GATE

ALL BRANCHES

ENGINEERING MATHEMATICS

Single Variable Calculus

Lecture No. 05

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TOPICS TO BE COVERED

01 Continuity of the function ✓ Done ✓

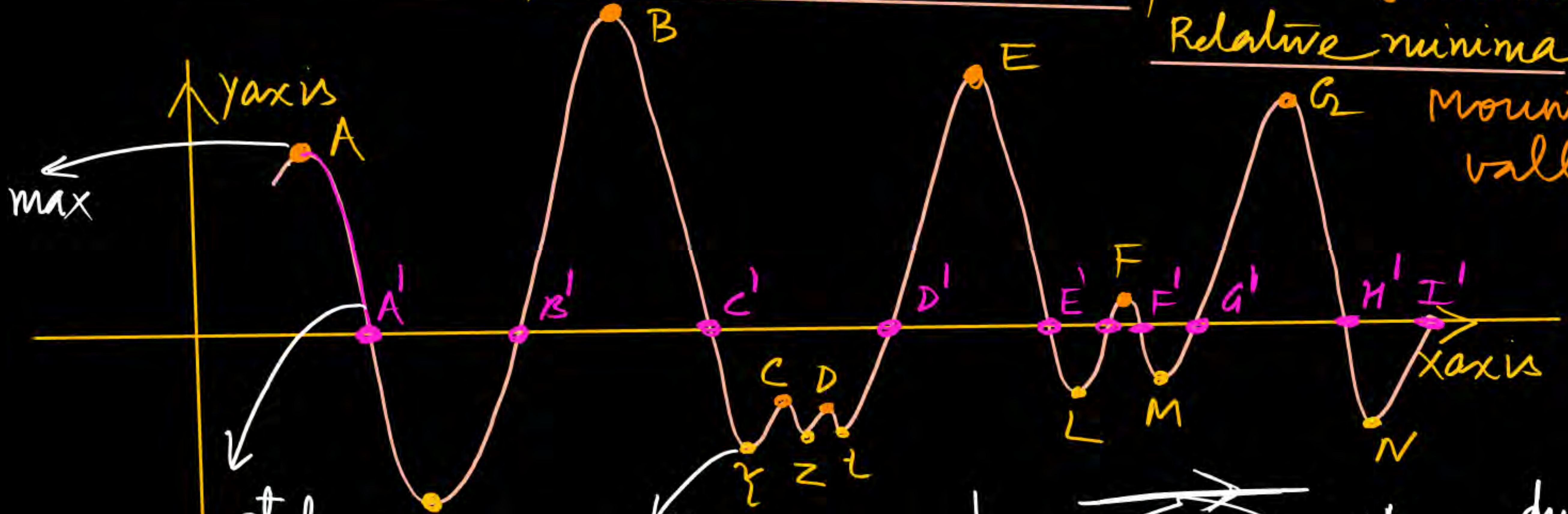
02 Differentiability of the function

03 Maxima and Minima with one variable

Max/min with one variable : [$y = f(x)$] "one variable function"

(A) Local maxima / Local min / Relative max / Relative minima

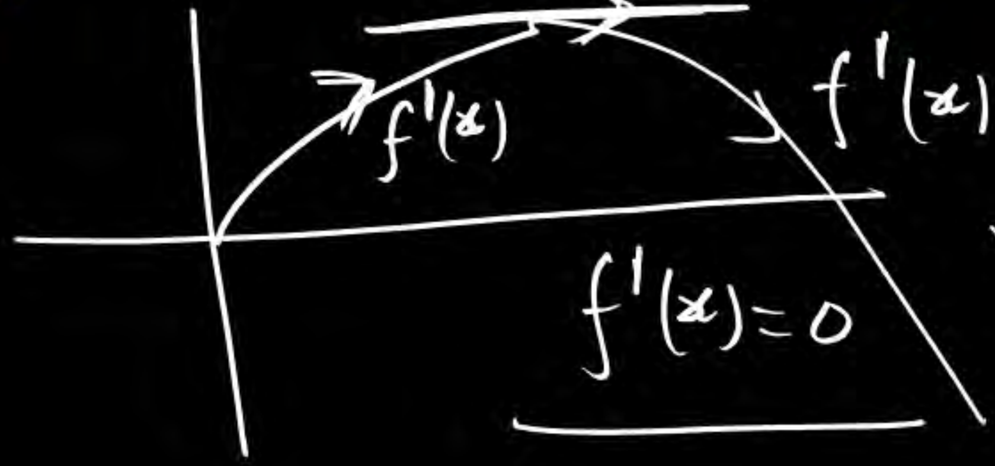
Mountain / valley



stationary Pt

min

✓ $\frac{dy}{dx} > 0$ (Increasing)



