GATE

ALL BRANCHES

ENGINEERING MATHEMATICS

Single Variable Calculus

Lecture No. 08

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01 Problems based on Definite Integrals

02 Property based problems

03 Area based problems

02 marks

Definite Integrals :

$$\int_a^b f(x)\,dx = \Big[F(x)\Big]_a^b = F(b) - F(a)$$

a = Lower Limit b = Upper Limit

Fundamental THEOREM of Calculus.

$\int_0^2 x^2\,dx = \left[\frac{x^3}{3}\right]_0^2 = \frac{(2)^3}{3} - \frac{(0)^3}{3} = \frac{2\times2\times2}{3} = \frac{8}{3}$

$$\int_0^1 \frac{dx}{\sqrt{1-x^2}} = \Big[\sin^{-1}x\Big]_0^1$$

$$= \sin^{-1}1 - \sin^{-1}0$$

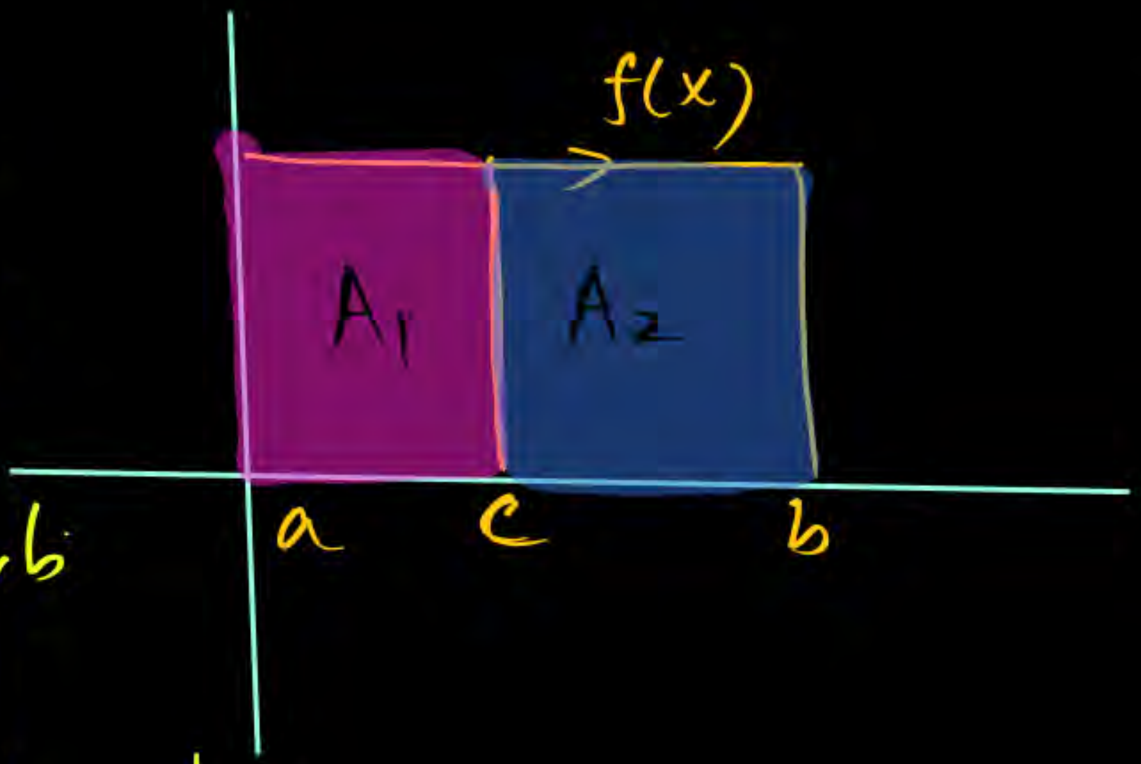
$$= \frac{\pi}{2} - 0 = \frac{\pi}{2} \text{ Ans}$$

