1. **Matrix** A matrix is an ordered rectangular array of numbers or functions. The numbers or functions are called the elements or the entries of the matrix.

2. **Order of a Matrix** If a matrix has m rows and n columns, then its order is written as $m \times n$. If a matrix has order $m \times n$, then it has mn elements.

   In general, a $m \times n$ matrix has the following rectangular array:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ a_{m1} & a_{m2} & a_{m3} & \dots & a_{mn} \end{bmatrix}_{m \times n} \quad \text{or} \quad A = [a_{ij}]_{m \times n},\ 1 \le i \le m, i \le j \le n; i, j \in N$$

NOTE We shall consider only those matrices, whose elements are real numbers or functions taking real values.

3. **Types of Matrices**

(i) **Column Matrix** A matrix which has only one column, is called a column matrix.

e.g. $\begin{bmatrix} 1 \\ 0 \\ -5 \end{bmatrix}$

In general, $A = [a_{ij}]_{m \times 1}$ is a column matrix of order $m \times 1$.

(ii) **Row Matrix** A matrix which has only one row, is called a row matrix.
e.g. $[1 \ \ 5 \ \ 9]$
In general, $A = [a_{ij}]_{1 \times n}$ is a row matrix of order $1 \times n$

(iii) **Square Matrix** A matrix which has equal number of rows and columns, is called a square matrix. e.g. $\begin{bmatrix} 3 & -1 \\ 5 & 2 \end{bmatrix}$

In general, $A = [a_{ij}]_{m \times m}$ is a square matrix of order m.

NOTE If $A = [a_{ij}]$ is a square matrix of order n, then elements $a_{11}, a_{22}, a_{33}, \ldots, a_{nn}$ are said to be constitute the diagonal of the matrix A.

4. **Equality of Matrices** Two matrices A and B are said to be equal, if

(i) order of A and B are same.

(ii) corresponding elements of A and B are same i.e. $a_{ij} = b_{ij}$, $\forall\, i$ and j.

e.g. $\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$ and $\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$ are equal matrices, but $\begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix}$ are not equal matrices.

5. **Operations on Matrices**

Between two or more than two matrices, following operations are defined below:

(i) **Addition and Subtraction of Matrices** Addition and subtraction of two matrices is defined, if order of both the matrices are same.

Addition of Matrix

If $A = [a_{ij}]_{m \times n}$ and $B = [b_{ij}]_{m \times n},$ then $A + B = [a_{ij} + b_{ij}]_{m \times n}$, $1 \le i \le m$, $1 \le j \le n$.

Subtraction of Matrix

If $A = [a_{ij}]_{m \times n}$ and $B = [b_{ij}]_{m \times n}$, then $A - B = [a_{ij} - b_{ij}]_{m \times n}$, $1 \le i \le m$, $1 \le j \le n$.

Properties of Multiplication of Matrices

(a) **Non-commutativity** Matrix multiplication is not commutative i.e. if AB and BA are both defined, then it is not necessary that $AB \neq BA$.

(b) **Associative law** For three matrices A, B and C, if multiplication is defined, then $A(BC) = (AB)C$.

(c) **Multiplicative identity** For every square matrix A, there exist an identity matrix of same order such that $IA = AI = A$.

NOTE For $A_{m \times m}$, there is only one multiplicative identity I_m.

(d) **Distributive law** For three matrices A, B and C,

- $A(B+C) = AB + AC$
- $(A+B)C = AC + BC$

whenever, both sides of equality are defined.

NOTE If A and B are two non-zero matrices, then their product may be a zero matrix.

e.g. Suppose $A = \begin{bmatrix} 0 & -1 \\ 0 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 5 \\ 0 & 0 \end{bmatrix}$, then $AB = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.

1. **Transpose of a Matrix** Let A be any matrix. Then, the matrix obtained by interchanging its rows and columns, is called the transpose of matrix A.

e.g. Let $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \\ 3 & 5 \end{bmatrix}$, then the transpose of $A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 3 & 5 \end{bmatrix}$.

Transpose of A is written as A' or A^T. Also, if matrix $A = [a_{ij}]_{m \times n}$, then its transpose is $A^T = [a_{ji}]_{n \times m}$.

Properties of Transpose of a Matrix Let A and B be any two matrices. Then, we have

(i) $(A')' = A$

(ii) $(A \pm B)' = A' \pm B'$

(iii) $(AB)' = B'A'$

(iv) $(kA)' = k \cdot A'$, where k is any constant.

(v) $(-A)' = -A'$

2. **Symmetric Matrix** A square matrix $A = [a_{ij}]$ is said to be a symmetric matrix, if $A' = A$ i.e. if $a_{ji} = a_{ij}, \forall\, i, j$.

e.g. $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ is a symmetric matrix as $A' = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} = A$.

NOTE Corresponding non-diagonal elements of symmetric matrix are equal.

3. **Skew-symmetric Matrix** A square matrix $A = [a_{ij}]$ is said to be a skew-symmetric matrix, if $A' = -A$ i.e. if $a_{ji} = -a_{ij}, \forall\; i, j$.

e.g. $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ is a skew-symmetric matrix as $A' = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = -A$.

NOTE Diagonal elements of a skew-symmetric matrix are always zero.

4. For a square matrix A with real number entries, $A + A'$ is a symmetric matrix and $A - A'$ is a skew-symmetric matrix.

5. Any square matrix can be expressed as the sum of a symmetric and a skew-symmetric matrices.

i.e. Let A be a square matrix, then it can be written as $A = \frac{1}{2}(A + A') + \frac{1}{2}(A - A')$.