GATE
ALL BRANCHES

ENGINEERING MATHEMATICS

Probability and Statistics

Lecture No. 11

BY- RAHUL SIR

TOPICS TO BE COVERED

01 Problems based on Probability Distribution

✓ Discrete Random var

✓ Continuous "

✓ Problems

✓ Prob. Distribution

finish

Normal

bernoulli

bionomal

Poisson

exponetial

Uniform

Discrete

Job

m

Bernoulli Trials:

(Discrete Distribution)

Arrival Pattern

- → Throwing A Die
- → Tossing A coin = HEAD
- → Tail
- → Picking A Ball 1, 2, 3, 4,
- → No. of Accidents
- → No. of insurance
- → Pick A card

Discrete Random variable (Distribution)

Arrival Pattern

Discrete Distribution → Arrival

$\boxed{X = \text{Number}}$

case

Continuous { How much Time required

H → H] Discrete
T   T

$\Rightarrow$ Discrete Distribution — Arrival Pattern $\longrightarrow$ How to get the success

Continuous Distribution — (Waiting Pattern)

(Claim Size) $a\,[\;\;]\,b$ Infinite pt

How much time required to get success.

$\Rightarrow$ Discrete $X = 0, 1, 2, 3, 4, 5 - - - -$

$\Rightarrow$ continuous $a \leq X \leq b$

Discrete Distribution :

Arrival $\begin{cases} \text{success} & 1 \\ \text{failure} & 0 \end{cases}$

Bernoulli Trials : Tossing A coin

HEAD   Tail

HEAD occurs — SUCCESS]
Tail occurs — failure] Independent Trials.

Throwing A Die

$\frac{1}{6} \longrightarrow$ SUCCESS → 6

$\frac{0}{6} \longrightarrow$ failure → 1, 2, 3, 4, 5

Pick A CARD

Ace $\longrightarrow$ SUCCESS ✓ 1

Not Ace $\longrightarrow$ failure ✓ 0

Bernoulli Trials always Independent

SUCCESS $\longrightarrow 1$

failure $\longrightarrow 0$

Indicator function $X = \begin{cases} 0 & \text{failure} \\ 1 & \text{success} \end{cases}$

✓ HEAD → present (1)   Tail → Not present (0)

Bernoulli trials (Independent) $\boxed{n=1}$

$\boxed{1 \mid 0}$

- H (success) 1
- T (failure) 0

$X =$ discrete (Arrival)

$X = \underbrace{0, 1}_{\text{success, failure}}$

$X = 0, 1$

Throwing A Die

- success (6) 1
- failure (1, 2, 3, 4, 5) 0

	success	failure
X	1	0
$P(X=x)$	$\frac{1}{2}$	$\frac{1}{2}$

$P(S) + P(F) = 1$

$X = 0, 1$

$X = 0, 1, 2, 3, 4, 5, 6, 7$ (Integer Arrival)

1 1 1 1 1 1 1

Total prob = 1

"$n = 1$ Single Trials"

Bernoulli Distribution

1	0

success
failure

brick

Prob. of success $= p$

Prob of failure $= 1 - p = q$

$$\boxed{\text{Prob}(S) + P(F) = 1}$$

X	1	0
$P(X=x)$	p	$(1-p) = q$

$(1 - \text{success}) = \text{failure}$

$E[X] = \text{mean}$

$$E[X] = \sum_{i=1}^{n} x_i p_i$$

$$= 1 \times p + 0 \times (1-p)$$

$$\boxed{E[X] = \mu = p}$$

$P(\text{red}) = \frac{2}{5}$

$P(\text{yellow}) = \frac{3}{5}$

bernoulli trials

$$\text{Var}(X) = E[X^2] - \left[E[X]\right]^2$$

$$\sigma_x^2 = V(x) = E[x^2] - [E[x]]^2$$

$$\sigma_x^2 = (1)^2 \times p + (0)^2 \times (1-p) - [p]^2$$

$$\sigma_x^2 = p - p^2 = p(1-p)$$

$$\boxed{\sigma_x^2 = p \cdot q}$$

Standard deviation $= \sqrt{pq}$

x	1	0
$P(X=x)$	p	$(1-p)$

bernoulli Trials - indep.

only one Parameter Are required → p.

