NCERT Exemplar Solutions

Class 13 – Mathematics

Chapter 13 – Probability

Objective Type Questions

Question 56. If $P(A)=\frac{4}{5}$ and $P(A\cap B)=\frac{7}{10}$, then $P(B/A)$ is equal to

(a) $\frac{1}{10}$

(b) $\frac{1}{8}$

(c) $\frac{7}{8}$

(d) $\frac{17}{20}$

Solution.(c)

Explanation: $\because\ P(A)=\frac{4}{5} P(A\cap B)=\frac{7}{10}$

$\therefore\ P(B/A)=\frac{P(A\cap B)}{P(A)}=\frac{7/10}{4/5}=\frac{7}{8}$

Question 57. If $P(A\cap B)=\frac{7}{10}$ and $P(B)=\frac{17}{20}$, then $P(A/B)$ equal to

(a) $\frac{14}{17}$

(b) $\frac{17}{20}$

(c) $\frac{7}{8}$

(d) $\frac{1}{8}$

Solution. (a)

Explanation: We have, $P(A\cap B)=\frac{7}{10}$ and $P(B)=\frac{17}{20}$

$\therefore\ P(A/B)=\frac{P(A\cap B)}{P(B)}=\frac{7/10}{17/20}=\frac{14}{17}$

Question 58. If $P(A)=\frac{3}{10}$, $P(B)=\frac{2}{5}$ and $P(A\cup B)=\frac{3}{5}$, then $P(A/B)+P(A/B)$ equal to

(a) $\frac{1}{4}$ (b) $\frac{1}{3}$

(c) $\frac{5}{12}$ (d) $\frac{7}{12}$

Solution. (a)

Explanation: Here, $P(A)=\frac{3}{10}$, $P(B)=\frac{2}{5}$ and $P(A\cup B)=\frac{3}{5}$

$$\begin{bmatrix} \because \quad P(A\cap B)=P(A)+P(B)-P(A\cap B) \\ i.e., P(A\cap B)=P(A)+P(B)-P(A\cap B) \end{bmatrix}$$

$$P(B/A)+P(A/B)=\frac{P(B\cap A)}{P(A)}+\frac{P(A\cap B)}{P(B)}$$

$$=\frac{\frac{3}{10}+\frac{2}{5}-\frac{3}{5}}{\frac{3}{10}}+\frac{\frac{3}{10}+\frac{2}{5}-\frac{3}{5}}{\frac{2}{5}}$$

$$=\frac{\frac{1}{10}}{\frac{3}{10}}+\frac{\frac{1}{10}}{\frac{2}{5}}=\frac{1}{3}+\frac{1}{4}=\frac{7}{12}$$

Question 59. If $P(A)=\frac{2}{5}$, $P(B)=\frac{3}{10}$ and $P(A\cap B)=\frac{1}{5}$ then $P(A'/B')\cdot P(B'/A')$ is equal to

(a) $\frac{5}{6}$ (b) $\frac{5}{7}$

(c) $\frac{25}{42}$ (d) 1

Solution. (a)

Explanation: Here, $P(A)=\frac{2}{5}$, $P(B)=\frac{3}{10}$ and $P(A\cup B)=\frac{1}{5}$

$$P(A'/B') = \frac{P(A' \cap B')}{P(B')} + \frac{1 - P(A \cap B)}{1 - P(B)}$$

$$= \frac{1 - [P(A) + P(B) - A(A \cap B)]}{1 - P(B)}$$

$$= \frac{1 - \left(\frac{2}{5} + \frac{3}{10} - \frac{1}{5}\right)}{1 - \frac{3}{10}}$$

$$= \frac{1 - \left(\frac{4+3-2}{10}\right)}{\frac{7}{10}} - \frac{1 - \frac{1}{2}}{\frac{7}{10}} = \frac{5}{7}$$

And $P(B'/A') = \frac{P(B' \cap A')}{P(A')} = \frac{1 - P(A \cup B)}{1 - P(A)}$

$$= \frac{1 - \frac{1}{2}}{1 - \frac{2}{5}} = \frac{1/2}{3/5} - \frac{5}{6} \qquad \left[\because P(A \cup B) = \frac{1}{2}\right]$$

$$\therefore P(A'/B') \cdot P(B'/A') = \frac{5}{7} \cdot \frac{5}{6} = \frac{25}{42}$$

Question 60. If A and B are two events such that $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$ and $P(A/B) = \frac{1}{4}$, then $P(A' \cap B')$ is equal to

