GATE
ALL BRANCHES
ENGINEERING MATHEMATICS

Probability and Statistics

Lecture No. 03

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01 Question Based on Probability

$$P(E) = \frac{n(E)}{n(S)} = \frac{\text{fav outcomes}}{\text{Total outcomes}}$$

- Without Replacement — Dependent
- With Replacement — Independent

Q. Questions

$\Rightarrow \frac{91}{216}$ ✓

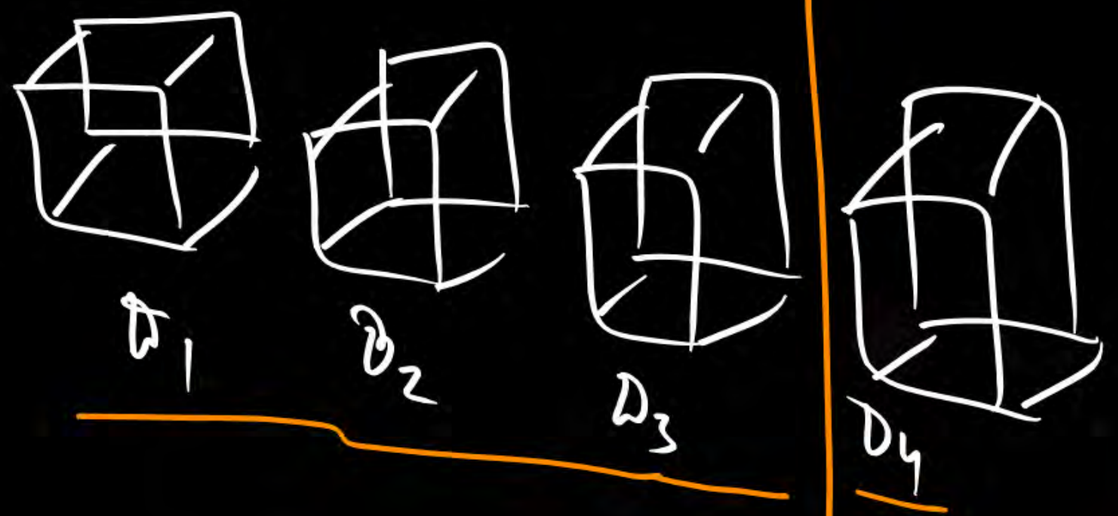
Four fair dice D_1, D_2, D_3, D_4, each having six faces numbered 1, 2, 3, 4, 5, 6 are rolled simultaneously. The probability that D_4 shows a number appearing on one of D_1, D_2 and D_3 is

(a) 91/216

(b) 108/216

(c) 125/216

(d) 127/216



D_1

D_2

D_3

D_4

✓ | a | b | c | d |
D_1 D_2 D_3 D_4

$S = \{1,2,3,4,5,6\}$

$a = (1,2,3,4,5,6)$
$b = (1,2,3,4,5,6)$
$c = (\quad)$
$d = (\quad)$

6 choice 6 choice 6 choice 6

D_1 D_2 D_3 D_4

Working Together

Total No. of choices $= 6 \times 6 \times 6 \times 6$
$= 6^4$ ✓

For chance :

1) $a=b=c$

d = 1 chance

$a=6$ ✓
$b=6$ ✓
$c=6$ ✓

$d=1$

D_1	D_2	D_3	D_4
a	b	c	d
6	1	1	1

$\Rightarrow 6\times1\times1\times1 = 6$

2) $a\neq b\neq c$

$D_1\neq D_2\neq D_3$

D_4 ← D_1 a, D_2 b, D_3 c

$6\times5\times4\times3 \Longrightarrow 6\times5\times4\times3 = 360$

$\begin{bmatrix}6\\5\\3\end{bmatrix}$ D_4 ← 6, 5, 3

$3=3\neq2$ D_4 ← 3, 2

3) $D_1=D_2\neq D_3$

$a=b\neq c$ D_4 ← a, c

$b=c\neq a$ D_4 ← a, b

$c=a\neq b$ D_4 ← c, a

$\Rightarrow 6\times1\times5\times2\times3$

180

$= 180+360+6$

$= 64$

1) $= D_1 = D_2 = D_3 =$

$D_4 = (D_1, D_2, D_3)$

1

$= \frac{6 \times 1 \times 1 \times 1}{6^4} = 6$ cases

$\rightarrow D_1 = D_2 = D_3$

✓ $5 = 5 = 5$

D_4

$\frac{6 \times 1 \times 1 \times 1}{6^4}$ | $D_4 = 5$

✓ $D_1 \neq D_2 \neq D_3$

$D_4 <$ D_1, D_2, D_3

$= \frac{6 \times 5 \times 4 \times 3}{6^4}$

✓ $D_1 = D_2 \neq D_3$

$D_2 = D_3 \neq D_1$

$D_1 = D_3 \neq D_2$

$D_4 <$ D_1, D_3

$D_4 <$ D_2, D_1

$D_4 <$ D_1, D_2

$= \frac{6 \times 1 \times 5 \times 2 \times 3}{6^4}$

Q. Questions

Three of the six vertices of a regular hexagon are chosen at random. The probability that a triangle with three vertices is equilateral.

