GATE
ALL BRANCHES
ENGINEERING MATHEMATICS

Probability and Statistics

Lecture No. 04

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TOPICS TO BE COVERED

01 Classification of Events

02 Questions Based on Events

Classification Events :

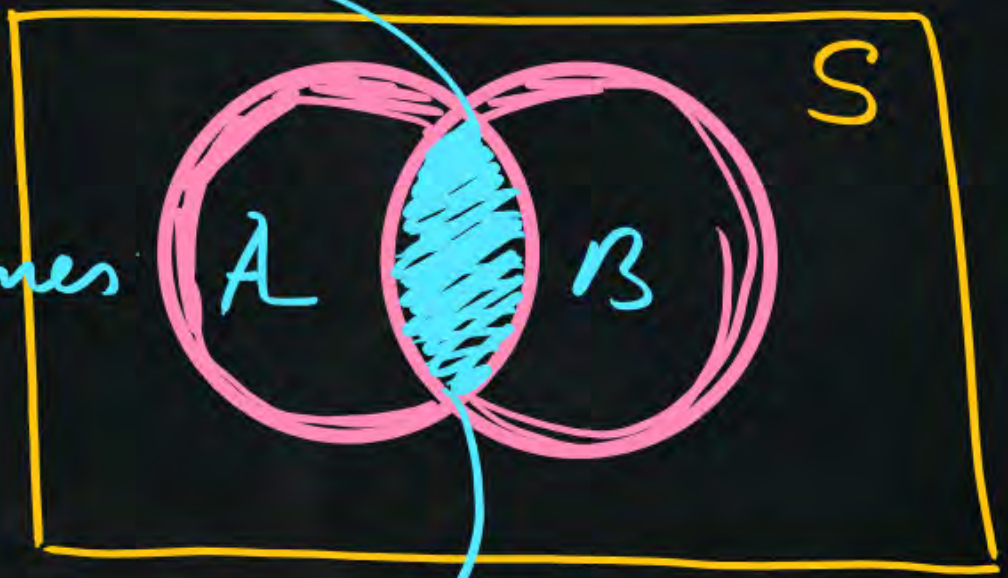
P(single event)

$$P(E) = \frac{n(E)}{n(S)}$$

compound events : Min Two events OR More Than Two events

CASE(A) something is Common (A, B)

for outcomes



A and B

$A \cap B$ (both occur)

$\cap$ = both A and B

$$P[A \text{ and } B] = P[A \cap B]$$
$$= P(\text{both Happening})$$

$$P(A \cap B) = P(\text{both occur})$$
$$= \frac{\text{No. of fav}}{\text{Total outcomes}}$$

