NCERT Exemplar Solutions

Class 12 – Mathematics

Chapter 2 – Inverse Trigonometric Functions

Objective Type Questions

Question 1.20:

Which of the following is the principal value branch of $\cos^{-1} x$?

(a) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

(b) $(0, \pi)$

(c) $[0, \pi]$

(d) $(0, \pi) - \left\{\frac{\pi}{2}\right\}$

Solution 1.20: (c)

The principal value branch of $\cos^{-1} x$ is $[0, \pi]$.

Question 1.21:

Which of the following is the principal value branch of $\operatorname{cosec}^{-1} x$?

(a) $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

(b) $[0, \pi] - \left\{\frac{\pi}{2}\right\}$

(c) $\left[\frac{\pi}{2}, \frac{\pi}{2}\right]$

(d) $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right] - [0]$

Solution 1.21: (d)

The principal value branch of $\operatorname{cosec}^{-1} x \left[\frac{-\pi}{2}, \frac{\pi}{2}\right] - 0.$

Question 1.22:

If $3\tan^{-1} x + \cot^{-1} x = \pi$ then x equals to

(a) 0 (b) 1 (c) – 1 (d) $\frac{1}{2}$

Solution 1.22: (b)

We are given that, $3\tan^{-1} x + \cot^{-1} x = \pi$

$\Rightarrow 2\tan^{-1} x + \tan^{-1} x + \cot^{-1} x = \pi$

$\Rightarrow 2\tan^{-1} x = \pi - \frac{\pi}{2}$ $\left[\because \tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}\right]$

$\Rightarrow 2\tan^{-1} x = \frac{\pi}{2}$

$\Rightarrow \tan^{-1} x = \frac{\pi}{4}$

$\Rightarrow \tan^{-1} x = \tan^{-1} 1$

$\Rightarrow x = 1$

Question 1.23:

The value of $\sin^{-1}\left[\cos\left(\frac{33\pi}{5}\right)\right]$ is

(a) $\frac{3\pi}{5}$

(b) $\frac{-7\pi}{5}$

(c) $\frac{\pi}{10}$

(d) $\frac{-\pi}{10}$

Solution 1.23: (d)

We have, $\sin^{-1}\left[\cos\left(\frac{33\pi}{5}\right)\right]$

$$= \sin^{-1}\left[\cos\left(6\pi + \frac{3\pi}{5}\right)\right]$$

$$= \sin^{-1}\left[\cos\left(\frac{3\pi}{5}\right)\right] \qquad [\because \cos(2n\pi + \theta) = \cos\theta]$$

$$= \sin^{-1}\left[\cos\left(\frac{\pi}{2} + \frac{\pi}{10}\right)\right]$$

$$= \sin^{-1}\left(-\sin\frac{\pi}{10}\right) \qquad \left[\because \cos\left(\frac{\pi}{2} + \theta\right) = -\sin\theta\right]$$

$$= -\sin^{-1}\left(\sin\frac{\pi}{10}\right) \quad [\because \sin^{-1}(-x) = -\sin^{-1}x]$$

$$= -\frac{\pi}{10} \qquad \left[\because \sin^{-1}(\sin x) = x,\ x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$$

Question 1.24:

The domain of the function $\cos^{-1}(2x - 1)$ is

(a) [0, 1]

(b) [– 1, 1]

(c) (–1, 1)

(d) [0, π]

Solution 1.24: (a)

We have, $\cos^{-1}(2x - 1)$

Now, we know that the domain of $\cos^{-1}(x)$ is $-1 \le x \le 1$

$\therefore\ -1 \le 2x - 1 \le 1$

Adding 1 to all terms, we get

$\Rightarrow 0 \le 2x \le 2$

Dividing all terms by 2, we get

$\Rightarrow 0 \le x \le 1$

$\therefore\ x \in [0, 1]$

Question 1.25:

The domain of the function defined by $f(x) = \sin^{-1}\sqrt{x-1}$ is

(a) [1,2]

(b) [-1,1]

(c) [0, 1]

(d) None of these

Solution 1.25: (a)

