

ENGINEERING MATHEMATICS

ALL BRANCHES



Single Variable Calculus

DPP 01 Discussion

(Part 02)



By - Rahul Sir

Q21. If $\int_0^\infty e^{-x^2} dx = \frac{\sqrt{x}}{2}$, then $\int_0^\infty e^{-ax^2} dx$ where $a > 0$ is

$\sqrt{x} \rightarrow \sqrt{\pi}$

$$I = \int_0^\infty e^{-ax^2} dx$$

Gamma Function

(a) $\frac{\sqrt{\pi}}{2}$

(b) $\frac{\sqrt{\pi}}{2a}$

(c) $2\frac{\sqrt{\pi}}{a}$

(d) $\frac{1}{2}\sqrt{\pi/a}$ ✓

$$I = \int_0^{\infty} e^{-ax^2} dx$$

put $ax^2 = t$

$$a \cdot 2x dx = dt$$

$$dx = \frac{dt}{2ax}$$

$$= \frac{dt}{2 \cdot a \cdot \frac{\sqrt{t}}{\sqrt{a}}}$$

$$= \frac{1}{2 \cdot \sqrt{t} \sqrt{a}}$$

$$dx = \frac{1}{2\sqrt{a}} t^{-1/2}$$

$$x = \sqrt{\frac{t}{a}} = \frac{\sqrt{t}}{\sqrt{a}}$$

$$I = \int_0^{\infty} e^{-t} \cdot \frac{1}{2\sqrt{a}} t^{-1/2} dt$$

$$\Rightarrow \frac{1}{2\sqrt{a}} \int_0^{\infty} e^{-t} t^{-1/2} dt$$

Gamma Function

$$\Rightarrow \frac{1}{2\sqrt{a}} \sqrt{\frac{1}{2}}$$

$$= \frac{\sqrt{\pi}}{2\sqrt{a}}$$

$$= \frac{1}{2} \sqrt{\frac{\pi}{a}}$$

$$\Gamma(\eta) = \int_0^{\infty} e^{-t} t^{\eta-1} dt$$

$$\eta-1 = -\frac{1}{2} \quad \eta = \frac{1}{2}$$

Q22. The expression $\frac{\int_0^n [x] dx}{\int_0^n \{x\} dx}$ is equal to

$$I_1 \Rightarrow \int_0^n [x] dx$$

$$I_2 = \int_0^n \{x\} dx$$

(where $[*]$ and $\{*\}$ denotes greatest integer function and fractional part function and $n \in \mathbb{N}$).

(a) $1/n - 1$

(b) $1/n$

(c) n

(d) $n - 1$

$$I_1 = \int_0^n [x] dx \xrightarrow{\text{Integer + Piecewise}}$$

$$= \int_0^1 0 dx + \int_1^2 1 dx + \int_2^3 2 dx + \int_3^4 3 dx + \dots + \int_{n-1}^n (n-1) dx$$

$$I_1 = \frac{n(n-1)}{2}$$

$$= 0 + 1(2-1) + 2(3-2) + \dots + (n-1) \text{ times}$$

$$= 1 + 2 + \dots + (n-1) \text{ times}$$

$$I_2 = \int_0^n \{x\} dx = \int_0^n (x - [x]) dx$$

$$x = [x] + \{x\}$$

$$\{x\} = x - [x]$$

$$\Rightarrow \int_0^1 (x-0) dx + \int_1^2 (x-1) dx + \dots + \int_{n-1}^n (x-(n-1)) dx$$

$$= \frac{n}{2} - \frac{n(n-1)}{2}$$

$$= \frac{n}{2} - \frac{n^2}{2} + \frac{n}{2}$$

$$I_2 = \left(n - \frac{n^2}{2} \right)$$

$$\frac{I_1}{I_2} = \left(\frac{\frac{n(n-1)}{2}}{\frac{n-n^2}{2}} \right) = \frac{(n-1)}{1}$$

Ans

Q23. Let $A = \int_0^1 \frac{e^t dt}{1+t}$ then $\int_{a-1}^a \frac{e^{-t}}{t-a-1} dt$ has the value

