# EQ2341 Pattern Recognition and Machine Learning Assignment 4

Ayushi Shah
ayushi@kth.se

Sri Janani Rengarajan
sjre@kth.se

May 16, 2020

In this assignment a Matlab function to perform the Backward Algorithm is implemented and verified.

## Backward Algorithm

Implementation of *backward* function from *MarkovChain* class

```
function betaHat=backward(mc,pX,c)
T=size(pX,2);%Number of observations
A = mc.TransitionProb;
N = mc.nStates;
betaHat=zeros(N,T);
if finiteDuration(mc)
    betaHat(:,T) = A(:,end)/(c(T)*c(T+1));
else
    betaHat(:,T) = 1/c(T);
end
for t = T-1:-1:1
    for i = 1:N
        betaHat(i,t) = sum(A(i,1:2)'.*pX(:,t+1).*betaHat(:,t+1))/c(t);
    end
end
end
```

## Verification

Backward algorithm implementation is verified for the following Markov chains using:

```
q = [1;0];
A = [0.9 0.1 0; 0 0.9 0.1];
mc=MarkovChain(q, A);%State generator
g1=GaussD("Mean",0,"StDev",1); %Distribution for state=1
g2=GaussD("Mean",3,"StDev",2); %Distribution for state=2
h=HMM(mc, [g1; g2]);
```

```
x = [-0.2,2.6,1.3];
pX = prob([g1,g2],x);
c = [1,0.1625,0.8266,0.0581];
betaHat=backward(mc,pX,c);
```

Finite duration:

$$q=\begin{pmatrix}1\\0\end{pmatrix};\ A=\begin{pmatrix}0.9 & 0.1 & 0\\0 & 0.9 & 0.1\end{pmatrix};\ B=\begin{pmatrix}\mathcal{N}(0,1)\\\mathcal{N}(3,2)\end{pmatrix}$$

Following is the output from the verification code:

```
Values from the code for finite duration

betaHat =

    1.0003    1.0393         0
    8.4182    9.3536    2.0822
```

Infinite duration:

$$q=\begin{pmatrix}1\\0\end{pmatrix};\ A=\begin{pmatrix}0.9 & 0.1\\0.1 & 0.9\end{pmatrix};\ B=\begin{pmatrix}\mathcal{N}(0,1)\\\mathcal{N}(3,2)\end{pmatrix}$$

Verification code for infinite duration

```
q = [1;0];
A= [0.9 0.1 ; 0.1 0.9];
mc=MarkovChain(q, A);%State generator
g1=GaussD("Mean",0,"StDev",1); %Distribution for state=1
g2=GaussD("Mean",3,"StDev",2); %Distribution for state=2
h=HMM(mc, [g1; g2]);
x = [-0.2,2.6,1.3];
pX = prob([g1,g2],x);
c = [1,0.1625,0.8266,0.0581];
betaHat=backward(mc,pX,c)
```

```
Values from the code for infinite duration

alfaHat =

    1.0000  0.3847  0.4591
    0       0.6153  0.5409
c =

    1.0000  0.1625  0.8881

betaHat =

    1.0000  6.7973  1.1260
    5.2223  5.7501  1.1260
```

Figure 1 shows the manual calculations for infinite duration HMM.

$T = 3$

$$\hat{\beta}_{i,3} = \begin{bmatrix} 1.1257 \\ 1.1257 \end{bmatrix}$$

$T = 2$

$$\hat{\beta}_{i,2} = \begin{bmatrix} 6.7944 \\ 5.7479 \end{bmatrix}$$

$T = 1$

$$\hat{\beta}_{i,1} = \begin{bmatrix} 1.1044 \\ 5.2130 \end{bmatrix}$$

Figure 1: Answers for manual calculation infinite duration HMM