$\int_0^{\frac{\pi}{2}} \sin x\,dx = \Big[-\cos x\Big]_0^{\pi/2}$

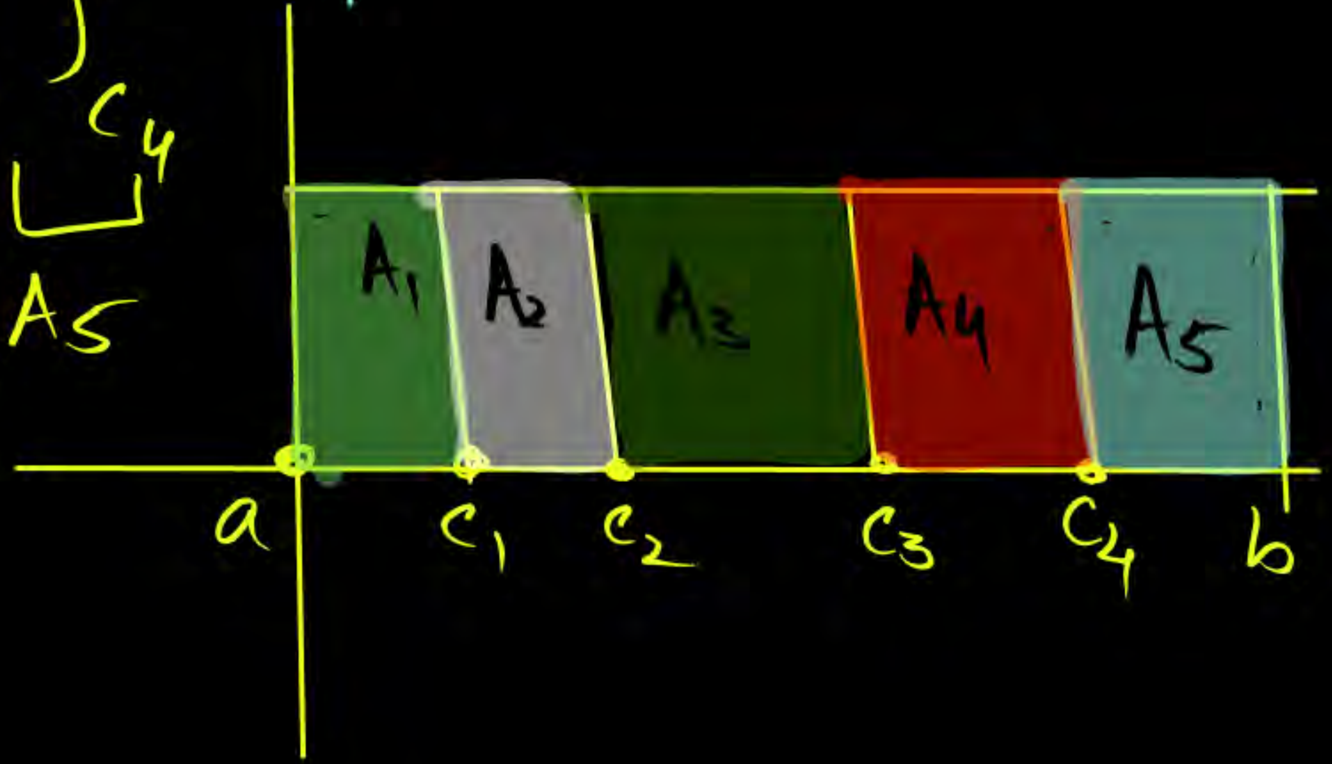
$$= -\cos\frac{\pi}{2} - (-\cos 0)$$

$$= 1$$

1) $$\int_a^b f(x)\,dx = \underbrace{\int_a^c f(x)\,dx}_{\text{region } R_1} + \underbrace{\int_c^b f(x)\,dx}_{\text{region } 2}$$



$$\int_a^b f(x)\,dx = \underbrace{\int_a^{c_1} f(x)\,dx}_{A_1} + \underbrace{\int_{c_1}^{c_2}}_{A_2} + \underbrace{\int_{c_2}^{c_3}}_{A_3} + \underbrace{\int_{c_3}^{c_4}}_{A_4} + \underbrace{\int_{c_4}^{b}}_{A_5}$$



