# Tutorial – 6
## DAA

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## Answer 1

Minimum Spanning Tree is a subset of edges of a connected edge-weighted undirected graph that connects all the vertices together without any cycles & with minimum possible edge-weight.

APPLICATIONS →

(i) Consider n stations are to be linked using a communication network and lying of communication link between any two stations involves a cost. The ideal solution would be ~~to~~ extract a subgraph termed as minimum cost spanning tree.

(ii) Designing LAN.

(iii) Suppose you meant or want to construct highways or railroads spanning several cities, then we can use concept of MST.

(iv) Laying pipeline connecting offshore drilling sites, refineries and consumer markets.

## Answer 2

⇒ Time Complexity of Prim's Algorithm = $O(|E|\log|V|)$

⇒ Space Complexity of Prim's Algorithm = $O(|V|)$

⇒ Time Complexity of Kruskal's Algorithm = $O(|E|\log|E|)$

⇒ Space Complexity of Kruskal's Algorithm = $O(|V|)$

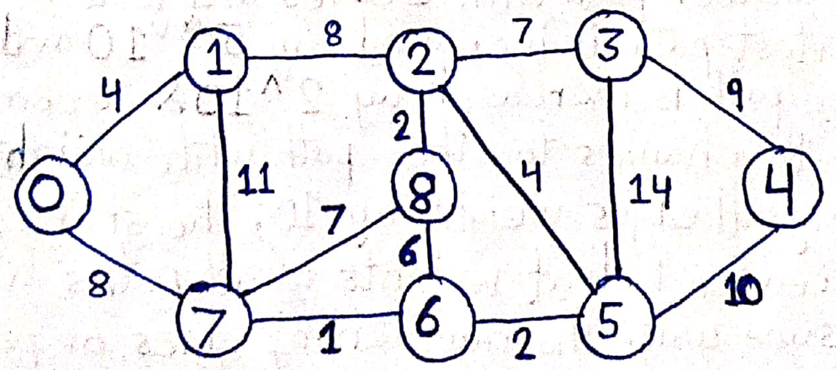
⇒ Time Complexity of Dijkstra's Algorithm = $O(V^2)$

⇒ Space Complexity of Dijkstra's Algorithm = $O(V^2)$

⇒ Time Complexity of Bellman Ford's Algorithm = $O(VE)$

⇒ Space Complexity of Bellman Ford's Algorithm = $O(E)$

## Answer 3



## Kruskal's Algorithm

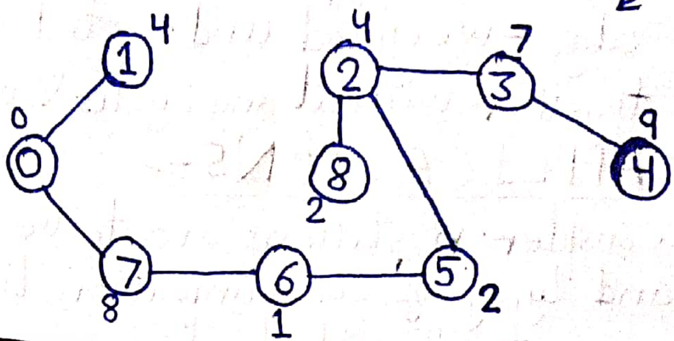
| O | V | W | |
|---|---|---|---|
| 6 | 7 | 1 | ✓ |
| 5 | 6 | 2 | ✓ |
| 2 | 8 | 2 | ✓ |
| 0 | 1 | 4 | ✓ |
| 2 | 5 | 4 | ✓ |
| 6 | 8 | 6 | ✗ |
| 2 | 3 | 7 | ✓ |
| 7 | 8 | 7 | ✗ |
| 0 | 7 | 8 | ✓ |
| 1 | 2 | 8 | ✗ |
| 4 | 3 | 9 | ✓ |
| 4 | 5 | 10 | ✗ |
| 1 | 7 | 11 | ✗ |
| 3 | 5 | 14 | ✗ |