Q. Questions

If P and Q are chosen randomly from the set {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} with replacement. Determine the probability that the roots of the equation

$x^2 + px + q = 0$ are real.

If p and q

$p = q = S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$P\left(p^2 - 4q \geq 0\right) = \dfrac{\left(p^2 - 4q \geq 0\right)}{\text{Total}}$

p: 1, 2, 3, ..., 10

q

$x^2 + px + q = 0$

Roots are real

$ax^2 + bx + c = 0$

$b^2 - 4ac \geq 0$

$\Rightarrow [p^2 - 4q \geq 0]$

$P[p^2 - 4q \geq 0] = \dfrac{\text{No of fav outcomes } p^2 - 4q \geq 0}{\text{Total outcomes}}$

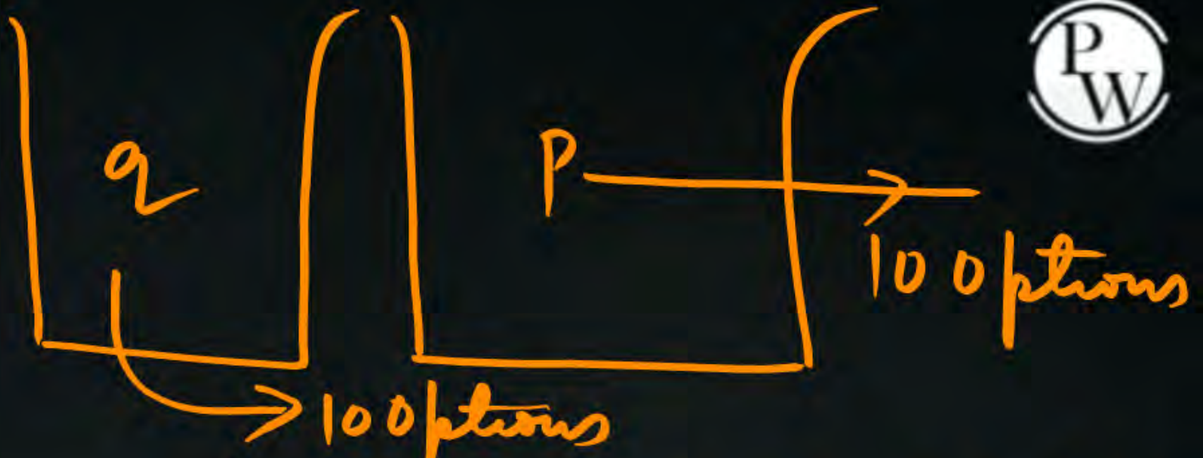
p: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10

q: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10

Total Numbers.

Total choices $= 10 \times 10$

$= 100$



$q = P$	1	2	3	4	5	6	7	8	9	10
P^2	1	4	9	16	25	36	49	64	81	100
$4q$	4	8	12	16	20	24	28	32	36	40

P	1	2	3	4	5	6	7	8	9	10
q	1	2	3	4	5	6	7	8	9	10

$p^2 - 4q \geq 0 \Rightarrow 0+1+2+4+6+9+10+10+10+10$

$\Rightarrow 62$

$n(p^2 - 4q \geq 0) = \underline{\underline{62}}$

$P(E) = \frac{62}{100}$ ✓

Q. Questions

Try yourself

The probability that two friends share the same birth month is _____.

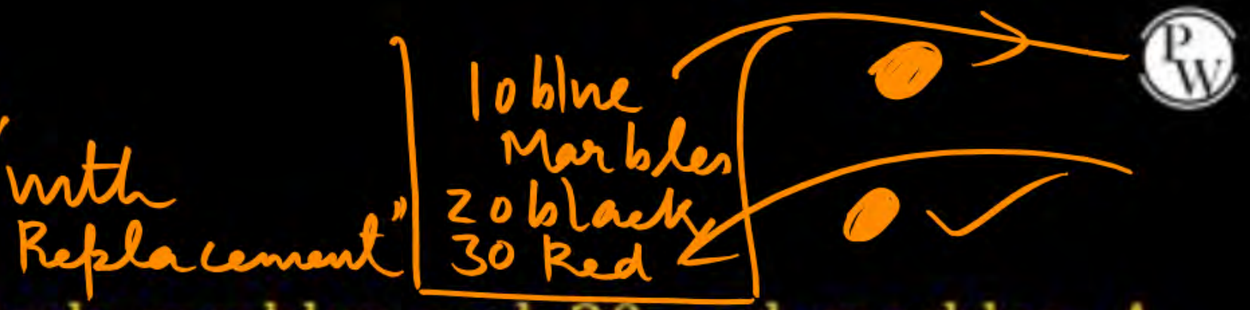
Q. Questions

Seven car accidents occurred in a week. What is the probability that they all occurred on the same day?

Q. Questions

A box contains 10 screw, 3 of which are defective. Two screws are drawn at random with replacement. The probability that none of the two screws is defective will be _____.

