NCERT Exemplar Solutions

Class 12 – Mathematics

Chapter 5 – Continuity and Differentiability

Objective Type Questions

Question 83: If $f(x) = 2x$ and $g(x) = \frac{x^2}{2} + 1$, then which of the following can be a discontinuous function?

(a) $f(x) + g(x)$

(b) $f(x) - g(x)$

(c) $f(x) \cdot g(x)$

(d) $\frac{g(x)}{f(x)}$

Solution: ***(d)***

We know that, if f and g be continuous functions, then

(a) $f + g$ is continuous

(b) $f - g$ is continuous.

(c) fg is continuous

(d) $\frac{f}{g}$ is continuous at these points, where $g(x) \neq 0$.

Now, we have $f(x) = 2x$ and $g(x) = \frac{x^2}{2} + 1$

$$\therefore \frac{g(x)}{f(x)} = \frac{\frac{x^2}{2} + 1}{2x} = \frac{x^2 + 2}{4x}$$

Now, $f(x) = 0$

$\Rightarrow 4x = 0$

$\Rightarrow x = 0$

Therefore, $\frac{g(x)}{f(x)}$ is discontinuous at $x = 0$.

Question 84: The function $f(x)=\frac{4-x^2}{4x-x^3}$ is

(a) discontinuous at only one point

(b) discontinuous at exactly two points

(c) discontinuous at exactly three points

(d) None of the above.

Solution: (c)

Consider, $f(x)=\frac{4-x^2}{4x-x^3}=\frac{(4-x^2)}{x(4-x^2)}$

$$=\frac{(4-x^2)}{x(2^2-x^2)}$$

$$=\frac{(4-x^2)}{x(2+x)(2-x)}$$

Now, $x(2+x)(2-x)=0$

$\Rightarrow x=0,-2$ and 2

Therefore, $f(x)$ is discontinuous at exactly three points $x = 0$, $x = -2$ and $x = 2$.

Question 85: The set of points where the function f given by $f((x) = |2x - 1| \sin x$ is differentiable is

(a) R

(b) $R-\left(\frac{1}{2}\right)$

(c) $(0, \infty)$

(d) None of these

Solution: *(c)*

Consider, $f(x)=|2x-1|\sin x$

Now, $2x-1=0$

$\Rightarrow x=\frac{1}{2}$

At $x=\frac{1}{2}, f(x)$ is not differentiable.

Hence, $f(x)$ is differentiable in $R-\left(\frac{1}{2}\right)$.

Now, $\text{RHL}_{x\to\frac{1}{2}^+} f(x)=\lim_{h\to 0}\dfrac{f\left(\frac{1}{2}+h\right)-f\left(\frac{1}{2}\right)}{h}$.

$$=\lim_{h\to 0}\frac{\left|2\left(\frac{1}{2}+h\right)-1\right|\sin\left(\frac{1}{2}+h\right)-0}{h}$$

$$=\lim_{h\to 0}\frac{|2h|-\sin\left(\frac{1+2h}{2}\right)}{h}=2\cdot\sin\frac{1}{2}$$

and $\text{LHL}_{x\to\frac{1}{2}^-}=\lim_{h\to 0}\dfrac{f\left(\frac{1}{2}-h\right)-f\left(\frac{1}{2}\right)}{-h}$

$$=\lim_{h\to 0}\frac{\left|2\left(\frac{1}{2}-h\right)-1\right|-\sin\left(\frac{1}{2}-h\right)-0}{-h}$$

$$=\lim_{h\to 0}\frac{|0-2h|-\sin\left(\frac{1}{2}-h\right)}{-h}=-2\sin\left(\frac{1}{2}\right)$$

$\because Rf\left(\frac{1}{2}\right)\neq Lf\left(\frac{1}{2}\right)$

$\therefore f(x)$ is differentiable at $R-\left(\frac{1}{2}\right)$.

Question 86: The function $f(x)$ = cot x is discontinuous on the set

(a)$\{x = n\pi : n \in Z\}$

(b) $\{x = 2n\pi : n \in Z\}$

(c) $\left\{x=(2n+1)\frac{\pi}{2}; n\in Z\right\}$

(d) $\left\{x=\frac{n\pi}{2};n\in Z\right\}$

Solution: ***(a)***

Consider, $f(x)=\cot x=\frac{\cos x}{\sin x}$

We know that, $\sin x = 0$ at $x = n\pi$, $n \in Z$]

Hence, $f(x) = \cot x$ is discontinuous on the set $\{x = n\pi : n \in Z\}$.

Question 87: The function $f(x)=e^{|x|}$ is

(a) continuous everywhere but not differentiable at $x = 0$

(b) continuous and differentiable everywhere

(c) not continuous at $x = 0$

(d) None of the above

Solution: ***(a)***

Let $u(x)=|x|$ and $v(x) = e^x$

$\therefore f(x) = vou(x) = v[u(x)]$

Since, $u(x)$ and $v(x)$ both are continuous functions.

