GATE
ALL BRANCHES

ENGINEERING MATHEMATICS

Probability and Statistics

Lecture No. 09

BY– RAHUL SIR

01 Problems based on Random Variables

Discrete
+
continuous Random variable

$\boxed{\text{Statistical Averages}}$

Statistical Averages:

Average → center value

OR

Center of mass

OR

Center of gravity

Balacing Pts

Average

1
2
3
4
5
6

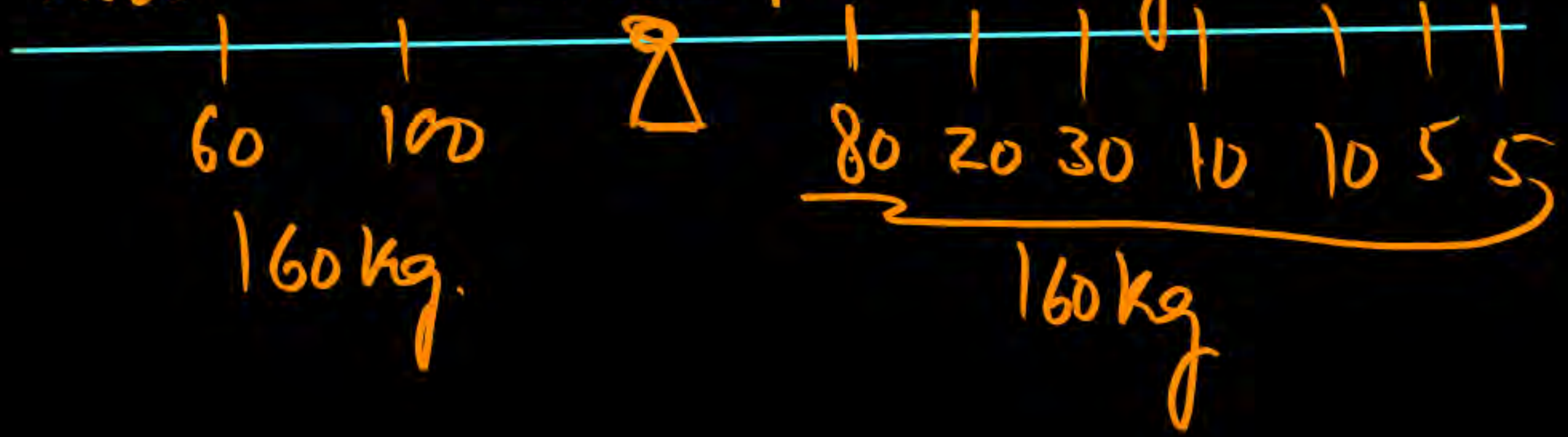
Average $= \frac{1+2+3+4+5+6}{6}$

$= 3.5$

$\Sigma x / N$

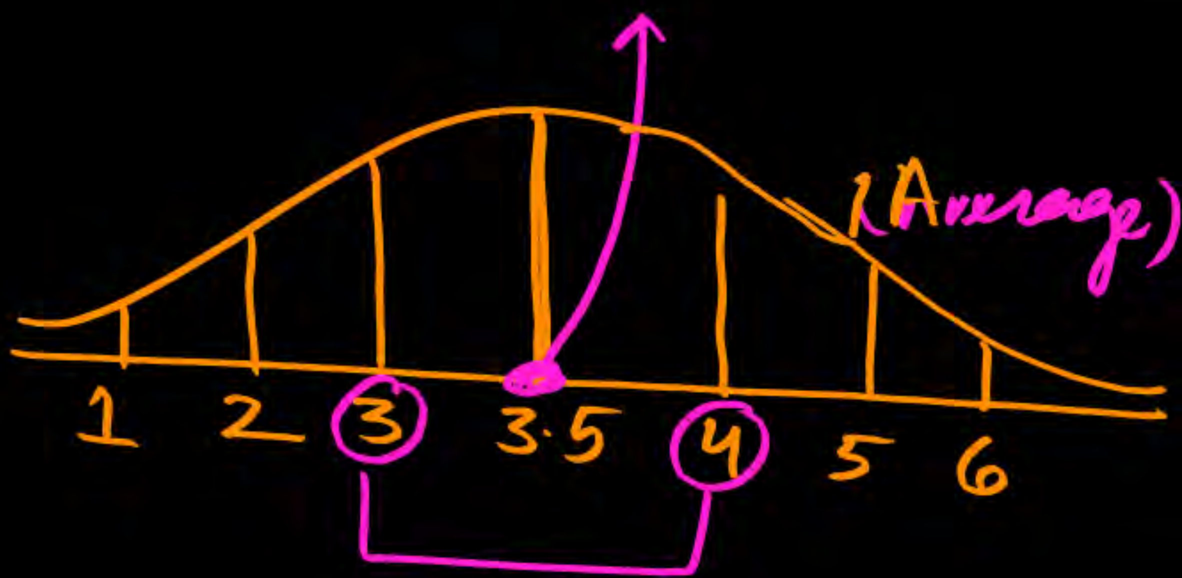
Rod

balance pt = Average = $\frac{\Sigma x}{n}$



Large No Trials.

Average = center of gravity