(a) $\frac{1}{12}$

(b) $\frac{3}{4}$

(c) $\frac{1}{4}$

(d) $\frac{3}{16}$

Solution. (c)

Explanation: Here, $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$ and $P(A/B) = \frac{1}{4}$

$$\because \qquad P(A/B)=\frac{P(A\cap B)}{P(B)}$$

$$\Rightarrow \quad P(A\cap B)=P(A/B)\cdot P(B)=\frac{1}{4}\cdot\frac{1}{3}=\frac{1}{12}$$

Now, $P(A'\cap B')=1-P(A\cup B)$

$$=1-\left[P(A)+P(B)-P(A\cap B)\right]$$

$$=1-\left[\frac{1}{2}+\frac{1}{3}-\frac{1}{12}\right]=1-\left[\frac{6+4-1}{12}\right]$$

$$=1-\frac{9}{12}=\frac{3}{12}=\frac{1}{4}$$

Question 61. If $P(A) = 0.4$, $P(B) = 0.8$ and $P(B/A) = 0.6$, then $P(A \cup B)$ is equal to

(a) 0.24

(b) 0.3

(c) 0.48

(d) 0.96

Solution. (d)

Explanation: Here, $P(A) = 0.4$, $P(B) = 0.8$ and $P(B/A) = 0.6$

$$\because \qquad P(B/A)=\frac{P(B\cap A)}{P(A)}$$

$$\Rightarrow \qquad P(B \cap A) = P(B/A)\cdot P(A)$$

$$\because \qquad P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= 0.4 + 0.8 - 0.24$$

$$= 1.2 - 0.24 = 0.96$$

Question 62. If A and B are two events and $A \neq \phi$, $B \neq \phi$, then

(a) $P(A/B) = P(A)\cdot P(B)$

(b) $P(A/B)=\frac{P(A\cap B)}{P(B)}$

(c) $P(A/B)\cdot P(B/A) = 1$

(d) $P(A/B) = P(A)/P(B)$

Solution. (b)

Explanation. If $A \neq \phi$ and $B \neq \phi$, then $P(A/B) = \dfrac{P(A \cap B)}{P(B)}$

Question 63. If A and B are events such that $P(A) = 0.4$, $P(B) = 0.3$ and $P(A \cup B) = 0.5$, then $P(B' \cap A)$ equals to

(a) $\frac{2}{3}$

(b) $\frac{1}{2}$

(c) $\frac{3}{10}$

(d) $\frac{1}{5}$

Solution. (d)

Explanation: Here, $P(A) = 0.4$, $P(B) = 0.3$ and $P(A \cup B) = 0.5$

$\because \quad P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow \quad P(A \cap B) = 0.4 + 0.3 - 0.5 = 0.2$

$\therefore \quad P(B' \cap A) = P(A) - P(A \cap B)$

$$= 0.4 - 0.2 = 0.2 = \frac{1}{5}$$

Question 64. If A and B are two events such that $P(B) = \frac{3}{5}$, $P(A/B) = \frac{1}{2}$ and $P(A \cup B) = \frac{4}{5}$, then P (A) equals to

(a) $\frac{3}{10}$

(b) $\frac{1}{5}$

(c) $\frac{1}{2}$

(d) $\frac{3}{5}$

Solution. (c)

Explanation: We have, $P(B) = \frac{3}{5}$, $P(A/B) = \frac{1}{2}$ and $P(A \cup B) = \frac{4}{5}$

$\because \quad P(A/B) = \dfrac{P(A \cap B)}{P(B)}$

$$\Rightarrow \quad \frac{1}{2} = \frac{P(A \cap B)}{3/5}$$

$$\Rightarrow \quad P(A \cap B) = \frac{3}{5} \times \frac{1}{2} = \frac{3}{10}$$

And $\quad P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\Rightarrow \quad \frac{4}{5} = P(A) + \frac{3}{5} - \frac{3}{10}$$

$$\therefore \quad P(A) = \frac{4}{5} - \frac{3}{5} + \frac{3}{10} = \frac{8-6+3}{10} = \frac{1}{2}$$

Question 65. In question 64 (above), $P(B/A')$ is equal to

(a) $\frac{1}{5}$

(b) $\frac{3}{10}$

(c) $\frac{1}{2}$

(d) $\frac{3}{5}$

Solution. (d)

