GATE
ALL BRANCHES

ENGINEERING MATHEMATICS

Probability & Statistics

Lecture 16

BY- RAHUL SIR

TOPICS TO BE COVERED

01 Problems based on Probability Distributions

Q. Questions

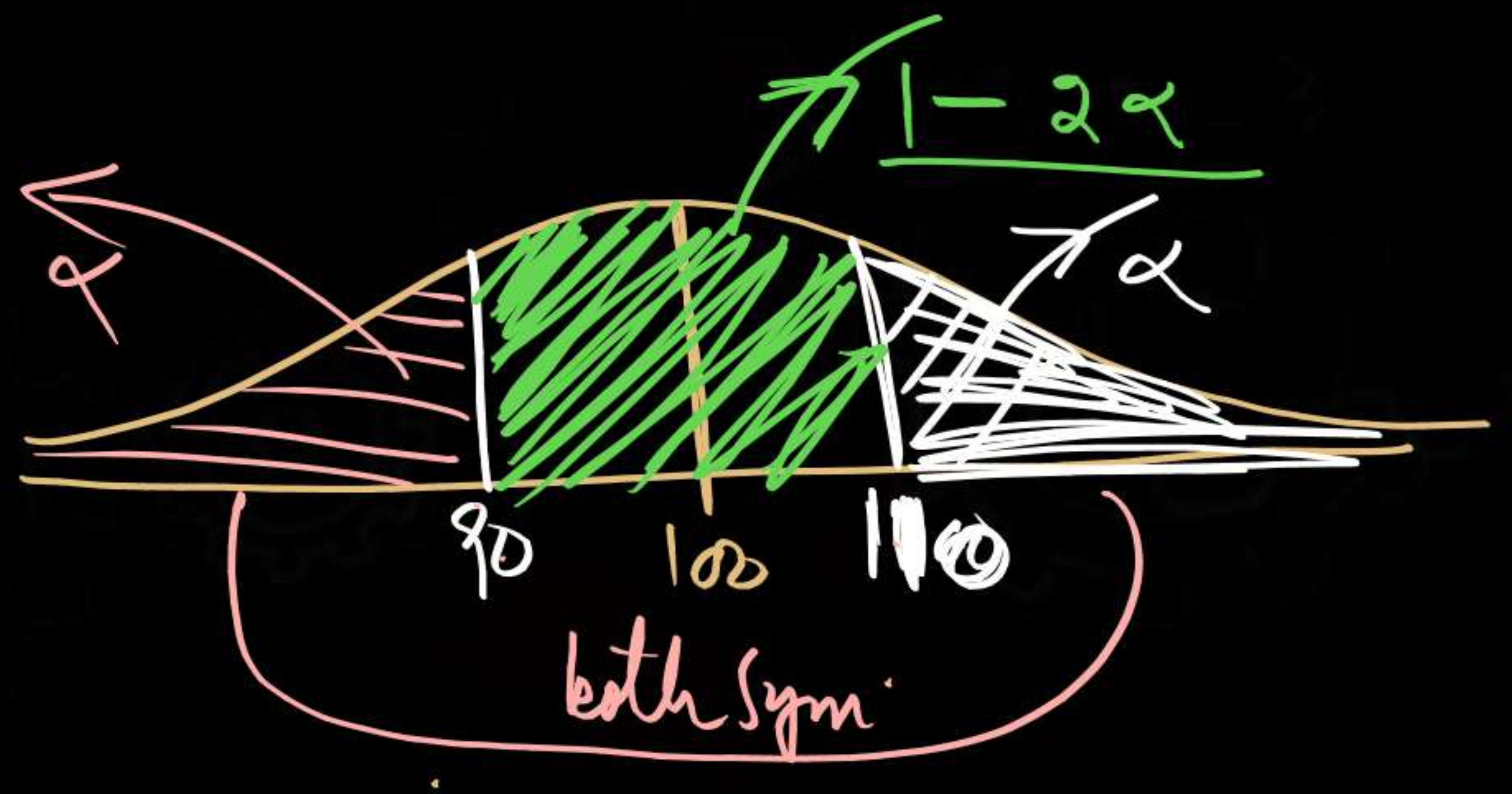
For a random variable $x(-\infty < x < \infty)$ following Normal distribution, the mean is $\mu = 100$. If the probability is $P = \alpha$ for $x \geq 110$. Then the probability x lying between 90 and 110 i.e., $P(90 \leq x \leq 110)$ is equal to

(a) $1 - 2\alpha$

(b) $1 - \alpha$

(c) $1 - \alpha/2$

(d) 2α



Q. Questions

The annual precipitation data of a city is normally distributed with mean and standard deviation as 1000 mm and 200 mm, respectively. The probability that the annual precipitation will be more than 1200 mm is

