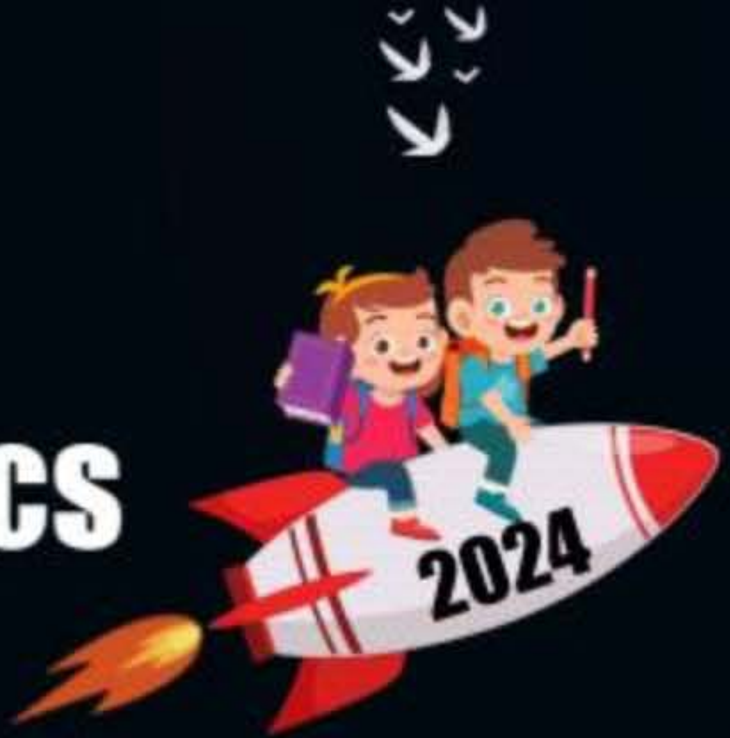
GATE-All BRANCHES
Engineering Mathematics



Linear Algebra

Lecture No.- 08

By- Rahul sir

Recap of previous lecture

Topic Question based on system of equations

Topic Span of vector space

Topics to be Covered

Topic Properties of matrices

Topic Question based on properties of the matrix

Topic Concept of eigen values and eigen vectors

Topic Problems based on eigen values and eigen vectors

Eigenvalues:

$$\boxed{AX = \lambda X}$$

λ = Scaler quantity

→ Eigenvalue Problem.

$A \rightarrow$ square matrix

X = vector

λ = eigen value / scalr quantity

$$AX = \lambda X$$

How to find the eigen value:

step ① $|AX - \lambda X| = 0$

$$|A - I\lambda|_{n \times n} = 0$$

$$\boxed{|A - I\lambda| = 0 = \text{charatistic eqn}}$$

$$\lambda^n + a_0\lambda^{n-1} + a_2\lambda^{n-2} + - + a_n = 0$$

Charaterstic Polynomial equation

$$A = \begin{bmatrix} 4 & 5 \\ 2 & 1 \end{bmatrix}_{2\times 2}$$ Square Matrix

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$ Identity Matrix

$|A - I\lambda| = 0$ or $\det(A - \lambda I) = 0$

$$\begin{vmatrix} 4-\lambda & 5 \\ 2 & 1-\lambda \end{vmatrix} = 0$$

$= \lambda^2 - 5\lambda + (4-10) = 0$

$= \lambda^2 - 5\lambda - 6 = 0$

$= \lambda^2 - 6\lambda + \lambda - 6 = 0$

$\Rightarrow \lambda(\lambda-6) + 1(\lambda-6) = 0$

$\lambda = -1$

$\lambda = 6$

λ_1 = eigen value $= -1$ ✓

λ_2 = eigen value $= 6$ ✓

$A_{n\times n} \longrightarrow n$ eigen values

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}_{3\times 3} \longrightarrow$$ 3 eigen values

Properties of Eigen values :

(A)