Explanation: $P(B/A') = \dfrac{P(B \cap A')}{P(A')} = \dfrac{P(B) - P(B \cap A)}{1 - P(A)}$

$$= \frac{\frac{3}{5} - \frac{3}{10}}{1 - \frac{1}{2}} = \frac{\frac{6-3}{10}}{\frac{1}{2}} = \frac{6}{10} = \frac{3}{5}$$

Question 66. If $P(B) = \frac{3}{5}$, $P(A/B) = \frac{1}{2}$ and $P(A \cup B) = \frac{4}{5}$, then $P(A \cup B)' + P(A' \cup B)$ is equal to

(a) $\frac{1}{5}$

(b) $\frac{4}{5}$

(c) $\frac{1}{2}$

(d) 1

Solution. (d)

Explanation: Here, $P(B)=\frac{3}{5},\ P(A/B)=\frac{1}{2}$ and $P(A\cup B)=\frac{4}{5}$

Since, $P(A/B)=\frac{P(A\cap B)}{P(B)}$

$\Rightarrow \quad P(A\cap B) = P(A/B)\cdot P(B)$

$$=\frac{1}{2}\times\frac{3}{5}=\frac{3}{10}$$

Also, $P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$\Rightarrow \quad \frac{4}{5}=P(A)+\frac{3}{5}-\frac{3}{10}$

$\Rightarrow \quad P(A)=\frac{4}{5}-\frac{3}{5}+\frac{3}{10}=\frac{1}{2}$

$\therefore\ P(A\cup B)' = 1-P(A\cup B)=1-\frac{4}{5}=\frac{1}{5}$

Also, we know that

$P(A'\cup B) = 1 - P(A - B) = 1 - [P(A) - P(A\cap B)]$

$$=1-\frac{1}{2}\cdot\frac{2}{5}=\frac{4}{5}$$

$\Rightarrow \quad P(A\cup B)'+P(A'\cup B)=\frac{1}{5}+\frac{4}{5}=\frac{5}{5}=1$

Question 67. If $P(A)=\frac{7}{13}, P(B)=\frac{9}{13}$ and $P(A\cap B)=\frac{4}{13}$ then $P(A'/B)$ is equal to

(a) $\frac{6}{13}$

(b) $\frac{4}{13}$

(c) $\frac{4}{9}$

(d) $\frac{5}{9}$

Solution. (d)

Explanation: Here, $P(A)=\frac{7}{13}, P(B)=\frac{9}{13}$ and $P(A\cap B)=\frac{4}{13}$

$$\therefore \qquad P(A'/B)=\frac{P(A'\cap B)}{P(B)}=\frac{P(B)-P(A\cap B)}{P(B)}$$

$$=\frac{\frac{9}{13}-\frac{4}{13}}{\frac{9}{13}}=\frac{\frac{5}{13}}{\frac{9}{13}}=\frac{5}{9}$$

Question 68. If A and B are such events that $P(A) > 0$ and $P(B) \neq 1$, then $P(A'/B')$ equals to

(a) $1 - P(A/B)$

(b) $1 - P(A'/B)$

(c) $\frac{1-P(A\cup B)}{P(B')}$

(d) $P(A')/P(B')$

Solution. (c)

Explanation: We have, $P(A) > 0$ and $P(B) \neq 1$

$$P(A'/B')=\frac{P(A'\cap B')}{P(B')}=\frac{1-P(A\cup B)}{P(B')}$$

Question 69. If A and B are two independent events with $P(A)=\frac{3}{5}$ and $P(B)=\frac{4}{9}$, then $P(A' \cap B')$ equals to

(a) $\frac{4}{15}$

(b) $\frac{8}{45}$

(c) $\frac{1}{3}$

(d) $\frac{2}{9}$

Solution. (d)

Explanation: $P(A' \cup B') = 1 - P(A \cap B)$

$$= 1 - [P(A) + P(B) - P(A \cap B)]$$

$$=1-\left[\frac{3}{5}+\frac{4}{9}-\frac{3}{5}\times\frac{4}{9}\right] \qquad [\because \quad P(A \cap B) = P(A)\cdot P(B)]$$

$$=1-\left[\frac{27+20-12}{45}\right]=1-\frac{35}{45}=\frac{10}{45}=\frac{2}{9}$$

Question 70. If two events are independent, then

(a) They must be mutually exclusive

(b) The sum of their probabilities must be equal to 1

(c) Both (a) and (b) are correct

(d) None of the above is correct

Solution. (d)

Explanation: Given that events A and B are independent, then we have:

$P(A \cap B) = P(A) \cdot P(B)$, $P(A) \neq 0$, $P(B) \neq 0$

Further, mutually exclusive events never have a common outcome.

