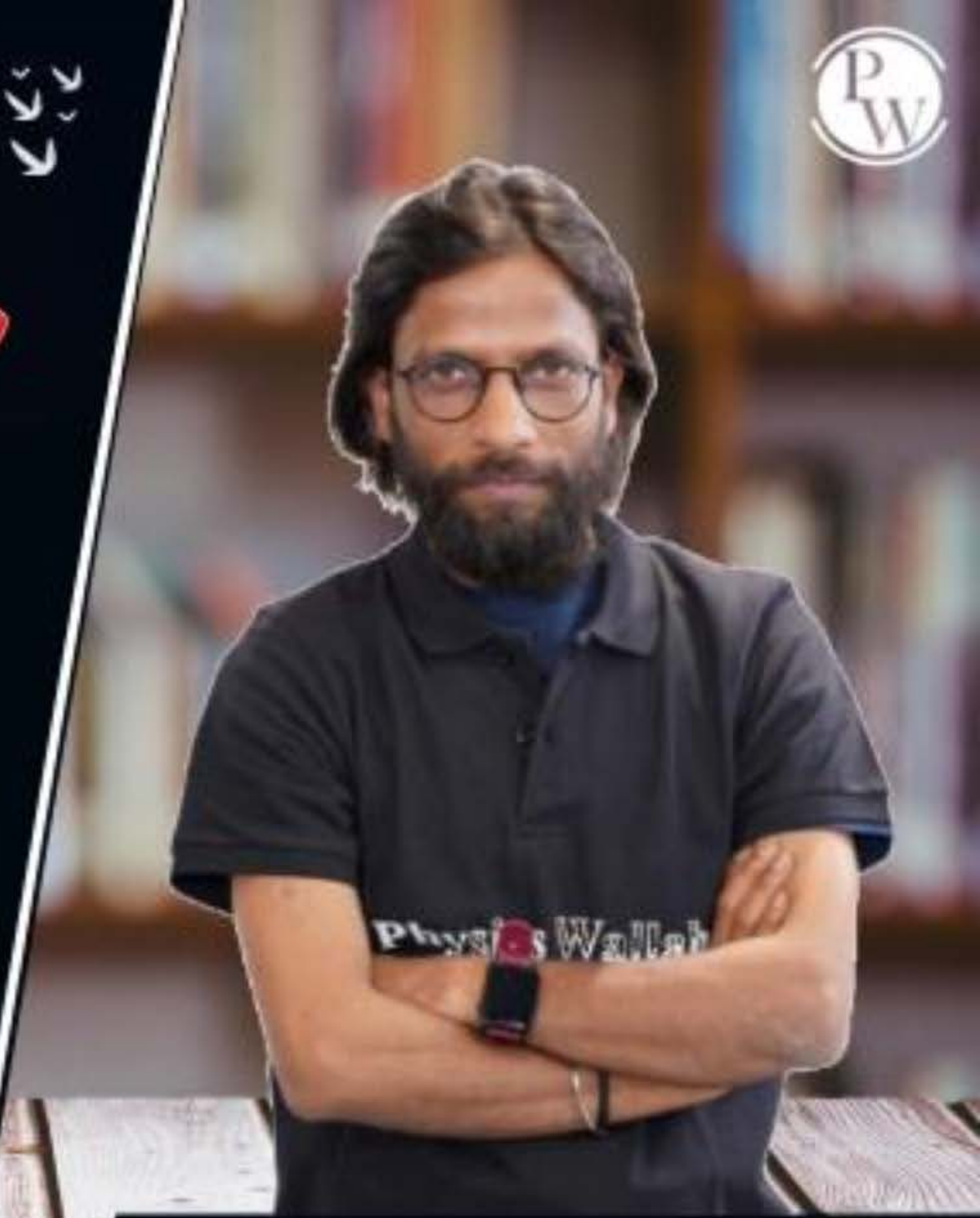
GATE-ALL BRANCHES

ENGINEERING MATHEMATICS

Linear Algebra

Lecture No.-02



By- Rahul sir

Recap of previous lecture

Topic Determinant of matrix

Topic Matrix properties

Topics to be Covered

Topic Determinant of matrix

Topic Matrix properties

Topic : Linear Algebra

#Q. If the determinant of the matrix $\begin{bmatrix} 1 & 3 & 2 \\ 0 & 5 & -6 \\ 2 & 7 & 8 \end{bmatrix}$ is 26, then the determinant of the matrix $\begin{bmatrix} 2 & 7 & 8 \\ 0 & 5 & -6 \\ 1 & 3 & 2 \end{bmatrix}$ is

$A = \begin{bmatrix} 1 & 3 & 2 \\ 0 & 5 & -6 \\ 2 & 7 & 8 \end{bmatrix} = 26$

first Row
Third Row

$A = \begin{bmatrix} 2 & 7 & 8 \\ 0 & 5 & -6 \\ 1 & 3 & 2 \end{bmatrix}$

$= \underline{-26}$

$\boxed{\text{A} \quad -26}$

B 26

C 0

D 52

Topic : Linear Algebra

CE/ME/EC/EE/CS

#Q. The determinant of the following matrix $\begin{bmatrix} 5 & 3 & 2 \\ 1 & 2 & 6 \\ 3 & 5 & 10 \end{bmatrix}$

$\det A \Rightarrow \underline{-28}$

$\boxed{|A| = -28}$

A – 76

B – 28

C 28

D 72

Slide 4

#Q. The determinant of the matrix given below

