Graph Theory Fall 2020

Assignment 2

Due at 5:00 pm on Tuesday, September 8

1. Among all simple graphs with 21 vertices, determine (with justification) the minimum possible and the maximum possible number of edges such a graph could have.

HW2

Q1) Simple graph : No of vertices = 21

A simple graph is one that contains no loops or multiple edges. The maximum number of edges in a simple graph is the 2x distinct vertices. It is also not a null graph.

a) The maximum number of possible edges a graph would have is given by the following combination function.

$$\Rightarrow {}^{21}C_2 = \frac{21!}{2!\,19!} =$$

With geometric progression, combination set is also computed by

$$= \frac{21(21-1)}{2} = \boxed{210}$$

Maximum no of edges with a graph of 21 vertices $\Rightarrow \boxed{210}$

b) The minimum no of edges such a graph would have is $(n-1)$ edges where $n =$ no. of vertices.

As in the graph, all the vertecies must be joined to form a path. In the question above, we have $n = 21$

$$(21-1) = \boxed{20}$$

Minimum no. of edges with a graph of 21 vertices = 20

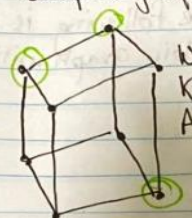
2. Suppose G is a simple graph (no loops, no parallel edges) with n vertices and m edges. Let H be the simple graph whose vertices take the form $(0, v)$ or $(1, v)$ for each vertex v of G. Two vertices (a, v) and (b, w) of H are adjacent if either of the following two conditions holds:

   - $a \neq b$ and $v = w$, or
   - $a = b$ and vw is an edge of G

   Later on, we will call this graph $K_2 \times G$. As an example, if G is K_4, then H is drawn below:

   In terms of n and m, how many vertices does H have and how many edges does H have?

G is a simple graph with n vertices and m edges. H takes (0,v) or (1,v)



We are given that H is a cross-product of $K_2 \times G$.
As K_2 has 2 vertices and G graph has n vectors. $\Rightarrow$ $\boxed{H = 2n \text{ Vertices}}$

H either take (0,v) or (1,v), Two vertices (a,v) and (b,w) are adjecent when,

a ≠ b and v=w or
a = b and vw is an edge.

This allows all vertices ~~with the~~ to be adjecent where vw is an edge in G. G has been given m edges.
This means there will $\boxed{2m}$ edges.

~~There will~~

3. Recall that a graph G is said to be cubic if it is 3-regular, i.e., every vertex has degree 3.
   a. Explain why a loopless cubic graph must have an even number of vertices.
   b. For each integer $n \geq 1$, construct a loopless cubic graph with $2n$ vertices.

c. For each integer $n \geq 3$, construct a simple cubic graph with $2n$ vertices. (You could apply question #2 to this purpose.)

~~Q3) a)~~

Q3) a) Loopless cubic graph must have an even number of vertices.

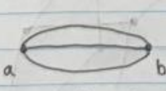
We know that for every cubic graph, the degree of vertices is given by the the following,

$$\sum \deg(v) = 2|E| \text{ (degree)}$$

This means that the sum of vertices is ~~two times~~ $2 \times$ Edges.
Any number which is multiplied by 2, is an Even Number.

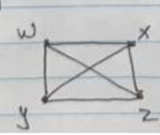
Example: if degree = 7 vertices
if degree of vertices $= 7 = (2(3)+1)$ (ODD)
This makes a contradiction to number of edges.
Therefore, a loopless cubic graph must have an even number of vertices.

b) For each integer $n \geq 1$, construct a loopless cubic graph with $2n$ vertices.
A loopless graph is one which does not have loops, but still allows multiple edges.
If we take $\boxed{n=1}$, the following graph G shows,