mid value

2) Median = mid value

n even $\frac{6}{2}$ th term, $\frac{6}{2}+1$ th term

3, 4

1 2 3 4 5 6

n = even: $\frac{n}{2}$ term, $\frac{n}{2}+1$ term

3) If n is odd No

median = $\frac{n+1}{2}$ th term

1, 2, 3, 4, 5

median = $\frac{5+1}{2}$ th term = 3th term

✓ Mode : Highest Frequent value.

1, 2, 2, 3, 4, 4, 4, 4, 5, 5, 2, 2, 2, 2, 3, 3, 3, 2, 2, 2, 2

Mode = 2 (Highest frequent value)

✓ Average = center value / center of mass / balance Pt.

median = mid value — n even / n odd

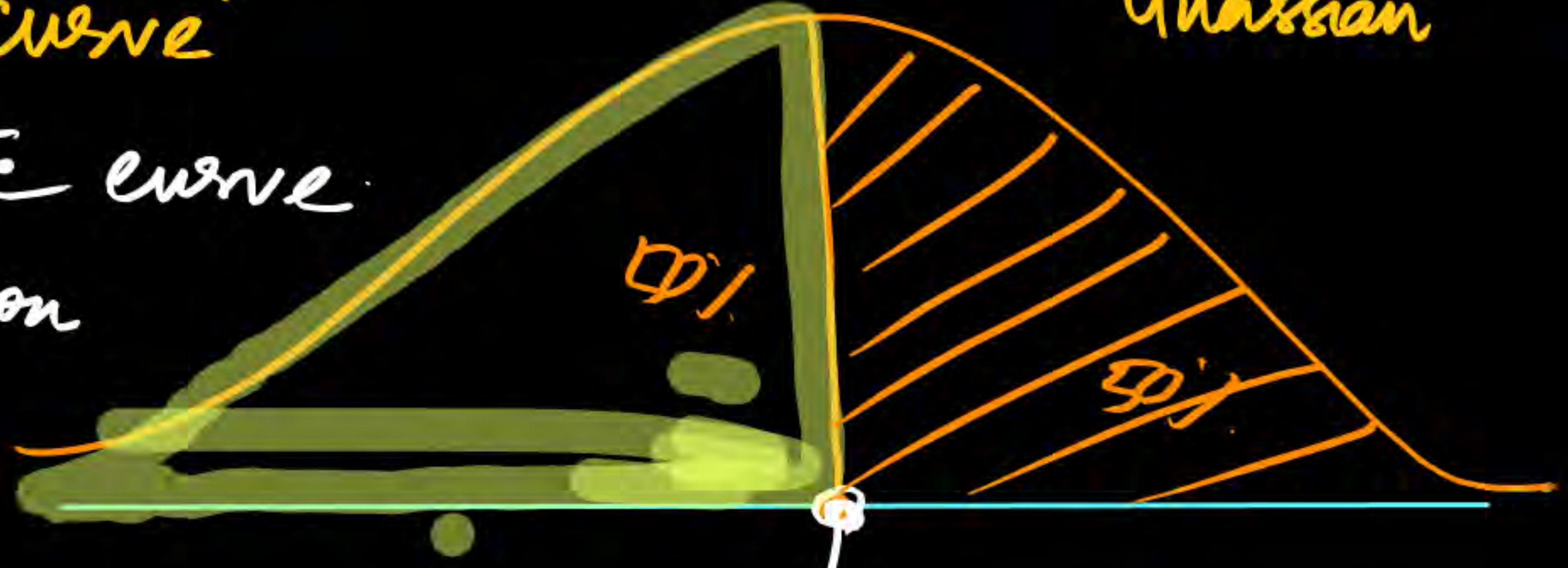
mode = most frequent value.

