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section - I
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Tutorial - 04.

1) $T(n) = 3T(\frac{n}{2}) + n^2$

$T(n) = aT(\frac{n}{b}) + f(n)$

$a \geq 1, b > 1$

On comparing

$a = 3, b = 2, f(n) = n^2$

$c = \log_b a = \log_2 3 = 1.584$

$n^c = n^{1.584} < n^2$

$f(n) > n^c$

$T(n) = O(n^2)$

2) $T(n) = 4T(n/2) + n^2$

$a \geq 1 \quad b > 1$

$a = 4 \quad b = 2 \quad f(n) = n^2$

$c = \log_2 4 = 2$

$n^c = n^2 = f(n) = n^2$

$T(n) = O(n^2 \log_2 n)$

3) $T(n) = T(n/2) + 2^n$

$a = 1 \quad b = 2$

$f(n) = 2^n$

$c = \log_b a = \log_2 1 = 0$

$n^c = n^0 = 1$

$f(n) > n^c$

$T(n) = O(2^n)$

4) $T(n) = 2^n (T) n/2) + n^n$

$a = 2^n$

$b = 2 \quad f(n) = n^n$

$c = \log_b a = \log_2 2^n = n$

$n^c = n^n$

$f(n) = n^c$

$T(n) = O(n^2 \log_2 n)$

5) $T(n) = 16T(n/4) + n$

$a = 16, b = 4$

$f(n) = n$

$c = \log_4 16 = \log_4 (4)^2 = 2$

$n^c = n^2$

$f(n) < n^c$

$T(n) = O(n^2)$

6) $T(n) = 2T(n/2) + n\log n$

$a = 2, b = 2$

$f(n) = n\log n$

$c = \log_2 2 = 1$

$n^c = n^1 = n$

$n^c = n^1 = n$

$n\log n > n$

$f(n) > n^c$

$T(n) = O(n\log n)$

7) $T(n) = 2T(n/2) + n/\log n$

$a = 2, b = 2, f(n) = n/\log n$

$c = \log_2 2 = 1$

$n^c = n^1 = 1$

since $\frac{n}{\log n} < n$

$f(n) < n^c$

$T(n) = O(n)$

8) $T(n) = 2T(n/4) + n^{0.51}$

$a = 2, b = 4, f(n) = 0.51$

$c = \log_b a = \log_4 2 = 0.5$

$n^c = n^{0.5}$

since $n^{0.5} < n^{0.51}$

$f(n) > n^c$

$T(n) = O(n^{0.51})$

9) $T(n) = 0.5T(n/2) + 1/n$

$a = 0.5,\ b = 2$

Since acc to master theorem $a \geq 1$ but here a is 0.5 so we cannot apply master theorem.

10 $T(n) = 16T(n/4) + n!$

$a = 16,\ b = 4,\ f(n) = n!$

$c = \log_b a = \log_4 16 = 2$

Now $n^c = n^2$

As $n! > n^2$

$T(n) = O(n!)$

11 $4T(n/2) + \log n$

$a = 4,\ b = 2\ f(n) = \log n$

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2$

$f(n) = \log n$

Since $\log n < n^2$

$f(n) < n^c$

$T(n) = O(n^c) = O(n^2)$

12) $T(n) = \text{sqrt}(n)T(n/2) + \log n$

$a = \sqrt{n},\ b = 2$

$c = \log_b a = \log_2 \sqrt{n} = \frac{1}{2}\log_2 n$

$\frac{1}{2}\log_2 n < \log n$

$f(n) > n^c$

$T(n) = O(f(n))$

$O(\log n)$

13 $T(n) = 3T(n/2) + n$

$a = 3,\ b = 2,\ f(n) = n$

$c = \log_b a = \log_2 3 = 1.5849$

$n^c = n^{1.5849}$

$n < n^{1.5849}$

$f(n) < n^c$

$T(n) = O(n^{1.5849})$

14 $T(n) = 3T(n/3) + \text{sqrt}(n)$

$a = 3,\ b = 3$

$c = \log_b a = \log_3 3 = 1$

$n^c = n^1 = n$

As $\text{sqrt}(n) < n$

$f(n) < n^c$

$T(n) = O(n)$

15) $T(n) = 4T(n/2) + cn$

$a = 4,\ b = 2$

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2$

$cn < n^2$ (for any const)

$f(n) < n^c$

$T(n) = O(n^2)$

16) $T(n) = 3T(n/4) + n\log n$

$a = 3,\ b = 4,\ f(n) = n\log n$

$c = \log_b a = \log_4 3 = 0.7\ldots$

$n^c = n^{0.789} < n\log n$

$T(n) = O(n\log n)$

17) $T(n) = 3T(n/3) + n/2$

$a = 3,\ b = 3\ \ c = \log_b a = 1$

$f(n) = n/2$

$n^c = n^1 = n > n/2$

$f(n) < n^c$

$T(n) = O(n)$

18) $T(n) = 6T(n/3) + n^2\log n$

$a = 6\ \ b = 3$

$c = \log_b a = \log_3 6 = 1.6309$

$n^c = n^{1.6309} < n^2\log n$

$T(n) = O(n^2\log n)$

19 $T(n) = 4T(n/2) + n\log n$

$a = 4$, $b = 2$, $f(n) = n\log n$

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2 > \frac{n}{\log n}$

$T(n) = O(n^2)$

21 $T(n) = 7T(n/3) + n^2$

$a = 7$, $b = 3$, $f(n) = n^2$

$c = \log_b a = \log_3 7 = 1.7712$

$n^c = n^{1.7712}$

$n^{1.7712} < n^2$

$T(n) = O(n^2)$.

20 $T(n) = 64T(n/8) - n^2\log n$

$a = 64$ $b = 8$

$c = \log_b a = \log_8 64 = 2$

$n^c = n^2$

$n^2\log n > n^2$

$T(n) = O(n^2\log n)$

22 $T(n) = T(n/2) + n(2 - \cos n)$

$a = 1$ $b = 2$

$c = \log_b a = \log_2 1 = 0$

$n^c = n^0 = 1$

$n(2 - \cos s) > n^c$

$T(n) = O(n(2 - \cos n))$