In other words, two independent events having - non-zero probabilities of occurrence cannot be mutually exclusive and conversely, *i.e.*, two mutually exclusive events having non - zero probabilities of outcome cannot be independent.

Question 71. If A and B be two events such that $P(A)=\frac{3}{8}, P(B)=\frac{5}{8}$ and $P(A \cup B)=\frac{3}{4}$, then $P(A/B) \cdot P(A'/B)$ is equal to

(a) $\frac{2}{5}$

(b) $\frac{3}{8}$

(c) $\frac{3}{20}$

(d) $\frac{6}{25}$

Solution. (d)

Explanation: Here, $P(A)=\frac{3}{8}, P(B)=\frac{5}{8}$ and $P(A \cup B)=\frac{3}{4}$

$\because \quad P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow \quad P(A \cap B)=\frac{3}{8}+\frac{5}{8}-\frac{3}{4}=\frac{3+5-6}{8}=\frac{2}{8}=\frac{1}{4}$

$\because \quad P(A/B)=\dfrac{P(A\cap B)}{P(B)}=\dfrac{1/4}{5/8}=\dfrac{8}{20}=\dfrac{2}{5}$

and $\quad P(A'/B)=\dfrac{P(A'\cap B)}{P(B)}=\dfrac{P(B)-P(A\cap B)}{P(B)}$

$$=\frac{\frac{5}{8}-\frac{1}{4}}{\frac{5}{8}}=\frac{\frac{5-2}{8}}{\frac{5}{8}}=\frac{3}{5}$$

$\therefore P(A/B)\cdot P(A'/B)=\dfrac{2}{5}\cdot\dfrac{3}{5}=\dfrac{6}{25}$

Question 72 If the events A and B are independent, then $P(A \cap B)$ is equal to

(a) $P(A) + P(B)$ (b) $P(A) - P(B)$

(c) $P(A)\cdot P(B)$ (d) $P(A)/P(B)$

Solution. (c)

Explanation: If A and B are independent, then $P(A \cap B) = P(A)\cdot P(B)$

Question 73. Two events E and F are independent. If $P(E) = 0.3$ and $P(E \cup F) = 0.5$, then $P(E/F) - P(F/E)$ equals to

Solution. (c)

Explanation: We have, $P(E) = 0.3$ and $P(E \cup F) = 0.5$

Also E and F are independent.

Now, $P(E \cup F) = P(E) + P(F) - P(E \cap F)$

$= P(E) + P(F) - P(E)\cdot P(F)$

$\Rightarrow \quad 0.5 = 0.3 + x - 0.3x$

$\Rightarrow \quad x=\dfrac{0.5-0.3}{0.7}=\dfrac{2}{7}=P(F)$

$\therefore \; P(E/F)-P(F/E)=\dfrac{P(E\cap F)}{P(F)}-\dfrac{P(F\cap E)}{P(E)}$

$$= \frac{P(E \cap F) \cdot P(E) - P(F \cap E) \cdot P(F)}{P(E) \cdot P(F)}$$

$$= \frac{P(E \cap F) \cdot [P(E) - P(F)]}{P(E \cap F)} = P(E) - P(F)$$

$$= \frac{3}{10} - \frac{2}{7} = \frac{21-20}{70} = \frac{1}{70}$$

Question 74. A bag contains 5 red and 3 blue balls. If 3 balls are drawn at random without replacement, them the probability of getting exactly one red ball is

(a) $\frac{45}{196}$

(b) $\frac{135}{392}$

(c) $\frac{15}{56}$

(d) $\frac{15}{29}$

Solution. (c)

Explanation: Probability of getting exactly one red (R) ball

$$= P_R \cdot P_{\bar{R}} \cdot P_{\bar{R}} + P_{\bar{R}} \cdot P_R \cdot P_{\bar{R}} + P_{\bar{R}} \cdot P_{\bar{R}} \cdot P_R$$

$$= \frac{5}{8} \cdot \frac{3}{7} \frac{2}{6} + \frac{3}{8} \cdot \frac{5}{7} \cdot \frac{2}{6} + \frac{3}{8} \cdot \frac{2}{7} \cdot \frac{5}{6}$$

$$= \frac{15}{4 \cdot 7 \cdot 6} + \frac{15}{4 \cdot 7 \cdot 6} + \frac{15}{4 \cdot 7 \cdot 6}$$

$$= \frac{5}{56} + \frac{5}{56} + \frac{5}{56} = \frac{15}{56}$$

Question 75. Refer to question 74 above. If the probability that exactly two of the three balls were red, then the first ball being red, is