✓ If all values Are Distinct Then mode does not exists bell shaped curve

Normal random / Guassian

Symmetric curve

Distribution



$\boxed{\text{mean} = \text{median} = \text{mode}}$

"should we Play this GAME"

YES/NO

$$\begin{cases} P(\text{winning}) = \frac{4}{6} = \frac{2}{3} \uparrow \\ P(\text{Loosing}) = \frac{2}{6} = \frac{1}{3} \downarrow \end{cases}$$

X	+1(W)	+2	−3(LOOSE
P(X=x)	$\frac{3}{6}$	$\frac{1}{6}$	$\frac{2}{6}$

"Playing A GAME"

Casino

Throwing A Die

$\boxed{1}$ $\boxed{2}$ $\boxed{3}$ $\boxed{4}$ $\boxed{5}$ $\boxed{6}$

Die	Pay off
1	−3 (LOOSE)
2	+1 (win)
3	+1 (win)
4	+1 (win)
5	+2 (win)
6	−3 (LOOSE)

Discreet

$$\frac{n}{2}+\frac{n}{3}-n$$
$$=-\frac{n}{6}$$

for Large No. of Trials

$$\frac{n}{2}+\frac{n}{3}-n = \frac{3n+2n-6n}{6} = -\frac{n}{6}$$

X	+1	+2	-3
P(X=x)	3/6	1/6	2/6

X = discrete

$$\underbrace{1\times\frac{3}{6}\times n}_{+1\text{(Rupee)}} + \underbrace{2\times\frac{1}{6}\times n}_{\text{2 Rupee }(+2)} + \underbrace{(-3)\times\frac{2}{6}\times n}_{-3\text{ LOOSE}} = \text{Total Payoff}$$

$$= x_1\times P[X=x_1]\times n + x_2 P[X=x_2]\times n + x_3\times P[X=x_3]\times n = -\frac{n}{6}$$

$$= \cancel{n}\left[x_1 P[X=x_1] + x_2 P[X=x_2] + x_3 P[X=x_3]\right] = -\frac{\cancel{n}}{6}$$

$$= \boxed{x_1 P[X=x_1] + x_2 P[X=x_2] + x_3 P[X=x_3] = -\frac{1}{6}}$$

n Large of trials

Quit this GAME

$$x_1 P[X=x_1] + x_2 P[X=x_2] + x_3 P[X=x_3] = -\frac{1}{6}$$

$$= \sum_{i=1}^{3} x_i P[X=x_i] = -\frac{1}{6}$$

MEAN / Expected value / Average / center of mass / center of gravity

$$E[X] = \sum_{i=1}^{n} x_i P[X=x_i]$$

discrete Random variable.

$$\boxed{E[X] = x_1 P_1 + x_2 P_2 + x_3 P_3 + \cdots + x_n P_n}$$

$X \sim$ Discrete Random variable

How to find The expected value: (MEAN / center of gravity / Average)

$X = x_1, x_2, x_3, x_4 \ldots x_n$

is Discrete

$P_0 + P_1 + P_2 + P_3 + \ldots + P_n = 1$

X	x_1	x_2	x_3		x_n
$P(X=x)$	P_1	P_2	P_3		P_n

$$\mu = E[X] = \frac{x_1P_1 + x_2P_2 + x_3P_3 + \ldots + x_nP_n}{P_1 + P_2 + P_3 + \ldots + P_n}$$