(a) Ae^{-a}

(b) $-Ae^{-a}$

(c) $-ae^{-a}$

(d) Ae^a

Manipulate

$$\Rightarrow - \int_1^0 \frac{e^{(z-a)} \cdot (-dz)}{1+z}$$

$$\Rightarrow \int_0^1 \frac{e^z \cdot e^{-a} (-dz)}{1+z}$$

$$= -e^{-a} \int_0^1 \frac{e^z}{1+z} dz = -Ae^{-a}$$

$$I = \int_{a-1}^a \frac{e^{-t}}{(t-a-1)} dt$$

$$= - \int_{a-1}^a \frac{e^{-t}}{1+(a-t)} dt$$

$$a-t=z$$

$$-dt=dz$$

$$a-(a-1)=z$$

$$1=z$$

$$a-a=z$$

$$z=0$$

$$f(2a-x) = f(x)$$

Q24. $\int_0^\pi x f(\sin x) dx$ is equal to

$$\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$$

$$\textcircled{I} = \int_0^\pi x f(\sin x) dx$$

$$\textcircled{I} \Rightarrow \int_0^\pi (\pi-x) f(\sin(\pi-x)) dx$$

Add $2I = \int_0^\pi \pi f(\sin x) dx$

(a) $\pi \int_0^\pi f(\sin x) dx$

(b) $\frac{\pi}{2} \int_0^{\pi/2} f(\sin x) dx$

$$I = \frac{\pi}{2} \int_0^{\pi/2} f(\sin x) dx$$

$\nearrow \sin(\frac{\pi}{2}-x)$

(c) $\pi \int_0^{\pi/2} f(\cos x) dx$

(d) $\pi \int_0^\pi f(\cos x) dx$

$$I = \pi \int_0^{\pi/2} f(\cos x) dx$$

Q25. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a differentiable function having

$f(2) = 6, f'(2) = \left(\frac{1}{48}\right)$. Then $\lim_{x \rightarrow 2} \int_6^{f(x)} \frac{4t^3}{x-2} dt$ equals

Newton
Leibniz Rule

- (a) 18
- (b) 12
- (c) 36
- (d) 24

$$\lim_{x \rightarrow 2} \left[\frac{4[f(x)]^3 \cdot f'(x)}{1} \right]$$

$$\lim_{x \rightarrow 2} \frac{1}{(x-2)} \left[\int_6^{f(x)} 4t^3 dt \right]$$

$$= 4[f(2)]^3 \cdot f'(2) = 4 \times (6)^3 \times \frac{1}{48} = 4 \times 6 \times 6 \times 6 \times \frac{1}{48} = 18$$

Q26. The value of $\int_0^{\pi/2} \frac{(\sin x + \cos x)^2}{\sqrt{1 + \sin 2x}} dx$ is

$$= \int_0^{\pi/2} \frac{(\sin x + \cos x)^2}{\sqrt{(\sin x + \cos x)^2}} dx$$

$$\frac{\sin^2 x + \cos^2 x + 2 \sin x \cos x}{(\sin x + \cos x)^2} \Rightarrow \int_0^{\pi/2} \frac{(\sin x + \cos x)^2}{|\sin x + \cos x|} dx \Rightarrow 0 \text{ to } \frac{\pi}{2}$$

(a) 0

(b) 1

(c) 2

(d) 3

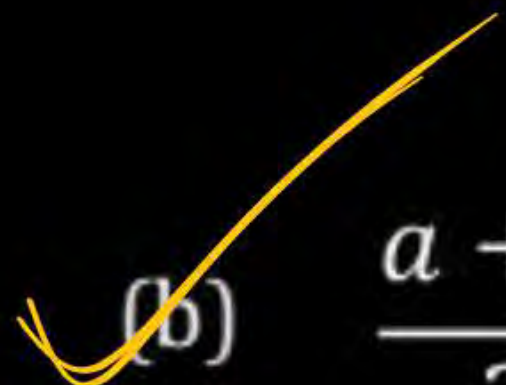
$\Rightarrow \int_0^{\pi/2} \frac{(\sin x + \cos x)^2}{\sin x + \cos x} dx$ Positive

