NCERT Exemplar Solutions

Class 12 – Mathematics
Chapter 9 – Differential Equations

Objective Type Questions

Question 34. The degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^2 = x\sin\left(\frac{dy}{dx}\right)$ is

(a) 1 (b) 2

(c) 3 (d) not defined

Solution. (d)

Explanation: The degree of above differential equation is not defined because on solving $\sin\left(\frac{dy}{dx}\right)$ we will get an infinite series in the increasing powers of $\frac{dy}{dx}$. Therefore its degree is not defined.

Question 35. The degree of the differential equation $\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2} = \frac{d^2y}{dx^2}$ is

(a) 4 (b) $\frac{3}{2}$

(c) not defined (d) 2

Solution. (d)

Explanation: Given is, $\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2} = \frac{d^2y}{dx^2}$

On squaring both sides, we get

$$\left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = \left(\frac{d^2y}{dx^2}\right)^2$$

So, the degree of differential equation is 2.

Question 36. The order and degree of the differential equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^{1/4} + x^{1/5} = 0$ respectively, are

(a) 2 and 4 (b) 2 and 2

(c) 2 and 3 (d) 3 and 3

Solution. (a)

Explanation: Given that, $\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^{1/4}=-x^{1/5}$

$$\Rightarrow \quad \left(\frac{dy}{dx}\right)^{1/4}=-\left(x^{1/5}+\frac{d^2y}{dx^2}\right)$$

On squaring both sides, we get

$$\left(\frac{dy}{dx}\right)^{1/2}=\left(x^{1/5}+\frac{d^2y}{dx^2}\right)^2$$

Again, squaring both sides, we get

$$\frac{dy}{dx}=\left(x^{1/5}+\frac{d^2y}{dx^2}\right)^4$$

Thus, order = 2, degree = 4

Question 37. If $y=e^{-x}(A\cos x+B\sin x)$, then y is a solution of

(a) $\frac{d^2y}{dx^2}+2\frac{dy}{dx}=0$

(b) $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+2y=0$

(c) $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+2y=0$

(c) $\frac{d^2y}{dx^2}+2y=0$

Solution. (c)

Explanation: Given that, $y=e^{-x}(A\cos x+B\sin x)$

On differentiating both sides w.r.t., x we get

$$\frac{dy}{dx}=-e^{-x}(A\cos x+B\sin x)+e^{-x}(-A\sin x+B\cos x)$$

$$\frac{dy}{dx}=-y+e^{-x}(-A\sin x+B\cos x)$$

Again, differentiating both sides w.r.t. x, we get

$$\frac{d^2y}{dx^2}=\frac{-dy}{dx}+e^{-x}(-\cos x-B\sin x)-e^{-x}(-A\sin x+B\cos x)$$

$$\Rightarrow \quad \frac{d^2y}{dx^2}=-\frac{dy}{dx}-y-\left[\frac{dy}{dx}+y\right]$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = -\frac{dy}{dx} - y - \frac{dy}{dx} - y$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = -2\frac{dy}{dx} - 2y$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 0$$

Question 38. The differential equation for $y = A \cos \alpha x + B \sin \alpha x$, where A and B are arbitrary constants is

(a) $\frac{d^2y}{dx^2} - \alpha^2 y = 0$

(b) $\frac{d^2y}{dx^2} + \alpha^2 y = 0$

(c) $\frac{d^2y}{dx^2} + \alpha y = 0$

(d) $\frac{d^2y}{dx^2} - \alpha y = 0$

Solution. (b)

Explanation: Given, $y = A\cos \alpha + B\sin \alpha$

On differentiating both sides w.r.t., x we get

$$\Rightarrow \quad \frac{dy}{dx} = -\alpha A \sin \alpha x + \alpha B \cos \alpha x$$

Again, differentiating both sides w.r.t. x, we get

$$\frac{d^2y}{dx^2} = -A\alpha^2 \cos \alpha x - \alpha^2 B \sin \alpha x$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = -\alpha^2 \left(A\cos \alpha x + B\sin \alpha x\right)$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = -\alpha^2 y$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} + \alpha^2 y = 0$$

Question 39. The solution of differential equation $xdy - ydx = 0$ represents

(a) a rectangular hyperbola

(b) parabola whose vertex is at origin

(c) straight line passing through origin

(d) a circle whose centre is at origin

Solution. (c)

Explanation: Given is, $xdy - ydx = 0$

$\Rightarrow \quad xdy = ydx$

$\Rightarrow \quad \dfrac{dy}{y} = \dfrac{dx}{x}$

On integrating both sides, we get

$\log y = \log x + \log C$

$\Rightarrow \quad \log y = \log Cx$

$\Rightarrow \quad y = Cx$

which represents is a straight line passing through origin.

