

Ans 1

$$T(n) = aT(n/b) + f(n^2)$$

$$a \geq 1, b \geq 1$$

Our comparing  
 $a = 3, b = 2, f(n) = n^2$   
 Now,

$$C = \log_b a = \log_2 3 = 1.584$$

$$n^2 = n^{1.584} < n^2$$

$$\because f(n) > n^C$$

$$\therefore T(n) = \Theta(n^2)$$

Ans 2

$$a \geq 1, b \geq 1$$

$$a = 4, b = 2, f(n) = n^2$$

$$C = \log_2 4 = 2$$

$$n^C = n^2 = f(n) = n^2$$

$$\therefore T(n) = \Theta(n^2 \log_2 n)$$

Ans 3

$$a = 1, b = 2$$

$$f(n) = 2^n$$

$$C = \log_b a = \log_2 1 = 0$$

$$n^C = n^0 = 1$$

$$f(n) > n^C$$

$$T(n) = \Theta(2^n)$$

Ans 4:-

$$a = 2^n$$

$$b = 2$$

$$f(n) = n^2$$

$$c = \log_b a = \log_2 2^n = n$$

$$n^c \Rightarrow n^n$$

$$f(n) = n^c$$

$$T(n) = \Theta(n^2 \log_2 n)$$

Ans 5:-

$$a = 16, b = 4$$

$$f(n) = n$$

$$c = \log_4 16 = \log_4 (4)^2 = 2 \log_4 4 = 2$$

$$n^c \Rightarrow n^2$$

$$f(n) < n^c$$

$$\therefore T(n) = \Theta(n^2)$$

Ans 6:-

$$a = 2, b = 2$$

$$f(n) = n \log n$$

$$c = \log_2 2 = 1$$

$$n^c = n^1 = n$$

$$n \log n > n$$

$$f(n) > n^c$$

$$T(n) = \Theta(n \log n)$$



Ans 7:-

$$a=2, b=2, f(n) = n / \log n$$

$$c = \log_2 2 = 1$$

$$n^c = n^1 = n$$

$$\therefore \frac{n}{\log n} < n$$

$$\therefore \cancel{f(n)} < n^c$$

$$\therefore T(n) = \Theta(n)$$

Ans 8:-

$$a=2, b=4, f(n) = n^{0.51}$$

$$c = \log_b a = \log_4 2 = \frac{1}{2} = 0.5$$

$$n^c = n^{0.5}$$

$$n^{0.5} < n^{0.51}$$

$$f(n) > n^c$$

$$\therefore T(n) = \Theta(n^{0.51})$$

Ans 9:-

$$a=0.5, b=2$$

$a > 1$  but here  $a$  is  $0.5$

So we cannot apply Master's Theorem



Ans 10:-

$$a=16, b=4, f(n)=n!$$

$$\therefore c = \log_b a = \log_4 16 = 2$$

$$n^c = n^2$$

$$\text{As } n! > n^2$$

$$\therefore T(n) = \Theta(n!)$$

Ans 11:-

$$a=4, b=2, f(n) = \log(n)$$

$$c = \log_b a = \log_2 4 = 2$$

$$n^c = n^2$$

$$f(n) = \log n$$

$$\log n < n^2$$

$$f(n) < n^c$$

$$T(n) = \Theta(n^c)$$

$$= \Theta(n^2)$$

Ans 12:-

$$a = \sqrt{n}$$

$$b = 2$$

$$c = \log_b a = \log_2 \sqrt{n} = \frac{1}{2} \log_2 n$$

$$\therefore \frac{1}{2} \log_2 n < \log(n)$$

$$\therefore f(n) > n^c$$

$$T(n) = \Theta(f(n))$$

$$= \Theta(\log(n))$$



Ans 13 :-

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$$a=3, b=2, f(n)=n$$

$$c = \log_b a = \log_2 3 = 1.5849$$

$$n^c = n^{1.5849}$$

$$n < n^{1.5849}$$

$$f(n) < n^c$$

$$T(n) = \Theta(n^{1.5849})$$

Ans 14 :-

$$a=3, b=3$$

$$c = \log_b a = \log_3 3 = 1$$

$$n^c = n^1 = n$$

$$\text{As } \sqrt{n} < n$$

$$f(n) < n^c$$

$$T(n) = \Theta(n)$$

Ans 15 :-

$$a=4, b=2$$

$$c = \log_b a = \log_2 4 = 2$$

$$n^c = n^2$$

$$n < n^2 \text{ (for any constant)}$$

$$f(n) < n^c$$

$$f(n) = \Theta(n^2)$$



Ans 16 :-

$$a=3, b=4, f(n)=n \log n$$

$$c = \log_b a = \log_4 3 = 0.792$$

$$n^c = n^{0.792}$$

$$n^{0.792} < n \log n$$

$$T(n) = \Theta(n \log n)$$

Ans 17 :-

$$a=3, b=3$$

$$c = \log_b a = \log_3 3 = 1$$

$$f(n) = n/2$$

$$\therefore n^c = n^1 = n$$

$$\text{As } n/2 < n$$

$$f(n) < n^c$$

$$\therefore \underline{\underline{T(n) = \Theta(n)}}$$

Ans 18 :-

$$a=6, b=3$$

$$c = \log_b a = \log_3 6 = 1.6309$$

$$n^c = n^{1.6309}$$

$$\text{As } n^{1.6309} < n^2 \log n$$

$$\therefore T(n) = \Theta(n^2 \log n)$$



Ans 19 :-

$$a=4, b=2, f(n) = \frac{n}{\log n}$$

$$C = \log_b a = \log_2 4 = 2$$

$$n^C = n^2$$

$$\frac{n}{\log n} < n^2$$

$$T(n) = \Theta(n^2)$$

Ans 20 :-

$$a=64, b=8$$

$$C = \log_b a = \log_8 64 = \log_8 8^2 = 2$$

$$C = 2$$

$$n^C = n^2$$

$$n^2 \log n > n^2$$

$$T(n) = \Theta(n^2 \log n)$$

Ans 21 :-

$$a=7, b=3, f(n) = n^2$$

$$C = \log_b a = \log_3 7 = 1.7712$$

$$n^C = n^{1.7712}$$

$$n^{1.7712} < n^2$$

$$T(n) = \Theta(n^2)$$

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Ans 22 :-

$$a = 1, b = 2$$

$$c = \log_b a = \log_2 1 = 0$$

$$n^c = n^0 = 1$$

$$n(2 - \cos x) > n^c$$

$$\underline{\underline{T(n) = \Theta(n(2 - \cos x))}}$$