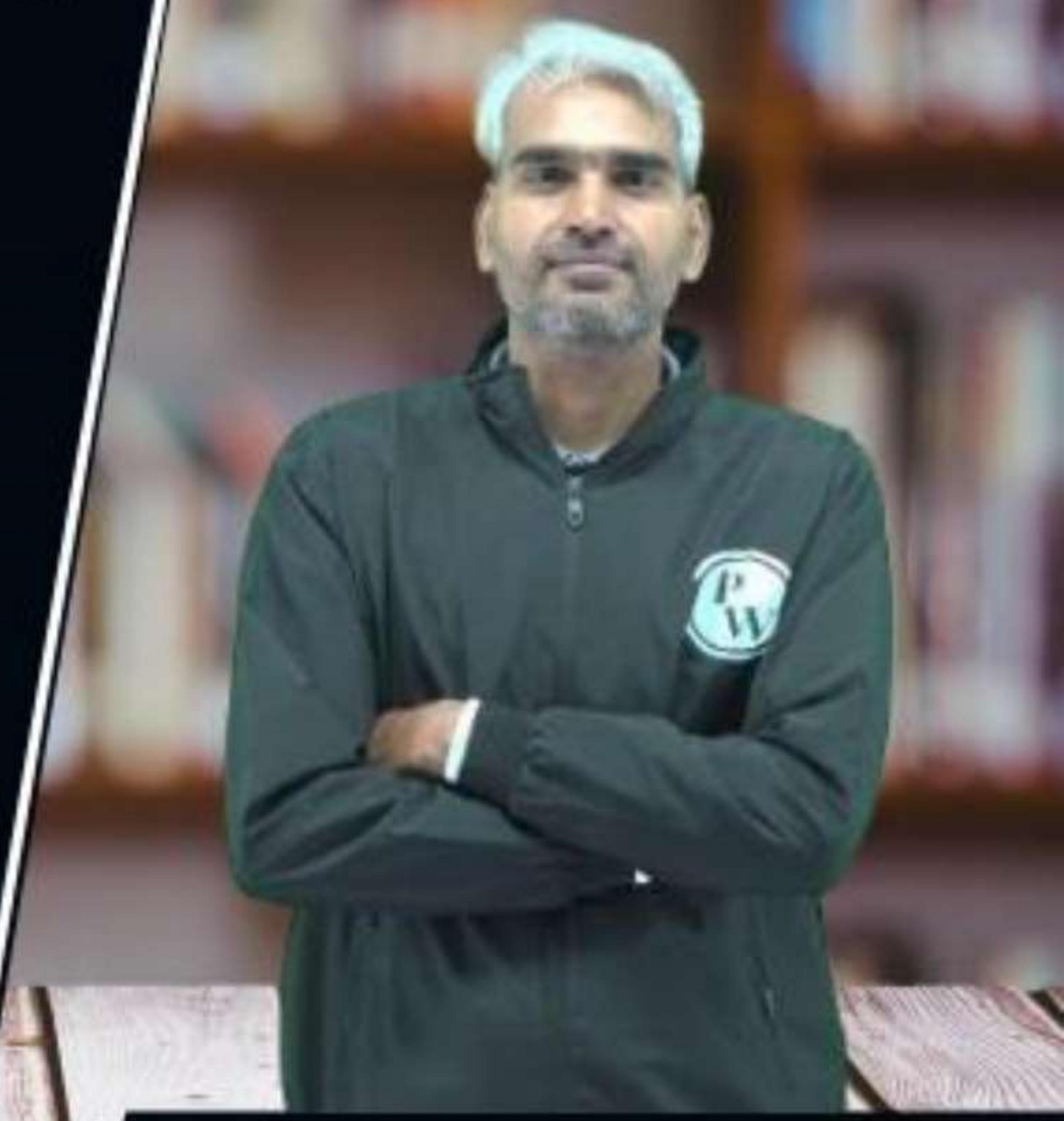
Computer Science & IT

Discrete Mathematics

Set Theory & Algebra

Lecture No. 09



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Recap of Previous Lecture

Topic — Reflexive Closure ✓

Topic — Symmetric Closure ✓

Topic — Transitive Closure ✓

Topic — Equivalence Relation ✓

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Topics to be Covered

Topic Partition of a set ✓

Topic Equivalence Class ✓

Topic Number of equivalence relation on a set ✓

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Topic : Equivalence Relation

Equivalence Relation = Reflexive + Symmetric + Transitive

eg: Let $A = \{1, 2, 3, 4, 5\}$

$R = \{\underbrace{(1,1)(2,2),(3,3)(4,4),(5,5)}, \boxed{(1,2),(2,1)}, \underbrace{(4,5),(5,4)}\}$

is an equivalence Relation

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Topic : Partition of a set

* Partition of a set is grouping of its elements into non-empty subsets, such that each element of the set is included in exactly one subset.

(Or)

Let A be any set, a sub-division of A into non-empty subsets $A_1, A_2, A_3, \ldots A_k$ is called partition of set A if and only if ① $A_i \cap A_j = \emptyset$, $\forall i,j \ (i \neq j)$

and ② $\bigcup_{i=1}^{k} A_i = A$ $\{$ i.e. $A_1 \cup A_2 \cup A_3 \cdots \cup A_k = A \}$

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* Partition of set A, is the set containing non-empty subsets of set A, such that for any two distinct subsets in the set their intersection must be empty, and union of of all the subsets in that set must result in A.