Question 40. The integrating factor of differential equation $\cos x \dfrac{dy}{dx} + y \sin x = 1$ is

(a) $\cos x$ (b) $\tan x$

(c) $\sec x$ (d) $\sin x$

Solution. (c)

Explanation: Given is, $\cos x \dfrac{dy}{dx} + y \sin x = 1$

$\Rightarrow \quad \dfrac{dy}{dx} + y \tan x = \sec x$

Here, $P = \tan x$ and $Q = \sec x$

$\text{IF} = e^{\int P dx} = e^{\int \tan x dx} = e^{\log \sec x}$

$\therefore \quad = \sec x$

Question 41. The solution of differential equation $\tan y \sec^2 x dx + \tan x \sec^2 y dy = 0$ is

(a) $\tan x + \tan y = k$ (b) $\tan x - \tan y = k$

(c) $\dfrac{\tan x}{\tan y} = k$ (d) $\tan x \cdot \tan y = k$

Solution. (d)

Explanation: Given is, $\tan y \sec^2 x dx + \tan x \sec^2 y dy = 0$

$\Rightarrow \quad \tan y \sec^2 x dx = - \tan x \sec^2 y dy$

$\Rightarrow \quad \dfrac{\sec^2 x}{\tan x} dx = \dfrac{-\sec^2 y}{\tan y} dy \quad \ldots(i)$

On integrating both sides, we get

$$\int \frac{\sec^2 x}{\tan x} dx = -\int \frac{\sec^2 y}{\tan y} dy$$

Put tan $x = t$

$\Rightarrow \quad \sec^2 x \, dx = dt$

Again put tan $y = u$

$\Rightarrow \quad \sec^2 y dy = du$

On substituting these values in equation (i), we get

$$\int \frac{dt}{t} = -\int \frac{du}{u}$$

$\Rightarrow \quad \log t = -\log u + \log k$

$\Rightarrow \quad \log(t \cdot u) = \log k$

$\Rightarrow \quad \log(\tan x \tan y) = \log k$

$\Rightarrow \quad \tan x \tan y = \log k$

Question 42. The family $y = Ax + A^3$ of curves is represented by differential equation of degree

(a) 1 (b) 2

(c) 3 (d) 4

Solution. (a)

Explanation: Given is, $y = Ax + A^3$

Differentiating both sides w.r.t. x, we get:

$$\frac{dy}{dx} = A$$

This equation can be differentiated only once because it has only one arbitrary constant.

$\therefore \quad$ Degree = 1

Question 43. The integrating factor of $\frac{xdy}{dx} - y = x^4 - 3x$ is

(a) x (b) $\log x$

(c) $\frac{1}{x}$ (d) $-x$

Solution. (c)

Explanation: Given is, $x\frac{dy}{dx} - y = x^4 - 3x$

$\Rightarrow \quad \frac{dy}{dx} - \frac{y}{x} = x^3 - 3$

Here, $\quad P = -\frac{1}{x}, Q = x^3 - 3$

$\therefore \quad \text{IF} = e^{\int P dx} = e^{-\int \frac{1}{x} dx} = e^{-\log x} = \frac{1}{x}$

Question 44. The solution of $\frac{dy}{dx} - y = 1$, $y(0) = 1$ is given by

(a) $xy = -e^x$

(b) $xy = -e^{-x}$

(c) $xy = -1$

(d) $y - 2e^x - 1$

Solution. (b)

Explanation: Given is, $\frac{dy}{dx} - y = 1$

$\Rightarrow \quad \frac{dy}{dx} = 1 + y$

$\Rightarrow \quad \frac{dy}{1+y} = dx$

On integrating both sides, we get

$\log(1 + y) = x + C \quad ...(i)$

When $x = 0$ and $y = 1$, then

$\log 2 = 0 + c$

$\Rightarrow \quad C = \log 2$

The required solution is

$\log (1 + y) = x + \log 2$

$\Rightarrow \quad \log\left(\frac{1+y}{2}\right) = x$

$\Rightarrow \quad \frac{1+y}{2} = e^x$

$\Rightarrow \quad 1 + y = 2e^x$

$\Rightarrow \quad y = 2e^x - 1$

Question 45. The number of solutions of $\frac{dy}{dx} = \frac{y+1}{x-1}$, when $y(1) = 2$ is

(a) none (b) one

(c) two (d) infinite

Solution. (b)

Explanation: Given is, $\frac{dy}{dx}=\frac{y+1}{x-1}$

$\Rightarrow \quad \frac{dy}{y+1}=\frac{dx}{x-1}$

On integrating both sides, we get

$\log(y + 1) = \log(x - 1) - \log C$

$C(y + 1) = (x - 1)$

$\Rightarrow \quad C=\frac{x-1}{y+1}$

When $x = 1$ and $y = 2$, then $C = 0$

So, the required solution is $x - 1 = 0$

Hence, only one solution exists.

Question 46. Which of the following is a second order differential equation?

