GATE-ALL BRANCHES

ENGINEERING MATHEMATICS

Linear Algebra

Lecture No.- 04

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Topics covered in previous lecture

Topic Problems on Determinant of a matrix

Topic Matrix multiplication and question based on matrices

Topic Adjoint and inverse of a matrix

Topics to be Covered

Topic Properties of matrices

Topic Question based on properties of the matrix

Properties of Adjoint:

(A) Adj (Null matrix) = [O]

$$\text{Adj}A \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(B) Adj (Diagonal) = Diagonal matrix

$$A = \begin{bmatrix} d_1 & 0 & 0 \\ 0 & d_2 & 0 \\ 0 & 0 & d_3 \end{bmatrix}$$

$$\text{Adj}\,A = \begin{bmatrix} d_2 d_3 & 0 & 0 \\ 0 & d_3 d_1 & 0 \\ 0 & 0 & d_1 d_2 \end{bmatrix}$$

C) $(\text{Adj}\,A)(\text{Adj}\,B) = (\text{Adj}\,B)(\text{Adj}\,A)$

Imp

Imp. D) $\boxed{\text{Adj}(\lambda A) = \lambda^{n-1}\,\text{adj}(A)}$

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

λ = scaler = constant

$10A = 10^{3-1}(\text{adj}\,A)$

E) $\boxed{|\text{Adj}\,A|_{n\times n} = |A|^{n-1}}$

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \quad |\text{Adj}\,A|$$

$|A|^{3-1}$ Ans

F) $\boxed{A(\text{adj}\,A) = (\text{adj}\,A)\,A = |A|\,I}$

Uses - determinant:

$AX = B$ (system of equations)

$X = A^{-1}B$

$A^{-1} = \frac{\text{adj } A}{|A|}$

X = vector

$X = \frac{\boxed{\text{adj } A \cdot B}}{|A|}$ $X = \frac{K}{|A|}$ $X \propto \frac{1}{|A|}$

1) If $|A| \uparrow$ $X \downarrow$ size of vector
$|A| \downarrow$ $X \uparrow$

2) If $|A|$ +
$|A|$ − vector orientation depend on determinant

$\swarrow$ clockwise $\searrow$ anticlockwise

Determinant of a matrix

#Q. Let A, B, C, D be n × n matrices, each with non-zero determinant. $(ABCD) = (I)$ then $B^{-1} =$

Let A, B, C, D → Non singular matrices.

$(ABCD)^{-1} = (I)^{-1}$

= Using Reversal law

$\underbrace{CDD^{-1}}_{I}C^{-1}B^{-1}A^{-1} = ICD$

$B^{-1}A^{-1}A = ICDA$

$\boxed{B^{-1} = CDA}$

$D^{-1}C^{-1}B^{-1}A^{-1} = I$

CDA CDA

$\boxed{B^{-1} = CDA}$

$X+3 = 5$

$X = 5-3$

$X+3-3 = 5-3$

(a) $D^{-1}C^{-1}A^{-1}$

(b) CDA

(c) ABC

(d) Does not exist

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Determinant of a matrix

H.W

#Q.The number of different $n \times n$ symmetric matrices with each elements being either 0 or 1 is

(a) 2^n

(b) 2^{n^2}

(c) $2^{\frac{n^2+n}{2}}$

(d) $2^{\frac{n^2-n}{2}}$

Slide

Determinant of a matrix

#Q.Let P be 2x2 real orthogonal matrix and $\bar{x}$ is a real vector $[x_1\ x_2]^T$ with length $\boxed{||\bar{x}|| = (x_1^2 + x_2^2)^{1/2}}$. Then which one of the following statement is correct?