This graph is a loopless cubic graph with $2n=2$ vertic

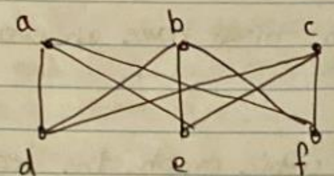
If we take $\boxed{n=2}$



Graph is a loopless cubic graph with 4 vertices

This works for graph and so on,

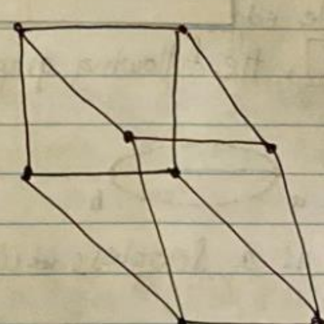
$\boxed{n=3}$

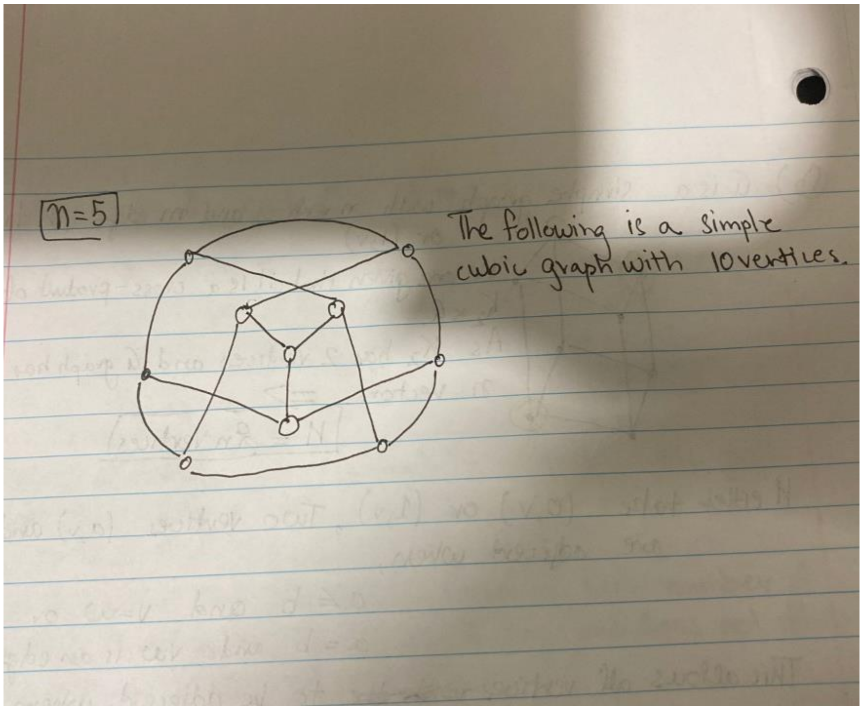


c) For each integer $n \geq 3$, construct a simple cubic graph with $2n$ vertices

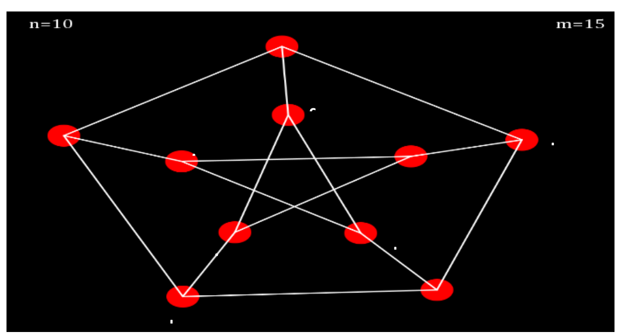
We know that all cubic graphs have an even number of vertices. A simple cubic graph is one that does not allow loops and ~~multiple~~ (parallel) edges.

$\boxed{n=4}$





4. Determine, with justification, whether the Petersen graph (drawn below, with vertex set $\boldsymbol{V} = \{\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}, \boldsymbol{d}, \boldsymbol{e}, \boldsymbol{f}, \boldsymbol{g}, \boldsymbol{h}, \boldsymbol{i}, \boldsymbol{j}\}$) is bipartite:



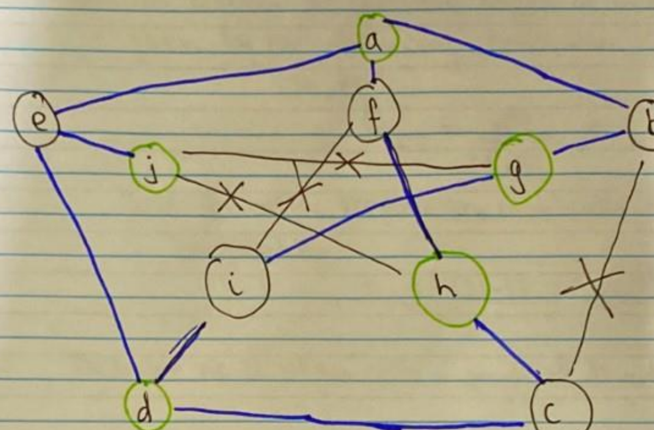
There is a well-known characterization of bipartite graphs. It states that a graph is bipartite if and only if it contains no odd cycles.

Q4) Determine wheather the Peterson graph is bipartite:

A graph is bipartike whose vertices can be partiored into two subsets (w) and (x), such that every edge joins a vertex to w to a vertex of x.

That implies that if 2 vertices are adjecent then one of ventices is in (w) and the other is in vertex (x)

Our Vertex Set = {a, b, c, d, e, f, g, h, i, j}



As we look at the graph above, ~~we~~ ~~can~~ if we make an edge from V_c to V_b, this adds a contradiction to the principles of a bipartite graph. The same would occour for V_i to V_f.
It also contains a cycle of odd length as the subgraphs above
This also counts for the fact that this graph contains odd cycles. Therefore, it is not bipartite.