$$\begin{array}{c l} \begin{bmatrix} 0 & 1 & 0 & 2 \\ -1 & 1 & 1 & 3 \\ 0 & 0 & 0 & 1 \\ 1 & -2 & 0 & 1 \end{bmatrix} & \begin{array}{l} - R_1 \\ - R_2 \\ - R_3 \\ - R_4 \end{array} \end{array}$$

4×4

Max zeroes – Row

$$-1 \begin{vmatrix} 0 & 1 & 0 \\ -1 & 1 & 1 \\ 1 & -2 & 0 \end{vmatrix}$$

$$= -1 \times (-1) \begin{vmatrix} -1 & 1 \\ 1 & 0 \end{vmatrix}$$

$$= +1 \begin{vmatrix} -1 & 1 \\ 1 & 0 \end{vmatrix}$$

$$= -1$$

A. -1

B. 0

C. 1

D. 2

Topic : Linear Algebra

#Q. The determinant of the matrix $\begin{bmatrix} 6 & -8 & 1 & 1 \\ 0 & 2 & 4 & 6 \\ 0 & 0 & 4 & 8 \\ 0 & 0 & 0 & -1 \end{bmatrix}$

Upper Triangular matrix

$= 6 \times 2 \times 4 \times -1$

$= \underline{-48}$ Ans.

A 11

B -48

C 0

D -24

Topic : Linear Algebra

#Q. The determinant of the matrix $\begin{bmatrix} 2 & 0 & 0 & 0 \\ 8 & 1 & 7 & 2 \\ 2 & 0 & 2 & 0 \\ 9 & 0 & 6 & 1 \end{bmatrix}$ is

det A = 4

$\boxed{|A| = 4}$

A 4

B 0

C 15

D 20

Slide 7

Topic : Linear Algebra

#Q. Which one of the following does NOT equal $\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$?

