BUCKLING ANALYSIS OF FUNCTIONALLY GRADED PLATE

BY

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This is to certify that the project entitled "**BUCKLING ANALYSIS OF FUNCTIONALLY GRADED PLATE"** been carried out by **AYUSH YADAV** (Admission Number 18MT0071) at the **Department of Mechanical Engineering Indian Institute of Technology (Indian School of Mines), Dhanbad** under the supervision in the partial fulfillment of the requirement of the award of the Degree of **Master of Technology** in Mechanical Engineering (specialization: Machine Design) during the academic session 2019-2020.

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DECLARATION

I hereby declare that the dissertation entitled "**Buckling analysis of Functionally Graded Plate"** submitted to INDIAN INSTITUTE OF TECHNOLOGY (INDIAN SCHOOL OF MINES), DHANBAD is a record of original work done by me under the guidance of **Dr. Md. Sikandar Azam**, (Assistant professor) Department of Mechanical Engineering and this work has not been presented anywhere fully or partially for the award of any degree.

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Abstract

In this work, the analysis of buckling of plates is conducted to learn the behavior of rectangular plates under biaxial thermo mechanical loading and subjected to different sets of boundary conditions within the framework of classical plate theory. Variational method of weighted residual is used to obtain the generalized form of differential equation. Trial functions denoting the displacement components are expressed in simple algebraic polynomial forms which can handle any sets of boundary conditions. The presented mathematical model omits the limitations regarding edge conditions and is accurate, more accessible, and computationally faster. Rayleigh–Ritz method is used to obtain the generalized solution of the differential equation. A parametric study has been thoroughly conducted focusing on the effects of aspect ratio, nano effect, power law exponent, elastic foundation and loading parameter on nondimensionalised buckling load of plate. It has been observed that the aspect ratio, elastic foundation and power law exponent have great influence on buckling load. New results for nondimensionalised buckling load are incorporated after performing a test of convergence. Comparison with the results from the existing literature are provided for validation.

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Chapter 1

Introduction

Plate is a two dimensional element, it means width is comparable to length but the thickness is not comparable to its width and length. Plate is one of the important structural element it has many engineering application such as air bridge, buildings, aircraft wings, big machines base and offshore structures. Most of the structural plates are stronger in tension as compared to compression. Generally compressed plate failure promote the tragic structural failure. So to avoid the failure of whole structure it is very important.

Functionally graded materials are a type of composite that made from two different materials of desires properties, which are impossible to find in a single material. In FGM, the variation of the material properties take place from one end of the material to another end. Now a days FGM used in many engineering field like mechanical engineering, civil, aerospace, nuclear and automotive. With the increasing demand of FGM material in various field, many researches have been done to analyse the behaviour of FGM under different conditions like environment, loading etc. Most of the research models are based on the displacement theories and to find the equilibrium equation they used principal of virtual work.

Based on some research model, the surrounding elastic medium such as Winkler-type and Pasternak-type elastic foundation has a great influence on the analysis of FGM plate. The FGM plate is treated to be rested on the elastic foundation of Winkler and Pasternak, and these parameter precisely considering both the spring and shear effect of elastic foundation. The results found by the researcher in their study using this Winkler and Pasternak foundation model are desired and accurate. Here, the Winkler and Pasternak coefficient represents the linear springs and the shear effect of the elastic foundation respectively .

1.1. Objective and Scope of Study

The buckling of rectangular FGM plate have been studied by several researches and various results are available based on some boundary conditions. However, a new FGM plate buckling problem where the FGM plate subjected to mechanical loading (biaxial compression), *thermal loading* with uniform and linear rise, FGM plate with size effect (nanoplate) and plate under thermal environment remains to be studied.

The aim of this work is to determine the nondimensional buckling load under thermos-mechanical loading with or without size effect. And this study also consists the effect of thermal environment on the buckling load. The plate have simply supported, clamp, free and mixed boundary condition. Classical plate theory and principal of virtual work is used to calculate the equilibrium equation. Further the study consider the effects of various plate parameters aspect ratios, foundation, size effect, temperature, power law exponent to vary the mechanical properties of FGM and different boundary condition .

Chapter 2

Literature Review

The following literature review will help to know the background information for this research. The literature review target on the thin isotropic FGM plate. In this part the focus will be on the investigation of FGM rectangular plates under different boundary conditions. Navier (1822) first derive the basic equilibrium equation of rectangular plate under lateral load. Saint-Venant (1883) include the axial edge forces and shearing forces and modified that equation. Now this equation become the basis for most of the work on plate stability of plates with various loads and different boundary condition.

Bryan (1891) first consider the plate under biaxial compressive loading with simply supported edge condition (SSSS) by written the deflection as a double Fourier series. Wang(1953) approached that problem by Finite-difference method. Timoshenko and Gere (1961) solved the plate under bi-axial loading with all clamped edge condition (CCCC) by the energy method. Bulson (1970) cited many research works on buckling of plate subjected to biaxial compression. One of them is proper analysis for ESFS plates by using the exact solution of the differential equation of equilibrium. And also added an extra term in equilibrium equation for transverse force. Along one side ranged between simply supported and clamped edge boundary condition it is found the effect of a restraint. In another example it is used two buckling forms, i.e. symmetric and anti-symmetric forms for the calculation of FSFS edge condition, with the help of this research work it is found that the load associated with symmetric buckling forms were much lower than the anti-symmetric form.

FGMs are appropriate for various engineering field and gained intense interest by several researches (Javaheri and Eslami, 2002; Chakraborty et al, 2003; Zenkour and Sobhy, 2010; Zhang, 2013; Pradhan and Chakraverty, 2013; Esfahani et al 2014; Wattanasakulpong and Chaikittiratana, 2015) .

Xiang et al. (2003) solved the problem of rectangular plate buckling with internal hinge under biaxial loads with the help of Ritz method. It is used to calculate the buckling factors for rectangular plates of various hinge location, boundary condition and aspect ratios. Hosseini-Hashemi et al. (2008) has worked on the buckling of isotropic plate. In this study they presented the non-dimensional critical buckling loads and mode shape for combination of four different boundary in which two opposite edges are simply supported under biaxial compression. V. Piscopo (2010) investigated the shimpi for thick plate and also recognized FEM as one of the most efficient method for thin plate buckling analysis under different boundaries conditions. Chee-Kiong Chin et al. (1993) conferred a finite element method using thin plate element. This method was capable to calculate the nondimensional buckling load for any arbitrarily shape thin plate member under general loading and boundary condition. MwisamMohammadi, et al. (2010) presented the explicit buckling analysis for thin FGM plate, and the equilibrium equations are obtained by using classical plate theory (CPT) and principal of minimum total potential energy. The resulting differentials are solved for different loading conditions and finally get the non dimensional buckling load .

Na and Kim (2004) examined three-dimensional (3D) thermal buckling analysis for FGM. They investigate the variation of material properties and temperature field in thickness direction using 18- node solid element by FEM. Shariat and Eslami (2005, 2006) performed thermal buckling analysis on geometrical imperfection rectangular FGM plate. They used classical plate theory to drive stability, compatibility and equilibrium equations. Naderi and

Saidi (2011) analysed the buckling of FGM annular sector plate resting on an elastic foundation and provide an analytical solution for that problem. They used Kirchoff 's plate theory and energy mathod to drive equilibrium and stability equation. Shariat and Eslami (2007) performed the buckling analysis of thick rectangular FGM plates under thermo- mechanical load using third-order shear deformation plate theory (HSDT). Zhao et al. (2009) used first- order shear deformation theory (FSDT) with the element- free *kp*-Ritz method to performed the buckling analysis under mechanical and thermo-mechanical loading. Lanhe (2004) used FSDT kinematics to drive the equilibrium and stability equations of a moderately thick FGM plate, which material properties varies in thickness direction subjected under thermal loading. Talha and Singh (2011) work on thermomechanical induced vibration characteristics of FGM plates based on modified higher-order shear deformation kinematics. They conclude the effect of volume fraction and temperature field on vibration characteristics of the FGM plates .

Nanoscale is a field of intense interest of several researches in the field of nanomechanics. Therefore, it is very important to consider the influence of small size in the mechanical analysis of nanostructures. The classical continuum elasticity theory is not capable of describing the size influence due to the lack of a material length scale. So, Eringen (Eringen, 1972 and 1983) proposed the nonlocal elasticity theory which is sizedependent continuum theory. According to this theory stress at a reference point is a function of strain of all points of the body. Hence this nonlocal theory (sizedependent) applied to analyse the mechanical response of nanoplates(Ressy , 2007; Prsdhan and Phadikar, 2009; Aydogdu, 2009; Murmu and Adhikari, 2010; Lim et al. 2010; Ke et al. 2014).

Reddy (2000) used third-order shear deformation plate theory and developed a FE model that accounts for the thermomechanical coupling for FGM plate. But, that analysis did not consists the change of material properties due to temperature distribution. Yang and Shen

(2002) presented the vibration characteristics and transient response analysis of FGM plate made of temperature-dependent material in thermal environment. According to the above-mentioned studies, one can notice the influence of various temperature environments on buckling behaviour of functionally graded nanoplate embedded in elastic foundation is not yet conducted .

Chapter 3

Mathematical modelling

This chapter consists of the derivation of the governing equation for rectangular FGM plate subjected under thermo- mechanical loading and Plate under thermal environment. We consider Classical plate theory and Hamilton principal to drive the equilibrium equation.

