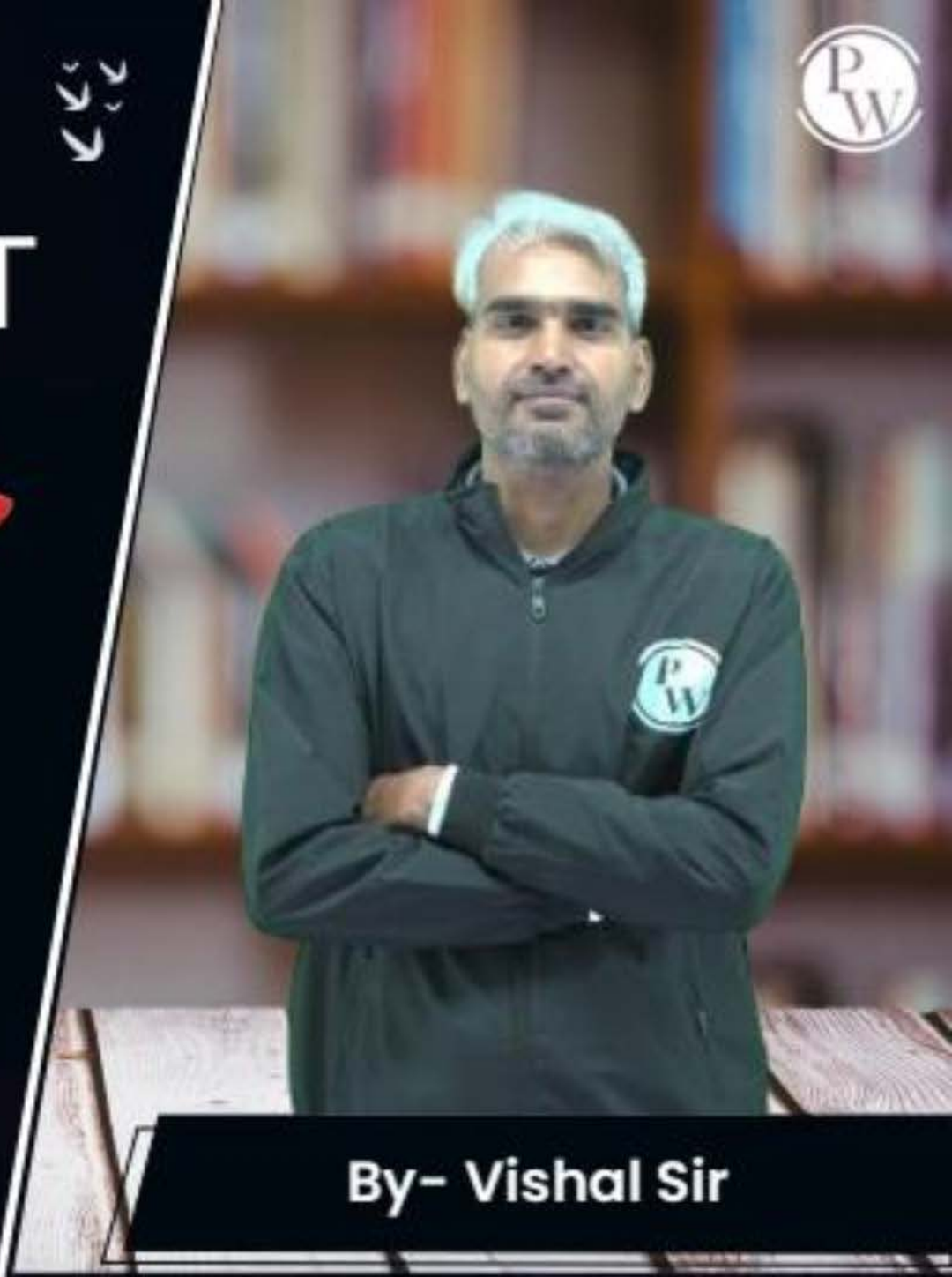
Computer Science & IT

Discrete Mathematics

Set Theory & Algebra

Lecture No. 16



By- Vishal Sir

Recap of Previous Lecture

Topic Types of Lattices

Topic Hasse Diagram

Completed
Sets & Relations

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Topics to be Covered

- Topic — Function
- Topic — Range of a function
- Topic — Injective (one-one) function
- Topic — Surjective (onto) function
- Topic — Bijective function

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Topic : Function

Every function is a relation, but every relation need not be a function

A relation from set A to set B is called a function from set A to set B if every element of set A relates with exactly one element of set B.

