Estimating Edge Effects on a parallel plate capacitor with a partially inserted dielectric plate

Applet Link (run on Google Colabs):

https://colab.research.google.com/drive/1etTNhwjF3KMj6kInsmujhoN1z0C3C6Ga?usp=sharing

Public Github Repo: (Note: This document will be attached to the repository for public reference)

https://github.com/AyushmaanAggarwal/Estimating-Edge-Effects-on-Parallel-Plate-Capacitor/blob/main/Estimate_Edge_Effects.ipynb

Description:

For a square parallel plate capacitor, the fringe effects on a parallel plate capacitor is non-negligible if the separation between the plates is not significantly smaller than the area of each plate. These fringe effects can be extremely complicated to calculate as conformal mapping from complex analysis is commonly invoked. These computations can become even more difficult if a partially inserted dielectric plate is inserted into the parallel plate capacitor

This applet uses an iterative approach to estimate the edge effects on a 10 by 10 grid of a parallel plate capacitor using Figure 1 as a reference.

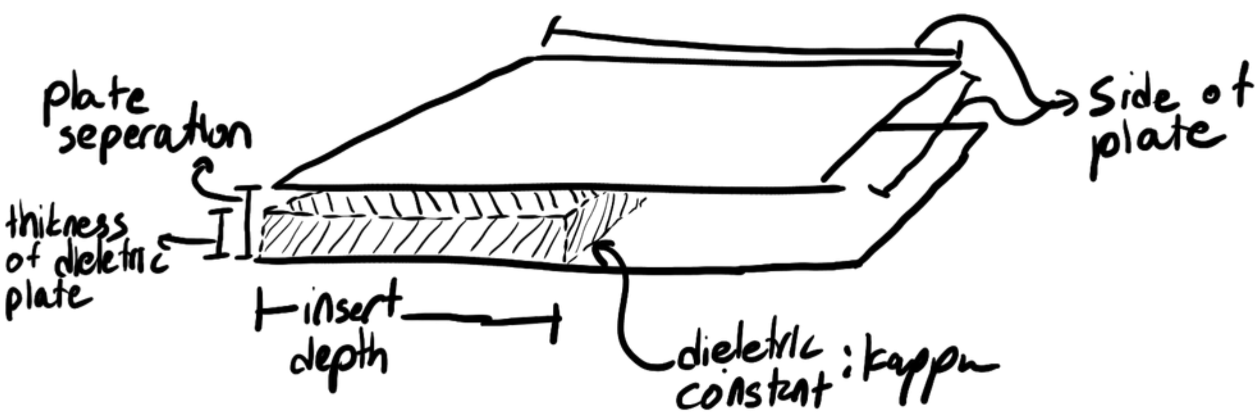

Figure 1: A drawing with all relevant parameters labeled in order to run applet

Running the applet:

A google account is needed to run the code on google servers, if desired, the code can also be run locally by downloading the ipynb file from Github and run using Jupyter Notebooks. Just follow the instructions at the top of the file.

Theory - Why fringe effects matter:

The voltage of a parallel plate capacitor can be characterized by equation (1) where this is the line integral beginning on one plate and ending on the other plate. The path can be chosen for the line integral along the z-axis from one plate to the next as seen in Figure 2 and equations (2,3).

$$V = \oint \vec{E} \cdot d\vec{l} \rightarrow V = \oint E_z dz \qquad \rightarrow V = \int E_z dz$$

(1,2)

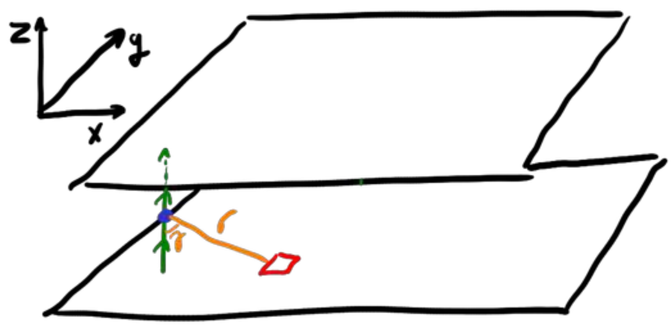

Figure 2: Parallel Plate Capacitors where path chosen for line integral is in z direction as indicated by the green line. The blue point represents an arbitrary point from which the electric field is measured with respect to a point charge in red which is distance r and angle gamma with respect to the vertical.

We can then calculate the electric field from an infinitesimal patch on the parallel plate to some arbitrary point on the plate as seen in Figure 1 by using equation (3). This can be generalized to equation (4) by using the surface integral across the entire surface of the parallel plates.

$$dE_z = k\frac{\sigma}{r^2}cos(\gamma) \qquad E_z = \iint \frac{k\sigma}{r^2}\frac{z}{r}dxdy$$

(3,4)

If we assume that the charge distribution is constant across the plate, the electric field for any given z height will be greater at the center of the plate than the edges and especially for the corners. This will directly impact the voltage drop and can be visualized by Figure 3 where the constant charge distribution yields a voltage drop distribution greatest in the center with a bright yellow and the smallest in the corners in dark blue.