3.1. Thin FGM plate subjected under biaxial compression

Consider a thin rectangular FGM plate of dimension, length a, width b, and height h is subjected under biaxial compressive load N_x and N_y as shown in figure 2.1. Taking reference plane as mid plane and adopting Cartesian coordinate system. The biaxial loading N_x and N_y are parallel to x and y axis respectively.

For thin plate all theories gives the same results so we are adopting classical plate theory here because it is a simplest plate theory.

Assumptions of classical plate theory are:

a. Thickness of the plate is small as compared to the thickness, it is a case of plain stress.
b. No stretching in z direction, it means plain strain condition.
c. Mid plane remain neutral i.e. no stretching in mid plane.
d. The plane section rotate during bending to remain normal to the neutral surface, and do not distort, so that stresses and strains are proportional to their distance from the neutral surface.
e. Effect of shearing load is neglected and the load is entirely resisted by the banding moment induced in plate element.

Derivation starts on the behalf of first two assumption, that no stretching in z direction. It means normal stress and strain in z direction are zero. Material properties of FG plate are assumed to vary along the thickness direction according to power-law form as:

$$p(z) = (p_c - p_m)\left(\frac{z}{h} + \frac{1}{2}\right)^k + p_m \qquad 3.1$$

Where p_m and p_c denote the value of the material properties of the metal and ceramic ingredient of the FG plate, respectively. Power law exponent (k) is a nonnegative variable parameter. At bottom surface $(z = -h/2)$ of FG plate is pure metal, although the top surface $(z = h/2)$ is pure ceramic and one can get the desirable properties by different value of k. In this case only young's modulus (E) considered as the variable property where as the value of poisson's ratio will remain constant with respect to thickness

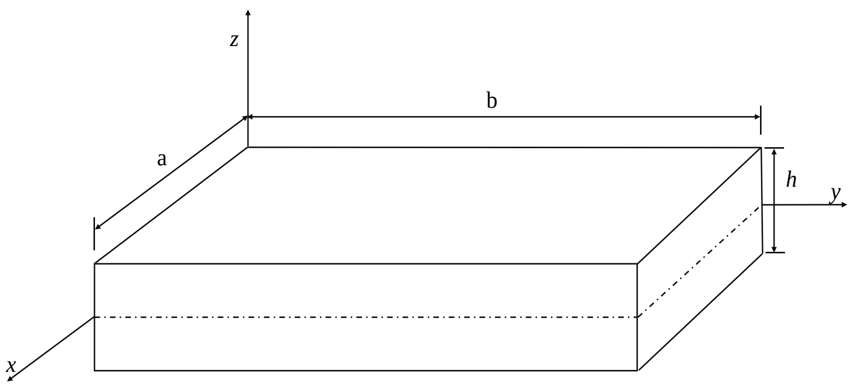


Figure 3.1. The schematic of the isotropic plate

According to the assumption, no stretching along the thickness,

$$\varepsilon_{zz} = 0 \qquad 3.2$$

$$\varepsilon_{zz} = \frac{\partial W}{\partial Z} = 0 \qquad 3.3$$

On integration,

W=f(x,y)

At the middle plate, z=0, and $W = W_0$ 3.4

Now assumption on shear strain of z plane considering equal to zero.

$$\gamma_{zx} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = 0$$

$$\frac{\partial u}{\partial z} = -\frac{\partial w}{\partial x} \tag{3.5}$$

On integration,

$$u = -z\frac{\partial w}{\partial x} + g(x, y) \tag{3.6}$$

And at mid plane z=0, so $u = u_0$

$$g(x, y) = u_o \tag{3.7}$$

$$u = u_0 - z\frac{\partial w}{\partial x} \tag{3.8}$$

Similarly,

$$v = v_0 - z\frac{\partial w}{\partial x} \tag{3.9}$$

The displacement field can be represented as

$$u = \begin{Bmatrix} u_x(x,y,t) \\ u_y(x,y,t) \\ u_z(x,y,z,t) \end{Bmatrix} = \begin{Bmatrix} u_0 - z\dfrac{\partial w(x,y)}{\partial x} \\ v_0 - z\dfrac{\partial w(x,y)}{\partial y} \\ w(x,y,t) \end{Bmatrix} \quad 3.10$$

Where u_x u_y and u_z are displacement along coordinates x, y and z, and w shows the deflection of an arbitrary point in the z-axis on the middle plane.

According to the nonlocal theory of elasticity, stress at a point in a body depends on strains at all other points of that body. According to this assumption, the nonlocal behaviour of a Hookean solid is introduced by the following transformed differential equation.

$$\sigma_{ij}^{(nl)} - \mu\nabla^2\sigma_{ij}^{(nl)} = \sigma_{ij}^{(l)} \quad 3.11$$

$$\alpha = \left(\frac{e_0 l_{\text{int}}}{l_{ext}}\right)^2 \quad 3.12$$

$$\mu = (l_0 l_{\text{int}})^2 \quad 3.13$$

Whereas $\sigma_{ij}^{(nl)}$ and $\sigma_{ij}^{(l)}$ represent the nonlocal and local stress tensor respectively. μ represent the nonlocal parameter.

According to Kirchhoff's assumption, the displacement associated with linear non zero strain is presented as

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u}{\partial x} \\ \dfrac{\partial v}{\partial y} \\ \dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x} \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u_0}{\partial x} - z\dfrac{\partial^2 w}{\partial x^2} \\ \dfrac{\partial v_0}{\partial y} - z\dfrac{\partial^2 w}{\partial y^2} \\ \left(\dfrac{\partial v_0}{\partial y} + \dfrac{\partial u_0}{\partial x} - 2z\dfrac{\partial^2 w}{\partial x \partial y}\right) \end{Bmatrix} \quad 3.14$$

where, ε_{xx} and ε_{yy} are normal strains and γ_{xy} is a shear strain. Following the generalized Hook's law, the stress-strain relationship is formulated as:

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{Bmatrix} = \begin{pmatrix} C_{11} & C_{12} & 0 \\ C_{21} & C_{22} & 0 \\ 0 & 0 & C_{66} \end{pmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} \qquad 3.15$$

where, σ_{xx} and σ_{yy} are normal stresses and τ_{xy} is shear stress in x – y plane. Further, $C_{11}, C_{12}, C_{22}, C_{66}$ are

$$C_{11} = \frac{E(Z)}{(1-\nu^2)} \qquad C_{12} = \frac{\nu E(Z)}{(1-\nu^2)} \qquad C_{66} = \frac{E(Z)}{2(1+\nu)} \qquad 3.16$$

Stress and moments are

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} dz$$

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \int_{-h/2}^{h/2} z \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix} dz \qquad 3.17$$

Using the principle of virtual work, the equation of motion of rectangular plate is derived and written as

$$\frac{\partial N_{xx}}{\partial x} + \frac{\partial N_{xy}}{\partial x} = 0$$

$$\frac{\partial N_{xx}}{\partial x} + \frac{\partial N_{xy}}{\partial x} = 0$$

$$\frac{\partial^2 M_x}{\partial^2 x^2} + \frac{\partial^2 M_y}{\partial^2 y^2} + \frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial}{\partial x}\left(N_x \frac{\partial w}{\partial x} \right) + \frac{\partial}{\partial y}\left(N_y \frac{\partial w}{\partial y} \right)$$
$$+ \frac{\partial}{\partial x}\left(N_{xy} \frac{\partial w}{\partial y} \right) + \frac{\partial}{\partial y}\left(N_{xy} \frac{\partial w}{\partial x} \right) + q - k_w + k_p \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) = 0 \qquad 3.18$$

Using equation (3.15) – (3.17), the bending moments resultants of plate in terms of flexural rigidities (bending stiffness) and displacement field are expressed as:

$$M_{ij}^{(nl)} - \mu\nabla^2 M_{ij}^{(nl)} = M_{ij}^{(l)}$$

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{21} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix} \begin{Bmatrix} \dfrac{\partial^2 w}{\partial x^2} \\ \dfrac{\partial^2 w}{\partial y^2} \\ \dfrac{\partial^2 w}{\partial xy} \end{Bmatrix} \qquad 3.19$$

The flexural rigidities is defined in terms of coefficient of stiffness as

$$D_{11} = \int_{\frac{-h}{2}}^{\frac{h}{2}} C_{11} Z^2 dz, \qquad D_{12} = \int_{\frac{-h}{2}}^{\frac{h}{2}} C_{12} Z^2 dz \quad \text{and} \quad D_{66} = \int_{\frac{-h}{2}}^{\frac{h}{2}} C_{66} Z^2 dz \qquad 3.20$$

With the help of equations (3.15), (3.17), (3.18) and (3.19), and assuming modal equation,

$w = W(x, y)e^{i\omega t}$, the following decoupled equilibrium equation will be obtained for rectangular plate.

$$\begin{aligned} & D_{11}\frac{\partial^4 W}{\partial x^4} + 2(D_{12} + 2D_{66})\frac{\partial^4 W}{\partial x^2 \partial y^2} + D_{22}\frac{\partial^4 W}{\partial y^4} \\ & - (1 - \mu\nabla^2)\left[q_0 - k_w W + k_p\left(\frac{\partial^2 W}{\partial x^2} + \frac{\partial^2 W}{\partial y^2}\right)\right] \\ & \left(\frac{\partial}{\partial x}\left(N_{xx}\frac{\partial W}{\partial x}\right) + \frac{\partial}{\partial y}\left(N_{yy}\frac{\partial W}{\partial y}\right) + \frac{\partial}{\partial x}\left(N_{xy}\frac{\partial W}{\partial y}\right) + \frac{\partial}{\partial y}\left(N_{yx}\frac{\partial W}{\partial x}\right)\right) = 0 \end{aligned} \qquad 3.21$$

3.2. Solution procedure

The calculus of variation is a branch of mathematics wherein the stationary property of function of function, namely, a functional, is studied.

