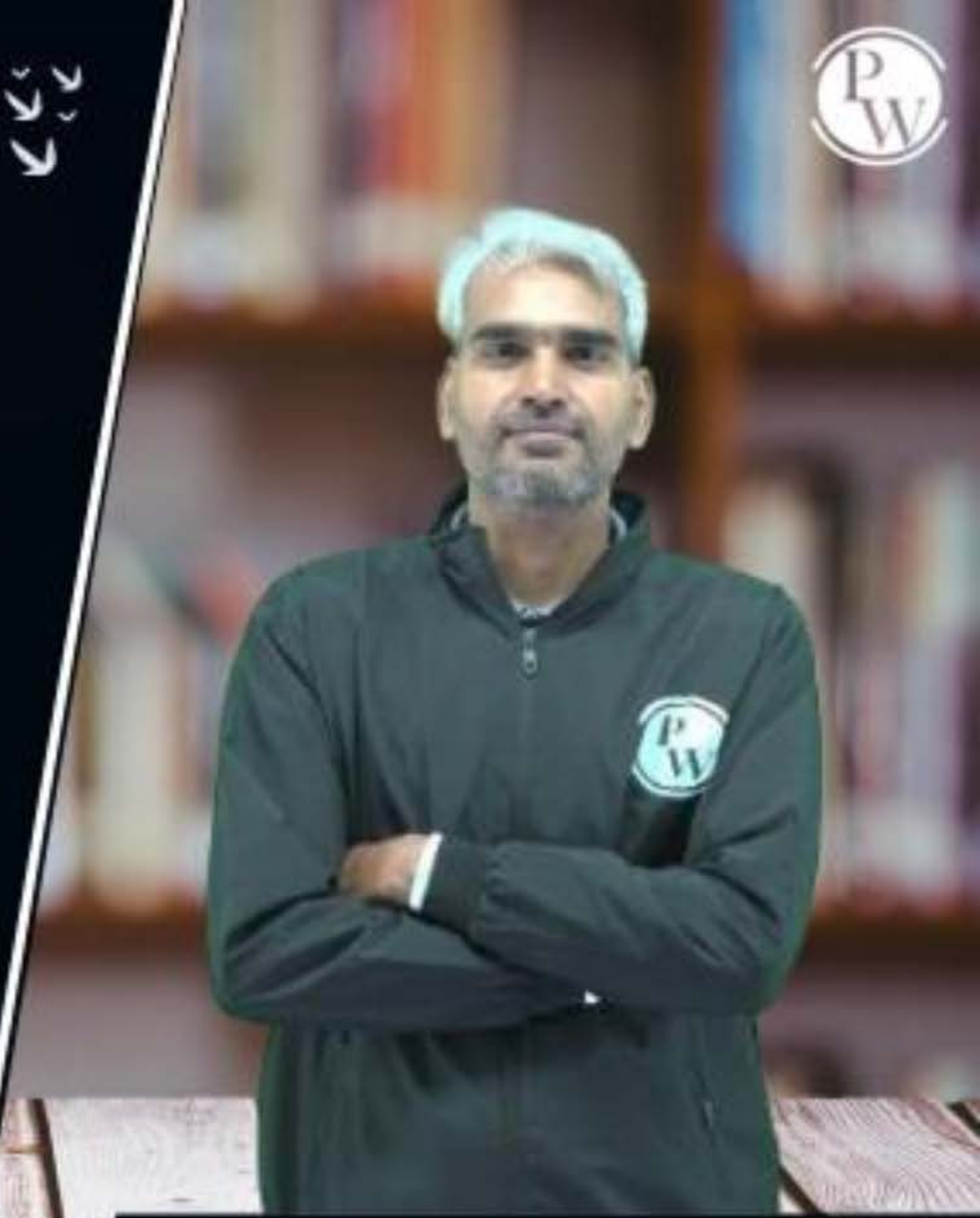
Computer Science & IT

Discrete Mathematics

Set Theory & Algebra

Lecture No. 18



By- Vishal Sir

Recap of Previous Lecture

Topic Number of onto functions

Topic Bijective function

$|A| = |B| = n$

$\#$ Bijection $= n!$

Topic Identity function

$I_A : f(x) = x \quad \forall x \in A$

Topic Constant function

$f(x) = c \quad \forall x \in$ domain

fixed

Topic Inverse of a function

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Topics to be Covered

- **Topic** Identical Functions
- **Topic** Function composition

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H.W.

