

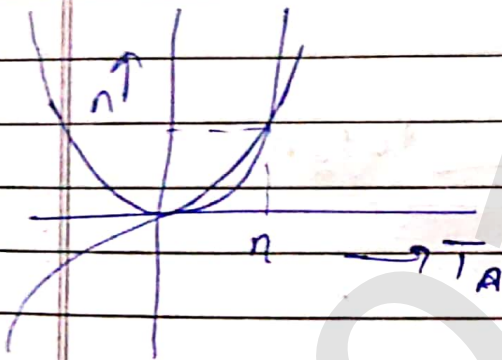
Assignment -

Ques 1) $O(n^2)$ for $n=10 \rightarrow 5 \text{ sec}$

$$\begin{aligned} 100 &\rightarrow 5 \\ 2500 &\rightarrow \frac{5 \times 2500}{100} = 125 \text{ sec} \end{aligned}$$

Ques 2)

$$\begin{aligned} T_A(n) &= n^3 \\ T_B(n) &= 2n^2 \end{aligned}$$



$$\begin{aligned} n^3 - 2n^2 &= 0 \\ n^2(n - 2) &= 0 \\ n &= 2 \end{aligned}$$

Ques 3)

$$\begin{aligned} f(n) &= n2^n \\ g(n) &= 4^n \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)}$$

$$\lim_{n \rightarrow \infty} \frac{n2^n}{4^n} = n \left(\frac{1}{2} \right)^n$$

$$\lim_{n \rightarrow \infty} \frac{n}{2^n} \Rightarrow \lim_{n \rightarrow \infty} = 0$$

$$O(f(n)) < O(g(n))$$

So the $n2^n$ is in $O(4^n)$

Ques (4)

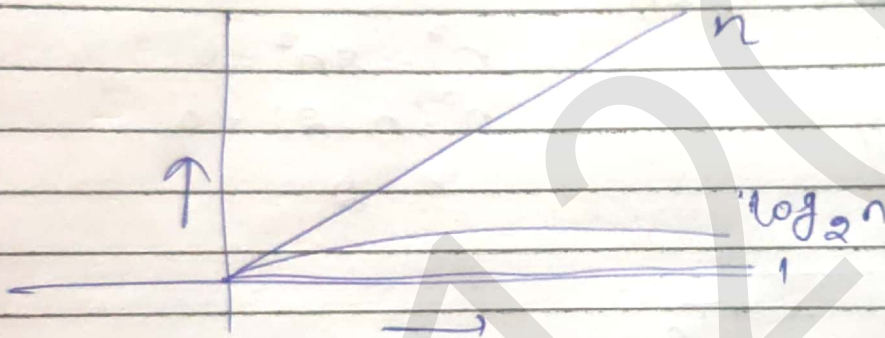
 $O(\log n)$

An algorithm's running time depends on its input size (n).

ex.

$\log_3 27$ is 3

Logarithmic function gets slightly slower as n grows. When n doubles the running time increased by constant



Ques (5) Worst case (Θ) is the function which performs the maximum number of steps on input data of size n .

ex. if we want to sort an array

Average case

of integers in ascending order then the worst case would be that array is already sorted in descending order.

Average case (Θ) is used when we refer to average number of input to

test the algorithm. It is the average resources used taking into all possible inputs

Q. (5) (b) O \rightarrow big-oh notation is an asymptotic notation notation for the worst case. It provides us with an asymptotic upper bound for the growth rate of the running of an algorithm.

$f(n)$ is $O(g(n))$

Ω - is an asymptotic notation for the best case. It provides us with an asymptotic lower bound for the growth rate of the running of an algorithm.

$f(n) > c g(n)$ for $(n > n_0)$

Q. (6) $n^4 + \log(n) + 17$
growth rate of constant

$$O(n^4) > O(\log n) > O(1)$$

$$\underline{\text{so}} \quad n^4 + \log(n) + 17 \\ = n^4 \quad O(n^4)$$

as we increase the input size the growth rate of logarithmic function is less than the polynomial function.

