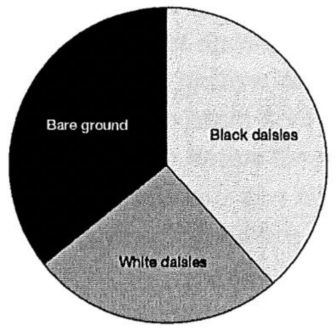
Greenhouse Effect in the Daisy World

EEPS 0250 Final Project

Ayushman Choudhury

December 7, 2022

Review of Daisy World



- Simple example of environmental self-regulation
- 3 surfaces: Barren Ground, White Daisies, Black Daisies
- Useful as a starting point for Earth modeling
- At each timestep, the following calculations determine how the populations change:

- Let Cg, Cw, and Cb be cover fractions, Let Ag, Aw, and Ab be the surface-type albedos
- Planet Albedo: A = (Cg * Ag) + (Cw * Aw) + (Cb * Ab)
- Let L = luminosity factor, S_0 = incoming solar energy, σ = Stefan-Boltzmann constant
- Atmospheric temperature: Ta = [L * S_0 * (1 - A) / (4σ)] ^ ¼
- Surface temperature: Ts = Ta * 2^¼
- Let K = homogeneity of the surface temperature
- Surface-type temperature: T(i) =(A - A(i)) * (1 - K) * L * S_0 / (4σ) + Ts^4
- Let b = birth sensitivity to temperature, T_0 = optimal growth temperature, D = death rate
- Birth Rate: B(i) = 1 - b * (T_0 - T(i))^2 and Death Rate: D(i) = D * C(i)

- Ag = 0.5, Aw = 0.8, Ab = 0.2
- L = 1, S_0 = 1380, σ = 5.6 * 10^{-8}
- K = 0.6
- B = 3.265 * 10^{-3}, T_0 = 293.7, D = 0.3

$$\frac{dC_i}{dt} = b_i C_i C_g - DC_i$$
$$= b_i[C_i - C_i^2 - C_j C_i] - DC_i$$

Add Greenhouse Effect

- 2014 article from Biosystems[1] describes a scheme using this equation:

$$\frac{dE}{dt} = a - bE(A_w + A_b)$$

- Variables
  - E is the alteration of the thermal balance by the Greenhouse Effect. Hold $E \geq 0$
  - a is the emission of greenhouse gases
  - b is the absorption of greenhouse gases

- Introduce into calculations
  - Greenhouse gases affect the entire planet's temperature
  - If E = 0, no effect. As E increases, the Greenhouse Effect increases and the temperature increases
  - New atmospheric temperature equation: Ta = **E +** [L * S_0 * (1 - A) / (4σ)] ^ ¼
  - All other equations can remain the same

- Discretization: Forward Euler

$$E(n+1) = E(n) + dt(a - bE(A_w + A_b))$$

[1] https://www.sciencedirect.com/science/article/pii/S0303264714001270#fig0005

Daisy World Discretization - 2-step Runge Kutta

$$Let\, f(t, C_i) = \frac{dC_i}{dt} = b_i C_i(t)[1 - C_i(t) - C_j(t)] - DC_i(t)$$

$$K_1 = \Delta t[b_i C_i(t)[1 - C_i(t) - C_j(t)] - DC_i(t)]$$

$$C_i(t + \star) = C_i(t) + K_1$$

$$K_2 = \Delta t[b_i C_i(t + \star)[1 - C_i(t + \star) - C_j(t)] - DC_i(t + \star)]$$

