

Aptitude

* Prime Numbers : 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

29 Prime nos
1-100

* Test of Prime No. : $\boxed{n^2 - P}$ $P = \text{given No}$
check P is divisible by any of prime no. less than or equal to n .
if no, then it is a prime no.

* Divisibility :

* Divisibility by 4 :- its last two digit is divisible by 4.

* Divisibility by 8 :- if hundred, ten and unit digit divisible by 8.

* Divisibility by 11 :- if difference b/w the sum of its digits at odd places and the sum of its digits at even places is either 0

or a number divisible by 11.

Series formulae :

$$(i) 1 + 2 + 3 + \dots + n = \boxed{\frac{1}{2} n(n+1)}$$

Series formulae

(i) $1 + 2 + 3 + \dots + n = \frac{1}{2} n(n+1)$

(ii) $1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{1}{6} n(n+1)(2n+1)$

(iii) $1^3 + 2^3 + 3^3 + \dots + n^3 = \frac{1}{4} n^2(n+1)^2$

(iv) Arithmetic progression (A.P) :- $a, a+d, a+2d, a+3d, \dots$

⇒ $n^{\text{th}} \text{ term} = a + (n-1)d$

⇒ sum of n terms = $\frac{n}{2} [2a + (n-1)d]$

⇒ sum of n terms = $\frac{n}{2} (a+l)$, $l = \text{last term}$

(v) Geometric progression (G.P) :- a, ar, ar^2, ar^3, \dots ,

⇒ $n^{\text{th}} \text{ term} = ar^{n-1}$

⇒ sum of n term = $\begin{cases} a(r^n - 1) \\ \frac{a(r^n - 1)}{(r-1)} \end{cases}$, if $r > 1$

Imp.

$$1 + r + r^2 + r^3 + \dots$$

(a=1)

* sum of infinite G.P. \Rightarrow

$$\text{Sum} = \frac{a}{1-r}$$

* $\boxed{\text{dividend} = (\text{divisor} \times \text{quotient}) + \text{remainder}}$

divisor / dividend (quotient)

remainder

* (x^n+1) will be divisible by $(x+1)$ only when n is odd

* (x^n-1) will be divisible by $(x+1)$ only when n is even.

* for every Natural no. n , (x^n-a^n) is always divisible by $(x-a)$

* when n is even, (x^n-a^n) is completely divisible by $(x+a)$

* when n is odd, (x^n-a^n) is always divisible by $(x+a)$

H.C.F = Highest common factor

L.C.M = least common multiple

H.C.F = Highest common factor

L.C.M = least common multiple

* Product of two number = product of their H.C.F and L.C.M

* H.C.F & L.C.M of fractions :-

$$\left(\text{H.C.F} = \frac{\text{H.C.F of Numerators}}{\text{L.C.M of Denominators}}, \text{ L.C.M} = \frac{\text{L.C.M of Numerator}}{\text{H.C.F of Denominator}} \right)$$

* Basic formulae 1

$$1) (a^2 - b^2) = (a+b)(a-b)$$

$$2) (a+b)^2 = a^2 + b^2 + 2ab \quad (a-b)^2 = a^2 + b^2 - 2ab$$

$$3) (a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$$

$$4) (a^3 + b^3) = (a+b)(a^2 - ab + b^2)$$

$$5) (a^3 - b^3) = (a-b)(a^2 + ab + b^2)$$

$$6) (a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$7) (a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

$$(x-y) = \sqrt{(x+y)^2 - 4xy}$$

* Distance b/w two points :- If going speed s_1 & return speed s_2 time taken $= t \text{ hr.}$

D = T.S

$$\boxed{\text{Distance} = T \left(\frac{s_1 + s_2}{s_1 s_2} \right)}$$

* BODMAS Rule :- correct sequence in which operations are executed,

B \rightarrow Brackets, D \rightarrow Division, M \rightarrow Multiplication, A \rightarrow Addition &

S \rightarrow Subtraction

* Remove 'in the order', () {} and []

* Modulus of a Real Number $|a| = \begin{cases} a & \text{if } a > 0 \\ -a & \text{if } a < 0 \end{cases}$

* Square Root $= \sqrt{x}$, $x^{\frac{1}{2}}$ Cube root $3\sqrt{x}$, $(x)^{\frac{1}{3}}$

* Average Speed :- A man covers a certain distance at x kmph and equal distance at y kmph. Then
~~avg~~ the average speed during the whole journey is $\boxed{\frac{2xy}{x+y}}$ kmph

Percentage :-

1) If the price of a commodity increase by $R\%$, then the reduction in consumption so as not to increase the expenditure is $\left[\frac{R}{(100+R)} \times 100 \right]\%$.