Norm || || = distance

$\bar{x} = [x_1\ x_2]^T$

(a) $||P\bar{x}|| \le ||\bar{x}||$ where at least one vector satisfies $||P\bar{x}|| < ||\bar{x}||$

(b) $||P\bar{x}|| = ||\bar{x}||$ for all vector $\bar{x}$

(c) $||P\bar{x}|| \ge ||\bar{x}||$ where at least one vector satisfies $||P\bar{x}|| > ||\bar{x}||$

(d) No relationship can be established between $||\bar{x}||$ and $||P\bar{x}||$

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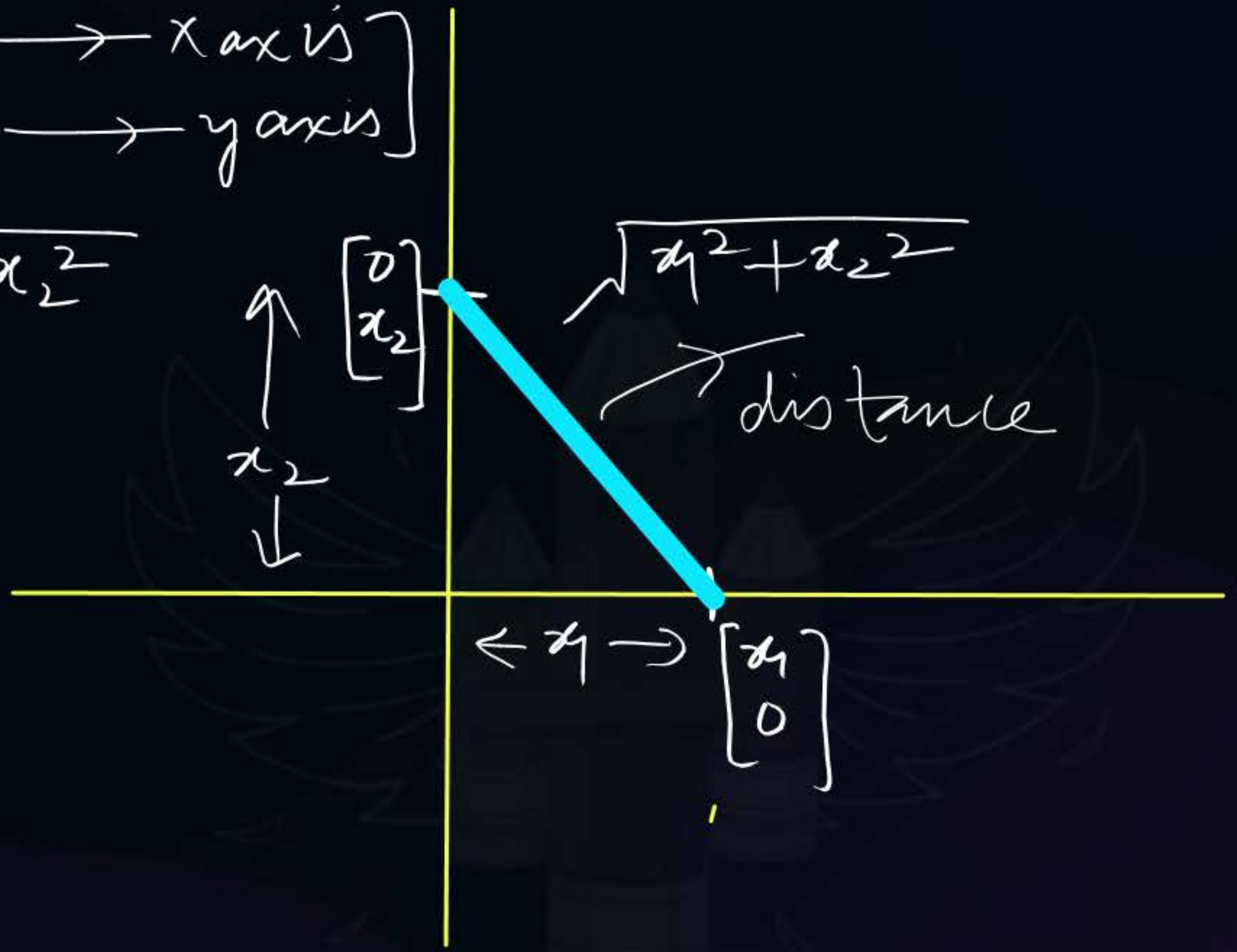
$$P = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{ or } \begin{bmatrix} \frac{3}{5} & \frac{4}{5} \\ -\frac{3}{5} & \frac{4}{5} \end{bmatrix} \text{ or } \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