$q = 1-p =$ failure

5 trials

bernoulli Trials [Independent Trials]

H → success 1

T → failure 0

A	1	0 ✓
B	1	0 ✓
C	1	0 ✓
D	1	0 ✓
E	1	0 ✓

✓ H H T T T

✓ T T H H T

✓ T T T H H

✓ T T T T T

Don't know

Not Decided

No. of trials are fixed

Prob. of success = p — (bernoulli Trials) only single Trial

Prob. of Failure = q

"What is The Prob. 3 HEADS occur)

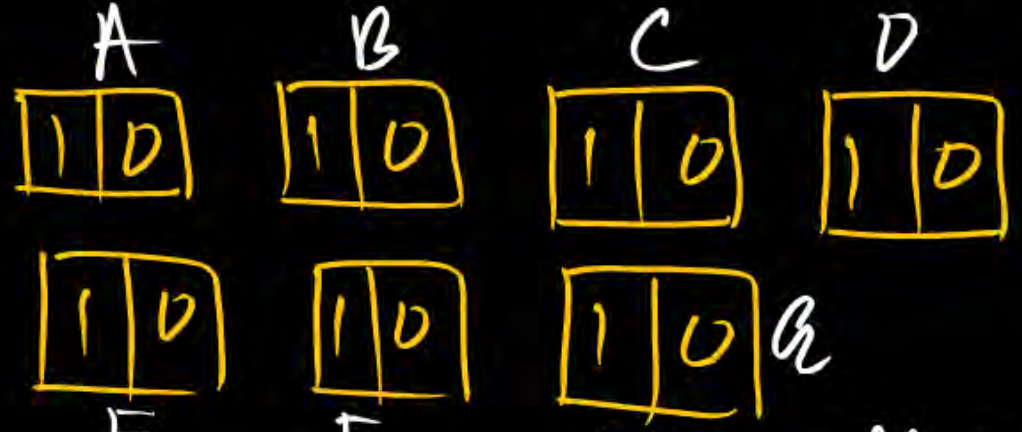
No. of trials = n (fixed)

Prob. of success = p

prob. of failure = q

No. of success = $r = \underline{0}, \underline{1}, \underline{2}, \underline{3}, \underline{4}, \underline{5}, \underline{6} \ldots$ → Mouse

(Arrival Pattern)