* A function 'f' from A to B is denoted by $f: A \rightarrow B$

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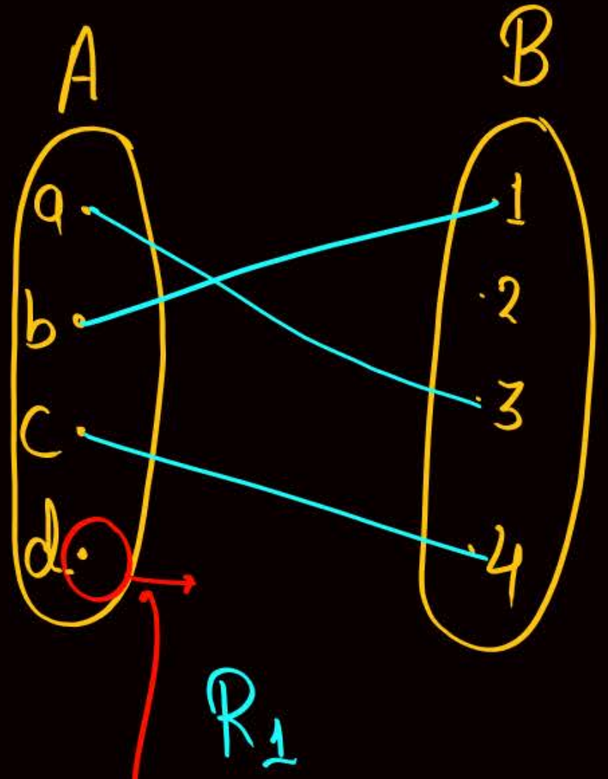
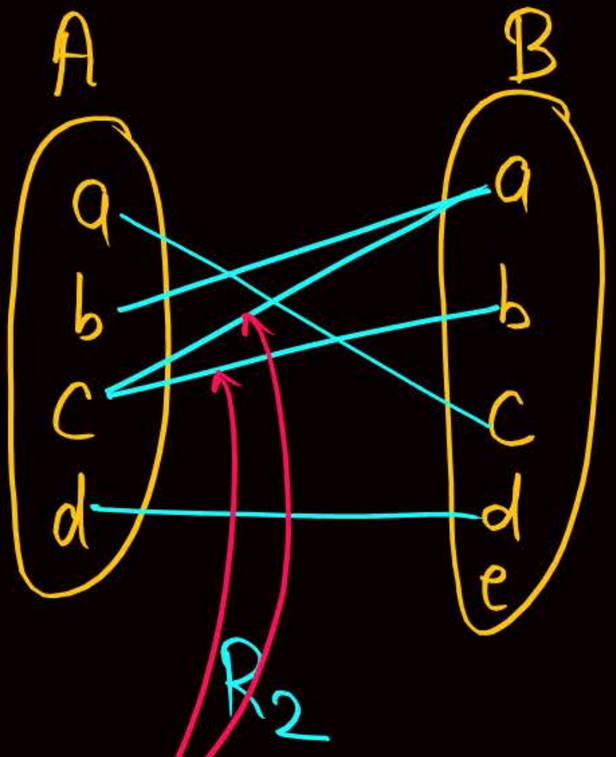


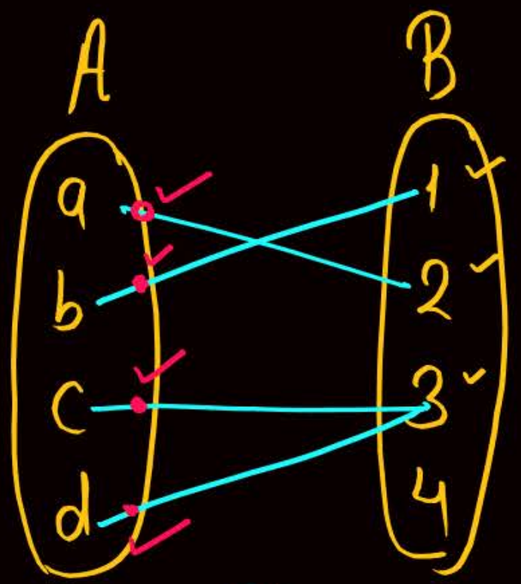
d is not related with any element of set B

