NCERT Exemplar Solutions

Class 12 – Mathematics

Chapter 11 – Three Dimensional Geometry

Objective Type Questions

Question 29: Distance of the point (α, β, γ) from *Y*-axis is

(a) β

(b) $|\beta|$

(c) |β| + |γ|

(d) $\sqrt{\alpha^2+\gamma^2}$

Solution. ***(d)***

The distance between the points (x_1, y_1, z_1) and (x_2, y_2, z_2) is given by

$$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$$

On *Y*-axis, the coordinates of the *X*-axis and *X*-axis are 0.

Therefore, the distance of the point (α, β, γ) from *Y*-axis is given by

$$\sqrt{(\alpha-0)^2+(\beta-\beta)^2+(\gamma-0)^2}$$
$$=\sqrt{\alpha^2+\gamma^2}$$

Question 30: If the direction cosines of a line are *k*, *k* and *k*, then

(a) $k > 0$

(b) $0 < k < 1$

(c) $k = 1$

(d) $k=\frac{1}{\sqrt{3}}$ or $-\frac{1}{\sqrt{3}}$

Solution. ***(d)***

We know that, If *l*, *m*, *n* are the direction cosines of a line, then
$l^2+m^2+m^2=1$

Here, $l=k, m=k$ and $n=k$

Substituting $l=k, m=k$ and $n=k$ in $l^2+m^2+m^2=1$, we get

$\Rightarrow k^2 + k^2 + k^2 = 1$

$\Rightarrow 3k^2 = 1$

$\Rightarrow k^2 = \frac{1}{3}$

$\Rightarrow k = \pm\frac{1}{\sqrt{3}}$

Question 31: The distance of the plane $\vec{\mathbf{r}}\left(\frac{2}{7}\hat{\mathbf{i}}+\frac{3}{7}\hat{\mathbf{j}}-\frac{6}{7}\hat{\mathbf{k}}\right)=1$ from the origin is

(a) 1

(b) 7

(c) $\frac{1}{7}$

(d) None of these

Solution. ***(a)***

The general equation of a plane in vector form is given by $\vec{\mathbf{r}}.\hat{n}=d$

Where d is the distance of the plane from the origin.

Comparing $\vec{\mathbf{r}}.\hat{n}=d$ and $\vec{\mathbf{r}}\left(\frac{2}{7}\hat{\mathbf{i}}+\frac{3}{7}\hat{\mathbf{j}}-\frac{6}{7}\hat{\mathbf{k}}\right)=1$, we get d = 1

Therefore, the distance of the plane $\vec{\mathbf{r}}\left(\frac{2}{7}\hat{\mathbf{i}}+\frac{3}{7}\hat{\mathbf{j}}-\frac{6}{7}\hat{\mathbf{k}}\right)=1$ from the origin is 1.

Question 32: The sine of the angle between the straight line $\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$ and the plane $2x - 2y + z = 5$ is

(a) $\frac{10}{6\sqrt{5}}$

(b) $\frac{4}{5\sqrt{2}}$

(c) $\frac{2\sqrt{3}}{5}$

(d) $\frac{\sqrt{2}}{10}$

Solution. ***(d)***

We can write the equation of the line and plane in vector form as

$$\vec{\mathbf{r}} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}} + \lambda\left(3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 5\hat{\mathbf{k}}\right) \text{ and } \vec{\mathbf{r}}.\left(2\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}}\right) = 5$$

$$\therefore \vec{\mathbf{b}} = 3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 5\hat{\mathbf{k}} \text{ and } \vec{\mathbf{n}} = 2\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$$

The angle between the line and the plane is given by $\sin\theta = \dfrac{|\vec{\mathbf{b}} \cdot \vec{\mathbf{n}}|}{|\vec{\mathbf{b}}| \cdot |\vec{\mathbf{n}}|}$

Substituting $\vec{\mathbf{b}} = 3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$ and $\vec{\mathbf{n}} = 2\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$ in $\sin\theta = \dfrac{|\vec{\mathbf{b}} \cdot \vec{\mathbf{n}}|}{|\vec{\mathbf{b}}| \cdot |\vec{\mathbf{n}}|}$, we get

$$\sin\theta = \frac{|(3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 5\hat{\mathbf{k}}) \cdot (2\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}})|}{\sqrt{3^2 + 4^2 + 5^2} \cdot \sqrt{4 + 4 + 1}}$$

$$= \frac{|6 - 8 + 5|}{\sqrt{50} \cdot 3}$$

$$= \frac{3}{15\sqrt{2}}$$

$$= \frac{1}{5\sqrt{2}}$$

$$\frac{\sqrt{2}}{10}$$

Question 33: The reflection of the point (α, β, γ) in the *XY*-plane is

(a) (α, β, 0)