$P(X > 1200)$

$\mu = 1000$

$\sigma = 200$

$P\left(Z > \frac{1200-1000}{200}\right)$

(a) $< 50\%$

(b) 50%

$P(Z > 1)$

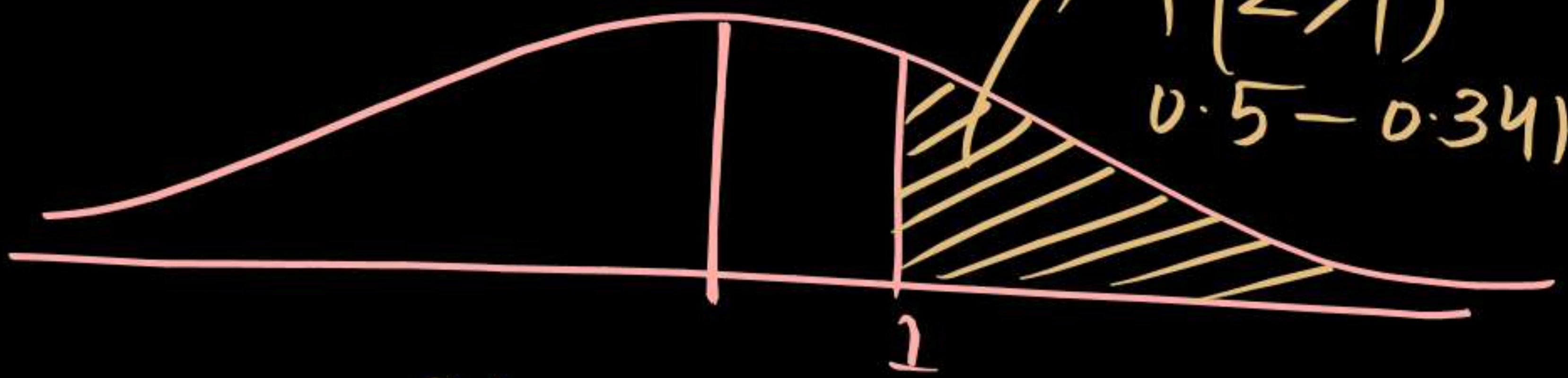
$P(X > 1200)$

$= P\left(\frac{X-\mu}{\sigma} > \frac{1200-\mu}{\sigma}\right)$

(c) 75%

(d) 100%

$P(Z>1)$
$0.5 - 0.3415$



$$P(Z>1) = 0.5 - 0.3415$$
$$= 0.1587$$
$$< 50\%$$ <u>Ans</u>

Q. Questions

Let X be a normal random variable with mean 1 and variance 4. The probability $P\{X < 0\}$ is

(a) 0.5

(b) Greater than zero less than 0.5

(c) Greater than 0.5 less than 1.0

(d) 1.0

Q. Questions

The lengths of a large stocks of titanium rods follow a normal distribution with a mean (μ) of 440 mm and a standard deviation (σ) of 1 mm. What is the percentage of rods whose lengths lie between 438 mm and 441 mm?

(a) 81.85%

(b) 68.4%

(c) 99.75%

(d) 86.64%

$\mu = 440$

$\sigma = 1\,mm$

$$P\left[\frac{438-\mu}{\sigma} < \frac{X-\mu}{\sigma} < \frac{441-\mu}{\sigma}\right]$$

$$P(438 < X < 441) = P\left(\frac{438-440}{1} < z < \frac{441-440}{1}\right)$$

$$= P(-2 < z < 1)$$

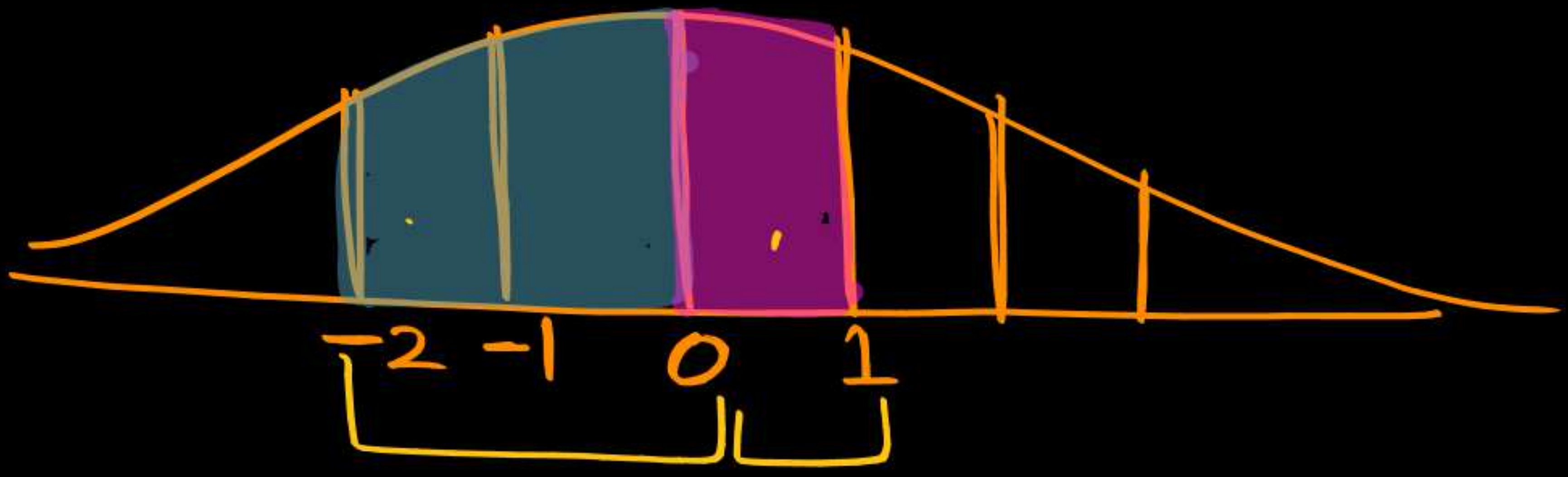
$$P(-2 < z < 1)$$

$$\Rightarrow P(-2 < z < 0) + P(0 < z < 1)$$

$$= 0.4774 + 0.3417$$

$$= 0.8185$$

$$= \underline{81.85\%}$$



Q. Questions

If X is a Gaussian Distributed Random variable with mean = 30 and standard deviation = 5, then find P (|X-30| < 5)