Q. Questions



A bag contains 10 blue marbles, 20 black marbles and 30 red marbles. A marble is drawn from the bag, its colour is recorded and it is put back in the bag. This process is repeated 3 times. The probability that no two of the marbles drawn have the same colour.

10 blue marbels
20 black
30 Red

3 times → 3 balls Draw

No Two Marbels Are The SAME Colour.

(1) → Blue Red Black → No Two marbels ARE SAME

" " " " " " Colour

$= 60$ ptons

OR $3! \times \frac{10}{60} \times \frac{30}{60} \times \frac{20}{60}$

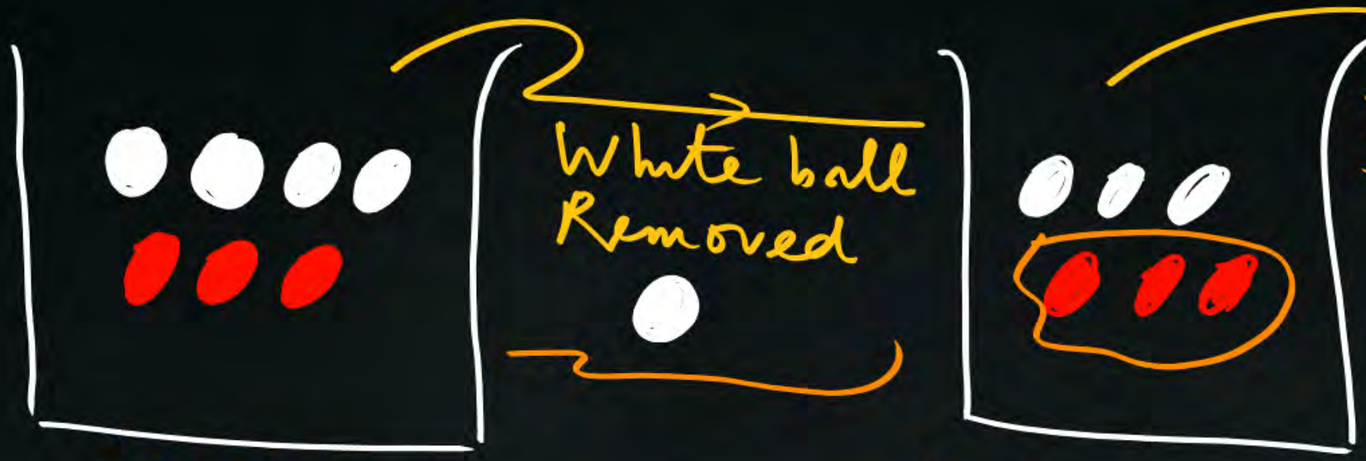
$= 6 \times$ (blue) (red) (white)

$= 6 \times \frac{1}{6} \times \frac{3}{6} \times \frac{2}{6} = \frac{1}{6}$

Q. Questions

A box contains 4 white balls and 3 red balls. In succession, two balls are randomly selected and removed from the box, given that the first removed ball is white, the probability that the second removed ball is red is _____.

4 White
3 Red



SAMPLE Space ①

Reduced SAMPLE Space

SECOND Removed ball

$= P(\text{SECOND removed ball red})$

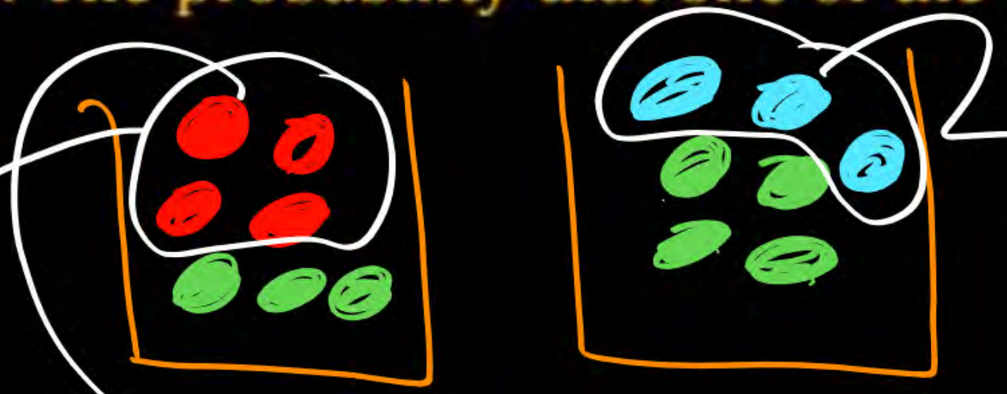
$= \frac{3}{6} = \frac{1}{2}$

Q. Questions

Individual work + working Together ×

$\frac{12}{49}$]

There are two containers, with one containing 4 Red and 3 Green balls and the other containing 3 Blue balls and 4 Green balls. One ball is drawn at random from each container. The probability that one of the balls is Red and the other is Blue will be ____.