(b) (0, 0, γ)

(c)(– α , – β , γ)

(d) (α, β, –γ)

Solution. ***(d)***

In *XY*-plane, only the sign of *z* coordinate of the point got changed after the reflection.

Therefore, the reflection of the point (α, β, γ) is (α, β, –γ).

Question 34: The area of the quadrilateral *ABCD* where *A* (0, 4, 1), *B*(2, 3, – 1), *C*(4, 5, 0), and *D*(2, 6, 2) is equal to

(a) 9 sq units

(b) 18 sq units

(c) 27 sq units

(d) 81 sq units

Solution. (*a*)

We have, *A* (0, 4, 1), *B*(2, 3, – 1), *C*(4, 5, 0) and *D*(2, 6, 2).

We can find $\overrightarrow{\mathbf{AB}}$ and $\overrightarrow{\mathbf{BC}}$ as

$$\overrightarrow{\mathbf{AB}} = (2-0)\hat{\mathbf{i}} + (3-4)\hat{\mathbf{j}} + (-1-1)\hat{\mathbf{k}} = 2\hat{\mathbf{i}} - \hat{\mathbf{j}} - 2\hat{\mathbf{k}}$$

$$\overrightarrow{\mathbf{BC}} = (4-2)\hat{\mathbf{i}} + (5-3)\hat{\mathbf{j}} + (0+1)\hat{\mathbf{k}} = 2\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$$

$$\therefore \text{ Area of quadrilateral } ABCD = |\overrightarrow{\mathbf{AB}} \times \overrightarrow{\mathbf{BC}}| = \left\| \begin{matrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 2 & -1 & -2 \\ 2 & 2 & 1 \end{matrix} \right\|$$

$$= \left| \hat{\mathbf{i}}(-1+4) - \hat{\mathbf{j}}(2+4) + \hat{\mathbf{k}}(4+2) \right|$$

$$= \left| 3\hat{\mathbf{i}} - 6\hat{\mathbf{j}} + 6\hat{\mathbf{k}} \right|$$

$$= \sqrt{9+36+36}$$

$$= \sqrt{81}$$

= 9sq units

Question 35: The Locus represented by $xy + yz = 0$ is

(a) a pair of perpendicular lines

(b) a pair of parallel lines

(c) a pair of parallel planes

(d) a pair of perpendicular planes

Solution. ***(d)***

We have, $xy + yz = 0$

$\Rightarrow xy = -yz$

Therefore, the locus represented by $xy + yz = 0$ is a pair of perpendicular planes.

Question 36: If the plane $2x - 3y + 6z - 11 = 0$ makes an angle $\sin^{-1}\alpha$ with X-axis, then the value of α is

(a) $\frac{\sqrt{3}}{2}$

(b) $\frac{\sqrt{2}}{3}$

(c) $\frac{2}{7}$

(d) $\frac{3}{7}$

Solution. ***(c)***

We are given that, $2x - 3y + 6z - 11 = 0$ makes an angle $\sin^{-1}\alpha$ with X-axis.

The equation of plane $2x - 3y + 6z - 11 = 0$ in vector form is given by $\vec{r}.\left(2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 6\hat{\mathbf{k}}\right) = 11$

$\therefore \vec{\mathbf{b}} = (\hat{\mathbf{i}} + 0\hat{\mathbf{j}} + 0\hat{\mathbf{k}})$ and $\vec{\mathbf{n}} = 2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 6\hat{\mathbf{k}}$

We know that, $\sin\theta = \frac{|\vec{\mathbf{b}} \cdot \vec{\mathbf{n}}|}{|\vec{\mathbf{b}}| \cdot |\vec{\mathbf{n}}|}$

$= \frac{|(\hat{\mathbf{i}}) \cdot (2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 6\hat{\mathbf{k}})|}{\sqrt{1}\sqrt{4+9+36}}$

$= \frac{2}{7}$