M.W

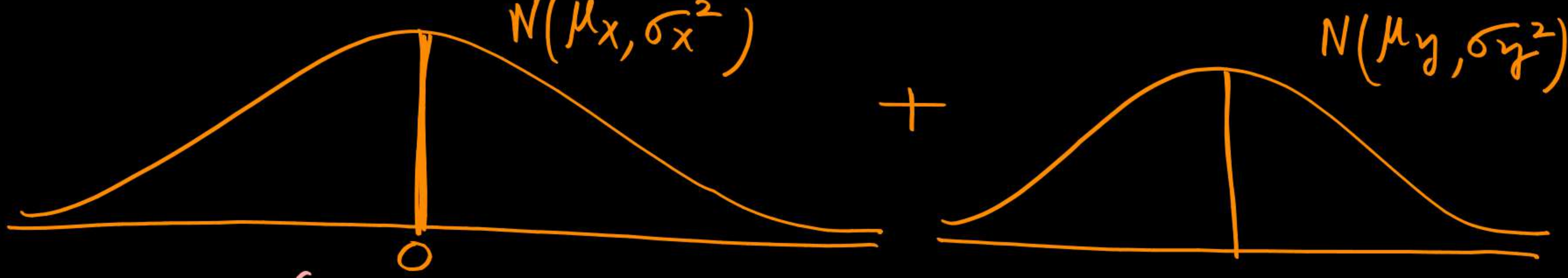
$= 2 \times .1587$

Q. Questions

Let x_1, x_2 and x_3 be independent and identically distribution random variables with the uniform distribution on [0, 1]. The probability p $\{x_1$ is the largest$\}$ is ________.

H.W

Property: SUM of Random Variable (Guassian)

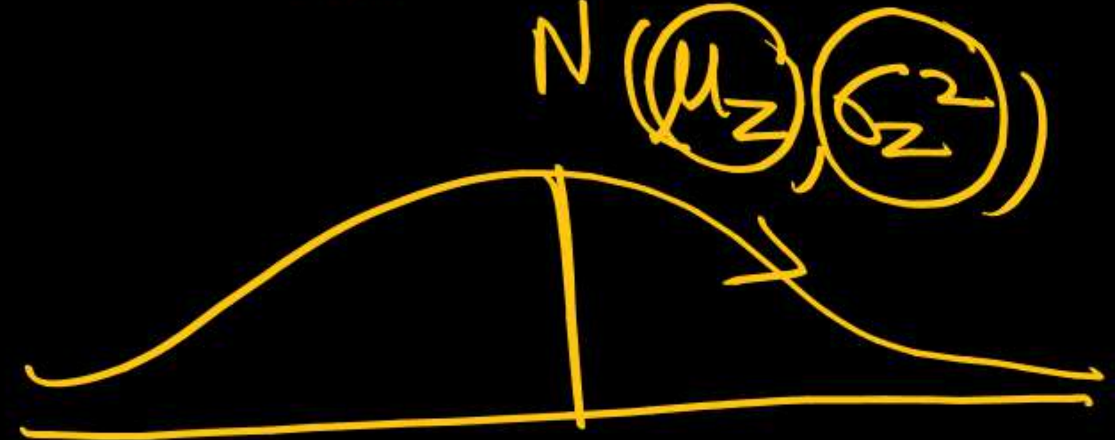


If X and Y Are Guassian Random variable

$$Z = X + Y$$

X and Y Independent

Z is also Guassian Random variable.