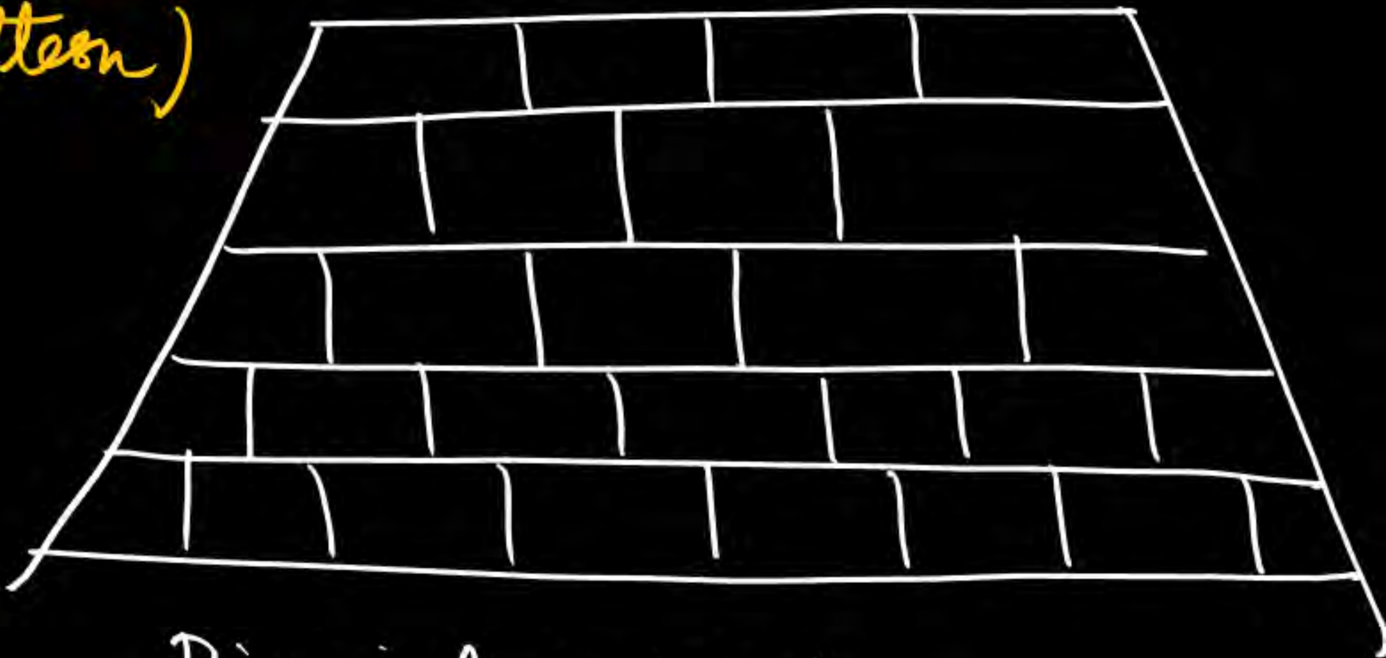
(Bernoulli Trials)

No. of Failure = $(n - r)$

prob. of success r times

$= p \cdot p \cdot p \cdot p \cdot p \ldots r$ times

$P(\text{success}) = p^r$



Binomial Distribution

Prob. of Failure $= (1-p)(1-p)(1-p) \cdots (n-r)$ times
$(n-r)$ times

$$\Rightarrow (1-p)^{n-r}$$

$$P[X = r \text{ success}] = {}^nC_r\, p^r (1-p)^{n-r}$$

In Binomial Distribution

$$\boxed{P[X = r \text{ success Arrival}] = {}^nC_r\, p^r q^{n-r}}$$

n = No. of trials
r = No. of success
$p = p(S)$ $q = P(F)$

$$P(S) + P(F) = 1$$

A B C D E

Re 1, Re 2, Re 3, Re 4, Re 5

$n = 5$ Trials (fixed)

$\underbrace{p = \frac{1}{2} \quad q = \frac{1}{2}}_{\text{Bernoulli Trials}}$ $r = 3$ ✓

Re 1, Re 2, Re 3, Re 4, Re 5

P(getting 3 HEAD) ✓
P(at least one HEAD)
P(at most one HEAD)

Using Binomial Distribution

$$P[X = 3\,\text{HEAD}] = {}^nC_r\, p^r q^{n-r}$$

$$= {}^5C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^{5-3}$$

$$= \frac{5 \times \cancel{4}^{2} \times \cancel{3}}{\cancel{3} \times \cancel{2} \times 1} \cdot \frac{1}{32}$$