So, $f(x)$ is also continuous function but $u(x)=|x|$ is not differentiable at $x = 0$, whereas $v(x) = e^x$ is differentiable at everywhere.

Hence, $f(x)$ is continuous everywhere but not differentiable at $x = 0$.

Question 88: If $f(x)=x^2\sin\frac{1}{x}$, where $x \neq 0$, then the value of the function f at $x = 0$, so that the function is continuous at $x = 0$, is

(a) 0

(b) – 1

(c) 1

(d) None of these

Solution: ***(a)***

The value of the function f at $x = 0$, so that it is continuous at $x = 0$ is 0.

Question 89: If $f(x) = \begin{cases} mx+1, & \text{if } x \le \frac{\pi}{2} \\ \sin x + n, & \text{if } x > \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$, then

(a) $m = 1, n = 0$

(b) $m = \frac{n\pi}{2} + 1$

(c) $n = \frac{m\pi}{2}$

(d) $m = n = \frac{\pi}{2}$

Solution: ***(c)***

We are given, $f(x) = \begin{cases} mx+1, & \text{if } x \le \frac{\pi}{2} \\ \sin x + n, & \text{if } x > \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$

Since, $f(x)$ is continuous at $x = \frac{\pi}{2}$.

Therefore, LHL = RHL

$$\Rightarrow \lim_{x \to \frac{\pi}{2}^-} (mx+1) = \lim_{x \to \frac{\pi}{2}^+} (\sin x + n)$$

$$\Rightarrow \lim_{h \to 0} \left[m\left(\frac{\pi}{2} - h\right) + 1 \right] = \lim_{h \to 0} \left[\sin\left(\frac{\pi}{2} + h\right) + n \right]$$

$$\Rightarrow \frac{m\pi}{2} + 1 = \lim_{h \to 0} \cos h + n$$

$$\Rightarrow \frac{m\pi}{2} + 1 = 1 + n$$

$\Rightarrow n = \dfrac{m\pi}{2}$

Question 90: If $f(x) = |\sin x|$, then

(a) f is everywhere differentiable

(b) f is everywhere continuous but not differentiable at $x = n\pi$, $n \in Z$

(c) f is everywhere continuous but not differentiable at $x = (2n+1)\dfrac{\pi}{2}, n \in Z$

(d) None of the above

Solution: ***(b)***

Let $u(x) = \sin x$ and $v(x) = |x|$

$\therefore f(x) = vou(x) = v[u(x)]$

Since, $u(x)$ and $v(x)$ both are continuous functions.

Hence, $f(x) = vo\,u(x)$ is also a continuous function but $v(x)$ is not differentiable at $x = 0$.

So, $f(x)$ is not differentiable where $\sin x = 0$

$\Rightarrow x = n\pi, n \in Z$

Hence, $f(x)$ is continuous everywhere but not differentiable at $x = n\pi$, $n \in Z$.

Question 91: If $y = \log\left(\dfrac{1-x^2}{1+x^2}\right)$, then $\dfrac{dy}{dx}$ is equal to

(a) $\dfrac{4x^3}{1-x^4}$

(b) $\dfrac{-4x}{1-x^4}$

(c) $\dfrac{1}{4-x^4}$

(d) $\dfrac{-4x^3}{1-x^4}$

Solution: ***(b)***

Consider, $y = \log\left(\dfrac{1-x^2}{1+x^2}\right)$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{\frac{1-x^2}{1+x^2}} \cdot \frac{d}{dx}\left(\frac{1-x^2}{1+x^2}\right)$$

$$= \frac{(1+x^2)}{(1-x^2)} \cdot \frac{(1+x^2)\cdot(-2x)-(1-x^2)\cdot 2x}{(1+x^2)^2}$$

$$= \frac{-2x[1+x^2+1-x^2]}{(1-x^2)\cdot(1+x^2)} = \frac{-4x}{1-x^4}$$

Question 92: If $y = \sqrt{\sin x + y}$ then $\frac{dy}{dx}$ is equal to

(a) $\frac{\cos x}{2y-1}$

(b) $\frac{\cos x}{1-2y}$

(c) $\frac{\sin x}{1-2y}$

(d) $\frac{\sin x}{2y-1}$

Solution: ***(a)*** $\because$

Consider, $y = (\sin x + y)^{½}$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}(\sin + y)^{-1/2} \cdot \frac{d}{dx}(\sin + y)$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2} \cdot \frac{1}{(\sin x + y)^{1/2}} \cdot \left(\cos x + \frac{dy}{dx}\right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2y}\left(\cos x + \frac{dy}{dx}\right) \qquad [\because (\sin x + y)^{1/2} = y]$$

$$\Rightarrow \frac{dy}{dx} - \frac{1}{2y}\frac{dy}{dx} = \frac{\cos x}{2y}$$

$$\Rightarrow \frac{dy}{dx}\left(1 - \frac{1}{2y}\right) = \frac{\cos x}{2y}$$

$$\Rightarrow \frac{dy}{dx}\left(\frac{2y-1}{2y}\right) = \frac{\cos x}{2y}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\cos x}{2y-1}$$