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eg: let $A = \{1, 2, 3, 4\}$

$P_1 = \{\{1,2\}, \{3\}, \{2,4\}\}$

[P_1 is not a partition of A because element 2 belongs to multiple subsets]

$\{1,2\} \cap \{2,4\} = 2 \neq \emptyset$

$P_2 = \{\{1,2\}, \{3\}\}$

[P_2 is not a partition of A, because element '4' is not included in any subset]

$P_3 = \{\{\ \}, \{1,2\}, \{3\}, \{4\}\}$

[Empty set can never be an element of the partition]

$$P_4 = \{\{1\}, \{2, 3\}, \{4\}\}$$

$$P_5 = \{\{1\}, \{2\}, \{3\}, \{4\}\}$$

$$P_6 = \{ \{1, 2, 3, 4\} \}$$

P_4, P_5 & P_6 all are partitions of set $A = \{1, 2, 3, 4\}$

Q: Let $A = \{\ \}$, then how many partitions of set A are possible

* Partition of set A = $\underbrace{\{\qquad\}}_{\text{in this example}}$

- Empty set is a partition
- Empty set is not an element of Partition

Note: if $|A| = 0$, then there will be only one partition of set A, and that Partition will be an empty set.

Q: Let $A = \{1\}$, then how many partitions of set A are possible.

$$1 = 1$$

Partition of A $= \{ \{1\} \}$

Note: If $|A| = 1$, then there will be only one partition of set A.

Q: let $A = \{1, 2\}$, then how many partitions of set A are possible?

$2 = 1 + 1$ $\quad$ 2

Partitions of set A are $= \{\{1\}, \{2\}\}$ and $\{\{1, 2\}\}$

$$\{\{1\}, \{2\}\} \equiv \{\{2\}, \{1\}\} \qquad \{\{1, 2\}\} \equiv \{\{2, 1\}\}$$

If $|A| = 2$, then number of partitions of set $A = 2$

Q:- let $A = \{1, 2, 3\}$, how many partitions of set A are possible.

$2 + 1$

$\equiv$

$3 = \quad 1 + 1 + 1 \qquad 1 + 2 \qquad 3$

Partitions of set A are $= \{\{1\}, \{2\}, \{3\}\}$

✓ $\{\{1\}, \{2, 3\}\}$

✓ $\{\{2\}, \{1, 3\}\}$

✓ $\{\{3\}, \{1, 2\}\}$

$\{\{1, 2, 3\}\}$

if $|A| = 3$, then number of partitions of set A $= 5$.

Q: let $A = \{1, 2, 3, 4\}$ how many partitions of set A are possible.

$4 = 1 + 1 + 1 + 1$

$\{\{1\}, \{2\}, \{3\}, \{4\}\}$

$\{\ \}, \{\ \}, \{\ \}, \{\ \}$

$\frac{4 * 3 * 2 * 1}{4!} = 1$ $= 1$

$= 1 + 1 + 2$

$\{\{1\}, \{2\}, \{3,4\}\}$

$\{\{2\}, \{1\}, \{3,4\}\}$

$\{\_\}\ \{\_\}\ \{\_,\_\}$

$\frac{4 * 3 * 1}{2!} = 6$ $\quad {}^4C_1 * {}^3C_1 * {}^2C_2 = 6$

$= 2 + 2$

$\{\{1,2\}, \{3,4\}\}$

$\{\_,\_\}\ \{\_,\_\}$

$\frac{{}^4C_2 * {}^2C_2}{2!} = 3$ $= 3$

$= 1 + 3$

$\{\{1\}, \{2,3,4\}\}$

$\{\_\}, \{\_,\_,\_\}$

${}^4C_1 * {}^3C_3 = 4$ $= 4$

$= 4$

$\{\{1,2,3,4\}\}$

$\{\_,\_,\_,\_\}$

${}^4C_4 = 1$ $= 1$

If $|A| = 4$, then 15 partitions are possible of set A

Q: Let $A = \{1,2,3,4,5\}$, how many partitions are possible of set A.

5 ⊝ $1+1+1+1+1 = \cancel{\dfrac{{}^5C_1 * {}^4C_1 * {}^3C_1 * {}^2C_1 * {}^1C_1}{5!}} \rightarrow 1$

⊝ $1+1+1+2 = \cancel{\dfrac{{}^5C_1 * {}^4C_1 * {}^3C_1 * {}^2C_2}{3!}} \rightarrow 10$

⊝ $\underline{1}+\underline{1}+3 = \cancel{\dfrac{{}^5C_1 * {}^4C_1 * {}^3C_3}{2!}} \rightarrow 10$

⊝ $1+4 = \cancel{{}^5C_1 * {}^4C_4} \rightarrow 5$

⊝ $1+2+2 = \cancel{\dfrac{{}^5C_1 * {}^4C_2 * {}^2C_2}{2!}} \rightarrow 15$

$\{1,2\}$ $\{3,4\}$

$\{3,4\}$ $\{1,2\}$

⊝ $5 = \cancel{\dfrac{{}^5C_5}{1!}} \rightarrow 1$

⊝ $2+3 = \cancel{{}^5C_2 * {}^3C_3} \rightarrow 10$

$1+10+10+5+15+1+10$
$= 52$

if $|A| = 5$, then number of partitions of set $A = \underline{\underline{52}}$

Q: Let $A = \{1, 2, 3, 4, 5, 6\}$

find number of partitions of set A.

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Q: Let $A = \{1, 2, 3, 4, 5\}$

Find the number of Partitions of set A such that number of subsets of A in the Partition are exactly two.

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2 mins Summary

Topic	Equivalence Relation
Topic	Partition of a set
Topic	Equivalence Class
Topic	Number of equivalence relation on a set

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THANK - YOU

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