NCERT Exemplar Solutions
Class 12 – Mathematics
Chapter 10 – Vector Algebra

Objective Type Questions

Question 19. The vector in the direction of the vector $\hat{\mathbf{i}}-2\hat{\mathbf{j}}+2\hat{\mathbf{k}}$ that has magnitude 9 is

(a) $\hat{\mathbf{i}}-2\hat{\mathbf{j}}+2\hat{\mathbf{k}}$ (b) $\frac{\hat{\mathbf{i}}-2\hat{\mathbf{j}}+2\hat{\mathbf{k}}}{3}$

(c) $3(\hat{\mathbf{i}}-2\hat{\mathbf{j}}+2\hat{\mathbf{k}})$ (d) $9(\hat{\mathbf{i}}-2\hat{\mathbf{j}}+2\hat{\mathbf{k}})$

Solution. (c)

Explanation: Let $\vec{\mathbf{a}}=\hat{\mathbf{i}}-2\hat{\mathbf{j}}+2\hat{\mathbf{k}}$

Unit vector in the direction of a vector $\vec{\mathbf{a}}$ = $\frac{\vec{\mathbf{a}}}{|\vec{\mathbf{a}}|}=\frac{\hat{\mathbf{i}}-2\hat{\mathbf{j}}+2\hat{\mathbf{k}}}{\sqrt{1^2+2^2+2^2}}=\frac{\hat{\mathbf{i}}-2\hat{\mathbf{j}}+2\hat{\mathbf{k}}}{3}$

$\therefore$ Vector in the direction of $\vec{\mathbf{a}}$ with magnitude $9=9\cdot\frac{\hat{\mathbf{i}}-2\hat{\mathbf{j}}+2\hat{\mathbf{k}}}{3}=3(\hat{\mathbf{i}}-2\hat{\mathbf{j}}+2\hat{\mathbf{k}})$

Question 20. The position vector of the point which divides the join of points $2\vec{\mathbf{a}}-3\vec{\mathbf{b}}$ and $\vec{\mathbf{a}}+\vec{\mathbf{b}}$ in the ratio 3 : 1, is

(a) $\frac{3\vec{\mathbf{a}}-2\vec{\mathbf{b}}}{2}$ (b) $\frac{7\vec{\mathbf{a}}-8\vec{\mathbf{b}}}{4}$

(c) $\frac{3\vec{\mathbf{a}}}{4}$ (d) $\frac{5\vec{\mathbf{a}}}{4}$

Solution. (d)

Explanation:

Let the given points be $A\left(2\vec{\mathbf{a}}-3\vec{\mathbf{b}}\right)$ and $B\left(\vec{\mathbf{a}}+\vec{\mathbf{b}}\right)$

Let *C* divides AB in ratio 3:1.

Now the position vector of a point *C* dividing the line segment joining the points *P* and *Q*, whose position vectors are p and q in the ratio *m* : *n* internally, is given by $\frac{m\vec{\mathbf{q}}+n\vec{\mathbf{p}}}{m+n}$

$\therefore$ Position vector $C = \dfrac{3(\vec{\mathbf{a}}+\vec{\mathbf{b}})+1(2\vec{\mathbf{a}}-3\vec{\mathbf{b}})}{3+1}$

$\Rightarrow \quad C = \dfrac{5\vec{\mathbf{a}}}{4}$

Question 21. The vector having initial and terminal points as (2, 5, 0) and (–3, 7, 4), respectively is

(a) $-\hat{\mathbf{i}}+12\hat{\mathbf{j}}+4\hat{\mathbf{k}}$
(b) $5\hat{\mathbf{i}}+2\hat{\mathbf{j}}-4\hat{\mathbf{k}}$
(c) $-5\hat{\mathbf{i}}+2\hat{\mathbf{j}}+4\hat{\mathbf{k}}$
(d) $\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}$

Solution. (c)

Explanation:

Given points are (2, 5, 0) and (–3, 7, 4).