It is well known that the basis of variational formulation is the principle of minimum total potential energy. The principle of minimum total potential energy is valid under the assumption that stress at a point can be uniquely defined in terms of the strain at that point.

Here an inverse approach is taken to reach to quadratic form of the governing equation.

This quadratic functional can be then used as a basis for different numerical solution such as Rayleigh- Ritz and finite element methods.

3.2.1. Weak formulation

The partial differential equation developed in equation (3.21) is the strong form of the governing system of equation for buckling of rectangular FG plate since obtaining the exact solution for a strong form of the system equation is usually difficult for practical engineering problems, because at the end of solution it forms inconsistence form of solution. A weak form of the system of equations is usually created for solution. The weak form is often an integral form and requires a weaker continuity on the field variables in contrast to the strong form. The weak form of the system of equations is usually created using one of the energy principles or weighted residual methods .

So by weighted residual method

$$\int_R k(\nabla^4 W + q) = 0 \qquad 3.22$$

K=weight function

$$\iint_R \left\{ \begin{array}{l} D_{11}\dfrac{\partial^4 W}{\partial x^4} + 2(D_{12} + 2D_{66})\dfrac{\partial^4 W}{\partial x^2 \partial y^2} + D_{22}\dfrac{\partial^4 W}{\partial y^4} \\ -(1-\mu\nabla^2)\left[q_0 - k_w W + k_p\left(\dfrac{\partial^2 W}{\partial x^2} + \dfrac{\partial^2 W}{\partial y^2}\right)\right] \\ \left(\left(\dfrac{\partial}{\partial x}\left(N_{xx}\dfrac{\partial W}{\partial x}\right) + \dfrac{\partial}{\partial y}\left(N_{yy}\dfrac{\partial W}{\partial y}\right) + \dfrac{\partial}{\partial x}\left(N_{xy}\dfrac{\partial W}{\partial y}\right) + \dfrac{\partial}{\partial y}\left(N_{yx}\dfrac{\partial W}{\partial x}\right)\right)\right) \end{array} \right\} K dx dy = 0 \qquad 3.23$$

With the help of Gauss and Green theorem breaking this residual form into integral of region and boundary.

$$\iint_R \dot{\Pi}(x,y)dxdy = 0$$

$$\begin{aligned}\dot{E} &= D_{11}\left(\frac{\partial^2 W}{\partial x^2}\frac{\partial^2 K}{\partial y^2}\right) + D_{12}\left(\frac{\partial^2 W}{\partial x^2}\frac{\partial^2 K}{\partial y^2} + \frac{\partial^2 W}{\partial y^2}\frac{\partial^2 K}{\partial x^2}\right) + D_{22}\left(\frac{\partial^2 W}{\partial y^2}\frac{\partial^2 K}{\partial y^2}\right) + \left(4D_{66}\frac{\partial^2 W}{\partial x\partial y}\frac{\partial^2 K}{\partial x\partial y}\right) \\ &+ k_w\left[WK + \mu\left(\frac{\partial W}{\partial x}\frac{\partial K}{\partial x} + \frac{\partial W}{\partial x}\frac{\partial K}{\partial x}\right)\right] \\ &+ k_p\left[\frac{\partial W}{\partial x}\frac{\partial K}{\partial x} + \frac{\partial W}{\partial x}\frac{\partial K}{\partial x} + \mu\left(\frac{\partial^2 W}{\partial x^2}\frac{\partial^2 K}{\partial x^2} + 2\frac{\partial^2 W}{\partial x\partial y}\frac{\partial^2 K}{\partial y\partial x} + \frac{\partial^2 W}{\partial y^2}\frac{\partial^2 K}{\partial y^2}\right)\right]\end{aligned} \qquad 3.24$$

$$\dot{F} = N_X\left(\frac{\partial W}{\partial x}\frac{\partial K}{\partial x} + \mu\left(\frac{\partial^2 W}{\partial x^2}\frac{\partial^2 K}{\partial x^2} + \frac{\partial^2 W}{\partial x\partial y}\frac{\partial^2 K}{\partial x\partial y}\right)\right) + N_y\left(\frac{\partial W}{\partial y}\frac{\partial K}{\partial y} + \mu\left(\frac{\partial^2 W}{\partial y^2}\frac{\partial^2 K}{\partial y^2} + \frac{\partial^2 W}{\partial x\partial y}\frac{\partial^2 K}{\partial x\partial y}\right)\right)$$

The Rayleigh Ritz functions obviate the tedious task of choosing the form of the infinite series or trigonometric or algebraic functions to suit the conditions of support along the edges. These functions are made from the product of a basic function and a set of orthogonal polynomials whose degree may be increased until the desired accuracy is achieved. The basic function is defined by the product of equations of the specified boundary shape of the plate raised to the power of either 0, 1 or 2 corresponding to free, simply supported and clamped edges, respectively.

$$W(x,y) = \sum C_j \phi_j(x,y) \qquad 3.25$$

C_j =unknown coefficient

ϕ_j =Algebraic function

It is well known that ‘The determination of the weak form is equivalent to the minimization of the quadratic function or minimization of the total potential energy .

$$B(K,W) = l(k) \qquad 3.26$$

To satisfy the essential boundary condition $K \longrightarrow \delta W$

Now this weak form become,

$$B(\delta W, W) = l(\delta W) \tag{3.27}$$

$$B(\delta W, W) = \int \left(\frac{\partial \delta W}{\partial x}\frac{\partial w}{\partial x} + \frac{\partial \delta W}{\partial x}\frac{\partial w}{\partial x} \right) d\Omega \tag{3.28}$$

$$B(\delta W, W) = \delta \int \frac{1}{2} \left(\left(\frac{\partial w}{\partial x} \right)^2 + \left(\frac{\partial w}{\partial x} \right)^2 \right) d\Omega \tag{3.29}$$

$$B(\delta W, W) = \frac{\delta}{2}\left(B(W, W) \right) \tag{3.30}$$

$$\frac{\delta}{2}\left(B(W, W) \right) - \delta l(W) = 0 \tag{3.31}$$

$$\frac{1}{2}\left[\left(B(W, W) \right) - l(W) \right] = 0 \tag{3.32}$$

$$\begin{aligned} E &= \frac{1}{2}[A + B + C] \\ A &= D_{11}\left(\frac{\partial^2 W}{\partial x^2} \right)^2 + 2D_{12}\left(\frac{\partial^2 W}{\partial x^2}\frac{\partial^2 W}{\partial y^2} \right) + D_{22}\left(\frac{\partial^2 W}{\partial y^2} \right)^2 + \left(4D_{66}\left(\frac{\partial^2 W}{\partial x \partial y} \right)^2 \right) \\ B &= k_w \left\{ W^2 + \mu \left(\left(\frac{\partial W}{\partial x} \right)^2 + \left(\frac{\partial W}{\partial y} \right)^2 \right) \right\} \\ C &= k_p \left\{ \left(\frac{\partial W}{\partial x} \right)^2 + \left(\frac{\partial W}{\partial y} \right)^2 + \mu \left(\left(\frac{\partial^2 W}{\partial x^2} \right)^2 + \left(\frac{\partial^2 W}{\partial y^2} \right)^2 + \left(\frac{\partial^2 W}{\partial x \partial y} \right)^2 \right) \right\} \end{aligned} \tag{3.33}$$

$$\begin{aligned} F &= \frac{1}{2}[D + G] \\ D &= N_x \left[\left(\frac{\partial W}{\partial x} \right)^2 + \mu \left(\left(\frac{\partial^2 W}{\partial x^2} \right)^2 + \left(\frac{\partial^2 W}{\partial x \partial y} \right)^2 \right) \right] \\ G &= N_y \left[\left(\frac{\partial W}{\partial y} \right)^2 + \mu \left(\left(\frac{\partial^2 W}{\partial y^2} \right)^2 + \left(\frac{\partial^2 W}{\partial x \partial y} \right)^2 \right) \right] \end{aligned} \tag{3.34}$$

Now put $W(x, y) = \sum C_j \phi_j (x, y)$

$$\phi_j(x,y) = \varphi\tau_j(x,y) \quad 3.35$$

$$\varphi = x^p y^q (x-a)^r (y-b)^s \quad 3.36$$

$$\tau_i(x,y) = \left(1, x, y, x^2, xy, y^2, x^3, x^2y, y^3, x^4, x^2y^2, y^4 .....\right) \quad 3.37$$

Where *p, q, r* and *s* are exponent, which controls different edge conditions, the parameter p= 0, 1 or 2 denotes that the edge is subjected to Free (F) S (simply supported) or C (clamped) support.

