GATE
ALL BRANCHES

ENGINEERING MATHEMATICS

Probability & Statistics

LECTURE-15

BY- RAHUL SIR

$$\boxed{1 - P(X=0) - P(X=1)}$$

01 Gaussian Distributions Daddy Dis

01 Problems based on gaussian Distributions

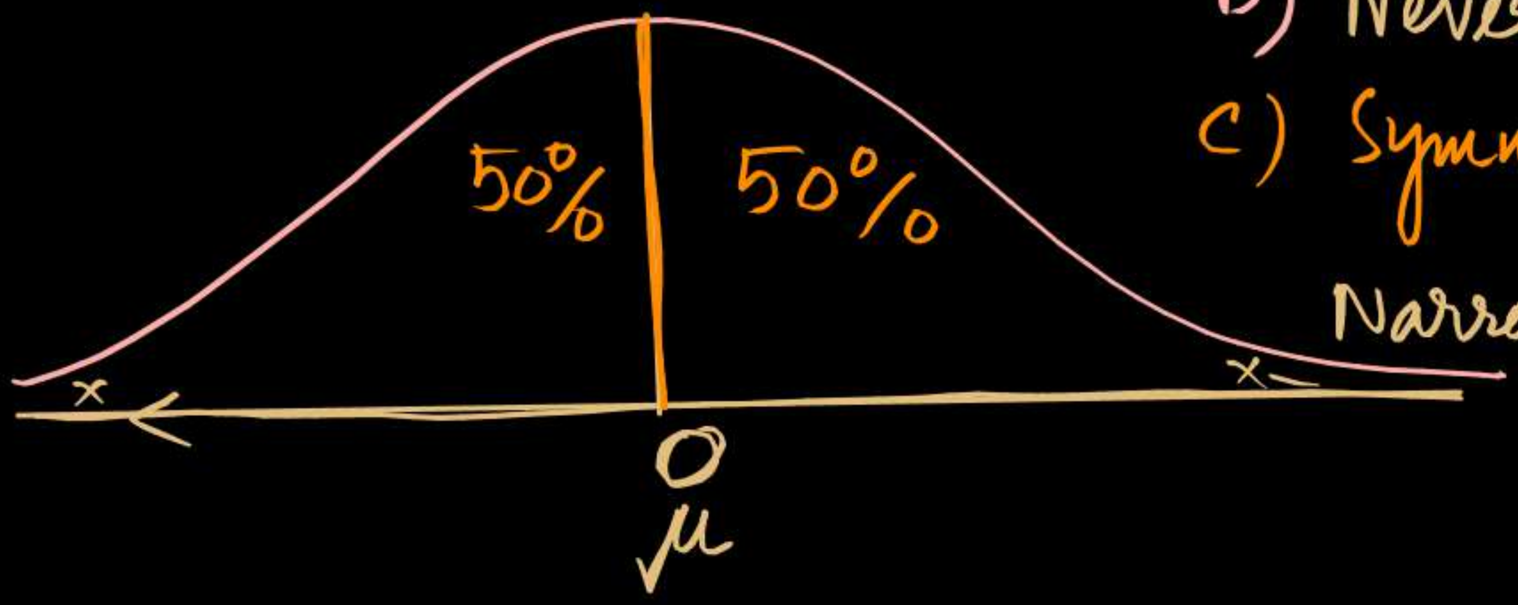
Guassian Distribution:

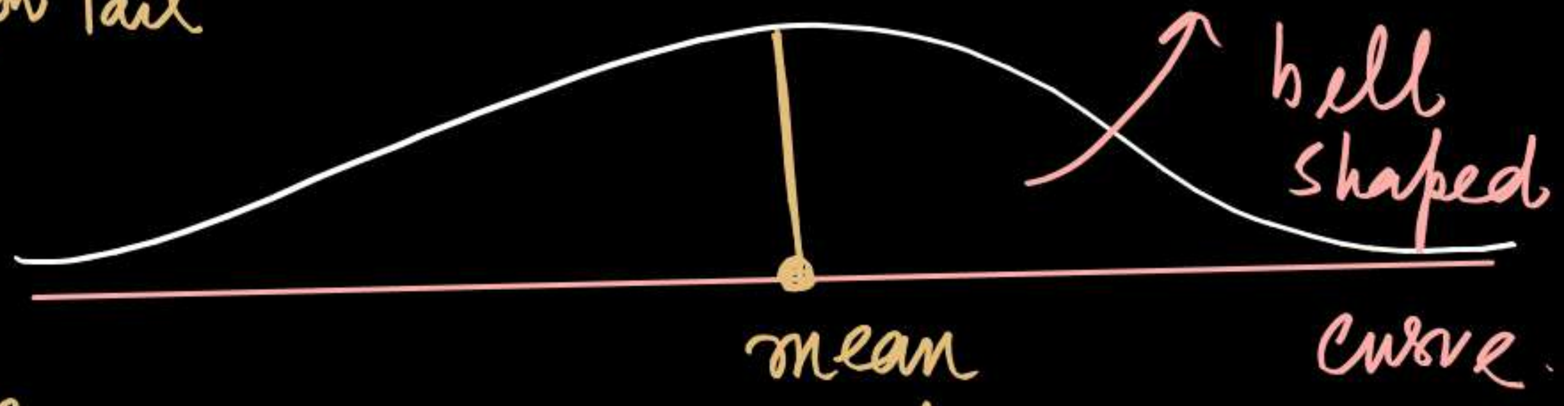
If n is Large No. of trials $\xrightarrow{n>30}$ Guassian Distribution

A) bell shaped curve

B) Never Touches or Crosses The Horizonatal line

C) Symmetric Distribution





= median
= mode.

