Computer Science & DA

Probability and Statistics

Probability

Lecture No. 01

By- Dr. Puneet Sharma Sir

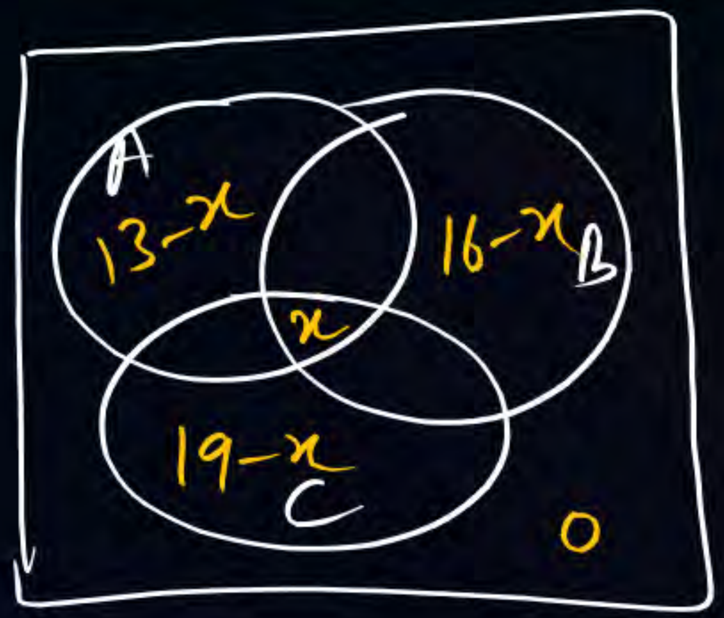
Recap of previous lecture

Topic Permutation and Combination-03

Topics to be Covered

Topic Probability-Basic definition

Q. 40 students watched films A, B and C over a week. Each student watched either only one film or all three. 13 students watched film A, 16 watched B, and 19 watched film C. How many students watched all three?



ATQ; $(13-x)+(16-x)+(19-x)+x=40$

$48-2x=40 \Rightarrow 2x=8 \Rightarrow x=4$

PROBABILITY (Possibility w.r.to Base 1)

RANDOM EXP → if outcome is not sure then such types of Exp are called R. Exp.

eg Tossing a coin, Throwing a die, A card is drawn at random from Playing Cards etc.

Sample Space → Total possible outcomes written in a set form known as S. Space.

Event → Any subset of of S. space is known as an event.

eg $D = \{1, 2, 3, 4, 5, 6\}$

$E_1 = \{1, 3, 5\} = \{\text{odd No.}\}$

$E_2 = \{2, 4, 6\} = \{\text{even No}\}$

$E_3 = \{1, 2, 3, 4\} = \{\text{No} \leq 4 \text{ is coming}\}$

is Total No of events $= 2^6 = 64$

(*) Total Number of events associated with S. Space S having cardinality n is?

$=$ Total No. of Subsets $= 2^n$

Cardinality → Counting of diff elements in a set is called it's Cardinality.

Special Events: ① Impossible Event (ϕ) $\rightarrow$ $\because \phi \subset S$ so ϕ is also an Event & $P(\phi) = 0$

② Sure Event (S) $\rightarrow$ $\because S \subseteq S$ so S is also an event & $P(S) = 1$
(Certain Event)