$f'(x) < 0$ $\frac{dy}{dx} = 0$

✓ $\frac{dy}{dx} < 0$ Decreasing

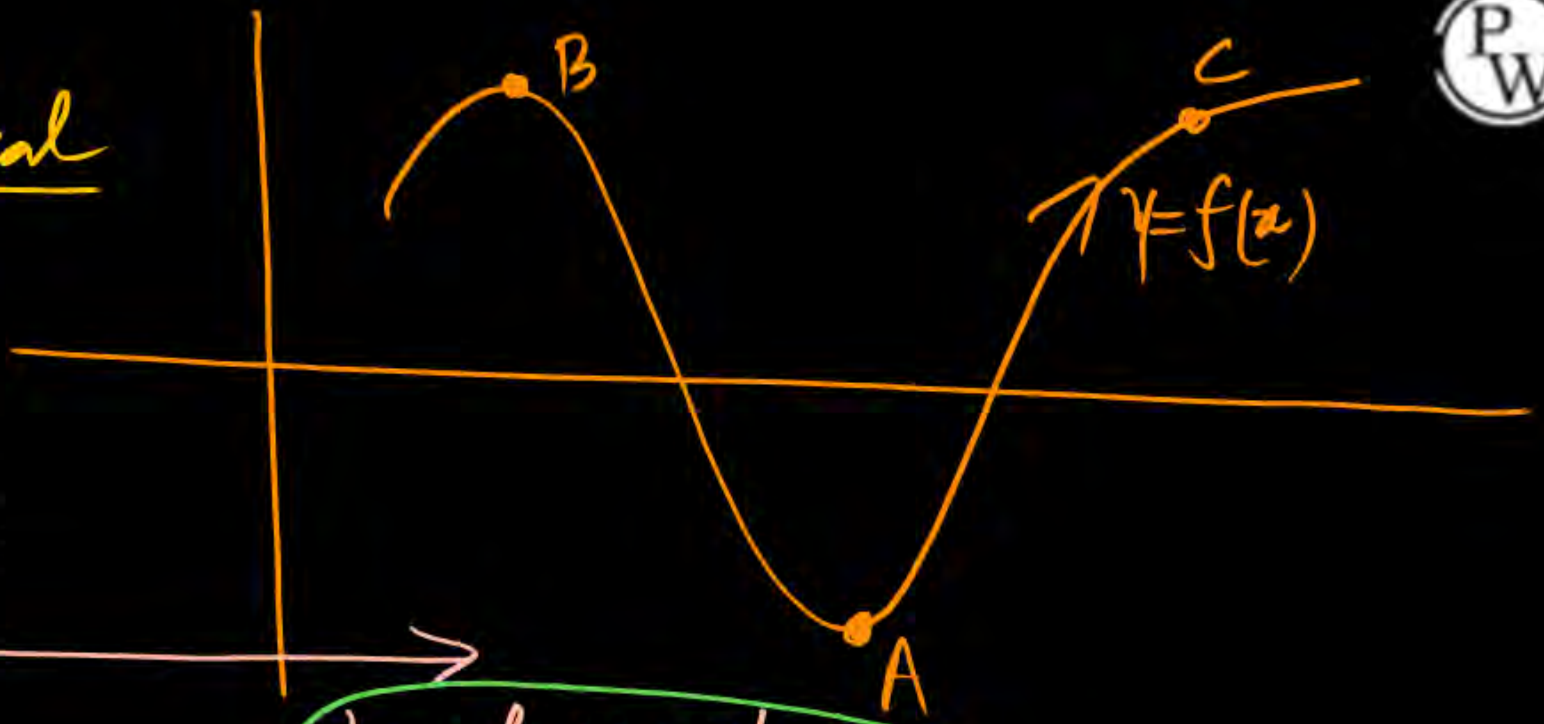
How to Calculate Local max / Local minima

