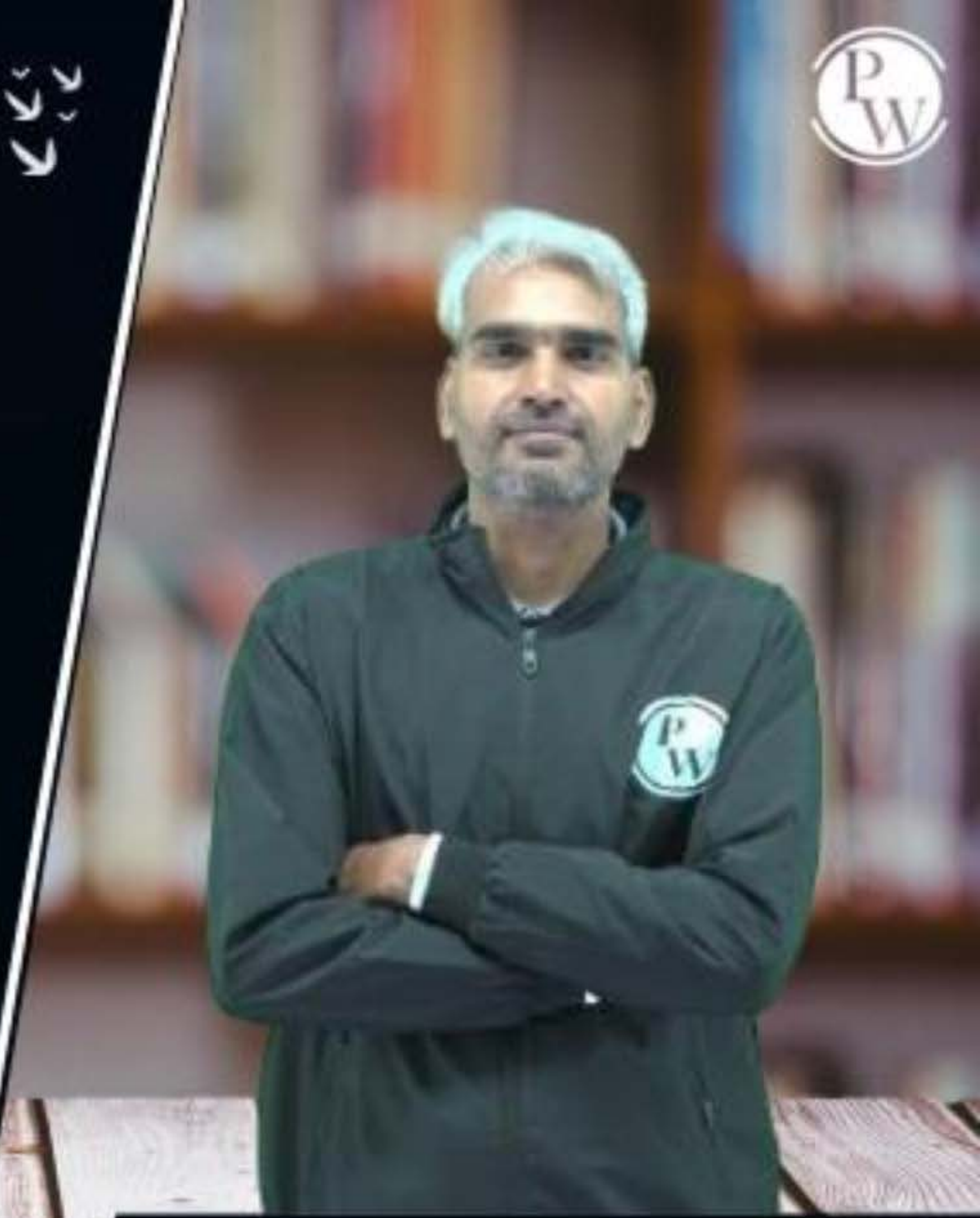
Computer Science & IT

Discrete Mathematics

Set Theory & Algebra

Lecture No. 17



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Recap of Previous Lecture

- **Topic** Function
- **Topic** Range of a function
- **Topic** Injective (one-one) function
- **Topic** Surjective (onto) function