P is a orthogonal matrix

$$\boxed{P \cdot \bar{x}} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \begin{array}{l} \rightarrow x \text{ axis} \\ \rightarrow y \text{ axis} \end{array}$$

$$= \sqrt{x_1^2 + x_2^2}$$

$$= \|\bar{x}\|$$



$$\boxed{\|P \cdot \bar{x}\| = \|\bar{x}\|}$$

B is correct

Determinant of a matrix

H.W

#Q.Let A be an $m \times n$ matrix and B an $n \times m$ matrix. It is given that determinant $(I_m + AB)$ = determinant $(I_n + \text{BA})$, where I_k is the $k \times k$ identity matrix. Using the above property, the determinant of the matrix given below

$$\begin{bmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{bmatrix}$$

is

(a) 2

(b) 5

(c) 8

(d) 16

Slide

Determinant of a matrix

#Q.The determinant of matrix A is 5 and the determinant of matrix B is 40.

The determinant of matrix AB is____.

$\det A = 5$

$\det B = 40$

$$\det(AB) = (\det A)(\det B)$$
$$= 5 \times 40$$
$$= \underline{200}$$

Slide

Determinant of a matrix

#Q. Given the matrices $J = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 6 \end{bmatrix}$ and $K = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$, the product $K^T JK$ is ____.

$= \boxed{23}$

$K^T J K$

$= \begin{bmatrix} 1 & 2 & -1 \end{bmatrix} \begin{bmatrix} 3 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 6 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$

$(1 \times 3) \times (3 \times 3)$

$(1 \times 3) \times (3 \times 1) = 1 \times 1$

Slide

Determinant of a matrix

#Q.The determinant of matrix $\begin{bmatrix} 0 & 1 & 2 & 3 \\ 1 & 0 & 3 & 0 \\ 2 & 3 & 0 & 1 \\ 3 & 0 & 1 & 2 \end{bmatrix}$ is

H.W

Slide

Determinant of a matrix

✓ Vrmg Property
✓ Vrmg ______

✓ Imp gate-2020

#Q. For $A = \begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$, the determinant of $A^T A^{-1}$ is

(a) $\sec^2 x$

(b) $\cos 4x$

(c) 1

(d) 0

$A = \begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$ $\quad |\underbrace{A^T A^{-1}}|$

$|A^T A^{-1}|$

$= |A^T| \cdot \frac{1}{|A|}$

$= \sec^2 x \cdot \frac{1}{\sec^2 x}$

$= 1$

$A^T = \begin{bmatrix} 1 & -\tan x \\ \tan x & 1 \end{bmatrix}$

$= \sec^2 x$

Slide

Determinant of a matrix

#Q.The matrix $A = \begin{bmatrix} a & 0 & 3 & 7 \\ 2 & 5 & 1 & 3 \\ 0 & 0 & 2 & 4 \\ 0 & 0 & 0 & b \end{bmatrix}$ has det(A) = 100 and trace (A) = 14.

The value of |a – b| is ____.

$b \begin{vmatrix} a & 0 & 3 \\ 2 & 5 & 1 \\ 0 & 0 & 2 \end{vmatrix}$

$ab = 10$

$a + 5 + 2 + b = 14$

$a + b = 7$

$2 \times b \begin{vmatrix} a & 0 \\ 2 & 5 \end{vmatrix}$

$= 2b \times 5a = 100$

$10ab = 100$

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$a+b=7$

$ab=10$

$(a-b)=\sqrt{(a+b)^2-4ab}$

$=\sqrt{49-40}$

$=\sqrt{9}$

$a-b=\pm 3$

$a+\cancel{b}=7$

$a-\cancel{b}=3$

$2a=10$

$a=5$

$a+b=7$

$5+b=7$

$b=2$

$|a-b|=|5-2|$

$=3$ Ans

Determinant of a matrix

H.W

#Q.Which one of the following matrices is singular?