Further, the partial derivative of Eqs (3.33) and (3.34) w.r.t unknown constant is taken which results in eq. (3.38)

$$\left([K]_{nxn} - \lambda_b [M]_{nxn}\right)\{\Delta\} = 0 \quad 3.38$$

where, $\{\Delta\}$ is a coefficient column vector, [K] and $[M]$ are stiffness matrix due to strain energy, stiffness matrix due to potential energy and λ_b denotes non-dimensional mechanical buckling parameters expressed as $\lambda_b = \frac{|N_x| a^2}{D_c}$. The uniform in-plane compressive loading $N_x = N_y = -N$ and $N_{xy}=0$. Where as the D_c is the flexural rigidity, $D_c = \frac{E_c h^3}{12\left(1-\nu^2\right)}$

3.39

3.3. Thermal Buckling

To find the critical buckling temperature of the FGM plate subjected to thermal loading, there is a need to compute the pre-buckling thermal forces. To achieve this, the method of Meyers and Hyer [35] are employed. The resultant of pre-buckling load of plate subject to thermal load is given by [36] as

$$N = \int_{-h/2}^{h/2} \left[\frac{E(z,T)}{1-\nu}\alpha(z,T)\Psi dz\right] \quad 3.40$$

Where α is a coefficient of thermal expansion and Φ is the type of temperature distribution through-thickness. For the different thermal distributions different functions of Φ are considered, which are given as

For Uniform Temperature rise

Let an FGM nanoplate at a reference temperature T_i, the temperature is uniformly raised to final value T_f, which the temperature change is

$$\Delta T = T_f - T_i$$
$$\Psi = \Delta T \qquad 3.41$$

For Linear Temperature rise

For a thin enough FGM plate the temperature distribution can be assumed to linear variation through the thickness as follows (Javaheri and Eslami, 2002): 3.42

$$\Psi = \frac{\Delta T}{h}\left(z + \frac{h}{2}\right) + T_m$$
$$\Delta T = T_C - T_m$$

Whereas T_c = Temperature of the ceramic rich surface. T_m = Temperature of the metal rich surface.

Following the procedure as above, ΔT can be obtained. The critical buckling temperature difference represented as ΔT_{cr} are reported here which are the lowest values

3.4. Buckling in thermal environment

The derivation of the generalised equation of thin FGM rectangular plate already discuss in the starting of this chapter. The Objective of this sub chapter is to study the power law variation of the temperature dependent material properties for the convergence of the results for the FG plate in thermal environment.

Here we are discuss the two different cases of the temperature variation and its effect on the mechanical properties on FG plate. The variation of temperature only occurs in thickness direction.

The material properties of FG material i.e. Young modulus, poisson ratio and thermal expansion coefficient are nonlinear function of temperature as study presented by J.N.Reddy (1998) and H.S.Shen (2002) .

$$P(T) = P^T{}_0\left(P_{-1}^T T^{-1} + 1 + P_1^T T^1 + P_2^T T^2 + P_3^T T^3\right) \qquad 3.43$$

Where $T = T_0 + \Delta T(z)$, denotes the environmental temperature and T_0 is the room temperature, and P_{-1}^T, P_0^T, P_1^T, P_2^T and P_3^T are the constant coefficients in the cubic expression of the respective material property.

We have already discuss the CPT and its assumptions in the starting of this chapter, in this sub-chapter we are presented the effect of temperature variation on the mechanical properties and these effects of buckling load parameter. So by Hooks law the stress strain relationship can be represented as:

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{Bmatrix} = \begin{pmatrix} C_{11} & C_{12} & 0 \\ C_{21} & C_{22} & 0 \\ 0 & 0 & C_{66} \end{pmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} \qquad 3.44$$

where, σ_{xx} and σ_{yy} are normal stresses and τ_{xy} is shear stress in x – y plane. Further, $C_{11}, C_{12}, C_{22}, C_{66}$ are

$$C_{11} = \frac{E(z,T)}{\left(1-\nu(z,T)^2\right)} \qquad C_{12} = \frac{\nu(z,T)E(z,T)}{\left(1-\nu(z,T)^2\right)} \qquad C_{66} = \frac{E(z,T)}{2(1+\nu(z,T))} \qquad 3.45$$

Now the expression of Strain energy and potential energy for the thin rectangular FG plate in thermal environment can be expressed as

$$
\begin{aligned}
E_{TH} &= \frac{1}{2}[A+B+C] \\
A &= A_1 + A_2 \\
A_1 &= \left(D^T{}_{11} + D^T{}_{12}\right)\left[\left(\frac{\partial^2 W}{\partial x^2}\right)^2 + \left(\frac{\partial^2 W}{\partial y^2}\right)^2 + 2\left(\frac{\partial^2 W}{\partial x \partial y}\right)^2\right] \\
A_2 &= \left(A^T{}_{11} + A^T{}_{12}\right)\left[\left(\frac{\partial^2 W}{\partial x^2}\right)^2 + \left(\frac{\partial^2 W}{\partial y^2}\right)^2\right] \\
B &= k_w\left\{W^2 + \mu\left(\left(\frac{\partial W}{\partial x}\right)^2 + \left(\frac{\partial W}{\partial y}\right)^2\right)\right\} \\
C &= k_p\left\{\left(\frac{\partial W}{\partial x}\right)^2 + \left(\frac{\partial W}{\partial y}\right)^2 + \mu\left(\left(\frac{\partial^2 W}{\partial x^2}\right)^2 + \left(\frac{\partial^2 W}{\partial y^2}\right)^2 + \left(\frac{\partial^2 W}{\partial x \partial y}\right)^2\right)\right\}
\end{aligned}
\qquad 3.46
$$

The flexural rigidities is defined in terms of coefficient of stiffness as:

$$
D_{11} = \int_{\frac{-h}{2}}^{\frac{h}{2}} C_{11} Z^2 \alpha \Delta T dz \quad \text{and} \quad D_{12} = \int_{\frac{-h}{2}}^{\frac{h}{2}} C_{12} Z^2 \alpha \Delta T dz \qquad 3.47
$$

$$
A_{11} = \int_{\frac{-h}{2}}^{\frac{h}{2}} C_{11} \alpha \Delta T dz \quad \text{and} \quad A_{12} = \int_{\frac{-h}{2}}^{\frac{h}{2}} C_{12} \alpha \Delta T dz \qquad 3.48
$$

$$
\begin{aligned}
F &= \frac{1}{2}[D+G] \\
D &= N_X\left[\left(\frac{\partial W}{\partial x}\right)^2 + \mu\left(\left(\frac{\partial^2 W}{\partial x^2}\right)^2 + \left(\frac{\partial^2 W}{\partial x \partial y}\right)^2\right)\right] \\
G &= N_y\left[\left(\frac{\partial W}{\partial y}\right)^2 + \mu\left(\left(\frac{\partial^2 W}{\partial y^2}\right)^2 + \left(\frac{\partial^2 W}{\partial x \partial y}\right)^2\right)\right]
\end{aligned}
\qquad 3.49
$$

Now we are considering the effect of both biaxial compression and thermal environment on the thin FG plate, so the total strain energy can be presented as:

$$
E_{eff} = E + E_{th} \qquad 3.50
$$

By equating the both differential equation of strain energy and potential energy we can get the Buckling load $'N'$ with the help of Ritz method.

$$\left([K]_{nxn} - \lambda_b [M]_{nxn}\right)\{\Delta\} = 0 \qquad 3.51$$

where, $\{\Delta\}$ is a coefficient column vector, [K] and $[M]$ are stiffness matrix due to strain energy, stiffness matrix due to potential energy and λ_b denotes non-dimensional mechanical buckling parameters expressed as $\lambda_b = \dfrac{|N_x| a^2}{D_c}$.

Chapter 4

Results and Discussions

We have considered different BCs for the FG plates. Here, the flow of BCs is demonstrated in Fig. 4.1. For example, if we consider the edge support SCSF, then 1 is meant for simply supported (S) and accordingly 2, 3 and 4 represent clamped (C), simply supported (S), completely free (F) edge conditions respectively. In Fig. 4.1 for Lévy plates, 1 and 3 are always kept to be simply supported and other edges 2and 4 vary as per different BCs.

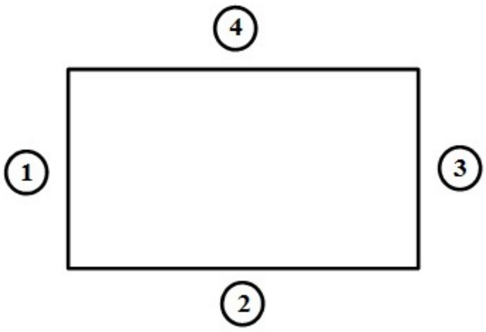


Figure 4. 1 The flow of various combinations of boundary supports in FG plate

The functionally graded plate is considered to be made up of Aluminium and Zirconium. The top and bottom layers of the plate are ceramics, and metallic respectively. The metal and ceramics that are employed as contributing materials of the FGEP are (table 1) .

Table 4.1 Material Properties of the constituents of FGEP

Properties	Aluminium (Metal)	ZrO_2 – I (Ceramic)
E	70 Gpa	151 Gpa
ν	0.3	0.3
ρ	2700 Kg/m^3	3000 Kg/m^3

4.1. Convergence

The convergence of non-dimensional buckling of plates (k = 0 in case of FG plates) has been performed in Table 4.2. The convergence of non-dimensional frequencies may be achieved by increasing the number of polynomials (n) in the displacement component.