* If price decrease by $R\%$, then the increase in consumption so as not to increase the expenditure is $\left[\frac{R}{(100-R)} \times 100 \right]\%$.

2) Result on population : Let the population of a town be P now and suppose it increase at the rate of $R\%$ per annum, then
① population after n years = $P \left(1 + \frac{R}{100} \right)^n$

② population n years ago = $P \left(1 - \frac{R}{100} \right)^n$

~~At a~~ At a ^{Actual} price P , the successive discount on d_1 and d_2 then the selling price

$$S.P = P \left(\frac{100-d_1}{100} \right) \left(\frac{100-d_2}{100} \right)$$

Result on Depreciation: Let present value of machine be P ,

suppose it depreciates at rate $R\%$ per annum then,

(I) value of machine after n year = $P \left(1 - \frac{R}{100} \right)^n$

(atq)

(II) value of machine n years ago = $\frac{P}{\left(1 - \frac{R}{100} \right)^n}$

4) If A is $R\%$ more than B , then B is less than A by

$$\left[\frac{R}{(100+R)} \times 100 \right]\%$$

If A is $R\%$ less than B , then B is more than A by

$$\left[\frac{R}{(100-R)} \times 100 \right]\%$$

* * * profit & loss \Rightarrow

$$\boxed{\text{Gain} = \text{S.P} - \text{C.P}}$$

S.P = selling price
C.P = cost price

* Loss or gain is always reckoned on C.P

$$* \boxed{\text{Gain \%} = \left(\frac{\text{Gain} \times 100}{\text{C.P}} \right)}, \quad \boxed{\text{Loss \%} = \left(\frac{\text{Loss} \times 100}{\text{C.P}} \right)}$$

$$* \boxed{\text{S.P} = \left(\frac{100 + \text{Gain \%}}{100} \right) \times \text{C.P}}$$

* If an article is sold at a gain of say, 35%, then

$$\text{S.P} = 135\% \text{ of C.P}$$

* If an article is sold at a loss of say 35%, then

$$\text{S.P} = 65\% \text{ of C.P}$$

* When a person sells two similar items, one at gain of say $x\%$ and the other at a loss of $x\%$, then the seller always incurs a loss given by

$$\boxed{\text{Loss \%} = \left(\frac{\text{common loss and gain}}{10} \right)^2 = \left(\frac{x}{10} \right)^2}$$

* If a trader professes to sell his goods at cost price, but uses false weights, then
Gain% = $\left[\frac{\text{Error}}{(\text{True value} - \text{Error})} \times 100 \right] \%$

* Proportion: $a:b = c:d$

if $\frac{a}{b} = \frac{c}{d}$ then $\left[\begin{array}{l} a+b = c+d \\ a-b = c-d \end{array} \right]$ (componendo and dividendo)

(A's share of profit = $\frac{x}{y}$) \leftarrow investment by A
B's share of profit \rightarrow by B

Chain rule :-

↳ Direct proportion :-

→ Cost is directly proportional to the number of articles
more articles, more cost

→ Work done is directly proportional to no. of men working on it.
more men, more work.

↗ Indirect proportion :-

→ More speed, less is the time to cover a distance
→ More persons, less is the time taken to finish a job

Chain Rule :-

1) Direct proportion :-

→ cost is directly proportional to the number of articles
more articles, more cost

→ work done is directly proportional to no. of men working on it.
more men, more work.