(a) $\frac{1}{3}$

(b) $\frac{4}{7}$

(c) $\frac{15}{28}$

(d) $\frac{5}{28}$

Solution. (b)

Explanation: Let E_1 = Event that first ball being red

and E_2 = Event that exactly two of the three balls being red

$$\therefore P(E_1) = P_R \cdot P_R \cdot P_R + P_R \cdot P_R \cdot P_{\bar{R}} + P_R \cdot P_{\bar{R}} \cdot P_R + P_R \cdot P_{\bar{R}} \cdot P_{\bar{R}}$$

$$= \frac{5}{8}\cdot\frac{4}{7}\cdot\frac{3}{6}+\frac{5}{8}\cdot\frac{4}{7}\cdot\frac{3}{6}+\frac{5}{8}\cdot\frac{4}{7}\cdot\frac{3}{6}+\frac{5}{8}\cdot\frac{4}{7}\cdot\frac{3}{6}$$

$$= \frac{60+60+60+30}{336} = \frac{210}{336}$$

$$P(E_1 \cap E_2) = P_R \cdot P_{\bar{R}} \cdot P_R + P_R \cdot P_R \cdot P_{\bar{R}}$$

$$= \frac{5}{8}\cdot\frac{3}{7}\cdot\frac{4}{6}+\frac{5}{8}\cdot\frac{4}{7}\cdot\frac{3}{6} = \frac{120}{336}$$

$$\therefore P(E_2/E_1) = \frac{P(E_1 \cap E_2)}{P(E_1)} = \frac{120/336}{210/336} = \frac{4}{7}$$

Question 76. Three persons *A*, *B* and *C*, fire at a target in turn, starting with *A*. Their probability of hitting the target are 0.4, 0.3 and 0.2, respectively. The probability of two hits is

(a) 0.024 (b) 0.188 (c) 0.336 (d) 0.452

Solution. (b)

Explanation: We have, $P(A) = 0.4, P(\bar{A}) = 0.6, P(B) = 0.3, P(\bar{B}) = 0.7,$

$P(C) = 0.2$ and $P(\bar{C}) = 0.8$

$\therefore$ Probability of two hits $= P_A \cdot P_B \cdot P_{\bar{C}} + P_A \cdot P_{\bar{B}} \cdot P_C + P_{\bar{A}} \cdot P_B \cdot P_C$

$= 0.4 \times 0.3 \times 0.8 + 0.4 \times 0.7 \times 0.2 + 0.6 \times 0.3 \times 0.2$

$= 0.096 + 0.056 + 0.036 = 0.188$

Question 77. Assume that in a family, each child is equally likely to be a boy or a girl. A family with three children is chosen at random. The probability that the eldest child is a girl given that the family has atleast one girl is

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{2}{3}$ (d) $\frac{4}{7}$

Solution. (d)

Explanation: Here, S = {(B, B, B), (G, G, G), (B, G, G), (G, B, G), (G, G, B), (G. B, B), (B, G, B), (B, B. G)}

E_1, = Event that a family has atleast one girl, then

E_1 = {(G, B, B), (B, G, B), (B, B. G), (G, G, B), (B, G, G), (G. B, G), (G, G, G),}

E_2 = Event that the eldest child is a girl, then

E_2 = {(G, B, B), (G, G, B), (G, B, G) (G, G, G)}

$\therefore \quad E_1 \cap E_2$ = {(G, B, B), (G, G, B), (G, B. G) (G, G, G)}

$$\therefore \quad P(E_2/E_1) = \frac{P(E_1 \cap E_2)}{P(E_1)} = \frac{4/8}{7/8} = \frac{4}{7}$$

Question 78. If a die is thrown and a card is selected at random from a deck of 52 playing cards, then the probability of getting an even number on the die and a spade card is

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$

(c) $\frac{1}{8}$ (d) $\frac{3}{4}$

Solution. (c)