E) Symmetric about The mean

F) continuous Distribution

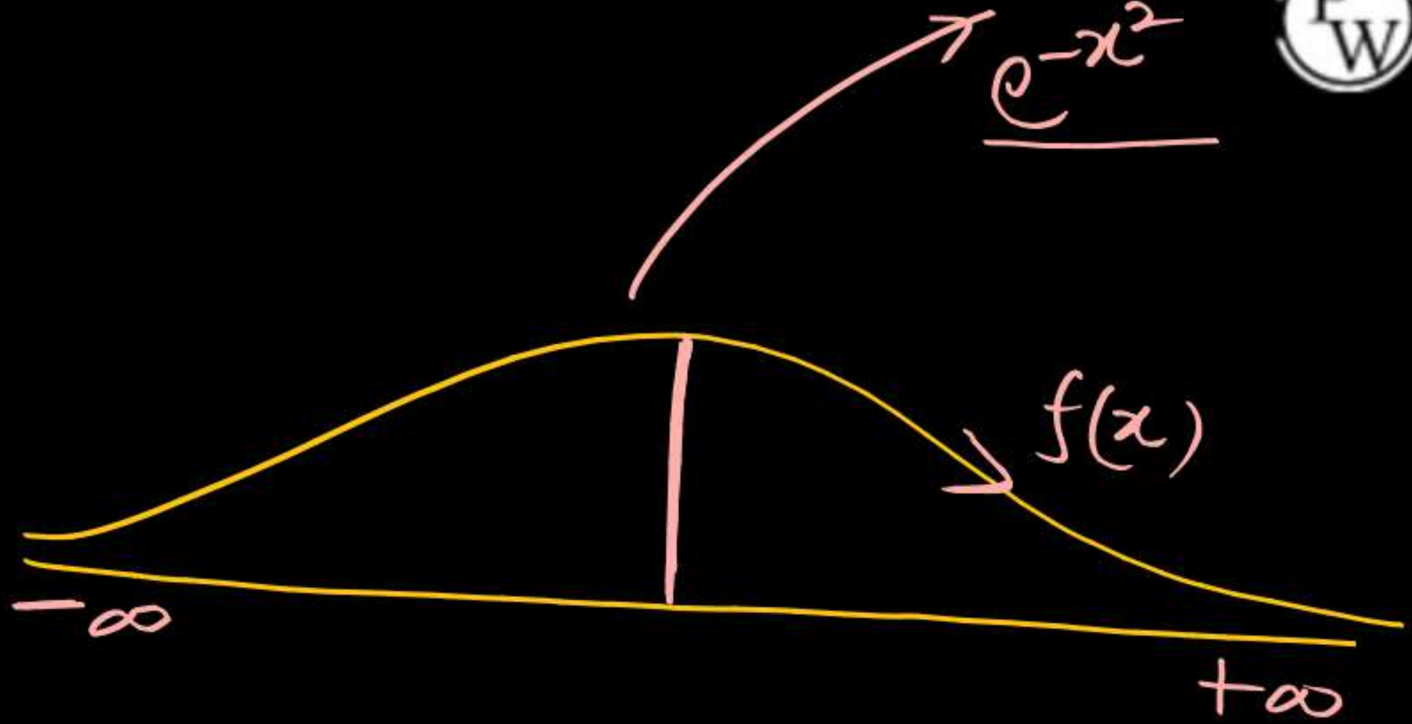
D) in Guassian Dist.
mean = median = mode

$$N(\mu,\sigma^2) = f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

μ = MEAN

σ = standard deviation

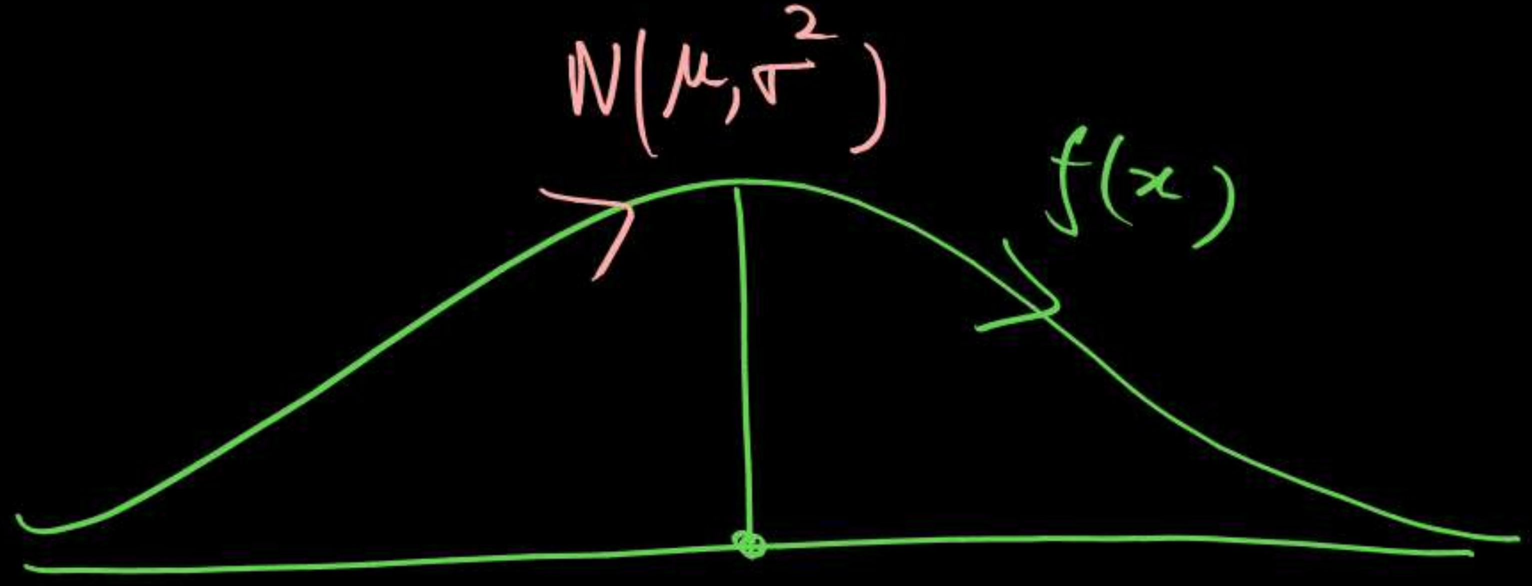
X = Random var



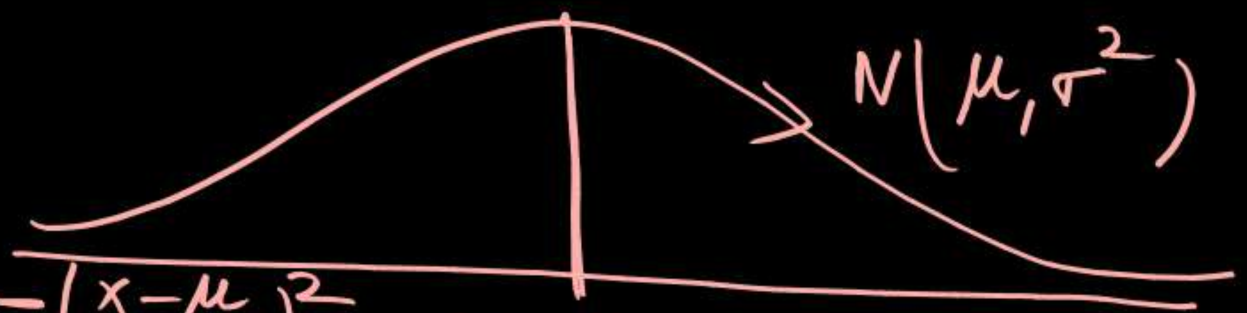
$$N(\mu,\sigma) = f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

If this is valid pdf

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$



$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$



$$\int_{-\infty}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx \Rightarrow \int_{-\infty}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-\left(\underbrace{\frac{x-\mu}{\sigma\sqrt{2}}}\right)^2}$$