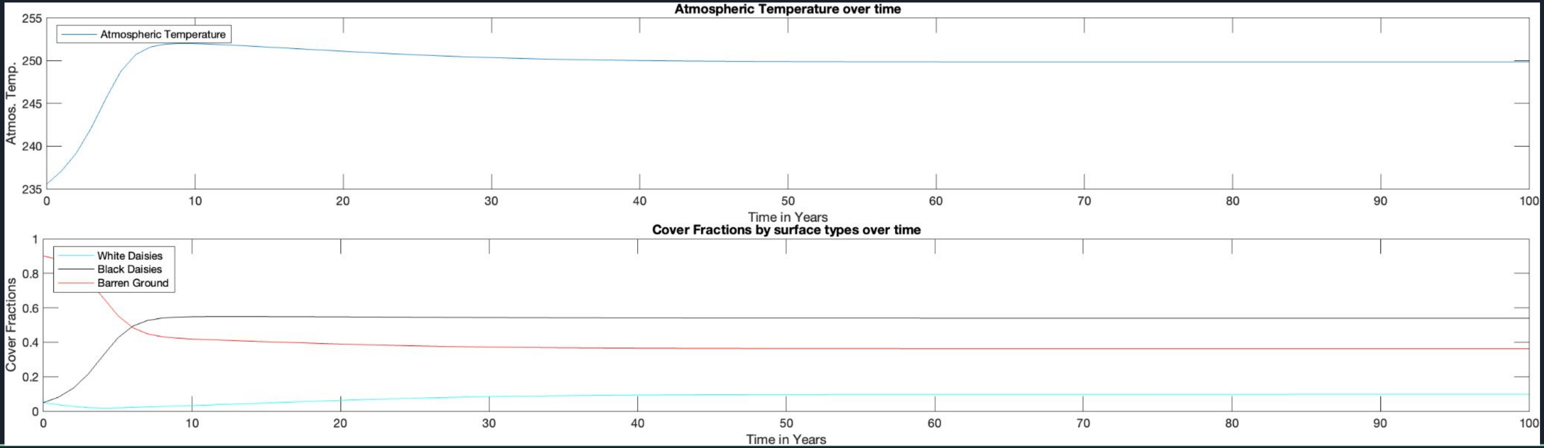
$$C_i(t + 1) = C_i(t) + \frac{K_1 + K_2}{2}$$

Development of Model and Results

Step 0: No Greenhouse Gases

$$\frac{dE}{dt} = 0$$

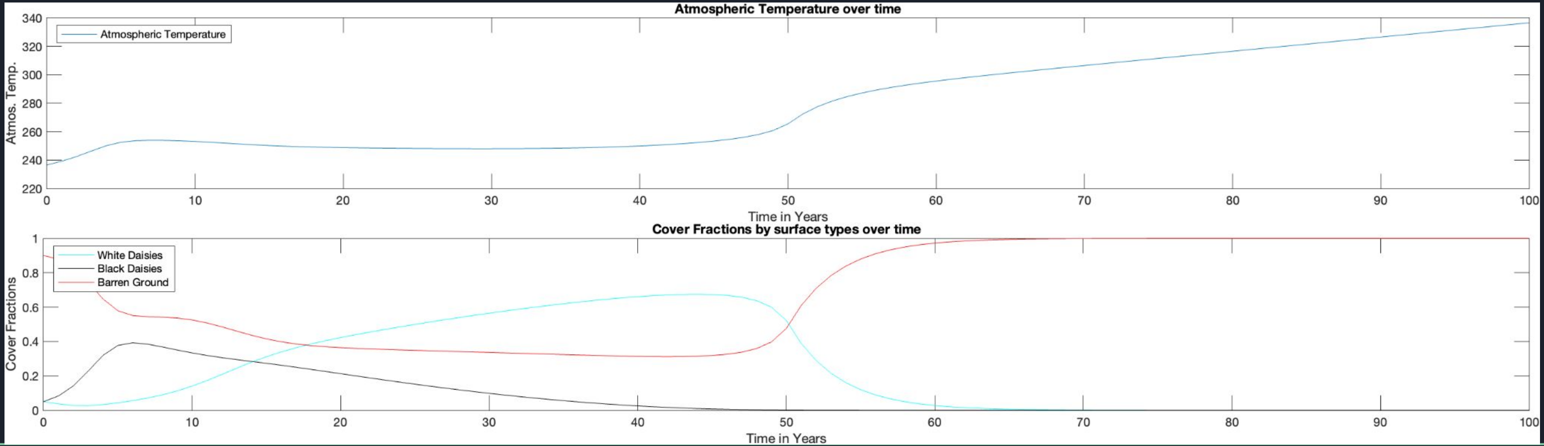
- Assume E starts at 0, so there is no Greenhouse Effect
- The atmospheric temperature equation is unchanged