(a) $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 4 \\ 3 & 6 \end{bmatrix}$ (d) $\begin{bmatrix} 4 & 3 \\ 6 & 2 \end{bmatrix}$

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Determinant of a matrix

#Q. If $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 1 \end{bmatrix}$ then $\det(A^{-1})$ is ____ (correct to two decimal places).

$|A| = 4$

$|A^{-1}| = \frac{1}{|A|}$

$= \frac{1}{4}$ Ans

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Do yourself

#Q. If $A = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$, then value of α for which $A^2 = B$, is

(a) 1

(b) –1

(c) 4

(d) No real values

$A = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}$

$A^2 = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}$

$A^2 = B$

$\begin{bmatrix} \alpha^2 & 0 \\ \alpha+1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$

$\alpha^2 = 1$ $\quad \alpha = \pm 1$

$\alpha + 1 = 5$ $\quad \alpha = 4$

$\alpha^2 = 1$

$\alpha = \pm 1$

Slide

Determinant of a matrix

H.W

#Q.Let $A = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$ and $B = A^3 - A^2 - 4A + 5I$, where I is the 3×3 identity matrix. The determinant of B is ____ (up to 1 decimal place).

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Determinant of a matrix

#Q. If $P=\begin{bmatrix}\frac{\sqrt{3}}{2} & \frac{1}{2}\\ -\frac{1}{2} & \frac{\sqrt{3}}{2}\end{bmatrix}$ and $A=\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$, $Q = PAP^T$ and $x = P^T Q^{2005} P$ then

x is equal to

(a) $\begin{bmatrix}1 & 2005\\ 0 & 1\end{bmatrix}$

(b) $\begin{bmatrix}4+2005\sqrt{3} & 6015\\ 2005 & 4-2005\sqrt{3}\end{bmatrix}$

(c) $\frac{1}{4}\begin{bmatrix}2+\sqrt{3} & 1\\ -1 & 2-\sqrt{3}\end{bmatrix}$

(d) $\frac{1}{4}\begin{bmatrix}2005 & 2-\sqrt{3}\\ 2+\sqrt{3} & 2005\end{bmatrix}$

Slide

#Q. If $\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix} = x + iy$, then

$= 6i\begin{vmatrix} 3i & -1 \\ 3 & i \end{vmatrix} + 3i\begin{vmatrix} 4 & -1 \\ 20 & i \end{vmatrix} + 1\begin{vmatrix} 4 & 3i \\ 20 & 3 \end{vmatrix}$

(a) x = 3, y = 1

(b) x = 0, y = 3

(c) x = 1, y = 3

(d) x = 0, y = 0

$i^2 = -1$

$= 6i(\underline{3i^2+3}) + 3i(4i+20) + 1(12-60i)$

$= 6i[0] + \cancel{12i^2} + \cancel{60i} - \cancel{60i} + \cancel{12}$

$= 0 = x + iy = 0 + i \cdot 0$

$x = 0$
$y = 0$

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Determinant of a matrix

skew sym. matrix
Diagonal $\rightarrow (0,0,0)$

skew symmetric
odd order
$\boxed{\det B = 0}$

#Q. Let k be a positive real number and let

$(2k+1)^3$

$$A = \begin{bmatrix} 2k-1 & 2\sqrt{k} & 2\sqrt{k} \\ 2\sqrt{k} & 1 & -2k \\ -2\sqrt{k} & 2k & -1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & 2k-1 & \sqrt{k} \\ 1-2k & 0 & 2\sqrt{k} \\ -\sqrt{k} & -2\sqrt{k} & 0 \end{bmatrix}_{3\times 3}$$

If $\boxed{\det(\text{adj } A) + \det(\text{adj } B) = 10^6}$, then [k] is equal to

greatest Integer

Note : adj M denotes the adjoint of square matrix M and [k] denotes the largest integer less than or equal k.