H.W

A $\begin{vmatrix} 1 & x(x+1) & x+1 \\ 1 & y(y+1) & y+1 \\ 1 & z(z+1) & z+1 \end{vmatrix}$

B $\begin{vmatrix} 1 & x+1 & x^2+1 \\ 1 & y+1 & y^2+1 \\ 1 & z+1 & z^2+1 \end{vmatrix}$

C $\begin{vmatrix} 0 & x-y & x^2-y^2 \\ 0 & y-z & y^2-z^2 \\ 1 & z & z^2 \end{vmatrix}$

D $\begin{vmatrix} 2 & x+y & x^2+y^2 \\ 2 & y+z & y^2+z^2 \\ 1 & z & z^2 \end{vmatrix}$

Topic : Linear Algebra

#Q. The determinant of the matrix $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 100 & 1 & 0 & 0 \\ 100 & 200 & 1 & 0 \\ 100 & 200 & 300 & 1 \end{bmatrix}$ is

det A = 1

Lower Triangular matrix

A 100

B 200

C 1

D 300

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#Q. X and Y are non-zero square matrices of size $n \times n$. If $XY = 0_{n \times n}$ then

If $\det(XY) = 0$

Then $\det X = 0$

$\det Y = 0$

A $|X| = 0$ and $|Y| \neq 0$

B $|X| \neq 0$ and $|Y| = 0$

C $|X| = 0$ and $|Y| = 0$

D $|X| \neq 0$ and $|Y| \neq 0$

Topic : Linear Algebra

#Q. The value of the following determinant is $\begin{bmatrix} 1 & 4 & 9 \\ 4 & 9 & 16 \\ 9 & 16 & 25 \end{bmatrix}$

$\boxed{\det A = -8}$

A 8

B 12

C -12

D -8

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Topic : Linear Algebra

#Q. The value of the determinant $\begin{bmatrix} 1 & 3 & 2 \\ 4 & 1 & 1 \\ 2 & 1 & 3 \end{bmatrix}$ is

H.W

do yourself

A. -28

B. -24

C. 32

D. 36

Topic : Linear Algebra

$2\begin{vmatrix} 1 & -1 \\ x^2 & x \end{vmatrix} - 1\begin{vmatrix} 1 & -1 \\ y & x \end{vmatrix} + 1\begin{vmatrix} 1 & 1 \\ y & x^2 \end{vmatrix}$

#Q. The equation $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & -1 \\ y & x^2 & x \end{bmatrix} = 0$ represents a parabola passing through the points

$\Rightarrow 2(x+x^2) - 1(x+y) + 1(x^2-y) = 0$

$\Rightarrow 2x + 2x^2 - x - y + x^2 - y = 0$

$\Rightarrow x + 3x^2 - 2y = 0$

$2y = x + 3x^2$

$y = \left(\frac{x+3x^2}{2}\right)$

$(0,0)(-1,1)(1,2)$

A (0, 1), (0, 2), (0, –1)

B (0, 0), (–1, 1), (1, 2) ✓

C (1, 1), (0, 0), (2, 2)

D (1, 2), (2, 1), (0, 0)

Topic : Linear Algebra

#Q. If $\Delta = \begin{bmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{bmatrix}$ then which of the following is a factor of Δ.

Solve - factor

elimination

$A = \begin{bmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{bmatrix} \begin{matrix} R_1 \\ R_2 \\ R_3 \end{matrix}$ $R_1 \to R_1 - R_2$

$A = \begin{bmatrix} 0 & a-b & -c(a-b) \\ 1 & b & ca \\ 1 & c & ab \end{bmatrix}$

$A = a-b \begin{vmatrix} 0 & 1 & -1\times c \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$

factor $(a-b)$

A. a + b

B. a − b ✓

C. ✗ abc

D. ✗ a + b + c

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Topic : Linear Algebra

#Q. The determinant $\begin{bmatrix} 1+b & b & 1 \\ b & 1+b & 1 \\ 1 & 2b & 1 \end{bmatrix}$ equals to

SAME column

$\begin{bmatrix} \boxed{\begin{matrix} 2+2b \\ 2+2b \\ 2+2b \end{matrix}} & \begin{matrix} b \\ 1+b \\ 2b \end{matrix} & \begin{matrix} 1 \\ 1 \\ 1 \end{matrix} \end{bmatrix}$