∴ Not a function from A to B

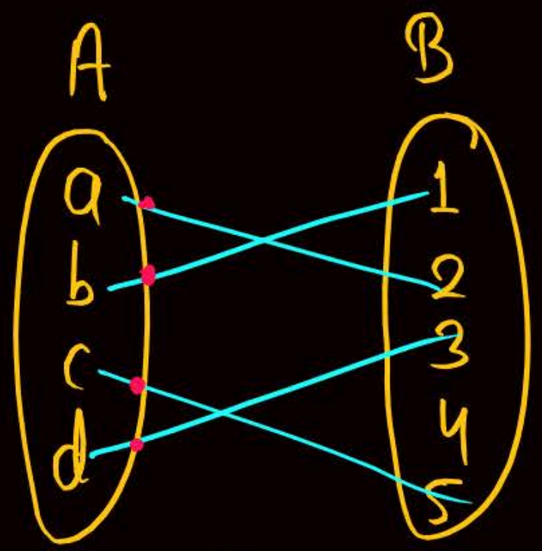


One element of set A relates with more than one elements of set B

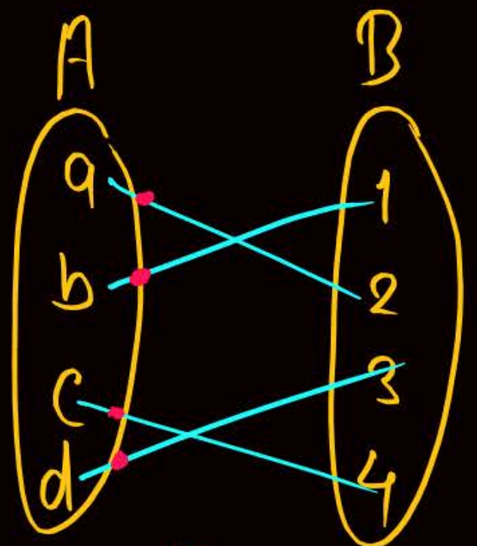
∴ R_2 is not a function from A to B



R_3



R_4



R_5

R_3, R_4 & R_5 are functions from A to B

Topic : Function

* If f is a function from A to B then,

A is called domain of the function f

and B is called Co-domain of the function f

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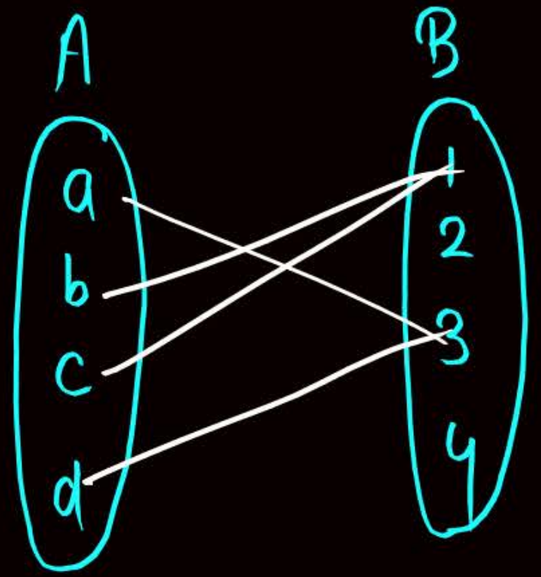

domain = {a, b, c, d}

Co-domain = {1, 2, 3, 4}

Topic : Range of a function

* Range of a function 'f' is the set of all the elements of Co-domain which are mapped by at-least one element of domain.