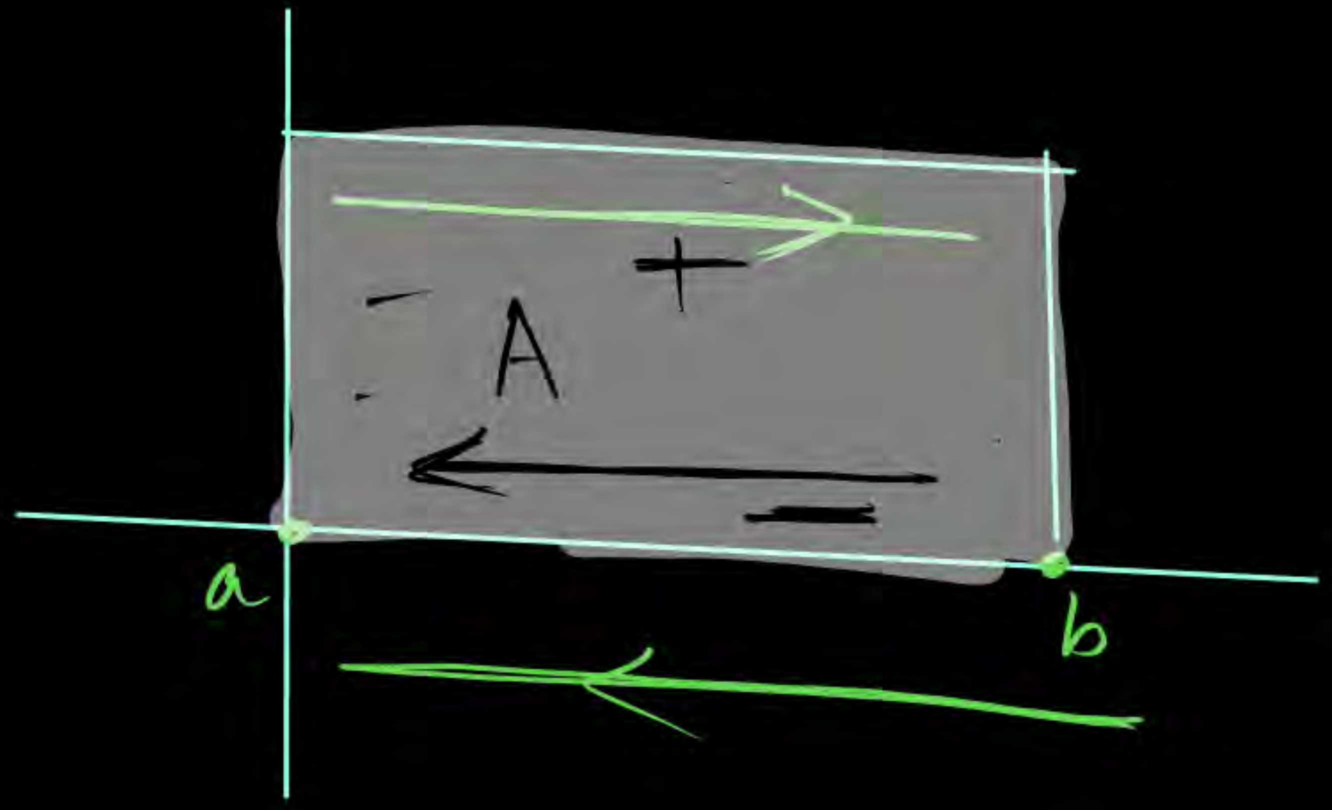
2) $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

$$\left.\begin{aligned}\int_0^1 x^2\,dx &= \frac{1}{3}\\ \int_1^0 x^2\,dx &= -\frac{1}{3}\end{aligned}\right]$$

$\xrightarrow{+}$

Orientation only Direction

$\xleftarrow[-]{}$



(3) $\int_a^b f(x)\,dx = \int_a^b f(\text{Lower}+\text{upper}-x)\,dx$ (Deafault Property)

$$\int_a^b f(x)\,dx \Rightarrow \int_a^b f(a+b-x)\,dx$$

Imp (4) $\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$ If $f(x)$ is even Function

