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1 2 3

APPOINTMENTS/MEETINGS

8AM

Homogeneous Coordinate System

- 9 To perform sequence of transformation such as
- 10 translation followed by rotation and
- 11 scaling, we need a sequence of process -

12 (i) Translate the coordinates

13 (ii) Rotate the translated coordinates & then

14 (iii) Scale the rotated coordinates to complete the composite transformation

15 To shorten this process, we have to use

16 3x3 transformation matrix instead of

17 2x2 transformation matrix.

18 6 To convert a (2x2) matrix to (3x3) matrix we add an extra dummy coordinate

In this way we can represent the point by 3 nos. instead of 2 nos, which is called homogeneous coordinate system.

We use homogeneous coordinate

b/c. addition creates problem if we want multiple transformation at the same time

JANUARY

WEEK 04

24

WEDNESDAY

024-341

APPOINTMENTS/MEETINGS

8 AM ie. writing down 2007

$$[R] = [t_{xy}] + [R_0][\theta][R][\theta]$$

In this system, we can represent all the transformation eqns in a matrix.

multiplication (normal)

Any cartesian point $P(x, y)$ can be converted to homogeneous coordinates by $P' (x_h, y_h, h)$.

① TRANSLATION

A translation moves an obj to a different position

$$(x, y) \rightarrow (x_h, y_h, h) \quad h = \text{homogeneous value}$$

depending on h , x and y can change new values

$$x_h = x_0 \cdot h - 1 \quad \text{any non zero value}$$

$$y_h = y_0 \cdot h - 1$$

DECEMBER '06						
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JANUARY '07						
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2007

WEEK 04

JANUARY

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APPOINTMENTS/MEETINGS

8AM

Back to 2D from ① & ②

9

$$x = x_n/a$$

10

$$y = y_n/h$$

11

To make calculate easy suppose $h=1$

12

$$\text{eq. } (2, 3) \text{ for } h=2$$

1PM

H.

$$(2 \times 2, 3 \times 2, 2)$$

H.

$$(4, 6, 2)$$

4

5

6

$$x' = x \cdot h \left(\frac{4}{2}, \frac{6}{2} \right) \quad \left. \begin{array}{l} \text{for 2-D. convert} \\ \text{by } h. \end{array} \right\}$$

$$\Rightarrow (2, 3)$$

→ modified w.r.t.
calculated value given

JANUARY

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FRIDAY

026-339

Conversion to homogeneous
(3D)

DECEMBER '06

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APPOINTMENTS/MEETINGS

8AM

Translation

$$9 \quad x' = x + tx$$

$$10 \quad y' = y + ty$$

$$11 \quad \rightarrow \begin{bmatrix} t_x \\ ty \end{bmatrix} + \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x' \\ y' \end{bmatrix}$$

$$12 \quad \begin{bmatrix} x' \\ y' \\ h \end{bmatrix} = \begin{bmatrix} 1 & 0 & tx \\ 0 & 1 & ty \\ 0 & 0 & h \end{bmatrix} \begin{bmatrix} x \\ y \\ h \end{bmatrix} = \begin{bmatrix} x+tx \\ y+ty \\ h \end{bmatrix}$$

$$13 \quad x + 0 + txh$$

$$14 \quad x + 0 + txh$$

$$15 \quad 0x + 0y + tyh$$

$$16 \quad 0x + 0y + tyh$$

$$17 \quad 0 + 0 + h \cdot h$$

$$18 \quad y + ty$$

$$\begin{bmatrix} x+tx \\ y+ty \\ h \end{bmatrix}$$

So

$$\begin{bmatrix} x' \\ y' \\ h \end{bmatrix} = \begin{bmatrix} x+tx \\ y+ty \\ h \end{bmatrix}$$

is the translation matrix after translation

14630 / 18370

2007

WEEK 04

JANUARY

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MARCH '07						
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SATURDAY

027-338

27

APPOINTMENTS / MEETINGS

8AM 2. Rotation

$$x' = x \cos \theta - y \sin \theta$$

$$y' = x \sin \theta + y \cos \theta$$

In matrix form

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

1PM

In homogeneous coordinates (anti clockwise θ, h)

$$\begin{bmatrix} x' \\ y' \\ h \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & h \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

5

6 ③ Scaling

$$x' = x \cdot s_x$$

$$y' = y \cdot s_y$$

$$\text{i.e. } \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x \cdot 0 & 0 & 0 \\ 0 & s_y \cdot 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