(a) $(y')^2 + x = y^2$ (b) $y'y'' + y = \sin x$

(c) $y''' + (y'')^2 + y = 0$ (d) $y' = y^2$

Solution. (b)

Explanation: The second order differential equation is $y'y'' + y = \sin x$.

Question 47. The integrating factor of differential equation $\left(1-x^2\right)\frac{dy}{dx}-xy=1$ is

(a) $-x$ (b) $\frac{x}{1+x^2}$

(c) $\sqrt{1-x^2}$ (d) $\frac{1}{2}\log(1-x^2)$

Solution. (c)

Explanation: Given is, $\left(1-x^2\right)\frac{dy}{dx}-xy=1$

$\Rightarrow \quad \frac{dy}{dx}-\frac{x}{1-x^2}y=\frac{1}{1-x^2}$

Which is a linear differential equation.

$\therefore \quad \text{IF} = e^{-\int \frac{x}{1-x^2}dx}$

Put $\quad 1-x^2 = t$

$\Rightarrow \quad -2xdx = dt$

$\Rightarrow \quad xdx = -\dfrac{dt}{2}$

Now, $\text{IF} = e^{\frac{1}{2}\int \frac{dt}{t}} = e^{\frac{1}{2}\log t} = e^{\frac{1}{2}\log(1-x^2)} = \sqrt{1-x^2}$

Question 48. $\tan^{-1} x + \tan^{-1} y = C$ is general solution of the differential equation

(a) $\dfrac{dy}{dx} = \dfrac{1+y^2}{1+x^2}$

(b) $\dfrac{dy}{dx} = \dfrac{1+x^2}{1+y^2}$

(c) $(1 + x^2)dy + (1 + y^2)dx = 0$

(d) $(1 + x^2)dx + (1 + y^2)dy = 0$

Solution. (c)

Explanation: Given is, $\tan^{-1} x + \tan^{-1} y = C$

On differentiating above eqaution w.r. t. x, we get

$$\frac{1}{1+x^2} + \frac{1}{1+y^2}\cdot\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{1}{1+y^2}\cdot\frac{dy}{dx} = -\frac{1}{1+x^2}$$

$$\Rightarrow \quad (1 + x^2)\, dy + (1 + y^2)\, dx = 0$$

Question 49. The differential equation $y\dfrac{dy}{dx} + x = C$ represents

(a) family of hyperbolas

(b) family of parabolas

(c) family of ellipses

(d) family of circles

Solution. (d)

Explanation: Given is, $y\dfrac{dy}{dx} + x = C$

$$\Rightarrow \quad y\frac{dy}{dx} = C - x$$

$\Rightarrow \quad y\,dy = (C - x)\,dx$

On integrating both sides, we get

$$\frac{y^2}{2} = Cx - \frac{x^2}{2} + K$$

$$\Rightarrow \quad \frac{x^2}{2} + \frac{y^2}{2} = Cx + K$$

$$\Rightarrow \quad \frac{x^2}{2} + \frac{y^2}{2} - Cx = K$$

Which represent family of circles.

Question 50. The general solution of $e^x \cos y dx - e^x \sin y dy = 0$ is

(a) $e^x \cos y = k$

(b) $e^x \sin y = k$

(c) $e^x = k \cos y$

(d) $e^x = k \sin y$

Solution. **(a)**

Explanation: Given is, $e^x \cos y dx - e^x \sin y dy = 0$

$\Rightarrow \quad e^x \cos y dx = e^x \sin y dy$

$$\Rightarrow \quad \frac{dx}{dy} = \tan y$$

$\Rightarrow \quad dx = \tan y dy$

On integrating both sides, we get

$x = \log \sec y + C$

$\Rightarrow \quad x - C = \log \sec y$

$\Rightarrow \quad \sec y = e^{x-C}$

$\Rightarrow \quad \sec y = e^x e^{-}$

$$\Rightarrow \quad \frac{1}{\cos y} = \frac{e^x}{e^C}$$

$\Rightarrow \quad e^x \cos y = K \qquad$ [where, $K = e^C$]

Question 51. The degree of differential equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 + 6y^5 = 0$ is

(a) 1

(b) 2

(c) 3

(d) 5

Solution. (a)

Explanation : $\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^3+6y^5=0$

We know that, the degree of a differential equation is exponent highest order derivative.