Question 93: The derivative of $\cos^{-1}(2x^2 - 1)$ w.r.t. $\cos^{-1} x$ is

(a) 2

(b) $\dfrac{-1}{2\sqrt{1-x^2}}$

(c) $\dfrac{2}{x}$

(d) $1 - x^2$

Solution: ***(a)***

Let $u = \cos^{-1}(2x^2 - 1)$ and $v = \cos^{-1} x$

$$\therefore \frac{dv}{dx} = \frac{-1}{\sqrt{1-\left(2x^2-1\right)^2}} \cdot 4x$$

$$= \frac{-4x}{\sqrt{1-\left(4x^4+1-4x^2\right)}}$$

$$= \frac{-4x}{\sqrt{-4x^4+4x^2}} = \frac{-4x}{\sqrt{4x^2(1-x^2)}}$$

$$= \frac{-2}{\sqrt{1-x^2}}$$

and $\dfrac{du}{dx} = \dfrac{-1}{\sqrt{1-x^2}}$

$$\therefore \frac{du}{dv} = \frac{du/dx}{dv/dx}$$

$$= \frac{\dfrac{-2}{\sqrt{1-x^2}}}{\dfrac{-1}{\sqrt{1-x^2}}}$$

$$= 2$$

Question 94: If $x = t^2$ and $y = t^3$, then $\frac{d^2y}{dx^2}$ is

(a) $\frac{3}{2}$

(b) $\frac{3}{4t}$

(c) $\frac{3}{2t}$

(d) $\frac{3}{2t}$

Solution: ***(b)***

We are given, $x = t^2$ and $y = t^3$

$\Rightarrow \frac{dx}{dt} = 2\text{t} \text{ and } \frac{dy}{dt} = 3t^2$

$\therefore \frac{dy}{dx} = \frac{dy/dt}{dx/dt}$

$\Rightarrow \frac{dy}{dx} = \frac{3t^2}{2t}$

$\Rightarrow \frac{dy}{dx} = \frac{3}{2}t$

$\Rightarrow \frac{d^2y}{dx^2} = \frac{3}{2} \cdot \frac{dt}{dx}$

$= \frac{3}{2} \cdot \frac{1}{2t}$ $\quad \left[\begin{array}{l} \because \frac{dx}{dt} = 2\text{t} \\ \Rightarrow \frac{dt}{dx} = \frac{1}{2t} \end{array} \right]$

$= \frac{3}{4t}$

Question 95: The value of c in Rolle's theorem for the function $f(x) = x^3 - 3x$ in the interval $\left[0, \sqrt{3}\right]$ is

(a) 1

(b) −i

(c) $\frac{3}{2}$

(d) $\frac{1}{3}$

Solution: ***(a)***

Consider, $f(x) = x^3 - 3x$

$\Rightarrow f'(x) = 3x^2 - 3$

$\Rightarrow f'(c) = 3c^2 - 3$

Now, $f'(c) = 0$

$\Rightarrow 3c^2 - 3 = 0$

$\Rightarrow c^2 = \frac{3}{3} = 1$

$\Rightarrow c = \pm 1$ where $1 \in \left(0, \sqrt{3}\right)$

Question 96: For the function $f\left(x\right) = x + \frac{1}{x}, x \in \left[1,3\right]$,], the value of c for mean value theorem is

(a) 1

(b) $\sqrt{3}$

(c) 2

(d) None of these

Solution: ***(b)***

We know that, $f'\left(c\right) = \frac{f\left(b\right) - f\left(a\right)}{b-a}$

$$\Rightarrow 1 - \frac{1}{c^2} = \frac{\left[3 + \frac{1}{3}\right] - \left[1 + \frac{1}{1}\right]}{3-1} \qquad \left[\because f'\left(x\right) = 1 - \frac{1}{x^2}, b = 3, a = 1\right]$$

$$\Rightarrow \frac{c^2 - 1}{c^2} = \frac{\frac{10}{3} - 2}{2}$$

$$\Rightarrow \frac{c^2 - 1}{c^2} = \frac{2}{3}$$

$\Rightarrow 3(c^2 - 1) = 2c^2$

$\Rightarrow 3c^2 - 2c^2 = 3$

$\Rightarrow c^2 = 3$

$\Rightarrow c = \pm\sqrt{3}$