Even Function

$f(-x) = f(x)$

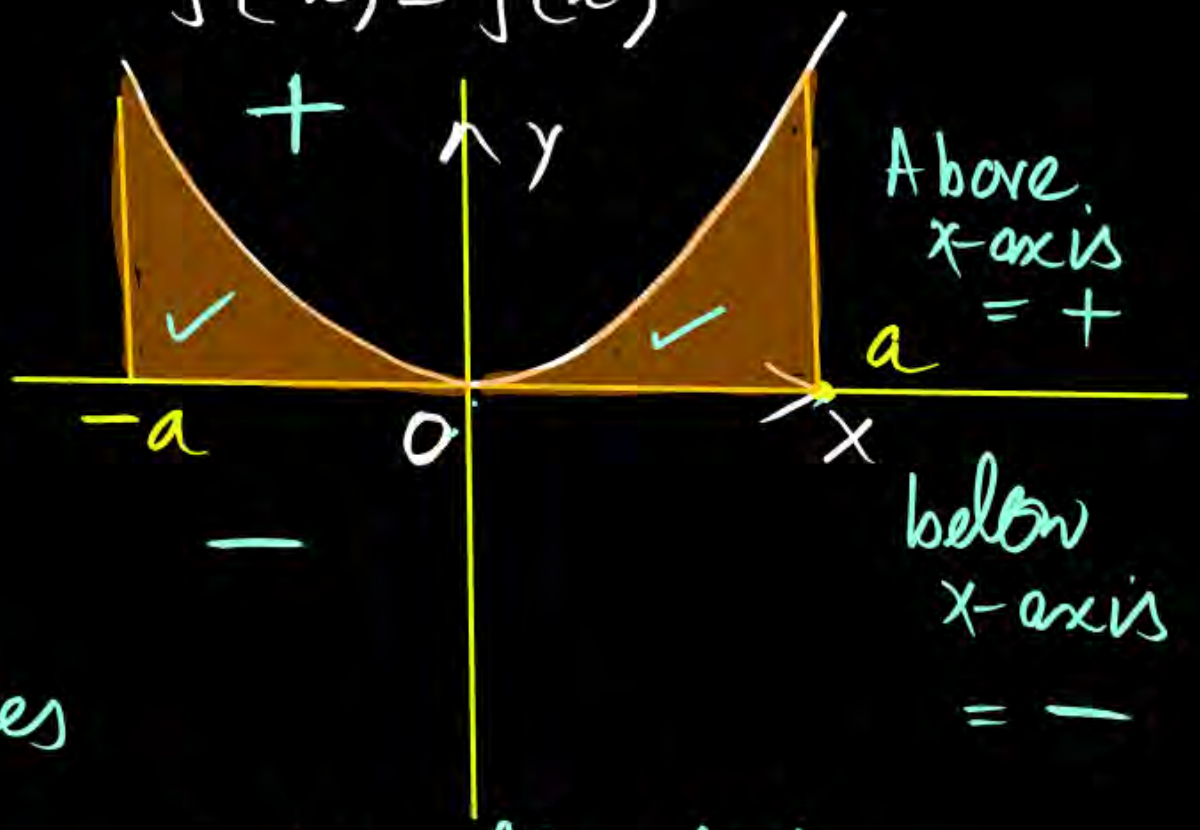
Even function $f(x) = x^2$

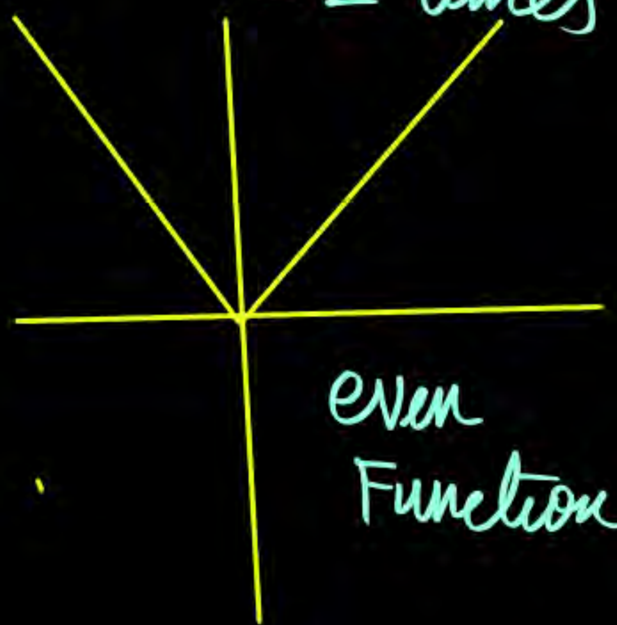
$f(-x) = (-x)^2 = x^2 = f(x)$

$f(x) = x^4$

$f(-x) = x^4$ = even function

$$\boxed{\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx}$$





= 2x right Half Area