Note ① $0 \le P(E) \le 1$

② P(Nothing occurs) = 0

③ P(something occurs) = 1

④ P(given statement) = 1

⑤ P(Death) = 1

⑥ P(GOD) = 1

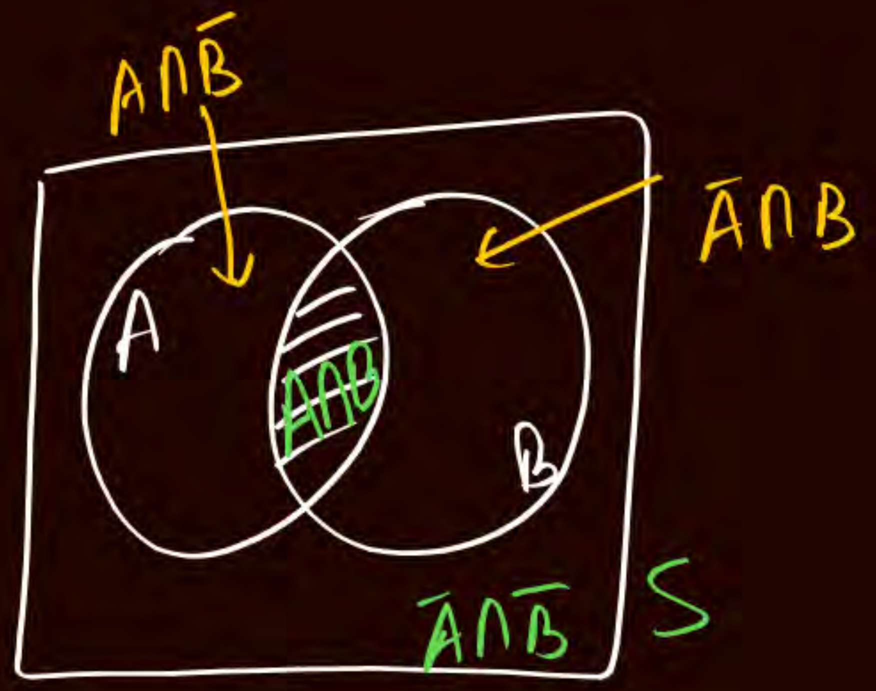
⑦ Prob $\simeq$ % $\simeq$ Proportion
(Base 1) (Base 100) (Base 1)

eg If in a mixture of M & W we have M : W = 2 : 5

then → Prop. of Milk $= \frac{2}{2+5} \simeq \frac{2}{7}$

→ " of Water $= \frac{5}{2+7} \simeq \frac{5}{7}$

Some More Important Points →



① Either A or B or Both = Atleast one of A or B = $A \cup B$

② Both A & B = Simultaneous occurrence of A & B = $A \cap B$

③ Neither A Nor B = None of A & B $= \bar{A} \cap \bar{B}$

④ (M-I) $(\bar{A}\cap B) \cup (A \cap \bar{B}) \cup (A \cap B) = A \cup B$

Not very easy to Calculate $A\cup B$ using this Approach.

(M-II) Addition Th$^{\underline{m}}$ → $\boxed{P(A\cup B) = P(A) + P(B) - P(A\cap B)}$

& for Three Events, $P(A\cup B\cup C) = P(A) + P(B) + P(C) - P(A\cap B) - P(B\cap C) - P(C\cap A) + P(A\cap B\cap C)$

⑤ Multiplication Th$^{\underline{m}}$ → $\boxed{P(A\cap B) = P(A/B)\, P(B)}$

⑥ $P(\text{Neither A Nor B}) = 1 - P(\text{Either A or B})$

$$\boxed{P(\bar{A} \cap \bar{B}) = 1 - P(A \cup B)}$$

⑦ Another Representation :→

$$P(A \cup B) = 1 - P(\bar{A} \cap \bar{B})$$

$$\boxed{P(\text{at least one of A or B}) = 1 - P(\text{None})}$$

⑧

	AND	OR	Ind	ME
P & C	Multiply	Add	–	—
Prob	Intersection (using M.Th)	Union (using A.Th)	$P(A \cap B) = P(A).P(B)$	$P(A \cap B) = 0$ $P(A \cup B) = P(A) + P(B)$

⑨ If A & B are Ind then

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= P(A) + P(B) - P(A).P(B)$$

Mutually Exclusive Events :-

if (two Events can't occur simultaneously) $\Longleftrightarrow$ then (Events are ME)

i.e

if A & B are ME $\Longleftrightarrow$ then $\boxed{A \cap B = \phi}$

eg $S = \{1, 2, 3, 4, 5, 6\}$

$A = \{1, 3, 5\}$, $B = \{2, 4, 6\}$, $C = \{1, 2, 3, 4\}$

$\because A \cap B = \phi \Rightarrow$ A & B are ME

$\because A \cap C \neq \phi \Rightarrow$ A & B are Not ME

& $B \cap C \neq \phi \Rightarrow$ B & C " Not ME

Independent Events :-

if occurence or Non occurence of one event does not alter the occurence or Non occurence of other event then Events are called Ind.