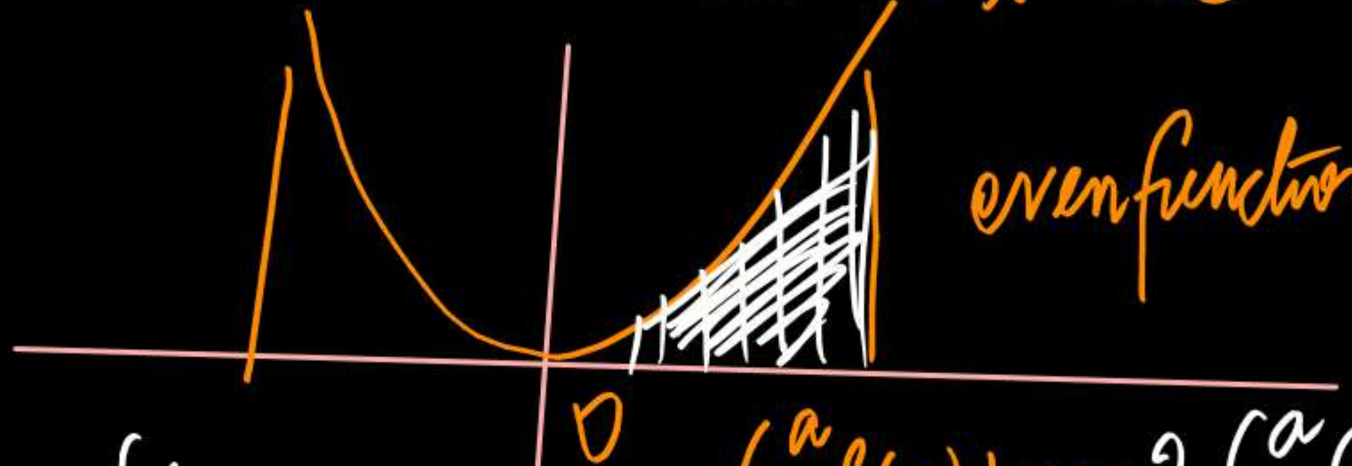
$$\frac{x-\mu}{\sigma\sqrt{2}} = t$$

Transform (substitute)

$$\Rightarrow \int_{-\infty}^{\infty} \frac{1}{\cancel{\sigma}\sqrt{\cancel{2}\pi}} e^{-t^2} \cancel{\sigma\sqrt{2}}\, dt$$

$$dx = \sigma\sqrt{2}\, dt$$

$$\Rightarrow \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-t^2} dt$$



even function

$$\Rightarrow \frac{1}{\sqrt{\pi}}\, 2\int_{0}^{\infty} e^{-t^2} dt$$

$$f(-x) = f(x)$$

$$\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dX$$

$$\Rightarrow \frac{1}{\sqrt{\pi}} 2 \underbrace{\int_0^{\infty} e^{-t^2} dt}_{\text{Gamma Function}}$$

$t^2 = z \quad 2t\,dt = dz$

$dt = \frac{dz}{2\sqrt{z}}$

$$\Rightarrow \frac{1}{\sqrt{\pi}} \cancel{2} \int_0^{\infty} e^{-z} \cdot \frac{dz}{\cancel{2}\sqrt{z}}$$

$$\Rightarrow \frac{1}{\sqrt{\pi}} \int_0^{\infty} e^{-z} \boxed{z^{-1/2}} dz$$

$z^{-1/2} = z^{n-1}$

$n-1 = -\frac{1}{2}$

$\boxed{n = \frac{1}{2}}$

$$\Gamma(n) = \int_0^{\infty} e^{-t} t^{n-1} dt$$

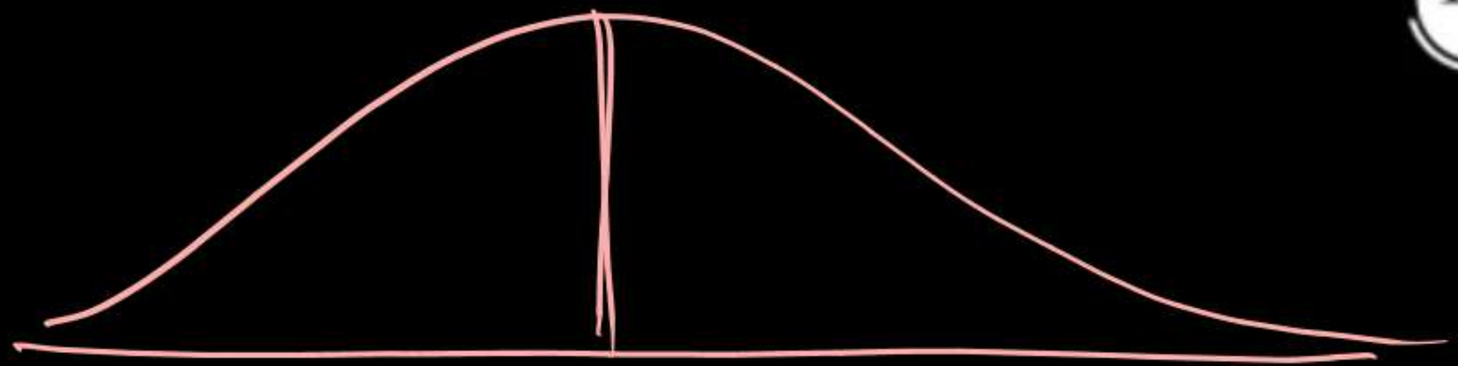
$$\left\{ \begin{array}{l} \Gamma(n) = (n-1)! \\ \Gamma\left(\frac{1}{2}\right) = \sqrt{\pi} \end{array} \right.$$

gamma function

$\Gamma\left(\frac{1}{2}\right)$

$$= \frac{1}{\sqrt{\pi}} \left[\underbrace{\int_0^{\infty} e^{-z} z^{-1/2} dz} \right] = \frac{\cancel{\sqrt{\pi}}}{\cancel{\sqrt{\pi}}} = 1$$

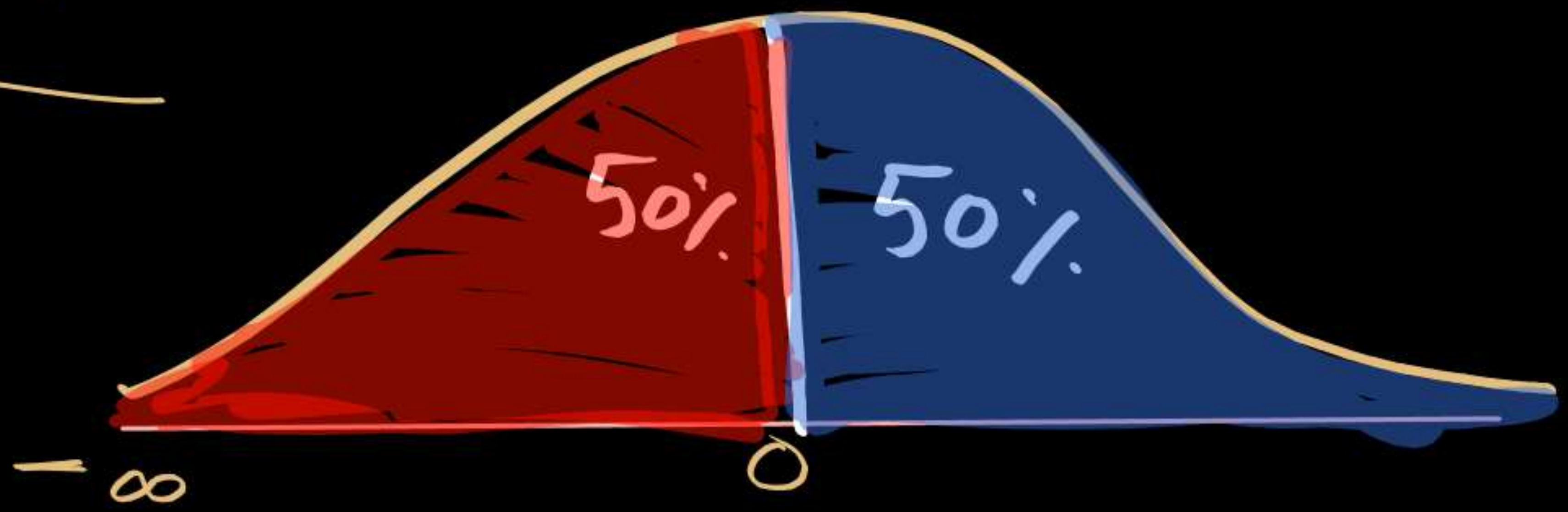
$$\boxed{\int_{-\infty}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx = 1}$$