Thus, the required vector $=(-3-2)\hat{\mathbf{i}}+(7-5)\hat{\mathbf{j}}+(4-0)\hat{\mathbf{k}} = -5\hat{\mathbf{i}}+2\hat{\mathbf{j}}+4\hat{\mathbf{k}}$

Question 22. The angle between two vectors $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ with magnitudes $\sqrt{3}$ and 4, respectively and $\vec{\mathbf{a}}\cdot\vec{\mathbf{b}} = 2\sqrt{3}$ is

(a) $\dfrac{\pi}{6}$
(b) $\dfrac{\pi}{3}$
(c) $\dfrac{\pi}{2}$
(d) $\dfrac{5\pi}{2}$

Solution. (b)

Explanation: We have: $|\vec{\mathbf{a}}| = \sqrt{3}, |\vec{\mathbf{b}}| = 4$ and $\vec{\mathbf{a}}\cdot\vec{\mathbf{b}} = 2\sqrt{3}$

We know that, $\vec{\mathbf{a}}\cdot\vec{\mathbf{b}} = |\vec{\mathbf{a}}||\vec{\mathbf{b}}|\cos\theta$

$\Rightarrow \quad 2\sqrt{3} = \sqrt{3}\cdot 4\cdot\cos\theta$

$\Rightarrow \quad \cos\theta = \dfrac{2\sqrt{3}}{4\sqrt{3}} = \dfrac{1}{2}$

$\therefore \quad \theta = \dfrac{\pi}{3}$

Question 23. Find the value of λ such that the vectors $\vec{\mathbf{a}} = 2\hat{\mathbf{i}}+\lambda\hat{\mathbf{j}}+\hat{\mathbf{k}}$ and $\vec{\mathbf{b}} = \hat{\mathbf{i}}+2\hat{\mathbf{j}}+3\hat{\mathbf{k}}$ are orthogonal.

(a) 0 (b) 1

(c) $\frac{3}{2}$ (d) $\frac{-5}{2}$

Solution. (d)

We know that any two non-zero vectors, are orthogonal, if their dot product is zero.

Here two non-zero vectors $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ are orthogonal

$\Rightarrow \quad \vec{\mathbf{a}}\cdot\vec{\mathbf{b}}=0$

$\therefore \quad (2\hat{\mathbf{i}}+\lambda+\hat{\mathbf{k}})\cdot(\hat{\mathbf{i}}+2\hat{\mathbf{j}}+3\hat{\mathbf{k}})=0$

$\Rightarrow \quad 2+2\lambda+3=0$

$\therefore \quad \lambda=\frac{-5}{2}$

Question 24. The value of λ for which the vectors $3\hat{\mathbf{i}}-6\hat{\mathbf{j}}+\hat{\mathbf{k}}$ and $2\hat{\mathbf{i}}-4\hat{\mathbf{j}}+\lambda\hat{\mathbf{k}}$ are parallel, is

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$

(c) $\frac{5}{2}$ (d) $\frac{2}{5}$

Solution. (a)

As the vectors $3\hat{\mathbf{i}}-6\hat{\mathbf{j}}+\hat{\mathbf{k}}$ and $2\hat{\mathbf{i}}-4\hat{\mathbf{j}}+\lambda\hat{\mathbf{k}}$ are parallel

$\therefore \quad \frac{3}{2}=\frac{-6}{-4}=\frac{1}{\lambda}$

$\Rightarrow \quad \lambda=\frac{2}{3}$

Question 25. The vectors from origin to the points A and B are $\vec{\mathbf{a}}=2\hat{\mathbf{i}}-3\hat{\mathbf{j}}+2\hat{\mathbf{k}}$ and $\vec{\mathbf{b}}=2\hat{\mathbf{i}}+3\hat{\mathbf{j}}+\hat{\mathbf{k}}$ respectively, then the area of ΔOAB is equal to