$\frac{4}{7}$

blue $= \frac{3}{7}$

$= \frac{4}{7} \times \frac{3}{7} = \frac{12}{49}$ ✓

$\frac{4}{7} \times \frac{3}{7} = \frac{12}{49}$ Ans

Q. Questions

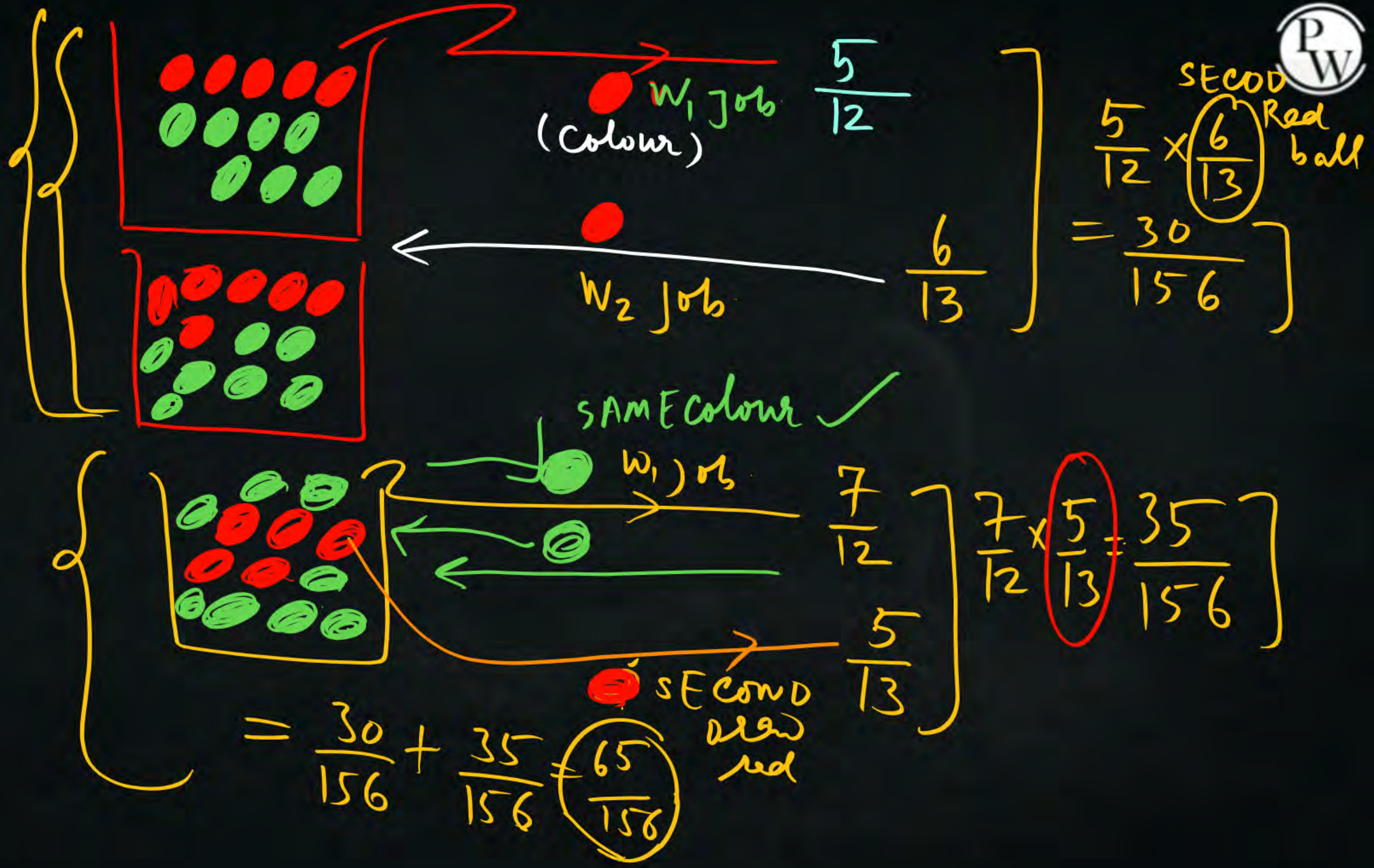
$\frac{65}{156}$

3 min

An urn contains 5 red and 7 green balls. A ball is drawn at random and its colour is noted. The ball is placed back into the urn along with another ball of the same colour. The probability of getting a red ball in the next draw is ____.

ball + Add

Colour Note → ball + Add


W1 Job
(Colour)
5/12
W2 Job
6/13
5/12 × 6/13 = 30/156
SECOND Red ball
SAME Colour ✓
W1 Job
7/12
5/13
7/12 × 5/13 = 35/156
SECOND Red ball
= 30/156 + 35/156 = 65/156

Q. Questions

$\frac{13}{32}$ ✓

3W 1B | 2W 2B | 1W 3B

If from each of the three boxes containing 3 white and 1 black balls, 2 white and 2 black balls, 1 white and 3 black balls, one ball is drawn at random, then the probability that 2 white and 1 black ball will be drawn is _____.