Table 4.2. The convergence of λ of plate having a/b=0.5, k=1, k_w=500, k_p=100.

BC	Polynomials	λ_1	λ_2	λ_3	λ_4
CCCC	4	7.2242	7.9788	9.9101	12.9987
	6	7.2135	7.8672	9.8977	12.9725
	8	7.2135	7.8358	9.8847	12.9571
	10	7.2135	7.8358	9.8847	12.9571
SSSS	4	3.8252	5.2214	3.9254	8.8838
	6	3.7121	5.0685	3.7388	8.6452
	8	3.7121	5.0607	7.4368	8.3002
	10	3.7121	5.0607	7.4368	8.3002

4.2. Comparison

The comparison of the computed result is done with the published one and presented in table 4.3. From the validation, it may be noticed that present results are in excellent agreement with the available literature, irrespective of the elastic foundations and boundary support considered.

Table 4.3. Comparison of λ of embedded FG square plate under biaxial loading

$\frac{K_w a^4}{D}$	$\frac{K_p a^4}{D}$	Source	Λ			
			CCCC	SCSC	SSSS	FCFC
0	0	Present	3.9031	2.7224	2.1010	1.2040
		Lam et al. (2000)	3.8300	2.6630	2.0000	1.1440
		Thai et al. (2013)	3.8299	2.6627	2.0000	1.1436
	100	Present	13.9824	12.8651	12.1803	11.3100
		Lam et al. (2000)	13.9600	12.8000	12.1300	11.2800
		Thai et al. (2013)	13.9620	12.7948	12.1321	11.2757
100	0	Present	4.2800	3.1320	2.5130	1.7620
		Lam et al. (2000)	4.2800	3.1320	2.5130	1.7620
		Thai et al. (2013)	4.2795	3.1320	2.5133	1.7617

	Present	14.4100	13.2600	12.6500	11.8900
100	Lam et al. (2000)	14.4100	13.2600	12.6500	11.8900
	Thai et al. (2013)	14.4116	13.2641	12.6454	11.8938

4.3. Numerical Results

4.3.1. Parameteric Effect

The effect of various parameters on buckling modes of FG plates resting on Winkler Pasternak Foundation may be observed in Tables 4.4 and 4.5 with different combination of boundary conditions. The important results in Tables 4.4 and 4.5 are summarized as below:

- Irrespective of the elastic foundations and edge support, it can easily be noticed that the buckling load follow the ascending behavior with increase in a/b as shown in the graph.
- Irrespective of k, k_w and kp assumed, it can also be observed that second non-dimensional buckling load for CCCC and SSSS square FG plates are coinciding with the respective third mode frequencies. But no coincidence can be observed for natural frequencies of rectangular FG plates (a/b≠1). This proposition may be true because of the identical edge supports at all edges of the FG square plate.
- Looking into these computations, we may draw the conclusions that the buckling loads are decreasing with the increasing values of k.
- It is evident that non-dimensional buckling load are increasing with increase in foundation parameters (k_w & k_p) regardless of the aspect ratio and power-law index considered. Although the effect of variation of k_p dominates over k_w .

Table 4.4. λ of simply supported embedded plate under biaxial loading

a/b	K	kw	kp	λ_1	λ_2	λ_3	λ_4	λ_1
0.5	0	0	50	3.1529	5.4119	9.0664	10.2572	14.1075
			100	3.8672	6.1262	9.7807	10.9715	14.8218
			500	9.5815	11.84	15.495	16.6858	20.5361

		100	50	3.2687	5.4816	9.1004	10.2974	14.1347
			100	3.983	6.1959	9.8147	11.0117	14.849
			500	9.6972	11.91	15.529	16.726	20.5633
		500	50	3.7318	5.7603	9.2364	10.4582	14.2436
			100	4.4461	6.4746	9.9507	11.1725	14.9579
			500	10.16	12.189	15.665	16.8868	20.6722
	1	0	50	2.4189	3.998	6.5526	7.38496	10.0764
			100	3.1332	4.7123	7.2669	8.09925	10.7907
			500	8.8475	10.427	12.981	13.8135	16.505
		100	50	2.5347	4.0677	6.5866	7.42517	10.1036
			100	3.249	4.782	7.3009	8.13945	10.8179
			500	8.9633	10.496	13.015	13.8537	16.5322
		500	50	2.9979	4.3464	6.7225	7.58599	10.2125
			100	3.7121	5.0607	7.4368	8.30028	10.9268
			500	9.4264	10.775	13.151	14.0146	16.6411
	2	0	50	2.277	3.7246	6.0665	6.82954	9.29691
			100	2.9913	4.4389	6.7808	7.54383	10.0112
			500	8.7056	10.153	12.495	13.2581	15.7255
		100	50	2.3928	3.7943	6.1005	6.86975	9.32412
			100	3.1071	4.5086	6.8147	7.58403	10.0384
			500	8.8213	10.223	12.529	13.2983	15.7527
		500	50	2.8559	4.073	6.2364	7.03057	9.43297
			100	3.5702	4.7873	6.9507	7.74485	10.1473
			500	9.2845	10.502	12.665	13.4591	15.8615
1	0	0	50	4.6156	10.615	13.934	20.8634	35.0863
			100	5.3299	11.329	14.649	21.5777	35.8006
			500	11.044	17.044	20.363	27.292	41.5149
		100	50	4.688	10.644	13.962	20.8804	35.0991
			100	5.4023	11.358	14.676	21.5947	35.8134
			500	11.117	17.073	20.39	27.309	41.5277
		500	50	4.9774	10.759	14.072	20.9485	35.1501
			100	5.6917	11.474	14.786	21.6627	35.8644
			500	11.406	17.188	20.5	27.377	41.5787
	1	0	50	3.4414	7.6352	9.9553	14.7989	24.741
			100	4.1557	8.3495	10.67	15.5132	25.4553
			500	9.87	14.064	16.384	21.2275	31.1696
		100	50	3.5138	7.6641	9.9828	14.8159	24.7538
			100	4.228	8.3784	10.697	15.5302	25.4681
			500	9.9423	14.093	16.411	21.2445	31.1824
		500	50	3.8032	7.7794	10.093	14.884	24.8048
			100	4.5175	8.4937	10.807	15.5983	25.5191
			500	10.232	14.208	16.521	21.3125	31.2334
	2		50	3.2143	7.059	9.1859	13.6262	22.7405

			100	3.9286	7.7732	9.9002	14.3405	23.4548
			500	9.6429	13.488	15.614	20.0548	29.1691
		100	50	3.2867	7.0878	9.2134	13.6432	22.7532
			100	4.001	7.8021	9.9277	14.3575	23.4675
			500	9.7153	13.516	15.642	20.0718	29.1818
		500	50	3.5762	7.2032	9.3233	13.7112	22.8043
			100	4.2904	7.9175	10.038	14.4255	23.5186
			500	10.005	13.632	15.752	20.1398	29.2328
2	0	0	50	10.469	17.242	38.886	49.3226	54.2873
			100	11.183	17.957	39.6	50.0369	55.0016
			500	16.897	23.671	45.314	55.7512	60.7159
		100	50	10.498	17.26	38.896	49.3306	54.2941
			100	11.212	17.975	39.61	50.0449	55.0084
			500	16.926	23.689	45.325	55.7592	60.7227
		500	50	10.613	17.332	38.936	49.3627	54.3213
			100	11.328	18.046	39.65	50.077	55.0356
			500	17.042	23.761	45.365	55.7913	60.7499
	1	0	50	7.5328	12.268	27.397	34.6925	38.1629
			100	8.2471	12.982	28.111	35.4067	38.8772
			500	13.961	18.696	33.826	41.121	44.5915
		100	50	7.5618	12.286	27.407	34.7005	38.1697
			100	8.276	13.1200	28.121	35.4148	38.884
			500	13.99	18.714	33.836	41.1291	44.5983
		500	50	7.6775	12.358	27.447	34.7326	38.1969
			100	8.3918	13.072	28.162	35.4469	38.9112
			500	14.106	18.786	33.876	41.1612	44.6255
	2	0	50	6.9651	11.306	25.175	31.8633	35.0448
			100	7.6794	12.02	25.89	32.5776	35.7591
			500	13.394	17.734	31.604	38.2919	41.4734
		100	50	6.994	11.324	25.185	31.8713	35.0516
			100	7.7083	12.038	25.9	32.5856	35.7659
			500	13.423	17.752	31.614	38.2999	41.4802
		500	50	7.1098	11.396	25.226	31.9034	35.0788
			100	7.8241	12.11	25.94	32.6177	35.7931
			500	13.538	17.824	31.654	38.332	41.5074

Table 4.5. λ of clamped embedded plate under biaxial loading

a/b	K	kw	kp	λ_1	λ_2	λ_3	λ_4
0.5	0	0	50	8.3694	9.4361	12.4968	17.0068
			100	9.0837	10.15	13.2111	17.7211
			500	14.798	15.865	18.9254	23.4354
		100	50	8.4566	9.4982	12.5405	17.0348