$= 2\int_{0}^{a} f(x)\,dx$

odd Function

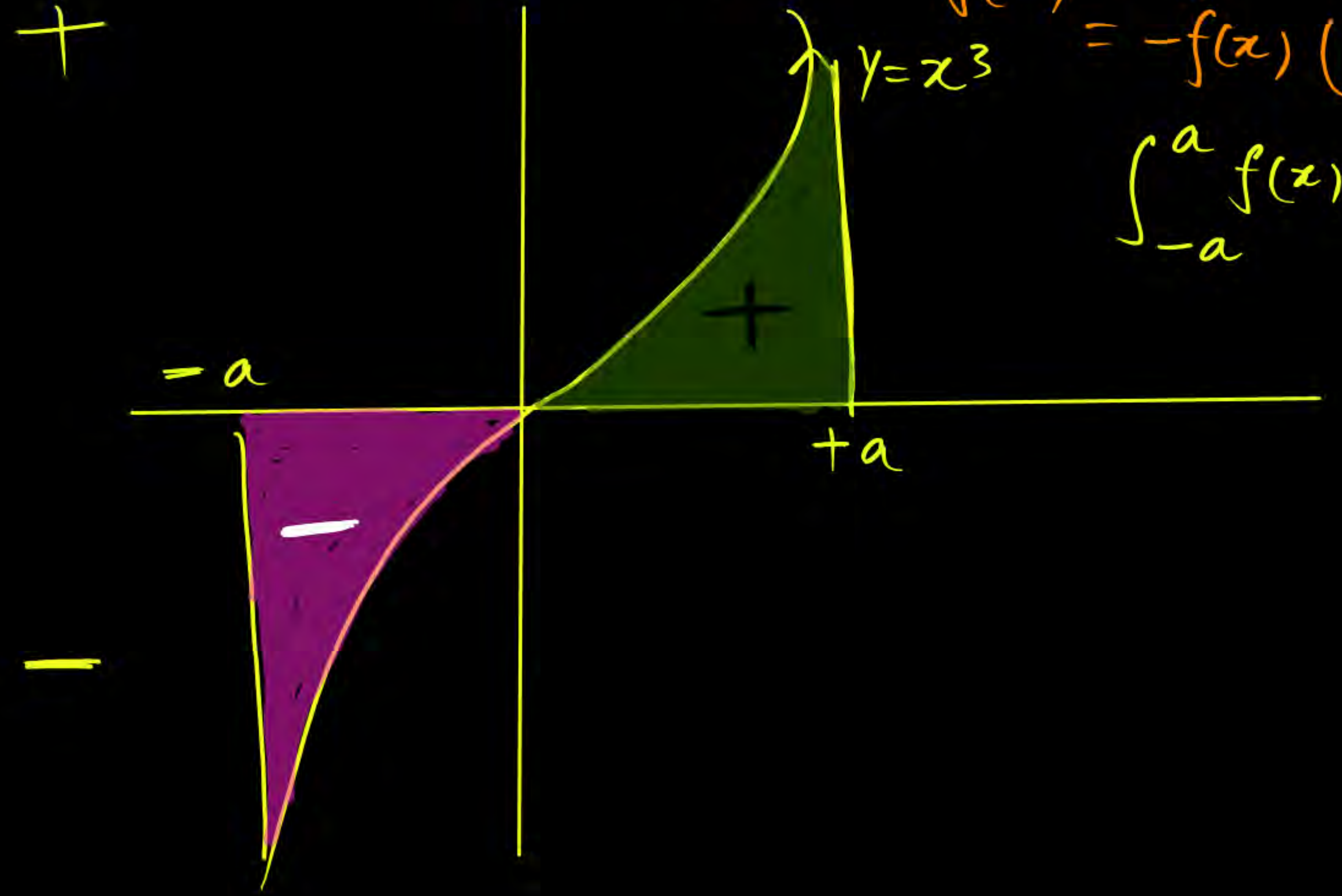
$f(-x) = -f(x)$

$f(x) = x^3$

$f(-x) = -x^3$

$= -f(x)$ (odd function)

$$\int_{-a}^{a} f(x)\,dx = 0 \text{ [odd function]}$$



Q. Questions

The value of the integral $I = \int_{0}^{\pi/2} \frac{\sqrt{\cot x}}{\sqrt{\cot x}+\sqrt{\tan x}}\,dx$ is