2White 1Black

P(2 white 1 black) balls Are Identical

I	II	III
W	W	B
B	W	W
W	B	W

$$\frac{3}{4}\times\frac{2}{4}\times\frac{3}{4}=\frac{18}{64}$$

$$\frac{1}{4}\times\frac{2}{4}\times\frac{1}{4}=\frac{2}{64}$$

$$\frac{3}{4}\times\frac{2}{4}\times\frac{1}{4}=\frac{6}{64}$$

$$\frac{n(E)}{n(S)}$$ — formula ↓

$$P(2\text{white }1\text{black})=\frac{18}{64}+\frac{2}{64}+\frac{6}{64}=\frac{26}{64}=\frac{13}{32}$$

Q. Questions

H.W

Urn A contains 6 red and 4 black balls and urn B contains 4 red and 6 black balls. One ball is drawn at random from urn A and placed in urn B. Then one ball is drawn from Urn B and placed in Urn A, if one ball is drawn at random from Urn A , the probability that it is found to be red, is ____

Do yourself

Q. Questions

Let ω be a complex cube root of unity with $\omega \neq 1$. A fair die is thrown three times. If r_1, r_2, r_3 are the numbers obtained on the die, then the probability that $\omega^{r_1} + \omega^{r_2} + \omega^{r_3} = 0$ is

