Graph Theory Fall 2020

Assignment 1

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I came across a result in a time series textbook the other day and have not been able to understand why it is true (the authors do not give a proof but just state it as true). I want to show that the eigenvalues of the matrix **G** given by

$$\mathbf{G} = \begin{pmatrix} \Phi_1 & \Phi_2 & \Phi_3 & \cdots & \Phi_{p-1} & \Phi_p \\ 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 1 & 0 & \cdots & 0 & 0 \\ \vdots & & & \ddots & 0 & 0 \\ 0 & 0 & \cdots & \cdots & 1 & 0 \end{pmatrix}$$

correspond to the reciprocal roots of the **AR(p)** characteristic polynomial

$$\boldsymbol{\Phi}(u) = 1 - \phi_1 u - \phi_2 u^2 - \cdots - \phi_p u^p$$

The one thing I was able to deduce is that the eigenvalues of **G** must satisfy

$$\lambda^P - \phi_1 \lambda^{p-1} - \phi_2 \lambda^{P-2} - \cdots - \phi_{P-1} - \phi_P = \mathbf{0}$$