(a) 340 (b) $\sqrt{25}$

(c) $\sqrt{229}$ (d) $\frac{1}{2}\sqrt{229}$

Solution. (d)

Area of $\Delta OAB = \frac{1}{2}|\overrightarrow{\mathbf{OA}} \cdot \overrightarrow{\mathbf{OB}}|$

$$= \frac{1}{2}|(2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}}) \times (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + \hat{\mathbf{k}})|$$

$$= \frac{1}{2}\begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 2 & -3 & 2 \\ 2 & 3 & 1 \end{vmatrix}$$

$$= \frac{1}{2}|[\hat{\mathbf{i}}(-3-6) - \hat{\mathbf{j}}(2-4) + \hat{\mathbf{k}}(6+6)]|$$

$$= \frac{1}{2}|-9\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 12\hat{\mathbf{k}}|$$

$$\Delta OAB = \frac{1}{2}\sqrt{(81+4+144)} = \frac{1}{2}\sqrt{229}$$

Question 26. For any vector $\vec{\mathbf{a}}$, the value of $(\vec{\mathbf{a}} \times \hat{\mathbf{i}})^2 + (\vec{\mathbf{a}} \times \hat{\mathbf{j}})^2 + (\vec{\mathbf{a}} \times \hat{\mathbf{k}})^2$ is

(a) $\vec{\mathbf{a}}^2$

(b) $3\vec{\mathbf{a}}^2$

(c) $4\vec{\mathbf{a}}^2$

(d) $2\vec{\mathbf{a}}^2$

Solution. (d)

Explanation: Let $\vec{\mathbf{a}} = x\hat{\mathbf{i}} + y\hat{\mathbf{i}} + z\hat{\mathbf{k}}$

$\therefore \quad \vec{\mathbf{a}}^2 = x^2 + y^2 + z^2$

$$\therefore \quad \vec{\mathbf{a}} \times \hat{\mathbf{i}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ x & y & z \\ 1 & 0 & 0 \end{vmatrix}$$

$$= \hat{\mathbf{i}}[0] - \hat{\mathbf{j}}[-z] + \hat{\mathbf{k}}[-y]$$

$$= z\hat{\mathbf{j}} - y\hat{\mathbf{k}}$$

$\therefore \quad (\vec{\mathbf{a}} \times \hat{\mathbf{i}})^2 = (z\hat{\mathbf{j}} - y\hat{\mathbf{k}})(z\hat{\mathbf{j}} - y\hat{\mathbf{k}}) = y^2 + z^2$

Similarly, $(\vec{\mathbf{a}} \times \hat{\mathbf{j}})^2 = x^2 + z^2$

And $\quad (\vec{\mathbf{a}} \times \hat{\mathbf{k}})^2 = x^2 + y^2$

$\therefore \quad (\vec{\mathbf{a}} \times \hat{\mathbf{i}})^2 + (\vec{\mathbf{a}} \times \hat{\mathbf{j}})^2 + (\vec{\mathbf{a}} \times \hat{\mathbf{k}})^2 = y^2 + z^2 + x^2 + z^2 + x^2 + y^2$

$$= 2(x^2 + y^2 + z^2) = 2\vec{\mathbf{a}}^2$$

Question 27. If $|\vec{\mathbf{a}}| = 10, |\vec{\mathbf{b}}| = 2$ and $\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = 12,$ then the value of |a × b| is

(a) 5 (b) 10

(c) 14 (d) 16

Solution. (d)

Explanation: Given that, $|\vec{\mathbf{a}}| = 10, |\vec{\mathbf{b}}| = 2$ and $\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = 12$

We know that, $\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = |\vec{\mathbf{a}}||\vec{\mathbf{b}}| \cos\theta$