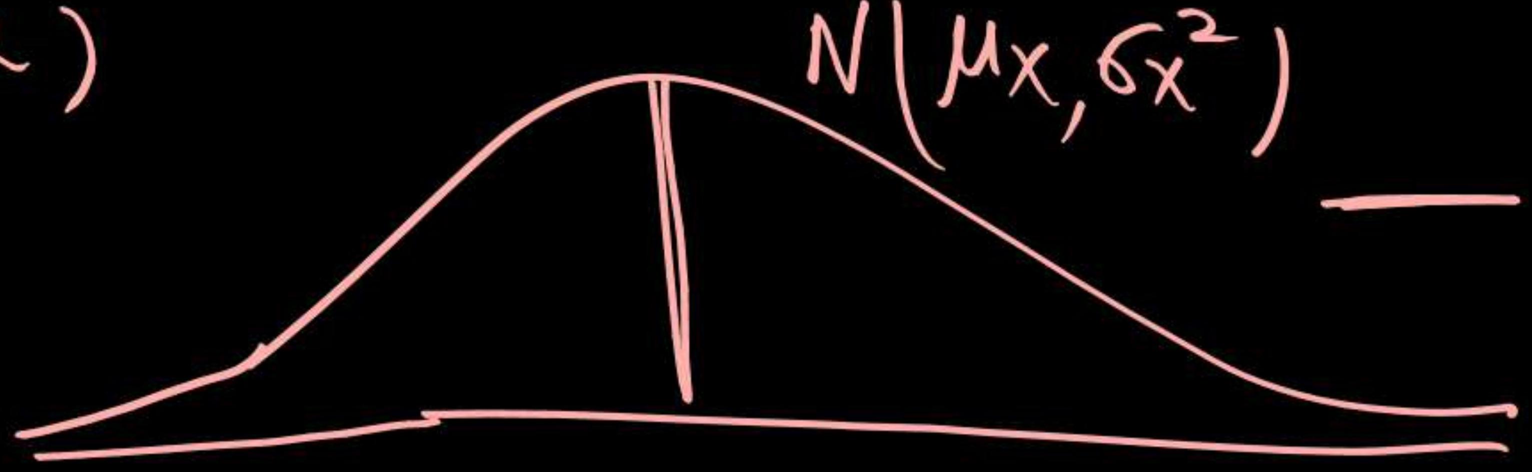
sum of combined mean OR variance

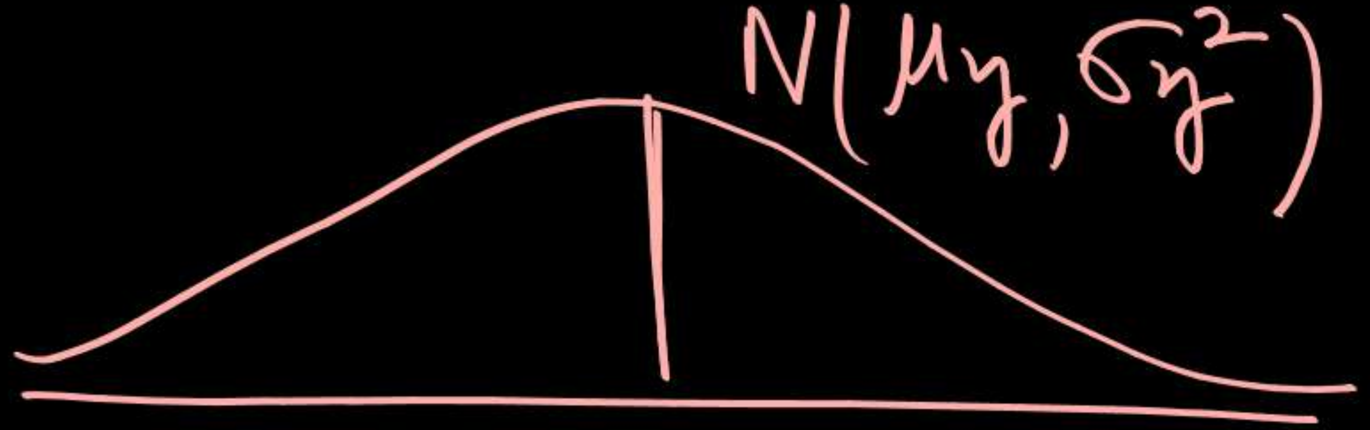
$$\begin{cases} \mu_Z = \mu_x + \mu_y \\ \sigma_Z^2 = \sigma_x^2 + \sigma_y^2 \end{cases}$$

$$\mu_N = \mu_X + \mu_Y + \mu_Z + \mu_t + \cdots$$

$$\sigma_N^2 = \sigma_X^2 + \sigma_y^2 + \sigma_Z^2 + \sigma_t^2 + \cdots$$

2)

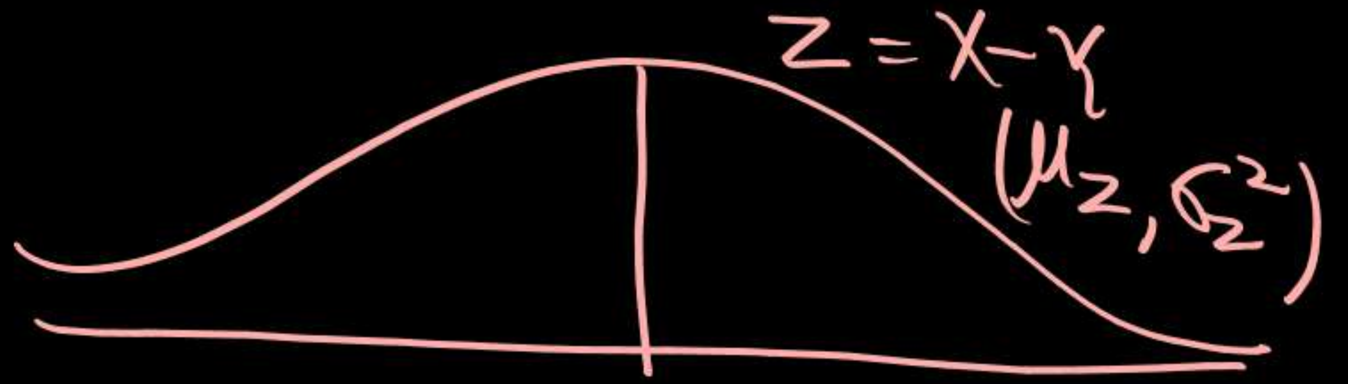




$$Z = X - Y$$

$$\mu_Z = \mu_X - \mu_Y$$

$$\sigma_Z^2 = \sigma_X^2 + \sigma_y^2$$



Q. Questions

$\mu_u = 0 \quad \mu_V = 0$

Let U and V be two independent zero mean Gaussian random variables of variances $\frac{1}{4}$ and $\frac{1}{9}$ respectively. The probability $P\{3V \geq 2U\}$ is

$\sigma_u^2 = \frac{1}{4}$

$\sigma_V^2 = \frac{1}{9}$

H.W

$P(3V - 2U \geq 0)$

$= \frac{1}{2}$

Q. Questions

$Y = X_1 - X_2$

$\mu_Y = 1 - 1 = 0$

let X_1, X_2 be two independent normal random variables with means μ_1, μ_2 and standard deviation σ_1, σ_2 respectively. Consider $Y = X_1 - X_2$; $\mu_1 = \mu_2 = 1$, $\sigma_1 = 1$, $\sigma_2 = 2$.