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Topics to be Covered

- **Topic** Number of onto functions
- **Topic** Bijective function
- **Topic** Identity function
- **Topic** Constant function
- **Topic** Inverse of a function

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Topic : Number of one-one function

If $|A| = m$ & $|B| = n$ s.t. $(m \le n)$

then,

No. of one-one function from A to B $= {}^nP_m = \dfrac{n!}{(n-m)!}$

if $|A| = |B| = n$

then no. of one-one functions from A to B $= n!$

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Topic : Number of onto functions

Case 1 : If $|A| = n$ & $|B| = 2$ $(n \geq 2)$

Number of onto functions from A to B $= 2^n - 2$

Case 2 : If $|A| = |B| = n$

No. of onto functions = No. of one-one functions $= n!$

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Case 3 : $|A| = n$ & $|B| = (n-1)$,

then number of onto functions

from A to B = ${}^{n}C_2 * (n-1)!$

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H.W. Case-4 : General Case :

if $|A| = m$ & $|B| = n$ s.t. $(m \geq n)$,

then number of onto functions possible from A to B

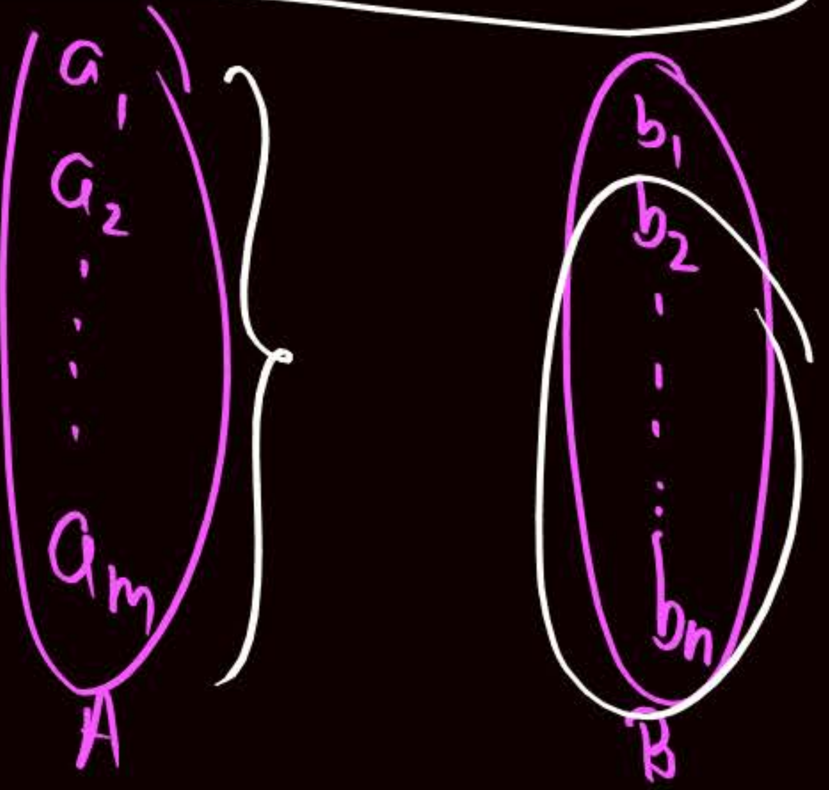
$$= n^m - {}^nC_1 * (n-1)^m + {}^nC_2 * (n-2)^m - {}^nC_3 * (n-3)^m + \cdots + (-1)^{n-1}\, {}^nC_{n-1}\, (n-(n-1))^m$$

$$\boxed{= \sum_{i=0}^{n-1} (-1)^i \cdot {}^nC_i \cdot (n-i)^m}$$

Principle of inclusion & Exclusion

- Number of Onto functions from A to B where $|A| = m$ & $|B| = n$
$(m \geq n)$

= Total no. of functions − no. of fn. in which exactly one element of co-domain is not used − Exactly two elements of co-domain are not used − − . . . − (Exactly $(n-1)$ elements of the co-domain are not used)