			100	9.1709	10.213	13.2548	17.7491
			500	14.885	15.927	18.9691	23.4634
		500	50	8.8041	9.7467	12.7163	17.1466
			100	9.5184	10.461	13.4306	17.8609
			500	15.233	16.175	19.1449	23.5752
	1	0	50	6.0654	6.811	8.9505	12.1031
			100	6.7796	7.5253	9.66479	12.8174
			500	12.494	13.24	15.3791	18.5317
		100	50	6.1526	6.8731	8.99425	12.1311
			100	6.8669	7.5874	9.70853	12.8453
			500	12.581	13.302	15.4228	18.5596
		500	50	6.4992	7.1216	9.17046	12.2429
			100	7.2135	7.8359	9.88475	12.9572
			500	12.928	13.55	15.599	18.6715
	2	0	50	5.6198	6.3034	8.26472	11.1548
			100	6.3341	7.0177	8.97901	11.8691
			500	12.048	12.732	14.6933	17.5834
		100	50	5.707	6.3655	8.30848	11.1828
			100	6.4213	7.0798	9.02276	11.8971
			500	12.136	12.794	14.737	17.6114
		500	50	6.0534	6.614	8.48483	11.2946
			100	6.7677	7.3282	9.19912	12.0089
			500	12.482	13.043	14.9134	17.7232
1	0	0	50	11.072	18.961	20.1861	27.1847
			100	11.786	19.675	20.9004	27.899
			500	17.501	25.39	26.6147	33.6133
		100	50	11.128	18.985	20.2117	27.201
			100	11.842	19.699	20.9259	27.9152
			500	17.556	25.414	26.6402	33.6295
		500	50	11.35	19.081	20.3137	27.2659
			100	12.064	19.795	21.028	27.9802
			500	17.779	25.509	26.7423	33.6945
	1	0	50	7.9545	13.469	14.3255	19.2177
			100	8.6688	14.184	15.0398	19.9319
			500	14.383	19.898	20.7541	25.6462
		100	50	8.0102	13.493	14.351	19.2339
			100	8.7245	14.207	15.0653	19.9482
			500	14.439	19.922	20.7796	25.6625
		500	50	8.2324	13.589	14.4531	19.2988
			100	8.9467	14.303	15.1673	20.0131
			500	14.661	20.017	20.8816	25.7274
	2	0	50	7.3517	12.407	13.1922	17.677
			100	8.066	13.122	13.9065	18.3913

			500	13.78	18.836	19.6208	24.1056
		100	50	7.4073	12.431	13.2177	17.6932
			100	8.1216	13.145	13.932	18.4075
			500	13.836	18.86	19.6463	24.1218
		500	50	7.6295	12.527	13.3197	17.7582
			100	8.3438	13.241	14.034	18.4725
			500	14.058	18.955	19.7483	24.1867
2	0	0	50	31.335	34.788	47.8443	70.8599
			100	32.049	35.502	48.5586	71.5742
			500	37.763	41.216	54.2729	77.2884
		100	50	31.357	34.802	47.8553	70.8675
			100	32.071	35.517	48.5696	71.5818
			500	37.785	41.231	54.2838	77.2961
		500	50	31.444	34.862	47.899	70.898
			100	32.158	35.576	48.6133	71.6123
			500	37.872	41.291	54.3275	77.3266
	1	0	50	22.119	24.532	33.6591	49.7475
			100	22.833	25.247	34.3734	50.4618
			500	28.547	30.961	40.0877	56.176
		100	50	22.14	24.547	33.6701	49.7551
			100	22.855	25.261	34.3844	50.4694
			500	28.569	30.976	40.0986	56.1837
		500	50	22.228	24.607	33.7138	49.7856
			100	22.942	25.321	34.4281	50.4999
			500	28.656	31.035	40.1424	56.2142
	2	0	50	20.336	22.549	30.916	45.6648
			100	21.051	23.263	31.6303	46.3791
			500	26.765	28.978	37.3446	52.0934
		100	50	20.358	22.564	30.927	45.6724
			100	21.072	23.278	31.6413	46.3867
			500	26.787	28.992	37.3555	52.101
		500	50	20.445	22.623	30.9707	45.7029
			100	21.16	23.338	31.685	46.4172
			500	26.874	29.052	37.3993	52.1315

4.3.2. Temperature Effect

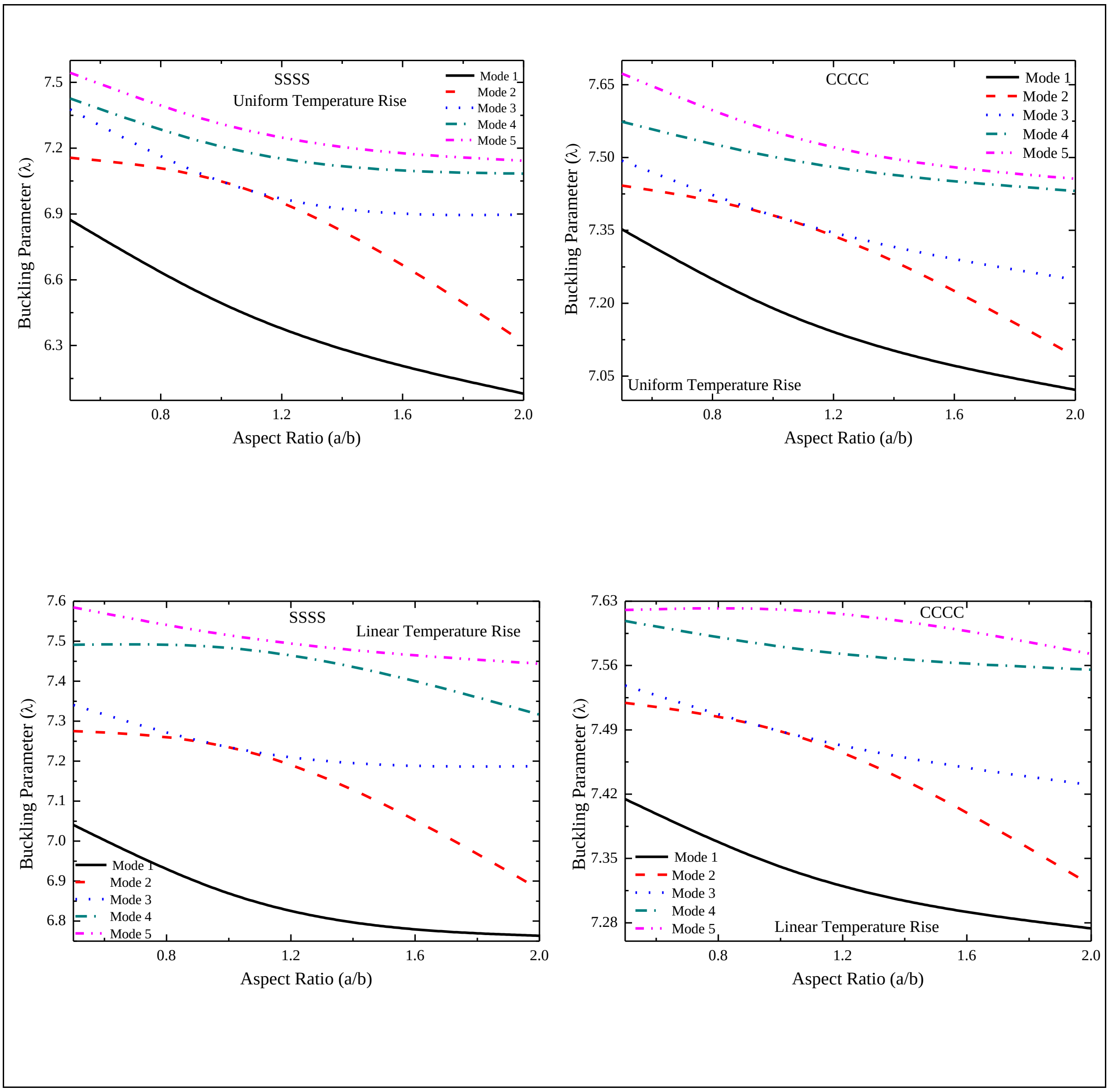


Figure 4.1 Variation of λ of embedded plate for different a/b with k=1, k_w =50 and k_p=100, ΔT=300.