$= (1)^2 + (2)^2 = 5$

(a) Y is normally distribution with mean 0 and variance 1

(b) Y is normally distribution with mean 0 and variance 5

(c) Y has mean 0 and variance 5, but is NOT normally distribution

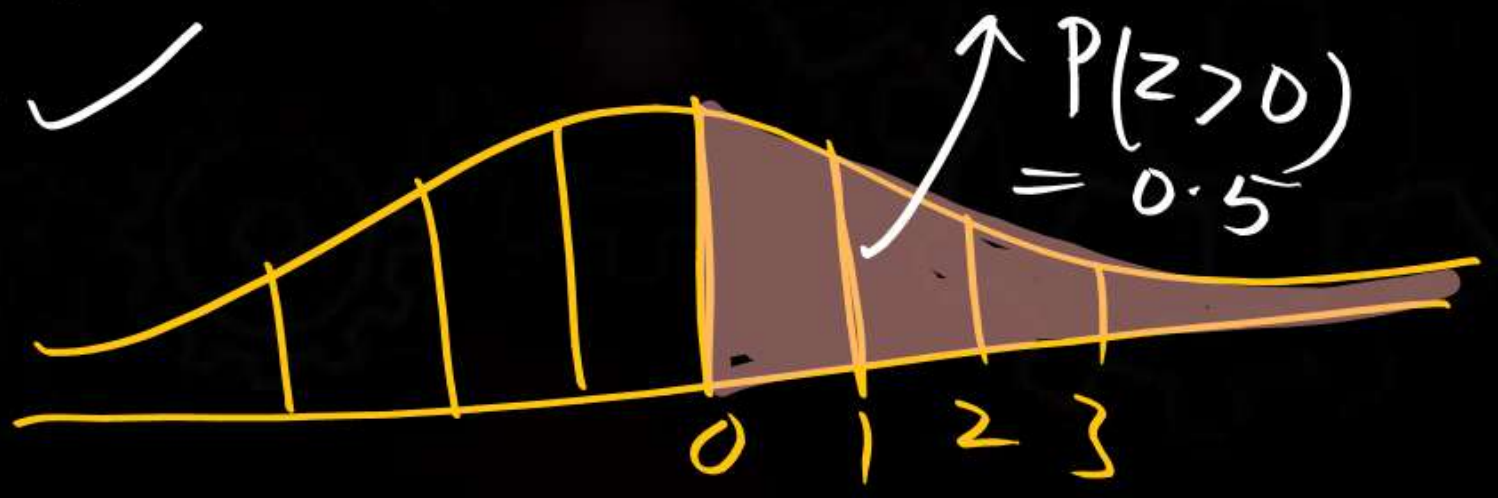
(d) Y has mean 0 and variance 1, but is NOT normally distribution

Q. Questions

$\mu = 500$
$\sigma = 50$

A nationalized bank has found the daily balance available in its saving accounts follows a normal distribution with a mean of Rs. 500 and a standard deviation of Rs. 50. The percentage of saving account holders, who maintain an average daily balance more than Rs. 500 is

$$P(X > 500)$$
$$= P\left(\frac{X-\mu}{\sigma} > \frac{500-\mu}{\sigma}\right)$$
$$= P\left(Z > \frac{500-500}{50}\right)$$
$$= P(Z > 0)$$
$$= 50\%$$
$$= \frac{1}{2}$$



Q. Questions

$n = 180$

$P(4) = \frac{1}{6} \quad P(\bar{4}) = \frac{5}{6}$

A Die is rolled 180 times using Gaussian random variable. Find the Probability that faces 4 will turn up at least 35 times.

$$\begin{bmatrix} n = 180 \; (\text{fixed}) \\ P(S) = \frac{1}{6} \\ P(F) = \frac{5}{6} \end{bmatrix}$$

Bernoulli

$$P(X > 35) = P\left(\frac{X-\mu}{\sigma} > \frac{35-\mu}{\sigma}\right)$$

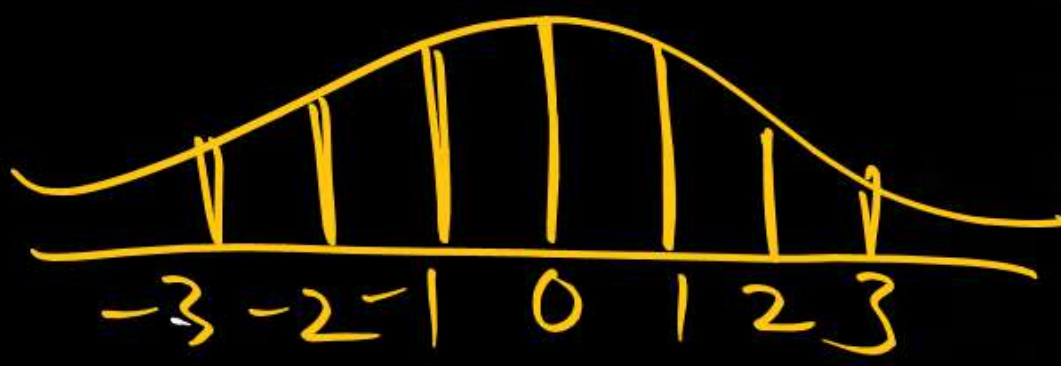
$$= P\left(Z > \frac{35-\mu}{\sigma}\right)$$

$$= P\left(Z > \frac{35-30}{5}\right)$$

$$= P(Z > 1)$$

$$= 0.5 - 0.3415$$

$$= 0.1587$$



$\mu = np = 180 \times \frac{1}{6} = 30$

$\sigma = \sqrt{npq} = 180 \times \frac{1}{6} \times \frac{5}{6} = 5$

Q. Questions

$1 \text{ feet} = 12 \text{ inches}$
$6 \text{ feet} = 72 \text{ inches}$

Assume Mean Height of the soldiers is 68.22 inches with the variance 10.8 inches. How many soldiers in Regiment of 1000 would you expected to be over (6 feet tall). Given that the Standard