(Interval is Not given)

Interval – open – Interval

✓ ( )

[ ]



Local max/min
Relative max/min

Procedure

1) $y = f(x)$

$\frac{dy}{dx} = 0$ to get the stationary Pt $x = x_0$

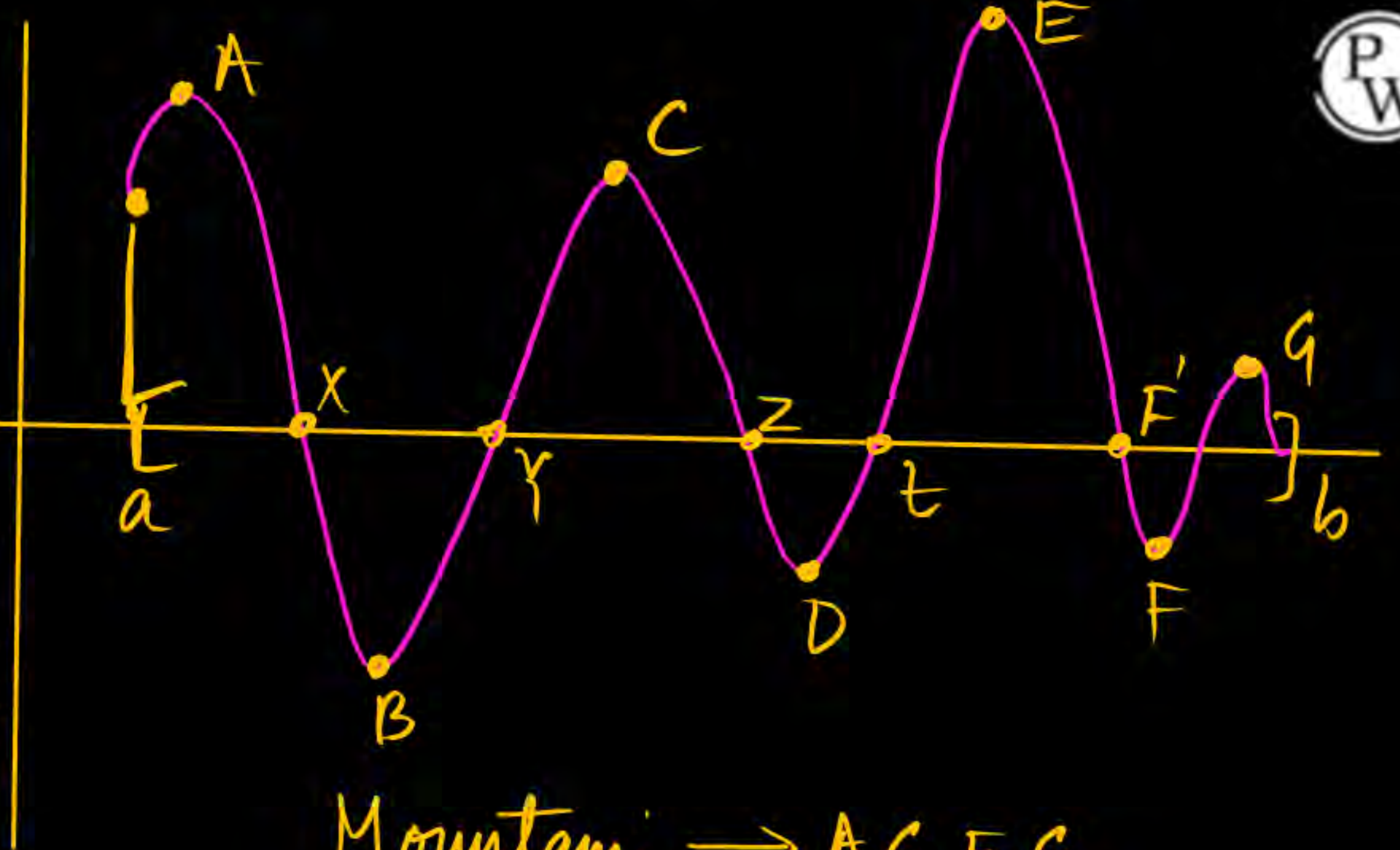
2) A) $\frac{d^2y}{dx^2}\left[f''(x_0)\right] > 0$ (minima) min value $y = f(x_0)$