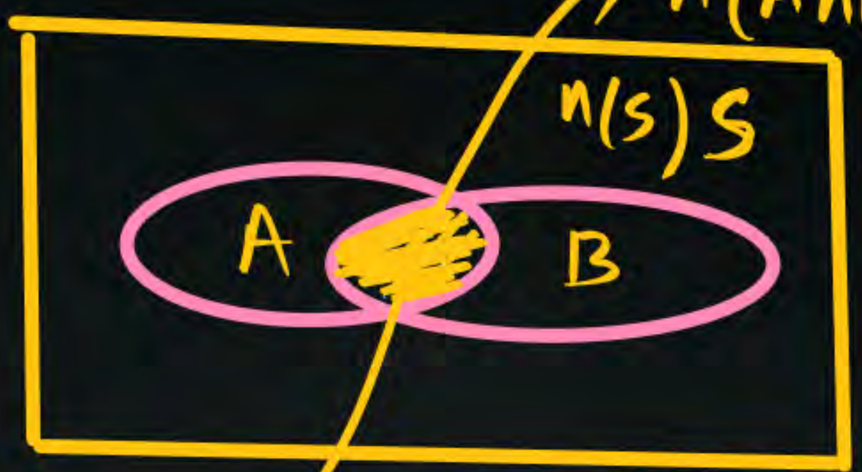
$P(\text{both occur}) = P(A \cap B)$

$$\boxed{P(A \cap B) = \frac{n(A \cap B)}{n(S)}}$$

A and B

both occur

$$\frac{\text{common element}}{\text{Total element}}$$



A and B Are simultaneously occur

Illustration 01

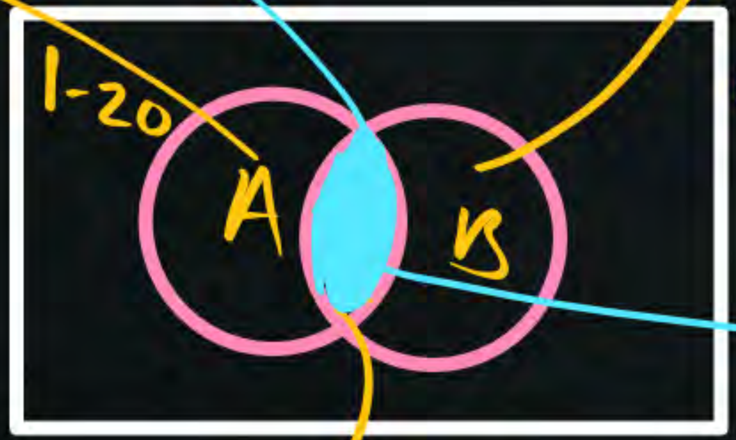
1 - 20 Digit

What is the Prob. P[div by 2 and 3]

A = div by 2   B = div by 3

div by 2

div by 3.



6, 12, 18 = 3

P[div by 2 and 3]

