Two Dimensional : Two Dimensional Transformation : Translation, Scaling, Rotation, Reflection, Shear, Homogenous coordinate system, composite transformations, Raster method of transformation. Two Dimensional Viewing: Window to Viewport coordinates transformation

---

2D Transformations

Transformations are the operations applied to geometrical description of an object to change its position, orientation or size are called geometric transformations.

Translation

It means changing some graphics into something else by applying rules. We can have various types of transformations such as

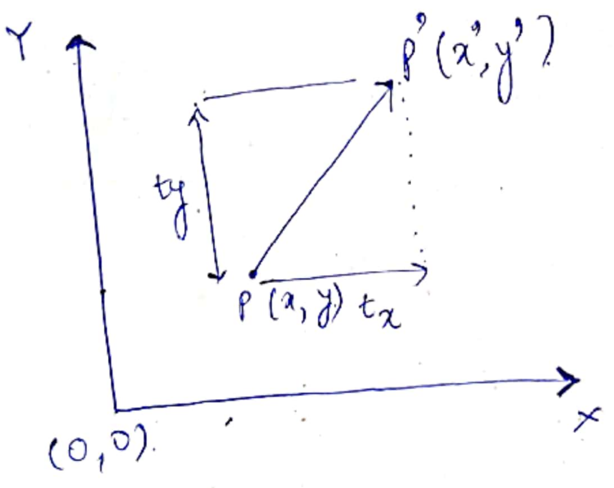
1. Translation
2. Scaling
3. Rotation
4. Reflection
5. Shear etc.
6.

When a transformation occurs / takes place on a 2D plane it is called 2D transformation.

Transformation plays an important role in computer graphics to reposition the graphics on the screen and change their size or orientation.

(1) Translation

A translation moves an object to a different position on screen. You can translate a point in 2D by adding translation coordinates (t_x, t_y) to the original coordinates (x, y) to get a new coordinate (x', y')



From this figure

$x' = x + t_x$

$y' = y + t_y$

The pair (t_x, t_y) is called as translation vector or shift vector.

The above equation is to be represented in matrix form. It can be represented either in column ~~matrix~~ vector or row matrix.

$$\cancel{P = \begin{bmatrix} x \\ y \end{bmatrix}} \quad \cancel{P' = \begin{bmatrix} x' \\ y' \end{bmatrix}} \quad \cancel{T = \begin{bmatrix} t_x \\ t_y \end{bmatrix}}$$

We can write it as

$$P' = P + T$$

$$[x' \ y'] = [x \ y] + [t_x \ t_y] \quad \text{row major}$$

or

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix} \quad \text{column major}$$

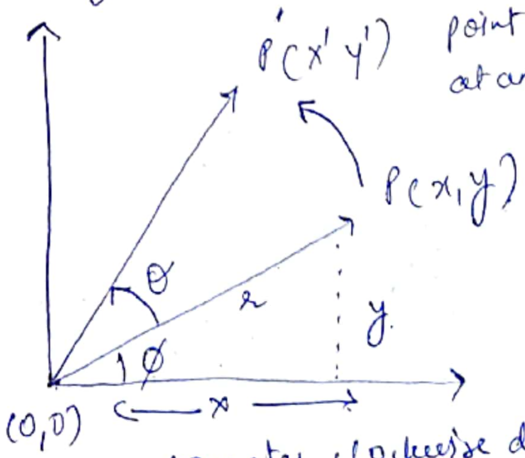
(2) Rotation -

In this we rotate the object at particular angle θ from its origin.

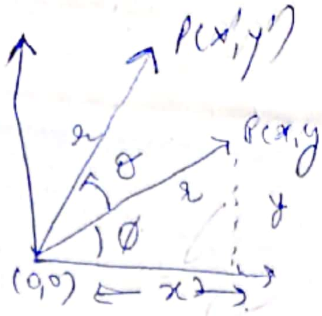
Let us suppose we want to rotate at an angle θ.

After rotating it to a new location, you will get a new point $P(x', y')$.