SUNDAY 28

Or

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

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029-336

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2007

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APPOINTMENTS/MEETINGS

8AM S. Shearing

x shear

$$\begin{bmatrix} x' \\ y' \\ h \end{bmatrix} = \begin{bmatrix} 1 & s_{hx} & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

1PM y shear

$$\begin{bmatrix} x' \\ y' \\ h \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

5 (16) Reflection

6 Reflection around x axes

$$\begin{bmatrix} x' \\ y' \\ h \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Reflection around origin

$$\begin{bmatrix} x' \\ y' \\ h \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

2007

WEEK 05

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MARCH '07						
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29	30	31				

TUESDAY

030-335

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APPOINTMENTS/MEETINGS

APPOINTMENTS/MEETINGS

8AM

Reflection around y-axis

9

$$\begin{bmatrix} x' \\ y' \\ w \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

10

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x' \\ y' \\ w \end{bmatrix}$$

11

Reflection about $y=x$

1PM

$$\begin{bmatrix} x' \\ y' \\ w \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

2

4

5

6

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x' \\ y' \\ w \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

JANUARY

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2007

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APPOINTMENTS/MEETINGS

8AM

Xumericals

9. Translate a polygon with coordinates
 10. $A(2, 5)$, $B(7, 10)$ and $C(10, 2)$ by
 11. 3 units in X direction and 4 units in
 12. Y direction.

1PM

$$A' = A + T$$

2

$$2 \begin{bmatrix} 2 \\ 5 \end{bmatrix} + \begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 5 \\ 9 \end{bmatrix}$$

4

$$B' = B + T$$

5

$$2 \begin{bmatrix} 7 \\ 10 \end{bmatrix} + \begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 10 \\ 14 \end{bmatrix}$$

6

$$C' = C + T$$

$$2 \begin{bmatrix} 10 \\ 2 \end{bmatrix} + \begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 13 \\ 7 \end{bmatrix}$$

FEBRUARY WEEK 05

1

THURSDAY

032-333

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FEBRUARY '07						
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8 AM 2. A point $(4, 3)$ is rotated counter clockwise by angle of 45° . find the rotation matrix and the resultant point.

$$R_2 \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos 45^\circ & \sin 45^\circ \\ -\sin 45^\circ & \cos 45^\circ \end{bmatrix}$$

$$= \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$

$$P' = \begin{bmatrix} 4, 3 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$

$$\therefore \begin{bmatrix} 4/\sqrt{2} - 3/\sqrt{2} & 4\sqrt{2} + 3\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$

2007

WEEK 05

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MARCH '07						
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FRIDAY

033-332

2

APPOINTMENTS / MEETINGS

8 AM Composite transformation

9 A composite member of transformations or sequence of transformations can be combined into single one called as composition.

10 The resulting matrix is called as composite matrix. The process of combining 11 is called as concatenation.

1 PM 12 A composite transformation is when two or more transformations are performed on a figure 13 to produce a new figure.

3 Q. Derive composite matrix

(A) Translation -

We will apply translation two times on an object (T_1 and T_2)

4 Suppose $P(x, y)$ is the object

5 after T_1 and T_2 is applied

the object will be $P'(x', y')$

$$\text{So } P' = T_2(x_2, y_2) [T_1(x_1, y_1), P]$$

$T_2(x_2, y_2)$
 $T_1(x_1, y_1)$
 P

P'
 $T_1(x_1, y_1)$

FEBRUARI

3

SATURDAY

034-331

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M	T	W	T	F	S	S
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8AM

As we know matrix multiplication is not commutative but it is associative
 $A \cdot B \neq B \cdot A$ where A & B are matrices

10

11

$$(A \cdot B) \cdot C = A \cdot (B \cdot C)$$

12

so similarly

$$P = T_2(x_2, y_2) \cdot [T_1(x_1, y_1), P]$$

can also be written as

$$\Rightarrow [T_2(x_2, y_2), T_1(x_1, y_1)] \cdot P$$

5

6

This is called composition of translation

matrices.

now let's see & solve

$$T_2(x_2, y_2) \cdot T_1(x_1, y_1)$$

04 SUNDAY

$$\Rightarrow \begin{bmatrix} 1 & 0 & x_2 \\ 0 & 1 & y_2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & x_1 \\ 0 & 1 & y_1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & x_1 + x_2 \\ 0 & 1 & y_1 + y_2 \\ 0 & 0 & 1 \end{bmatrix}$$