Step 1: Constant rate of emission

$$\frac{dE}{dt} = a$$

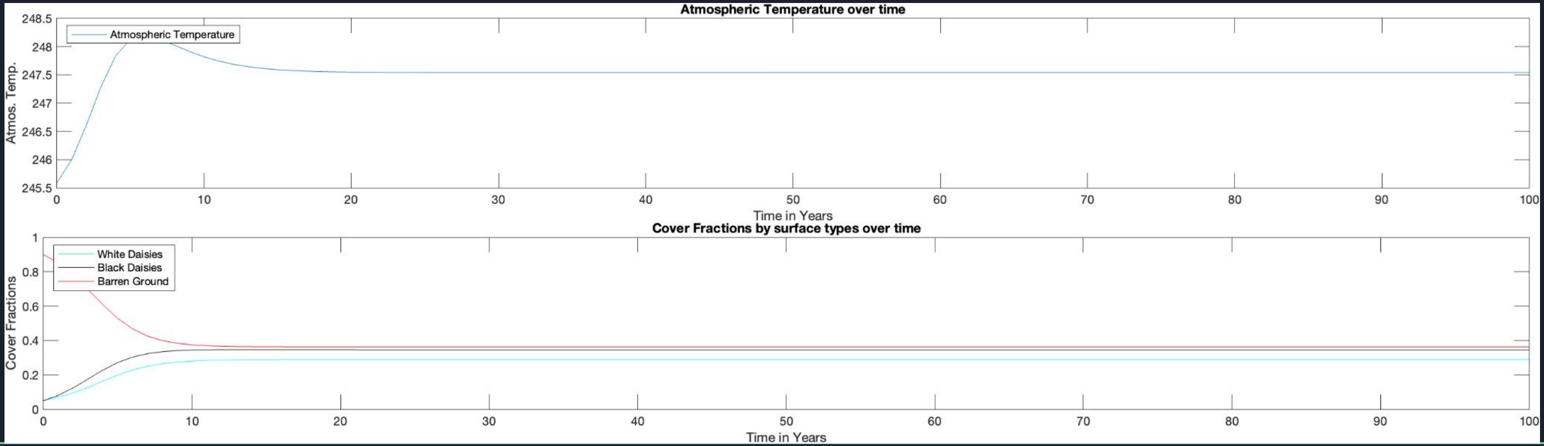
- Let a = 1, initial E = 0
- dE/dt = 1, so the greenhouse gas effect linearly increases over time
- As temperature increases, Black daisies die off, White daisies grow more
- However, once the temperature grows too much, White daisies die too



Step 2: Equal Emission and Absorption

$$\frac{dE}{dt} = a - bE(A_w + A_b)$$

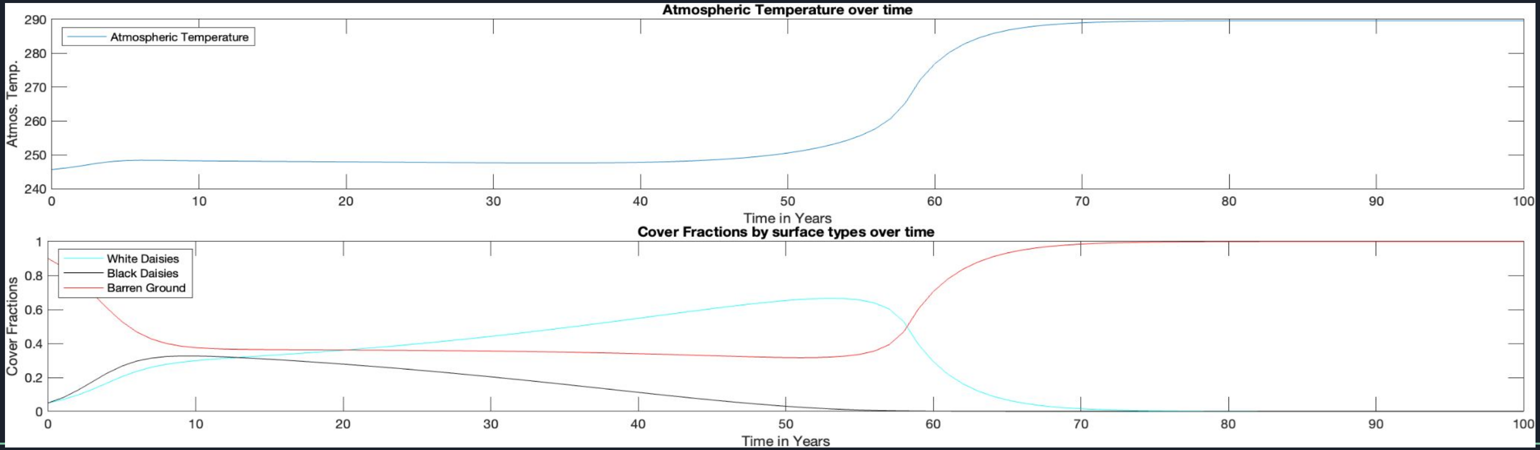
- Let a = 5, b = 0.5, initial E = 10, Aw = 0.8, Ab = 0.2
- dE/dt = 0 so the greenhouse gas effect is a constant 10, added to the baseline temperature
- The daisy populations reach a steady state again, but at a lower cover fraction
- This is the equation described in the paper I read