2) Indirect proportion :-

→ more speed, less is the time to cover a distance
→ more persons, less is the time taken to finish a job

Time and Work :-

* if A can do a piece of work in n days, then

$$\boxed{\text{A's 1 day's work} = \frac{1}{n}}$$

* If A is thrice as good a workman as B, then

$$\text{Ratio of work done by A and B} = 3 : 1$$

$$\text{Ratio of time taken by A and B to finish a work} = 1 : 3$$

Pipes and cisterns ↗

(1) If a pipe can fill a tank in x hours, then,
part filled in 1 hour = $\boxed{\frac{1}{x}}$

(2) If a pipe can empty a full tank in y hours, then,
part emptied in 1 hour = $\boxed{-\frac{1}{y}}$

(3) If a pipe can fill a tank in x hours and another pipe can
empty the full tank in y hours (where $y > x$),
then on opening both the pipes, the part filled
in 1 hour = $\boxed{\frac{1}{x} - \frac{1}{y}}$

Time and Distance :-

$$\boxed{\text{Speed} = \frac{\text{Distance}}{\text{time}}}$$

$$\boxed{\frac{1000}{3600} \times \frac{5}{18} x \text{ km/hr} = \left(x \times \frac{5}{18} \right) \text{ m/sec}}$$

* If the ratio of the speeds of A and B is $a:b$, then

* the ratio of the times taken by them to cover the same distance is $\left(\frac{1}{a} : \frac{1}{b} \right)$ or $(b:a)$

Problem on Trains :-

* Time taken by a train of length L meters to pass a pole or a standing man or a signal post is equal to the time taken by the train to covers L meters.

* Time taken by a train of length L meters to pass a stationary object of length B meters is the time taken by the train to cover $(L+B)$ meters.

* suppose two trains or two bodies are moving in the same direction at u m/s and v m/s. where $u > v$
then their relatives speed = $(u-v)$ m/s

* suppose two trains or two bodies are moving in opposite directions at u m/s and v m/s. then their relatives speed = $(u+v)$ m/s

* If two trains of length a meters and b metres are moving in opposite directions at u m/s and v m/s, then time taken by the trains to cross each other = $\frac{(a+b)}{(u+v)}$ sec

if moving in same direction = $\frac{(a+b)}{(u-v)}$ sec

~~Emp~~ two trains start at the same time from points A and B towards each other and after crossing they take a and b sec in reaching B and A respectively, then

$$\boxed{A's \text{ speed} : B's \text{ speed} = \sqrt{b} : \sqrt{a}}$$

Alligation or Mixture :-

* Rule of Alligation :- If two ingredients are mixed, then

$$\frac{\text{Quantity of cheaper}}{\text{Quantity of dearer}} = \frac{(\text{CP of dearer}) - (\text{mean price})}{(\text{mean price}) - (\text{CP of cheaper})}$$

CP of cheaper c $\frac{c}{d}$, CP of a unit quantity
of dearer m $\frac{m}{d}$

mean price

$$(dm) \quad (\text{CP of mixture}) \quad (m-c)$$

$$(\text{cheaper quantity}) : (\text{dearer quantity}) = (d-m) : (m-c)$$

cheaper

dearer

* Suppose a container contains x units of liquid from which y units are taken out and replaced by water.

After n operations, the quantity of pure liquid

$$= \boxed{x \left(1 - \frac{y}{x}\right)^n}$$

Interest

P → Principal

* simple interest: $\left[S.I = \frac{P \times R \times T}{100} \right]$ R → Rate, R%, per annum
T → Time, years.

$$A = P + S.I$$

$$A = P + \left(\frac{P \times R \times T}{100} \right)$$

Remark: The day on which money is deposited is not counted while the day on which money is withdrawn is counted.

* Compound Interest

When interest is compounded Annually :

$$\text{Amount} = P \left(1 + \frac{R}{100} \right)^n$$

When interest is compounded Half-Yearly :-

$$A = P \left(1 + \frac{R/2}{100} \right)^{2n}$$

When interest is compounded quarterly :-

$$A = P \left(1 + \frac{R/4}{100} \right)^{4n}$$

* When rates are different for different years, then

$$\text{Amount} = P \left(1 + \frac{R_1}{100} \right) \left(1 + \frac{R_2}{100} \right) \left(1 + \frac{R_3}{100} \right)$$

