Tutorial-4
DAA

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① $T(n) = 3T(n/2) + n^2$

Ans $a = 3, b = 2, f(n) = n^2$

$n^{\log_b a} = n^{\log_2 3}$

Comparing $n^{\log_2 3}$ and n^2 → $n^{\log_2 3} < n^2$ (Case 3)

∴ According to Master's Theorem → $\boxed{T(n) = \Theta(n^2)}$

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② $T(n) = 4T(n/2) + n^2$

Ans $a = 4, b = 2, f(n) = n^2$

⇒ $n^{\log_b a} = n^{\log_2 4} = n^2 = f(n)$ (Case 2)

∴ According to Master's Theorem → $\boxed{T(n) = \Theta(n^2 \log n)}$

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③ $T(n) = T(n/2) + 2^n$

Ans $a = 1, b = 2, f(n) = 2^n$

⇒ $n^{\log_2 1} = n^0 = 1$ and $1 < 2^n$ (Case 3)

∴ According to Master's Theorem → $\boxed{T(n) = \Theta(2^n)}$

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④ $T(n) = 2^n T(n/2) + n^n$

Ans ∴ Master's Theorem not applicable as **a** is a function of **n**.

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⑤ $T(n) = 16T(n/4) + n$

Ans $a = 16, b = 4, f(n) = n$

⇒ $n^{\log_b a} = n^{\log_4 16} = n^2$ and $n^2 > f(n)$ (Case 1)

∴ According to Master's Theorem → $\boxed{T(n) = \Theta(n^2)}$

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⑥ $T(n) = 2T(n/2) + n\log n$

Ans $a = 2, b = 2, f(n) = n\log n$

⇒ $n^{\log_b a} = n^{\log_2 2} = n$ and $f(n) > n$ (Case 3)

∴ According to Master's Theorem → $\boxed{T(n) = \Theta(n \log n)}$

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⑦ $T(n) = 2T(n/2) + \frac{n}{\log n}$

$a = 2, b = 2, f(n) = \frac{n}{\log n}$

⇒ $n^{\log_b a} = n^{\log_2 2} = n$

And $n > f(n)$ (Case 1)

∴ According to Master's Theorem → $\boxed{T(n) = \Theta(n)}$

⑧ $T(n) = 2T(n/4) + n^{0.51}$

Ans $a=2, b=4, f(n)=n^{0.51}$

$\Rightarrow n^{\log_b a} = n^{\log_4 2} = n^{0.5}$ and $n^{0.5} < f(n)$ (Case 3)

$\therefore$ According to Master's Theorem → $\boxed{T(n) = \Theta(n^{0.51})}$

or → ~~$T(n) = \Theta(\sqrt{n})$~~

⑨ $T(n) = 0.5T(n/2) + 1/n$

Ans $\therefore$ Master's Theorem is not applicable as $a < 1$.

⑩ $T(n) = 16T(n/4) + n!$

Ans $a=16, b=4, f(n)=n!$

$\Rightarrow n^{\log_b a} = n^{\log_4 16} = n^2$ and $n^2 < f(n)$ (Case 3)

or $n^2 < n!$

$\therefore$ According to Master's Theorem → $\boxed{T(n) = \Theta(n!)}$

⑪ $T(n) = 4T(n/2) + \log n$

Ans $a=4, b=2, f(n)=\log n$

$\Rightarrow n^{\log_b a} = n^{\log_2 4} = n^2$ and $n^2 > f(n)$ (Case 1)

$\therefore$ According to Master's Theorem → $\boxed{T(n) = \Theta(n^2)}$

⑫ $T(n) = \text{sqrt}(n)\, T(n/2) + \log n$

Ans $\therefore$ Master's Theorem is not applicable as **a** is not constant.

⑬ $T(n) = 3T(n/2) + n$

Ans $a=3, b=2, f(n)=n$

$\Rightarrow n^{\log_b a} = n^{\log_2 3} = n^{1.58}$ and $n^{1.58} > n$ (Case 1)

$\therefore$ According to Master's Theorem → $\boxed{T(n) = O(n^{\log_2 3})}$

⑭ $T(n) = 3T(n/3) + \sqrt{n}$

Ans $a=3, b=3, f(n)=\sqrt{n}$

$\Rightarrow n^{\log_b a} = n^{\log_3 3} = n$ and $n > \sqrt{n}$ (Case 1)

$\therefore$ According to Master's Theorem → $\boxed{T(n) = \Theta(n)}$

⑮ $T(n) = 4T(n/2) + Cn$

Ans $a=4, b=2, f(n)=c*n$

$\Rightarrow n^{\log_b a} = n^{\log_2 4} = n^2$ and $n^2 > c*n$ (Case 1)

$\therefore$ According to Master's Theorem → $\boxed{T(n) = \Theta(n^2)}$

(16) $T(n) = 3T(n/4) + n\log n$

Ans $a=3, b=4, f(n) = n\log n$

$\Rightarrow n^{\log_b a} = n^{\log_4 3} = n^{0.79}$

And $n^{0.79} < n\log n$ (Case 3)

or $n^{0.79} < f(n)$

$\therefore$ According to Master's Theorem → $\boxed{T(n) = \Theta(n\log n)}$

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(17) $T(n) = 3T(n/3) + n/2$

Ans $a=3, b=3, f(n) = n/2$

$\Rightarrow n^{\log_b a} = n^{\log_3 3} = n$ (Case 2)

And $\Theta(n) = \Theta(n/2)$ (since both are order of n)

$\therefore$ According to Master's Theorem → $\boxed{T(n) = \Theta(n\log n)}$

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(18) $T(n) = 6T(n/3) + n^2\log n$

Ans $a=6, b=3, f(n) = n^2\log n$

$\Rightarrow n^{\log_b a} = n^{\log_3 6} = n^{1.63}$ (Case 3)

And $n^{1.63} < n^2\log n$

$\therefore$ According to Master's Theorem → $\boxed{T(n) = \Theta(n^2\log n)}$

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(19) $T(n) = 4T(n/2) + n/\log n$

Ans $a=4, b=2, f(n) = n/\log n$

$\Rightarrow n^{\log_b a} = n^{\log_2 4} = n^2$

And $n^2 > f(n)$ (Case 1)

$\therefore$ According to Master's Theorem → $\boxed{T(n) = \Theta(n^2)}$

(20) $T(n) = 64T(n/8) - n^2 \log n$ (4)

Ans Master's Theorem is not applicable as $f(n)$ ~~does~~ is not an increasing function.

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(21) $T(n) = 7T(n/3) + n^2$

Ans $a = 7, b = 3, f(n) = n^2$

$\Rightarrow n^{\log_b a} = n^{\log_3 7} = n^{1.7}$ (Case 3)

And $n^{1.7} < f(n)$

or $n^{1.7} < n^2$

$\therefore$ According to Master's Theorem $\rightarrow$ $\boxed{T(n) = \Theta(n^2)}$

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(22) $T(n) = T(n/2) + n(2 - \cos n)$

Ans Master's Theorem isn't applicable since regularity condition is isolated in case 3.