Step 3: Emitters and Absorbers

$$\frac{dE}{dt} = a(C_w) - b(C_b)$$

- Let a = 2, b = 1, initial E = 10, initial Cover Fractions = 0.05
- dE/dt initially grows, then shrinks, then grows, before shrinking again to 0
- First the Black Daisies grow faster, then die off as White Daisies grow, which die off too

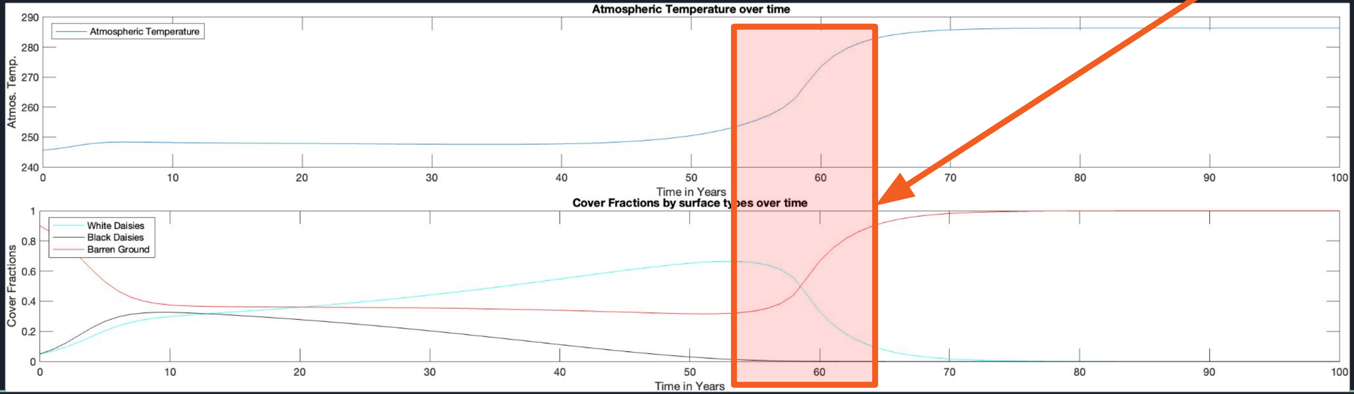


Step 4: Emitters learn to emit less

$$\frac{dE}{dt} = a(1-R)(C_w) - b(C_b)$$

- Let a = 2, b = 1, initial E = 10, initial Cover Fractions = 0.05
- R measures the emitter's attempt to reduce their emissions
    - Initial R = 0
    - If the emitter sees its population declining, R += 0.1
    - If the emitter population stops declining, R = 0
- Expected: greenhouse gas emissions decrease, populations stabilize