$\Rightarrow \int_0^{\pi/2} (\sin x + \cos x) dx = 2 \checkmark$

Q27. If $f(a + b - x) = f(x)$, then $\int_a^b xf(x)dx$ is equal to

using this Property

(a) $\frac{a+b}{2} \int_a^b f(b-x)dx$

 (b) $\frac{a+b}{2} \int_a^b f(x)dx$

(c) $\frac{b-a}{2} \int_a^b f(x)dx$

(d) $\frac{a+b}{2} \int_a^b f(a+b+x)dx$

Q28. The value of $\lim_{x \rightarrow 0} \frac{\int_0^{x^2} \sec^2 t \, dt}{x \sin x}$ is

$\lim_{x \rightarrow 0} \frac{\int_0^{x^2} \sec^2 t \, dt}{x \sin x} = \frac{\rightarrow 0}{\rightarrow 0}$ form $\frac{0}{0}$
 Using L-Hospital Rule

- (a) 3
- (b) 2
- (c) 1
- (d) -1

$$= \lim_{x \rightarrow 0} \frac{\sec^2 x \cdot 2x}{x \cos x + \sin x}$$

$$= \lim_{x \rightarrow 0} \frac{2 \sec^2 x^2}{\frac{x}{x} \cos x + \frac{\sin x}{x}}$$

$$= \lim_{x \rightarrow 0} \frac{2 \sec^2 x^2}{\cos x + \frac{\sin x}{x}} = 1$$

$$= \lim_{x \rightarrow 0} \frac{2 \sec^2 x^2}{2} = \boxed{1}$$

Q29. $\int_{\sin x}^1 t^2 f(t) dt = 1 - \sin x \quad \forall x \in (0, \pi/2)$, then $f\left(\frac{1}{\sqrt{3}}\right)$ is $\xrightarrow{\sin x = \frac{1}{\sqrt{3}}}$

apply L-Hospital Rule:

(a) \checkmark 3 $= \frac{1}{\sin^2 x} f(\sin x) \cos x = 0 / \cos x$

(b) $\sqrt{3}$ $\sin^2 x f(\sin x) \cos x = \cos x$

(c) $1/3$ $f(\sin x) = \frac{1}{\sin^2 x} \checkmark$

(d) None of these

$\sin x = \frac{1}{\sqrt{3}}$

$f\left(\frac{1}{\sqrt{3}}\right) = \frac{1}{\left(\frac{1}{\sqrt{3}}\right)^2} = \sqrt{3} = 3$

Q30. $\lim_{x \rightarrow 0} \frac{\int_0^{x^2} \cos t^2 dt}{x \sin x}$ is equal to

$\frac{0}{0}$
L-Hospital
Rule

Using Newton Leibnitz Rule

$$\lim_{x \rightarrow 0} \frac{\cos x^4 \cdot 2x}{x \cos x + \sin x} \Rightarrow 1$$

(a) -1

(b) 1

(c) 2

(d) -2

Q31. If $\int_{\ln 2}^x \frac{dx}{\sqrt{e^x - 1}} = \frac{\pi}{6}$, then $x =$

(a) 4

(b) $\ln 8$

(c) $\ln 4$

(d) None of these

$$\int_{\ln 2}^x \frac{dx}{\sqrt{e^x - 1}} = \frac{\pi}{6} \quad \text{Square Root}$$

$$= \int_1^{\sqrt{e^x - 1}} \frac{2t \, dt}{(t^2 + 1) \sqrt{t^2}}$$

$$= 2 \int_1^{\sqrt{e^x - 1}} \frac{1}{(t^2 + 1)} dt = \frac{\pi}{6}$$

$$2 \left[\tan^{-1} t \right]_1^{\sqrt{e^x - 1}} = \frac{\pi}{6}$$

$$2 \left[\tan^{-1} \sqrt{e^x - 1} - \tan^{-1} 1 \right] = \frac{\pi}{6}$$

$$\boxed{e^x - 1 = t^2}$$

$$e^x = t^2 + 1$$

$$e^x dx = 2t \, dt$$

$$dx = \frac{2t \, dt}{e^x}$$

$$= \frac{2t \, dt}{t^2 + 1}$$

Change
The
limit
 $e^{\ln 2} - 1 = t^2$
 $2 - 1 = t^2$
 $t = \pm 1$

$$2 \left[\tan^{-1} \sqrt{e^x - 1} - \tan^{-1} 1 \right] = \frac{\pi}{6}$$

$$= 2 \left[\tan^{-1} \sqrt{e^x - 1} - \frac{\pi}{4} \right] = \frac{\pi}{6}$$

$$\tan^{-1} \sqrt{e^x - 1} = \frac{\pi}{12} + \frac{\pi}{4} = \frac{16\pi}{48} = \frac{\pi}{3}$$