if (A & B are Ind) $\Longleftrightarrow$ then $\boxed{P(A \cap B) = P(A) \cdot P(B)}$

eg $C = \{H, T\}$, $D = \{1, 2, 3, 4, 5, 6\}$

$A = \{H\}$, $B = \{1, 2, 3, 4\}$

$P(A) = \frac{1}{2}$, $P(B) = \frac{4}{6}$

then $P(A \cap B) = ? = \frac{1}{2} \times \frac{4}{6} = \frac{1}{3}$

Note: ① ME Events are associated with same S space while Ind Events are associated with different S spaces

② i.e In case of Ind Events if we take $A \cap B = \phi$ then it is senseless

and we can find the prob of their simultaneous occurrence by Multiplying the individual Events i.e

if A & B are Ind then $\Longleftrightarrow$ $P(A \cap B) = P(A) \cdot P(B)$

Similarly if A, B & C are Ind then $\Longleftrightarrow$ $\boxed{P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C)}$

③ if A & B are ME then $\boxed{A \cap B = \phi}$ $\to$ $P(A \cap B) = 0$

$\to$ $P(A \cup B) = P(A) + P(B) - 0$

Imp

* Nature of Elements in a Sample Space

if R. Exp is repeated n times then elements of sample space are in the form of ordered n-tuple

eg (i) if Dice is thrown once then $S = \{1, 2, 3, 4, 5, 6\}$

& $n(S) =$ six

eg (ii) if Dice is thrown twice then

$$S = \left\{\begin{matrix} (11)(12)(13)(14)(15)(16) \\ (21)(22)(23) \ldots\ldots (26) \\ (31)(32) \ldots\ldots (36) \\ \ldots\ldots\ldots (65), (66) \end{matrix}\right\}$$

$$\Rightarrow n(S) = \underset{D_1}{6} \times \underset{D_2}{6} = 36 \text{ - O. pair}$$

eg (iii) A coin is tossed five times then

$$S = \left\{\begin{matrix} (HHHHH), (HHHHT), (HHHTT) \\ \ldots\ldots\ldots (HTTTT), (TTTTT) \end{matrix}\right\}$$

$$n(S) = \underset{C_1}{2} \times \underset{C_2}{2} \times \underset{C_3}{2} \times \underset{C_4}{2} \times \underset{C_5}{2} = 2^5 = 32 \text{ O-five tuples}$$

eg (iv) A couple has 3 children then

$$S = \left\{\begin{matrix} (BBB), (BBG), (BGB), (BGG) \\ (GBB), (GBG), (GGB), (GGG) \end{matrix}\right\}$$

$$n(S) = \underset{C_1}{2} \times \underset{C_2}{2} \times \underset{C_3}{2} = 8 \text{ (O-Triplets)}$$

Note → Methods of Solving Questions →

App 1 → By listing all the elements of S. Space (S) & Fav Event (E) & then $P(E) = \frac{n(E)}{n(S)}$

App 2 → if it is not easy to write S space and Fav Event then directly Calculate fav No. of Cases and Total No of Cases by using the concept of P & C

then Req Prob = $\frac{\text{fav Cases}}{\text{Total Cases}}$ Note if App ① fails then only we will follow App 2

App 3 → By using Some Standard Results & Standard def^n.

Note ① if in a Quest, given information is in the form of Prob then we use App 3.

② favourable Cases → which is Required should be assumed as fav Cases.

(*) $(a,b) \neq (b,a)$ while $\{a,b\} = \{b,a\}$

$\downarrow$ O-Pair $\quad \downarrow$ O-Pair

$(a,b,c,d) \simeq$ O-Quadruple

$(a,b,c,d,e) \simeq$ O-Five tuple

(*) A Dice is thrown thrice $\cong$ Three Dice are thrown simultaneously

$\Rightarrow$ In both the situation S.Space would be same.