Det A = 0

Ans = 0

A. 0 ✓

B. 2b (b – 1)

C. 2 (1 – b) (1 + 2b)

D. 3b (1 + b)

Adjoint of matrix:

$$[A_{ij}] = \underset{3\times3}{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{3\times3}$$

Adj A → cofactors → sign scheme → Transpose

$(-1)^{i+j}$

Adjoint of matrix (05–10 min)

Shortcut method For 3X3 matrix:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & -1 \\ -2 & 1 & 5 \end{bmatrix}_{3\times3} \begin{array}{l} -R_1 \\ -R_2 \\ -R_3 \end{array} \quad \boxed{\begin{array}{l} R_3 \\ R_1 \\ R_2 \end{array}} \text{ (repeat)}$$

every square 3X3 matrix is valid

$$\text{Adj } A = \begin{bmatrix} 1 & -1 & 2 & 1 \\ 1 & 5 & -2 & 1 \\ 2 & 3 & 1 & 2 \\ 1 & -1 & 2 & 1 \end{bmatrix}^T$$

$$= \begin{bmatrix} 6 & -8 & 4 \\ -7 & 11 & -5 \\ -5 & 7 & -3 \end{bmatrix}^T$$

Ans

Trans

Row → co

#

$$A = \begin{array}{c} \begin{bmatrix} 1 & 0 & -1 \\ 2 & -3 & -2 \\ 1 & 2 & 1 \end{bmatrix}_{3\times 3} \end{array} \begin{array}{l} — R_3 \\ — R_1 \\ — R_2 \end{array}$$

Calulate Adj A

$$\text{Adj}A = \begin{bmatrix} -3 & -2 & 2 & -3 \\ 2 & 1 & 1 & 2 \\ 0 & -1 & 1 & 0 \\ -3 & -2 & 2 & -3 \end{bmatrix}$$

$$\text{Adj}A = \begin{bmatrix} 1 & -4 & 7 \\ -2 & 2 & -2 \\ -3 & 0 & -3 \end{bmatrix}^T = \begin{bmatrix} 1 & -2 & -3 \\ -4 & 2 & 0 \\ 7 & -2 & -3 \end{bmatrix}$$ Ans

#

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}_{2\times 2}$$

$$\text{Adj}\,A = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}_{2\times 2}$$

$$A = \begin{bmatrix} 1 & 4 \\ 3 & 5 \end{bmatrix}$$

$$\text{Adj}\,A = \begin{bmatrix} 5 & -4 \\ -3 & 1 \end{bmatrix}$$

Diagonal matrix

$$A = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix} \qquad \text{Adj } A = \begin{bmatrix} bc & 0 & 0 \\ 0 & ca & 0 \\ 0 & 0 & ab \end{bmatrix} = \underline{\text{Trick}}$$

scalar / Identity / Unit

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 3 \end{bmatrix} \qquad \text{Adj } A = \begin{bmatrix} 18 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