$\boxed{\text{Normal Curve } X = 0 \text{ to } 1.15 = 0.3746.}$

$\mu = 68.22$ inches

$\sigma^2 = 10.8$

$$P(X > 72)$$

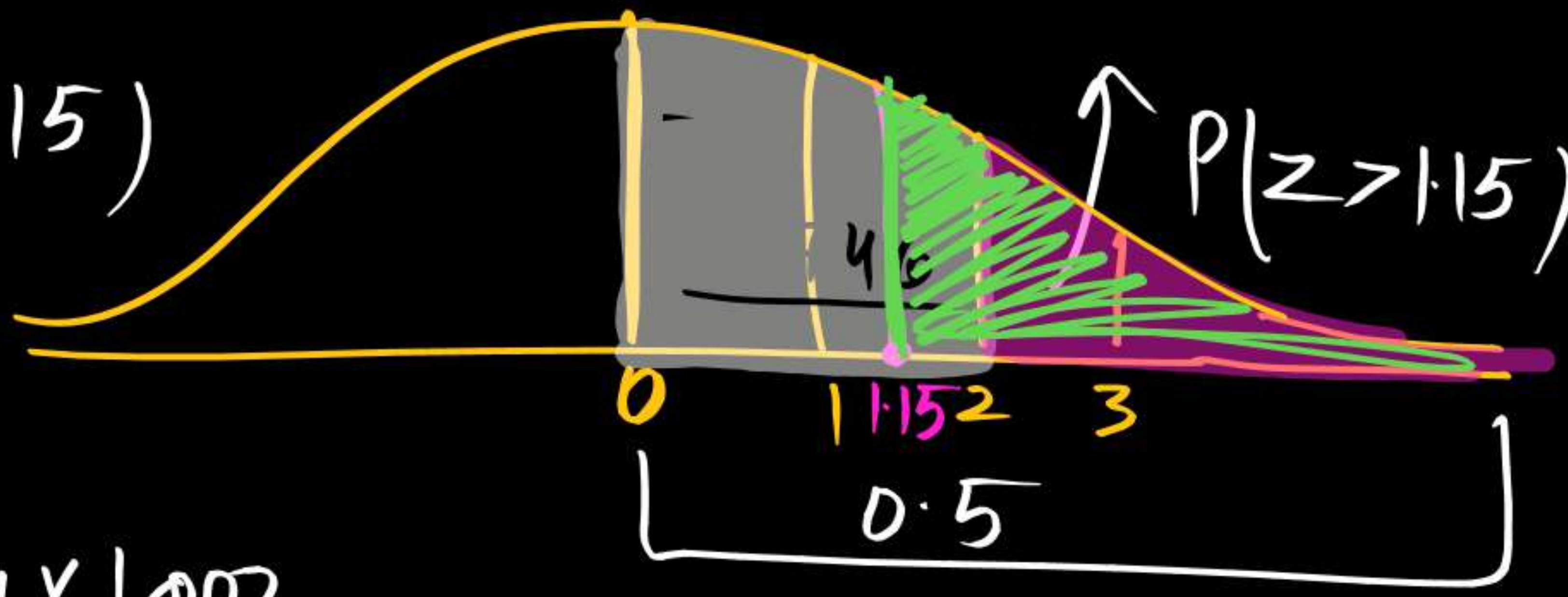
$$= P\left(\frac{X-\mu}{\sigma} > \frac{72-\mu}{\sigma}\right)$$

$$= P\left(Z > \frac{72-68.22}{\sqrt{10.8}}\right) = P(Z > 1.15)$$

$P(Z > 1.15) = 0.5 - P(0 \le Z \le 1.15)$

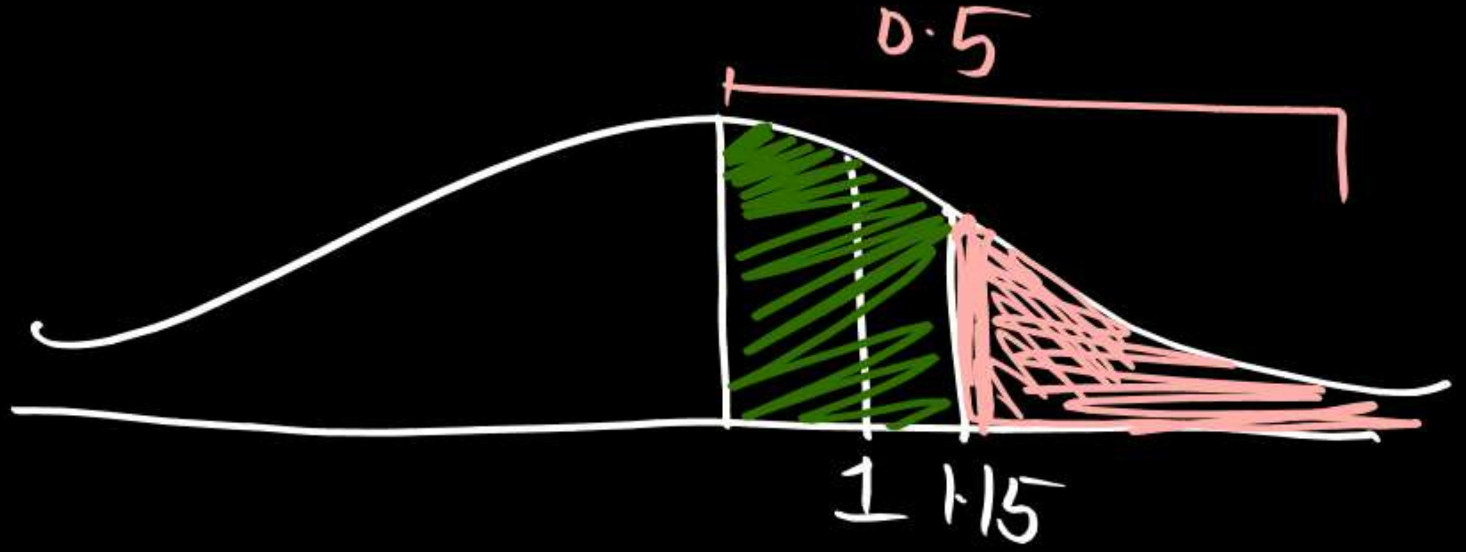
$= 0.5 - 0.3746$

$= 0.1254$



Total Solders $= 0.1254 \times 1000$

$= \underline{125.4 \text{ solders}}$ (125 solders OR 126)