✓ $$\int_{-\infty}^{0} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx = 0.5$$

or

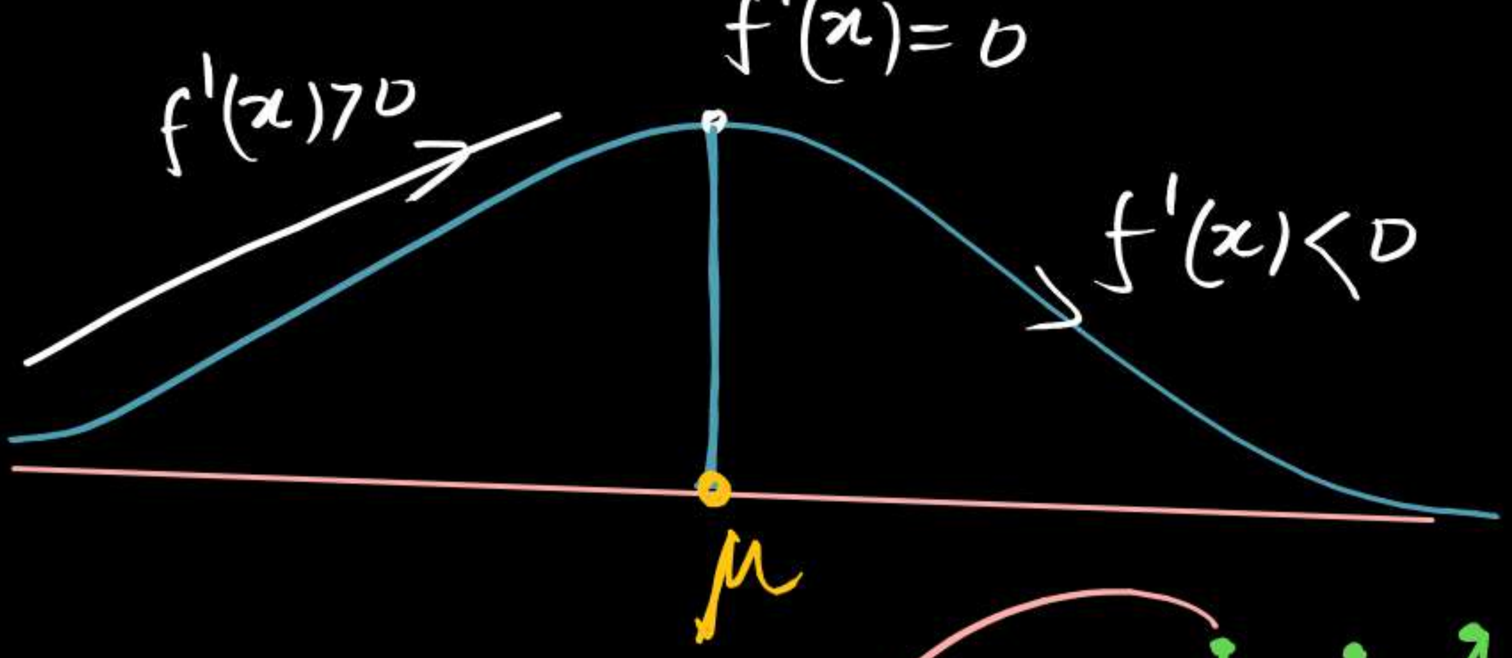
✓ $$\int_{0}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx = 0.5$$



$$N(\mu, \sigma^2) = y = f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$f'(x) = 0$$

$f'(x) = 0$ to get the stationary pt $x = \mu$



$$f''(x) = 0$$

$$= -\frac{1}{\sigma^3\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \left[1 - \frac{(x-\mu)^2}{\sigma^2}\right] = 0$$

$$1 - \frac{(x-\mu)^2}{\sigma^2} = 0 \quad \boxed{x = \mu \pm \sigma}$$

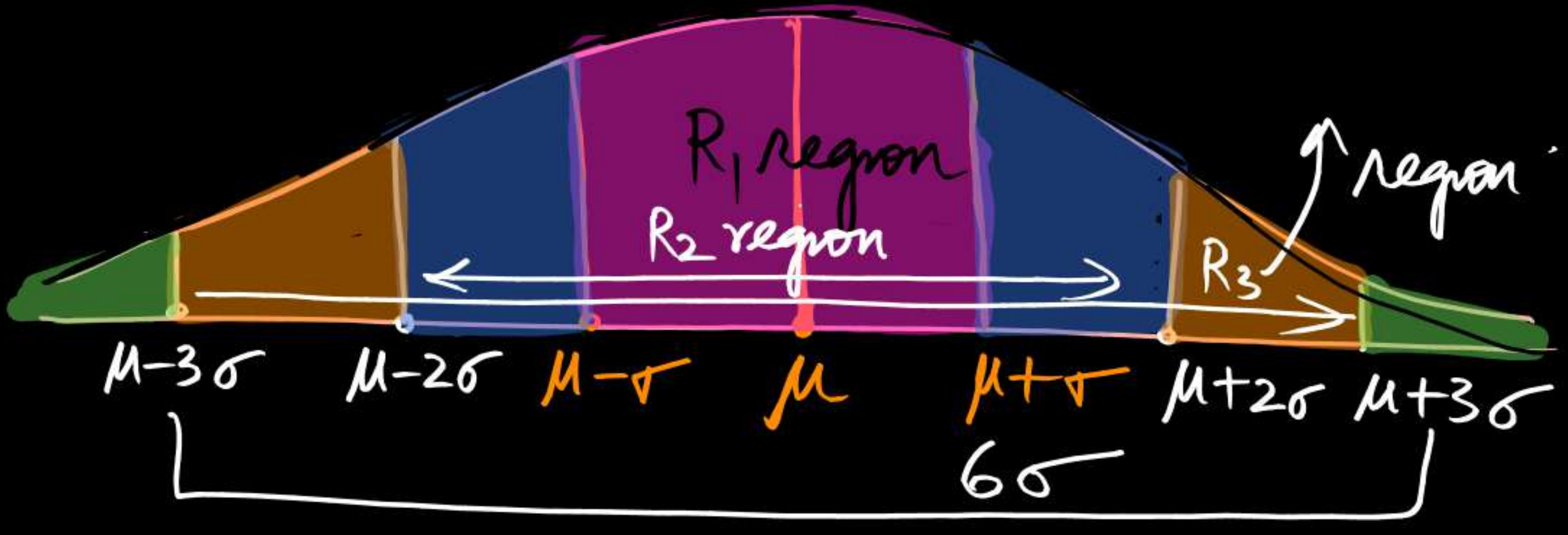
Point of Inflection
OR
Point of contraflexure