Ques (5)

```

k = 1 → 0(1)
while (k ≤ n) → 0(n)
{
    let assume
    k = k + 1 ; → 0(n)
}

```

$N = 5$

$N = 5$ number of time of looping
5

→ $0(1) + 0(n) + 0(n)$
→ $0(n)$ time complexity

for ($i = 1$ to $n-1$) → $0(n)$

{

for ($j = i+1$ to n) → $0(n-1)$

for $n = 5$

{

swap → $c(n-1)$

}

}

Time $0(n) + 0(n-1) + c(n-1)$
= $0(n)$

Q. (8) quadratic algorithm

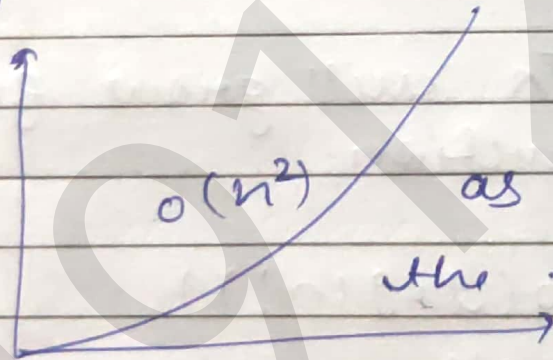
$$f(n) = n^2 + 2n + 4$$

as we see the growth of scale

$$O(n^2) > O(n) > O(1)$$

$$f(n) \approx O(n^2)$$

as increase the input size double then the running time will become larger



as the graph shows

as the input size increases the time also increases

Q. (9)

$$T_A = 100^n$$

$$T_B = n^4$$

$$\lim_{n \rightarrow \infty} \frac{T_A}{T_B} = \frac{100^n}{n^4}$$

$$= \frac{(10)^{2n}}{n^4}$$

$$100^n < n^4$$

$$T_B(n) < T_A(n)$$

$$n^4 < 100^n$$

$$\lim_{n \rightarrow \infty} \frac{100^n}{n^4}$$

L-hospital rule

$$\frac{n(100)^{n-1}}{4n^3} = \infty$$

Thus $f(n)$ is in $O(g(n))$

thus n^4 is an upper bound of $(100)^n$

as $n \rightarrow \infty$ T_A will grow faster than T_B function.

Ex. (10)

$$n \log n \in O(\log(n!))$$

as $n = \text{infinite}$

$$\log(n!) = \log(1) + \log(2) + \dots + \log(n-1) + \log(n)$$

$$\begin{aligned} \log(n!) &= \log(n \times (n-1)!) \\ &= \log n + \log(n-1)! \end{aligned}$$

$$n \log n \rightarrow \log 1 + 2 \log 2 + 3 \log 3 \dots$$

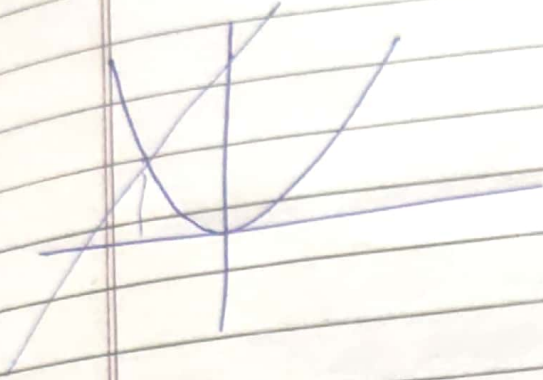
$\sim n \log n$

Qu (12)

$$2^{n-1} + 4^{n-1}$$

$$T_A = n^2$$

$$T_B = n + 9$$



$$n^2 = n + 2$$

$$n^2 - n - 2 = 0$$

$$n^2 - 2n + n - 2 = 0$$

$$\boxed{n=2}$$

Breaking point $n=2$

Qu (14) total $\infty \rightarrow O(1)$
 for $(i=0 \text{ to } n-1) \rightarrow O(n-1)$

{ sum = sum + A[i] $\rightarrow C(O(n))$
 return sum; $\rightarrow C(O(n))$
 }

$T_{sum} = O(1) + O(n-1) + C(O(n-1)) + C(O(n))$
 $\rightarrow O(n)$

Qu (15) int count = 0; $\rightarrow O(1)$
 for $(i=0 \text{ to } n-1: i++) \rightarrow O(n)$

{ for $(j=0 \text{ to } i: j++) \rightarrow O(O(n))$

{ count++ $\rightarrow C(O(n))$
 }