Q. Questions

Let X_1, X_2, X_3 be three independent and identically distribution random variables with Uniform Distribution on [0, 1]. Find the probability

$P[X_1 + X_2 \le X_3]$

Let $x_1, x_2, x_3 \longrightarrow U[0,1]$ uniform

$P(x_1 + x_2 \le x_3) = P(\underbrace{x_1 + x_2 - x_3} \le 0)$

$U[0,1]$

$\boxed{X = x_1 + x_2 - x_3}$

$x_1 + x_2 \le x_3$

$x_1 + x_2 - x_3 \le x_3 - x_3$

$(x_1 + x_2 - x_3 \le 0)$

$\Rightarrow P(X \le 0) = P\left(\frac{X-\mu}{\sigma} \le \frac{0-\mu}{\sigma}\right)$

Z SCORE $= P\left(Z \le \frac{0-\mu}{\sigma}\right)$

$$X = X_1 + X_2 - X_3$$

$$E(X) = E[X_1] + E[X_2] - E[X_3]$$

$$E[X_1] = \frac{0+1}{2} = \frac{1}{2}$$

$$E[X_2] = \frac{0+1}{2} = \frac{1}{2}$$

$$E[X_3] = \frac{0+1}{2} = \frac{1}{2}$$

$E[X]$

$$\mu = \frac{1}{2} + \cancel{\frac{1}{2}} - \cancel{\frac{1}{2}}$$

$$\boxed{\mu = \frac{1}{2}}$$

$$E[X+Y] = E[X] + E[Y]$$

$$E[X] = \frac{a+b}{2} = \frac{0+1}{2} = \frac{1}{2}$$

$V[a,b)$

$$V(X) = \frac{(b-a)^2}{12}$$

$V[a,b)$

$[0,1]$ $$V(X) = V(X_1) + V(X_2) + V(X_3)$$

$$V(X_1) = \frac{(0-1)^2}{12}$$

$$\boxed{V(X_1) = \frac{1}{12}}$$

$$= \frac{1}{12} + \frac{1}{12} + \frac{1}{12}$$

$$= \frac{3}{12} = \frac{1}{4}$$

$$\sigma = \frac{1}{2}$$

$$S.D = \sqrt{\sigma^2}$$

$$S.D = \sqrt{\frac{1}{4}} = \frac{1}{2}$$