Rotate a point P at an angle θ



CASE 1: counter clockwise direction rotation



find $\cos\phi = \frac{B}{H} = \frac{x}{r}$

$\Rightarrow x = r\cos\phi$ — ①

$\sin\phi = \frac{P}{H} = \frac{y}{r}$

$y = r\sin\phi$ — ②

$\frac{P}{H}$ $\frac{B}{H}$ $\frac{P}{B}$

$P = y$ $B = x$
$H = r$

$P'(x' y')$ can be represented as

$x' = r\cos(\overset{A}{\theta} + \overset{B}{\phi})$

Applying formula

$\Rightarrow r\cos\theta\cos\phi - r\sin\theta\sin\phi$ — ③

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

$\sin(A+B) = \sin A\cos B + \cos A\sin B$

$y' = r\sin(\theta+\phi)$

$\Rightarrow r\sin(\overset{A}{\phi} + \overset{B}{\theta})$

$\Rightarrow r\cos\phi\sin\theta + r\sin\phi\cos\theta$ — ④

put ①, ② in ③ and ④

$\therefore x' = x\cos\theta - y\sin\theta$

$y' = x\sin\theta + y\cos\theta$

$\Rightarrow$ Representation in matrix form.

$$[x' \ y'] = [x \ y]\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$ Row major

$P' = P \cdot R$ → Rotation matrix

Rotation in column major

$x' = x\cos\theta - y\sin\theta$

$y' = x\sin\theta + y\cos\theta$

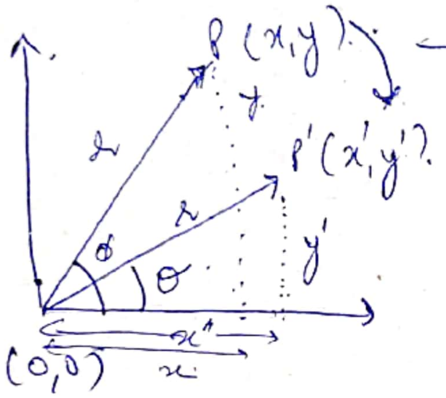
In matrix form

column major

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$P' = R \,.\, P$$

Case 2 clockwise direction



→ clockwise rotation

initial angle = ϕ

new angle, $\phi - \theta$.

$x = r\cos\phi$

$y = r\sin\phi$

$x' = r\cos(\phi - \theta)$

$y' = r\sin(\phi - \theta)$

$x' = r\cos\phi\cos\theta + r\sin\phi\sin\theta$

$\boxed{x' = x\cos\theta + y\sin\theta}$

$y' = r\sin\phi\,.\cos\theta - r\cos\phi\sin\theta$

$\boxed{y' = y\cos\theta - x\sin\theta}$

$x' = x\cos\theta + y\sin\theta$

$y' = y\cos\theta - x\sin\theta$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$ clockwise direction

row major

column major

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

③ Scaling

To change the size of an object, scaling transformation is used.

In this scaling process, we can either expand or compress the dimensions of the object.

It can be achieved by multiplying the original coordinates of the objects with the scaling factor to get the desired result.

Let us assume that the original coordinates are (x, y), the scaling factor (S_x, S_y) and the produced (new) coordinates are (x', y').

This can be mathematically represented as.

$x' = x\,S_x$ — ① and $y' = y.S_y$ — ②

If S_x, S_y lies between 0 and 1 size of the object decreases

If S_x, S_y is greater than 1 the size of the object increases

If $S_x = S_y$ uniform scaling size does not increase.

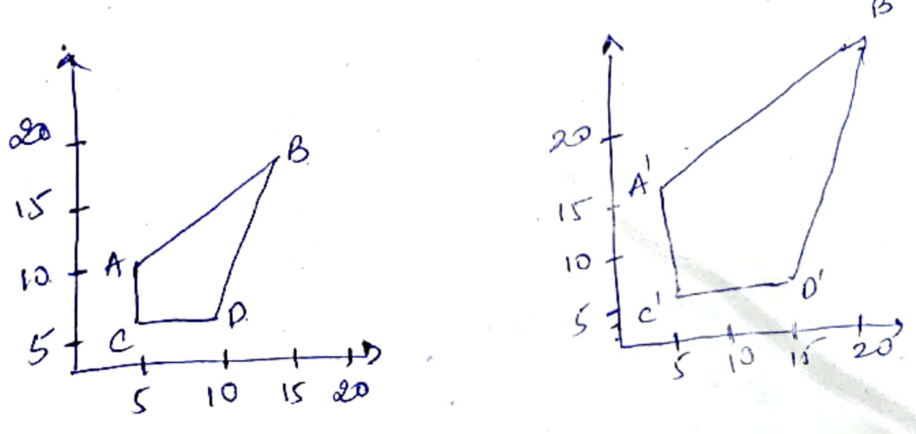
The scaling factors S_x and S_y scales the object in x and y direction respectively.