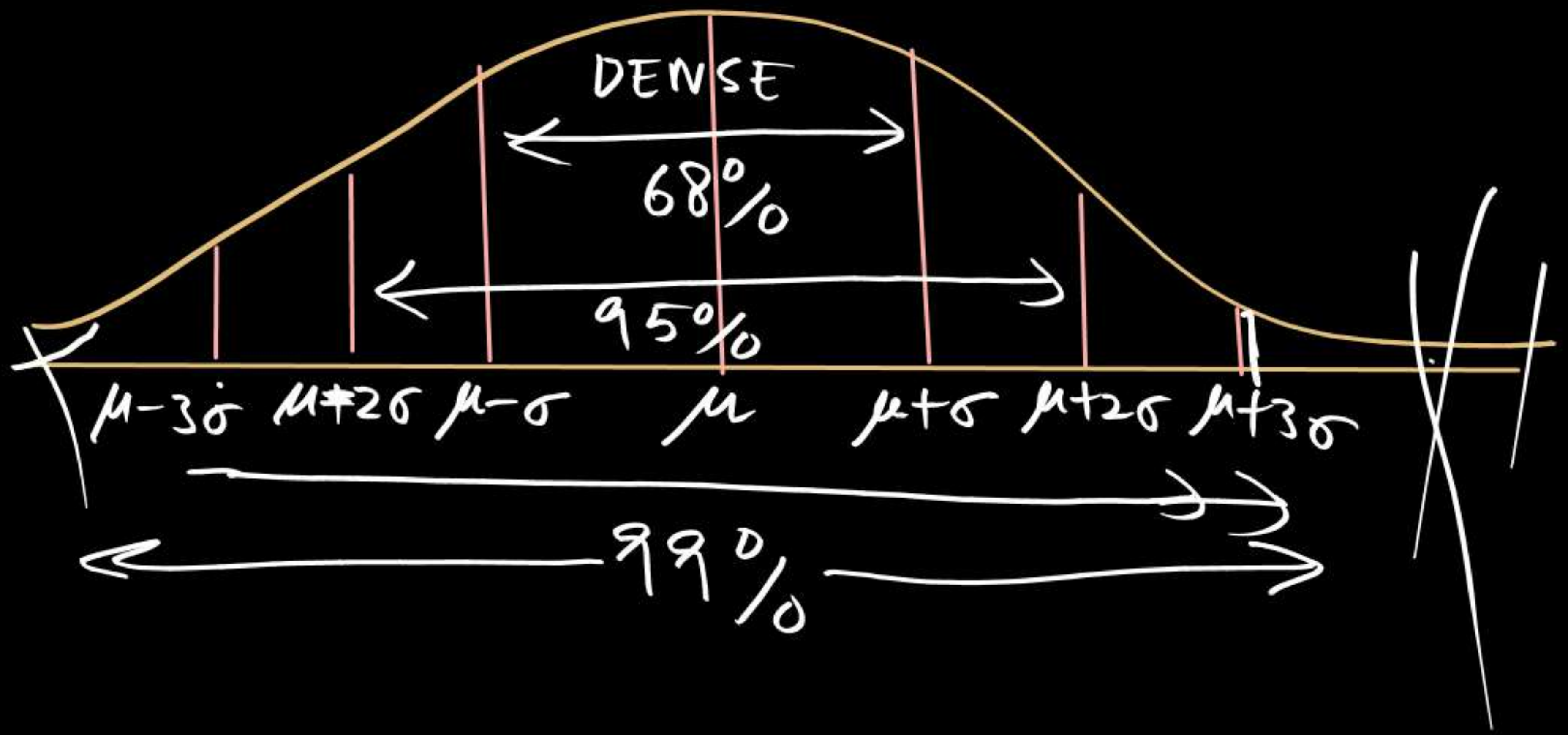
$\frac{d^2y}{dx^2} = 0$ (graph sudden change of Nature)

$X = \mu \pm 2\sigma$

$X = \mu \pm 3\sigma$

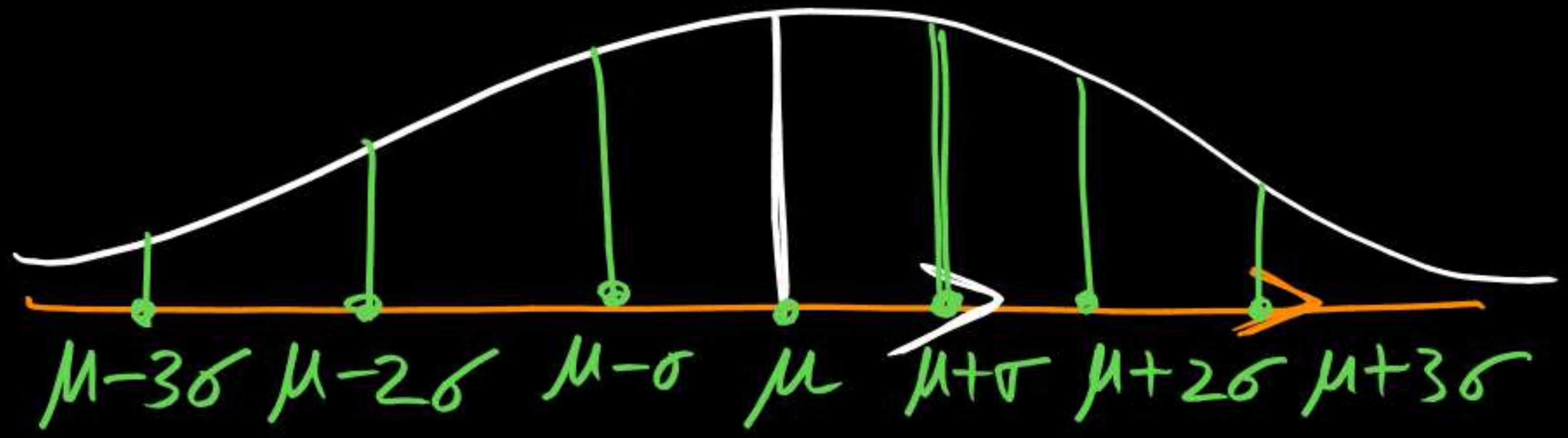
$$\begin{cases} R_1 \text{ region} = 68\% \\ R_2 \text{ region} = 95\% \\ R_3 \text{ region} = 99\% \end{cases}$$





Z SCORE → Parameter

$$Z = \frac{X-\mu}{\sigma} = \frac{\text{Random var} - \text{mean}}{S.D}$$



$$f(x) = N(\mu, \sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\left(\frac{x-\mu}{2\sigma^2}\right)^2}$$

$$\boxed{N(\mu, \sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{Z^2}{2}}}$$

