

1. **Event** A subset of the sample space associated with a random experiment is called an event or a case.  
e.g. In tossing a coin, getting either head or tail is an event.
2. **Equally Likely Events** The given events are said to be equally likely, if none of them is expected to occur in preference to the other.  
e.g. In throwing an unbiased die, all the six faces are equally likely to come.
3. **Mutually Exclusive Events** A set of events is said to be mutually exclusive, if the happening of one excludes the happening of the other, i.e. if  $A$  and  $B$  are mutually exclusive, then  $(A \cap B) = \phi$ .  
e.g. In throwing a die, all the 6 faces numbered 1 to 6 are mutually exclusive, since if anyone of these faces comes, then the possibility of others in the same trial is ruled out.
4. **Exhaustive Events** A set of events is said to be exhaustive, if the performance of the experiment always results in the occurrence of atleast one of them.  
If  $E_1, E_2, \dots, E_n$  are exhaustive events, then  $E_1 \cup E_2 \cup \dots \cup E_n = S$ .  
e.g. In throwing of two dice, the exhaustive number of cases is  $6^2 = 36$ . Since, any of the numbers 1 to 6 on the first die can be associated with any of the 6 numbers on the other die.

5. **Complement of an Event** Let  $A$  be an event in a sample space  $S$ , then complement of  $A$  is the set of all sample points of the space other than the sample point in  $A$  and it is denoted by  $A'$  or  $\bar{A}$ .

i.e.  $A' = \{n : n \in S, n \notin A\}$

- NOTE**
- (i) An operation which results in some well-defined outcomes, is called an **experiment**.
  - (ii) An experiment in which the outcomes may not be same even if the experiment is performed in identical condition is called **random experiment**.

## 6. Probability of an Event

If a trial result is  $n$  exhaustive, mutually exclusive and equally likely cases and  $m$  of them are favourable to the happening of an event  $A$ , then the **probability** of happening of  $A$  is given by

$$P(A) = \frac{\text{Number of favourable cases to } A}{\text{Number of exhaustive cases}} = \frac{n(A)}{n(S)} = \frac{m}{n}$$

### NOTE

- (i)  $0 \leq P(A) \leq 1$
- (ii) Probability of impossible event is zero.
- (iii) Probability of certain event (possible event) is 1.
- (iv)  $P(A \cup A') = P(S)$
- (v)  $P(A \cap A') = P(\phi)$
- (vi)  $P(A')' = P(A)$
- (vii)  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

7. **Conditional Probability** Let  $E$  and  $F$  be two events associated with the same sample space of a random experiment. Then, probability of occurrence of event  $E$ , when the event  $F$  has already occurred, is called conditional probability of event  $E$  over  $F$  and is denoted by  $P(E/F)$ .

$$P(E/F) = \frac{P(E \cap F)}{P(F)}, \text{ where } P(F) \neq 0$$

Similarly, conditional probability of event  $F$  over  $E$  is given as

$$P(F/E) = \frac{P(F \cap E)}{P(E)}, \text{ where } P(E) \neq 0$$

8. **Properties of Conditional Probability** If  $E$  and  $F$  are two events of sample space  $S$  and  $G$  is an event of  $S$  which has already occurred such that  $P(G) \neq 0$ , then

- (i)  $P[(E \cup F)/G] = P(E/G) + P(F/G) - P[(E \cap F)/G]$ ,  $P(G) \neq 0$
- (ii)  $P[(E \cup F)/G] = P(E/G) + P(F/G)$ , if  $E$  and  $F$  are disjoint events.
- (iii)  $P(E'/G) = 1 - P(E/G)$
- (iv)  $P(S/E) = P(E/E) = 1$

9. **Multiplication Theorem** If  $E$  and  $F$  are two events associated with a sample space  $S$ , then the probability of simultaneous occurrence of the events  $E$  and  $F$  is

$$P(E \cap F) = P(E) \cdot P(F/E), \text{ where } P(E) \neq 0$$

or 
$$P(E \cap F) = P(F) \cdot P(E/F), \text{ where } P(F) \neq 0$$

This result is known as multiplication rule of probability.

10. **Multiplication Theorem for More than Two Events** If  $E$ ,  $F$  and  $G$  are three events of sample space, then

$$P(E \cap F \cap G) = P(E) \cdot P\left(\frac{F}{E}\right) \cdot P\left(\frac{G}{E \cap F}\right)$$

11. **Independent Events** Two events  $E$  and  $F$  are said to be independent, if probability of occurrence or non-occurrence of one of the events is not affected by that of the other. For any two independent events  $E$  and  $F$ , we have the relation

(i)  $P(E \cap F) = P(E) \cdot P(F)$

(ii)  $P(F/E) = P(F), P(E) \neq 0$

(iii)  $P(E/F) = P(E), P(F) \neq 0$

Also, their complements are independent events.

i.e. 
$$P(\bar{E} \cap \bar{F}) = P(\bar{E}) \cdot P(\bar{F})$$

**NOTE** If  $E$  and  $F$  are dependent events, then  $P(E \cap F) \neq P(E) \cdot P(F)$ .

12. Three events  $E$ ,  $F$  and  $G$  are said to be mutually independent, if

(i) $P(E \cap F) = P(E) \cdot P(F)$	(ii) $P(F \cap G) = P(F) \cdot P(G)$
(iii) $P(E \cap G) = P(E) \cdot P(G)$	(iv) $P(E \cap F \cap G) = P(E) \cdot P(F) \cdot P(G)$

If atleast one of the above is not true for three given events, then we say that the events are not independent.