$\therefore$ Degree = 1

Question 52. The solution of $\frac{dy}{dx}+y=e^{-x}, y(0)$ is

(a) $y = e^x(x - 1)$ (b) $y = xe^{-x}$

(c) $y = xe^{-x} + 1$ (d) $y = (x + 1)\, e^{-x}$

Solution. (b)

Explanation: Given is, $\frac{dy}{dx}+y=e^{-x}$

Here, $P = 1$, $Q = e^{-x}$

$\text{IF} = e^{\int p\,dx} = e^{\int dx} = e^x$

Thus the general solution is

$$y\cdot e^x=\int e^{-x}e^x dx + C$$

$$\Rightarrow \quad y\cdot e^x=\int dx + C$$

$$\Rightarrow \quad y\cdot e^x = x + C \quad \text{...(i)}$$

When $x = 0$ and $y = 0$, then equation (i) gives:

$$0 = 0 + C$$

$$\Rightarrow \quad C = 0$$

Putting the value of C in equation (i), we get:

$$y\cdot e^x = x$$

$$\Rightarrow \quad y = x\,e^{-x}$$

Question 53. The integrating factor of differential equation $\frac{dy}{dx}+y\tan x-\sec x=0$ is

(a) $\cos x$ (b) $\sec x$

(c) $e^{\cos x}$ (d) $e^{\sec x}$

Solution. (b)

Explanation: Given, $\frac{dy}{dx} + y \tan x - \sec x = 0$

Here, $P = \tan x$, $Q = \sec x$

$\text{IF} = e^{\int P dx} = e^{\int \tan x dx} = e^{(\log \sec x)} = \sec x$

Question 54. The solution of differential equation $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$ is

(a) $y = \tan^{-1} x$ (b) $y - x = k(1 + xy)$

(c) $x = \tan^{-1} y$ (d) $\tan(xy) = k$

Solution. (b)

Explanation: Given that, $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$

$\Rightarrow \quad \frac{dy}{1+y^2} = \frac{dx}{1+x^2}$

On integrating both sides, we get

$\tan^{-1} y = \tan^{-1} x + C$

$\Rightarrow \quad \tan^{-1} y - \tan^{-1} x = C$

$\Rightarrow \quad \tan^{-1}\left(\frac{y-x}{1+xy}\right) = C$

$\Rightarrow \quad \frac{y-x}{1+xy} = \tan C$

$\Rightarrow \quad y - x = \tan C\,(1 + x\,y)$

$\Rightarrow \quad y - x = C'\,(1 + x\,y)$

Where, $C' = \tan C$

Question 55. The integrating factor of differential equation $\frac{dy}{dx} + y = \frac{1+y}{x}$ is

(a) $\frac{x}{e^x}$ (b) $\frac{e^x}{x}$

(c) xe^x (d) e^x

Solution. (b)

Explanation: Give is, $\frac{dy}{dx} + y = \frac{1+y}{x}$

$$\Rightarrow \quad \frac{dy}{dx}=\frac{1+y}{x}-y$$

$$\Rightarrow \quad \frac{dy}{dx}=\frac{1+y-xy}{x}$$

$$\Rightarrow \quad \frac{dy}{dx}=\frac{1}{x}+\frac{y(1-x)}{x}$$

$$\Rightarrow \quad \frac{dy}{dx}-\left(\frac{1-x}{x}\right)y=\frac{1}{x}$$

Here, $P=\dfrac{-(1-x)}{x}, Q=\dfrac{1}{x}$

$$\text{IF}=e^{\int Pdx}=e^{-\int \frac{1-x}{x}dx}=e^{\int \frac{x-1}{x}dx}=e^{\int \left(1-\frac{1}{x}\right)dx}$$

$$=e^{\int x-\log x}=e^{x}\cdot e^{\log\left(\frac{1}{x}\right)}=e^{x}\cdot\frac{1}{x}$$

Question 56. $y = ae^{mx} + be^{-mx}$ satisfies which of the following differential equation?

(a) $\dfrac{dy}{dx}+my=0$ (b) $\dfrac{dy}{dx}-my=0$

(c) $\dfrac{d^2y}{dx^2}-m^2y=0$ (d) $\dfrac{d^2y}{dx^2}+m^2y=0$

Solution. (c)

Explanation: Given is, $y = ae^{mx} + be^{-mx}$

On differentiating both sides w.r.t x, we get

$$\frac{dy}{dx}=mae^{mx}-bme^{-mx}$$

Again, differentiating both sides w.r.t. x, we get

$$\frac{d^2y}{dx^2}=m^2ae^{mx}-bm^2e^{-mx}$$

$$\Rightarrow \quad \frac{d^2y}{dx^2}=m^2\left(ae^{mn}+be^{-mn}\right)$$

$$\Rightarrow \quad \frac{d^2y}{dx^2}=m^2y$$

$\Rightarrow \quad \frac{d^2y}{dx^2} - m^2 y = 0$

Question 57. The solution of differential equation $\cos x \sin y \, dx + \sin x \cos y dy = 0$ is

(a) $\frac{\sin x}{\sin y} = C$ (b) $\sin x \sin y = C$

(c) $\sin x + \sin y = C$ (d) $\cos x \cos y = C$

Solution. (b)

