1. **Rate of Change of Quantities** Let $y = f(x)$ be a function of x. Then, $\frac{dy}{dx}$ represents the rate of change of y with respect to x. Also, $\left[\frac{dy}{dx}\right]_{x=x_0}$ represents the rate of change of y with respect to x at $x = x_0$.

2. If two variables x and y are varying with respect to another variable t, i.e. $x = f(t)$ and $y = g(t)$, then

$$\frac{dy}{dx} = \frac{dy}{dt} \Big/ \frac{dx}{dt}, \text{ where } \frac{dx}{dt} \neq 0 \text{ (by chain rule)}$$

In other words, the rate of change of y with respect to x can be calculated using the rate of change of y and that of x both with respect to t.

NOTE $\frac{dy}{dx}$ is positive, if y increases as x increases and it is negative, if y decreases as x increases.

3. **Marginal Cost** Marginal cost represents the instantaneous rate of change of the total cost at any level of output.

   If $C(x)$ represents the cost function for x units produced, then marginal cost (MC) is given by

$$\mathrm{MC} = \frac{d}{dx}\{C(x)\}$$

4. **Marginal Revenue** Marginal revenue represents the rate of change of total revenue with respect to the number of items sold at an instant.

   If $R(x)$ is the revenue function for x units sold, then marginal revenue (MR) is given by

$$\mathrm{MR} = \frac{d}{dx}\{R(x)\}$$

5. Let I be an open interval contained in the domain of a real valued function f. Then, f is said to be

(i) **increasing** on I, if $x_1 < x_2$ in $I \Rightarrow f(x_1) \leq f(x_2), \forall\, x_1, x_2 \in I$.

(ii) **strictly increasing** on I, if $x_1 < x_2$ in $I \Rightarrow f(x_1) < f(x_2), \forall\, x_1, x_2 \in I$.

(iii) **decreasing** on I, if $x_1 < x_2$ in $I \Rightarrow f(x_1) \geq f(x_2), \forall\, x_1, x_2 \in I$.

(iv) **strictly decreasing** on I, if $x_1 < x_2$ in $I \Rightarrow f(x_1) > f(x_2), \forall\, x_1, x_2 \in I$.

6. Let x_0 be a point in the domain of definition of a real valued function f, then f is said to be increasing, strictly increasing, decreasing or strictly decreasing at x_0, if there exist an open interval I containing x_0 such that f is increasing, strictly increasing, decreasing or strictly decreasing, respectively in I.

NOTE If for a given interval $I \subseteq R$, function f increase for some values in I and decrease for other values in I, then we say function is neither increasing nor decreasing.

7. Let f be continuous on $[a, b]$ and differentiable on the open interval (a, b). Then,

   (i) f is increasing in $[a, b]$, if $f'(x) > 0$ for each $x \in (a, b)$.

   (ii) f is decreasing in $[a, b]$, if $f'(x) < 0$ for each $x \in (a, b)$.

   (iii) f is a constant function in $[a, b]$, if $f'(x) = 0$ for each $x \in (a, b)$.

NOTE (i) f is strictly increasing in (a,b), if $f'(x) > 0$ for each $x \in (a,b)$.

(ii) f is strictly decreasing in (a,b), if $f'(x) < 0$ for each $x \in (a,b)$.

8. **Monotonic Function** A function which is either increasing or decreasing in a given interval I, is called monotonic function.

9. **Approximation** Let $y = f(x)$ be any function of x. Let Δx be the small change in x and Δy be the corresponding change in y.

   i.e. $\Delta y = f(x + \Delta x) - f(x)$. Then, $dy = f'(x)\,dx$ or $dy = \dfrac{dy}{dx} \cdot \Delta x$

   is a good approximation of Δy, when $dx = \Delta x$ is relatively small and we denote it by $dy \approx \Delta y$.

1. **Binary Operation** A binary operation * on set X is a function $*: X \times X \to X$. It is denoted by $a * b$.
2. **Commutative Binary Operation** A binary operation * on set X is said to be commutative, if $a * b = b * a, \forall\, a, b \in X$.
3. **Associative Binary Operation** A binary operation * on set X is said to be associative, if $a * (b * c) = (a * b) * c, \forall\, a, b, c \in X$.

NOTE For a binary operation, we can neglect the bracket in associative property. But in absence of associative property, we cannot neglect the bracket.