1. **Inverse Trigonometric Functions** Trigonometric functions are many-one functions but we know that inverse of function exists, if function is bijective. If we restrict the domain of trigonometric functions, then these functions become bijective and inverse of trigonometric functions are defined within the restricted domain. Inverse of $f$ is denoted by '$f^{-1}$'.

Let $y = f(x) = \sin x$, then its inverse is $x = \sin^{-1} y$.

2. **Domain and Range of Inverse Trigonometric Functions**

| Function | Domain | Range (Principal value branch) |
|---|---|---|
| $\sin^{-1} x$ | $[-1, 1]$ | $\left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$ |
| $\cos^{-1} x$ | $[-1, 1]$ | $[0, \pi]$ |
| $\tan^{-1} x$ | $R$ | $\left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$ |
| $\cot^{-1} x$ | $R$ | $(0, \pi)$ |
| $\mathrm{cosec}^{-1} x$ | $R-(-1,1)$ | $\left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]-\{0\}$ |
| $\sec^{-1} x$ | $R-(-1,1)$ | $[0, \pi]-\left\{\dfrac{\pi}{2}\right\}$ |

3. (i) $\sin^{-1}(\sin\theta) = \theta; \forall\theta \in \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$     (ii) $\cos^{-1}(\cos\theta) = \theta; \forall\theta \in [0, \pi]$

(iii) $\tan^{-1}(\tan\theta) = \theta; \forall\theta \in \left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$     (iv) $\mathrm{cosec}^{-1}(\mathrm{cosec}\,\theta) = \theta; \forall\theta \in \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right], \theta \neq 0$

(v) $\sec^{-1}(\sec\theta) = \theta; \forall\,\theta \in [0, \pi], \theta \neq \dfrac{\pi}{2}$     (vi) $\cot^{-1}(\cot\theta) = \theta; \forall\,\theta \in (0, \pi)$

(vii) $\sin(\sin^{-1} x) = x, \forall\, x \in [-1,1]$     (viii) $\cos(\cos^{-1} x) = x; \forall x \in [-1,1]$

(ix) $\tan(\tan^{-1} x) = x, \forall\, x \in R$                    (x) $\operatorname{cosec}(\operatorname{cosec}^{-1} x) = x, \forall x \in (-\infty, -1] \cup [1, \infty)$

(xi) $\sec(\sec^{-1} x) = x, \forall x \in (-\infty, -1] \cup [1, \infty)$    (xii) $\cot(\cot^{-1} x) = x, \forall x \in R$

**NOTE** $\sin^{-1}(\sin\theta) = \theta$ ; $\sin^{-1} x$ should not be confused with $(\sin x)^{-1} = \dfrac{1}{\sin x}$ or $\sin^{-1} x = \sin^{-1}\left(\dfrac{1}{x}\right)$ and similarly for other trigonometric functions.

4. The value of an inverse trigonometric function, which lies in the range of principal value branch, is called the principal value of the inverse trigonometric function.

   **NOTE** Whenever no branch of an inverse trigonometric function is mentioned, it means we have to consider the principal value branch of that function.

5. **Properties of Inverse Trigonometric Functions**

   (a) (i) $\sin^{-1}\left(\dfrac{1}{x}\right) = \operatorname{cosec}^{-1} x; x \geq 1 \text{ or } x \leq -1$    (ii) $\cos^{-1}\left(\dfrac{1}{x}\right) = \sec^{-1} x \;;\; x \geq 1 \text{ or } x \leq -1$

   (iii) $\tan^{-1}\left(\dfrac{1}{x}\right) = \begin{cases} \cot^{-1} x \;; x > 0 \\ -\pi + \cot^{-1} x; x < 0 \end{cases}$

   (b) (i) $\sin^{-1}(-x) = -\sin^{-1} x; x \in [-1, 1]$    (ii) $\tan^{-1}(-x) = -\tan^{-1} x \;; x \in R$

   (iii) $\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1} x \;; |x| \geq 1$

   (c) (i) $\cos^{-1}(-x) = \pi - \cos^{-1} x \;; x \in [-1, 1]$    (ii) $\sec^{-1}(-x) = \pi - \sec^{-1} x \;; |x| \geq 1$

   (iii) $\cot^{-1}(-x) = \pi - \cot^{-1} x \;; x \in R$

   (d) (i) $\sin^{-1} x + \cos^{-1} x = \dfrac{\pi}{2}; x \in [-1, 1]$    (ii) $\tan^{-1} x + \cot^{-1} x = \dfrac{\pi}{2} \;; x \in R$

   (iii) $\operatorname{cosec}^{-1} x + \sec^{-1} x = \dfrac{\pi}{2} \;; |x| \geq 1$

   (e) (i) $\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\dfrac{x+y}{1-xy}\right); xy < 1$

   (ii) $\tan^{-1} x - \tan^{-1} y = \tan^{-1}\left(\dfrac{x-y}{1+xy}\right); xy > -1$

   (f) (i) $2\tan^{-1} x = \sin^{-1}\left(\dfrac{2x}{1+x^2}\right); |x| \leq 1$    (ii) $2\tan^{-1} x = \cos^{-1}\left(\dfrac{1-x^2}{1+x^2}\right); x \geq 0$

   (iii) $2\tan^{-1} x = \tan^{-1}\left(\dfrac{2x}{1-x^2}\right); -1 < x < 1$

   (iv) $2\sin^{-1} x = \sin^{-1}(2x\sqrt{1-x^2}); \dfrac{-1}{\sqrt{2}} \leq x \leq \dfrac{1}{\sqrt{2}}$

   (v) $2\cos^{-1} x = \sin^{-1}(2x\sqrt{1-x^2}); \dfrac{-1}{\sqrt{2}} \leq x \leq 1 \text{ or } 2\cos^{-1} x = \cos^{-1}(2x^2 - 1); 0 \leq x \leq 1$

   (g) (i) $\sin^{-1} x + \sin^{-1} y = \sin^{-1}(x\sqrt{1-y^2} + y\sqrt{1-x^2})$

   (ii) $\sin^{-1} x - \sin^{-1} y = \sin^{-1}(x\sqrt{1-y^2} - y\sqrt{1-x^2})$

   (iii) $\cos^{-1} x + \cos^{-1} y = \cos^{-1}(xy - \sqrt{1-x^2}\sqrt{1-y^2})$

   (iv) $\cos^{-1} x - \cos^{-1} y = \cos^{-1}(xy + \sqrt{1-x^2}\sqrt{1-y^2})$