2007

WEEK 06

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APRIL '07						
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MONDAY

036-329

5

APPOINTMENTS/MEETINGS

8 AM This composite translation matrix is additive in nature so it can also be written as

$$P' = T(x_1 + x_2, y_1 + y_2) \cdot P$$

② Rotation

We will apply rotation two times on an object $R(\theta_2) \cdot R(\theta_1)$ $R(\theta_2) \leftarrow \theta_2 - \theta_1$

Suppose the point is $P(x, y)$ (object)

after θ_1 and θ_2 is applied the object will be P' .

$$\text{so } P' = R(\theta_2) [R(\theta_1) \cdot P]$$

it can also be written as

$$P' = [R(\theta_2) \cdot R(\theta_1)] P$$

This is called composite rotation matrix.

FEBRUARY WEEK 100

6

TUESDAY

037-328

APPOINTMENTS/MEETINGS

8 AM

$$R(\theta) = \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

9

11
for

$$R(\theta_2) \cdot R(\theta_1)$$

1 PM

$$\Rightarrow \begin{bmatrix} \cos \theta_2 & -\sin \theta_2 & 0 \\ \sin \theta_2 & \cos \theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \theta_1 & -\sin \theta_1 & 0 \\ \sin \theta_1 & \cos \theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

2

$$\Rightarrow \begin{bmatrix} \cos \theta_2 \cos \theta_1 & -\sin \theta_2 \sin \theta_1 & 0 \\ \sin \theta_2 \cos \theta_1 & \cos \theta_2 \sin \theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

3

$$\Rightarrow \begin{bmatrix} \cos(\theta_1 + \theta_2) & -\sin(\theta_1 + \theta_2) & 0 \\ \sin(\theta_1 + \theta_2) & \cos(\theta_1 + \theta_2) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

composite rotation matrix is additive in nature

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Basic rotation matrix

2007

WEEK 06

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29	30	31	*	23	24	25

APPOINTMENTS/MEETINGS

8 AM
so $P' = R(\theta_2) \cdot R(\theta_1) \cdot P$

9
 $\Rightarrow R(\theta_1 + \theta_2) \cdot P$

③ Scaling

12 We will apply scaling values on an object. P. published no trade related

2 after s_x and s_y is applied the new object
3 will be P'

4 so $P' = s_2 \cdot (s_{x_2} s_{y_2}) [s_1 (s_{x_1} s_{y_1}) \cdot P]$

5 $= [s_2 (s_{x_2} s_{y_2}) s_1 (s_{x_1} s_{y_1})], P$

6 This is composite scaling matrix

2 $\begin{bmatrix} s_{x_2} & 0 \\ 0 & s_{y_2} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} s_{x_1} & 0 \\ 0 & s_{y_1} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

2) $\begin{bmatrix} s_{x_2} s_{x_1} & 0 & 0 \\ 0 & s_{y_2} s_{y_1} & 0 \\ 0 & 0 & 1 \end{bmatrix}$

FEBRUARY

WEEK 06

8

THURSDAY

039-326

APPOINTMENTS/MEETINGS

8AM

$$P' = S(s_{x_1}, s_{x_2}, s_{y_1}, s_{y_2}) \cdot P$$

composite scaling matrix

11

Exercise:

Now suppose we want to perform rotation about an arbitrary point, then we can't perform it by a sequence of transformations.

1. Translation

2. Rotation

3. Reverse Translation

The ordering sequence of these numbers of transformations must not be changed.

Steps:

①. Translate fixed point to origin.

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FEBRUARY '07						
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2007

WEEK 06

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FRIDAY

040-325

9

APPOINTMENTS / MEETINGS

8 AM

$$T_v = \begin{bmatrix} 1 & 0 & -h \\ 0 & 1 & -k \\ 0 & 0 & 1 \end{bmatrix}$$

 \rightarrow translation matrix.

12 we have to decrease h points along x axis

1 PM and k points along y axis.

2 so $t_x = -h$ and $t_y = -k$ substituting the

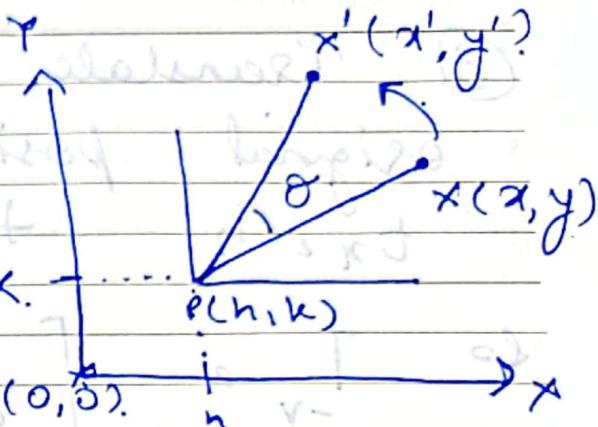
3 value in translation matrix.