Inverse of a matrix

$$A^{-1} = \frac{\text{adj } A}{|A|}$$

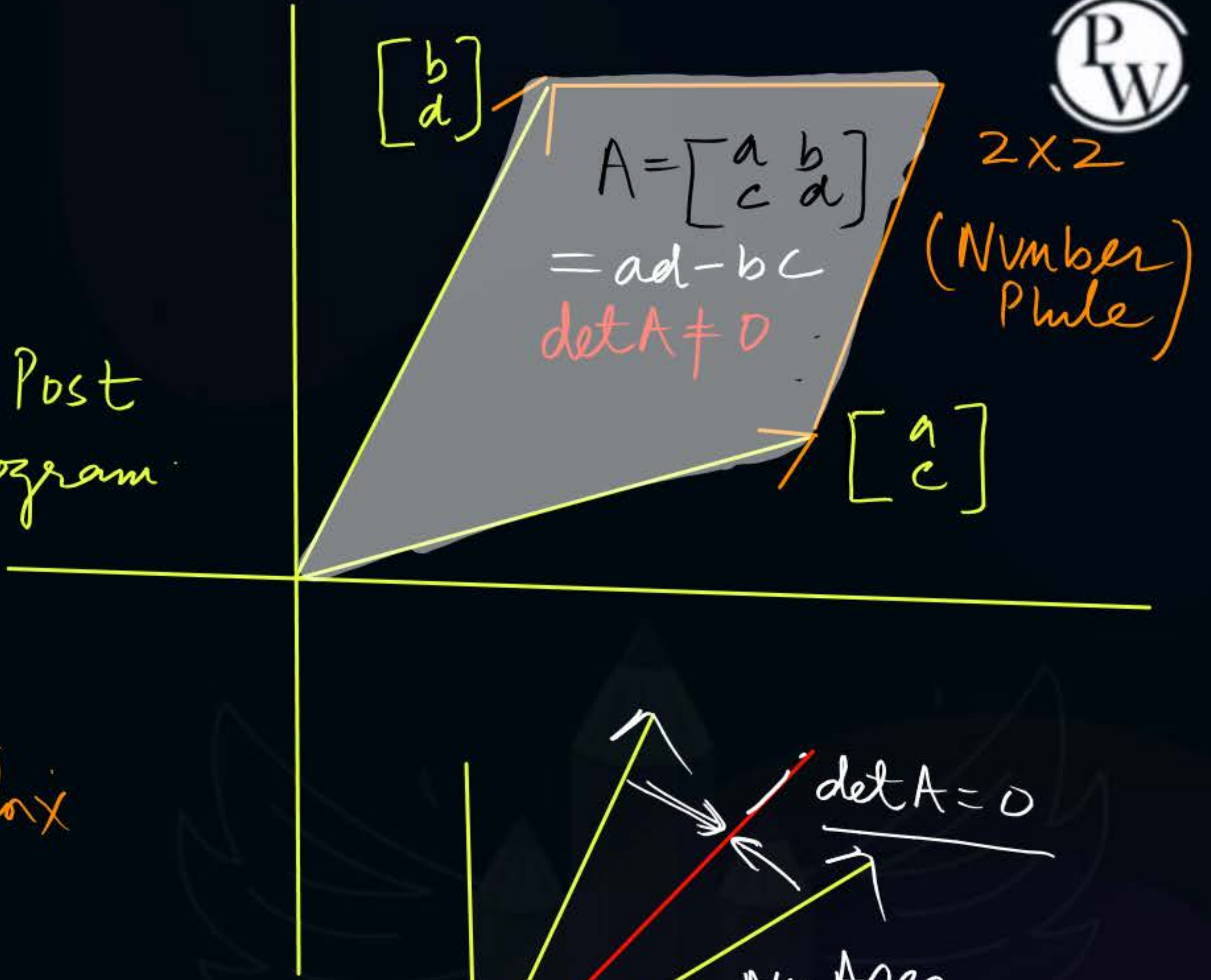
If $\det A = 0$ (singular matrix) A^{-1} does Not exists

If $\det A \neq 0$ (Non Singular matrix) A^{-1} exists

$$A^{-1} = \frac{\operatorname{adj} A}{|A|} \qquad |A| \neq 0$$

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 2 \\ 3 & 1 & 2 \end{bmatrix}_{3\times 3}$$

solution Post on Telegram.