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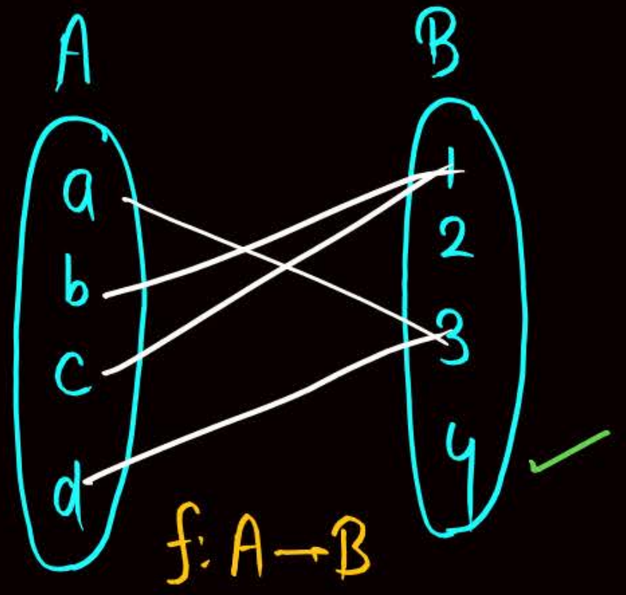


In general,

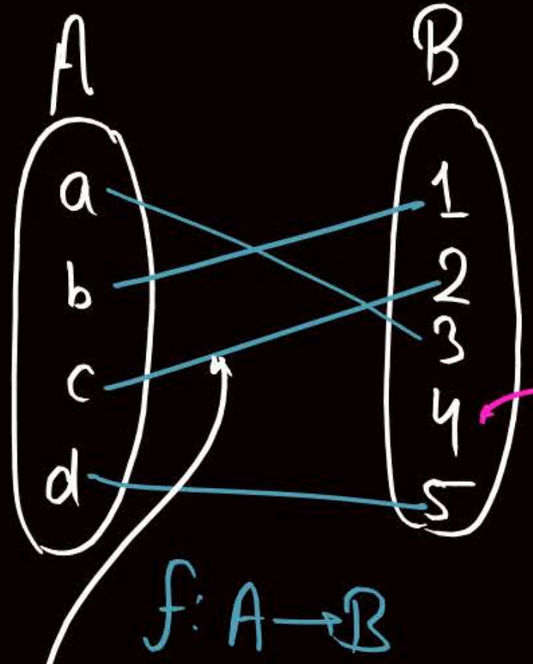
Range $\subseteq$ Co-domain

domain = {a, b, c, d}

Co-domain = {1, 2, 3, 4}

Range of function f = {1, 3}

Note :-



there is no pre image of 4 w.r.t. function 'f'.

$f(c) = 2$

image of 'c' w.r.t. function 'f' is '2'

and 'c' is the pre-image of 2.

Note:- A function 'f' from set A to set A itself is called a function on set A.