No. of onto functions from A to B

$$= n^m - {}^nC_1(n-1)^m + {}^nC_2 * (n-2)^m - {}^nC_3(n-3)^m + {}^nC_4(n-4)^m - \ldots$$

total no. of function

Exactly 4 left

Exactly 4 left

Exactly 4 element left

$$- \underbrace{4}_{{}^4C_1} + \underbrace{6}_{{}^4C_2} - \underbrace{4}_{{}^4C_3} = -2$$

A

1
2
3
4
5
6
7
8

m

$^{4}C_{2} = 6$

B

a
b
c
d
e
f

o	x	o	o
o	o	o	x
o	o	x	o
x	o	o	o

$^{4}C_{3} = 4$

$^{n}C_{1} \, (n-1)^{m}$

Gate 21

Q. There are 6 jobs with distinct difficulty levels, and 3 computers with distinct processing speeds. Each job is assigned to a computer such that:

H.W.

- The fastest computer gets the toughest job and the slowest computer gets the easiest job.
- Every computer gets at least one job. The number of ways in which this can be done is_____.

Slide

Topic : Bijective Function

A function 'f' from set A to set B is called bijective function if 'f' is one-one as well as onto.

f is one-one $\therefore |A| \leq |B|$
& f is onto $\therefore |A| \geq |B|$

Both can be true only if $|A| = |B|$

Note :- A bijection from set A to Set B is possible only if $|A| = |B|$

Slide

* A bijective function is said to have One-to-one Correspondance.

* If there exists a one-one Correspondance between set A & set B, then it means $|A| = |B|$.

* Let $|A| = |B| = n$, then

how many bijection (i.e. bijective functions) are possible from A to B = ?

Ans = $n!$

if $|A| = |B|$, then all one-one functions are onto.

i.e. all one-one function are bijection

Note:- For a function "f", inverse of function "f" exists if and only if "f" is a bijective function.

Topic : Identity Function

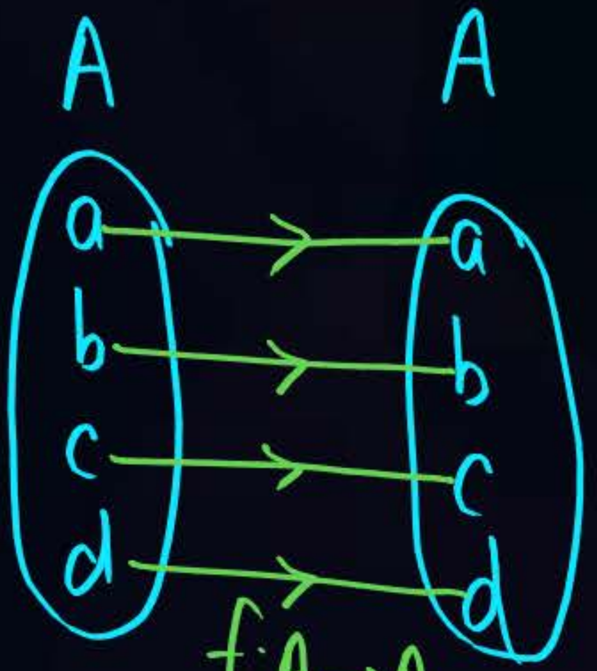
i.e. a function f on set A.

A function $f: A \to A$ is called an identity function

Only if, $f(x) = x, \ \forall x \in A$

- Identity function on set A is denoted by I_A.

eg.



$f: A \to A$ is identity function

Note: Every identity function is a bijection, and inverse of identity function is the identity function itself.