* Present worth of Rs x due n year :-

$$\left[\text{Present worth} = x \cdot \left(1 + \frac{R}{100} \right)^{-n} \right]$$

$$\log_a(a^m) = m \log_a a$$

where a is constant.

$$a^m = x$$

Logarithms :- $m = \log_a x$ if a is positive real number.

$$\log_a (xy) = \log_a x + \log_a y$$

$$\log_a x = 1$$

$$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$$

$$\log_a x^p = p \log_a x$$

$$\log_a 1 = 0$$

$$\log_a x = \frac{1}{\log_a a}$$

$$\log_a x = \log_b x = \frac{\log x}{\log b}$$

$$\log_a a = \log_a a$$

* Note :- When base is not mentioned, it is taken as 10

CALNDAR :-

* Odd days :- The no. of days more than the complete weeks are called odd days.

- * Leap Year :- (i) Every leap year is divisible by 4, if it is not a century
 - (ii) Every 4th century is a leap year and no other century is a leap year.

CALENDAR

- * odd days :- the No. of days more than the complete weeks are called odd days.
- * Leap year :- (i) every leap year is divisible by 4, if it is not a century
(ii) Every 4th century is a leap year and no other century is a leap year.

* A leap year has [366 days]

- Example (i) Each of years 1948, 2004, 1676 etc is a leap year
- (ii) Each of the years 1900, 800, 1200, 1600, 2000 etc is a leap year
- * ordinary year :- The year which is not a leap year is called an ordinary year. An ordinary year has 365 days.

* Counting of odd days :-

$$(i) \text{ 1 ordinary year} = 365 \text{ day} = (52 \text{ weeks} + 1 \text{ day})$$

∴ 1 ordinary year has 1 odd day

(ii) 1 leap year = 366 day = (52 weeks + 2 day)

∴ 1 leap year has 2 odd days.

(iii) 100 years = 76 ordinary years + 24 leap years
= $(76 \times 1 + 24 \times 2)$ odd days = 124 odd days
= ... (17 weeks + 5 days) = 5 odd days

$$\left[\frac{100}{4} = 25 \text{ but, } 100 \text{ is first century, so} \right]$$

* Number of odd days in 100 years = 5

* Number of odd days in 200 years = $(5 \times 2)^{100} = 3$ odd days

* Number of odd days in 300 year = $(5 \times 3)^{150} = 1$ odd day.

* Number of odd days in 400 year = $(5 \times 4 + 1)^{200}$ = 0 odd day

for $\frac{1}{4}$ leap year, 2 ordinary days.

* Similarly, each one of 800 years, 1200 years, 1600 years, 2000 years etc

has 0 odd day.

* Day of week related to odd days :-

| No. of days | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|-------------|-----|-----|------|------|-------|-----|------|
| Day | Sun | Mon | Tues | Weds | Thurs | Fri | Sat. |

| Month | Jan | Feb | March | April | May | June | July | Aug. | Sept. |
|-------|-----|-----|-------|-------|-----|------|------|------|-------|
| days | 31 | 29 | 28 | 31 | 30 | 31 | 31 | 30 | 31 |

If leap

year

31 30 31

Oct

Nov

Dec

Clocks :-

* Short hand (smaller one) :- hour hand

* long hand (larger one) :- minute hand

- (i) In 60 minute, the minute hand gains 55 minutes on the hour hand
 - (ii) In every hour, both the hands coincide once.
 - (iii) The hands are in the same straight line when they are coincident or opposite to each other. $1 \text{ day} = 12 \times 12 = 24 \text{ times}$
 - (iv) When the two hands are at right angles, they are 15 minute spaces apart.
 - (v) When the hands are in opposite directions, they are 30 minute spaces apart.
 - (vi) Angle traced by hour hand in 12 hrs = 360°
 - (vii) Angle traced by minute hand in 60 min = 360°
- ~~Too fast :- If watch indicate 8:15, correct time 8, it is 15 minutes too fast.~~
- ~~Too slow :- If watch indicate 7:45, correct time 8, it is 15 minutes too slow,~~

factorial,

$$n! = n(n-1)(n-2) \dots 3.2.1$$

$n = \text{positive integer}$

$$\boxed{0! = 1}, \boxed{1! = 1}, \boxed{2! = 2}, \boxed{3! = 6}, \boxed{4! = 24}, \boxed{5! = 120}$$