(a) $\frac{\pi}{4}$

(b) $\frac{\pi}{2}$

(c) π

(d) None of these

$$\int_0^{\pi/2} \frac{\sqrt{\cot x}}{\sqrt{\tan x}+\sqrt{\cot x}}\,dx$$

Using Default

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

$$I = \int_0^{\pi/2} \frac{\sqrt{\cot\left(\frac{\pi}{2}-x\right)}}{\sqrt{\cot\left(\frac{\pi}{2}-x\right)}+\sqrt{\tan\left(\frac{\pi}{2}-x\right)}}\,dx$$

$(a+b-x)$

$\left(0+\frac{\pi}{2}-x\right)$

$$I = \int_0^{\pi/2} \frac{\sqrt{\tan x}}{\sqrt{\tan x}+\sqrt{\cot x}}\,dx$$

$$2I = \int_0^{\pi/2} \frac{\cancel{\sqrt{\tan x}+\sqrt{\cot x}}}{\cancel{\sqrt{\tan x}+\sqrt{\cot x}}}$$

$$2I = \int_0^{\pi/2} dx$$

$$I = \frac{\pi}{4}$$

$$\boxed{I = \int_a^b \frac{f(x)}{f(a+b-x)+f(x)}\,dx} = \frac{(b-a)}{2} \quad \underline{\underline{\text{Ans}}}$$

$$I = \int_3^5 \frac{\sqrt{x}}{\sqrt{8-x}+\sqrt{x}}\,dx = \frac{5-3}{2} = 1$$

$$I = \int_2^5 \frac{\sqrt{x}}{\sqrt{7-x}+\sqrt{x}}\,dx = \frac{3}{2}$$

Q. Questions

Let $f(x) = \int_1^x \sqrt{2 - t^2}\,dt$ Then the real roots of the equation $x^2 - f'(x) = 0$ are

[2002 S]

(a) ± 1

(b) $\pm 1/\sqrt{2}$

(c) $\pm 1/2$

(d) 0 and 1

Q. Questions

The value of the integral $\int_0^1 \sqrt{\frac{1-x}{1+x}}\,dx$ is

Do yourself

(a) $\frac{\pi}{2}+1$

(b) $\frac{\pi}{2}-1$

(c) -1

(d) 1

Property Based Questions

The value of $\displaystyle\int_{\pi/4}^{3\pi/4} \frac{\phi}{1+\sin\phi}\,d\phi$ is ....

Q. Questions

$$\frac{(b-a)}{2} = \frac{3-2}{2} = \frac{1}{2}$$

The value of $\int_{2}^{3} \frac{\sqrt{x}}{\sqrt{5-x}+\sqrt{x}} dx$ is ....

Q. Questions

The value of $\int_{1}^{e^{37}} \frac{\pi \sin(\pi \ln x)}{x} dx = \ldots\ldots\ldots$

Q. Questions

Let $\frac{d}{dx}F(x) = \frac{e^{\sin x}}{x}, x > 0$. If $\int_1^4 \frac{2e^{\sin x^2}}{x} dx = F(k) - F(1)$

Then one of the possible values of k is ....

Q. Questions

The value of $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x} dx, \quad a > 0$ is

(a) π

(b) $a\pi$

(c) $\pi/2$

(d) 2π

Q. Questions

The integral $\int_{-1/2}^{1/2} \left([x] + \ln\left(\frac{1+x}{1-x}\right) \right) dx$ equal to

(a) $-1/2$

(b) 0

(c) 1

(d) $2 \ln(1/2)$

Q. Questions

$$\int_{-2}^{0} \{x^3 + 3x^2 + 3x + 3 + (x+1)\cos(x+1)\}dx$$

is equal to

$$I = \int_{-2}^{0} (x^3 + 3x^2 + 3x + 3 + (x+1)\cos(x+1)\,dx$$

$$\int_{-2}^{0} \left[\boxed{x^3+3x^2+3x+1} + 2 + (x+1)\cos(x+1)\right]dx$$

$$\Rightarrow \int_{-2}^{0} (x+1)^3 + 2 + (x+1)\cos(x+1)\,dx$$

$$\Rightarrow \int_{-1}^{1} \left[t^3 + 2 + t\cos t\right]dt$$

$x+1 = t$

$dx = dt$

$-2+1 = t$

$t = -1$

$0+1 = t$

$t = 1$

$$\cancel{\int_{-1}^{1} t^3\,dt}^{\,0} + \int_{-1}^{1} 2\,dt + \cancel{\int_{-1}^{1} t\cos t\,dt}^{\,0}$$