$$\tan^{-1} \sqrt{e^x - 1} = \frac{\pi}{6}$$

$$\sqrt{e^x - 1} = \sqrt{3}$$

$$e^x - 1 = 3$$

$$e^x = 4$$

$$e^x = 4$$

$$x = \ln 4$$

Q32. $\lim_{x \rightarrow \infty} \left(\frac{x^2 - 2x + 1}{x^2 - 4x + 2} \right)^x =$

$\lim_{x \rightarrow \infty} \left(\frac{x^2 - 2x + 1}{x^2 - 4x + 2} \right)^x \xrightarrow{\text{Ans}} \infty$
 $\text{Using } (1 \rightarrow \infty) \rightarrow e^A$

Where $A = \lim_{x \rightarrow \infty} (f(x) - 1)g(x)$

(a) 1

(b) 2

(c) e^2

(d) e

Q33. If α and β be the roots of $ax^2 + bx + c = 0$ then $\lim_{x \rightarrow \alpha} (1 + ax^2 + bx + c)^{\frac{1}{x-\alpha}}$ is $\rightarrow \alpha, \beta$

$$ax^2 + bx + c = 0$$

$$a(x-\alpha)(x-\beta) = 0$$

Two Roots

$$\lim_{x \rightarrow \alpha} (1 + ax^2 + bx + c)^{\frac{1}{x-\alpha}}$$

where $A = \lim_{x \rightarrow \alpha} [f(x) - 1]g(x) = \lim_{x \rightarrow \alpha} [1 + a(x-\alpha)(x-\beta)]^{\frac{1}{x-\alpha}}$

$$= \lim_{x \rightarrow \alpha} [1 + a(x-\alpha)(x-\beta)]^{\frac{1}{x-\alpha}}$$

$$= a(\alpha - \beta)$$

$$\text{Ans} = e^{a(\alpha - \beta)}$$

$\rightarrow \infty$ form

$$= e^A$$

(a) $a(\alpha - \beta)$

(b) $\ln |a(\alpha - \beta)|$

(c) $e^{a(\alpha - \beta)}$ \rightarrow mod

(d) $e^{a|\alpha - \beta|}$

Q34. $\lim_{x \rightarrow \pi/2} \frac{2^{-\cos x} - 1}{x(x - \pi/2)} =$

$\lim_{x \rightarrow a} \frac{a^x - 1}{x} = \ln a$

$$\lim_{x \rightarrow \frac{\pi}{2}} \frac{2^{-\cos x} - 1}{x(x - \frac{\pi}{2})} \times \frac{2^{\cos x}}{2^{\cos x}}$$

$$= \lim_{x \rightarrow \frac{\pi}{2}} \frac{1 - 2^{\cos x}}{x(x - \frac{\pi}{2}) 2^{\cos x}}$$

(a) $\frac{2 \ln 2}{\pi}$

(b) $\ln 2 \Rightarrow \lim_{x \rightarrow \frac{\pi}{2}} \frac{2^{\cos x} - 1}{x(\frac{\pi}{2} - x) 2^{\cos x} \frac{(2^x - 1)}{x} = \ln 2}$

(c) $2/\pi$

(d) Does not exist $\lim_{x \rightarrow \frac{\pi}{2}} \frac{2^{\sin(\frac{\pi}{2} - x)} - 1}{x(\frac{\pi}{2} - x) 2^{\cos x}}$

$\frac{\ln 2 \times x}{\pi} = \frac{2 \ln 2}{\pi}$

Q35. Limit $\frac{(1-\tan\frac{x}{2})(1-\sin x)}{(1+\tan\frac{x}{2})(\pi-2x)^3}$ is