## Prim's Algorithm

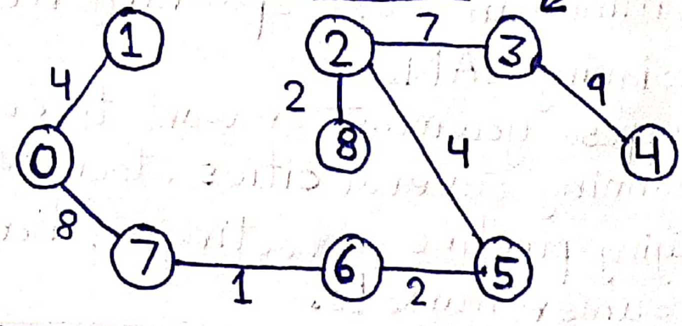
Weight = 4+8+1+2+4+2+7+9

⇒ $\boxed{\text{Weight} = 37}$

Prim's MST



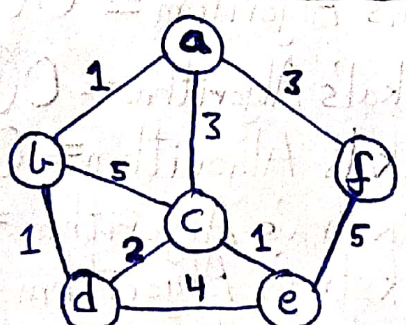

Kruskal's MST




Weight = 1+2+2+4+4+7+8+9

⇒ $\boxed{\text{Weight} = 37}$

## Answer 4

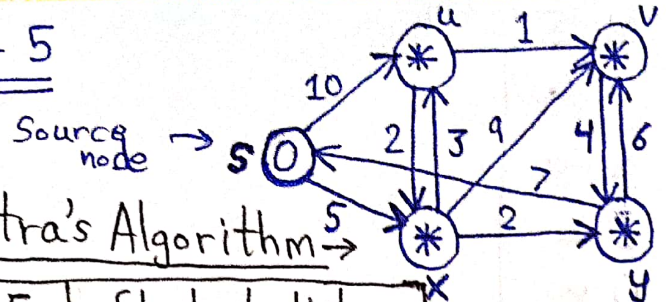



(i) The shortest path may change. The reason that is there may be a different no. of edges in different paths from 's' to 't'.
For example – let the shortest path of weight 15 and has edges 5. Let there be another path with 2 edges and total weight 25. The weight of shortest path is increased by 5*10 and becomes 15+50. Weight of other path is increased by 2*10 & becomes 25+20. So, the shortest path changes to other path with weight as 45.

(ii) If we multiply all edges weight by 10, the shortest path does not change. The reason is that weights of all paths from 's' to 't' gets multiplied by same unit. The number of edges or path doesn't matter.

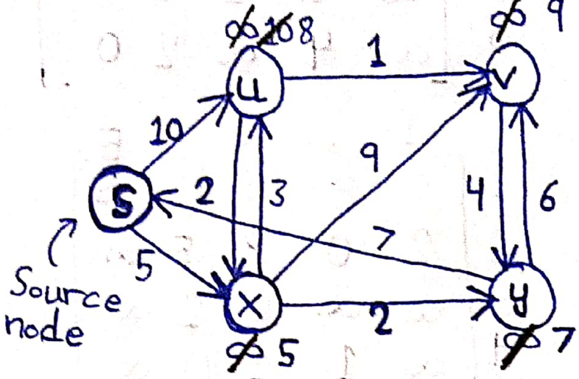
# Answer 5






## Dijkstra's Algorithm →

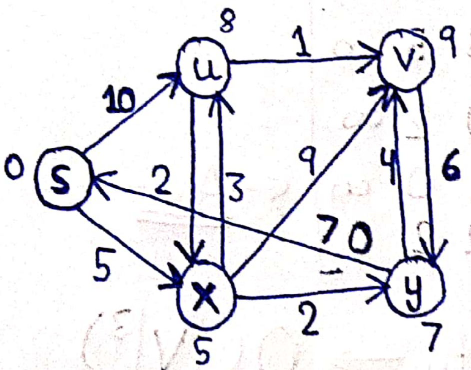
| NODE | Shortest distance from source node |
|---|---|
| u | 8 |
| x | 5 |
| v | 9 |
| y | 7 |