A. 1/18

B. 1/9

C. 2/9

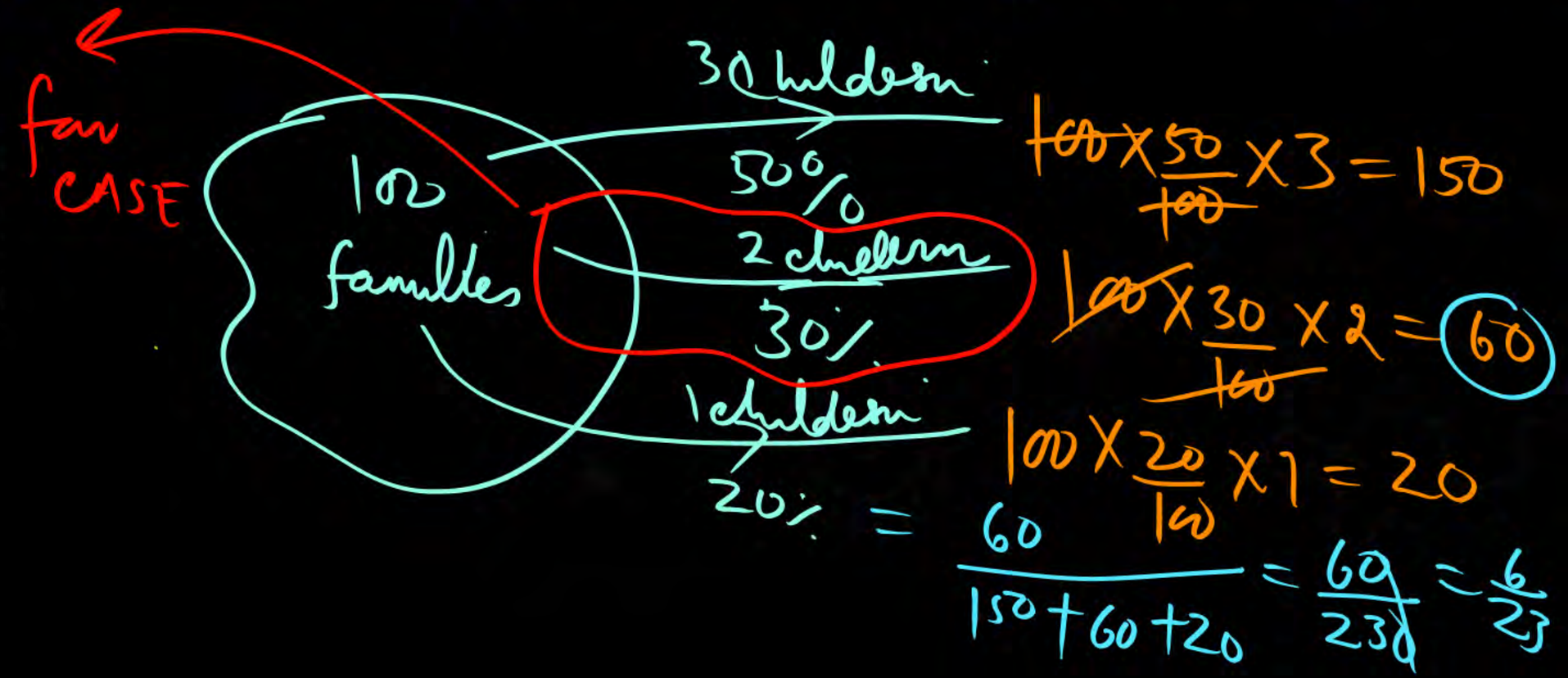
D. 1/36

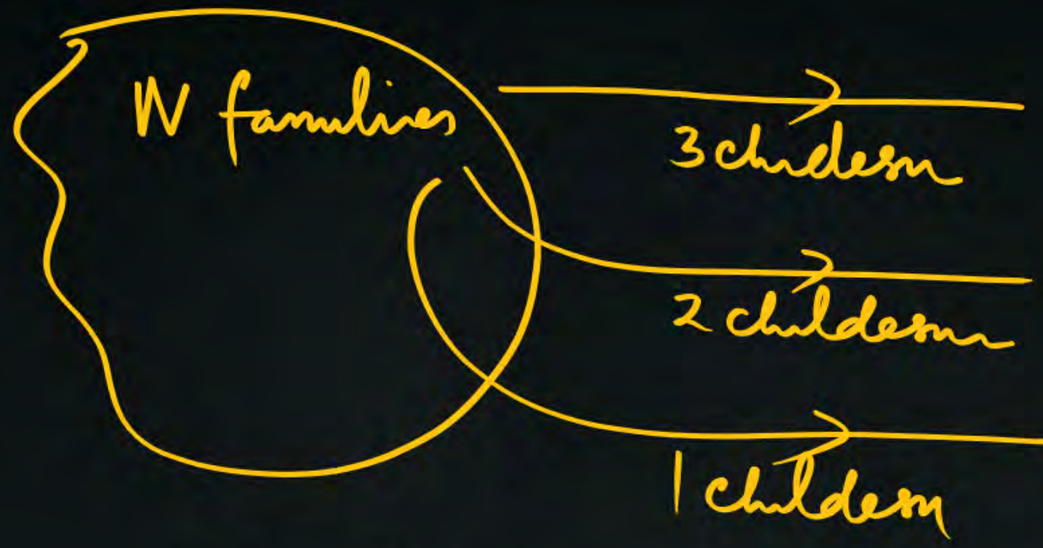
Q. Questions

Rahul Sir PW

In a population of N families, 50% of the families have three children, 30% of families have two children and the remaining families have one child. What is the probability that a randomly picked child belongs to a family with two children?

(a) 3/23
(b) 6/23
(c) 3/10
(d) 3/5





$N \times 3 \times \frac{50}{100} = \frac{3N}{2}$

$N \times 2 \times \frac{30}{100} = \frac{3N}{5}$

$N \times 1 \times \frac{20}{100} = \frac{N}{5}$

$$P(2 \text{ chld}) = \frac{\frac{3N}{5}}{\frac{3N}{2} + \frac{3N}{5} + \frac{N}{5}}$$

$$= \frac{6}{23}$$

Q. Questions

$w_1 \times w_2 \frac{1}{2} \times \frac{1}{4} + \frac{1}{2} \times \frac{1}{6}$

2, 4, 6, 8 — A

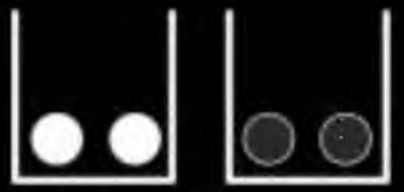
1, 2, 3, 4, 5, 6 — B

P(box select + Tkt on 4)

Two white and two black balls, kept in two bins, are arranged in four ways as shown below. In each arrangement a bin needs to be picked randomly from the chosen bin. Which one of the following arrangements has the highest probability for getting a white ball picked?

(a) 

$\frac{1}{2} \times 1 + \frac{1}{2} \times \frac{1}{2} = \frac{1}{2}$

(b) 

$\frac{1}{2} \times \frac{2}{2} + \frac{1}{2} \times 0 = \frac{1}{2}$

(c) 

$= \frac{1}{2} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$

(b) 

