1. **Continuity at a Point** A function $f(x)$ is said to be continuous at a point $x=a$, if

Left hand limit of $f(x)$ at $(x=a)$ = Right hand limit of $f(x)$ at $(x=a)$ = Value of $f(x)$ at $(x=a)$

i.e. if at $x=a$ $\quad$ LHL = RHL = $f(a)$

where, LHL = $\lim\limits_{x \to a^-} f(x)$ and RHL = $\lim\limits_{x \to a^+} f(x)$

NOTE To evaluate LHL of a function $f(x)$ at $(x=a)$, put $x=a-h$ and to find RHL, put $x=a+h$.

2. **Continuity in an Interval** A function $y=f(x)$ is said to be continuous in an interval (a, b), where $a<b$ if and only if $f(x)$ is continuous at every point in that interval.

3. (i) Every identity function is continuous.
(ii) Every constant function is continuous.
(iii) Every polynomial function is continuous.
(iv) Every rational function is continuous.
(v) All trigonometric functions are continuous in their domain.

4. **Standard Results of Limits**

(i) $\lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1}$

(ii) $\lim_{x \to 0} \frac{\sin x}{x} = 1$

(iii) $\lim_{x \to 0} \frac{\tan x}{x} = 1$

(iv) $\lim_{x \to 0} \frac{e^x - 1}{x} = 1$

(v) $\lim_{x \to \infty} \frac{1}{x^p} = 0, p \in (0, \infty)$

(vi) $\lim_{x \to 0} \frac{\log(1 + x)}{x} = 1$

(vii) $\lim_{x \to 0} \frac{a^x - 1}{x} = \log_e a$

(viii) $\lim_{x \to 0} \frac{\sin^{-1} x}{x} = 1$

(x) $\lim_{x \to 0} (1 + x)^{1/x} = e$

(xi) $\lim_{x \to \infty} \frac{\sin x}{x} = 0$ (xii) $\lim_{x \to \infty} \left(1+\frac{1}{x}\right)^x = e$

(xiii) $\lim_{x \to \infty} \sin x = \lim_{x \to \infty} \cos x =$ lies between -1 to 1.

5. Algebra of Continuous Functions

I. Suppose f and g are two real functions, continuous at real number c. Then,

(i) $f+g$ is continuous at $x=c$. (ii) $f-g$ is continuous at $x=c$.

(iii) $f \cdot g$ is continuous at $x=c$. (iv) cf is continuous, where c is any constant.

(v) $\left(\frac{f}{g}\right)$ is continuous at $x=c$, [provided $g(c) \neq 0$]

II. Suppose f and g are two real valued functions such that (fog) is defined at c. If g is continuous at c and f is continuous at $g(c)$, then (fog) is continuous at c.

III. If f is continuous, then $|f|$ is also continuous.

1. **Differentiability** A function $f(x)$ is said to be differentiable at a point $x = a$, if

Left hand derivative at $(x = a)$ = Right hand derivative at $(x = a)$

i.e. LHD at $(x = a)$ = RHD (at $x = a$), where

Right hand derivative, $Rf'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

Left hand derivative, $Lf'(a) = \lim_{h \to 0} \frac{f(a-h) - f(a)}{-h}$

NOTE Every differentiable function is continuous but every continuous function is not differentiable.

2. **Differentiation** The process of finding derivative of a function is called differentiation.

3. **Rules of Differentiation**

(i) **Sum and Difference Rule** Let $y = f(x) \pm g(x)$. Then, by using sum and difference rule, it's derivative is written as $\frac{dy}{dx} = \frac{d}{dx} f(x) \pm \frac{d}{dx} g(x)$.