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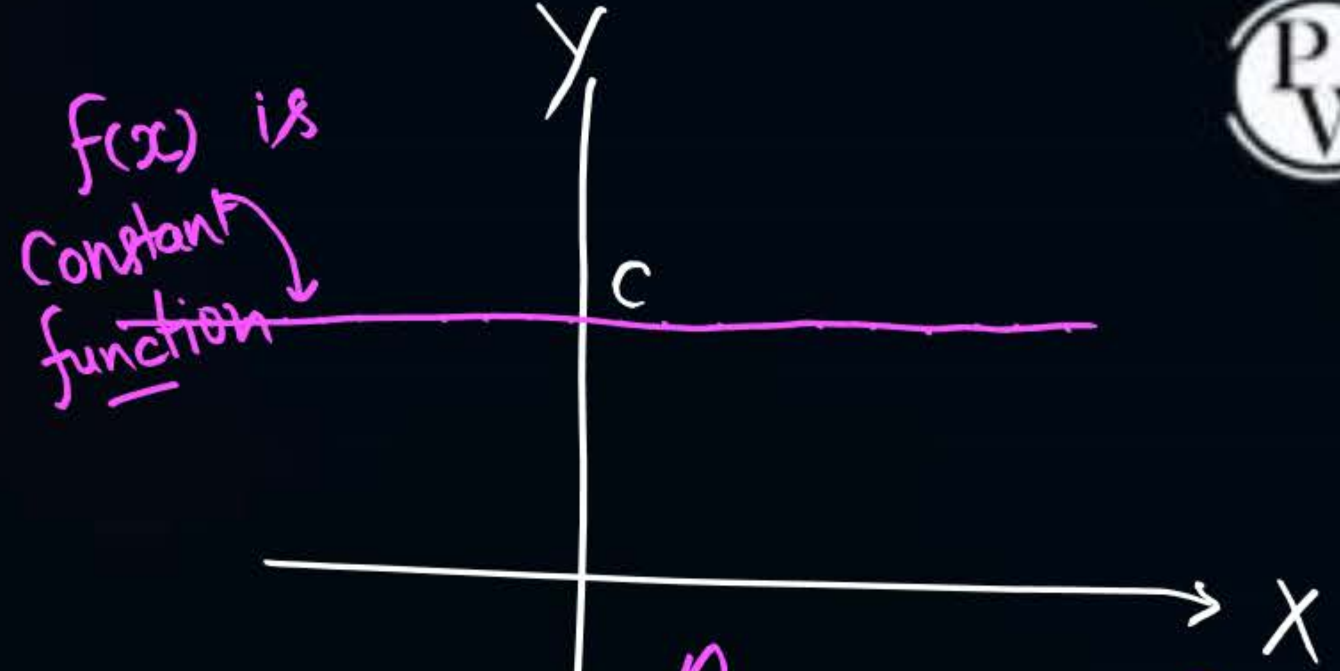
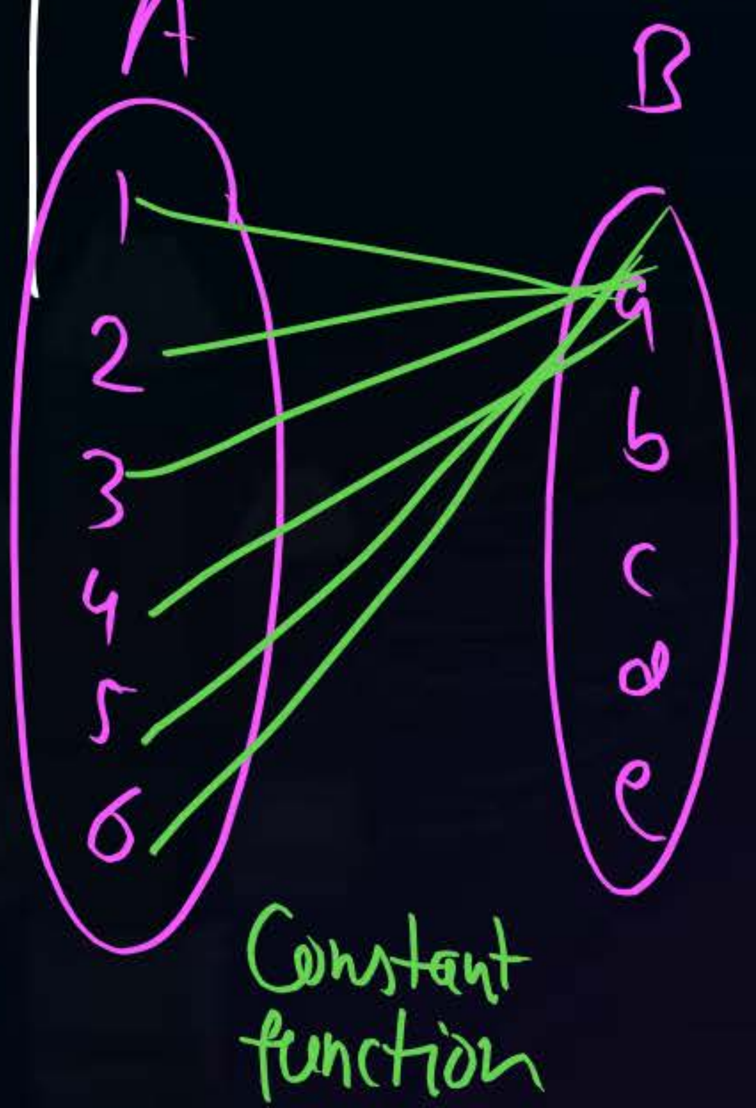
Topic : Constant Function