Explanation: Let E_1, = Event for getting an even number on the die

and E_2 = Event that a spade card is selected

$$\therefore \; P(E_1) = \frac{3}{6} = \frac{1}{2} \text{ and } P(E_2) = \frac{13}{52} = \frac{1}{4}$$

Then, $P(E_1 \cap E_2) = P(E_1) \cdot P(E_2) = \frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8}$

Question 79. A box contains 3 orange balls, 3 green balls and 2 blue balls. Three balls are drawn at random from the box without replacement. The probability of drawing 2 green balls and one blue ball is

(a) $\frac{3}{28}$ (b) $\frac{2}{21}$

(c) $\frac{1}{28}$ (d) $\frac{167}{168}$

Solution. (a)

Explanation: Probability of drawing 2 green balls and one blue ball

$$= P_G \cdot P_G \cdot P_B + P_B \cdot P_G \cdot P_G + P_G \cdot P_B \cdot P_G$$

$$= \frac{3}{8}\cdot\frac{2}{7}\cdot\frac{2}{6}+\frac{2}{8}\cdot\frac{3}{7}\cdot\frac{2}{6}+\frac{3}{8}\cdot\frac{2}{7}\cdot\frac{2}{6}$$

$$= \frac{1}{28}+\frac{1}{28}+\frac{1}{28}=\frac{3}{28}$$

Question 80. A flashlight has 8 batteries out of which 3 are dead. If two batteries are selected without replacement and tested, then probability that both are dead is

(a) $\frac{33}{56}$ (b) $\frac{9}{64}$

(c) $\frac{1}{14}$ (d) $\frac{3}{28}$

Solution. (d)

Explanation: Required probability $= P_D \cdot P_D = \frac{3}{8}\cdot\frac{2}{7} = \frac{3}{28}$

Question 81. If eight coins are tossed together, then the probability of getting exactly 3 heads is

(a) $\frac{1}{256}$ (b) $\frac{7}{32}$

(c) $\frac{5}{32}$ (d) $\frac{3}{32}$

Solution. (b)

Explanation: We know that, probability distribution $P(X = r) = {}^nC_r, (p)^r q^{n-r}$

Here, $n = 8, r = 3, p = \frac{1}{2}$ and $q = \frac{1}{2}$

$\therefore$ Required probability $= {}^8C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^{8-3} = \frac{8!}{5!3!}\left(\frac{1}{2}\right)^8$

$$= \frac{8 \cdot 7 \cdot 6}{3 \cdot 2} \cdot \frac{1}{16 \cdot 16} = \frac{7}{32}$$

Question 82. Two dice are thrown. If it is known that the sum of numbers on the dice was less than 6, the probability of getting a sum 3, is

(a) $\frac{1}{18}$

(b) $\frac{5}{18}$

(c) $\frac{1}{5}$

(d) $\frac{2}{5}$

Solution. (c)

Explanation: Let E_1 = Event that the sum of numbers on the dice was less than 6

And E_2 = Event that the sum of numbers on the dice is 3

$\therefore$ $E_1 = \{(1. 4). (4, 1), (2, 3), (3, 2), (2,2), (1, 3), (3,1), (1, 2), (2,1), (1,1)\}$

$\Rightarrow$ $n(E_1) = 10$

And $E_2 = \{(1,2), (2,1)\}$

$\Rightarrow$ $n(E_2) = 2$

$\therefore$ Required Probability $= \frac{2}{10} = \frac{1}{5}$

Question 83. Which one is not a requirement of a Binomial distribution?

(a) There are 2 outcomes for each trial

(b) There is a fixed number of trials

(c) The outcomes must be dependent on each other

(d) The probability of success must be the same for all the trials

Solution. (c)

Explanation: We know that, in a Binomial distribution,

(i) There are 2 outcomes for each trial.

(ii) There is a fixed number of trial.

(iii) The probability of success must be the same for all the trials.