$$\text{Var}(X) > 0$$

$$S.D = \sqrt{\text{vari}}$$

$$V(-X_3) = (-1)^2 \text{var}(X)$$

D.E If X, Y, Z Are Independent Random Variable

$E[X+Y+Z] = E[X]+E[Y]+E[Z]$ — superposition

$E[X+Y] = E[X]+E[Y]$

X = Tossing A coin   Y: Throwing A Die

X	H(1)	T(0)
$P(X=x)$	$\frac{1}{2}$	$\frac{1}{2}$

1	2	3	4	5	6
$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

$E[X] = 1\times\frac{1}{2}+0\times\frac{1}{2}$

$E[X] = 0.5$

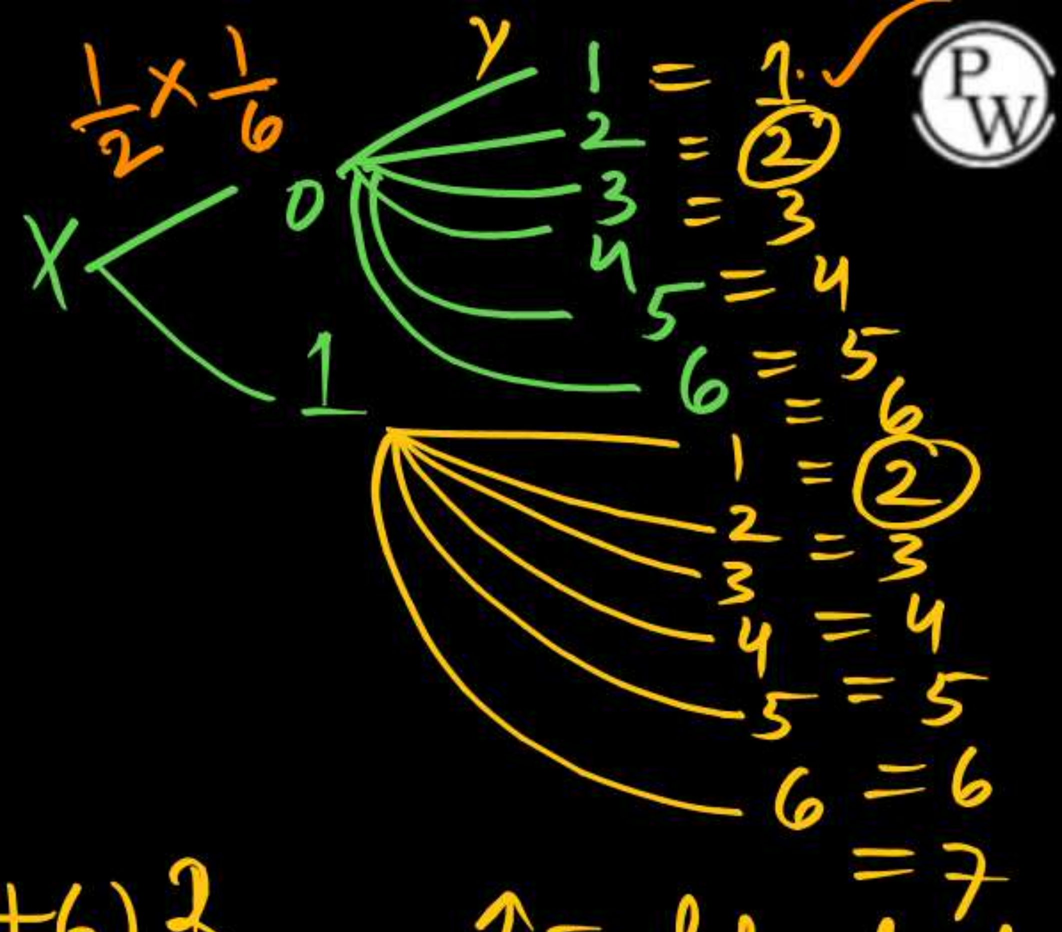
$E[Y] = (1+2+3+4+5+6)\frac{1}{6}$

$E[Y] = \frac{21}{6} = 3.5$

both are Independent

$$X+Y = 1, 2, 3, 4, 5, 6, 7$$

$X+Y$	1	2	3	4	5	6	7
$P(X+Y = x_i+y_j)$	$\frac{1}{12}$	$\frac{2}{12}$	$\frac{2}{12}$	$\frac{2}{12}$	$\frac{2}{12}$	$\frac{2}{12}$	$\frac{1}{12}$



$$E[X+Y] = (1+7)\times\frac{1}{12} + (2+3+4+5+6)\frac{2}{12}$$

$$E[X+Y] = \frac{48}{12} = 4$$

$E[X] = 0.5$

$E[Y] = 3.5$

$E[X+Y] = 4$

Independent

$P(\underbrace{0 \wedge 1}) \quad P(A \wedge B) = P(A)\,P(B)$

$P(0)\,P(1)$

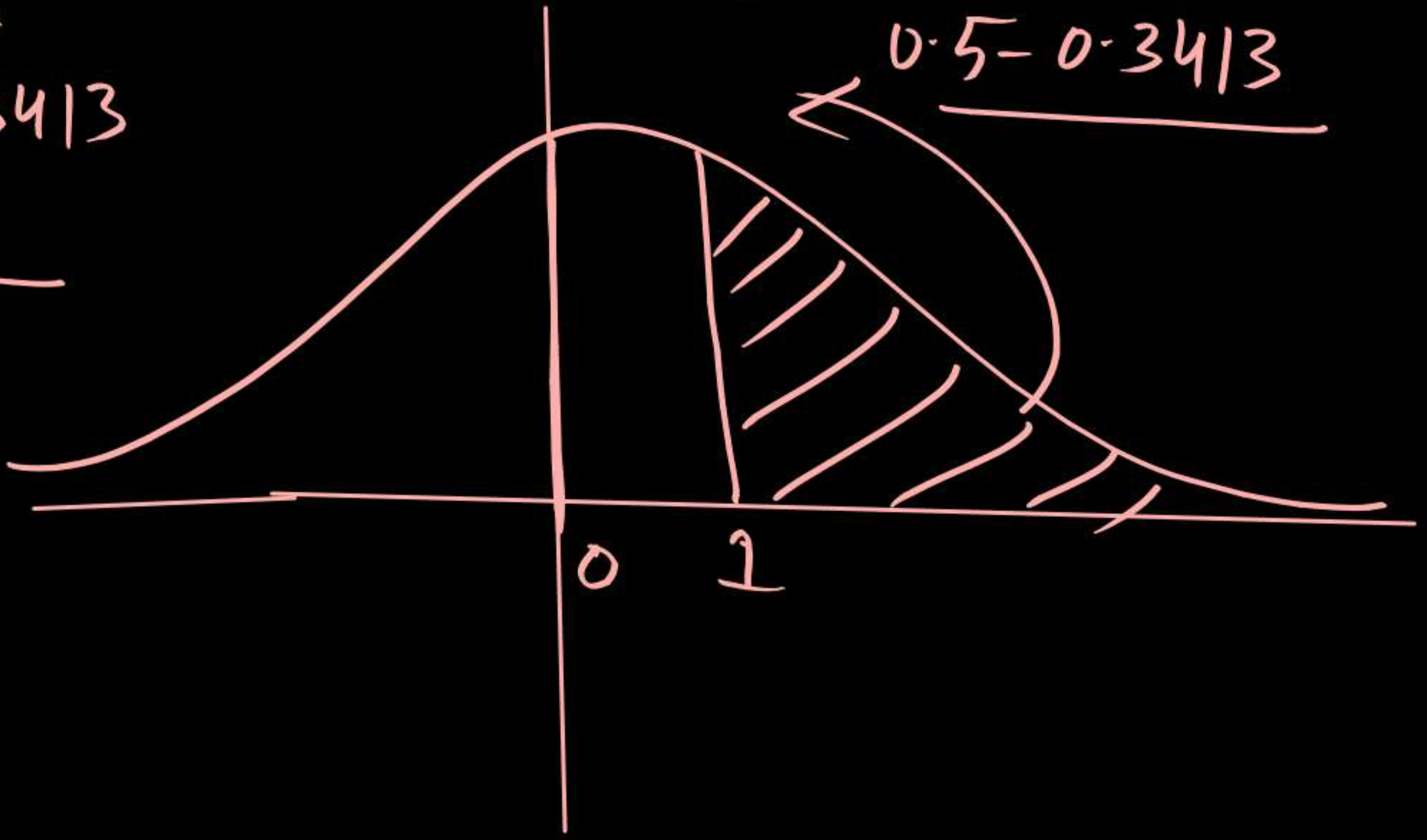
$= \frac{1}{2}\times\frac{1}{6} = \frac{1}{12}$

$\mu = \frac{1}{2} \quad \sigma = \frac{1}{2}$

$$P\left[Z \le \frac{0-\frac{1}{2}}{\frac{1}{2}}\right] = P(Z \le -1) \text{ OR } P(Z \ge 1)$$

$$= 0.5 - 0.3413$$

$$= \underline{0.1587}$$



Thank You!

GW Soldiers