A function $f: A \rightarrow B$ is called a constant function only if every element of domain maps to the same element of co-domain.

i.e. $f(x) = c \quad \forall x \in \text{domain } (A)$

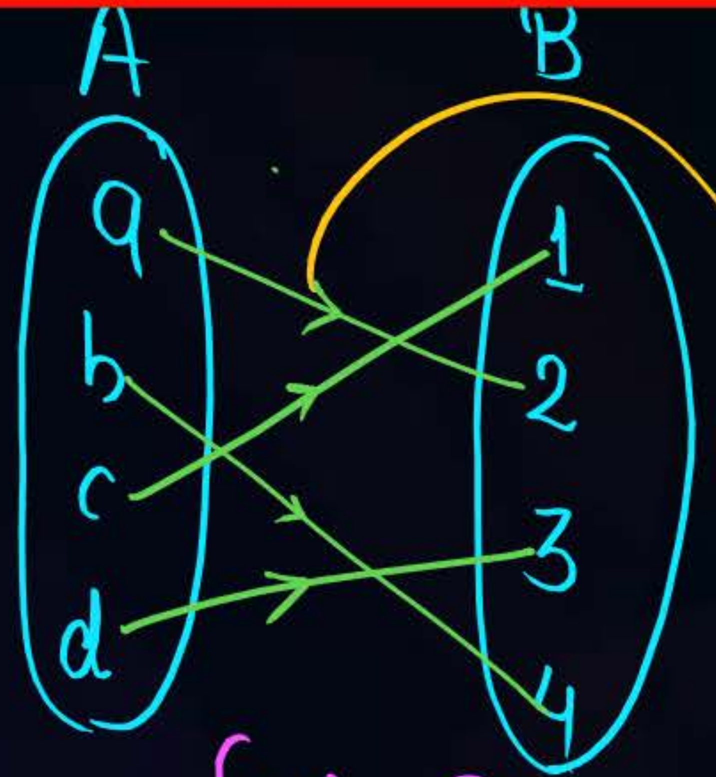
fixed $\{c \in B\}$





Topic : Inverse of a Function

- Inverse of function "f" exists if and only if function 'f' is a bijective function.



$f : A \to B$

if f is a bijective function from set A to B,
then f^{-1} will exist, and it will be a function from B to A