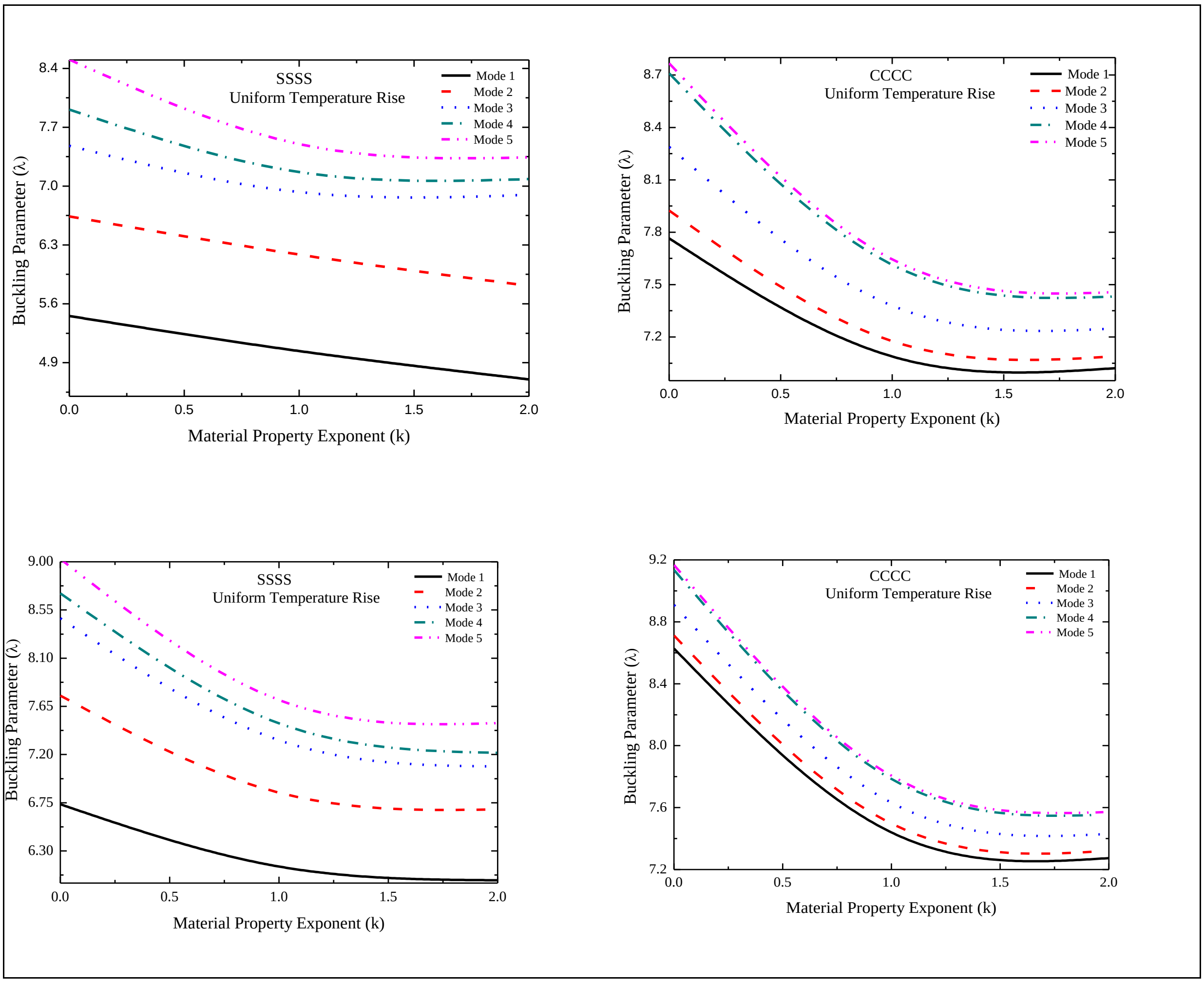
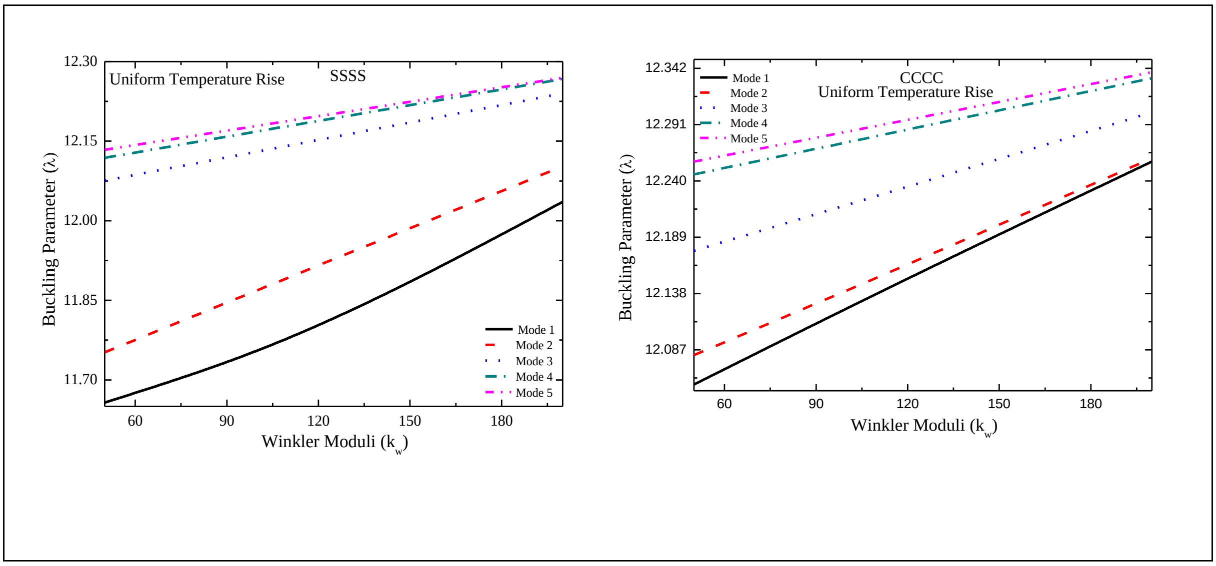


Figure 4.2. Variation of λ of embedded plate for different k with a/b=2, k_w =50 and k_p=100, ΔT=300.



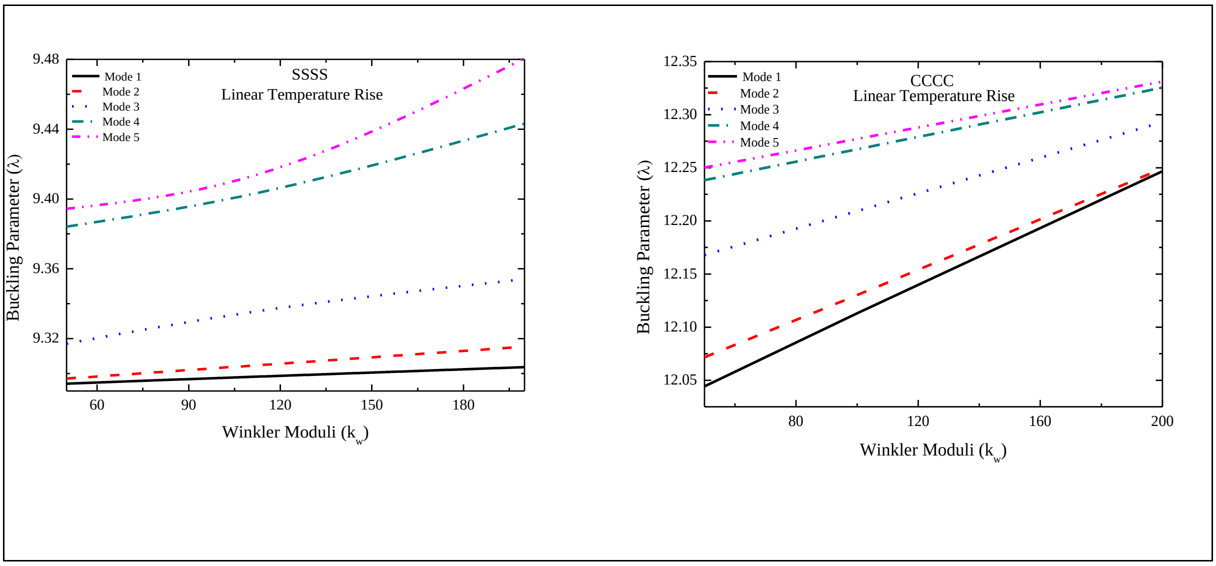


Figure 4.3. Variation of λ of embedded plate with k_w for with k=1, a/b=2 and k_p=100, ΔT=300.