Topic : Total number of functions

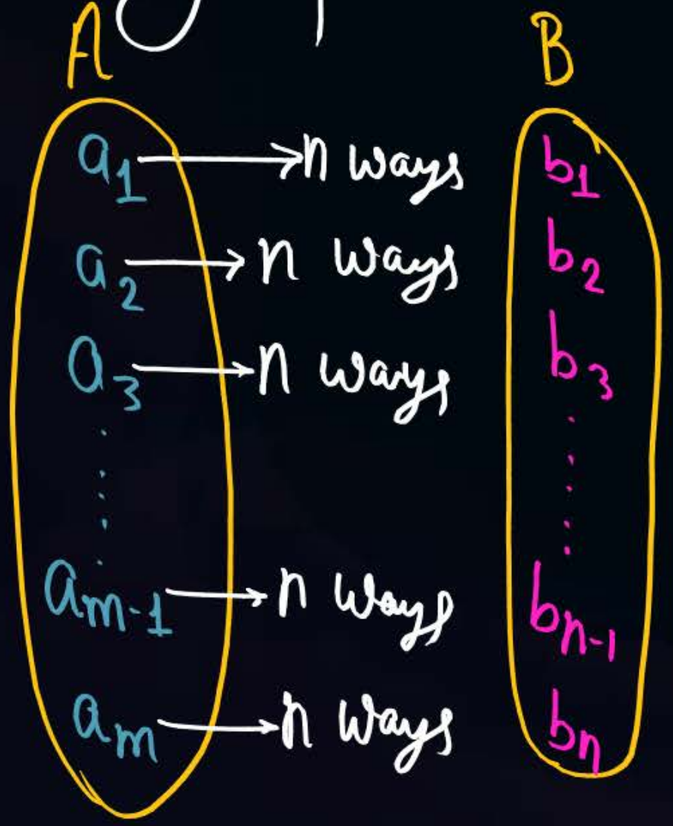
Co-domain

domain

* Let $|A| = m$ and $|B| = n$,
then how many functions are possible from A to B

$= \underbrace{n * n * n * \cdots * n}_{m \text{ times}}$

$= n^m$



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Note:-

let A and B are two sets, then

number of functions possible from A to B

$$= (\text{Cardinality of } B)^{(\text{Cardinality of } A)}$$

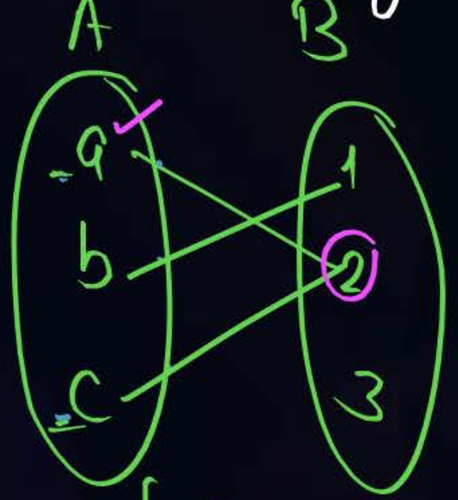
$$= |B|^{|A|}$$

$$= (\text{Size of co-domain})^{(\text{Size of domain})}$$

Topic : Injective (one-one) function

* A function $f: A \to B$ is called an injective (one-one) function if distinct elements of domain have distinct images in Co-domain. {i.e. every element of Co-domain have at most one pre-image}

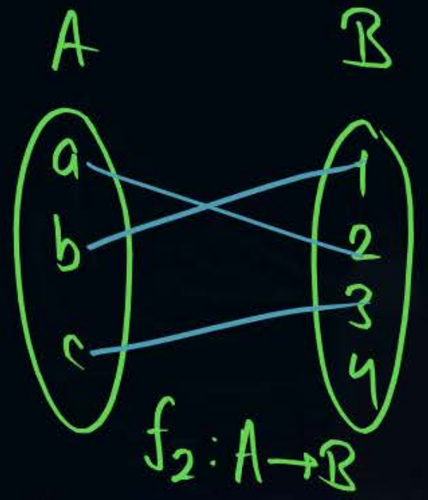
eg.



$f_1: A \to B$

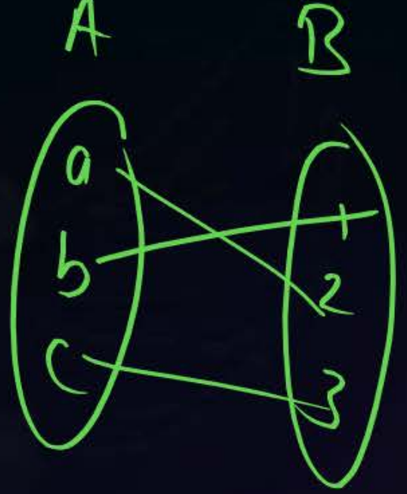
$f(a) = f(c) = 2$

and $a \neq c \therefore$ Not one-one



$f_2: A \to B$

it is one-one function



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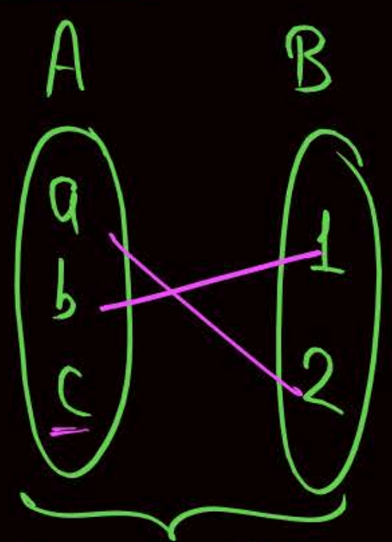
* **Note:-** In a one-to-one function

if $f(a) = f(b)$, then $a = b$

* **Note:-** A one-one function from set A to set B is Possible only if $|A| \leq |B|$