Explanation: Given differential equation is

$\cos x \sin y dx + \sin x \cos y dy = 0$

$\Rightarrow \quad \cos x \sin y dx = - \sin x \cos y dy$

$\Rightarrow \quad \frac{\cos x}{\sin x} dx = -\frac{\cos y}{\sin y} dy$

$\Rightarrow \quad \cot x \, dx = - \cot y dy$

On integrating both sides, we get

$\log \sin x = - \log \sin y + \log C$

$\Rightarrow \quad \log \sin x \sin y = \log C$

Carrying the exponent on both sides, we get

$\Rightarrow \quad \sin x \sin y = C$

Question 58. The solution of $x\frac{dy}{dx} + y = e^x$ is

(a) $y = \frac{e^x}{x} + \frac{k}{x}$ (b) $y = xe^x + Cx$

(c) $y = xe^x + k$ (d) $x = \frac{e^y}{y} + \frac{k}{y}$

Solution. (a)

Explanation: Given is, $x\frac{dy}{dx} + y = e^x$

$\Rightarrow \quad \frac{dy}{dx} + \frac{y}{x} = \frac{e^x}{x}$

This is a linear differential equation.

$\therefore \quad \text{IF} = e^{\int \frac{1}{x} dx} = e^{(\log x)} = x$

Thus, the general solution is $y \cdot x = \int \left(\frac{d^x}{x} \cdot x \right) dx$

$\Rightarrow \quad y \cdot x = \int e^x dx$

$\Rightarrow \quad y \cdot x = e^x + k$

$\Rightarrow \quad y = \frac{e^x}{x} + \frac{k}{x}$

Question 59. The differential equation of the family of curves $x^2 + y^2 - 2ay = 0$, where o is arbitrary constant, is

(a) $\left(x^2 - y^2\right)\frac{dy}{dx} = 2xy$ (b) $2\left(x^2 + y^2\right)\frac{dy}{dx} = xy$

(c) $2\left(x^2 - y^2\right)\frac{dy}{dx} = xy$ (d) $\left(x^2 + y^2\right)\frac{dy}{dx} = 2xy$

Solution. (a)

Explanation: Given equation is, $x^2 + y^2 - 2ay = 0$

$\Rightarrow \quad \frac{x^2 + y^2}{y} = 2a$

On differentiating both sides w.r.t. x, we get

$$\frac{y\left(2x + 2y\frac{dy}{dx}\right) - \left(x^2 + y^2\right)\frac{dy}{dx}}{y^2} = 0$$

$\Rightarrow \quad 2xy + 2y^2\frac{dy}{dx} - \left(x^2 + y^2\right)\frac{dy}{dx} = 0$

$\Rightarrow \quad \left(2y^2 - x^2 = y^2\right)\frac{dy}{dx} = -2xy$

$\Rightarrow \quad \left(y^2 - x^2\right)\frac{dy}{dx} = -2xy$

$\Rightarrow \quad \left(x^2 - y^2\right)\frac{dy}{dx} = 2xy$

Question 60. The family $Y = Ax + A^3$ of curves will correspond to a differential equation of order

(a) 3 (b) 2

(c) 1 (d) not defined

Solution. (c)

Explanation: Given family of curves is $y = Ax + A^3$ ...(i)

$\Rightarrow \quad \frac{dy}{dx} = A$

Replacing A by $\frac{dy}{dx}$ in equation (i) we get

$$y = \frac{dy}{dx} + \left(\frac{dy}{dx}\right)^3$$

$\therefore \quad$ Order = 1

Question 61. The general solution of $\frac{dy}{dx} = 2xe^{x^2} - y$ is

(a) $e^{x^2-y} = C$ (b) $e^{-y} + e^{x^2} = C$

(c) $e^y = e^{x^2} + C$ (d) $e^{x^2+y} = C$

Solution. (c)

Explanation: Given is, $\frac{dy}{dx} = 2xe^{x^2-y}$

Or $\quad \frac{dy}{dx} = 2xe^{x^2} \cdot e^{-y}$

$\Rightarrow \quad e^y \frac{dy}{dx} = 2xe^{x^2}$

$\Rightarrow \quad e^y\, dy = 2xe^{x2}\, dx$

On integrating both sides, we get

$$\int e^y dy = 2\int xe^{x^2} dx \qquad \text{...(i)}$$

Put $x^2 = t$

$\Rightarrow \quad 2x\, dx = dt$

Putting these values in equation (i), we get:

$$\int e^y dy = \int e^t dt$$

$\Rightarrow \quad e^y = e^t + C$

$\Rightarrow \quad e^y = e^{x2} + C$

Question 62. The curve for which the slope of the tangent at any point is equal to the ratio of the abcissa to the ordinate of the point is

(a) an ellipse (b) parabola

(c) circle (d) rectangular hyperbola

Solution. (d)

Explanation: Slope of tangent to a curve $=\frac{dy}{dx}$

And ratio of abscissa to the ordinate $=\frac{x}{y}$

Now, According to the question,

$$\frac{dy}{dx}=\frac{x}{y}$$

$\Rightarrow \quad yd\,y = xd\,x$

On integrating both sides, we get

$$\frac{y^2}{2}=\frac{x^2}{2}+C$$

$$\Rightarrow \quad \frac{y^2}{2}-\frac{x^2}{2}=C$$

$$\Rightarrow \quad y^2-x^2=2C$$

This is an equation of rectangular hyperbola.