$P(A \cap B) = P(\text{div by 2 and 3})$

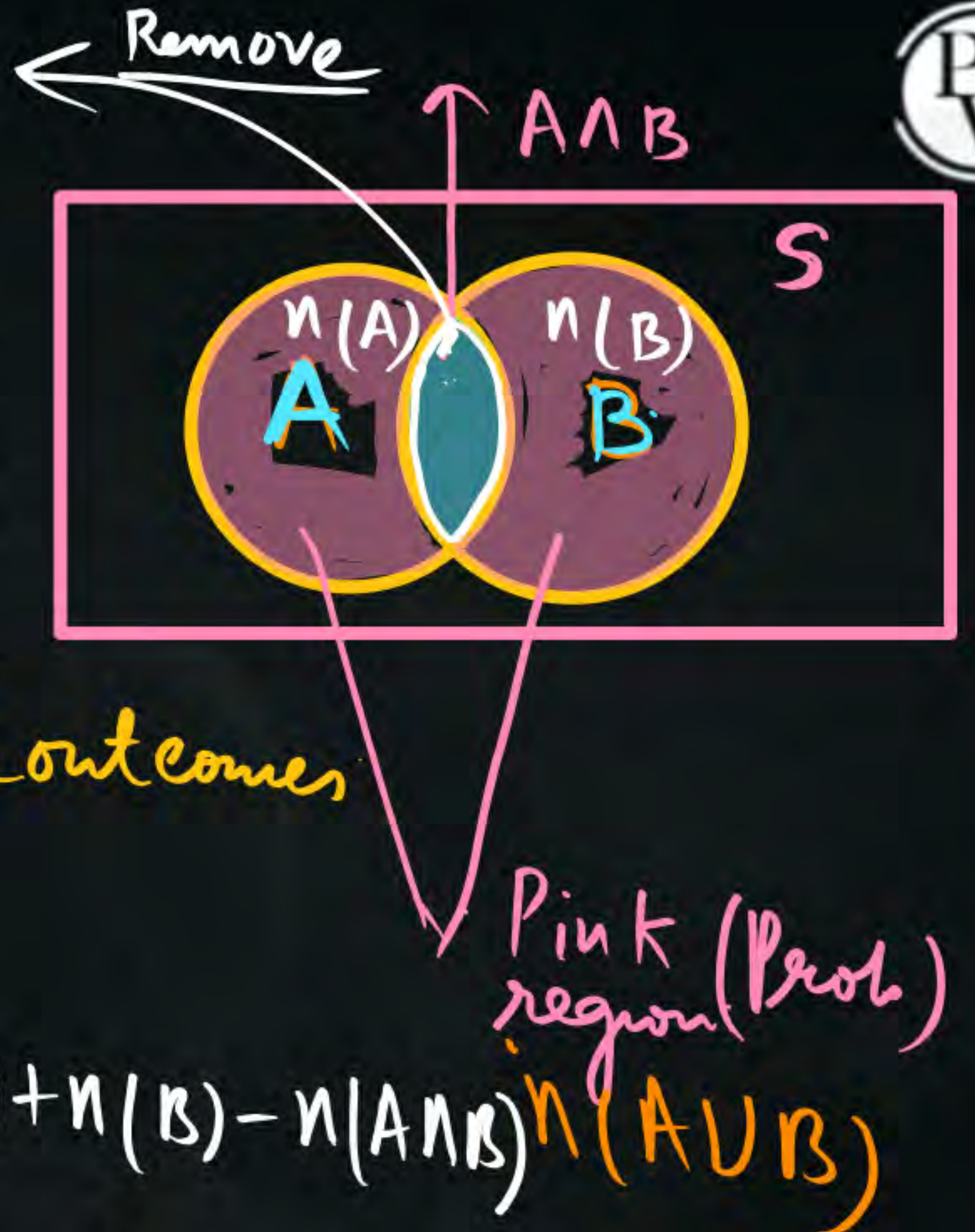
$\Rightarrow \frac{3}{20}$ Ans

Case(B) (something is common)

$P(A \text{ or } B) = P(A \cup B) = P(\text{either } A \text{ or } B)$

$P(A \text{ or } B) = \dfrac{\text{No. of Favourable}}{\text{Total No. of Possible outcomes}}$

$\Rightarrow \dfrac{n(A \cup B)}{n(S)}$



Using Venn Diagram $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$P(A \text{ or } B) = P(A \cup B) = \dfrac{n(A) + n(B) - n(A \cap B)}{n(S)}$