Question 84. If two cards are drawn from a well shuffled deck of 52 playing cards with replacement, then the probability that both cards are queens, is

(a) $\frac{1}{13} \cdot \frac{1}{13}$

(b) $\frac{1}{13} + \frac{1}{13}$

(c) $\frac{1}{13} \cdot \frac{1}{17}$

(d) $\frac{1}{13} \cdot \frac{4}{51}$

Solution. (a)

Explanation: Required probability = $\frac{4}{52} \cdot \frac{4}{52} = \frac{1}{13} \times \frac{1}{13}$ [with replacement]

Question 85. The probability of guessing correctly atleast 8 out of 10 answers on a true false type examination is

(a) $\frac{7}{64}$

(b) $\frac{7}{128}$

(c) $\frac{45}{1024}$

(d) $\frac{7}{41}$

Solution. (b)

Explanation: We know that, $P(X = r) = {}^nC_r, (p)^r\ (q)^{n-1}$

Here, $n = 10, p = \frac{1}{2}, q = \frac{1}{2}$

and $r \geq 8$ *i.e.*, $r = 8, 9, 10$

$\Rightarrow \quad P(X = r) = P(r = 8) + P(r = 9) + P(r = 10)$

$$= {}^{10}C_8 \left(\frac{1}{2}\right)^8 \left(\frac{1}{2}\right)^{10-8} + {}^{10}C_9 \left(\frac{1}{2}\right)^9 \left(\frac{1}{2}\right) + {}^{10}C_{10} \left(\frac{1}{2}\right)^{10} \cdot \left(\frac{1}{2}\right)^0$$

$$= \frac{10!}{8!2!}\left(\frac{1}{2}\right)^{10} + \frac{10!}{9!1!}\left(\frac{1}{2}\right)^{10} + \left(\frac{1}{2}\right)^{10}$$

$$= \left(\frac{1}{2}\right)^{10} \cdot [45+10+1] = \left(\frac{1}{2}\right)^{10} \cdot 56$$

$$= \frac{1}{16 \cdot 64} \cdot 56 = \frac{7}{128}$$

Question 86. If the probability that a person is not a swimmer is 0.3, then the probability that out of 5 persons 4 are swimmers is

(a) ${}^5C_4(0.7)^4$ (0.3)

(b) ${}^5C_1$, (0.7) $(0.3)^4$

(c) ${}^3C_4$ (0.7) $(0.3)^4$

(d) $(0.7)^4$ (0.3)

Solution. (*a*)

Explanation: Here, $\bar{p} = 0.3 \Rightarrow p = 0.7$ and $q = 0.3$, $n = 5$ and $r = 4$

$\therefore$ Required probability = ${}^5C_4$ $(0.7)^4$ (0.3)

Question 87. The probability distribution of a discrete random variable *X* is given below

X	2	3	4	5
$P(X)$	$\frac{5}{k}$	$\frac{7}{k}$	$\frac{9}{k}$	$\frac{11}{k}$

The value of *k* is

(a) 8

(b) 16

(c) 32

(d) 48

Solution. (c)

Explanation: We know that, $\sum P(X) = 1$

$\Rightarrow \quad \frac{5}{k}+\frac{7}{k}+\frac{9}{k}+\frac{11}{k}=1$

$\Rightarrow \quad \frac{32}{k}=1$

$\therefore \quad k = 32$

Question 88. For the following probability distribution.

X	-4	-3	-2	-1	0
$P(X)$	0.1	0.2	0.3	0.2	0.2

$E(X)$ is equal to

(a) 0 (b) – 1 (c) – 2 (d) – 1.8

Solution. (d)

Explanation: $E(X) = \sum X\, P(X)$

$= -4 \times (0.1) + (-3 \times 0.2) + (-2 \times 0.3) + (-1 \times 0.2) + (0 \times 0.2)$

$= -0.4 - 0.6 - 0.6 - 0.2 = -1.8$

Question 89. For the following probability distribution.

X	1	2	3	4
$P(X)$	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{3}{10}$	$\frac{2}{5}$

$E(X^2)$ is equal to

(a) 3 (b) 5

(c) 7 (d) 10

Solution. (d)

Explanation: $E(X)^2 = \sum X^2 P(X) = 1\cdot\frac{1}{10} + 4\cdot\frac{1}{5} + 9\cdot\frac{3}{10} + 16\cdot\frac{2}{5}$

$$= \frac{1}{10} + \frac{4}{5} + \frac{27}{10} + \frac{32}{5}$$

$$= \frac{1+8+27+64}{10} = 10$$

Question 90. Suppose a random variable X follows the Binomial distribution with parameters n and p, where $0 < p < 1$. If $P(x = r)/\,P(x = n - r)$ is independent of n and r, then p equals to

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{1}{5}$ (d) $\frac{1}{7}$

Solution (a)

Explanation: We know that $P(X = r) = {}^nC_r\,(p)^r\,(q)^{n-r}$

$$= \frac{n!}{(n-r)!\,r!}(p)^r(1-p)^{n-1} \quad [\because q = 1 - p] \quad ...(i)$$