Question 63. The general solution of differential equation $\frac{dy}{dx}=e^{\frac{x^2}{2}}+xy$ is

(a) $y=Ce^{-x^2/2}$ (b) $y=Ce^{x^2/2}$

(c) $y=(x+C)e^{x^2/2}$ (d) $y=(C-x)e^{x^2/2}$

Solution. (c)

Explanation: Given is, $\frac{dy}{dx}=e^{x^2/2}+xy$

$$\Rightarrow \quad \frac{dy}{dx}-xy=e^{x^2/2}$$

This is a linear differential equation.

Here, $\quad P=-x, Q=e^{x^2/2}$

$$\text{IF}=\text{e}^{\int -x\,dx}=e^{-x^2/2}$$

$\therefore$ The general solution is

$$y \cdot e^{-x^2/2} = \int \left(e^{-x^2/2} . e^{x^2/2}\right) dx + C$$

$$\Rightarrow \quad y \cdot e^{-x^2/2} = \int 1 dx + C$$

$$\Rightarrow \quad y \cdot e^{-x^2/2} = x + C$$

$$\Rightarrow \quad y = xe^{x^2/2} + Ce^{+x^2/2}$$

$$\Rightarrow \quad y = (x+C)e^{x^2/2}$$

Question 64. The solution of equation $(2y - 1)\, dx - (2x + 3)\, dy = 0$ is

(a) $\dfrac{2x-1}{2y+3} = k$ (b) $\dfrac{2y+1}{2x-3} = k$

(c) $\dfrac{2x+3}{2y-1} = k$ (d) $\dfrac{2x-1}{2y-1} = k$

Solution. (c)

Explanation: Given is, $(2y - 1)\, dx - (2x + 3) d\, y = 0$

$$\Rightarrow \quad (2y - 1)dx = (2x + 3)dy$$

$$\Rightarrow \quad \frac{dx}{2x+3} = \frac{dy}{2y-1}$$

On integrating both sides, we get

$$\frac{1}{2}\log\, (2x+3) = \frac{1}{2}\log(2y - 1) + \log C$$

$$\Rightarrow \quad \frac{1}{2}\log\, \cdot (2x+3) - \log(2y - 1) = \log C$$

$$\Rightarrow \quad \frac{1}{2}\log\left(\frac{2x+3}{2y-1}\right) = \log C$$

$$\Rightarrow \quad \left(\frac{2x+3}{2y-1}\right)^{1/2} = C$$

$$\Rightarrow \quad \frac{2x+3}{2y-1} = C^2$$

$$\Rightarrow \quad \frac{2x+3}{2y-1} = k, \text{ where } k = C^2$$

Question 65. The differential equation for which y = *a* cos *x* + *b* sin *x* is a solution, is

(a) $\frac{d^2y}{dx^2}+y=0$ (b) $\frac{d^2y}{dx^2}-y=0$

(c) $\frac{d^2y}{dx^2}+(a+b)y=0$ (d) $\frac{d^2y}{dx^2}+(a-b)y=0$

Solution. **(a)**

Explanation: Given equation is, $y= a\cos x + b\sin x$

On differentiating both sides w.r.t. *x*. we get

$$\frac{dy}{dx}=-a\sin x+b\cos x\ dx$$

Again, differentiating w.r.t. *x*. we get

$$\frac{d^2y}{dx^2}=-a\sin x+b\cos x\ dx$$

$$\Rightarrow \quad \frac{d^2y}{dx^2}=-y$$

$$]\Rightarrow \quad \frac{d^2y}{dx^2}=-y=0$$

Question 66. The solution of $\frac{dy}{dx}+y=e^{-x}, y(0)=0$ is

(a) $y=e^{-x}(x-1)$ (b) $y = xe^{x}$

(c) $y = xe^{-x} + 1$ (d) $y = xe^{-x}$

Solution. **(d)**

Explanation: Given is, $\frac{dy}{dx}+y=e^{-x}$

which is a linear differential equation

Here, $P = 1$ and $Q = e^{-x}$

$$\text{IF}=\text{e}^{\int dx}=e^{x}$$

The general solution is

$$y\cdot e^{x}=\int e^{-x}\cdot e^{x}dx+C$$

$$\Rightarrow \quad ye^{x}=\int dx+C$$

$\Rightarrow \quad ye^x = x + C \quad$ ...(i)

When $x = 0$ and $y = 0$ then equation (i) becomes

$0 = 0 + C$

$\Rightarrow \quad C = 0$

Putting value of C in equation (i) we get:

$y \cdot e^x = x$

$\Rightarrow \quad y = xe^{-x}$

Question 67. The order and degree of differential equation $\left(\frac{d^3y}{dx^3}\right)^2 - 3\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^4 = y^4$ are

(a) 1, 4 (b) 3, 4

(c) 2, 4 (d) 3, 2

Solution. (d)

Explanation: Given that, $\left(\frac{d^3y}{dx^3}\right)^2 - 3\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^4 = y^4$

We know that the order of a differential equation is the order of the highest order derivative and the degree of a differential equation is the power of the highest order derivative.