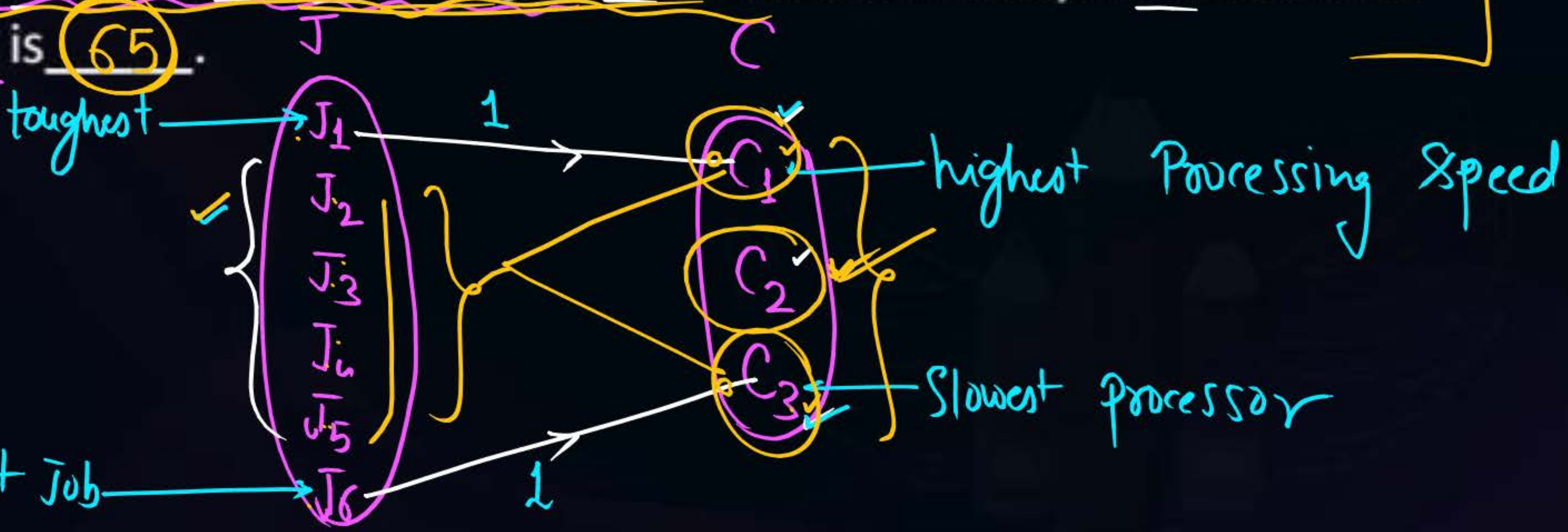
Q. There are 6 jobs with distinct difficulty levels, and 3 computers with distinct processing speeds. Each job is assigned to a computer such that:

- The fastest computer gets the toughest job and the slowest computer gets the easiest job.
- Every computer gets at least one job. The number of ways in which this can be done is 65.

$3^4 - 2^4$

$81 - 16$

$= 65$



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Topic : Identical function

Two functions are said to be identical if

1. They have same domain
2. They have same range
3. Both the functions should produce the same value for all the elements in the domain

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* Two functions f & g are said to be identical if