$\Rightarrow \quad 12 = 10 \times 2 \cos\theta$

$\Rightarrow \quad \cos\theta = \frac{12}{20} = \frac{3}{5}$

$\Rightarrow \quad \sin\theta = \sqrt{1-\cos^2\theta}\sqrt{1-\frac{9}{25}}$

$\sin\theta = \pm\frac{4}{5}$

Now, $|\vec{\mathbf{a}} \times \vec{\mathbf{b}}| = |\vec{\mathbf{a}}||\vec{\mathbf{b}}||\sin\theta| = 10 \times 2 \times \frac{4}{5} = 16$

Question 28. The vectors $\lambda\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}}, \hat{\mathbf{i}} + \lambda\hat{\mathbf{j}} - \hat{\mathbf{k}}$ and $2\hat{\mathbf{i}} - \hat{\mathbf{j}} + \lambda\hat{\mathbf{k}}$ are coplanar, if

(a) λ = –2 (b) λ = 0

(c) λ = 1 (d) λ = –1

Solution. (a)

Explanation: Let $\vec{\mathbf{a}} = \lambda\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}}, \vec{\mathbf{b}} = \hat{\mathbf{i}} + \lambda\hat{\mathbf{j}} - \hat{\mathbf{k}}$ and $\vec{\mathbf{c}} = 2\hat{\mathbf{i}} - \hat{\mathbf{j}} + \lambda\hat{\mathbf{k}}$

For $\vec{\mathbf{a}}, \vec{\mathbf{b}}$ and $\vec{\mathbf{c}}$ to be coplanar,

$$\begin{vmatrix} \lambda & 1 & 2 \\ 1 & \lambda & -1 \\ 2 & -1 & \lambda \end{vmatrix} = 0$$

$\Rightarrow \quad \lambda(\lambda^2 - 1) - 1(\lambda + 2) + 2(-1 - 2\lambda) = 0$

$\Rightarrow \quad \lambda^3 - \lambda - \lambda - 2 - 2 - 4\lambda = 0$

$\Rightarrow \quad \lambda^3 - 6\lambda - 4 = 0$

$\Rightarrow \quad (\lambda + 2)(\lambda^2 - 2\lambda - 2) = 0$

$\Rightarrow \quad \lambda = -2 \text{ or } \lambda = \dfrac{2 \pm \sqrt{12}}{2}$

$\Rightarrow \quad \lambda = -2 \text{ or } \lambda = 1 \pm \sqrt{3}$

Question 29. If $\vec{\mathbf{a}}, \vec{\mathbf{b}}$ and $\vec{\mathbf{c}}$ are unit vectors such that $\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}} = \vec{0}$, then the value of $\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} + \vec{\mathbf{b}} \cdot \vec{\mathbf{c}} + \vec{\mathbf{c}} \cdot \vec{\mathbf{a}}$ is

(a) 1

(b) 3

(c) $-\dfrac{3}{2}$

(d) None of these

Solution. (c)

Explanation: We have, $\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}} = 0$

$\Rightarrow \quad (\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}})(\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}}) = 0$

$\Rightarrow \quad \vec{\mathbf{a}}^2 + \vec{\mathbf{a}} \cdot \vec{\mathbf{b}} + \vec{\mathbf{a}} \cdot \vec{\mathbf{c}} + \vec{\mathbf{b}} \cdot \vec{\mathbf{a}} + \vec{\mathbf{b}}^2 + \vec{\mathbf{b}} \cdot \vec{\mathbf{c}} + \vec{\mathbf{c}} \cdot \vec{\mathbf{a}} + \vec{\mathbf{c}} \cdot \vec{\mathbf{b}} + \vec{\mathbf{c}}^2 = 0$