**NOTE** Independent and mutually exclusive events do not have same meaning.

1. **Partition of Sample Space** A set of events  $E_1, E_2, \dots, E_n$  is said to represent a partition of the sample space  $S$ , if it satisfies the following conditions:

- (i)  $E_i \cap E_j = \emptyset; i \neq j; i, j = 1, 2, \dots, n$       (ii)  $E_1 \cup E_2 \cup \dots \cup E_n = S$   
(iii)  $P(E_i) > 0, \forall i = 1, 2, \dots, n$

2. **Theorem of Total Probability** Let events  $E_1, E_2, \dots, E_n$  form a partition of the sample space  $S$  of an experiment. If  $A$  is any event associated with sample space  $S$ , then

$$P(A) = P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2) + \dots + P(E_n) \cdot P(A/E_n) = \sum_{j=1}^n P(E_j) \cdot P(A/E_j)$$

3. **Baye's Theorem** If  $E_1, E_2, \dots, E_n$  are  $n$  non-empty events which constitute a partition of sample space  $S$ , i.e.  $E_1, E_2, \dots, E_n$  are pairwise disjoint,  $E_1 \cup E_2 \cup \dots \cup E_n = S$  and  $P(E_i) > 0$ , for all  $i = 1, 2, \dots, n$ . Also, let  $A$  be any non-zero event, then probability

$$P(E_i/A) = \frac{P(E_i) \cdot P(A/E_i)}{\sum_{i=1}^n P(E_i) \cdot P(A/E_i)}, \forall i = 1, 2, 3, \dots, n$$

4. **Random Variable** A random variable is a real valued function, whose domain is the sample space of a random experiment. Generally, it is denoted by capital letter  $X$ .

**NOTE** More than one random variables can be defined on the same sample space.

5. **Probability Distributions** The system in which the values of a random variable are given along with their corresponding probabilities is called probability distribution.

Let  $X$  be a random variable which can take  $n$  values  $x_1, x_2, \dots, x_n$ . Let  $p_1, p_2, \dots, p_n$  be the respective probabilities. Then, a probability distribution table is given as follows:

<b><math>X</math></b>	$x_1$	$x_2$	$x_3$	...	$x_n$
<b><math>P</math></b>	$p_1$	$p_2$	$p_3$	...	$p_n$

such that  $p_1 + p_2 + p_3 + \dots + p_n = 1$



**NOTE** If  $x_i$  is one of the possible values of a random variable  $X$ , then statement  $X = x_i$  is true only at some point(s) of the sample space. Hence, the probability that  $X$  takes value  $x_i$  is always non-zero, i.e.  $P(X = x_i) \neq 0$ .

6. **Mean and Variance of a Probability Distribution** Mean of a probability distribution is  $\sum_{i=1}^n x_i \cdot p_i$ . It is also called expectation of  $X$ , i.e.  $E(X) = \mu = \sum_{i=1}^n x_i p_i$ .

Variance is given by  $V(X)$  or  $\sigma_x^2$  and defined as  $\sigma^2 = \sum_{i=1}^n x_i^2 \cdot p(x_i) - \left( \sum_{i=1}^n x_i \cdot p(x_i) \right)^2$

or  $E(X^2) - [E(X)]^2$

or Equivalently  $V(X) = E(X - \mu)^2$  or  $\sum_{i=1}^n (x_i - \mu)^2 p(x_i)$

Standard deviation is given by,  $\sigma_x = \sqrt{v(x)} = \sqrt{\sum_{i=1}^n (x_i - \mu)^2 p(x_i)}$

7. **Bernoulli Trial** Trials of random experiment are called bernoulli trials, if they satisfy the following conditions:
- (i) There should be a finite number of trials.
  - (ii) The trials should be independent.
  - (iii) Each trial has exactly two outcomes, success or failure.
  - (iv) The probability of success remains the same in each trial.
8. **Binomial Distribution** The probability distribution of numbers of successes in an experiment consisting  $n$  Bernoulli trials obtained by the binomial expansion  $(p+q)^n$ , is called binomial distribution.

Let  $X$  be a random variable which can take  $n$  values  $x_1, x_2, \dots, x_n$ . Then, by binomial distribution, we have  $P(X=r) = {}^nC_r p^r q^{n-r}$

where,

$n$  = Total number of trials in an experiment

$p$  = Probability of success in one trial

$q$  = Probability of failure in one trial

$r$  = Number of success trial in an experiment

Also,  $p + q = 1$

Binomial distribution of the number of successes  $X$  can be represented as

$X$	0	1	2	... r ...	$n$
$P(X)$	${}^nC_0 p^0 q^n$	${}^nC_1 p^1 q^{n-1}$	${}^nC_2 p^2 q^{n-2}$	${}^nC_r p^r q^{n-r}$	${}^nC_n p^n$

#### 9. Mean and Variance of Binomial Distribution

(i) Mean  $(\mu) = \sum x_i p_i = np$  (ii) Variance  $(\sigma^2) = \sum x_i^2 p_i - \mu^2 = npq$

(iii) Standard deviation  $(\sigma) = \sqrt{\text{Variance}} = \sqrt{npq}$

**NOTE** Mean > Variance