① domain of f = domain of g

② Range of f = Range of g

③ $f(x) = g(x) \quad \forall x \in \text{domain}$

Q: let $f(x) = x$

$g(x) = \sqrt{(x)^2}$

False S1: f & g are identical ✓

True S2: f & g are not identical ✓

Range of f ≠ Range of g

$\sqrt{a}$

domain $a \geq 0$

$f(x) = x$

✓ domain of $f(x)$ = Set of all real numbers

Range of $f(x)$ = Set of all Real numbers

$g(x) = \sqrt{x^2}$

✓ domain of $g(x) \Rightarrow (x)^2 \geq 0$

$\therefore x \in R$

Range of $g(x) = R^+ \cup \{0\}$

Q: Let $f = \log x^2$

$g = 2 \log x$

domain of $f \neq$ domain of g

$\therefore$ Not identical

$f = \log x^2$

log is defined for +ve Real No.s

$\therefore x^2 > 0$

$\therefore x \in R - \{0\}$

domain of f $= R - \{0\}$

$g = 2 \log x$

$x > 0$

$\therefore x \in R^+$

domain of g $= R^+$

Q: domain of function $\dfrac{1}{\sqrt{|x|-x}}$

$(-\infty, 0]$

$R^- \cup \{0\}$

$\Rightarrow$ function is not defined when $\sqrt{|x|-x} = 0$

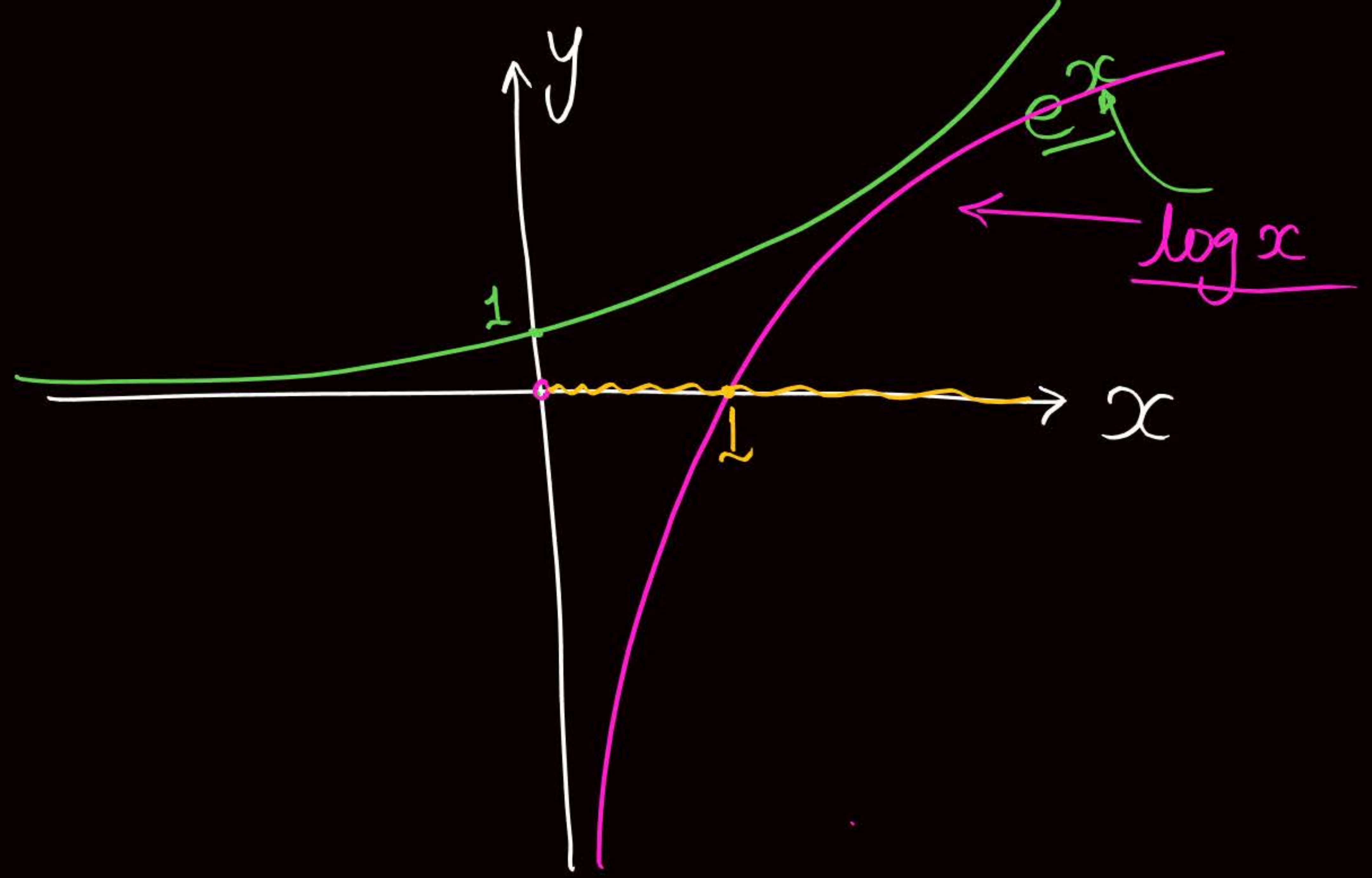
i.e. $|x| - x = 0$

$$|x| - x \begin{cases} -x - x = -2x & \text{if } x < 0 \\ x - x = 0 & \text{if } x \geq 0 \end{cases}$$

$$|x| = \begin{cases} -x & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$$

i.e function is not defined when $x \geq 0$

$\therefore$ domain $= R^- = (-\infty, 0)$



y
1
x
1
e^x
log x

Topic : Function composition

- Function Composition will Combine two functions 'f' and 'g' in the given order
- for two functions f & g, Composition of f & g can be defined as 'fog' or 'gof'
- In general,

$$f \circ g \neq g \circ f$$

{ i.e. function Composition is not Commutative }

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$$(\overset{f_1}{f} \circ \overset{f_2}{g})(x) = f(\underbrace{g(x)}) \quad \Big| \quad (g \circ f)(x) = g(f(x))$$

<u>Note</u>: $((\underline{f \circ g}) \circ \underline{h})(x)$

$$(\underbrace{f}_{f_1} \circ \underbrace{(g \circ h)}_{f_2})(x)$$

$$(f \circ g)(h(x)) \qquad f((g \circ h)(x))$$

$$\underbrace{f(g(h(x)))} = f(g(h(x)))$$

$\therefore$ function Composition is associative.