Transform

ZERO mean
OR
Unit variance

standard Normal distribution

$\mu = 0$ ✓

$\sigma = 1$ ✓

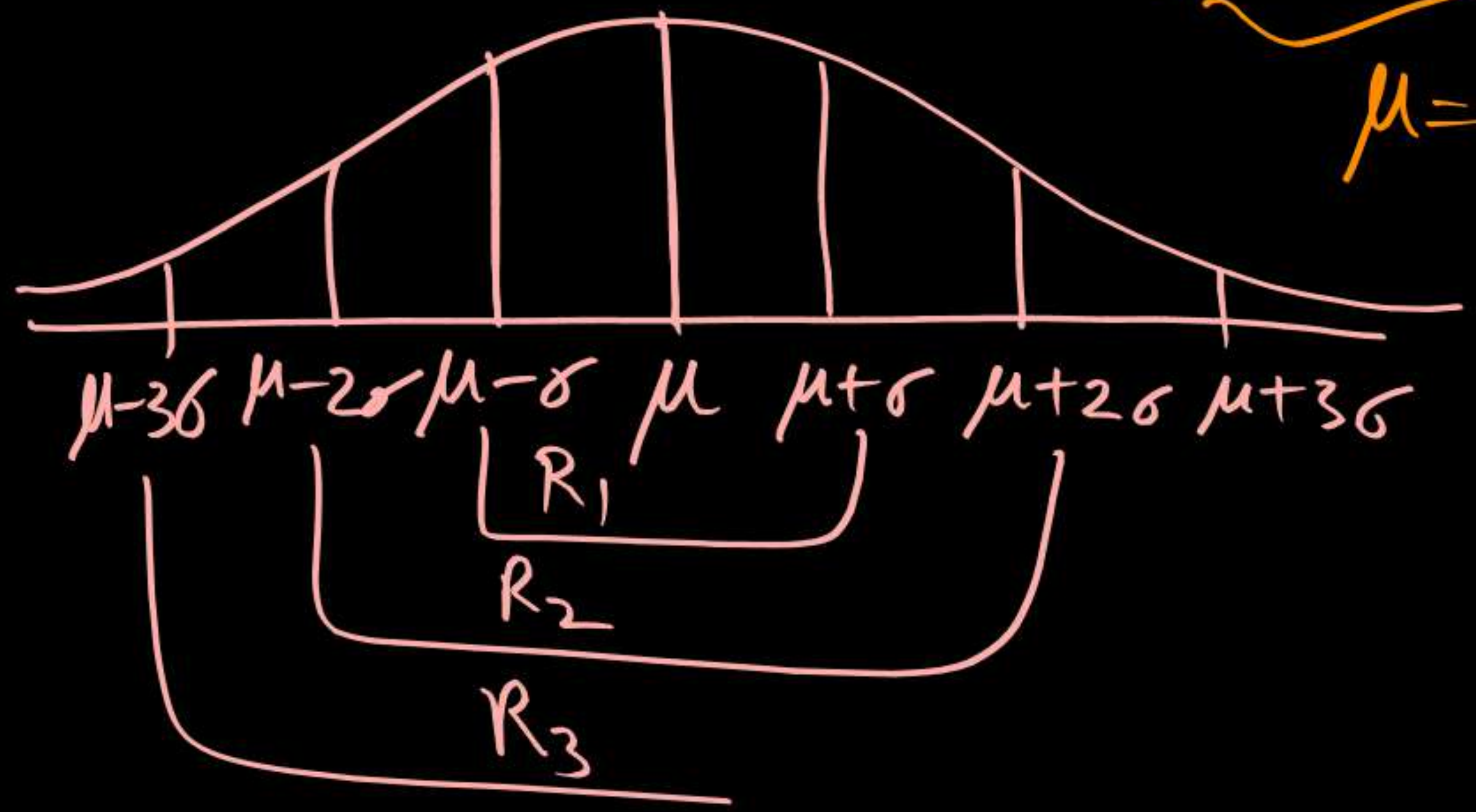
$$N(0,1) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