② Now perform Rotation (R_θ).5 We are rotating anti-clockwise through 6 so θ will be positive.

$$R_\theta =$$

$$\begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- Rotation matrix



FEBRUARY

WEEK 00

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- ③ Translate the point back to original position
 $\rightarrow \text{tx} = h$ $\text{ty} = k$
- ④ T_{-v}
- ⑤ R_o
- ⑥ T_v
- ⑦ $T_{-v} R_o T_v$
- ⑧ $T_{-v} R_o T_v$

Together the three steps we have to make a Transformation matrix.

$$\begin{bmatrix} T_{-v} \\ R_o \\ T_v \end{bmatrix}$$

The operation precedence is from right to left

left
middle
right

Object

11 SUNDAY New Transformation matrix

2) T_o old object matrix

3) \cdot Answer.

2007

2007

WEEK 07

FEBRUARY

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12	13	14	15	16	17	18
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26	27	28	29	30	31	*

APRIL '07						
M	T	W	T	F	S	S
30	*	*	*	*	*	1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29

MONDAY

043-322

12

APPOINTMENTS/MEETINGS

8AM

Question 1

- 9 Perform 45° rotation of a $\Delta A(0,0)$,
 $B(1,1)$ & $C(5,2)$ about the point $(-1, -1)$.

11



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FEBRUARY

13

TUESDAY

044-321

APPOINTMENTS/MEETINGS

8 AM

Q. consider a square $A(1,0)$, $B(0,0)$, $C(0,1)$, $D(1,1)$. Rotate pt by 45° clockwise about the point $A(1,0)$.

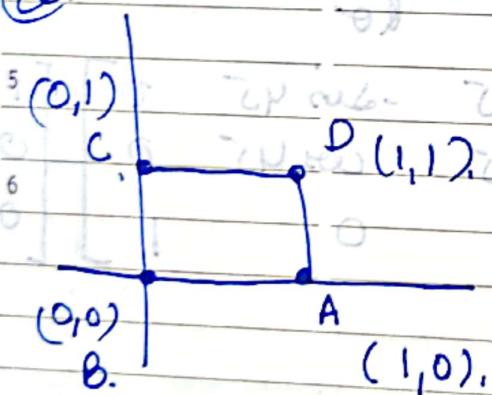
Soln. clockwise rotation so $\theta = -45^\circ$

12

① We have to rotate about a fixed point. We have to shift to origin

$$\text{so } T_{vz} \begin{bmatrix} 1 & 0 & b_x \\ 0 & 1 & b_y \\ 0 & 0 & 1 \end{bmatrix} z \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

②



$$R_{\theta}^2 \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$2) \begin{bmatrix} \cos(-45) & -\sin(-45) & 0 \\ \sin(-45) & \cos(-45) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

2007

WEEK 07

FEBRUARY

MARCH '07						
M	T	W	T	F	S	S
•	•	•	1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	*

APRIL '07						
M	T	W	T	F	S	S
30	*	*	*	*	*	1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29

APPOINTMENTS / MEETINGS

8 AM

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*

sin	All
tan	cos

045-320

14

(3) Agais translation

Tx = 1

Ty = 0.

$$Tv = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

new object matrix after transformation

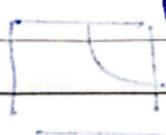
$$\text{new matrix} = [R^T \quad -v \quad R_o \quad Tv] \cdot X_o \quad (\text{object matrix})$$

new object matrix = new matrix * object matrix

$$\Rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

all better in respects to rotation all

$$\begin{bmatrix} 1 & 0 & 1/\sqrt{2} \\ 0 & 1 & -1/\sqrt{2} \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



FEBRUARY

WEEK 07

15

THURSDAY

046-319

APPOINTMENTS/MEETINGS

8 AM

A

 $P(\text{odd})$

1, 0

9

B

0, 0

10

C

0, 1

11

D

1, 1

12

1 PM

PA Live

 $P'(\text{new})$

1, 0

 $-\sqrt{2}, \sqrt{2}$

Ans

1, $\sqrt{2}$ $1 + \sqrt{2}, \sqrt{2}$

2 Clipping -

The technique of hiding that part of the

drawing (one) is not interested in is

called clipping. We can clip a point

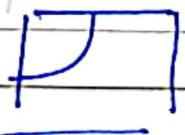
line, polygon or a curve

The portion left outside the region of

the window in graphics is called the

clipped part and the process of displaying

the inside image is called clipping.