## Bellman Ford's Algorithm →

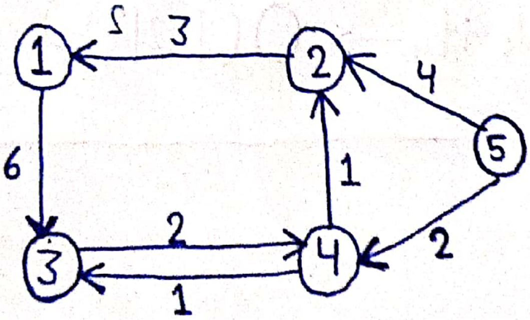
| | S | u | v | x | y |
|---|---|---|---|---|---|
| 1st → | 0 | ~~10~~ ∞ | ∞ | ~~∞~~ 5 | ∞ |
| 2nd → | 0 | 10 | ~~∞~~ 11 | 5 | ∞ |
| 3rd → | 0 | ~~10~~ 8 | ~~11~~ 9 | 5 | ~~∞~~ 7 |
| 4th → | 0 | 8 | 9 | 5 | 7 |

Graph does not have negative cycle.




→ Final Graph

# Answer 6



$$\begin{array}{cc} & \begin{array}{ccccc}1&2&3&4&5\end{array} \\ \begin{array}{c}1\\2\\3\\4\\5\end{array} & \begin{bmatrix} 0 & \infty & 6 & 3 & \infty \\ 2 & 0 & \infty & \infty & \infty \\ \infty & \infty & 0 & 2 & \infty \\ \infty & 1 & 1 & 0 & \infty \\ \infty & 4 & \infty & 2 & 0 \end{bmatrix}\end{array}$$

$$\begin{array}{cc} & \begin{array}{ccccc}1&2&3&4&5\end{array} \\ \begin{array}{c}1\\2\\3\\4\\5\end{array} & \begin{bmatrix} 0 & \infty & 6 & 3 & \infty \\ 2 & 0 & 8 & 5 & \infty \\ \infty & \infty & 0 & 2 & \infty \\ \infty & 1 & 1 & 0 & \infty \\ \infty & 4 & \infty & 2 & 0 \end{bmatrix}\end{array}$$

$$\begin{array}{cc} & \begin{array}{ccccc}1&2&3&4&5\end{array} \\ \begin{array}{c}1\\2\\3\\4\\5\end{array} & \begin{bmatrix} 0 & \infty & 6 & 3 & \infty \\ 2 & 0 & 8 & 5 & \infty \\ \infty & \infty & 0 & 2 & \infty \\ 3 & 1 & 1 & 0 & \infty \\ 6 & 4 & 12 & 2 & 0 \end{bmatrix}\end{array}$$

$$\begin{array}{cc} & \begin{array}{ccccc}1&2&3&4&5\end{array} \\ \begin{array}{c}1\\2\\3\\4\\5\end{array} & \begin{bmatrix} 0 & \infty & 6 & 3 & \infty \\ 2 & 0 & 8 & 5 & \infty \\ \infty & \infty & 0 & 2 & \infty \\ 3 & 1 & 1 & 0 & \infty \\ 6 & 4 & 12 & 2 & 0 \end{bmatrix}\end{array} \leftarrow \underline{\text{Ans}}$$

Time Complexity $\rightarrow O(|V|^3)$

Space Complexity $\rightarrow O(|V|^2)$