We are given that, $f(x) = \sin^{-1}\sqrt{x-1}$

Now, we know that the domain of $\sin^{-1}(x)$ is $-1 \le x \le 1$

$\therefore\ -1 \le \sqrt{x-1} \le 1$

Squaring all the terms, we get

$0 \le x - 1 \le 1$

Adding 1 to all terms, we get

$\Rightarrow 1 \le x \le 2$

$\therefore\ x \in [1, 2]$

Question 1.26:

If $\cos\left(\sin^{-1}\frac{2}{5} + \cos^{-1}x\right) = 0$, then x is equal to

(a) $\frac{1}{5}$

(b) $\frac{2}{5}$

(c) 0

(d) 1

Solution 1.26: (b)

We are given that, $\cos\left(\sin^{-1}\frac{2}{5} + \cos^{-1}x\right) = 0$

$$\Rightarrow \sin^{-1}\frac{2}{5}+\cos^{-1}x=\cos^{-1}0$$

$$\Rightarrow \sin^{-1}\frac{2}{5}+\cos^{-1}x=\cos^{-1}\left(\cos\frac{\pi}{2}\right)$$

$$\Rightarrow \sin^{-1}\frac{2}{5}+\cos^{-1}x=\frac{\pi}{2} \qquad \left[\because \cos^{-1}(\cos x)=x,\ x\in[0,\pi]\right]$$

$$\Rightarrow \cos^{-1}x=\frac{\pi}{2}-\sin^{-1}\frac{2}{5}$$

$$\Rightarrow \cos^{-1}x=\cos^{-1}\frac{2}{5} \qquad \left[\begin{array}{l}\because\ \cos^{-1}x+\sin^{-1}x=\dfrac{\pi}{2}\\ \Rightarrow \dfrac{\pi}{2}-\sin^{-1}x=\cos^{-1}x\end{array}\right]$$

$$\Rightarrow x=\frac{2}{5}$$

Question 1.27:

The value of $\sin[2\tan^{-1}(0.75)]$ is

(a) 0.75

(b) 1.5

(c) 0.96

(d) sin 1.5

Solution 1.27: (c)

We have, $\sin\left[2\tan^{-1}(0.75)\right]=\sin\left(2\tan^{-1}\frac{3}{4}\right) \qquad \left[\because\ 0.75=\frac{75}{100}=\frac{3}{4}\right]$

$$=\sin\left(\sin^{-1}\frac{2\cdot\frac{3}{4}}{1+\frac{9}{16}}\right) \qquad \left[\because 2\tan^{-1}x=\sin^{-1}\frac{2x}{1+x^2}\right]$$

$$=\sin\left[\sin^{-1}\frac{3/2}{25/16}\right]$$

$$= \sin\left[\sin^{-1}\left(\frac{48}{50}\right)\right]$$

$$= \sin\left[\sin^{-1}\left(\frac{24}{25}\right)\right]$$

$$= \frac{24}{25}$$

$$= 0.96$$

Question 1.28:

The value of $\cos^{-1}\left(\cos\frac{3\pi}{2}\right)$ is

(a) $\frac{\pi}{2}$

(b) $\frac{3\pi}{2}$

(c) $\frac{5\pi}{2}$

(d) $\frac{7\pi}{2}$

Solution 1.28: (a)

We have, $\cos^{-1}\left(\cos\frac{3\pi}{2}\right)$

$$= \cos^{-1}\cos\left(2\pi - \frac{\pi}{2}\right)$$

$$= \cos^{-1}\cos\left(\frac{\pi}{2}\right) \qquad \left[\because \cos(2\pi-\theta) = \cos\theta\right]$$

$$= \frac{\pi}{2} \qquad \left[\because \cos^{-1}(\cos x) = x,\ x \in [0,\pi]\right]$$

Question 1.29:

The value of $2\sec^{-1}2 + \sin^{-1}\left(\frac{1}{2}\right)$ is

(a) $\frac{\pi}{6}$

(b) $\frac{5\pi}{6}$

(c) $\frac{7\pi}{6}$

(d) 1

Solution 1.29: (b)