B) $\frac{d^2y}{dx^2}\left[f''(x_0)\right] < 0$ (maxima) max value $y = f(x_0)$

C) $\frac{d^2y}{dx^2} = 0$ (Pts of Inflection) (Neither max Nor min)

Absolute max / absoulte min
OR
Global max / global min

[a, b] closed Interval



Procedure: (A) $y = f(x)$ is given $[a, b]$

$\frac{dy}{dx}$ or $f'(x) = 0$

to get the stationary Points

Mountain $\rightarrow$ A, C, E, G

Valley $\rightarrow$ B, D, F

global max $\Rightarrow$ Max value of $[f(A), f(C), f(E), f(G) - -]$

global min $\Rightarrow$ min value of $[f(B), f(D), f(F)]$

$[a, b]$

local

( ) —

( ] ✓

✓ ✓

Q. Questions

The function $y = x^2 + \frac{250}{x}$ at $x = 5$ attains

Local min / max

$\frac{1}{x^2} = x^{-2}$

$= -2x^{-3}$

$= \frac{-2}{x^3}$

$y = x^2 + \frac{250}{x}$

$\frac{dy}{dx} = 0$

$2x - \frac{250}{x^2} = 0$

$\frac{2x^3 - 250}{x^2} = 0$

$2x^3 - 250 = 0$

$x^3 - 125 = 0$

$x = 5$ stationary Pt.

$f''(x) = 2 + 250 \cdot \frac{2}{x^3}$

$= 2 + \cancel{250}^{2} \times \frac{2}{\cancel{125}}$

$\boxed{f''(x) = 6}$

> 0

$f''(x) > 0$

minima at $x = 5$

(a) Maximum

(b) Minimum

(c) Neither

(d) 1

Q. Questions

Local max at $x = ?$

For the function $f(x) = x^2e^{-x}$, the maximum occurs when x is equal to

$f(x) = \overset{\mathrm{I}}{x^2}\,\overset{\mathrm{II}}{e^{-x}}$

Step (A) $f'(x) = 0$

$\frac{d}{dx}(\mathrm{I} \times \mathrm{II}) = \mathrm{I}\frac{d}{dx}\mathrm{II} + \mathrm{II}\cdot\frac{d}{dx}(\mathrm{I})$

(a) 2

(b) 1

(c) 0

(d) –1

$f'(x) = x^2\frac{d}{dx}(e^{-x}) + e^{-x}\frac{d}{dx}(x^2)$

$= x^2(-e^{-x}) + e^{-x}(2x)$

$\boxed{f'(x) = -x^2e^{-x} + e^{-x}2x}$

$e^{-x}(2x - x^2) = 0$

$2x - x^2 = 0$

$x^2 - 2x = 0$

$\Rightarrow x(x-2) = 0$

$\Rightarrow \left.\begin{array}{l} x = 0 \\ x = 2 \end{array}\right]$ stationary Pts

$$f'(x) = -x^2e^{-x} + e^{-x}2x$$

$$f''(x) = -[-x^2e^{-x} + e^{-x}2x] + 2[-xe^{-x} + e^{-x}\cdot 1]$$

$$= x^2e^{-x} - 2xe^{-x} - 2xe^{-x} + 2e^{-x}$$

$$f''(x) = x^2e^{-x} - 4xe^{-x} + 2e^{-x} \qquad x_0 = 2, 0$$

$$f''(0) = 0 - 0 + 2e^0 = 2$$

$\boxed{f''(0) > 0 \text{ min point}}$

$$f''(2) = 4e^{-2} - 4\times 2e^{-2} + 2e^{-2}$$

$$= 4e^{-2} - 8e^{-2} + 2e^{-2}$$

$$= -2e^{-2}$$

$f''(2) < 0$ max at $x = 2$

Max value at $x = 2$

$f(x) = x^2e^{-x} = (2)^2e^{-2}$

max value: local max $= 4e^{-2}$

Q. Questions

Closed – global

Absoulte max | global max

The maximum value of $f(x) = x^3 - 9x^2 + 24x + 5$ in the interval $\boxed{[1, 6]}$ is

