \in R;S$ if and only if there is an element $y \in Y$ such that $(x,y) \in R$ and $(y,z) \in S$

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2 mins Summary

Topic Types of Relations

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THANK - YOU

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