$\therefore \quad$ Order = 3

And $\quad$ degree = 2

Question 68. The order and degree of differential equation $\left[1 + \left(\frac{dy}{dx}\right)^2\right] = \frac{d^3y}{dx^2}^4$ are

(a) $2, \frac{2}{3}$ (b) 2, 3

(c) 2, 1 (d) 3, 4

Solution. (c)

Explanation: Given is, $\left[1 + \left(\frac{dy}{dx}\right)^2\right] = \frac{d^3y}{dx^2}^4$

We know that the order of a differential equation is the order of the highest order derivative and the degree of a differential equation is the power of the highest order derivative.

$\therefore \quad$ Order = 2

And degree = 1

Question 69. The differential equation of family of curves $y^2 = 4a(x + a)$ is

(a) $y^2 = 4\frac{dy}{dx}\left(x + \frac{dy}{dx}\right)$ (b) $2y\frac{dy}{dx} = 4a$

(c) $y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 0$ (d) $2x\frac{dy}{dx} + 4\left(\frac{dy}{dx}\right)^2 - y = 0$

Solution. (d)

Explanation: Given is, $y^2 = 4a(x + a)$ ...(i)

On differentiating both sides w.r.t. x we get

$$\Rightarrow \quad 2y\frac{dy}{dx} = 4a$$

$$\Rightarrow \quad 2y\frac{dy}{dx} = 4a$$

$$\Rightarrow \quad y\frac{dy}{dx} = 2a$$

$$\Rightarrow \quad a = \frac{1}{2}y\frac{dy}{dx}$$

On putting the value of a equation (i), we get

$$y^2 = 2y\frac{dy}{dx}\left(x + \frac{1}{2}y\frac{dy}{dx}\right)$$

$$\Rightarrow \quad y^2 = 2xy\frac{dy}{dx} + y^2\left(\frac{dy}{dx}\right)^2$$

$$\Rightarrow \quad 2x\frac{dy}{dx} + y^2\left(\frac{dy}{dx}\right)^2 - y = 0$$

Question 70. Which of the following is the general solution of $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$?

(a) $y = (Ax + B)e^x$ (b) $y = (Ax + B)e^{-x}$

(c) $y = Ae^x + Be^{-x}$ (d) $y = A\cos x + B\sin x$

Solution. (a)

Explanation: Given is, $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$

Or $D^2y - 2Dy + y = 0$, where $D = \frac{d}{dx}$

$\Rightarrow$ $(D^2 - 2D + 1)\, y = 0$,

The auxiliary equation is

$m^2 - 2m + 1 = 0$

$(m - 1)^2 = 0$

$\Rightarrow$ $m = 1, 1$

Since, the roots are real and equal

$\therefore$ $\text{CF} = (Ax + B)e^x$

Now, if the roots of an auxiliary equation are real and equal say (m), then $CF = (C_1 x + C_2)e^{mx}$

$\Rightarrow$ $y = (Ax + B)e^x$

Question 71. The general solution of $\frac{dy}{dx} + y \tan x = \sec x$ is

(a) $y \sec x = \tan x + C$ (b) $y \tan x = \sec x + C$

(c) $\tan x = y \tan x + C$ (d) $x \sec x = \tan y + C$

Solution. (a)

Explanation: Given differential equation is

$$\frac{dy}{dx} + y \tan x = \sec x$$

This is a linear differential equation

Here, $p = \tan x$, $Q = \sec x$,

$\therefore$ $\text{IF} = e^{\int \tan x d x} = e^{\log|\sec x|} = \sec x$

Thus, the general solution is

$$y \cdot \sec x = \int \sec x \cdot \sec x + C$$

$\Rightarrow$ $y \cdot \sec x = \int \sec^2 x \, dx + C$

$\Rightarrow$ $y \cdot \sec x = \tan x + C$

Question 72. The solution of differential equation $\frac{dy}{dx} + \frac{y}{x} = \sin x$ is

(a) $x\,(y + \cos x) = \sin x + C$ (b) $x\,(y - \cos x) = \sin x + C$

(c) $xy \cos x = \sin x + C$ (d) $x\,(y + \cos x) = \cos x + C$

Solution. (a)

Explanation: Given differential equation is

$$\frac{dy}{dx} + y\frac{1}{x} = \sin x$$

This is a linear differential equation.