- # **PERMUTATIONS** : The different arrangements of a given no. of things by taking some or all at a time, are called permutations.
- * Number of permutations of n things, taken r at a time

$${}^n P_r = n(n-1)(n-2) \dots (n-r+1) = \frac{n!}{(n-r)!} \quad {}^n P_r = n!$$

- * No. of all permutations of n things, taken all at a time = $n!$
- * If n objects of which P_1 are alike one kind, P_2 are alike another kind, then No. of permutations = $\frac{n!}{P_1! P_2! \dots}$

- # **COMBINATIONS** : Selection which can be formed by taking some or all of a number of objects, is called a combination.
Note → that ab and ba are two different permutations but they represent the same combination.

COMBINATIONS: Selection which can be formed by taking some or all of a number of objects, is called a combination.

* Note that ab and ba are two different permutations but they represent the same combination

$${}^n C_r = \frac{n!}{(n-r)! r!}$$

$$= \frac{n(n-1)(n-2) \dots to r factors}{r!}$$

* Note that:

$${}^n C_n = 1$$

$${}^n C_0 = 1$$

*

Probability

- * Sample space: the set S of all possible outcomes
- * probability of occurrence of an Event.

Let S be the sample space and let E be an event.

$$P(E) = \frac{n(E)}{n(S)}$$

$$* P(S) = 1 ; P \leq P(E) \leq 1 ; P(\emptyset) = 0$$

$$* P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

* If A denotes (not A) then

$$P(\bar{A}) = 1 - P(A)$$

Heights & Distance

ΕΛΛΗΝΙΚΑ ΔΟΚΥ

~~5~~ Since ~~coso~~ ~~nameo~~

四

四

1

K K A

$$\csc \theta = \frac{1}{\sin \theta}$$

四

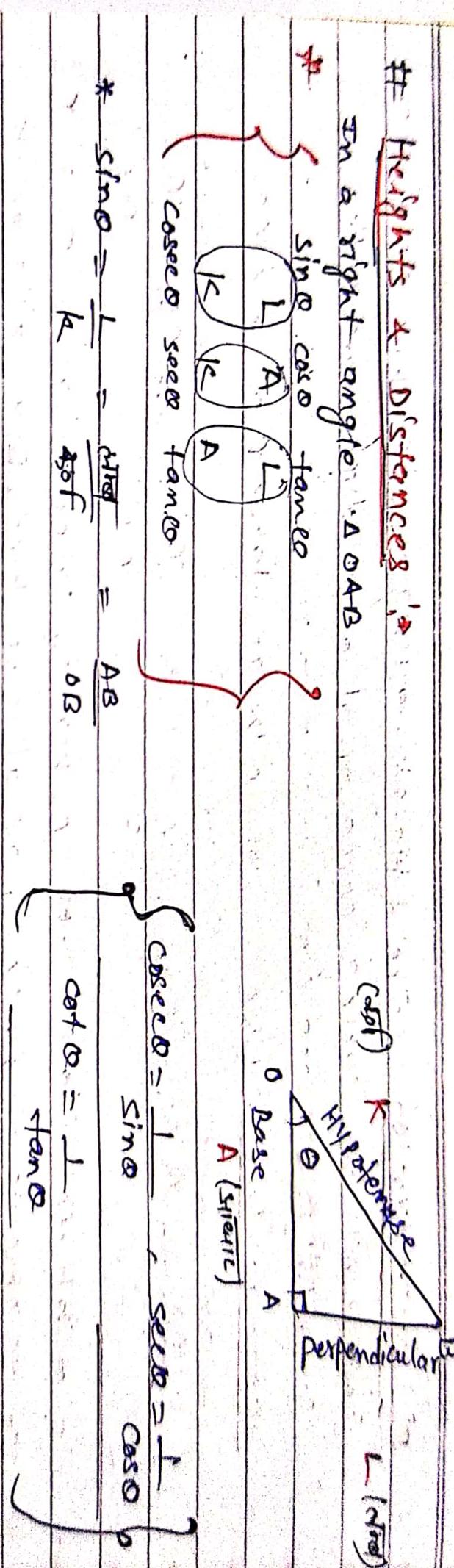
$$* \text{ } S_{\text{ino}} = \frac{L}{f} = \frac{\text{Mass}}{\text{AP}} = \frac{AB}{B}$$