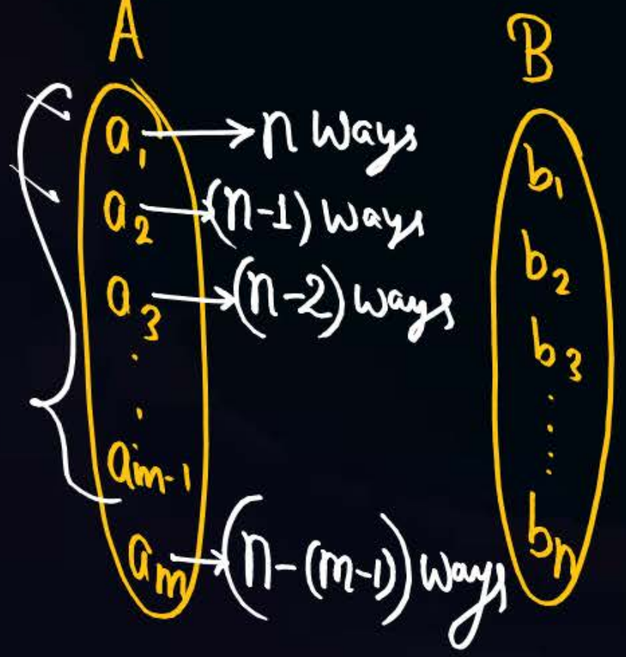


one-one function from A to B can not be defined if $|A| > |B|$

Topic : Number of one-one function

* Let $|A| = m$ & $|B| = n$ such that $(m \leq n)$,

How many one-one functions are possible from A to B = $n * (n-1) * (n-2) * \cdots * (n-(m-1))$



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$|A| = m$, $|B| = n$

number of functions from A to B $= n * (n-1) * \cdots (n-(m-1)) * \dfrac{(n-m)*(n-m-1)\cdots 3*2*1}{(n-m)(n-m-1)*\cdots 3*2*1}$

$= \dfrac{n(n-1)(n-2)*\cdots(n-m+1)*(n-m)*(n-m-1)*\cdots 3*2*1}{(n-m)*(n-m-1)*\cdots *3*2*1}$

$= \dfrac{n!}{(n-m)!} = {}^{n}P_{m}$

* Note:- Let $|A| = n$,

How many functions are possible on set A = ?

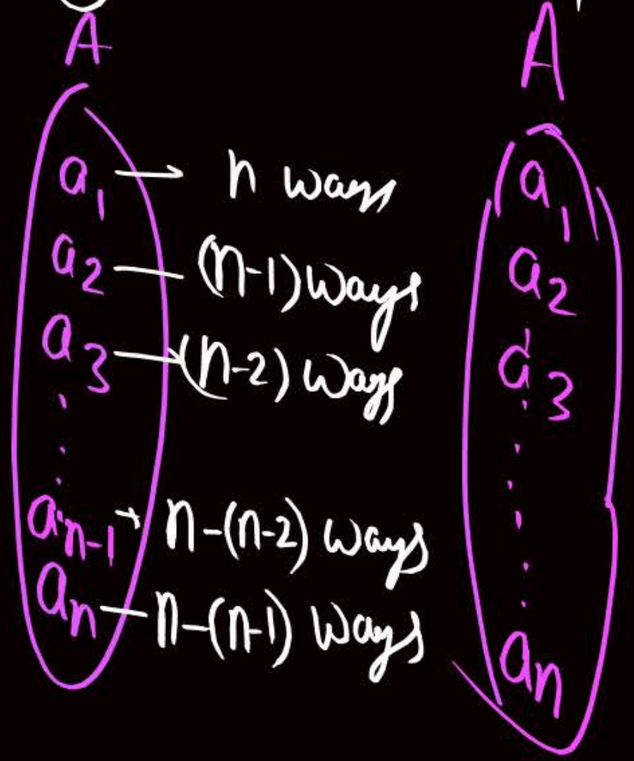
from A to A = $|A|^{|A|}$

$= n^n$

Note:

Let $|A| = n$.