$$N(\mu, \sigma^2) = N(0,1)$$

ASSUME Transform $[N(0,1)]$

$\mu = 0 \quad \sigma = 1$

$$f(x) = N(0,1) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

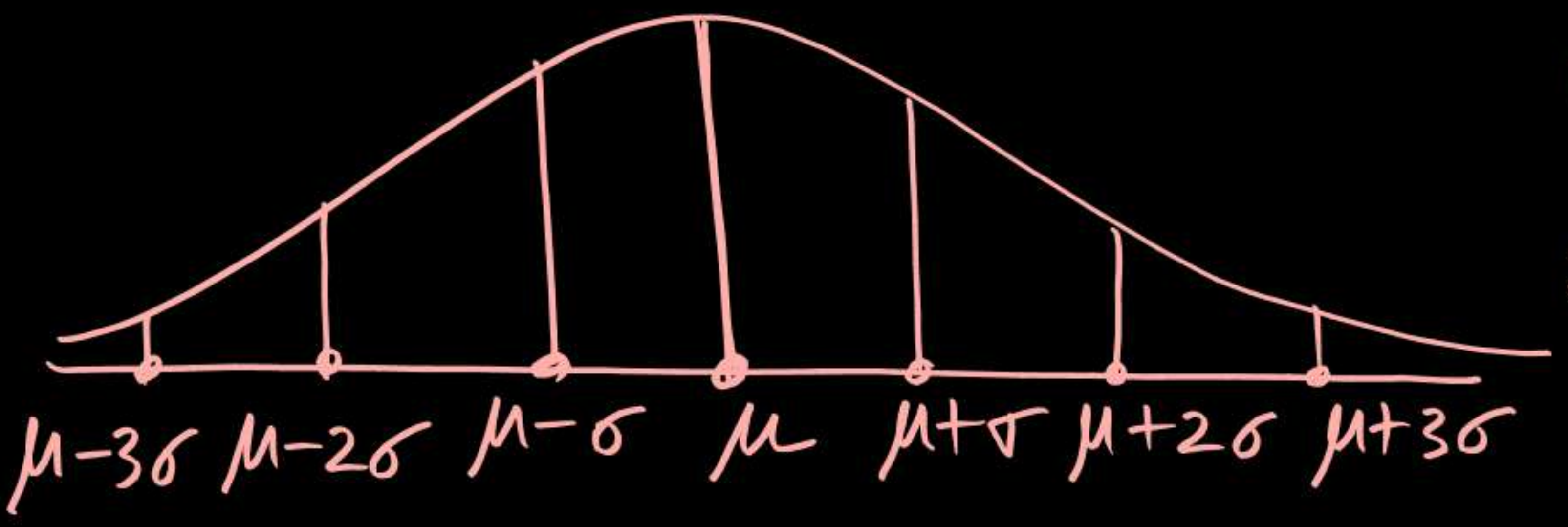


standard Normal

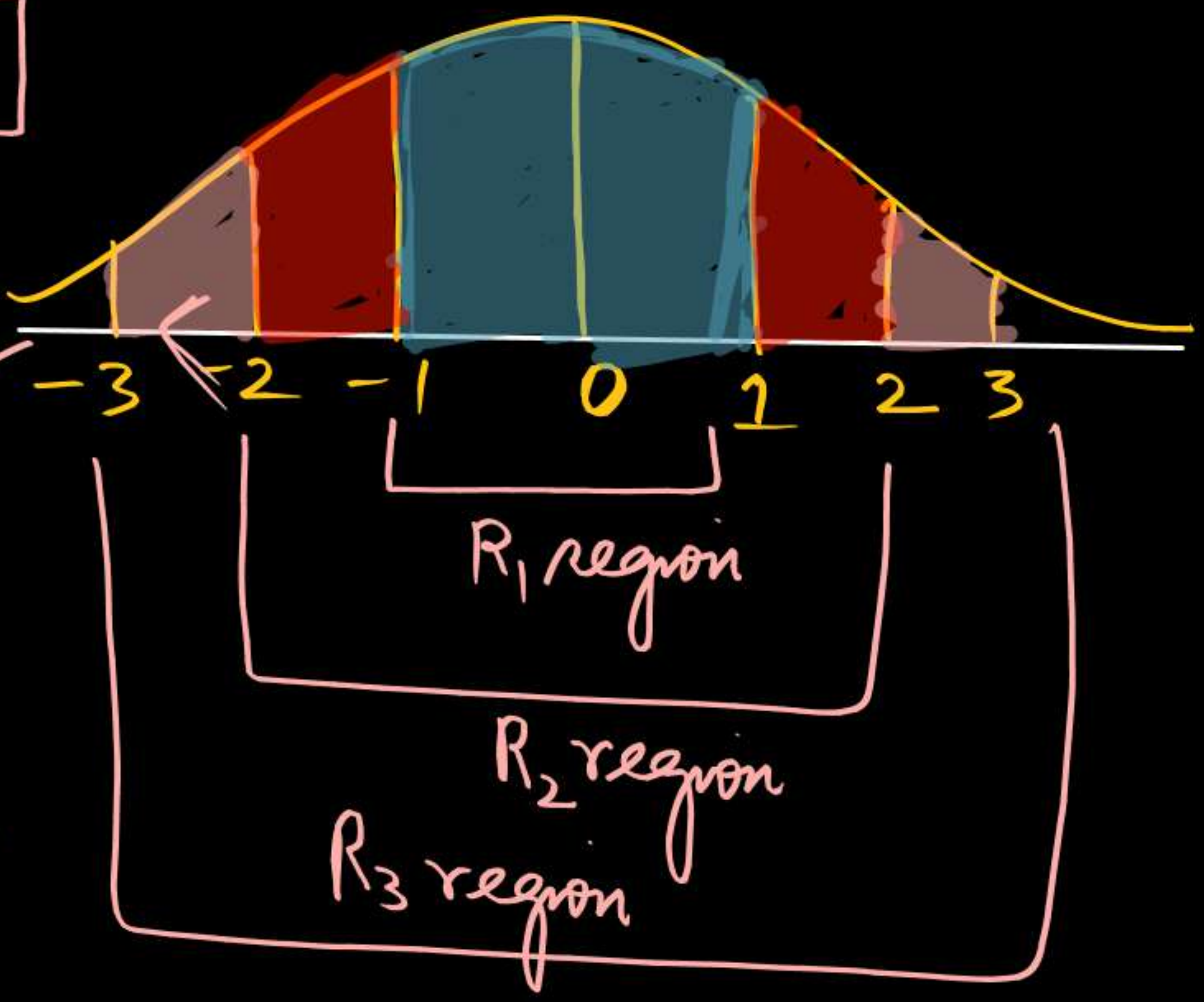
$\boxed{\mu = 0 \quad \sigma = 1}$

standard Normal

(Condition for)