(ii) **Product Rule** Let $y = f(x)\, g(x)$. Then, by using product rule, it's derivative is written as

$$\frac{dy}{dx} = \left[\frac{d}{dx}(f(x))\right] g(x) + \left[\frac{d}{dx}(g(x))\right] f(x).$$

(iii) **Quotient Rule** Let $y = \frac{f(x)}{g(x)}$; $g(x) \neq 0$, then by using quotient rule, it's derivative is written as

$$\frac{dy}{dx} = \frac{g(x) \times \frac{d}{dx}[f(x)] - f(x) \times \frac{d}{dx}[g(x)]}{[g(x)]^2}.$$

6. **Second order Derivative** It is the derivative of the first order derivative.

i.e. $$\frac{d^2 y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$$

7. **Some Standard Derivatives**

(i) $\frac{d}{dx}(\sin x) = \cos x$

(ii) $\frac{d}{dx}(\cos x) = -\sin x$

(iii) $\frac{d}{dx}(\tan x) = \sec^2 x$

(iv) $\frac{d}{dx}(\sec x) = \sec x \tan x$

(v) $\frac{d}{dx}(\text{cosec } x) = -\text{cosec } x \cot x$

(vi) $\frac{d}{dx}(\cot x) = -\text{cosec}^2 x$

(vii) $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$

(viii) $\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$

(ix) $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$

(x) $\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$

(xi) $\frac{d}{dx}(\text{cosec}^{-1} x) = \frac{-1}{x\sqrt{x^2-1}}$

(xii) $\frac{d}{dx}(\cot^{-1} x) = \frac{-1}{1+x^2}$

(xiii) $\frac{d}{dx}(x^n) = nx^{n-1}$

(xiv) $\frac{d}{dx}(\text{constant}) = 0$

(xv) $\frac{d}{dx}(e^x) = e^x$

(xvi) $\frac{d}{dx}(\log_e x) = \frac{1}{x}, x > 0$

(xvii) $\frac{d}{dx}(a^x) = a^x \log_e a, a > 0$

(iv) **Chain Rule** Let $y = f(u)$ and $u = f(x)$, then by using chain rule, we may write

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}, \text{ when } \frac{dy}{du} \text{ and } \frac{du}{dx} \text{ both exist.}$$

4. **Logarithmic Differentiation** Let $y = [f(x)]^{g(x)}$ ...(i)

So by taking log (to base e) we can write Eq. (i) as $\log y = g(x) \log f(x)$. Then, by using chain rule

$$\frac{dy}{dx} = [f(x)]^{g(x)} \left[\frac{g(x)}{f(x)} f'(x) + g'(x) \log f(x) \right]$$

5. **Differentiation of Functions in Parametric Form**

A relation expressed between two variables x and y in the form $x = f(t)$, $y = g(t)$ is said to be parametric form with t as a parameter, when

$$\frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} \qquad \left(\text{whenever } \frac{dx}{dt} \neq 0\right)$$

NOTE dy/dx is expressed in terms of parameter only without directly involving the main variables x and y.

8. **Rolle's Theorem** Let $f:[a, b] \to R$ be continuous on $[a, b]$ and differentiable on (a, b) such that $f(a) = f(b)$, where a and b are some real numbers. Then, there exists atleast one number c in (a, b) such that $f'(c) = 0$.

9. **Mean Value Theorem** Let $f:[a, b] \to R$ be continuous function on $[a, b]$ and differentiable on (a, b). Then, there exists atleast one number c in (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

NOTE Mean value theorem is an expansion of Rolle's theorem.

10. **Some Useful Substitutions for Finding Derivatives**

Expression	Substitution
(i) $a^2 + x^2$	$x = a\tan\theta$ or $x = a\cot\theta$
(ii) $a^2 - x^2$	$x = a\sin\theta$ or $x = a\cos\theta$
(iii) $x^2 - a^2$	$x = a\sec\theta$ or $x = a\,\text{cosec}\,\theta$
(iv) $\sqrt{\frac{a-x}{a+x}}$ or $\sqrt{\frac{a+x}{a-x}}$	$x = a\cos 2\theta$
(v) $\sqrt{\frac{a^2-x^2}{a^2+x^2}}$ or $\sqrt{\frac{a^2+x^2}{a^2-x^2}}$	$x^2 = a^2\cos 2\theta$