180

* Trigonometrical Identities:

$$\theta = \sec^2 \alpha$$

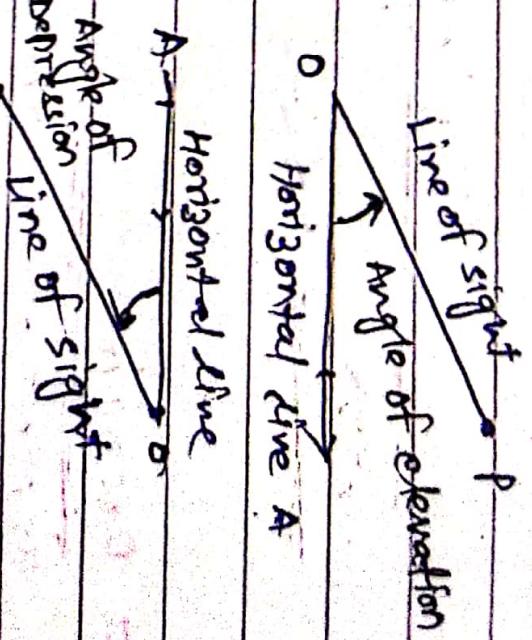
| θ | 0° | 30° ($\pi/6$) | 45° ($\pi/4$) | 60° ($\pi/3$) | 90° ($\pi/2$) |
|---------------|----|----------------------|----------------------|----------------------|-----------------|
| $\sin \theta$ | 0 | $\frac{1}{2}$ | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{3}}{2}$ | 1 |
| $\cos \theta$ | 1 | $\frac{\sqrt{3}}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | 0 |
| $\tan \theta$ | 0 | $\frac{\sqrt{3}}{3}$ | 1 | $\sqrt{3}$ | ∞ |



* Angle of Elevation :-

If object P placed above the level of eye.

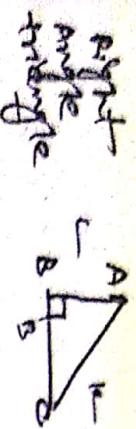
* Angle of Depression :-
If object P placed below the level of eye.



Pie-charts: (Data Interpretation)

Sum of all the central angles is $= 360^\circ$

$$\left\{ \begin{array}{l} \text{Central angle of the component} = \left(\frac{\text{Value of the component}}{\text{Total value}} \times 360^\circ \right) \\ \text{Component} \end{array} \right\}$$



Pythagoras theorem

$$AC^2 = AB^2 + BC^2$$

AREA :-

* sum of angles of a triangle = 180°

Result on Triangles :-

1) \rightarrow sum of any two sides of a triangle is greater than the third side.

2) The line joining the mid-point of a side of a triangle to the opposite vertex called the median.

3) The point where the three medians of a triangle meet is called centroid. The centroid divides each of the median in the ratio 2:1

4) $\frac{1}{4}$ th area of triangle

Result on Quadrilaterals :-

→ diagonal

square

rectangle

+ Diagonals of a Rhombus are unequal and bisect each other at right angles.

* Of all the parallelogram of given sides, the parallelogram which is a rectangle has the greatest area.

* Important formulae -

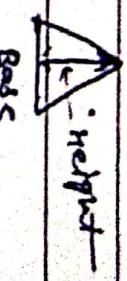
2) Area of a Rectangle = (length x Breadth)

Perimeter of a Rectangle = 2(L+B)

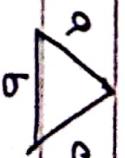
3) Area of a square (orf) = (side)² = $\frac{1}{2}$ (diagonal)²