$f(a) = 2$

$\therefore f^{-1}(2) = a$

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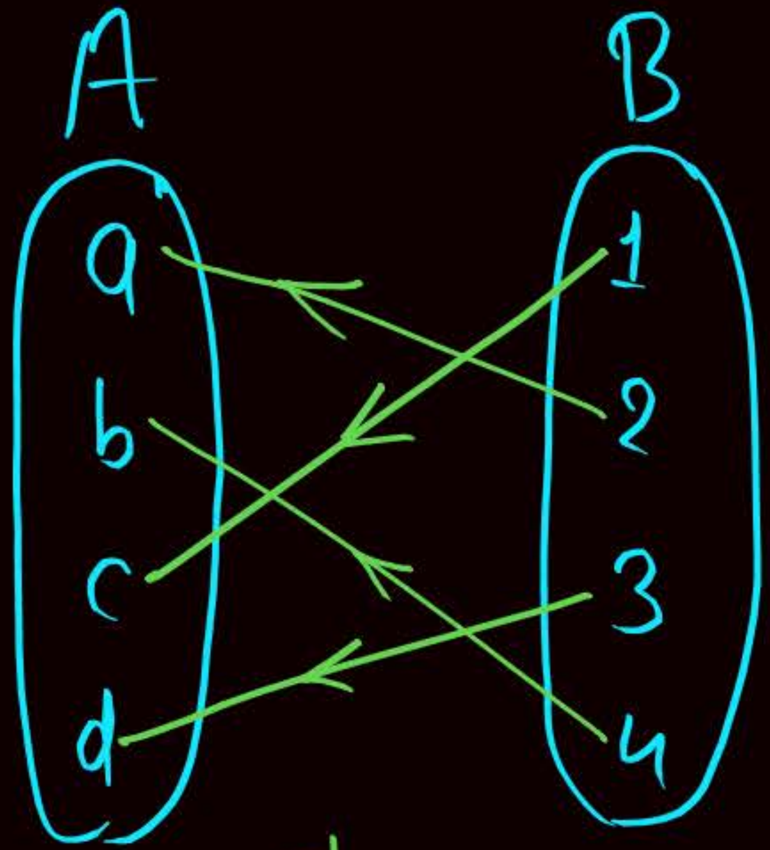

$f^{-1}: B \to A$

Inverse function :-

let $f: A \rightarrow B$ is a relation from A to B such that 'f' is a bijection, then inverse relation of relation 'f' i.e f^{-1} will be the inverse function of function f.

Topic : Inverse of a Function

Q: Let $f(x) = 2x+3$ is a bijective function, find the inverse function $f^{-1}(x)$.

$f(a) = 1$

$\therefore f^{-1}(1) = a$

let $y = f(x)$

$\therefore f^{-1}(y) = x$

$f^{-1}(y)$ will be a function of y, $\therefore x$ must be represented in the form of y

$y = f(x)$

$\Downarrow$

$y = 2x+3$

$\Rightarrow x = \frac{(y-3)}{2}$

$f^{-1}(y) = x$

$f^{-1}(y) = \frac{(y-3)}{2}$

$f^{-1}(x) = \frac{x-3}{2}$

Slide

Q:- Let R denote the set of real numbers, Let $f: R \times R \to R \times R$ be a bijective function defined by $f(x,y) = (x+y,\ x-y)$. The inverse of function f is.

(a) $f^{-1}(x,y) = \left(\frac{1}{x+y}, \frac{1}{x-y}\right)$

(b) $f^{-1}(x,y) = (x-y,\ x+y)$

(c) $f^{-1}(x,y) = \left(\frac{x+y}{2}, \frac{x-y}{2}\right)$

(d) $f^{-1}(x,y) = (2(x-y),\ 2(x+y))$

Let $f(x,y) = (a,b)$

$f^{-1}(a,b) = \underbrace{(x,y)}$

output must be in terms of a & b

$f(x,y) = (x+y,\ x-y) = (a,b)$

$\left.\begin{array}{l} a = x+y \\ b = x-y \end{array}\right\} \Rightarrow x = \frac{a+b}{2}$

$y = \frac{a-b}{2}$

$\therefore f^{-1}(a,b) = \left(\frac{a+b}{2}, \frac{a-b}{2}\right)$

$\therefore f^{-1}(x,y) = \left(\frac{x+y}{2}, \frac{x-y}{2}\right)$

$$
\begin{aligned}
x+y &= a \quad \text{—} \; ① \\
+ \quad x-y &= b \quad \text{—} \; ② \\
\hline
2x-0 &= a+b
\end{aligned}
$$

$$x = \frac{a+b}{2}$$

$$
\begin{aligned}
x+y &= a \quad \text{—} \; ① \\
- \quad x-y &= b \quad \text{—} \; ② \\
\hline
0+2y &= a-b
\end{aligned}
$$

$$y = \frac{a-b}{2}$$

2 mins Summary

Topic Number of onto functions

Topic Bijective function

Topic Identity function

Topic Constant function

Topic Inverse of a function

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THANK - YOU

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