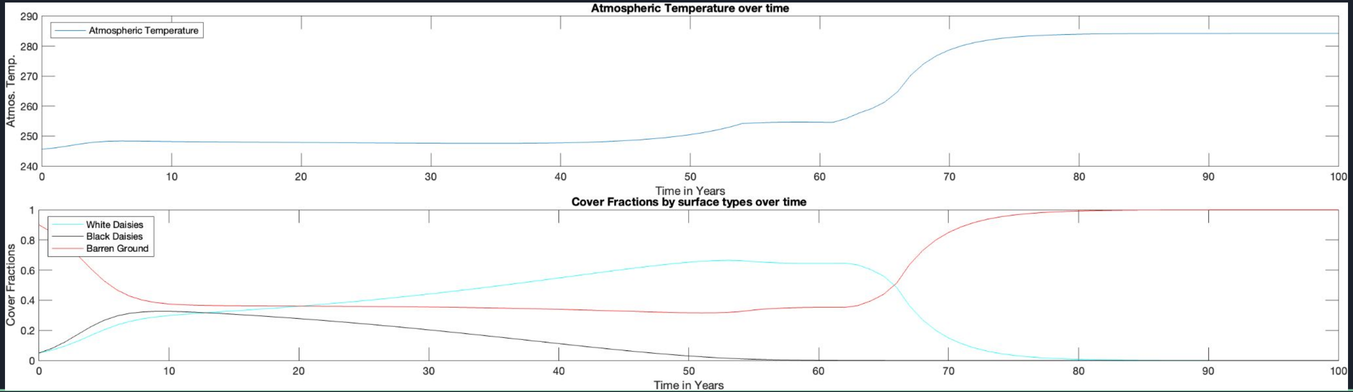
By the time the emitters act, the absorbers are extinct.



Step 4b: Emitters learn slowly, stop quickly

$$\frac{dE}{dt} = a(1 - R)(C_w) - b(C_b)$$

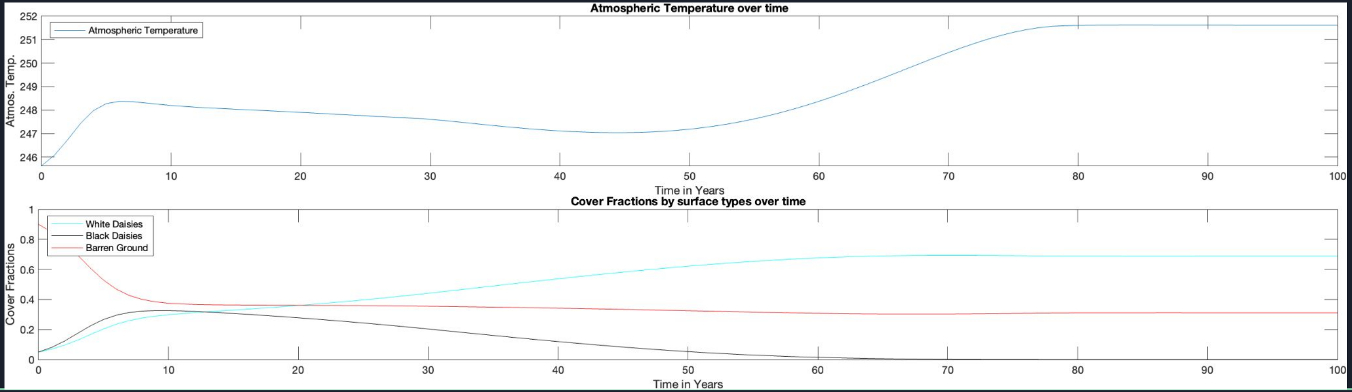
- Let a = 2, b = 1, initial E = 10, initial Cover Fractions = 0.05
- R measures the emitter's attempt to reduce their emissions
  - Initial R = 0
  - If the emitter sees its population declining, R = 1
  - If the emitter population stops declining, R = 0