$= \dfrac{n(A)}{n(S)} + \dfrac{n(B)}{n(S)} - \dfrac{n(A \cap B)}{n(S)}$

$$\boxed{P(A\cup B) = P(A) + P(B) - P(A\cap B)}$$

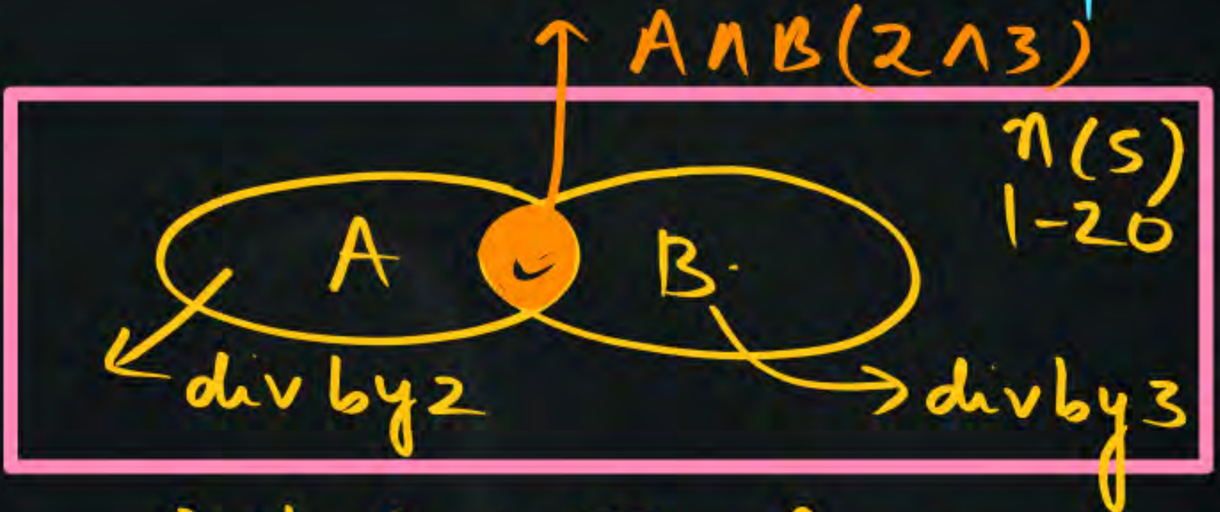
Prob Addition THEOREM

$$P(A\cup B) = P(A) + P(B) - P(A\cap B)$$

$$P(\text{div by } 2\cup 3) = P(2) + P(3) - P(2\cap 3)$$

$$= \frac{10}{20} + \frac{6}{20} - \frac{3}{20}$$

$$\boxed{P(\text{div by } 2 \text{ or } 3) = \frac{13}{20}}$$



What is The Prob

$P(\text{div by } 2 \text{ or div by } 3)$

✓ CASE 03

dependent events :: (without replacement)

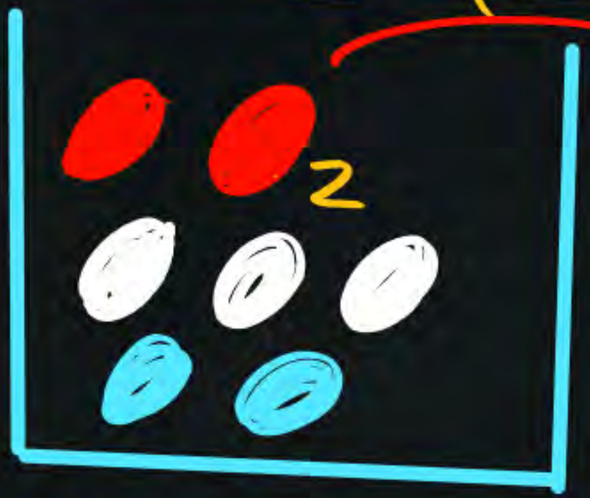
Stage (1)

Box − 1

$P(\text{Red } 1) = \frac{3}{8}$

Ist draw

Stage (2)



$P\left(\frac{\text{Red } 2}{\text{Red } 1}\right) = \frac{2}{7}$

Red 1 is already Happened

R_2 is Happeining

2nd Draw

Stage (3)

$P\left[\frac{\text{Red } 3}{\text{Red } 1 \cap \text{Red } 2}\right] = \frac{1}{6}$

"What is The Prob P(3 Red balls Are drawn at random one at a time)

$$P(\underbrace{3\text{ red Balls}}) = P(\text{Red }1)\,P\left(\frac{\text{Red }2}{\text{Red }1}\right)P\left(\frac{\text{Red }3}{\text{Red }1\cap\text{Red }2}\right)$$

Red 1
Red 2
Red 3] Three events Are working Together

$$P(\text{Red }1\cap\text{Red }2\cap\text{Red }3)$$

$$\Rightarrow \boxed{P(R_1\cap R_2\cap R_3) = P(R_1)\,P\left(\frac{R_2}{R_1}\right)P\left(\frac{R_3}{R_1\cap R_2}\right)}$$ Dependent events

$R_1 \to A \quad R_2 \to B \quad R_3 \to C$

$$\boxed{P(A\cap B\cap C) = P(A)\,P\left(\frac{B}{A}\right)P\left(\frac{C}{A\cap B}\right)}$$ → Events Are Dependent ✓

If n Events (Dependence of events)

$$P(A \cap B \cap C \cap D \cap E - - -) = P(A)\, P\left(\frac{B}{A}\right) P\left(\frac{C}{A \cap B}\right) P\left(\frac{D}{A \cap B \cap C}\right) - - -$$

If Two events Are dependent

$$\boxed{P(A \cap B) = P(A)\, P\left(\frac{B}{A}\right)}$$

A) SAMPLE space Are Reduced

B) Next prob effected on Previous probability

CASE 04 Independent events : (with replacement)

$P(Y_1) = \frac{3}{5}$

$P\left(\frac{y_2}{y_1}\right) = \frac{3}{5} = P(y_2)$

$P\left(\frac{y_3}{y_1 \cap y_2}\right) = \frac{3}{5} = P(y_3)$

"What is the prob 3 yellow balls are Drawn at random one at a time".