3) Area of 4 walls of a room = 2 (length + Breadth) x Height

4) Area of a triangle = $\frac{1}{2}$ x base x height



Area of a triangle = $\sqrt{s(s-a)(s-b)(s-c)}$



Base

Height

$s = \frac{1}{2}(a+b+c)$

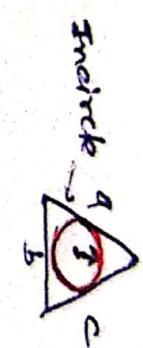
$s = \text{semi-perimeter}$

* Area of a equilateral triangle =

$$\frac{\sqrt{3}}{4} \times (\text{side})^2$$

square

$$\begin{aligned} AC^2 &= AB^2 + BC^2 \\ AC^2 &= x^2 + x^2 \\ AC^2 &= 2x^2 \\ AC &= \sqrt{2}x \end{aligned}$$



$$s = \frac{1}{2}(a+b+c)$$

* Radius of incircle of an equilateral triangle of side $a = \frac{a}{2\sqrt{3}}$

* Radius of inradius of an triangle of area A and semi-perimeter $s = \frac{A}{s}$

* Area of a parallelogram = (Base x Height)

Area of a rhombus = $\frac{1}{2}$ (product of diagonals)

Area of a trapezium = $\frac{1}{2}$ (sum of parallel sides) \times (distance b/w them)

* 6) Area of a circle = πR^2

\circlearrowright $R = \text{Radius}$

* Circumference of a circle = $2\pi R$

* Length of an Arc = $\frac{\alpha \pi R}{360}$, α is the central angle

Volume and Surface Area ↗

→ Cuboid : Volume = $(l \times b \times h)$

$$\text{Surface Area} = 2(lb + bh + lh)$$

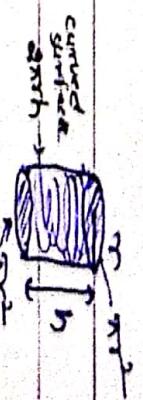
$$\text{Diagonal} = \sqrt{l^2 + b^2 + h^2}$$



→ Cube : Volume = a^3

$$\text{Surface Area} = 6a^2$$

$$\text{Diagonal} = \sqrt{3}a$$



→ Cylinder : Volume = $\pi r^2 h$

* * Curved Surface Area = $2\pi rh$

$$\text{Total Surface Area} = (2\pi rh + 2\pi r^2) = 2\pi r(h+r)$$

→ Sphere : Volume = $\frac{4}{3} \pi r^3$

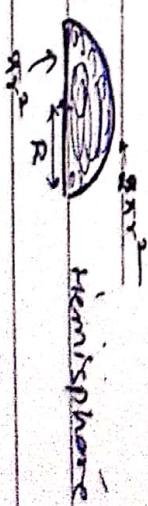
$$\text{Surface Area} = 4\pi r^2$$



→ Hemisphere : Volume = $\frac{2}{3} \pi r^3$

$$\text{Curved Surface Area} = 2\pi r^2$$

$$\text{Total Surface Area} = (2\pi r^2 + \pi r^2) = 3\pi r^2$$



g) **Sphere** :- Volume = $\frac{4}{3} \pi r^3$

* surface area = $4\pi r^2$



sphere

h) **Hemisphere** :- volume = $\frac{2}{3} \pi r^3$

curved surface area = $2\pi r^2$

$$\text{total surface area} = (2\pi r^2 + \pi r^2)$$

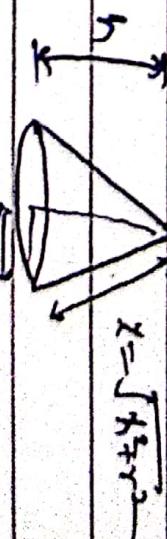
$$= 3\pi r^2$$



Hemisphere

i) **Cone** :- slant height? = $\ell = \sqrt{h^2 + r^2}$

$$\text{Volume} = \frac{1}{3} \pi r^2 h$$



h

$$\ell = \sqrt{h^2 + r^2}$$

curved surface area = $\pi r \ell$

$$\text{total surface area} = (\pi r \ell + \pi r^2)$$

* $1 \text{ litre} = 1000 \text{ cm}^3$

$$1 \text{ m}^3 = 1000 \text{ litre}$$

$$1 \text{ lit} = 10^{-3} \text{ m}^3$$