1. **Relation** A relation R from set X to a set Y is defined as a subset of the cartesian product $X \times Y$. We can also write it as $R \subseteq \{(x, y) \in X \times Y : xRy\}$.

NOTE If $n(A) = p$ and $n(B) = q$ from set A to set B, then $n(A \times B) = pq$ and number of relations $= 2^{pq}$.

2. **Types of Relation**
   (i) **Empty Relation** A relation R in a set X, is called an empty relation, if no element of X is related to any element of X,
   i.e. $$R = \phi \subset X \times X$$
   (ii) **Universal Relation** A relation R in a set X, is called universal relation, if each element of X is related to every element of X,
   i.e. $$R = X \times X$$
   (iii) **Reflexive Relation** A relation R defined on a set A is said to be reflexive, if
   $$(x, x) \in R, \forall\, x \in A$$
   or
   $$xRx, \forall\, x \in R$$
   (iv) **Symmetric Relation** A relation R defined on a set A is said to be symmetric, if
   $$(x, y) \in R \Rightarrow (y, x) \in R, \forall\, x, y \in A$$
   or
   $$xRy \Rightarrow yRx, \forall\, x, y \in R.$$
   (v) **Transitive Relation** A relation R defined on a set A is said to be transitive, if
   $(x, y) \in R$ and $(y, z) \in R \Rightarrow (x, z) \in R, \forall\, x, y, z \in A$
   or $xRy, yRz \Rightarrow xRz, \forall\, x, y, z \in R$.

3. **Equivalence Relation** A relation R defined on a set A is said to be an equivalence relation, if R is reflexive, symmetric and transitive.

4. **Equivalence Classes** Given an arbitrary equivalence relation R in an arbitrary set X, R divides X into mutually disjoint subsets A_i called **partitions** or **sub-divisions** of X satisfying
   (i) all elements of A_i are related to each other, for all i.
   (ii) no element of A_i is related to any element of $A_j, i \neq j$.
   (iii) $A_i \cup A_j = X$ and $A_i \cap A_j = \phi, i \neq j$. The subsets A_i and Aj are called **equivalence classes.**

5. **Function** Let X and Y be two non-empty sets. A function or mapping f from X into Y written as $f : X \to Y$ is a rule by which each element $x \in X$ is associated to a unique element $y \in Y$. Then, f is said to be a function from X to Y.

   The elements of X are called the domain of f and the elements of Y are called the codomain of f. The image of the element of X is called the range of X which is a subset of Y.

NOTE Every function is a relation but every relation is not a function.

6. **Types of Functions**
   (i) **One-one Function or Injective Function** A function $f : X \to Y$ is said to be a one-one function, if the images of distinct elements of x under f are distinct, i.e. $f(x_1) = f(x_2) \Leftrightarrow x_1 = x_2, \forall\, x_1, x_2 \in X$
   A function which is not one-one, is known as many-one function.
   (ii) **Onto Function or Surjective Function** A function $f : X \to Y$ is said to be onto function or a surjective function, if every element of Y is image of some element of set X under f, i.e. for every $y \in y$, there exists an element X in x such that $f(x) = y$.
   In other words, a function is called an onto function, if its range is equal to codomain.
   (iii) **Bijective or One-one and Onto Function** A function $f : X \to Y$ is said to be a bijective function, if it is both one-one and onto.

7. **Composition of Functions** Let $f : X \to Y$ and $g : Y \to Z$ be two functions. Then, composition of functions f and g is a function from X to Z and is denoted by fog and given by $(fog)(x) = f[g(x)], \forall\, x \in X$.

NOTE
(i) In general, $fog(x) \neq gof(x)$.
(ii) In general, gof is one-one implies that f is one-one and gof is onto implies that g is onto.
(iii) If $f : X \to Y$, $g : Y \to Z$ and $h : Z \to S$ are functions, then $ho(gof) = (hog)of$.

8. **Invertible Function** A function $f : X \to Y$ is said to be invertible, if there exists a function $g : Y \to X$ such that $gof = I_X$ and $fog = I_Y$. The function g is called inverse of function f and is denoted by f^{-1}.

NOTE
(i) To prove a function invertible, one should prove that, it is both one-one or onto, i.e. bijective.
(ii) If $f : X \to Y$ and $g : Y \to Z$ are two invertible functions, then gof is also invertible with $(gof)^{-1} = f^{-1} o\, g^{-1}$.