Here, $p = \frac{1}{x}$ and $Q = \sin x$

$\therefore$ $\text{IF} = e^{\int \frac{1}{2} dx} = e^{\log x} = x$

Thus, the general solution is

$$y \cdot x = \int x.\sin x \, dx + C \qquad \ldots\text{(i)}$$

Take $I = \int x \sin x \, dx$

$$= -x\cos x - \int -\cos x \, dx$$

$$= -x \cos x + \sin x$$

Putting the value of I in equation (i), we get

$$xy = -x \cos x + \sin x + C$$

$\Rightarrow$ $x(y + \cos x) = \sin x + C$

Question 73. The general solution of differential equation $(e^x + 1)\, ydy = (y + 1)\, e^x\, dx$ is

(a) $(y + 1) = k(e^x + 1)$

(b) $y + 1 = e^x + 1 + k$

(c) $y = \log\{k(y + 1)(e^x + 1)\}$

(d) $y = \log\left\{\frac{e^x + 1}{y + 1}\right\} + k$

Solution. (c)

Explanation: Given differential equation

$$(e^x + 1)\, ydy = (y + 1)e^x\, dx$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{e^x(1+y)}{(e^x+1)y} \Rightarrow \frac{dx}{dy} = \frac{(e^x+1)y}{e^x(1+y)}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{e^x y}{e^x(1+y)} \frac{y}{e^x(1+y)}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{y}{1+y} + \frac{y}{(1+y)e^x}$$

$\Rightarrow \quad \dfrac{dy}{dx} = \dfrac{y}{1+y}\left(1+\dfrac{1}{e^x}\right)$

$\Rightarrow \quad \dfrac{dy}{dx} = \dfrac{y}{1+y}\left(\dfrac{e^x+1}{e^x}\right)$

$\Rightarrow \quad \left(\dfrac{y}{1+y}\right)dy = \left(\dfrac{e^x}{e^x+1}\right)dx$

On integrating both sides, we get

$$\int \frac{y}{1+y}\,dy = \int \frac{e^x}{1+e^x}\,dx$$

$\Rightarrow \quad \displaystyle\int \frac{1+y-1}{1+y}\,dy = \int \frac{e^x}{1+e^x}\,dx$

$\Rightarrow \quad \displaystyle\int 1dy - \int \frac{y}{1+y}\,dy = \int \frac{e^x}{1+e^x}\,dx$

$\Rightarrow \quad y - \log|(1+y) = \log|(1+e^x) + \log(k)$

$\Rightarrow \quad y = \log\{k(1+y)(1+e^x)\}$

Question 74. The solution of differential equation $\dfrac{dy}{dx} = e^{x-y} + x^2e^{-y}$ is

(a) $y = e^{x-y} - x^2 e^{-y} + C$

(b) $e^y - e^x = \dfrac{x^3}{3} + C$

(c) $e^x + e^y = \dfrac{x^3}{3} + C$

(d) $e^x - e^y = \dfrac{x^3}{3} + C$

Solution. (b)

Explanation: Given is, $\dfrac{dy}{dx} = e^{x-y} + x^2e^{-y}$

$\Rightarrow \quad \dfrac{dy}{dx} = e^xe^{-y} + x^2e^{-y}$

$\Rightarrow \quad \dfrac{dy}{dx} = \dfrac{e^x + x^2}{e^y}$

$\Rightarrow \quad e^y\,dy = (e^x + x^2)\,dx$

On integrating both sides, we get

$$\int e^y dy = \int \left(e^x + x^2\right)dx$$

$$\Rightarrow \quad e^y = e^x + \frac{x^3}{3} + C$$

$$\Rightarrow \quad e^y - e^x = \frac{x^3}{3} + C$$

Question 75. The solution of differential equation $\frac{dy}{dx} + \frac{2xy}{1+x^2} = \frac{1}{(1+x^2)^2}$ is

(a) $y(1+x^2) = C + \tan^{-1} x$ (b) $\frac{y}{1+x^2} = C + \tan^{-1} x$

(c) $y \log(1+x^2) = C + \tan^{-1} x$ (d) $y(1+x^2) = C + \sin^{-1} x$

Solution. (a)

Explanation: Given is, $\frac{dy}{dx} + \frac{2xy}{1+x^2} = \frac{1}{(1+x^2)^2}$

Here, $P = \frac{2x}{1+x^2}$ and $Q = \frac{1}{(1+x^2)^2}$

This is a linear differential equation.

$$\therefore \quad \text{IF} = e^{\int \frac{2x}{1+x^2} dx}$$

Put $1 + x^2 = t \Rightarrow 2x\, dx = dt$

$$\therefore \quad \text{IF} = e^{\int \frac{dt}{t}} = e^{\log t} = e^{\log(1+x^2)} = 1 + x^2$$

Thus, the general solution is

$$y \cdot (1+x^2) = \int (1+x^2) \frac{1}{(1+x^2)^2} + C$$

$$\Rightarrow \quad y(1+x^2) = \int \frac{1}{1+x^2} dx + C$$

$$\Rightarrow \quad y(1+x^2) = \tan^{-1} x + C$$