In matrix form eq. can be written as

$\cancel{\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix}}$ or $\boxed{P' = P.S.}$

where S. is scaling matrix

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

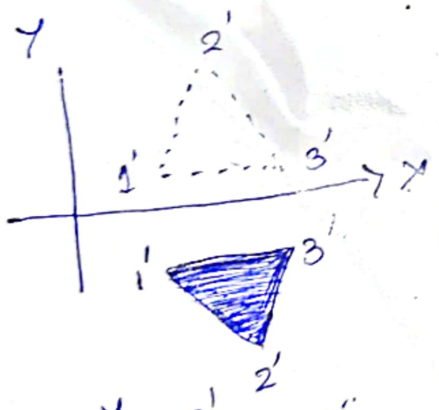


If we have the value of scaling factor less than 1, then we reduce the size of the object

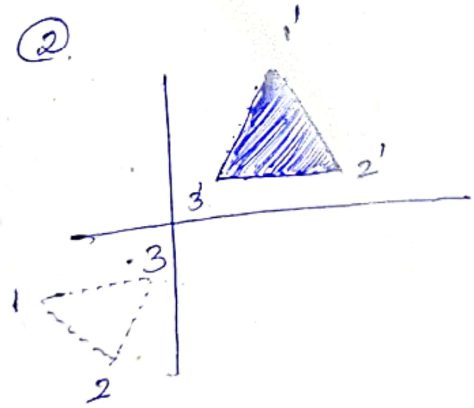
If we provide values of scaling factor greater than 1 than we can increase the size of the object.

④. <u>Reflection</u> - Reflection is Mirror image of original object or it is a rotation operation with 180°. The size of the object does not change in reflection.

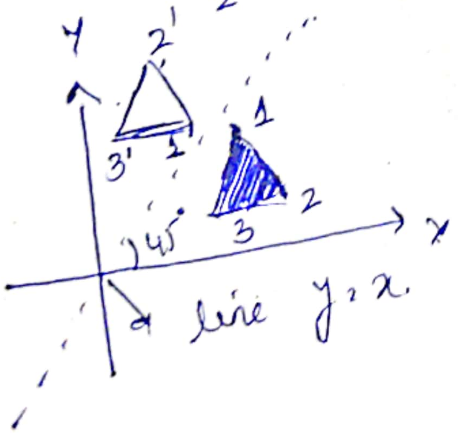
①



②

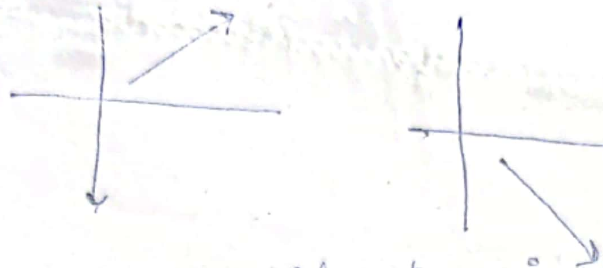


③.



Cases of Reflection

(1)

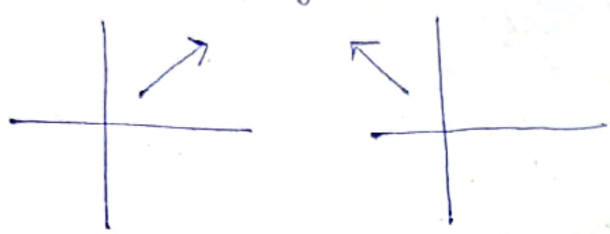


$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

about x axis.

x coordinate same
y " negative

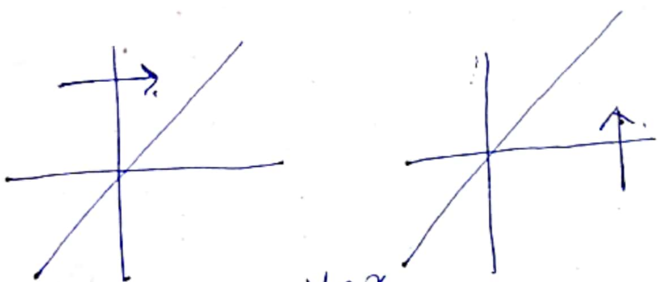
(2)



$$\begin{array}{cc} x & \\ \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \\ & y \end{array}$$

about y axis

(3)



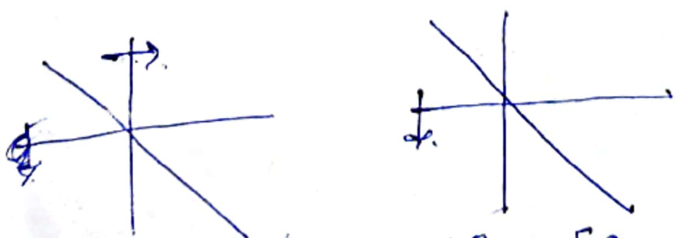
$y = x$

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$x = y$ axis

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} y \\ x \end{bmatrix} \quad \begin{array}{l} x' = y \\ y' = x \end{array}$$