$$P(3 \text{ yellow balls}) = P(y_1 \cap y_2 \cap y_3) = P(y_1)\, P\left(\frac{y_2}{y_1}\right) P\left(\frac{y_3}{y_1 \cap y_2}\right)$$

$$P\left(\frac{y_2}{y_1}\right) = P(y_2) \qquad P\left(\frac{y_3}{y_1 \cap y_2}\right) = P(y_3)$$

$$\boxed{P(y_1 \cap y_2 \cap y_3) = P(y_1)\, P(y_2)\, P(y_3)} \rightarrow \text{"Independent events"}$$

$y_1 \rightarrow A \quad y_2 \rightarrow B \quad y_3 \rightarrow C$

$$\boxed{P(A \cap B \cap C) = P(A)\, P(B)\, P(C)} \longrightarrow \text{Independence of events}$$

for two events

$$\boxed{P(A \cap B) = P(A)\, P(B)}$$

for 'n' events $P(A \cap B \cap C \cap D \cap E \ldots) = P(A)\, P(B)\, P(C)\, P(D) \ldots$

"something is common"

CASE 05 : $P(\text{only } A)$, $P(\text{only } B)$, $P(\text{exactly one})$

If A and B both are Independent events

$$P(\text{only } A) = P(A) - P(A \cap B)$$

$$P(\text{only } B) = P(B) - P(A \cap B)$$

only A

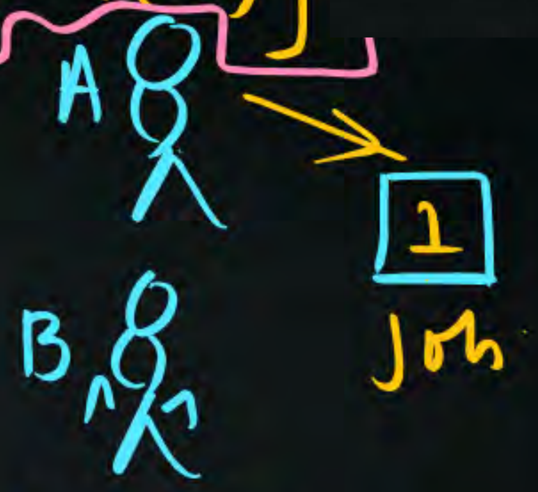
$$P(\text{only } A) = P(A) - P(A)P(B)$$

$$\begin{cases} P(\text{only } A) = P(A)[1 - P(B)] \\ P(\text{only } B) = P(B)[1 - P(A)] \end{cases}$$

only B

✓ $P(\text{only } A) = P(A \cap \bar{B}) = P(A)\,P(\bar{B}) = \boxed{P(A)[1 - P(B)]}$

$\therefore$ If A and B Are Independent events $\longrightarrow$ $\bar{A}$ $\bar{B}$ also Independent