$\frac{1}{2} \times 0 + \frac{1}{2} \times \frac{2}{3} = \frac{1}{3}$

Q. Questions

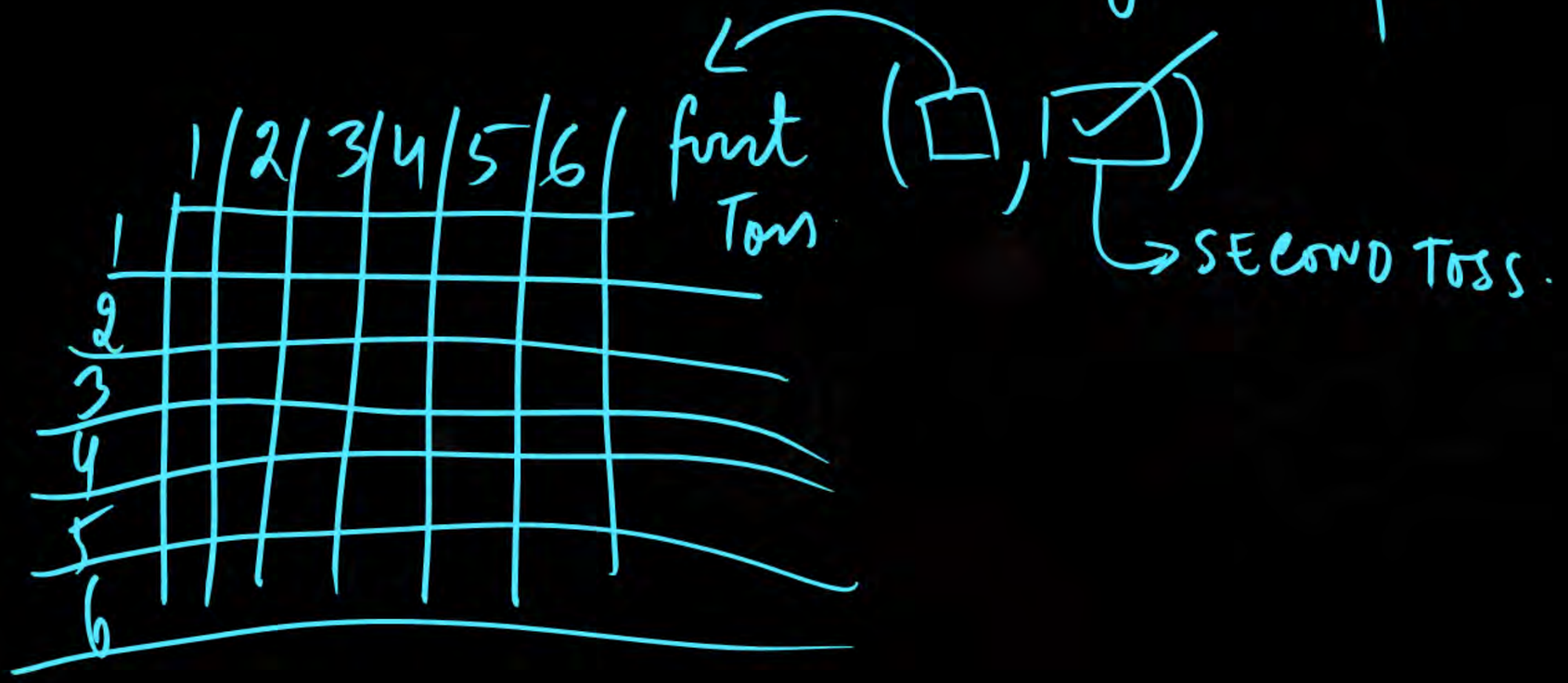
H.W

A fair dice is tossed two times. The probability that the 2nd toss result in a value that is higher than the first toss is

(a) 2/36
(b) 2/6
(c) 5/12 ✓
(d) 1/2

1st

Do yourself.



Q. Questions

box1 | 3, 6, 9, 12, 15 | 6, 11, 16, 21, 26 | box2

The box 1 contains chips numbered 3, 6, 9, 12 and 15. The box 2 contains chips numbered 6, 11, 16, 21 and 26. Two chips, one from each box are drawn at the random. The numbers written on these chips are multiplied. The probability for the product to be even number is

P(product is even)

product even → (E X E)
O X E
E X O

(a) 6/25

(b) 2/5

(c) 3/5

(d) 19/25

Product is even = P[(EXE) + (EXO) + (OXE)]

W1 (EXE): Even — 3, 6, 9, 12, 15 — 5 options

W2 (EXO): Even — 6, 11, 16, 21, 26 — 5 options

W3 (OXE): Even

W1, W2, W3 Performed Individual

= W1 + W2 + W3

I^st case ⇒ (E X E) = 2 X 3 = 6
(I) X (II)

II^nd case ⇒ 2 X 2 = 4
E X O

III^rd case O X E = 3 X 3 = 9

fav case = 6 + 4 + 9
= 19

Total case = 5 X 5 = 25

P(E) = 19/25

Q. Questions

A mapping is selected at random from the set of all mappings of the set $A = \{1,2,.....n\}$ into itself. The probability that the mapping selected is bijective is:

(a) $1/n!$

(b) $1/n^n$

(c) $n!/2^n$

(d) $n!/n^n$

Q. Questions

One mapping is selected at random from all the mappings of the set. A = {1,2,3.....n) into itself. The probability that the mapping selected is one to one is given is:

(a) $1/n^n$

(b) $1/n!$

(c) $n!/n^n$

(d) $\frac{(n-1)!}{n^{n-1}}$

Q. Questions

Integer without replacement = x: 0 1 2 3 4 5 6 7 8 9 10; y: 0 1 2 3 4 5 6 7 8 9,10

Two distinct Integers x and y are chosen, without replacement. at random from the set $\{x, y \mid 0 \le x \le 10, 0 \le y \le 10, x \text{ and } y \text{ are integers}\}$ the probability that $|x - y| \le 5$ is