→ Using L-Hospital Rule.
Two times
 $L = \frac{1}{32}$

(a) $1/16$

(b) $-1/16$

(c) $1/32$

(d) $-1/32$

Q36. $\lim_{x \rightarrow 0} (\cos mx)^{\frac{n}{x^2}}$

$$\cos mx = \cos^2 \frac{mx}{2} - \sin^2 \frac{mx}{2}$$

$$= 1 - 2 \sin^2 \frac{mx}{2}$$

$$= 2 \cos^2 \frac{mx}{2} - 1$$

(a) $e^{-m^2 n/4}$

(b) $e^{-m^2 n/2}$

(c) $e^{-mn^2/2}$

(d) $e^{-mn^2/4}$

$$\lim_{x \rightarrow 0} (\cos mx)^{\frac{n}{x^2}}$$

Ans = e^A

Limit $\rightarrow \infty$ form

Where $A = \lim_{x \rightarrow 0} [f(x) - 1]g(x)$

$$= \lim_{x \rightarrow 0} [\cos mx - 1] \frac{n}{x^2}$$

$$= \frac{-m^2 n}{2}$$

$$L = e^{-\frac{m^2 n}{2}}$$

$$\lim_{x \rightarrow 0} \frac{n(1 - \cos mx)}{x^2}$$

$$\lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \underline{\underline{\log a}}$$



Q37. Limit $\lim_{x \rightarrow 0} \frac{(4^x - 1)^3}{\sin\left(\frac{x}{p}\right) \ln\left(1 + \frac{x^2}{3}\right)} =$

- (a) $9p(\log 4)$
- (b) $3p(\log 4)^3$
- (c) $12p(\log 4)^3$
- (d) $27p(\log 4)^2$

$$\lim_{x \rightarrow 0} \frac{(4^x - 1)^3 \cdot x^3 \cdot 1}{x^3 \cdot \left(\frac{\sin\left(\frac{x}{p}\right)}{\left(\frac{x}{p}\right)} \right) \cdot \left(\frac{\ln\left(1 + \frac{x^2}{3}\right)}{\frac{x^2}{3}} \right) \cdot \left(\frac{1}{3}\right)}$$

$$= (\log 4)^3 \cdot 3 \times p = \underline{\underline{3p(\log 4)^3}}$$

Q39. $\lim_{x \rightarrow \infty} \left(\frac{x^2 + 5x + 3}{x^2 + x + 3} \right)^x$ is equal to

- (a) ☒ e^4
- (b) ☐ e^2
- (c) ☐ e^3
- (d) ☐ e

$$\lim_{x \rightarrow \infty} \left(\frac{x^2 + 5x + 3}{x^2 + x + 3} \right)^x \rightarrow 1^\infty$$

$$\text{Ans} = e^A$$

Where $A = \lim_{x \rightarrow \infty} [f(x) - 1]g(x)$

$$\text{Ans} = e^4$$

Q40. Let α and β be the distinct roots of $ax^2 + bx + c = 0$,

Then $\lim_{x \rightarrow \alpha} \frac{1 - \cos(ax^2 + bx + c)}{(x - \alpha)^2}$ is equal to