How many one-one functions are possible on set A = ?



$= n * (n-1) * (n-2) \cdots - 3 * 2 * 1$

$= n!$

Q: Let $|A| = |B| = n$. then

How many one-one functions are possible from set A to set B = $n!$

Topic : Surjective (onto) function

A function from set A to set B is called onto (Surjective) function if every element of Co-domain is mapped by at least one element of the domain

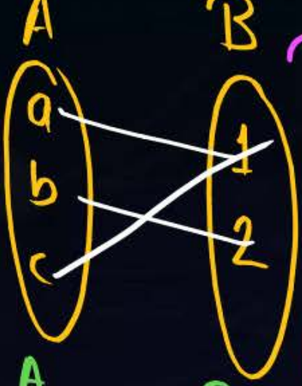
i.e. In an onto function

Range = Co-domain

eg:

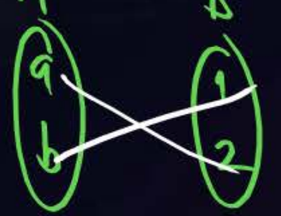
if $|B| \geq |A|$ then onto function from A to B is not possible

eg:



Both are onto

eg:



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Note:

* Onto function may be possible from set A to set B only if $|A| \geq |B|$

Topic : Number of onto functions

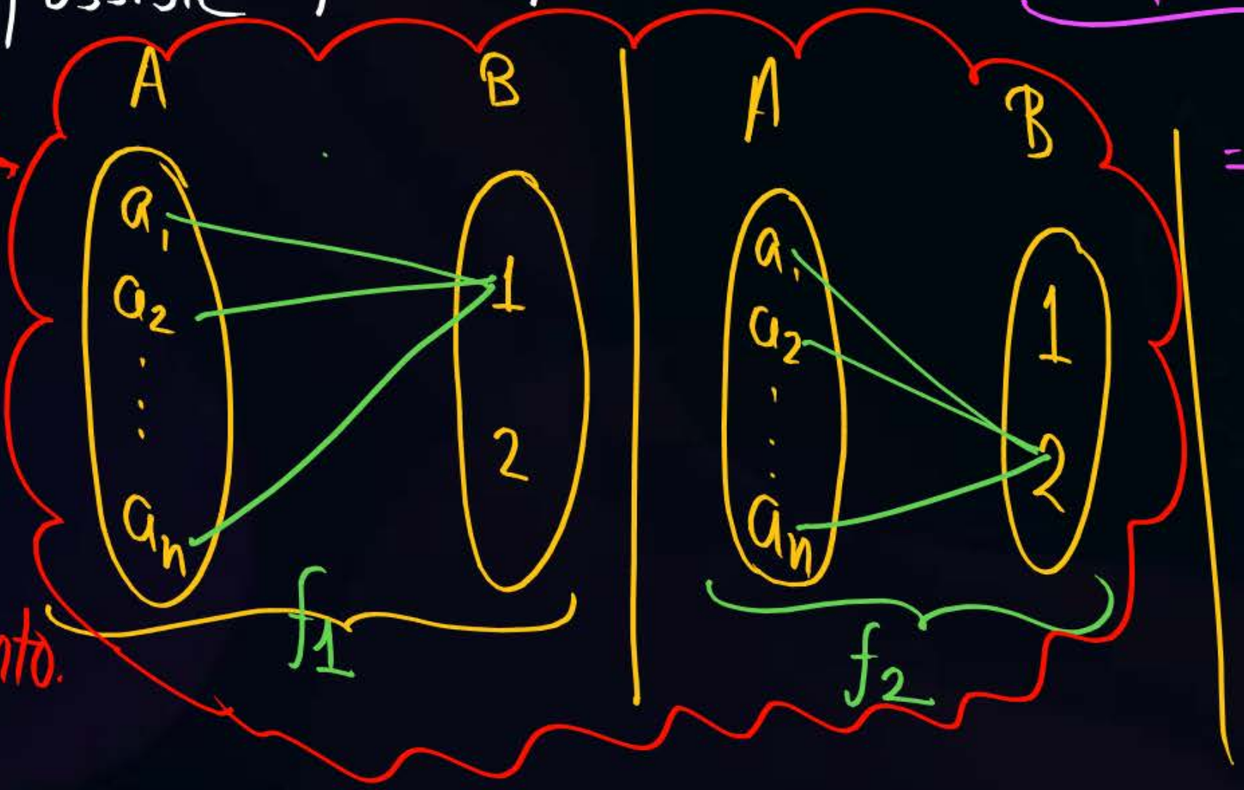
IMP

let $|A| = n$ and $|B| = 2$, then

Number of onto functions Possible from A to B = Total no. of functions from A to B − The number of functions from A to B, which are not onto

$= 2^n - 2$

f_1 and f_2 are the only two functions from A to B which are not onto.



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Topic : Number of onto functions

Note:- If $|A| = |B|$, then Every one-one function from A to B is onto, and every onto function from A to B is one-one

$\therefore$ If $|A| = |B|$, then

Number of onto functions from A to B = No. of one-one function from A to B

i.e. if $|A| = |B| = n$,

then No. of one-one function = No. of onto functions = $n!$