$$\boxed{\mu = E[X] = x_1P_1 + x_2P_2 + x_3P_3 + \ldots + x_nP_n}$$

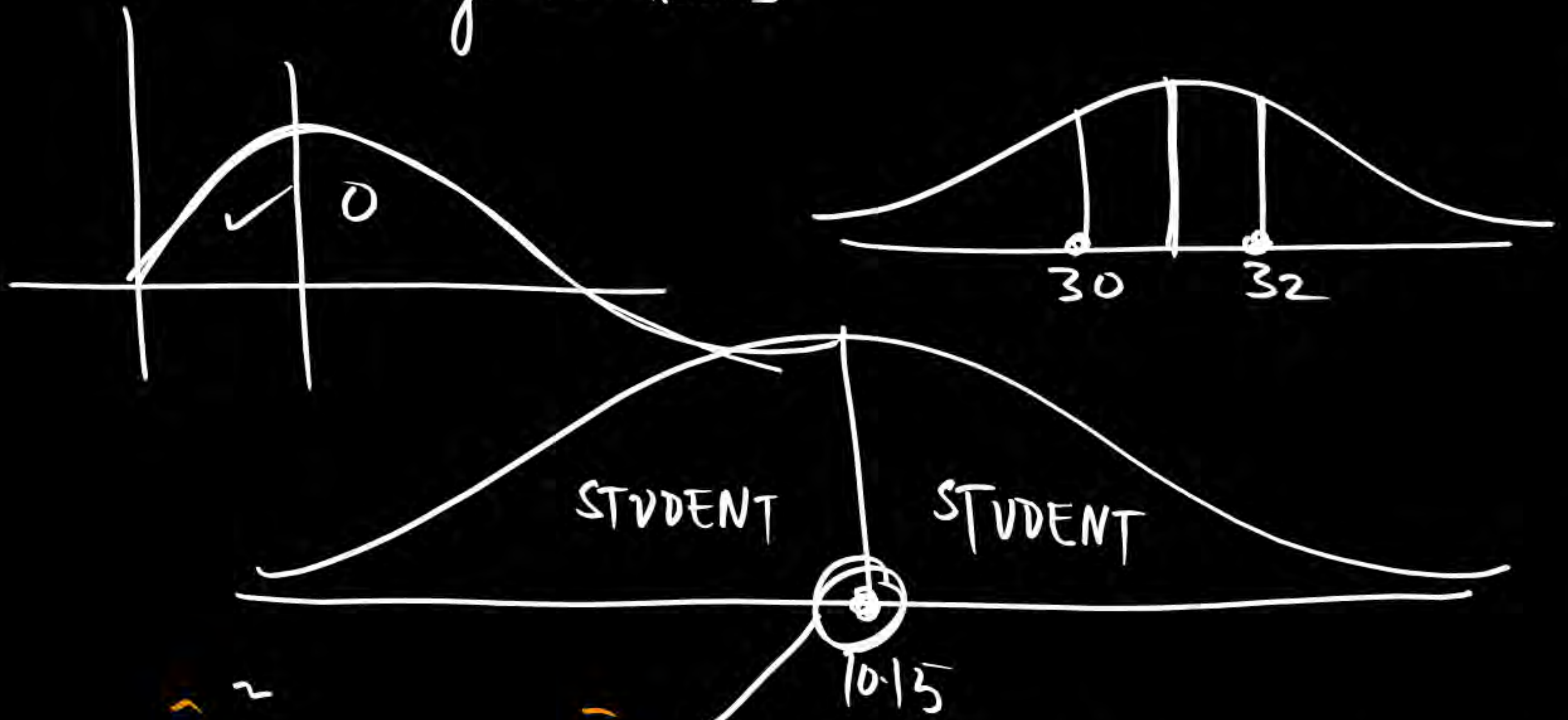
1000 students mean

Average $= \frac{\sum x}{n} = 10.15$

Class Performance

SOM marks $\underline{30}$

Large No. Trials



Single No – Average – whole class. Performance

I_1	10
I_2	12
I_3	13
I_4	14
	18
	19
	05
	06
	10
	12
	05
	06
I_{13}	02

Average

$\boxed{10.15}$

I

← 10.15 →

$$V(X) = \sigma_x^2 = \underbrace{E[X^2]}_{} - [E[X]]^2$$

$$V(X) = \sigma_x^2 = E[X^2] - [\text{Average}]^2$$

For discrete Random variable.

X	x_0	x_1	x_2	x_3	x_4		x_n
$P(X=x)$	P_0	P_1	P_2	P_3	P_4		P_n

$$E[X^2] = [x_0^2 P_0 + x_1^2 P_1 + x_2^2 P_2 + \dots + x_n^2 P_n]$$

$$E[X] = \sum_{i=0}^{n} x_i p_i$$

$$\boxed{\text{standard deviation} = \sqrt{\text{variance}}}$$

$E[X] = \mu$

1, 2, 3, 4, 5, 6 → 3.5

$(1-3.5)^2$

$(2-3.5)^2$

$(3-3.5)^2$

$(4-3.5)^2$

$(5-3.5)^2$

$(6-3.5)^2$

deviation

variance of random variable.