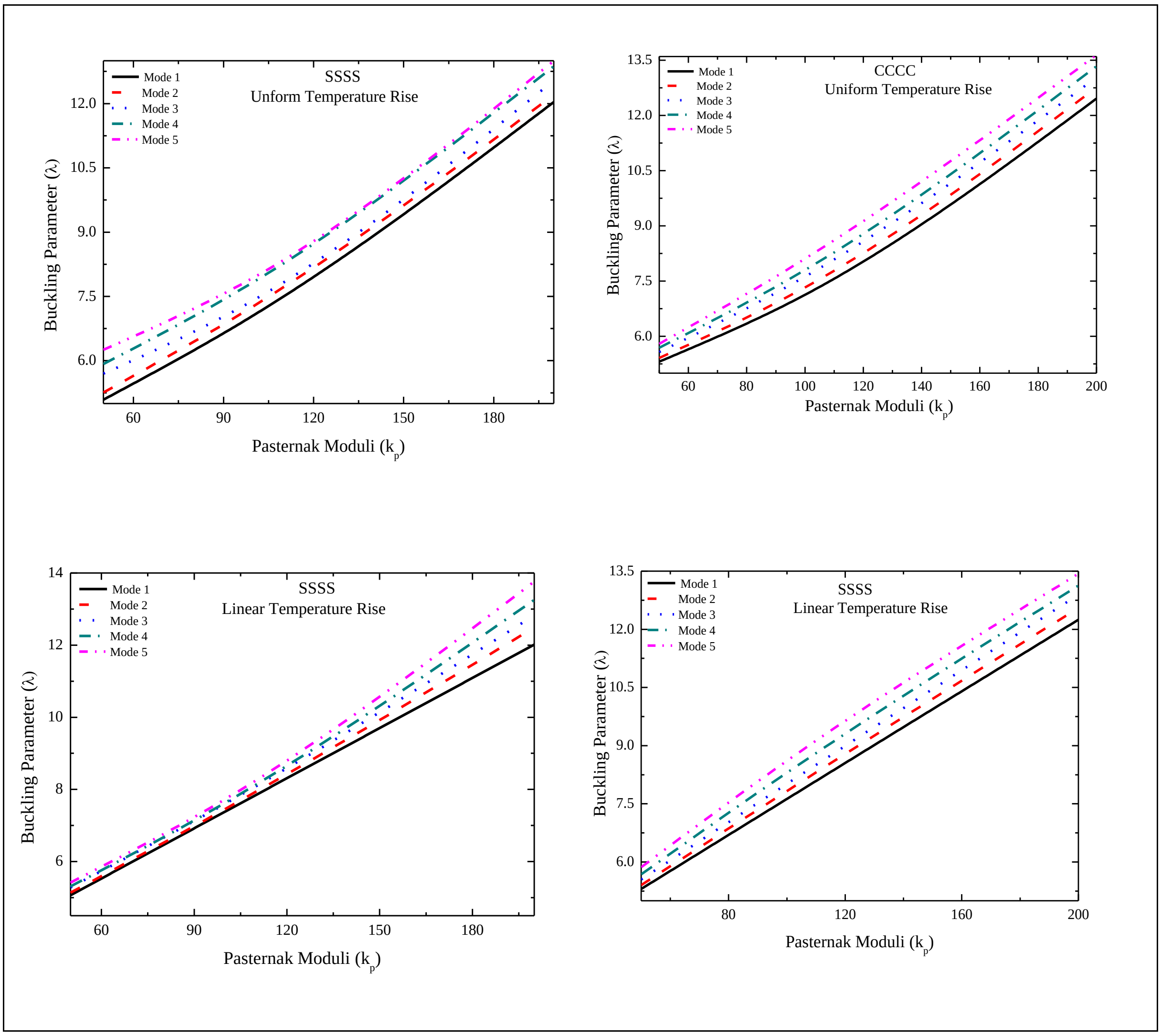


Figure 4.4 Variation of λ of embedded plate with k_w for k=1, a/b=2 and k_w=100, ΔT=300.

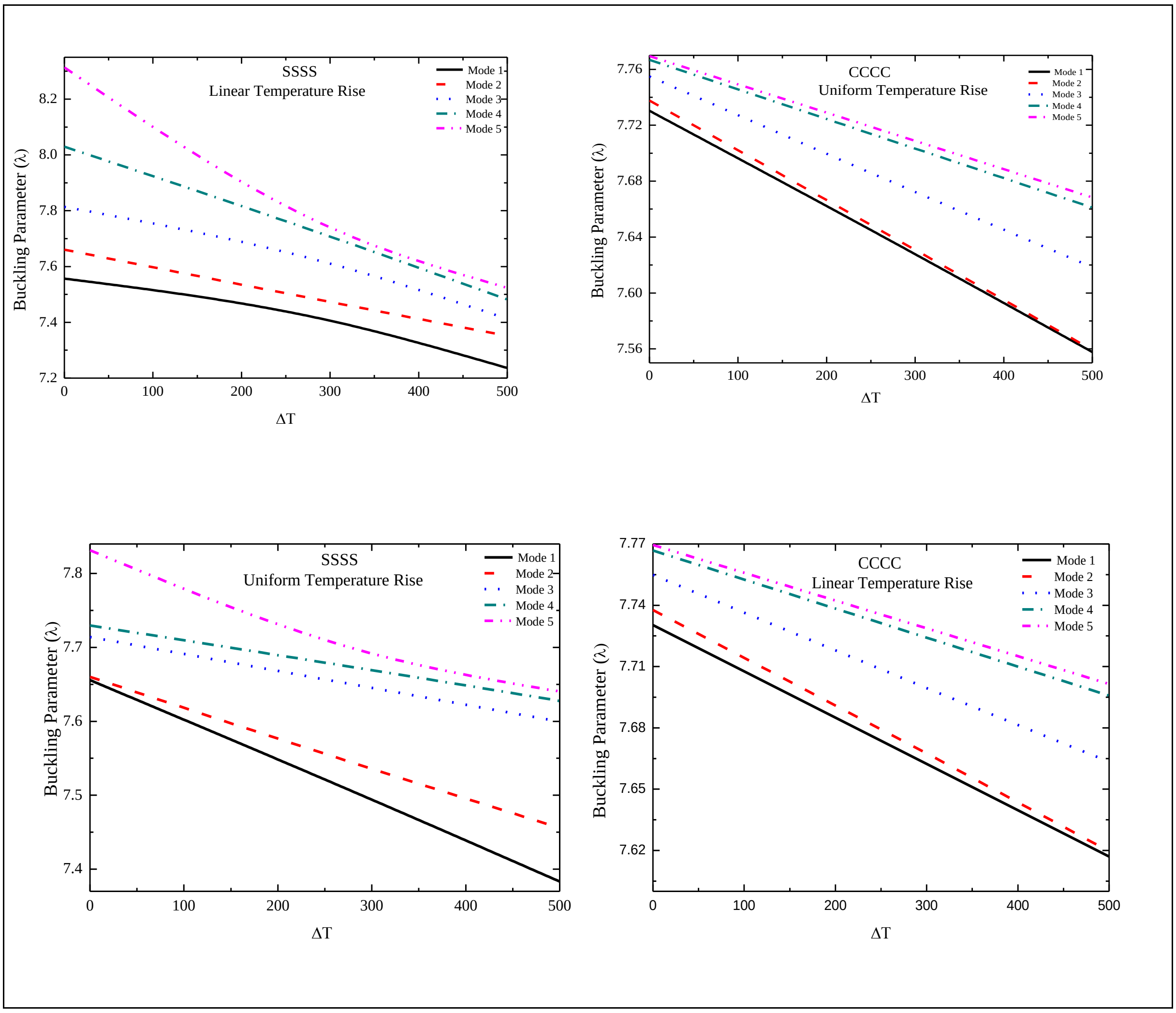


Figure 4.5. Variation of λ of embedded plate with ΔT for a/b=2, k=1, k_w =50 and k_p=100.

In this section, effect of temperature on buckling parameter (λ) and frequency parameter of FGP in are plotted in figures 4.1 – 4.5. CCCC and SSSS edge constraints are taken into account for the analysis. Effect of the thermal environment are summarized as.

1. Irrespective of the edge support and thermal state, λ of an embedded plate decrease with an increase in the aspect ratio. The reason is that the plate loses it's stiffness in a thermal environment.
2. The second nondimensional buckling load coincides with the third for a FG square plate irrespective of the thermal condition but no coincidence can be observed for natural frequencies of rectangular FG plates.

Chapter 5

Conclusions

The thesis is concerned with the buckling of FG rectangular plate resting on the elastic foundation. The plate has four edges with simply supported, clamped, free and their combination. The tedious task of solving the infinite series of trigonometric functions for edge constraints is obviated by the Rayleigh-Ritz and orthogonal algebraic polynomials. These polynomials are capable of handling any configurations of edge constraints simultaneously and efficiently. Also, there is no requirement of generation of mesh, and thus large degrees of freedom can be attained.

The buckling solution constitute of different solution combination depending on various boundary condition, aspect ratios, effect of nano scale, the power law exponent, the temperature and foundation and the influence of these factors are presented in graphs. It is found that in thermal environment the less buckling load required as compared with normal environment, it is obvious because on heating the material getting soft. Our research should be useful for the engineers designing the plated walls under heating environment and subjected to various boundary conditions.

Furthermore, from this analysis, the following can be concluded:

- With increasing a/b of plate, the thermal buckling load of plate increases.
- The 2^{nd} and 3^{rd} mode of mechanical buckling parameters are same for the case of square plate under clamped and simply supported boundary condition.
- Effect of temperature rise is nominal
- On increasing k buckling load decreases.

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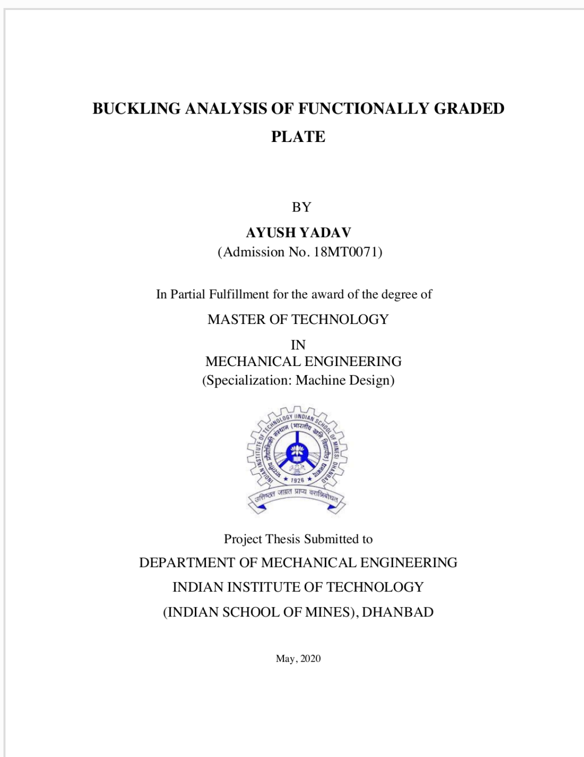

BUCKLING ANALYSIS OF FUNCTIONALLY GRADED PLATE

BY

AYUSH YADAV

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