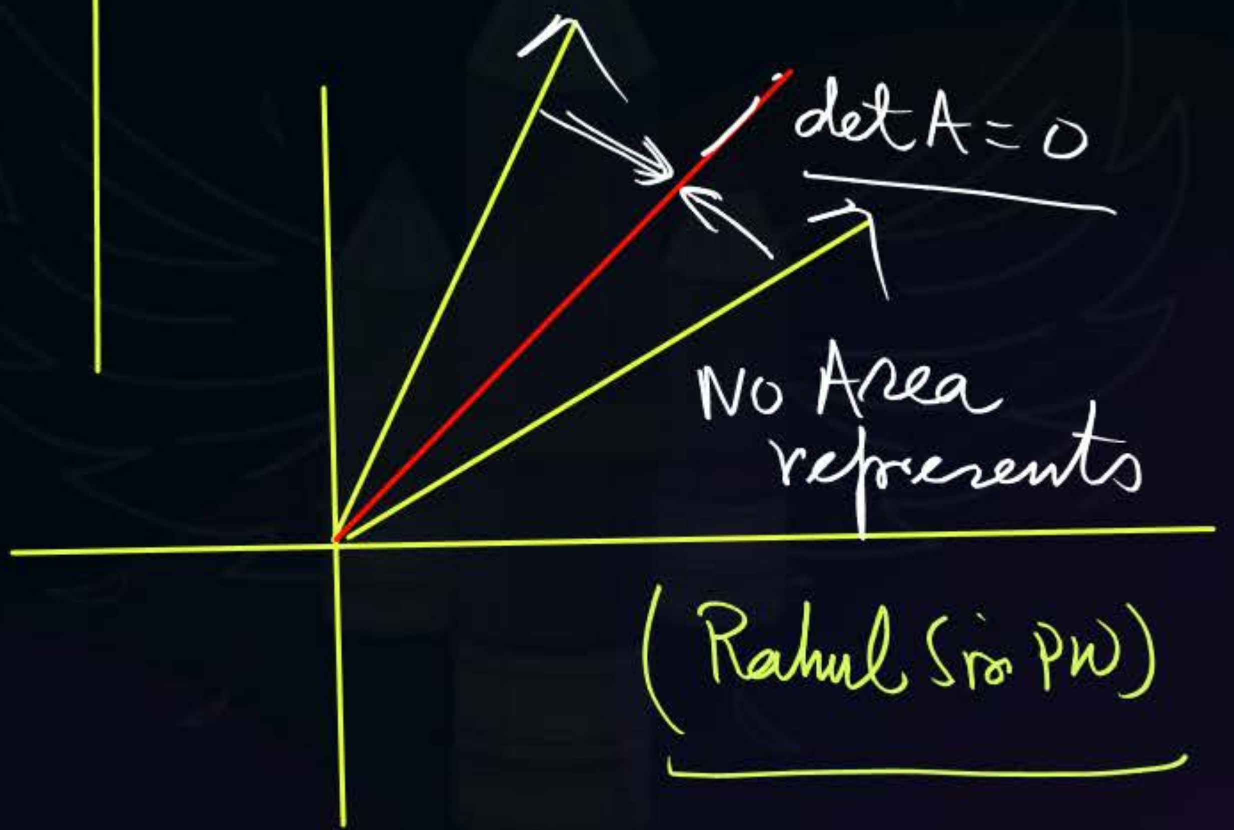


If A matrix is Diagonal

$$A = \begin{bmatrix} d_1 & 0 & 0 \\ 0 & d_2 & 0 \\ 0 & 0 & d_3 \end{bmatrix}$$

or scaler matrix
or Identity matrix
or unit matrix

$$A^{-1} = \begin{bmatrix} \frac{1}{d_1} & 0 & 0 \\ 0 & \frac{1}{d_2} & 0 \\ 0 & 0 & \frac{1}{d_3} \end{bmatrix}$$



(Rahul Sir PW)

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}_{2\times 2}$$

Number Rule

$$\boxed{A^{-1} = \frac{1}{(ad-bc)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}}$$

$$A = \begin{bmatrix} 1 & 2 \\ 5 & 8 \end{bmatrix} \qquad A^{-1} = -\frac{1}{2} \begin{bmatrix} 8 & -2 \\ -5 & 1 \end{bmatrix}$$

Ans

THANK - YOU