JANUARY '07

M	T	W	T	F	S	S
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

M	T	W	T	F	S	S
.	.	.	1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28				

2007

(MARCH '07)						
S	M	T	W	T	F	S
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31	1*	2*	3*

APPOINTMENTS/MEETINGS

(APRIL '07)						
S	M	T	W	T	F	S
	1*	2*	3*	4*	5*	6*
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31	1*	2*	3*

WEEK 07

FEBRUARY

FRIDAY

047-318

16

Window and Viewport

8AM ~~designed~~ and ~~introduction~~ ~~existing~~ ~~with~~ ~~the~~ ~~basic~~
 9 2D Viewing ~~language~~ ~~of~~ ~~beginner~~ ~~with~~ ~~the~~ ~~basic~~
 10 A graphics package allows a user to specify
 11 which part of a defined picture is to be
 12 displayed and where that part is to be
 1PM displayed on the display device using a
 2 concept called clipping.

3 ~~subject~~ all ~~a~~ ~~first~~ ~~most~~ ~~problems~~ ~~not~~
 4 * Any conventional cartesian coordinate system
 5 referred to as the world coordinate
 6 reference frame can be used to define the
 picture

* for a 2D picture a view is selected
 by specifying a sub area of the total picture
 area

max X
min X
max Y
min Y
where to be specified

point
area
subarea defined

FEBRUARY

17

SATURDAY

048-317

JANUARY '07

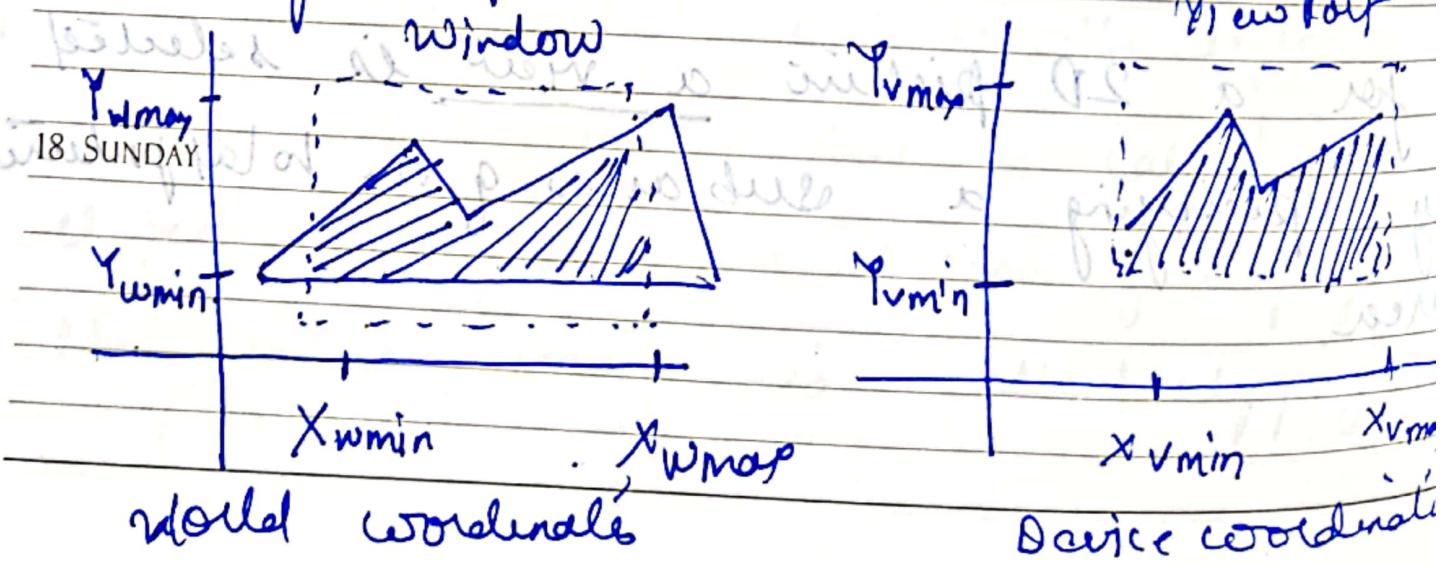
M	T	W	T	F	S	S
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

M	T	W	T	F	S	S
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19	20	21	22	23	24	25
26	27	28				

APPOINTMENTS/MEETINGS

8 AM

- * The picked parts within the selected areas are then mapped to specific areas of the device. coordinates.
- * Transformation from world \rightarrow device coordinates involves translation, rotation, scaling operations as well as procedures for deleting those part of the picked that are outside the limits of a selected display area.