Throwing A Die:

x	1	2	3	4	5	6
$P(X=x)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

$$E[X] = 1\times\frac{1}{6} + 2\times\frac{1}{6} + 3\times\frac{1}{6} + 4\times\frac{1}{6} + 5\times\frac{1}{6} + 6\times\frac{1}{6}$$

$$= \frac{1+2+3+4+5+6}{6} = \frac{21}{6} = 3.5$$

$$V(X) = E[X^2] - [E[X]]^2$$

$$E[X^2] = 1^2\times\frac{1}{6} \times 2^2\times\frac{1}{6} + 3^2\times\frac{1}{6} + 4^2\times\frac{1}{6} + 5^2\times\frac{1}{6} + 6^2\times\frac{1}{6}$$

$$E[X^2] = \frac{1+4+9+16+25+36}{6} = 15.1$$

$$= 15.1 - (3.5)^2 = 2.85$$

Standard deviation [RMS value]

$$= \sqrt{\text{variance}}$$

$$= \sqrt{2.85} = 1.68$$

Discrete Random Variable	Continuous Random variable
1) MEAN $= \sum_{i=0}^{n} x_i p_i$	MEAN $= \int_a^b x f(x)\,dx$ $\quad a \le X \le b$ $\qquad F(x) = \int_a^b f(x)\,dx$ If Interval is Not given mean $= \int_{-\infty}^{\infty} x f(x)\,dx \quad -\infty \le X \le \infty$
2) variance $= E[x^2] - [E[x]]^2$ $= \sum_{i=0}^{n} x_i^2 p_i - \left[\sum_{i=0}^{n} x_i p_i\right]^2$	2) variance $= E[x^2] - [E[x]]^2$ $= \int_a^b x^2 f(x)\,dx - \left[\int_a^b x f(x)\,dx\right]^2$ $E[x^3] = \int_a^b x^3 f(x)\,dx \qquad E[x^4] = \int_a^b x^4 f(x)\,dx$

STANDARD DEV

Discrete	Continuous
1) $S.D = \sqrt{var(X)}$	1) $S.D = \sqrt{V(X)}$
2) $\sum_{i=0}^{n} P[X=x_i] = 1$	2) $\int_a^b f(x)\,dx = 1$

Q. Questions

In the following table, x is a discrete random variable and $p(x)$ is the probability density.

The standard deviation of x is

X	1	2	3
P(x)	0.3	0.6	0.1

(a) 0.18

(b) 0.36

(c) 0.54

(d) 0.6

x	1	2	3
p(x)	0.3	0.6	0.1

Standard deviation $= \sqrt{var(x)}$

$var(x) = (1)^2 \times 0.3 + (2)^2 \times 0.6 + (3)^2 \times 0.1 - (1 \times 0.3 + 2 \times 0.6 + 3 \times 0.1)^2$

$var(x) = 0.3 + 2.4 + 0.9 = (1.8)^2$

$= 3.6 - (3.24) = 0.36$

$S.D = \sqrt{0.36} = \underline{0.6}$

Q. Questions

$X = 0, 1, 2$

X	0	1	2
$P(X=x)$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{1}{6}$

A machine produces 0, 1 or 2 defective pieces in a day with associated probability of 1/6, 2/3 and 1/6, respectively. Then mean value and the variance of the number of defective pieces produced by

(a) 1 and 1/3

(b) 1/3 and 1

(c) 1 and 4/3

(d) 1/3 and 4/3

Mean value $E[X] = \sum_{i=1}^{n} x_i p_i = 0 \times \frac{1}{6} + 1 \times \frac{2}{3} + 2 \times \frac{1}{6} = 1$

$V(X) = E[X^2] - [E[X]]^2$

$= (0)^2 \times \frac{1}{6} + (1)^2 \frac{2}{3} + (2)^2 \frac{1}{6} - (1)^2$

$= \frac{2}{3} + \frac{4}{6} - (1)^2 = \frac{1}{3}$

$V(X) = \frac{1}{3}$

$E[X] = \mu = 1$

$\boxed{\text{Rahul Sir PW}}$ 10 'o clock Questions 15 questions

Thank You!

GW Soldiers