$$= \frac{10}{32} = \frac{5}{16}$$

5 HEAD → 3 chose

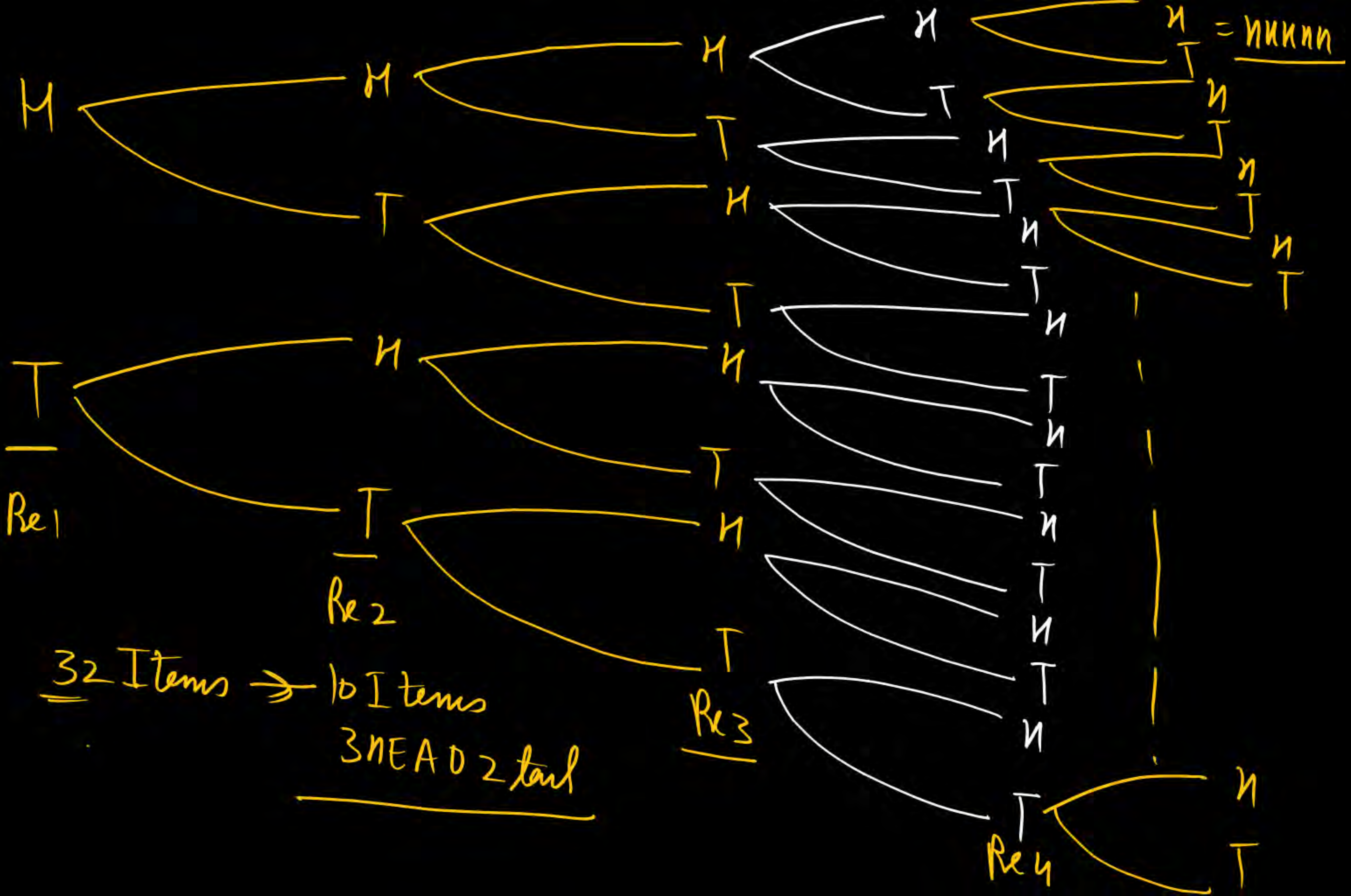
$$= {}^nC_r = {}^5C_3$$

$$= \frac{n!}{(n-r)!\,r!}$$

$\underbrace{{}^nC_r\, p^r q^{n-r}}$

$p \downarrow$ S, $q \downarrow$ F (SAME time) → independent

$P(A \cap B) = P(A)P(B)$



32 Items → 10 Items

3HEAD 2 tail

$P(\text{atleast one HEAD}) = P(X \geq 1)$

Re 1 — A → H 1H
Re 2 — B → H 2H
Re 3 — C → H 3H
Re 4 — D → H 4H
Re 5 — E → H 5H

n

$$P(X \geq 1) = \underbrace{P(X=1)}_{r=1} + \underbrace{P(X=2)}_{r=2} + \underbrace{P(X=3)}_{r=3} + \underbrace{P(X=4)}_{r=4} + \underbrace{P(X=5)}_{r=5}$$