Step 5a: Emitters learn soon, act slow

$$\frac{dE}{dt} = a(1 - R)(C_w) - b(C_b)$$

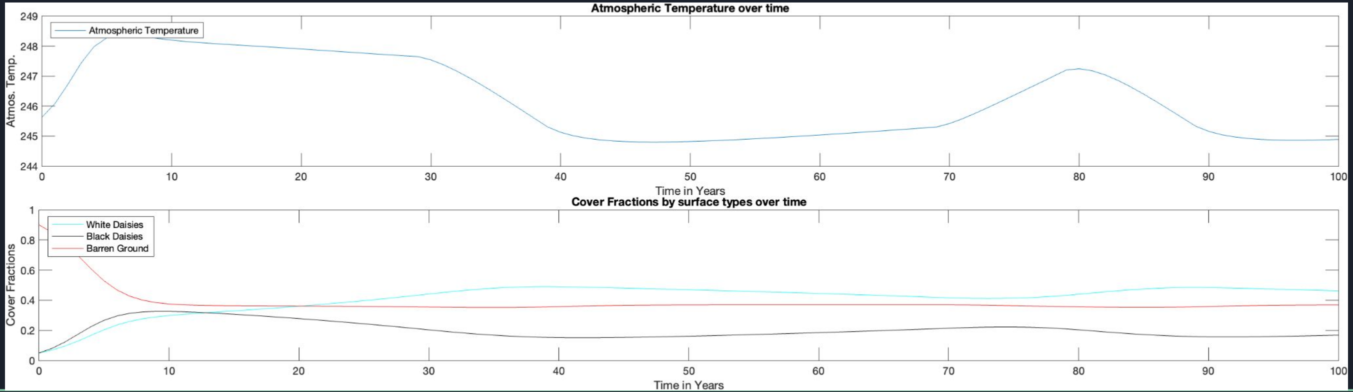
- Let a = 2, b = 1, initial E = 10, initial Cover Fractions = 0.05
- R measures the emitter's attempt to reduce their emissions
  - Initial R = 0
  - If the emitter sees its population is more than double the absorber population, R += 0.02
  - If this condition is no longer true, R -= 0.02
- White daisies stabilize, Black daisies die out



Step 5b: Emitters learn soon, take action

$$\frac{dE}{dt} = a(1-R)(C_w) - b(C_b)$$

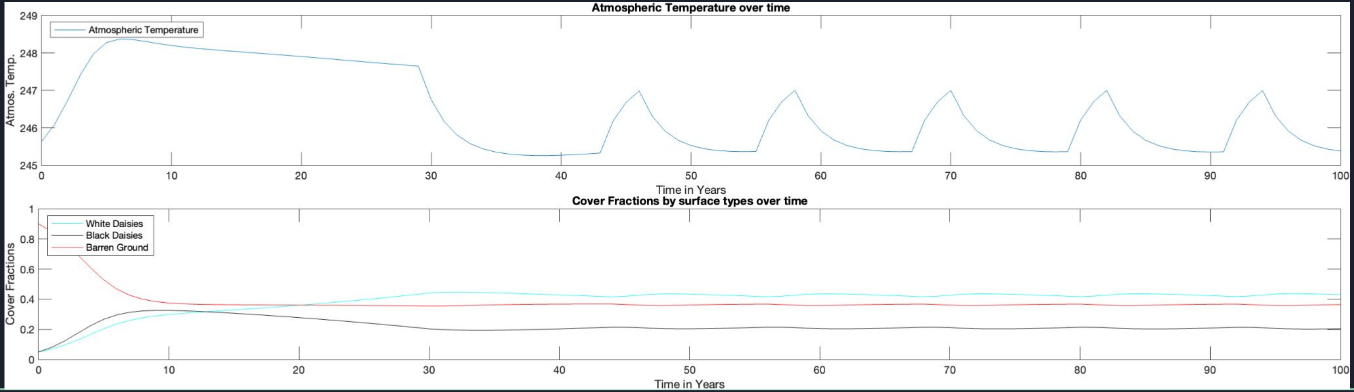
- Let a = 2, b = 1, initial E = 10, initial Cover Fractions = 0.05
- R measures the emitter's attempt to reduce their emissions
    - Initial R = 0
    - If the emitter sees its population is more than double the absorber population, R += 0.1
    - If this condition is no longer true, R -= 0.1
- Both populations eventually recover



Step 5c: Emitters learn soon, act urgently

$$\frac{dE}{dt} = a(1 - R)(C_w) - b(C_b)$$

- Let a = 2, b = 1, initial E = 10, initial Cover Fractions = 0.05
- R measures the emitter's attempt to reduce their emissions
    - Initial R = 0
    - If the emitter sees its population is more than double the absorber population, R += 1
    - If this condition is no longer true, R -= 1
- Both populations recover and oscillate