$\Rightarrow \quad \vec{\mathbf{a}}^2 + \vec{\mathbf{b}}^2 + \vec{\mathbf{c}}^2 + 2(\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} + \vec{\mathbf{b}} \cdot \vec{\mathbf{c}} + \vec{\mathbf{c}} \cdot \vec{\mathbf{a}}) = 0 \; [\because \vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = \vec{\mathbf{b}} \cdot \vec{\mathbf{a}}, \vec{\mathbf{b}} \cdot \vec{\mathbf{c}} = \vec{\mathbf{c}} \cdot \vec{\mathbf{b}} \text{ and } \vec{\mathbf{c}} \cdot \vec{\mathbf{a}} = \vec{\mathbf{a}} \cdot \vec{\mathbf{c}}]$

$\Rightarrow \quad 1 + 1 + 1 + 2(\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} + \vec{\mathbf{b}} \cdot \vec{\mathbf{c}} + \vec{\mathbf{c}} \cdot \vec{\mathbf{a}}) = 0$

$\Rightarrow \quad \vec{\mathbf{a}} \cdot \vec{\mathbf{b}} + \vec{\mathbf{b}} \cdot \vec{\mathbf{c}} + \vec{\mathbf{c}} \cdot \vec{\mathbf{a}} = -\dfrac{3}{2}$

Question 30. The projection vector of $\vec{\mathbf{a}}$ on $\vec{\mathbf{b}}$ is

(a) $\left(\dfrac{\vec{\mathbf{a}} \cdot \vec{\mathbf{b}}}{|\vec{\mathbf{b}}|}\right)\vec{\mathbf{b}}$

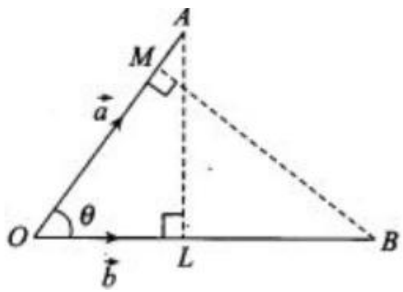
(b) $\dfrac{\vec{\mathbf{a}} \cdot \vec{\mathbf{b}}}{|\vec{\mathbf{b}}|}$

(c) $\dfrac{\vec{\mathbf{a}} \cdot \vec{\mathbf{b}}}{|\vec{\mathbf{a}}|}$

(d) $\left(\dfrac{\vec{\mathbf{a}} \cdot \vec{\mathbf{b}}}{|\vec{\mathbf{a}}|^2}\right)\hat{\mathbf{b}}$

Solution. (a)

Explanation: Let the two vectors $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ be represented by $\overrightarrow{\mathbf{OA}}$ and $\overrightarrow{\mathbf{OB}}$ respectively.



$$\text{Now}, \vec{\mathbf{a}}.\vec{\mathbf{b}} = |\vec{\mathbf{a}}||\vec{\mathbf{b}}|\cos\theta$$
$$= |\vec{\mathbf{b}}|(|\vec{\mathbf{a}}|\cos\theta)$$
$$= |\vec{\mathbf{b}}|(OA\cos\theta)$$
$$= |\vec{\mathbf{b}}|(OL)$$

$$\Rightarrow \quad OL = \frac{\vec{\mathbf{a}}.\vec{\mathbf{b}}}{|\vec{\mathbf{b}}|}$$

Thus, the projection vector of $\vec{\mathbf{a}}$ on $\vec{\mathbf{b}}$ is given by $= \frac{\vec{\mathbf{a}}.\vec{\mathbf{b}}}{|\vec{\mathbf{b}}|}.\frac{\vec{\mathbf{b}}}{|\vec{\mathbf{b}}|} = \frac{\vec{\mathbf{a}}\cdot\vec{\mathbf{b}}}{|\vec{\mathbf{b}}|^2}\vec{\mathbf{b}}$

Question 31. If $\vec{\mathbf{a}}, \vec{\mathbf{b}}$ and $\vec{\mathbf{c}}$ are three vectors such that