Viewing Pipe line

2007

Windows -viewport

MARCH '07							APRIL '07				
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5	6	7	8	9	10	11	2	3	4	5	6
12	13	14	15	16	17	18	9	10	11	12	13
19	20	21	22	23	24	25	16	17	18	19	20
26	27	28	29	30	31	•	23	24	25	26	27

APPOINTMENTS / MEETINGS

- 8AM ~~Several meetings today & working~~
- 1) A world coordinate area selected for display is called a window.
 - 2) An area on a display device to which a window is mapped is called viewport.
 - 3) Window defines - What is to be viewed.
 - 4) Viewport - where it is to be displayed.
- Mapping of a part of a world coordinate scene to device coordinates is referred to as Viewing Transformation.
- Sometimes, the 2D viewing transformation is simply referred to as the window to viewport transformation or the WINDOWING TRANSFORMATION.

What is it?

WEEK 08

FEBRUARY

050-315

19

FEBRUARY

WEEK 08

20

TUESDAY

051-314

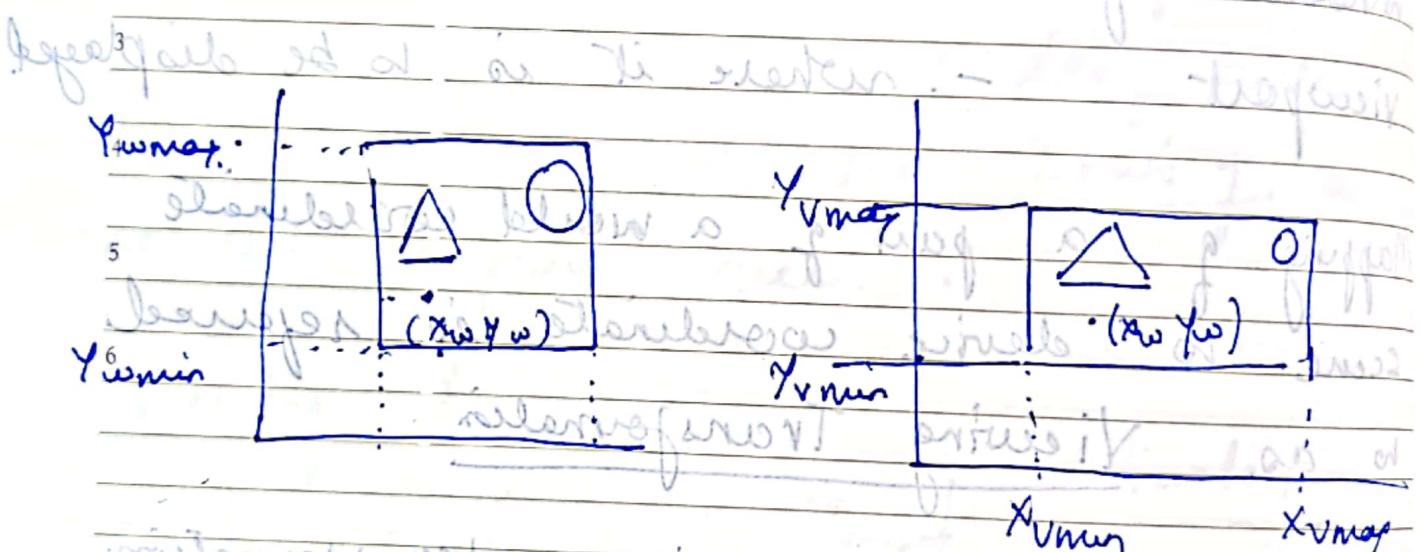
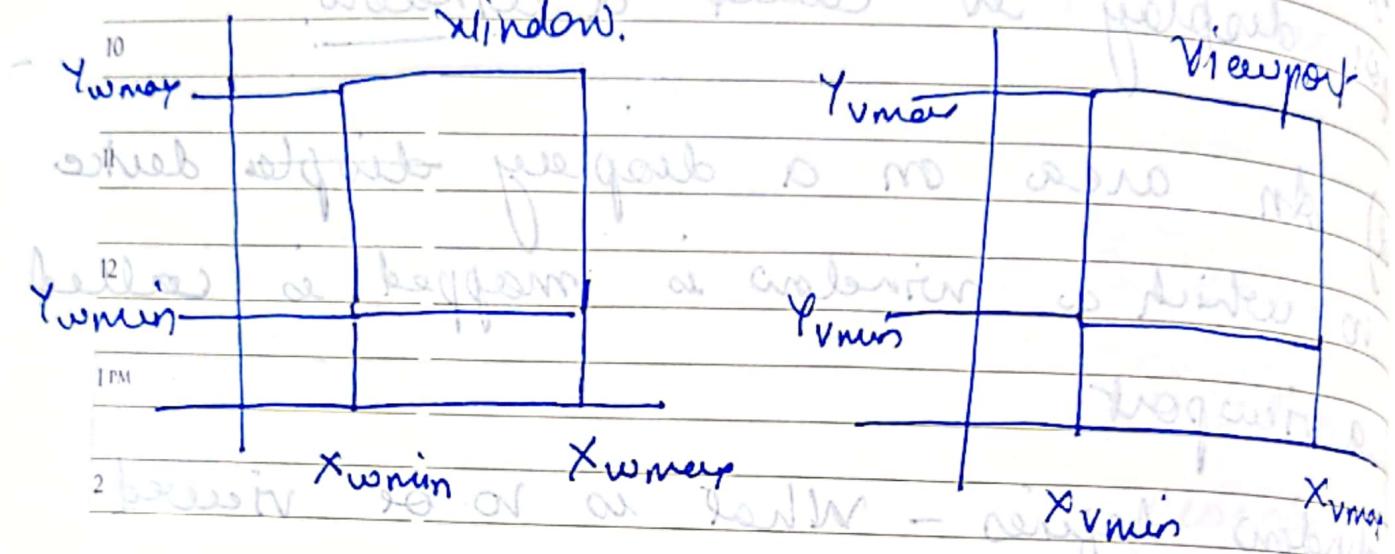
JANUARY '07