$P(X = 0) = (1 - p^n)$

And $P(X = n - r) = {}^nC_{n-r}\,(p)^{n-r}\,(q)^{n-(n-r)}$

$$= \frac{n!}{(n-r)!\,r!}(p)^{n-r}(1-p)^{+r} \quad [\because q = 1 - p]\ [\because {}^nC_r = {}^nC_{n-r}] \quad ...(ii)$$

Now, $$\frac{P(x=r)}{P(x=n-r)} = \frac{\frac{n!}{(n-r)!\,r!}p^r(1-p)^{n-r}}{\frac{n!}{(n-r)!\,r!}p^{n-r}(1-p)^{+r}} \quad \text{[Using Eqs. (i) and (ii)]}$$

$$= \left(\frac{1-p}{p}\right)^{n-r} \times \frac{1}{\left(\frac{1-p}{p}\right)^r}$$

Above expression is independent of n and r, if

$$\frac{1-p}{p}=1 \Rightarrow \frac{1}{p}=2 \Rightarrow p=\frac{1}{2}$$

Question 91. In a college, 30% students fail in Physics, 25% fail in Mathematics and 10% fail in both. One student is chosen at random. The probability that she fails in Physics, if she has failed in Mathematics is

(a) $\frac{1}{10}$

(b) $\frac{2}{5}$

(c) $\frac{9}{20}$

(d) $\frac{1}{3}$

Solution. (b)

Explanation: Here, $P(Ph)=\frac{30}{100}=\frac{3}{10}, P(M)=\frac{25}{100}=\frac{1}{4}$

And $P(M \cap Ph)=\frac{10}{100}=\frac{1}{10}$

$$\therefore \quad P\left(\frac{Ph}{M}\right)=\frac{P(Ph \cap M)}{P(M)}=\frac{1/10}{1/4}=\frac{2}{5}$$

Question 92 *A* and *B* are two students. Their chances of solving a problem correctly are $\frac{1}{3}$ and $\frac{1}{4}$, respectively. If the probability of their making a common error is, $\frac{1}{20}$ and they obtain the same answer, then the probability of their answer to be correct is

Solution. (d)

Explanation: Let E_1, = Event that both *A* and *B* solve the problem

$$\therefore \quad P(E_1)=\frac{1}{3}\times\frac{1}{4}=\frac{1}{12}$$

Let E_2 = Event that both *A* and *B* got incorrect solution of the problem

$$P(E_2)=\frac{2}{3}\times\frac{3}{4}=\frac{1}{12}$$

Let *E* = Event that they got same answer

Here, $P(E/E_1)=1, P(E/E_2)=\frac{1}{20}$

$$P(E_1/E)=\frac{P(E_1\cap E)}{P(E)}=\frac{P(E_1)\cdot P(E/E_1)}{P(E_1)\cdot P(E/E_1)+P(E_2)P(E/E_2)}$$

$$=\frac{\frac{1}{12}\times 1}{\frac{1}{12}\times 1+\frac{1}{2}\times\frac{1}{20}}=\frac{1/12}{\frac{10+3}{120}}=\frac{120}{12\times 13}=\frac{10}{13}$$

Question 93. If a box has 100 pens of which 10 are defective, then what is the probability that out of a sample of 5 pens drawn one by one with replacement atmost one is defective?

(a) $\left(\frac{9}{10}\right)^5$

(b) $\frac{1}{2}\left(\frac{9}{10}\right)^4$

(c) $\frac{1}{2}\left(\frac{9}{10}\right)^4$

(d) $\left(\frac{9}{10}\right)^5+\frac{1}{2}\left(\frac{9}{10}\right)^4$

Solution. (d)

Explanation: Here, n = 5, $p=\frac{10}{100}=\frac{1}{10}$ and $q=\frac{9}{10}$

Now, $r\le 1$

$\Rightarrow$ r = 0, 1

Also, $P(X = r) = {}^nC_r\, p^r\, q^{n-1}$

$\therefore$ $P(X = r) = P(r = 0) + P(r = 1)$

$$={}^5C_0\left(\frac{1}{10}\right)^0\left(\frac{9}{10}\right)^5+5C_1\left(\frac{1}{10}\right)^1\left(\frac{9}{10}\right)^4$$

$$=\left(\frac{9}{10}\right)^5+5\cdot\frac{1}{10}\cdot\left(\frac{9}{10}\right)^4$$