$$(f \circ g)(x) = f\big(g(x)\big) \qquad (g \circ f)(x) = g\big(f(x)\big)$$

domain of g

co-domain of f

(from co-domain of g)

$f : A \longrightarrow A$

$g : A \longrightarrow A$

$(f \circ g)(x)$: domain of g $\longrightarrow$ co-domain of f

: $A \longrightarrow A$

$f : A \longrightarrow A$

$g : A \longrightarrow A$

$(g \circ f)(x)$: domain of f $\longrightarrow$ co-domain of g

: $A \longrightarrow A$

$$(f\circ g)(x) = f(g(x))$$

domain of g

(from co-domain of g)

co-domain of f

$$f: A \to B$$
$$g: B \to A$$

$(f\circ g)(x)$: domain of g $\to$ co-domain of f

: $B \to B$

$$(g\circ f)(x) = g(f(x))$$

$$f: A \to B$$
$$g: B \to A$$

$(g\circ f)(x)$: domain of f $\to$ co-domain of g

: $A \to A$

$(f \circ g)(x) = f(g(x))$ domain of g | $(g \circ f)(x) = g(f(x))$

from co-domain of g

co-domain of f

$f : A \to B$

$g : B \to C$

$(f \circ g)(x)$: need not be defined

In this case $(f \circ g)(x)$ may be defined only if Range of $g \subseteq$ domain of f

$f : A \to B$

$g : B \to C$

$(g \circ f)(x)$: domain of $f \to$ co-domain of g

$: A \to C$

$I_A : A \to A$

Topic : Function composition

* Let $f : A \to A$

$(f \circ I_A) = f \Rightarrow (f \circ I_A)(x) = f(\underbrace{I_A(x)}) = f(x)$

$(I_A \circ f) = f \Rightarrow (I_A \circ f)(x) = I_A(\underbrace{f(x)}_{\text{from set } A})$

$= f(x)$

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* Let $f : A \to B$

$(f \circ I_A) = f$

$(I_A \circ f) =$ Need not be defined

Let $f : A \to B$

$(f \circ I_B)$: Need not be defined

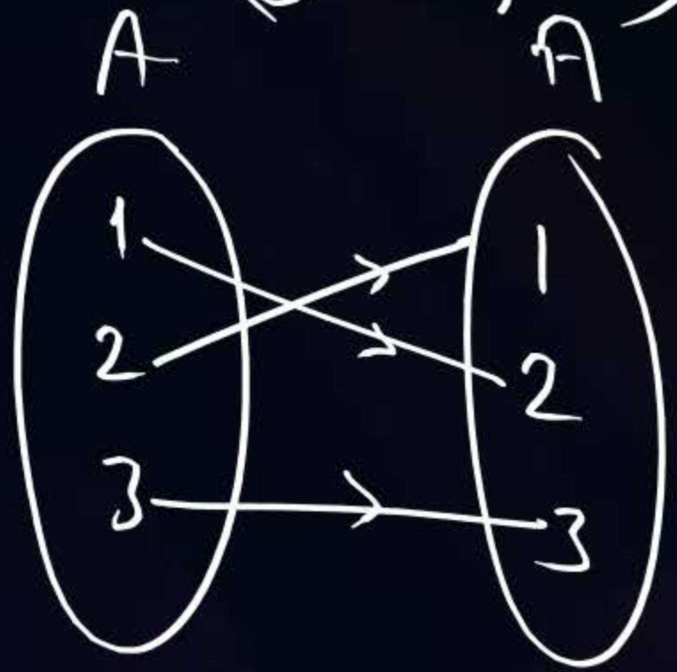
$(I_B \circ f) = f$

$: A \to B$

Topic : Function composition

* Let $f : A \to A$ is a bijective function

$$(f \circ f^{-1}) = I_A = (f^{-1} \circ f)$$



$$(f \circ f^{-1})(2) = f(f^{-1}(2)) = f(1) = 2$$

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* Let $f: A \to B$ is a bijective function

$$(f^{-1} \circ f) = I_A \qquad (f \circ f^{-1}) = I_B$$

Topic : Function composition

$$\boxed{(f o g)^{-1} = g^{-1} o f^{-1}}$$

- for any binary opn $*$ $(f*g)^{-1} = g^{-1} * f^{-1}$ will always hold true, irrespective of the fact whether "$*$" is Commutative or not.

$(f*g)^{-1} = f^{-1} * g^{-1}$ only if "$*$" is Commutative

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2 mins Summary

Topic Identical Function

Topic Function Composition

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THANK - YOU

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