M	T	W	T	F	S	S
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
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M	T	W	T	F	S	S
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

APPOINTMENTS/MEETINGS

Window \rightarrow Viewport Transformation formula
derivation



position (Relative position) will not change
only size will change according to windows
& viewport.

TRANSFORMATION

PHOTOSHOP

WEEK 08

FEBRUARY

WEDNESDAY

052-313

21

2007

MARCH '07						
M	T	W	T	F	S	S
1	2	3	4	5	6	7
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22	23	24	25	26	27	28
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APRIL '07						
M	T	W	T	F	S	S
30	•	•	•	•	•	1
1	2	3	4	5	6	7
8	9	10	11	12	13	14
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22	23	24	25	26	27	28
29	30	31				

APPOINTMENTS/MEETINGS

Let us have $(x_w y_w)$ given we have to find $(x_v y_v)$

$$\frac{x_w - x_{w\min}}{x_{w\max} - x_{w\min}} = \frac{x_v - x_{v\min}}{x_{v\max} - x_{v\min}} \quad \text{because relative pos is same}$$

$$\frac{y_w - y_{w\min}}{y_{w\max} - y_{w\min}} = \frac{y_v - y_{v\min}}{y_{v\max} - y_{v\min}}$$

From eq 1, 2

$$x_v - x_{v\min} = \frac{(x_{v\max} - x_{v\min})}{(x_{w\max} - x_{w\min})} \cdot (x_w - x_{w\min})$$

$$x_v - x_{v\min} = \frac{(x_w - x_{w\min})}{x_{w\max} - x_{w\min}} \cdot (x_{v\max} - x_{v\min})$$

$$x_v - x_{v\min} = \frac{(x_w - x_{w\min})}{x_{w\max} - x_{w\min}} \cdot \left[\frac{(x_{v\max} - x_{v\min})}{(x_{w\max} - x_{w\min})} \right]$$

Scaling

FEBRUARY WEEK 08
22 THURSDAY
053-312

JANUARY '07

M	T	W	T	F	S	S
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31	*	*	*	*

M	T	W	T	F	S	S
*	*	*	1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	*	*	*	*