Total Possible choices
$= 11 \times 10$
$= 110$ choices

(a) 87/121

(b) 88/121

(c) 91/121

(d) 101/121

$\frac{8}{11} \times \frac{11}{11}$

$\frac{88}{121}$

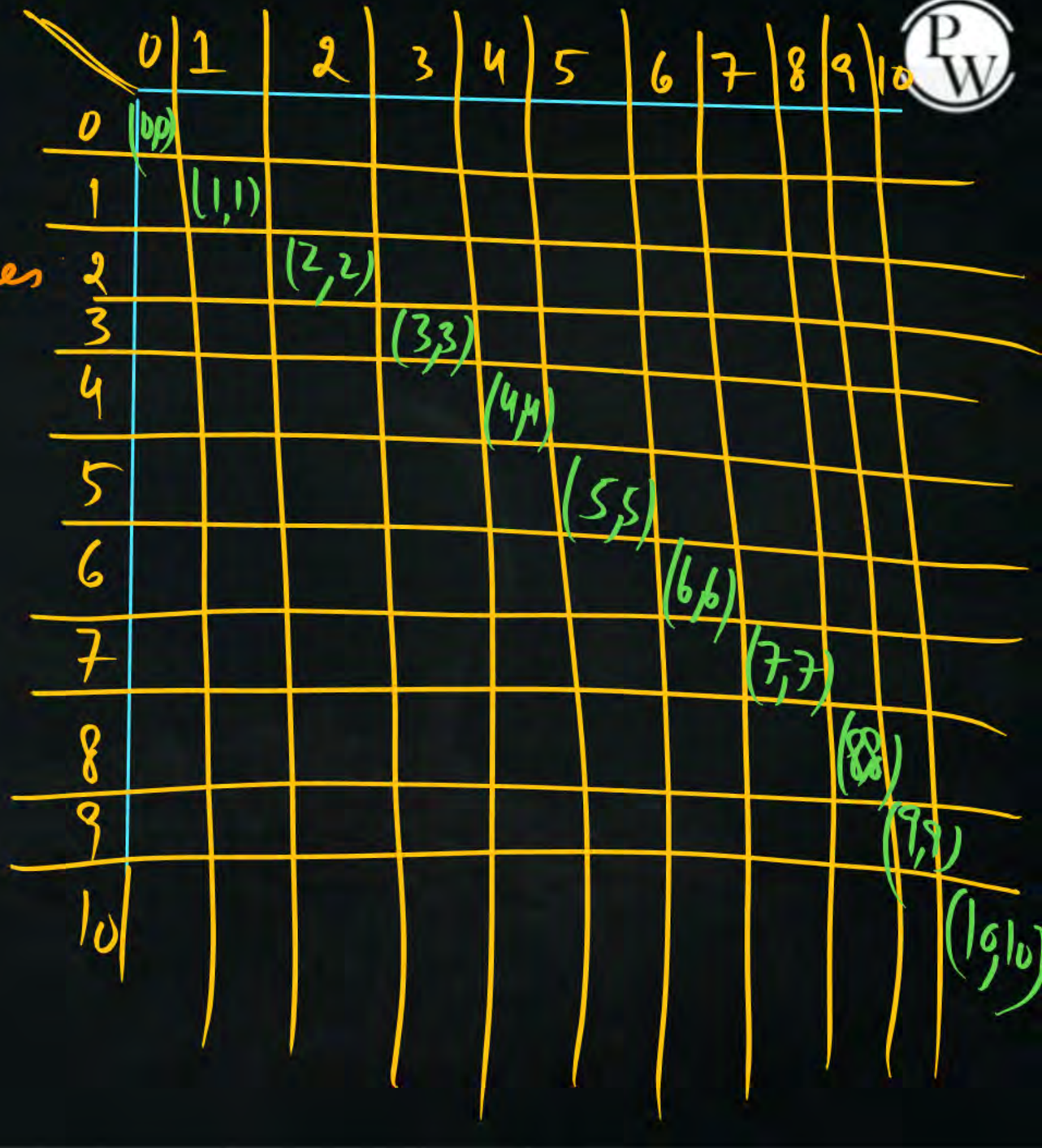
✓ Total choices =

with replacement = $11 \times 11 = 121$ choices

✓ without replacement

$= 11 \times 10$

$= \underline{110}$



$$P[|X-Y| \le 5] = P(-5 \le (x-y) \le 5]$$

with replacement

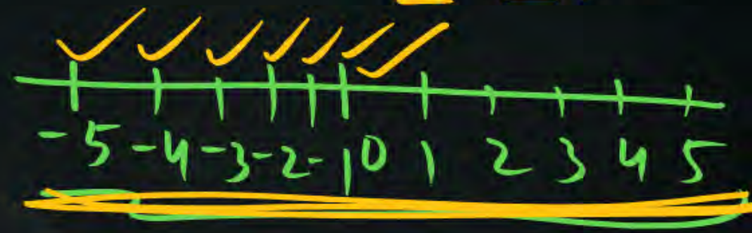
X	Y
0	0
1	1
2	2
3	3
4	4
5	5
6	6
7	7
8	8
9	9
10	10

$-5 \le x-y \le 5$

6 options — 0
7 options — 1
8 options — 2
9 option — 3
10 options — 4
11 options — 5
10 options — 6
9 options — 7
8 options — 8
7 options — 9
6 options — 10

$|x| \le 5$

$-5 \le x \le 5$



$= 2(6+7+8+9+10) + 11 - 11$

$= 2(40) + 11 - 11$ with replacement

$= 80$

$$P(E) = \frac{8\cancel{0}}{11\cancel{0}} \times \frac{11}{11} = \frac{88}{121}$$

Thank You!

GW Soldiers