✓ $P(\text{only } B) = P(\bar{A} \cap B) = P(\bar{A})\,P(B)$

$P(\text{only } B) = \boxed{P(B)[1 - P(A)]}$

A select B Not $\rightarrow$ only A

B select A Not $\rightarrow$ only B

CASE 03

$P(\text{exactly one}) =$

$P(\text{exactly one}) = P(\text{only } A) + P(\text{only } B)$

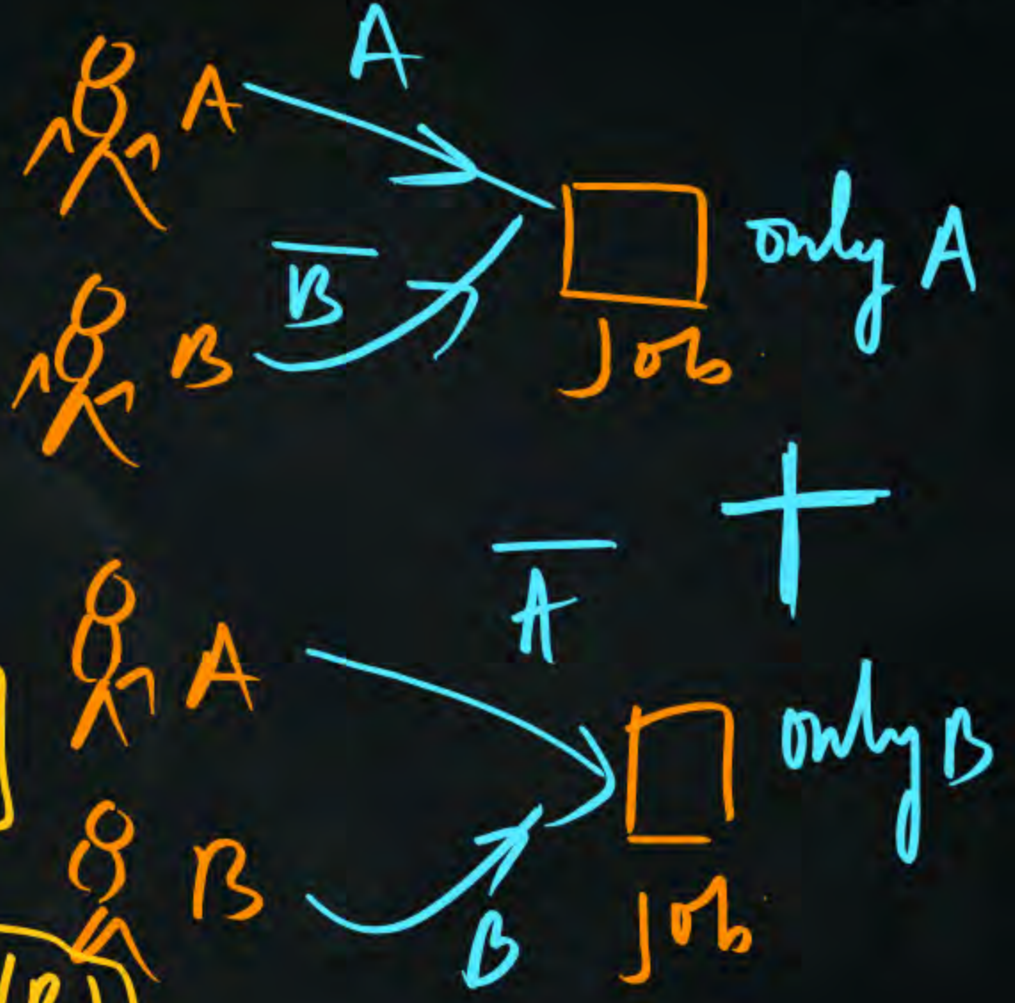
$P[(A \cap \bar{B}) \cup (\bar{A} \cap B)] = P(A \cap \bar{B}) + P(B \cap \bar{A})$

$= P(A)[1 - P(B)]$

$+ P(B)[1 - P(A)]$

$$\boxed{P(\text{exactly one}) \Rightarrow P(A) + P(B) - 2P(A)P(B)}$$

$\Rightarrow \underline{P(A) + P(B) - 2P(A \cap B)}$



Event A — A, $\overline{A}$

event B — B, $\overline{B}$

$$A \begin{cases} B = A \cap B \text{ both} \\ \overline{B} = A \cap \overline{B} \text{ Only A} \end{cases}$$

$$\overline{A} \begin{cases} B = \overline{A} \cap B \text{ only B} \\ \overline{B} = \overline{A} \cap \overline{B} \text{ (Neither A Nor B)} \end{cases}$$

"A and B Independent events"

	A (Happen)	$\overline{A}$ (does Not Happen)
B (Happen)	$A \cap B$ (both occur)	$\overline{A} \cap B$ (only B)
$\overline{B}$ (does Not)	$A \cap \overline{B}$ (only A)	($\overline{A} \cap \overline{B}$) Neither A Nor B

$\overline{A} \cap B$ (only B) + $A \cap \overline{B}$ (only A) → P(exactly one)

Q. Questions

H.W Homework

Let E and F be two independent events. The probability that exactly one of them occurs is 11/25 and the probability of none of them occurring is 2/25. If P(T) denotes the probability of occurrence of the event T, then:

(a) $P(E)=\frac{4}{5}, P(F)=\frac{3}{5}$

(b) $P(E)=\frac{4}{5}, P(F)=\frac{2}{5}$

(c) $P(E)=\frac{2}{5}, P(F)=\frac{1}{5}$

(d) $P(E)=\frac{3}{5}, P(F)=\frac{4}{5}$

Q. Questions

$6 = 2 \times 3 = 48$

$8 = 2 \times 4$

Lcm $2 \times 3 \times 4 = 24$

6 or 8: $(6 \vee 8)$ $\quad \frac{200}{6} = 33 \quad \frac{200}{24} = 8$

An integer is chosen at random from 200 positive integers. Find the probability that the integer chosen is divided by 6 or 8

$n(S) = 200$

(a) 1/4

(b) 1/6

(c) 1/2

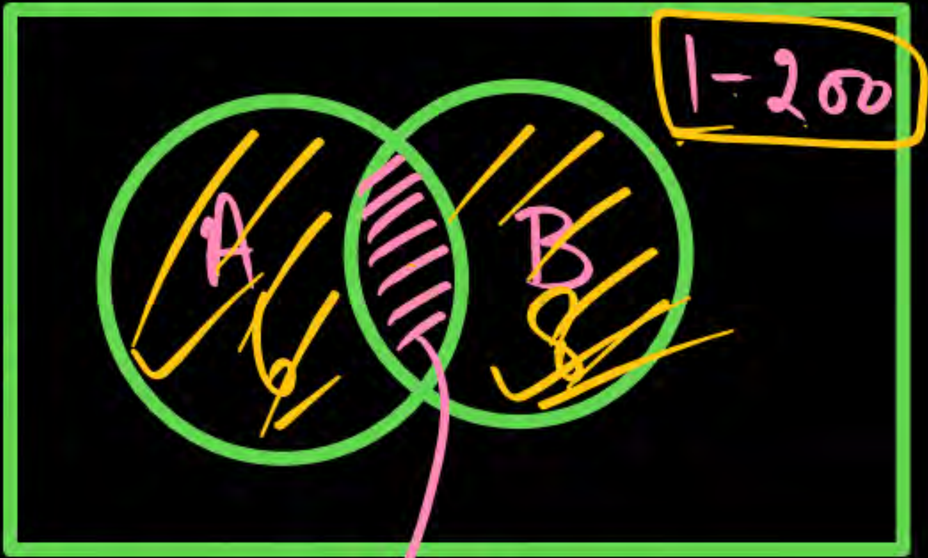
(d) N.O.T.

$P(6 \vee 8) = P(6) + P(8) - P(6 \text{ And } 8)$

$= \frac{n(6)}{200} + \frac{n(8)}{200} - \frac{n(6 \cap 8)}{200}$

$= \frac{33}{200} + \frac{25}{200} - \frac{8}{200} = \frac{50}{200} = \frac{1}{4}$

and 6, 8 Lcm 24 24



$(A \cap B)$

Q. Questions

✓ 26 Red < Dim, Heart
✓ 26 black < club, spade
4K, 4Q, 4J, 4A

$\boxed{52 \text{ cards}}$ → 2 cards

P(R ∩ K)

Two cards are drawn from pack of 52 cards. What is the probability that either of the cards, both are red or both are kings

P(R ∪ K)

(a) ✓ 660/2652

(b) 660/1352

(c) 330/2652

(d) 1/52×53

| R | K | → 2 cards

2 cards

Red ← | 26 | 4 | → King

52 cards

$P(\text{Red cards}) = \frac{26}{52} \times \frac{25}{51}$

$P(\text{King}) = \frac{4}{52} \times \frac{3}{51}$

$P(\text{red} \cap \text{King}) = \frac{2}{52} \times \frac{1}{51}$

$P(R \cup K) = P(R) + P(K) - P(R \cap K)$

$= \frac{26 \times 25 + 4 \times 3 - 2 \times 1}{52 \times 51}$

$= \frac{660}{2652}$

52 cards

↓ colour Division

(26) Red Colour	Black colour (26)
Photo 2	
Diamond (13), HEART (13)	Club (13), Spade (13)