We have, $2\sec^{-1}2+\sin^{-1}\frac{1}{2}$

$=2\sec^{-1}\left(\sec\frac{\pi}{3}\right)+\sin^{-1}\sin\frac{\pi}{6}\qquad\left[\because \sec\frac{\pi}{3}=2 \text{ and } \sin\frac{\pi}{6}=\frac{1}{2}\right]$

$=2\cdot\frac{\pi}{3}+\frac{\pi}{6}$ $[\because \sec^{-1}(\sec)\,x = x$ and $\sin^{-1}(\sin x) = x]$

$=\frac{4\pi+\pi}{6}$

$=\frac{5\pi}{6}$

Question 1.30:

If $\tan^{-1}x+\tan^{-1}y=\frac{4\pi}{5}$, then $\cot^{-1}x+\cot^{-1}y$ equals to

(a) $\frac{\pi}{5}$

(b) $\frac{2\pi}{5}$

(c) $\frac{3\pi}{5}$

(d) π

Solution 1.30: (a)

We have, $\tan^{-1}x+\tan^{-1}y=\frac{4\pi}{5}$

$$\Rightarrow \frac{\pi}{2}-\cot^{-1}x+\frac{\pi}{2}-\cot^{-1}y=\frac{4\pi}{5} \qquad \left[\begin{array}{l}\because \tan^{-1}x+\cot^{-1}x=\frac{\pi}{2}\\ \Rightarrow \tan^{-1}x=\frac{\pi}{2}-\cot^{-1}x\end{array}\right]$$

$$\Rightarrow \pi-\left(\cot^{-1}x+\cot^{-1}y\right)=\frac{4\pi}{5}$$

$$\Rightarrow \pi-\frac{4\pi}{5}=\cot^{-1}x+\cot^{-1}y$$

$$\Rightarrow \cot^{-1}x+\cot^{-1}y=\frac{\pi}{5}$$

Question 1.31:

If $\sin^{-1}\left(\frac{2a}{1+a^2}\right)+\cos^{-1}\left(\frac{1-a^2}{1+a^2}\right)=\tan^{-1}\left(\frac{2x}{1-x^2}\right)$, where a, x $\in$]0, 1[, then the value of x is

(a) 0

(b) $\frac{a}{2}$

(c) a

(d) $\frac{2a}{1-a^2}$

Solution 1.31: (d)

We have, $\sin^{-1}\left(\frac{2a}{1+a^2}\right)+\cos^{-1}\left(\frac{1-a^2}{1+a^2}\right)=\tan^{-1}\left(\frac{2x}{1-x^2}\right)$

$$\Rightarrow 2\tan^{-1}a+2\tan^{-1}a=\tan^{-1}\left(\frac{2x}{1-x^2}\right) \qquad \left[\because 2\tan^{-1}a=\sin^{-1}\left(\frac{2a}{1+a^2}\right)=\cos^{-1}\left(\frac{1-a^2}{1+a^2}\right)\right]$$

$$\Rightarrow 4\tan^{-1}a=2\tan^{-1}x \qquad \left[\because 2\tan^{-1}x=\tan^{-1}\left(\frac{2x}{1-x^2}\right)\right]$$

$$\Rightarrow 2\tan^{-1}a=\tan^{-1}x \qquad \left[\because 2\tan^{-1}x=\tan^{-1}\left(\frac{2x}{1-x^2}\right)\right]$$

$$\Rightarrow \tan^{-1}\left(\frac{2a}{1-a^2}\right) = \tan^{-1} x \quad \left[\because 2\tan^{-1} a = \tan^{-1}\left(\frac{2a}{1-a^2}\right)\right]$$

$$\Rightarrow x = \frac{2a}{1-a^2}$$

Question 1.32:

The value of $\cot\left[\cos^{-1}\left(\frac{7}{25}\right)\right]$ is

(a) $\frac{25}{24}$

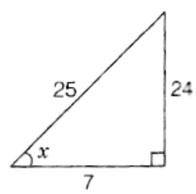
(b) $\frac{25}{7}$

(c) $\frac{24}{25}$

(d) $\frac{7}{24}$

Solution 1.32: (d)