Q. A Dice is thrown twice then write it's S.space

$$S = \left\{ \begin{matrix} (11)(12)(13) \dots (16) \\ (21),(22), \dots (66) \end{matrix} \right\} \Rightarrow n(S) = 36 \text{ pair}$$

① find the Prob that sum of outcomes is 8 ?

App 1

$A = \{\text{Sum is } 8\} = \{(26),(62),(35),(53),(44)\}$

$n(A) = 5 \Rightarrow P(A) = \frac{n(A)}{n(S)} = \frac{5}{36}$

② $P(\text{Sum of outcomes is } 9) = ?$

App 1

$B = \{\text{Sum is } 9\} = \{(36)(63),(54),(45)\}$

$n(B) = 4 \Rightarrow P(B) = \frac{n(B)}{n(S)} = \frac{4}{36}$

③ $P(\text{Sum is Both } 8 \text{ and } 9) = ?$

App③ $P(A \cap B) = P(\phi) = 0$

$\because A \cap B = \phi \Rightarrow A$ & B are M.E

④ $P(\text{Sum is either } 8 \text{ or } 9) = ?$

App③ $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{5}{36} + \frac{4}{36} - 0 = \frac{1}{4}$

⑤ $P(\text{Sum is Neither } 8 \text{ Nor } 9) = ?$

App③ $P(\bar{A} \cap \bar{B}) = 1 - P(A \cup B)$

$= 1 - \frac{1}{4} = \frac{3}{4}$

GATE

(6) Find the Prob the all the outcomes are identical = ?

C

AppI

$C = \{(1\,1), (2\,2), (3\,3), (4\,4), (5\,5), (6\,6)\}$

$n(C) =$ six so $P(C) = \frac{n(C)}{n(S)} = \frac{6}{36}$

GATE

(7) Find the Prob that Product on the upper faces will be perfect square = ?