$$\left[\begin{array}{l} \mu = 0 \\ \sigma = 1 \end{array}\right]$$



standard Normal

$$Z = \frac{X - \mu}{\sigma}$$

R_1 region -1 to 1

R_2 ———— -2 to 2

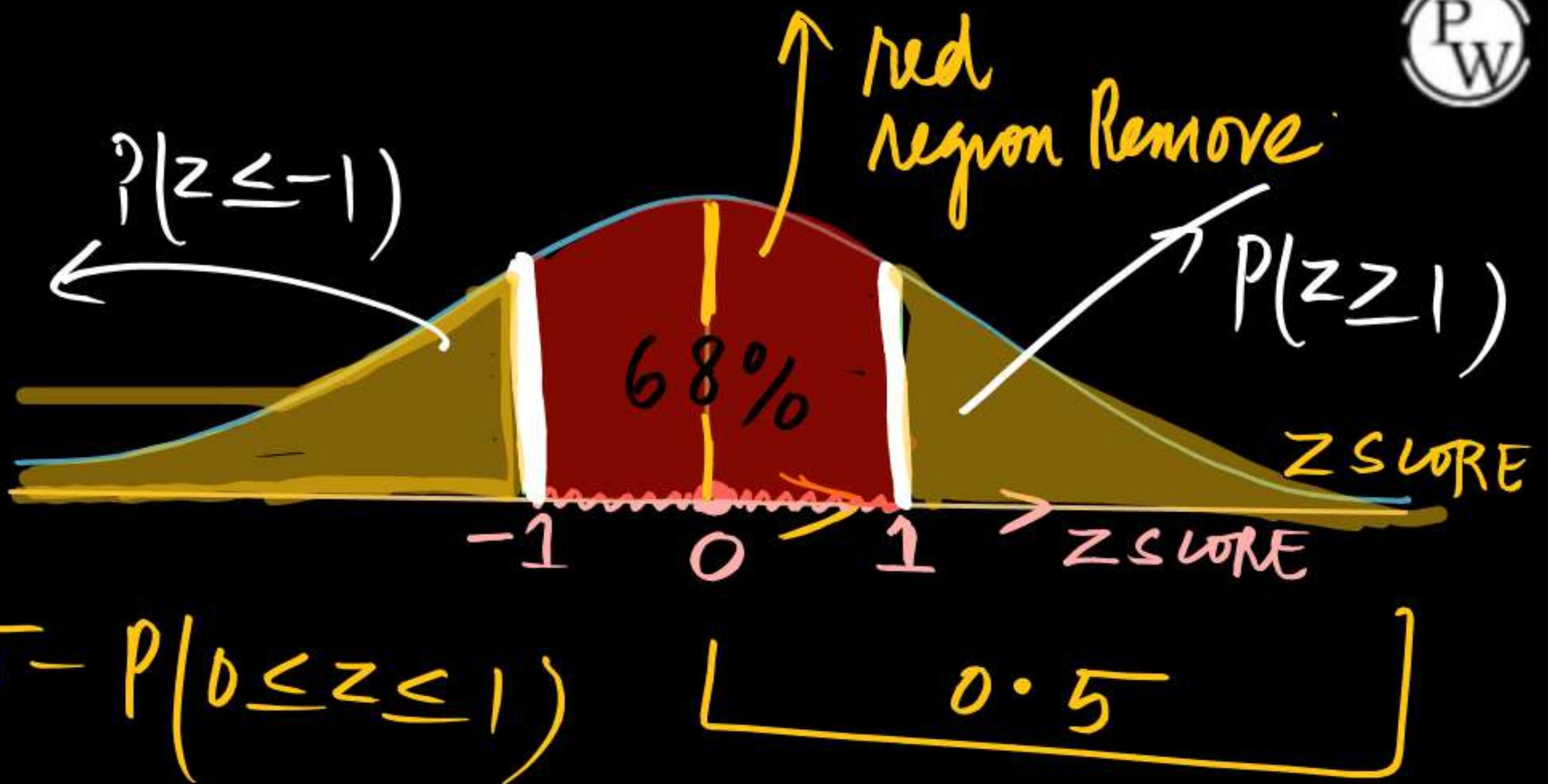
R_3 ———— -3 to 3

1) # $P(-1 \le z \le 1) = 0.6837$

$P(-1 \le z \le 0) = P(0 \le z \le 1)$

$= 0.3418$

$P(z \ge 1)$ or $P(z \le -1) = 0.5 - P(0 \le z \le 1)$

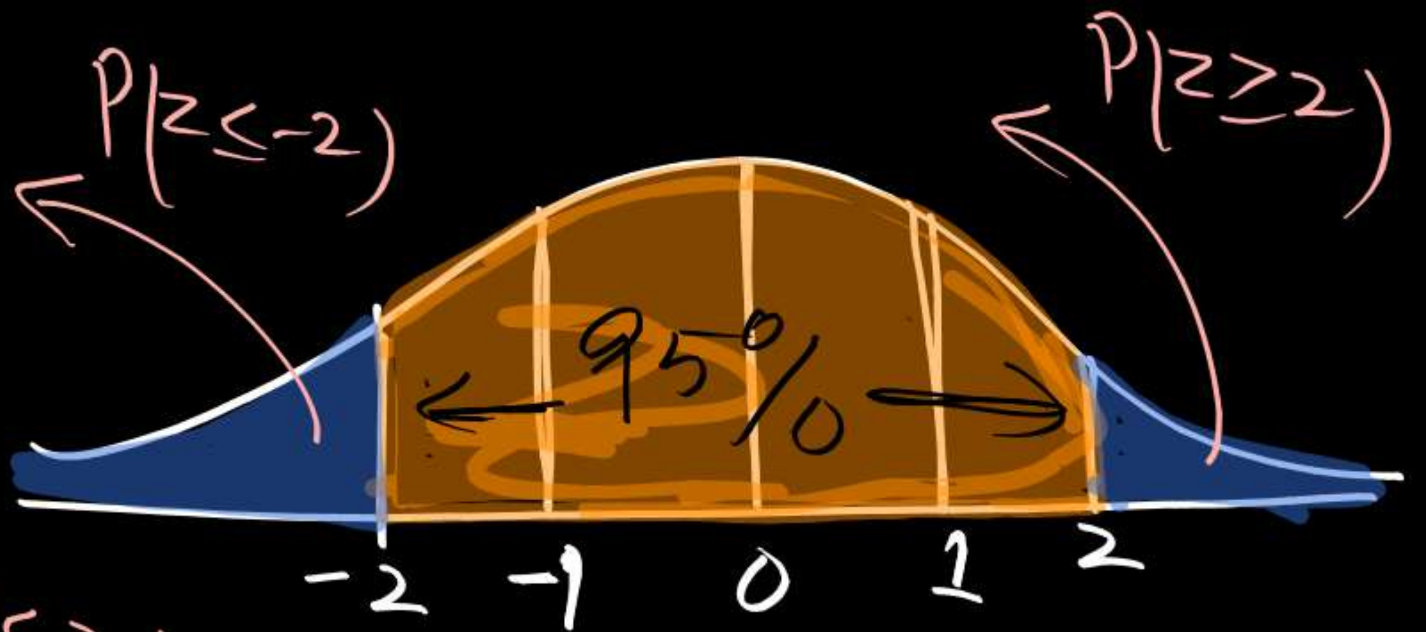


2) $P(-2 \le z \le 2) = 0.9545$

$P(-2 \le z \le 0)$ or $P(0 \le z \le 2)$

$= 0.4772$

$P(z \ge 2)$ or $P(z \le -2) = 0.5 - P(0 \le z \le 2)$

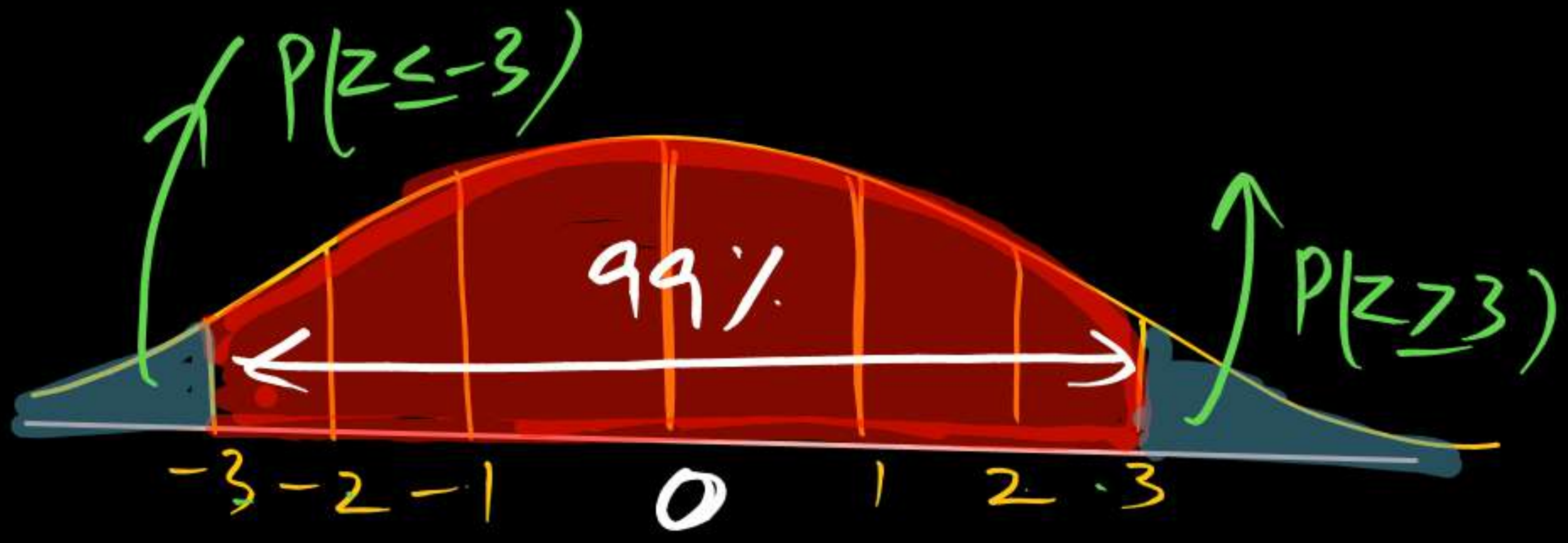


3) $P(-3 \le z \le 3) = 0.9971$

$P(0 \le z \le 3) = 0.4985$

$P(z \le -3)$ OR $P(z \ge 3)$

$= 0.5 - P(0 \le z \le 3)$



2) $P(X \ge a) = P\left(\frac{X-\mu}{\sigma} \ge \frac{a-\mu}{\sigma}\right)$

(3) $P(a \le X \le b) = P\left[\frac{a-\mu}{\sigma} \le \frac{X-\mu}{\sigma} \le \frac{b-\mu}{\sigma}\right]$

given

Guassian random

$= P\left[\frac{a-\mu}{\sigma} \le z \le \frac{b-\mu}{\sigma}\right]$

Thank You!

GW Soldiers