APPOINTMENTS / MEETINGS

$$x_v - x_{vmin} = (x_w - x_{wmin}) s_x$$

$$x_v = x_{vmin} \pm (x_w - x_{wmin}) s_x \quad \rightarrow ①A$$
~~$$x_v = x_{vmin} \pm (x_w - x_{wmin}) s_x$$

$$x_v = x_{vmin} + (x_w - x_{wmin}) s_x$$

$$x_v = x_{vmin}$$~~

Summary

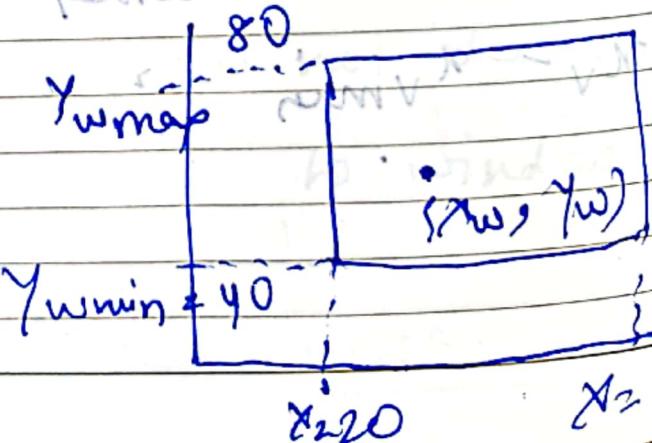
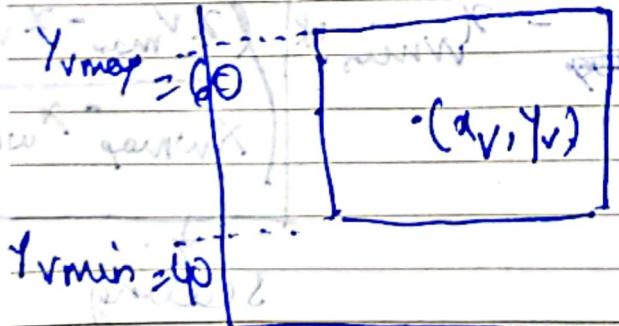
$$y_v = y_{vmin} + (y_w - y_{wmin}) s_y$$

where s_x and s_y - scaling factor

$$x_v = x_{vmin} + (x_w - x_{wmin}) s_x$$

e.g. Viewport

window



2007

WEEK 08

FEBRUARY

MARCH '07						
M	T	W	T	F	S	S
•	•	1	2	3	4	
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	*

APRIL '07						
M	T	W	T	F	S	S
30	*	1	2	3	4	5
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	*	1	2	3	4	5

FRIDAY

054-3II

23

APPOINTMENTS / MEETINGS

8 AM

$$S_x = \frac{x_{v_{max}} - x_{v_{min}}}{x_{w_{max}} - x_{w_{min}}} \quad 08 = \checkmark$$

$$\frac{y_2 - y_{v_{min}}}{y_{w_{max}} - y_{w_{min}}} \quad 18 = \checkmark$$

$$\left| \frac{y_2 - y_{v_{max}}}{y_{w_{max}} - y_{w_{min}}} \right| \quad 18 = \checkmark$$

$$x_{v_{min}} = 30.$$

$$x_{v_{max}} = 60$$

$$y_{v_{min}} = 40$$

$$y_{v_{max}} = 60$$

$$(x_v, y_v) = ? \quad \text{Find out.}$$

$$x_{w_{min}} = 20$$

$$x_{w_{max}} = 80$$

$$y_{w_{min}} = 40$$

$$y_{w_{max}} = 80$$

$$x_w, y_w = (30, 60)$$

$$\frac{x_v - x_{v_{min}}}{x_{v_{max}} - x_{v_{min}}} = \frac{x_w - x_{w_{min}}}{x_{w_{max}} - x_{w_{min}}}$$

$$\frac{x_v - 30}{60 - 30} = \frac{30 - 20}{80 - 20}$$

$$\Rightarrow \frac{x_v - 30}{60 - 30} = \frac{30 - 20}{80 - 20}$$

FEBRUARY

WEEK 6

24

SATURDAY

055-310

APPOINTMENTS/MEETINGS

8 AM

$$x_v = 30 = \left(\frac{10}{60} \right) 30$$

9

$$x_v = 35$$

10

11

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1 PM

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25 SUNDAY

JANUARY '07

M	T	W	T	F	S	S
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31	•	•	•	•

FEBRUARY '07						
•	•	•	1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	•