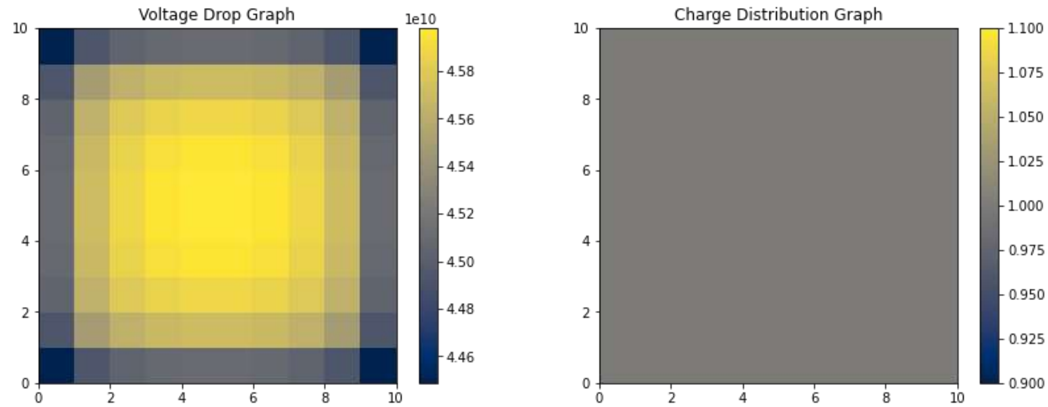

Figure 4: The charge distribution is a constant value(left) and the voltage drop is greatest in the corners and decreases the farther out the value is(right). All the values in these 2 graphs are arbitrary and relative, there are no real physical units that should be attached to these graphs as a scaling factor is needed to represent a real life experiment.

This model is physically impossible to create in real life as the voltage drop across a metal plate has to be 0 whereas this indicates there must be a range of voltages across the plate. The most clear solution to this problem is that the charge distribution is not uniform, but rather fluid and adapts to create a uniform voltage drop across the 2 plates at any point in space. By developing a machine learning inspired algorithm, we can incrementally modify the charge distribution on the plates. The algorithm begins by normalizing the voltage drops across the plate between 1 and -1. This is then multiplied by a learning rate and subtracted from the charge distribution in order to decrease the charge distribution in all the locations where the voltage drop is above the average and increase the charge on the surface where the voltage drop is below the average voltage drop on the plate. After running this model roughly 12 times, we can achieve a standard deviation of voltage drops that is 0.33% of the original standard deviation of voltage drops with a constant charge distribution. Figure 4 shows that final charge distribution centered around the corners. In the graph, the voltage drops in Figure 4 may appear non-uniform, however, after comparing the scale of the voltage drop distribution in Figure 3 to Figure 4, the change in voltage drops across the plate is nearly negligible with a range of 0.0004.

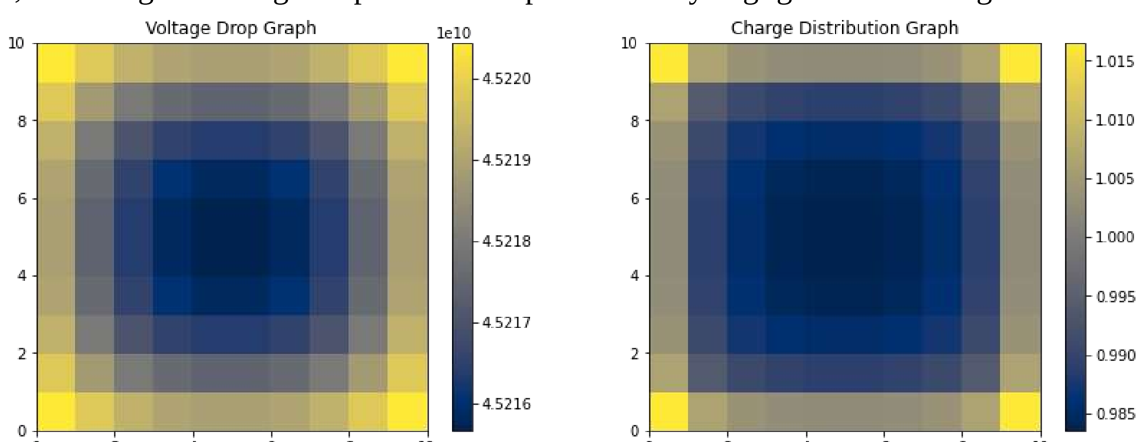

Figure 4: Voltage Drops and Charge Distribution after training

We could then use equation (10) in order to calculate the final capacitance as any scaling factor for the charge distribution would be canceled out by the division of Q and V.