$|\text{Adj } A| = |A|^{n-1}$

$\det A = (2k+1)^3$

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$$\det(\operatorname{Adj} A) + \det(\operatorname{Adj} B) = 10^6$$

Using Property

$$= |A|^{n-1} + \underline{|B|^{n-1}} = 10^6$$

skew Sym odd order $= |B| = 0$

$$= |A|^{n-1} = 10^6 \qquad n = 3$$

$$= |A|^{3-1} = 10^6$$

$$\boxed{\det A = (2K+1)^3}$$

$$= \left[(2K+1)^3\right]^2 = 10^6$$

$$= (2K+1)^6 = 10^6$$

$$2K+1 = 10$$

$$2K = 9$$

$$K = 4.5$$

$[K] \rightarrow$ greatest Integer

$[4.5] = 4$

Determinant of a matrix

#Q. Let $M^4 = I$ (where I denotes the identity matrix) and $M \neq I$, $M^2 \neq I$ and $M^3 \neq I$. Then, for any natural number k, M^{-1} equals:

(a) M^{4k+1}

(b) M^{4k+2}

(c) M^{4k+3}

(d) M^{4k}

M^{4k+3}

$M \neq I,\ M^2 \neq I,\ M^3 \neq I$

$M^4 = I$

both sides Square It

$M^8 = I$

$M^7 = M^{-1}$

$M^{11} = M^{-1}$

$M^{15} = M^{-1}$

7, 11, 15 $= 4k+3$, $k = 1$

$M^4 = I$

$(M^4)^2 = I$

$M^8 = I$

Multiply via M^{-1}

$M^7 = M^{-1}$

$M^{-1} = M^{4k+3}$

Slide

Determinant of a matrix

#Q. For the given orthogonal matrix Q, $Q = \begin{bmatrix} 3/7 & 2/7 & 6/7 \\ -6/7 & 3/7 & 2/7 \\ 2/7 & 6/7 & -3/7 \end{bmatrix}$ The inverse is

Q is orthogonal matrix

$QQ^T = I$

$\boxed{Q^T = Q^{-1}}$

$Q^T = \begin{bmatrix} \frac{3}{7} & -\frac{6}{7} & \frac{2}{7} \\ \frac{2}{7} & 3/7 & 6/7 \\ 6/7 & 2/7 & -\frac{3}{7} \end{bmatrix}$

(a) $Q = \begin{bmatrix} 3/7 & 2/7 & 6/7 \\ -6/7 & 3/7 & 2/7 \\ 2/7 & 6/7 & -3/7 \end{bmatrix}$

(b) $Q = \begin{bmatrix} -3/7 & -2/7 & 6/7 \\ 6/7 & -3/7 & -2/7 \\ -2/7 & -6/7 & 3/7 \end{bmatrix}$

(c) ✓ $Q = \begin{bmatrix} 3/7 & -6/7 & 2/7 \\ 2/7 & 3/7 & 6/7 \\ 6/7 & 2/7 & -3/7 \end{bmatrix}$

(d) $Q = \begin{bmatrix} -3/7 & 6/7 & -2/7 \\ -2/7 & -3/7 & -6/7 \\ -6/7 & -2/7 & 3/7 \end{bmatrix}$

Slide

Determinant of a matrix

#Q. If P is a 3 × 3 matrix such that $P^T = 2P + I$, where P^T is the transpose of P and I is the 3 × 3 identity matrix, then there exists a column matrix $\boxed{X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}}$ such that

(a) $PX = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

(b) $PX = 2X$

(c) $PX = X$

(d) $PX = -X$ ✓

$P \longrightarrow 3 \times 3$

$P^T = (2P + I)$ both sides Transpose

$(P^T)^T = 2P^T + I^T$

$P = 2P^T + I$

$P = 2(2P + I) + I$

$= \underline{4P} + 2I + I$

$P = 4P + 3I$

$P - 4P = 3I$

$-3P = 3I$

$-PX = IX$

$PX = -X$

$P = 4P + 2I + I$

$-3P = 3I$

$\boxed{-P = I}$

multiply X

$PX = -X$

Slide

THANK - YOU