We have, $\cot\left[\cos^{-1}\left(\frac{7}{25}\right)\right]$



Let us suppose, $\cos^{-1}\frac{7}{25} = x$

$$\Rightarrow \cos x = \frac{7}{25}$$

Now, $\sin x = \sqrt{1-\cos^2 x}$

$$= \sqrt{1-\left(\frac{7}{25}\right)^2}$$

$$= \sqrt{\frac{625-49}{625}}$$

$$= \frac{24}{25}$$

Also, $\cot x = \dfrac{\cos x}{\sin x}$

$$\Rightarrow \cot x = \frac{\frac{7}{25}}{\frac{24}{25}} \qquad \left[\because \cos x = \frac{7}{25} \text{ and } \sin x = \frac{24}{25}\right]$$

$$\Rightarrow \cot x = \frac{7}{24}$$

$$\Rightarrow x = \cot^{-1}\frac{7}{24}$$

$$\Rightarrow \cot^{-1}\frac{7}{24} = \cos^{-1}\frac{7}{25} \qquad \left[\because \cos^{-1}\frac{7}{25} = x\right]$$

$$\therefore \quad \cot\left(\cos^{-1}\frac{7}{25}\right) = \cot\left(\cot^{-1}\frac{7}{24}\right) \qquad \left[\because \cot^{-1}\frac{7}{24} = \cos^{-1}\frac{7}{25}\right]$$

$$= \frac{7}{24}$$

Question 1.33:

The value of $\tan\left(\dfrac{1}{2}\cos^{-1}\dfrac{2}{\sqrt{5}}\right)$ is

(a) $2+\sqrt{5}$

(b) $\sqrt{5}-2$

(c) $\dfrac{\sqrt{5}+2}{2}$

(d) $5+\sqrt{2}$

Solution 1.33: (b)

We have, $\tan\left(\dfrac{1}{2}\cos^{-1}\dfrac{2}{\sqrt{5}}\right)$

Let us suppose, $\dfrac{1}{2}\cos^{-1}\dfrac{2}{\sqrt{5}} = \theta$

$$\Rightarrow \cos^{-1}\frac{2}{\sqrt{5}} = 2\theta$$

$$\Rightarrow \cos 2\theta = \frac{2}{\sqrt{5}}$$

$$\Rightarrow 1-2\sin^2\theta = \frac{2}{\sqrt{5}} \qquad \left[\because \cos 2\theta = 1-2\sin^2\theta\right]$$

$$\Rightarrow 2\sin^2\theta = 1-\frac{2}{\sqrt{5}}$$

$$\Rightarrow \sin^2\theta = \frac{1}{2}-\frac{1}{\sqrt{5}}$$

$$\Rightarrow \sin\theta = \sqrt{\frac{1}{2}-\frac{1}{\sqrt{5}}}$$

Now, $\cos^2\theta = 1 - \sin^2\theta$

$$\Rightarrow \cos^2\theta = 1-\frac{1}{2}+\frac{1}{\sqrt{5}}$$

$$\Rightarrow \cos^2\theta = \frac{1}{2}+\frac{1}{\sqrt{5}}$$

$$\Rightarrow \cos\theta = \sqrt{\frac{1}{2}+\frac{1}{\sqrt{5}}}$$

We know that, $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$$\Rightarrow \tan\theta = \sqrt{\frac{\frac{1}{2}-\frac{1}{\sqrt{5}}}{\frac{1}{2}+\frac{1}{\sqrt{5}}}}$$

$$\Rightarrow \tan\theta = \sqrt{\frac{\sqrt{5}-2}{\sqrt{5}+2}}$$

$$\Rightarrow \tan\theta = \sqrt{\frac{\sqrt{5}-2}{\sqrt{5}+2}\cdot\frac{\sqrt{5}-2}{\sqrt{5}-2}}$$

$$\Rightarrow \tan\theta = \sqrt{\frac{\left(\sqrt{5}-2\right)^2}{5-4}}$$

$$\Rightarrow \tan\theta = \sqrt{5}-2$$

$$\Rightarrow \theta = \tan^{-1}\left(\sqrt{5}-2\right)$$

$$\Rightarrow \theta = \tan^{-1}\left(\sqrt{5}-2\right)$$

$$\Rightarrow \tan^{-1}\left(\sqrt{5}-2\right) = \frac{1}{2}\cos^{-1}\frac{2}{\sqrt{5}}$$

$$\therefore\ \tan\left(\frac{1}{2}\cos^{-1}\frac{2}{\sqrt{5}}\right) = \tan\left[\tan^{-1}\left(\sqrt{5}-2\right)\right] = \sqrt{5}-2$$