$A = \begin{bmatrix} & 3\times3 & \end{bmatrix}$

$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{3\times3}$

SUM of the eigen values

$\Rightarrow a_{11} + a_{22} + a_{33}$

$$\boxed{\sum_{i=1}^{n} \lambda_i = \sum_{i=j} a_{ij} = \text{Trace}}$$

(B) Product of eigen values

$\lambda_1 \lambda_2 \cdots \lambda_n = \text{Det } A$

$\lambda_1 \lambda_2 \lambda_3 \cdots \lambda_n = |A|$

$$\boxed{\prod_{i=1} \lambda_i = |A|}$$

$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}_{3\times3}$

$$\boxed{\lambda_1 \lambda_2 \lambda_3 = |\det A|}$$

$$\boxed{\lambda^2-(\lambda_1+\lambda_2)\lambda+\det A=0}$$

$\alpha+\beta=2$

$\alpha\beta=-2$

$x^2-\underline{(\alpha+\beta)}\,x+\underline{\det A}=0$

$\alpha\cdot\beta$

$$A=\begin{bmatrix}4 & 5\\ 2 & 1\end{bmatrix}_{2\times 2}\qquad |A-I\lambda|=0$$

SUM of Eigenvalues

$$\left\{\begin{aligned}\lambda_1+\lambda_2&=4+1=5\\ \lambda_1\lambda_2&=4-10=-6\end{aligned}\right]$$

SHEAR STRESS

$$A=\begin{bmatrix}1 & 0 & -1\\ 2 & 3 & 5\\ 1 & 0 & 2\end{bmatrix}\qquad |A-I\lambda|=0$$

SUM $\lambda_1+\lambda_2+\lambda_3=1+3+2=6$

$\lambda_1\lambda_2\lambda_3=\det A$

$$=1\begin{vmatrix}3 & 5\\ 0 & 2\end{vmatrix}-0\begin{vmatrix}2 & 5\\ 1 & 2\end{vmatrix}+1\begin{vmatrix}2 & 3\\ 1 & 0\end{vmatrix}$$

$$\underline{\lambda^3-(\text{Trace})\lambda^2+(A_{11}+A_{22}+A_{33})\lambda-\det A=0}$$

Inductor

Solenodal flow

core reason

$\to \underline{\text{Div}\,\vec{F}=0}$

Crude

(2)

✓ Upper Triangular matrix

$$\begin{bmatrix} a & d & e \\ 0 & b & f \\ 0 & 0 & c \end{bmatrix}$$

OR

✓ Lower Triangular matrix

$$\begin{bmatrix} a & 0 & 0 \\ d & b & 0 \\ e & f & e \end{bmatrix}$$

OR

✓ Diagonal matrix

$$\begin{bmatrix} d_1 & 0 & 0 \\ 0 & d_2 & 0 \\ 0 & 0 & d_3 \end{bmatrix}$$

✓ OR

Identity matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

OR

✓ Unit matrix

OR

✓ Scaler matrix

$$\begin{bmatrix} K & 0 & 0 \\ 0 & K & 0 \\ 0 & 0 & K \end{bmatrix}$$

Eigen values = Diagonal elements

A) $a, b, c = \lambda$

B) $\lambda = a, b, c$
$\lambda_1, \lambda_2, \lambda_3$

C) $\lambda = d_1, d_2, d_3$

D) $\lambda = 1, 1, 1$

E) $\lambda = K, K, K$

If $A_{n\times n} \longrightarrow \lambda_1, \lambda_2, \lambda_3 \cdots \lambda_n$

A) $KA \rightarrow K\lambda_1, K\lambda_2, K\lambda_3 \cdots K\lambda_n$	$A \rightarrow 1, 2, 3$ $10A \rightarrow 10\times1, 10\times2, 10\times3, \Rightarrow 10, 20, 30$
B) $A^{-1} \rightarrow \frac{1}{\lambda_1}, \frac{1}{\lambda_2} \cdots \frac{1}{\lambda_n}$	$A \rightarrow 1, 2, 3$ $A^{-1} \longrightarrow \frac{1}{1}, \frac{1}{2}, \frac{1}{3} = 1, \frac{1}{2}, \frac{1}{3}$
C) $A^m \longrightarrow \lambda_1^m, \lambda_2^m, \lambda_3^m \cdots \lambda_n^m$	$A \rightarrow 1, 2, 3$ $A^2 \longrightarrow (1)^2, (2)^2, (3)^2 = 1, 4, 9$ $A^3 \longrightarrow (1)^3, (2)^3, (3)^3 = 1, 8, 27$
D) $\underline{A^n + a_0A^{n-1} + a_1A^{n-2} + \cdots + a_{n-1} = 0}$	$\underline{\text{satisfied } \lambda}$ $A = \lambda$ $A \rightarrow 1, 2$ $A^2 + 2A + I = (1)^2 + 2\times1 + 1, (2)^2 + 2\times2 + 1$ $= \underline{4, 9}$