Ghol max $= 41$

global min $= 21$

(a) 21

(b) 25

(c) 41

(d) 46

$f'(x) = 3x^2 - 18x + 24 = 0$

$3(x^2 - 6x + 8) = 0$

$x^2 - 6x + 8 = 0$

$x^2 - 4x - 2x + 8 = 0$

$x(x-4) - 2(x-4) = 0$

$\left.\begin{array}{l} \checkmark\ x = 2 \\ \checkmark\ x = 4 \end{array}\right]$ stationary Point

1 2 4 6

$f(1)$ $f(2)$ $f(4)$ $f(6)$

$+5$ } Accepted

-7 } Ignore It

$f(1) = 1^3 - 9(1)^2 + 24 \times 1 + 5 = 21$

$f(2) = 2^3 - 9(2)^2 + 24 \times 2 + 5 = 25$

$f(4) = 4^3 - 9(4)^2 + 24 \times 4 + 5 = 21$

$f(6) = 6^3 - 9 \times 6^2 + 24 \times 6 + 5 = 41$

max value $= 41$

Q. Questions

open — local

For $0 \le t < \infty$, the maximum value of the function $f(t) = e^{-t} - 2e^{-2t}$ occurs at

(a) $t = \log_e 4$

(b) $t = \log_e 2$

(c) $t = 0$

(d) $t = \log_e 8$

$f'(t) = -e^{-t} + 4e^{-2t} = 0$

$-e^{-t} + 4e^{-2t} = 0$

$\Rightarrow e^{-t}[-1 + 4e^{-t}] = 0$

$-1 + 4e^{-t} = 0$

$-1 = -4e^{-t}$

$\frac{1}{4} = e^{-t}$

$e^{t} = 4 \quad t = \ln 4$

$t = \ln 4$

stationary pt.

$f''(t) < 0$ max

$f''(\ln 4) < 0$ max

Q. Questions

The maximum value of the function

$f(x) = \ln(1 + x) - x$ (where $x > -1$) occurs at $x =$ ____.

Do yourself

Q. Questions

Do yourself

Global max

The maximum value of $\boxed{f(x) = 2x^3 - 9x^2 + 12x - 3}$ in the interval $0 \leq x \leq 3$ is ____.

Q. Questions

Let $f(x) = xe^{-x}$. The maximum value of the function in the interval $(0, \infty)$ is

(a) e^{-1}

(b) e

(c) $1 - e^{-1}$

(d) $1 + e^{-1}$

Max value

$= f(x) = xe^{-x}$

$f(1) = 1 \times e^{-1}$

$= \frac{1}{e}$

$f(x) = xe^{-x}$

$f'(x) = 0$

$-xe^{-x} + e^{-x} = 0$

$e^{-x}(x-1) = 0$

$x = 1$

$f''(1) < 0$ max

Q. Questions

Minimum of the real valued function $f(x) = (x - 1)^{2/3}$ occurs at x equal to

Do yourself

(a) $-\infty$

(b) 0

(c) 1

(d) ∞

Q. Questions

The maximum value attained by the function $f(x) = x(x-1)(x-2)$ in the interval $[1,2]$ is.

Do yourself

Q. Questions

The minimum value of the function $f(x)=\frac{1}{3}x\left(x^2-3\right)$ in the interval $-100 \le x \le 1000$ occurs at $x =$ ____.

✓ Do yourself

Q. Questions

Do yourself

Let $f(x) = 3x^3 - 7x^2 + 5x + 6$. The maximum value of $f(x)$ over the interval $[0,2]$ is ____________. (upto 1 decimal place)

Funda mental
Integration ① ② ③ 10 Tricks

✓ Taylor SERIES
✓ Max/min – word Problem

CS – Prob.
– single
– Linear Algebra

civil → (complex Analysis) X

Thank You!

GW Soldiers

ESE – full subject
GATE CE — complex X

EC/EE/ME – whole syllabus.
CE – gate – complex X
ESE — whole