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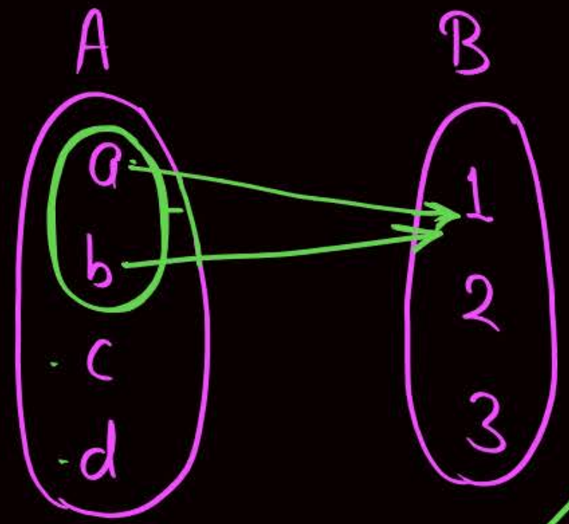
Q: Let $|A|=4$ and $|B|=3$, then how many onto functions are possible from A to B.



In this case Exactly two elements of set A will map with same element of set B

$^4C_2$ * 3! = 6 * 6 = 36

$^4C_2$: Those two elements can be Choosen in $^4C_2$ ways And Can be Combined into a Single element

3!: after that there will be logically 3 elements in set A & 3 elements in set B,

∴ No. of onto functions Possible = 3! When $|A|=|B|=3$

Topic : Number of onto functions

Note:- Let $|A| = n$ and $|B| = (n-1)$, then

Number of onto functions possible from A to B $= {}^{n}C_{2} * (n-1)!$

Case: when

Size of domain = 1 + Size of Co-domain

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Topic : Number of onto functions

H.W. Let $|A|=m$ & $|B|=n$ where $(m \geq n)$, then number of onto functions Possible from A to B

$$= n^m - {}^nC_1 \cdot (n-1)^m + {}^nC_2 \cdot (n-2)^m - {}^nC_3 \cdot (n-3)^m + \cdots + (-1)^{n-1} \cdot {}^nC_{n-1} \cdot (n-(n-1))^m$$

$$= {}^nC_0 \cdot (n-0)^m - {}^nC_1 \cdot (n-1)^m + {}^nC_2 \cdot (n-2)^m - {}^nC_3 \cdot (n-3)^m + \cdots + (-1)^{n-1} \cdot {}^nC_{n-1} \cdot (n-(n-1))^m$$

$$= \sum_{i=0}^{n-1} (-1)^i \cdot {}^nC_i \cdot (n-i)^m$$

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2 mins Summary

Topic Function

Topic Domain, Co-domain and Range of function

Topic One-one function

Topic Onto function

Topic Bijective function

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THANK - YOU