$$0 + \int_{-1}^{1} 2\,dt + 0$$

$$\Rightarrow \quad 2\int_{0}^{1} 2\,dt$$

$$= 2\times 2\int_{0}^{1} dt$$

$$= 4\left[t\right]_0^1$$

$$= 4[1-0] = 4$$

$f(t) = t^3 =$ odd

$f(t) = 2 =$ ✓

$f(t) = t\cos t =$ odd Function

$f(-t) = -t\cos(-t)$

$= \underline{-t\cos t}$ odd function

Q. Questions

The value of $\int_{\sqrt{\ln 2}}^{\sqrt{\ln 3}} \frac{x \sin x^2}{\sin x^2 + \sin(\ln 6 - x^2)} dx$ is

(a) $\frac{1}{4}\ln\frac{3}{2}$

(b) $\frac{1}{2}\ln\frac{3}{2}$

(c) $\ln\frac{3}{2}$

(d) $\frac{1}{6}\ln\frac{3}{2}$

$\boxed{\ln 2 + \ln 3 = \ln 5}$

$x \longrightarrow t$

$x^2 = t$

$2x\,dx = dt \quad x\,dx = \frac{dt}{2}$

$(\sqrt{\ln 2})^2 = t = \ln 2$

$(\sqrt{\ln 3})^2 = t = \ln 3$

$\frac{\ln 3 - \ln 2}{2} \times \frac{1}{2}$

$I \Rightarrow \frac{1}{2}\int_{\ln 2}^{\ln 3} \frac{\sin t}{\sin t + \sin(\ln 6 - t)} dt$

$I = \frac{\sin(\ln 2 + \ln 3 - t)}{\sin(\ln 2 + \ln 3 - t) + \sin t}$ $\quad \ln(2\times 3) - t$

$I = \int_{\ln 2}^{\ln 3} \frac{\sin(\ln 6 - t)}{\sin(\ln 6 - t) + \sin t}$

$I = \frac{1}{2}\left[\int_{\ln 2}^{\ln 3} \frac{\sin t + \sin(\ln 6 - t)}{\sin t + \sin(\ln 6 - t)}\right] dt$

$= \frac{\ln 3 - \ln 2}{2 \times 2} = \frac{1}{2\times 2}\ln\left(\frac{3}{2}\right)$

$\Rightarrow \frac{1}{2}\int_{\ln 2}^{\ln 3} dt$

$= \frac{1}{2\times 2}(\ln 3 - \ln 2) = \frac{\ln 3 - \ln 2}{2} = \frac{1}{4}\ln\frac{3}{2}$

Q. Questions

The value of the integral $\int_{-\pi/2}^{\pi/2}\left(x^2+\ln\frac{\pi+x}{\pi-x}\right)\cos x\,dx$ is

(a) 0

(b) $\frac{\pi^2}{2}-4$

(c) $\frac{\pi^2}{2}+4$

(d) $\frac{\pi^2}{2}$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x^2\cos x+\underbrace{\ln\left(\frac{\pi+x}{\pi-x}\right)\cos x}\,dx$$

$$f(x)=\ln\left(\frac{\pi+x}{\pi-x}\right)\cos x=\underline{\text{odd function}} = 0$$

$$f(-x)=\ln\left(\frac{\pi-x}{\pi+x}\right)\cos(-x)=\cos x\ln\left(\frac{\pi-x}{\pi+x}\right)=-\left[\cos x\ln\left(\frac{\pi+x}{\pi-x}\right)\right]$$

$$f(x) = x^2\cos x$$

$$f(-x) = (-x)^2\cos(-x) = \text{even Function}$$

$$= x^2\cos x$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x^2\cos x\,dx = 2\int_0^{\frac{\pi}{2}} x^2\cos x\,dx = \frac{\pi^2}{2} - 4$$

x^2	$+$	$\cos x$
$2x$	$-$	$+\sin x$
2	$+$	$-\cos x$
0		$-\sin x$

Why what
Where
Whon

Thank You!

GW Soldiers