Question 1.34:

If $|x| \le 1$, then $2\tan^{-1}x + \sin^{-1}\left(\frac{2x}{1+x^2}\right)$ is equal to

(a) $4\tan^{-1}x$

(b) 0

(c) $\frac{\pi}{2}$

(d) π

Solution 1.34: (a)

We have, $2\tan^{-1}x + \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

$$= 2\tan^{-1}x + 2\tan^{-1}x \qquad \left[\because 2\tan^{-1}x = \sin^{-1}\frac{2x}{1+x^2}\right]$$

$$= 4\tan^{-1}x$$

Question 1.35:

If $\cos^{-1}\alpha + \cos^{-1}\beta + \cos^{-1}\gamma = 3\pi$, then $\alpha(\beta + \gamma) + \beta(\gamma + \alpha) + \gamma(\alpha + \beta)$ equals to

(a) 0

(b) 1

(c) 6

(d) 12

Solution 1.35: (c)

The domain of $\cos^{-1}x$ is $[0, \pi]$

We are given that, $\cos^{-1}\alpha + \cos^{-1}\beta + \cos^{-1}\gamma = 3\pi$

Which is possible only when $\alpha = \beta = \gamma = \cos \pi$ or -1

Now, $\alpha(\beta + \gamma) + \beta(\gamma + \alpha) + \gamma(\alpha + \beta)$

$= -1(-1-1) - 1(-1-1) - 1(-1-1)$

$= 2 + 2 + 2$

$= 6$

Question 1.36:

The number of real solutions of the equation

$$\sqrt{1+\cos 2x} = \sqrt{2}\cos^{-1}(\cos x) \text{ in } \left[\frac{\pi}{2}, \pi\right] \text{ is}$$

(a) 0

(b) 1

(c) 2

(d) ∞

Solution 1.36: (a)

We are given that, $\sqrt{1+\cos 2x} = \sqrt{2}\cos^{-1}(\cos x), \left[\frac{\pi}{2}, \pi\right]$

$\Rightarrow \sqrt{2\cos^2 x} = \sqrt{2}\cos^{-1}(\cos x)$ $\quad \left[\because 1+\cos 2x = 2\cos^2 x\right]$

$\Rightarrow \sqrt{2}\cos x = \sqrt{2}\cos^{-1}(\cos x)$

$\Rightarrow \cos x = \cos^{-1}(\cos x)$

$\Rightarrow \cos x = x$ $\quad [\because \cos^{-1}(\cos x) = x]$

which is not true for any real value of *x*.

Hence, there is no solution possible for the given equation.

Question 1.37:

If $\cos^{-1} x > \sin^{-1} x$, then

(a) $\frac{1}{\sqrt{2}} < x \le 1$

(b) $0 \le x < \frac{1}{\sqrt{2}}$

(c) $-1 \le x < \frac{1}{\sqrt{2}}$

(d) $x > 0$

Solution 1.37: (c)

We have, $\cos^{-1} x > \sin^{-1} x$

$$\Rightarrow \frac{\pi}{2} - \sin^{-1} x > \sin^{-1} x \qquad \left[\begin{array}{l} \because \sin^{-1} x + \cos^{-1} x = \frac{\pi}{2} \\ \Rightarrow \cos^{-1} x = \frac{\pi}{2} - \sin^{-1} x \end{array}\right]$$

$$\Rightarrow \frac{\pi}{2} > 2\sin^{-1} x$$

$$\Rightarrow \frac{\pi}{4} > \sin^{-1} x$$

$$\Rightarrow \sin\left(\frac{\pi}{4}\right) > x$$

$$\Rightarrow \frac{1}{\sqrt{2}} > x$$

$$\Rightarrow -1 \le x < \frac{1}{\sqrt{2}} \qquad \left[\because \sin^{-1} x \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]\right]$$