13 cards

- face card Q, J, K
- Non face card A, 2 -- 10

Q. Questions

Independent $A \;\text{—}\; 3/4$, $B \;\text{—}\; 1/2$, $C \;\text{—}\; 5/8$

[Algebra for circuits]

3 person A, B, C independently try to hit a target, if probability of hitting a target by A, B, C are 3/4, 1/2, 5/8 then the probability that target hit by A or B but not C

$P[(A \vee B \wedge \bar{C})]$

$= P(A \vee B)\, P(\bar{C})$

$= (P(A) + P(B) - P(A \wedge B)]\,[1 - P(C)]$

$= \left[\frac{3}{4} + \frac{1}{2} - P(A)\cdot P(B)\right]\left[1 - \frac{5}{8}\right]$

$= \left[\frac{3}{4} + \frac{1}{2} - \frac{3}{4} \times \frac{1}{2}\right]\left[1 - \frac{5}{8}\right] = \frac{21}{64}$

$P[(A \vee B) \wedge \bar{C}]$

$P[(A \text{ or } B) \text{ but Not } C]$

$= P(A \vee B) \cdot P(\bar{C})$

(a) ✓ 21/64

(b) 1/64

(c) 7/64

(d) 3/64

Q. **Questions**

$P(E)P(F) = \frac{1}{12}$ $P(E \cap F) = \frac{1}{12}$ ✓

$P(\bar{E})P(\bar{F}) = \frac{1}{2}$ $P(\bar{E} \cap \bar{F}) = \frac{1}{2}$ $\frac{P(E)}{P(F)}$

Let E and F be two independent events. The probability of both E and F happens is 1/12 and neither E nor F happens is 1/2 then the value $\frac{P(E)}{P(F)} = ?$

(a) 4/5

(b) 3/2

(c) 1/5

(d) N.O.T.

$P(E \cap F) = P(E)P(F) = \frac{1}{12}$ $P(\bar{E})\,P(\bar{F}) = \frac{1}{2}$

$P(E)\,P(F) = \frac{1}{12}$ $[1-P(E)][1-P(F)] = \frac{1}{2}$

$\Rightarrow \quad x \cdot y = \frac{1}{12}$ — (1) $(1-x)(1-y) = \frac{1}{2}$ — (2)

In Equ^n (2)

$(1-x)(1-y) = \frac{1}{2}$

$= (1 - y - x + xy) = \frac{1}{2}$

$= 1 - y - x + \frac{1}{12} = \frac{1}{2}$

$= 1 - \frac{1}{2} + \frac{1}{12} = x + y$

$\boxed{x + y = \frac{7}{12}}$

$$\left\{\begin{array}{l} x + y = \frac{7}{12} \\ xy = \frac{1}{12} \end{array}\right.$$

Quadratic equ^n

P(both Happen) $= \frac{1}{12}$

P[Neither E Nor F] $= \frac{1}{2}$

✓ $P(E) = x$

$P(F) = y$

If something is common

$P(E) = x$ $P(F) = y$

If Nothing is common

$P(E) = x$

$P(F) = 1 - x$

$x+y=\frac{7}{12}$ $xy=\frac{1}{12}$

$x-y=\pm\sqrt{(x+y)^2-4xy}$

$=\pm\sqrt{\left(\frac{7}{12}\right)^2-4\times\frac{1}{12}}$

$=\pm\sqrt{\frac{49}{144}-\frac{4}{12}}$

$=\pm\sqrt{\frac{1}{144}}=\frac{1}{12}$

$x-y=\frac{1}{12}$

$\begin{bmatrix}\alpha+\beta\\ \alpha\beta\end{bmatrix}$

$\alpha-\beta=\sqrt{(\alpha+\beta)^2-4\alpha\beta}$

$x+\cancel{y}=\frac{7}{12}$

$x-\cancel{y}=\frac{1}{12}$

$2x=\frac{8}{12}$ $x=\frac{1}{3}$

$y=\frac{1}{4}$

$\frac{P(E)}{P(F)}=\frac{\frac{1}{3}}{\frac{1}{4}}=\frac{4}{3}$

Thank You!

GW Soldiers