(4)



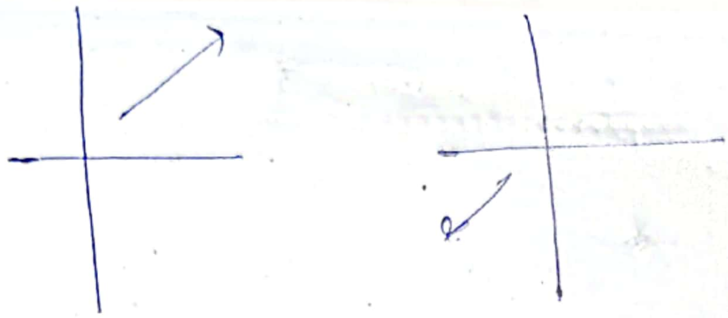
$$\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$$

$y = -x$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -y \\ -x \end{bmatrix}$$

$(x' = -y,\ y' = -x)$

(5). about origin.



$$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

x	y
0	0
-1	1
-2	2.

(5). **Shear**

A transformation that slants the shape of an object is called shear transformation

They are of 2 types

x shear

y shear.

Suppose if books are kept in a rack and you take one book out all the books are slanted. This is shear.

If it is x shear. - x coordinate values is shifted

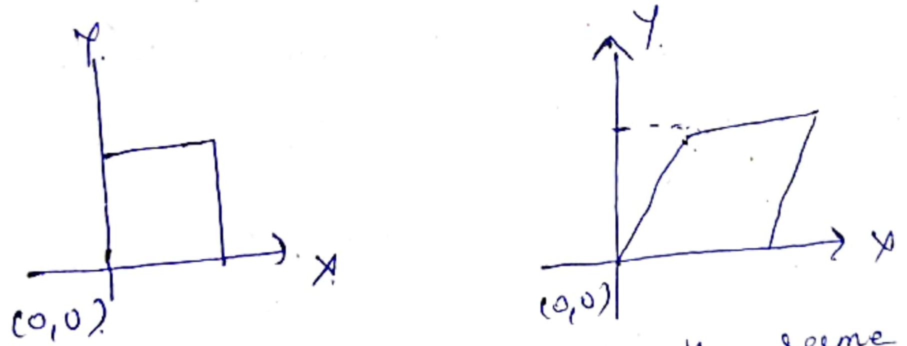
it is y shear - y coordinate values is shifted

However in both the cases only 1 coordinate changes its coordinates and the other preserves it values.

Note: shearing is also termed as skewing.

(a) X Shear

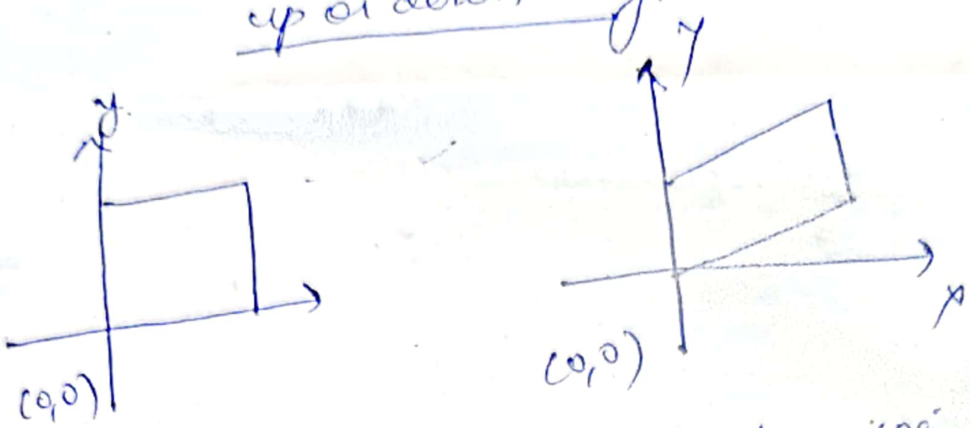
It preserves the y coordinate and changes are made to the x coordinate which causes the verticle line to tilt right or left as shown in the fig



y remains the same
x changes

(b) Y-shear - changes made in Y coordinate
it preserves x coordinate
This causes the horizontal lines to transform into lines which slopes up or down as shown in the fig.

up or down shift



x is preserved and y is changing