Conclusions

Summary of models

$$E_0 = 0, \frac{dE}{dt} = 0$$

$$E_0 = 0, \frac{dE}{dt} = 1$$

$$\boxed{E_0 = 10, (C_w)_0 = (C_b)_0 = 0.05}$$

$$\frac{dE}{dt} = 5 - 0.5E(A_w + A_b)$$

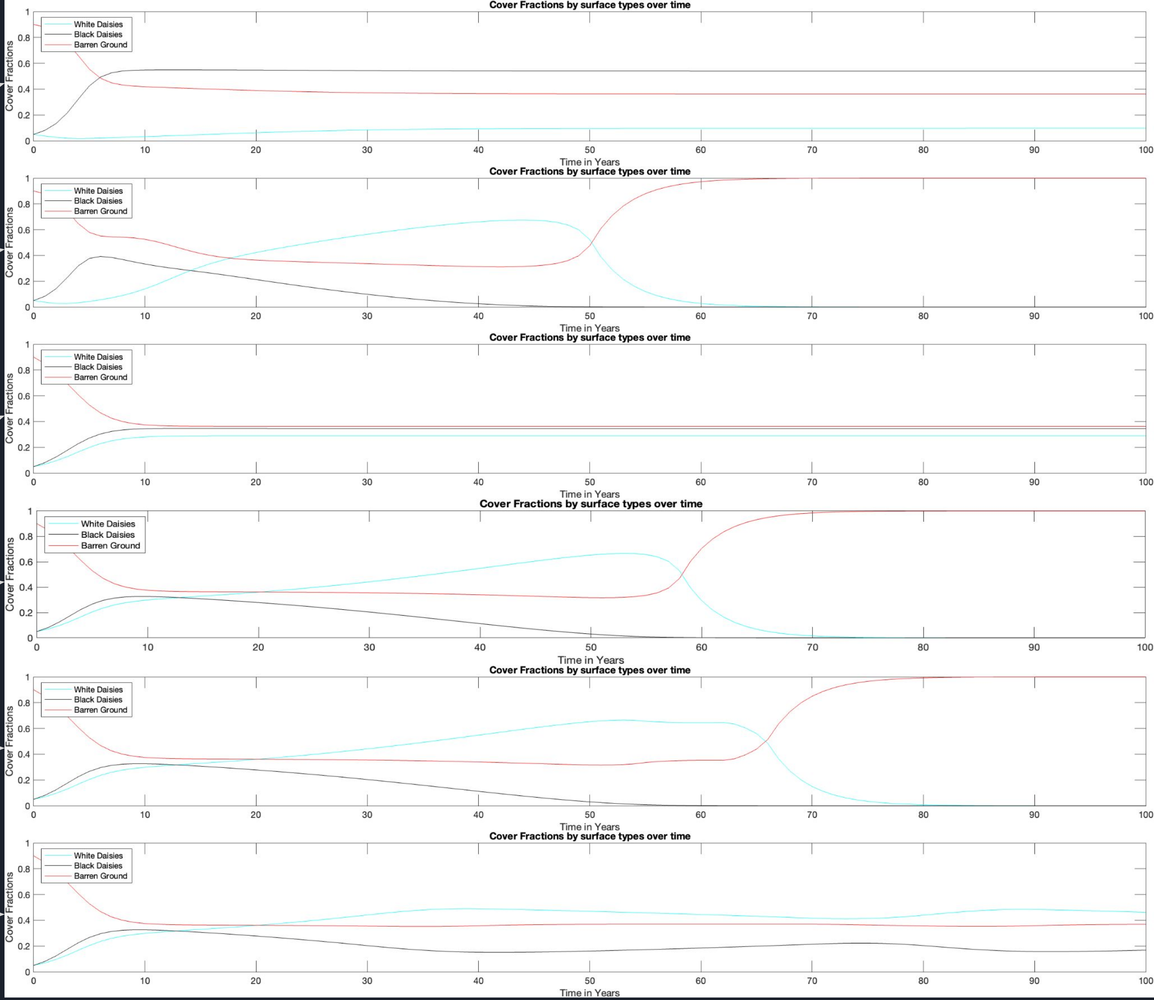
$$\frac{dE}{dt} = 2(C_w) - 1(C_b)$$

$$\frac{dE}{dt} = 2(1 - R)(C_w) - 1(C_b)$$

Reduction increment = 1, Revert to 0.

$$\frac{dE}{dt} = 2(1 - R)(C_w) - 1(C_b)$$

Reduction increment, decrement = 0.1.



Real-life parallels

- Climate Change is a serious threat
- Early and decisive action is important

Future improvements

- Finding specific greenhouse gases and modeling their impacts
- Model gases that help/hurt the other species' growth
- Introduce more species (different ideal temps, gas interactions)

Thank you!