$\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}} = \vec{0}$ and $|\vec{\mathbf{a}}| = 2$, $|\vec{\mathbf{b}}| = 3$ and $|\vec{\mathbf{c}}| = 5$, then the value of $\vec{\mathbf{a}}\cdot\vec{\mathbf{b}} + \vec{\mathbf{b}}\cdot\vec{\mathbf{c}} + \vec{\mathbf{c}}\cdot\vec{\mathbf{a}}$ is

(a) 0 (b) 1

(c) –19 (d) 38

Solution. (c)

Explanation: Here, $\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}} = \vec{0}$ and $\vec{\mathbf{a}}^2 = 4$, $\vec{\mathbf{b}}^2 = 9$, $\vec{\mathbf{c}}^2 = 25$

$$\therefore \quad (\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}})\cdot(\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}}) = \vec{0}$$

$$\Rightarrow \quad \vec{\mathbf{a}}^2 + \vec{\mathbf{a}}\cdot\vec{\mathbf{b}} + \vec{\mathbf{a}}\cdot\vec{\mathbf{c}} + \vec{\mathbf{b}}\cdot\vec{\mathbf{a}} + \vec{\mathbf{b}}^2 + \vec{\mathbf{b}}\cdot\vec{\mathbf{c}} + \vec{\mathbf{c}}\cdot\vec{\mathbf{a}} + \vec{\mathbf{c}}\cdot\vec{\mathbf{b}} + \vec{\mathbf{c}}^2 = \vec{0}$$

$$\Rightarrow \quad \vec{\mathbf{a}}^2 + \vec{\mathbf{b}}^2 + \vec{\mathbf{c}}^2 + 2(\vec{\mathbf{a}}\cdot\vec{\mathbf{b}} + \vec{\mathbf{b}}\cdot\vec{\mathbf{c}} + \vec{\mathbf{c}}\cdot\vec{\mathbf{a}}) = 0 \qquad [\because \vec{\mathbf{a}}\cdot\vec{\mathbf{b}} = \vec{\mathbf{b}}\cdot\vec{\mathbf{a}}]$$

$$\Rightarrow \quad 4 + 9 + 25 + 2(\vec{\mathbf{a}}\cdot\vec{\mathbf{b}} + \vec{\mathbf{b}}\cdot\vec{\mathbf{c}} + \vec{\mathbf{c}}\cdot\vec{\mathbf{a}}) = 0$$

$$\Rightarrow \quad \vec{\mathbf{a}}\cdot\vec{\mathbf{b}} + \vec{\mathbf{b}}\cdot\vec{\mathbf{c}} + \vec{\mathbf{c}}\cdot\vec{\mathbf{a}} = \frac{-38}{2} = -19$$

Question 32. If $|\vec{\mathbf{a}}| = 4$ and $-3 \le \lambda \le 2$, then the range of $|\lambda\vec{\mathbf{a}}|$ is

(a) [0, 8] (b) [–12, 8]

(c) [0, 12]

(d) [8, 12]

Solution. (c)

Explanation: We have, $|\vec{\mathbf{a}}| = 4$ and $-3 \leq \lambda \leq 2$

Since, $-3 \leq \lambda \leq 2$

$$0 \leq |\lambda| \leq 3$$

$$\Rightarrow \quad 0 \leq 4|\lambda| \leq 12$$

$$\Rightarrow \quad 0 \leq |\lambda\vec{\mathbf{a}}| \leq 12$$

$$\therefore \quad |\lambda\vec{\mathbf{a}}| \in [0, 12]$$

Question 33. The number of vectors of unit length perpendicular to the vectors $\vec{\mathbf{a}} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$ and $\vec{\mathbf{b}} = \hat{\mathbf{j}} + \hat{\mathbf{k}}$ is

(a) one

(b) two

(c) three

(d) infinite

Solution. (b)

The number of vectors of unit length perpendicular to the vectors $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ given as,

$$\vec{\mathbf{c}} = \pm(\vec{\mathbf{a}} \times \vec{\mathbf{b}})$$

So, there will be two vectors of unit length perpendicular to the vectors $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$.