$$= {}^5C_1\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^{5-1} + {}^5C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^{5-2} + {}^5C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^{5-3}$$

$$+ {}^5C_4\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^{5-4}$$

$$+ {}^5C_5\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^{5-5}$$

$=$ Ans

Re 1, Re 2, Re 3, Re 4, Re 5

0H -
1H
2H
3H
4H
5H

$$P(0H) + \underbrace{P(1H) + P(2H) + P(3H) + P(4H) + P(5H)}_{P(X \geq 1) = 1 - P(X=0)} = 1$$

$$P(X \geq 1) = 1 - P(X = 0)$$

$$= 1 - \underbrace{{}^5C_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^{5-0}}$$

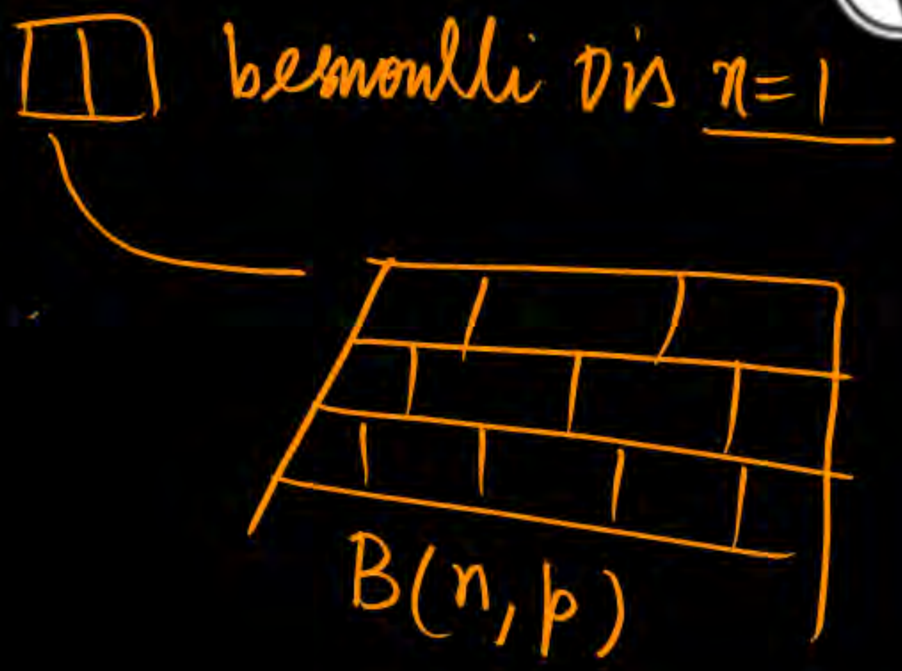
$$= 1 - \frac{1}{32} = \frac{31}{32} \text{ Ans}$$

3) $P[X \leq 2 \text{ HEAD}] = P(0H) + P(1H) + P(2H)$

$$= {}^5C_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^{5-0} + {}^5C_1\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^{5-1} + {}^5C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^{5-2}$$

$$= \frac{16}{32} \text{ Ans}$$

In binomial Distribution → Two Parametres Are Involve



bernoulli → binomial B(n, p)

Trials 1 ↑

MEAN: $E[X] = np$

variance: $V(X) = \sigma_X^2 = npq$

S.D $S.D = \sqrt{npq}$

where n = No. of trials

Thank You!

GW Soldiers