(a) $\frac{1}{2}(\alpha - \beta)^2$

(b) $-\frac{a^2}{2}(\alpha - \beta)^2$

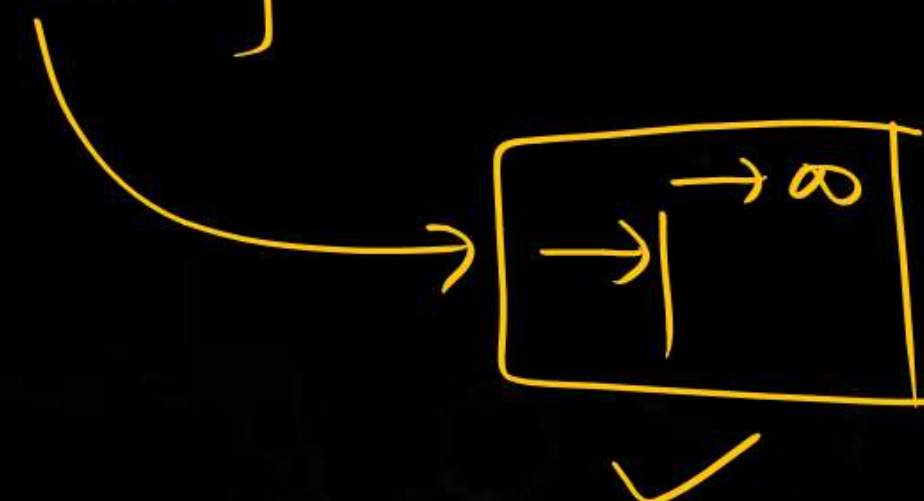
(c) 0

✓ (d) $\frac{a^2}{2}(\alpha - \beta)^2$

$$\begin{aligned} & \lim_{x \rightarrow \alpha} \frac{1 - \cos a(x - \alpha)(x - \beta)}{(x - \alpha)^2} \\ &= \lim_{x \rightarrow \alpha} \frac{1 - 1 + 2 \sin^2 \frac{a(x - \alpha)(x - \beta)}{2}}{(x - \alpha)^2} \\ &= \lim_{x \rightarrow \alpha} \frac{2 \sin^2 \frac{a(x - \alpha)(x - \beta)}{2}}{(x - \alpha)^2} \\ &= \frac{a^2}{2} (\alpha - \beta)^2 \end{aligned}$$

Q41. $\lim_{x \rightarrow 0} \left(\cot \left(\frac{\pi}{4} + x \right) \right)^{\operatorname{cosec} x} =$

$$\begin{aligned} & \lim_{x \rightarrow 0} \left[\cot \left(\frac{\pi}{4} + x \right) \right]^{\operatorname{cosec} x} = e^{-2} \\ & = \lim_{x \rightarrow 0} \left[\frac{1 - \tan x}{1 + \tan x} \right]^{\frac{1}{\sin x}} \rightarrow e^{-2} \end{aligned}$$



(a) e^{-1}

(b) e^2

(c) e^{-2}

(d) e^1

Q42. $\lim_{x \rightarrow \infty} \left(\sin \frac{1}{x} + \cos \frac{1}{x} \right)^x$ is $\rightarrow 1 \rightarrow \infty$

$y \rightarrow \frac{1}{x} \quad x \rightarrow \infty \quad y \rightarrow 0$

$$= \lim_{y \rightarrow 0} \left[\sin y + \cos y \right]^{\frac{1}{y}} \left(\rightarrow 1 \rightarrow \infty \right) \text{ form}$$

Where $A = \lim_{y \rightarrow 0} \left[\sin y + \cos y - 1 \right]^{\frac{1}{y}} = e^A$

$$= \lim_{y \rightarrow 0} \left[\frac{\sin y + \cos y - 1}{y} \right] \left(\frac{\rightarrow 0}{\rightarrow 0} \right) \text{ form}$$

L-Hospital Rule

Ans = e^A

$A = 1$

$= e^1 = e$

(a) ☒ e

(b) ☐ e^2

(c) ☐ $1/e$

(d) ☐ Does not exist

Q43. $\lim_{x \rightarrow \infty} \frac{\sin(6x^2)}{\ln \cos(2x^2 - x)} =$ L-Hospital Rule — Two times

misprint $x \rightarrow 0$

$$L = \lim_{x \rightarrow 0} \frac{\sin(6x^2)}{\ln \cos(2x^2 - x)} = \underline{\underline{-12}}$$

(a) 12

(b) ☒ -12

(c) 6

(d) -6

Q44. $\lim_{x \rightarrow 0} \frac{e^{-x^2/2} - \cos x}{x^3 \sin x} =$

Two Times
 $L = \frac{1}{12}$

Using L-Hospital Rule

- (a) $1/4$
- (b) $1/6$
- (c) ✓ $1/12$
- (d) $1/8$

✓ Leibnitz Rule:

$$f(x) = \int_{\phi(x)}^{\psi(x)} f(t) dt$$

remove
The Integral

$$f(x) = \int_{x^2}^{x^3} \sin t dt$$

$$= \sin x^3 \cdot 3x^2 - \sin x^2 \cdot 2x$$

$$\frac{d}{dx} f(x) = \frac{d}{dx} \int_{\phi(x)}^{\psi(x)} f(t) dt$$

$$\Rightarrow f[\psi(x)] \frac{d}{dx} \psi(x) -$$

remove

The
Integral

$$f[\phi(x)] \frac{d}{dx} [\phi(x)]$$

Thank You!

GW Soldiers