(E) $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}_{2\times 2}$

Characteristic Polynomial

$$\lambda^2 - (\text{Trace})\lambda + \det A = 0$$

$$\Rightarrow \boxed{\lambda^2 - (a+d)\lambda + (ad-bc) = 0}$$

Ex:- $A = \begin{bmatrix} 1 & 5 \\ 6 & 8 \end{bmatrix}$ $|A - I\lambda| = 0$

$$\lambda^2 - \underbrace{(1+8)}_{\text{Trace}}\lambda + \underbrace{(8-30)}_{\det} = 0$$

$$\boxed{\lambda^2 - 9\lambda - 22 = 0}$$

(F) $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{3\times 3}$

$$\lambda^3 - (\text{Trace})\lambda^2 + (\underbrace{A_{11} + A_{22} + A_{33}}_{\substack{\text{cofactors} \\ \det}})\lambda - \det A = 0$$

$A = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 2 & 0 \\ 5 & 6 & 3 \end{bmatrix}$ $\begin{bmatrix} 2 & 0 \\ 6 & 3 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 \\ 4 & 2 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 \\ 5 & 3 \end{bmatrix}$ =

$$\lambda^3 - (1+2+3)\lambda^2 + [6+3+2]\lambda - \det A = 0$$

(G) If THREE eigen values Are NON-ZERO $\lvert A\rvert \neq 0$	$\lvert A\rvert \neq 0$
(H) If one eigen value is ZERO Then matrix is Singular.	$\lvert A\rvert = 0$
(I) $A \longrightarrow \lambda_1, \lambda_2, \lambda_3 - - - \lambda_n$ $\text{Adj } A \longrightarrow \frac{\lvert A\rvert}{\lambda_1}, \frac{\lvert A\rvert}{\lambda_2} - - \frac{\lvert A\rvert}{\lambda_n}$	$A = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 2 & 0 \\ 5 & 6 & 3 \end{bmatrix}$ eigen values $= 1, 2, 3$ $\det A = 6$ $\text{Adj } A = \frac{\lvert A\rvert}{\lambda_1}, \frac{\lvert A\rvert}{\lambda_2}, \frac{\lvert A\rvert}{\lambda_3}$ $= \frac{6}{1}, \frac{6}{2}, \frac{6}{3} = 6, 3, 2 \rightarrow$ eigenvalues

Topic: Eigen values

#Q. The two eigen values of the matrix $\begin{bmatrix} 2 & 1 \\ 1 & p \end{bmatrix}$ have a ratio of 3:1 for p=2.

What is another value of "p" for which the eigen values have the same ratio of 3:1?

(a) -2

(b) 1

(c) 7/3

(a) 14/3

Topic : Eigen values

#Q. The eigen values of the matrix $A = \begin{bmatrix} 1 & -1 & 5 \\ 0 & 5 & 6 \\ 0 & -6 & 5 \end{bmatrix}$

A) -1,5,6

B) 1,-5,+6i,-6i

C) 1,5,+6i,-6i

D) 1,5,5

Topic : Eigen values

#Q. Consider the following 2×2 matrix A where two elements are unknown and are marked by a and b . The eigenvalues of this matrix are -1 and 7. What are the values of a and b? $A=\begin{pmatrix}1 & 4\\ b & a\end{pmatrix}$

A. a=6,b=4

B. a=4,b=6

C. a=3,b=5

D. a=5,b=3

Topic : Eigen values

#Q. The smallest and the largest eigen values of the following matrix are:

$$\begin{bmatrix} 3 & -2 & 2 \\ 4 & -4 & 6 \\ 2 & -3 & 5 \end{bmatrix}$$

$\lambda^3 - (\text{Trace})\lambda^2 + (A_{11} + A_{22} + A_{33})\lambda - \det A = 0$

✓ $\boxed{\lambda = 1, 1, 2}$

$\lambda^3 - (3 - 4 + 5)\lambda^2 + (5)\lambda - (+2) = 0$

$\lambda^3 - 4\lambda^2 + 5\lambda - 2 = 0$

$\lambda = 1, 1, 2$

A. 1.5 and 2.5

B. 0.5 and 2.5

C. 1.0 and 3.0

D. 1.0 and 2.0

THANK - YOU