D

AppI

$D = \{(1\,1), (2\,2), (3\,3), (4\,4), (5\,5), (6\,6), (1\,4), (4\,1)\}$

$\Rightarrow n(D) = 8 \Rightarrow P(D) = \frac{8}{36} = \frac{2}{9}$

(8) $P(\text{Sum is divisible by } 4) = ?$

$= P(\text{Sum} = 4 \text{ or } 8 \text{ or } 12) = \frac{3+5+1}{36}$

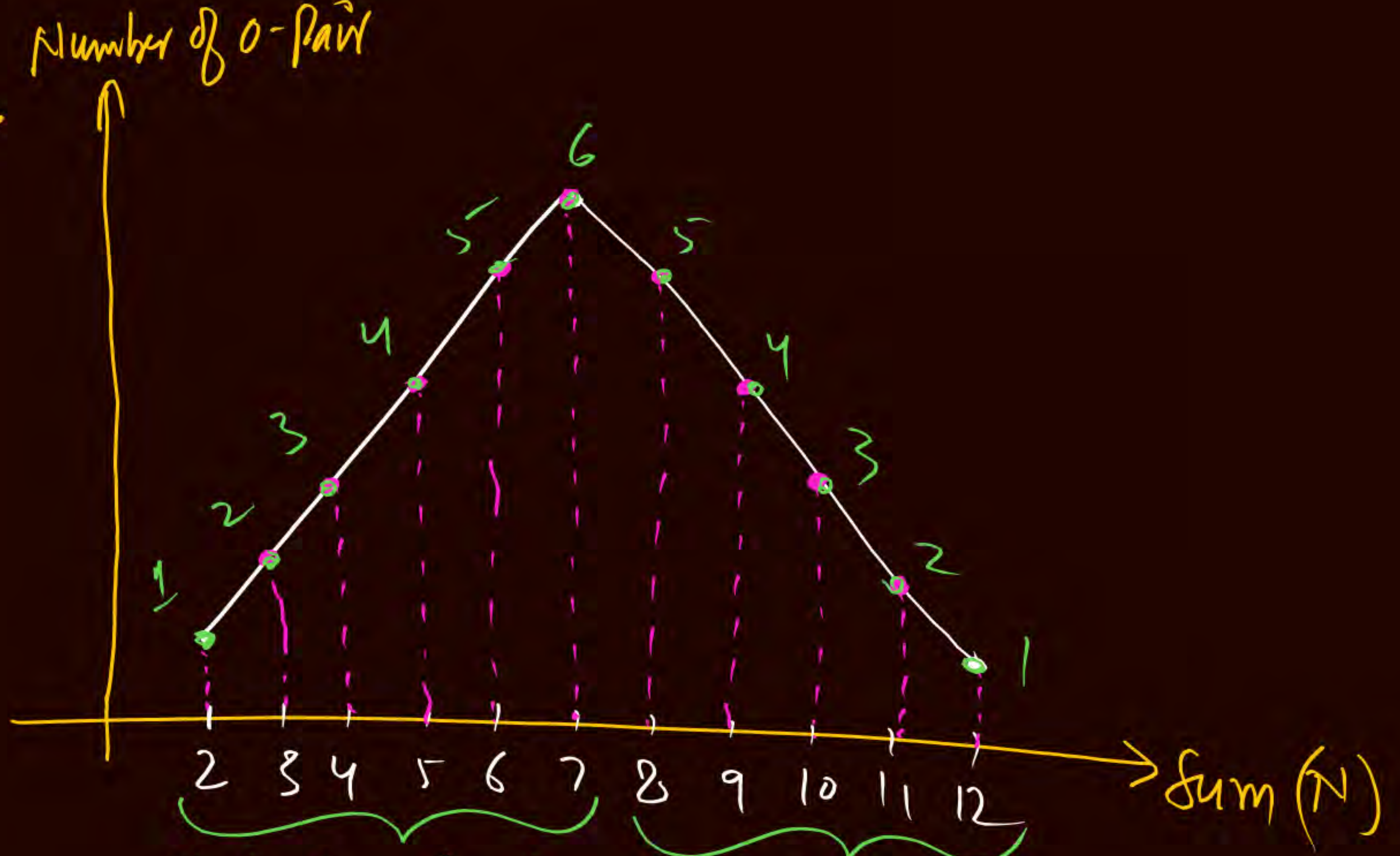
(9) $P(\text{Sum is a prime Number}) = ?$

$= P(\text{Sum} = 2 \text{ or } 3 \text{ or } 5 \text{ or } 7 \text{ or } 11) = \frac{1+2+4+6+2}{36}$

(10) $P(\text{Sum exceeds } 9) = ?$

$= P(\text{Sum} = 10 \text{ or } 11 \text{ or } 12) = \frac{3+2+1}{36}$

Shortcut



No. of O-Pair $= (N-1)$

No. of O-Pair $= (13-N)$

P-18

Q- four dices are thrown simultaneously then find the prob that sum is 22 ?

Sol: $S = \begin{Bmatrix} (1111), (1112), (1113), (1114), (1115), (1116) \\ (2111) (21112) \text{ ------- } (2116) \\ (3111), (3112) \text{ -------- } (6666) \end{Bmatrix} \Rightarrow n(S) = \underset{D_1}{6} \times \underset{D_2}{6} \times \underset{D_3}{6} \times \underset{D_4}{6} = 6^4$

App I

$= 1296$ O-Quadruples

fav outcomes $= \{\text{sum is } 22\}$

$= \begin{Bmatrix} (6664), (6646), (6466), (4666) \\ (6655), (6565), (6556), (5566) (5656), (5665) \end{Bmatrix} = 10$ Quadruples

App II – Total $= 6^4 = 1296$ & fav $= \begin{Bmatrix} (6664) \text{-----} \\ (6655) \text{----} \end{Bmatrix}$ $\begin{matrix} 4!/3! = 4 \\ \frac{4!}{2!.2!} = 6 \end{matrix} \Bigg\} = 10$

So Req. Prob $= \frac{f}{T} = \frac{10}{1296}$

THANK - YOU