NCERT Exemplar Solutions

Class 12 – Mathematics

Chapter 1 – Relations and Functions

Objective Type Questions

Question 1.28:

Let T be the set of all triangles in the Euclidean plane and let a relation R on T be defined as aRb, if a is congruent to $b, \forall a, b \in T$ Then, R is

(a) reflexive but not transitive

(b) transitive but not symmetric

(c) equivalence

(d) None of these

Solution 1.28: (c)

Consider that aRb, if a is congruent to $b, \forall a, b \in T$.

Since, every triangle is congruent to itself.

Hence, R is reflexive. ...(1)

Let $aRb \Rightarrow a \cong b$

$\Rightarrow b \cong a \Rightarrow b \cong a$

$\Rightarrow bRa$

If first triangle is congruent to second triangle, then the second triangle is also congruent to first triangle.

Hence, R is symmetric. ...(2)

Let aRb and bRc

$\Rightarrow$ $a \cong b$ and $b \cong c$

$\Rightarrow$ $a \cong c \Rightarrow aRc$

Hence, R is transitive. ...(3)

Therefore, R is equivalence relation.

Question 1.29:

Consider the non-empty set consisting of children in *a* family and *a* relation R defined as aRb, if *a* is brother of *b*. Then, R is

(a) symmetric but not transitive

(b) transitive but not symmetric

(c) neither symmetric nor transitive

(d) both symmetric and transitive

Solution 1.29: (b)

We are given that a relation R defined $aRb \Rightarrow a$ is brother of b

$aRa \Rightarrow a$ is brother of a, which is not true.

Hence, R is not reflexive.

$aRb \Rightarrow a$ is brother of b.

This does not mean b is also *a* brother of a and b can be *a* sister of a.

Hence, it is not symmetric.

$aRb \Rightarrow a$ is brother of b

and $bRc \Rightarrow b$ is *a* brother of c.

So, a is brother of c.

Hence, R is transitive.

Question 1.30:

The maximum number of equivalence relations on the set $A = \{1, 2, 3\}$ is

(*a*) 1

(*b*) 2

(*c*) 3

(*d*) 5

Solution 1.30: (d)

We are given that, $A = \{1, 2, 3\}$

The maximum number of equivalence relations on the set $A = \{1, 2, 3\}$ is 5, which are given below:

$R_1 = \{(1,1), (2,2), (3,3)\}$

$R_2 = \{(1,1), (2,2), (3,3), (1,2), (2,1)\}$

$R_3\{(1,1),(2,2),(3,3),(1,3),(3,1)\}$

$R_4 = \{(1,1),(2,2),(3,3),(2,3),(3,2)\}$

$\therefore$ Maximum number of equivalence relation en the set $a = \{1,2,3\} = 5$

Question 1.31:

If a relation R on the set $\{1,2,3\}$ be defined by $R = \{(1,2)\}$, then R is

(a) reflexive

(b) transitive

(c) symmetric

(d) None of these

Solution 1.31: (b)

We are given a relation R on the set $\{1,2,3\}$ be defined by $R = \{(1,2)\}$

It is clear that R is transitive.

Question 1.32:

Let us define a relation R in R as $aRb \; if \; a \geq b$. Then, R is

(a) an equivalence relation

(b) reflexive, transitive but not symmetric

(c) symmetric, transitive but not reflexive

(d) neither transitive true nor reflexive but symmetric

Solution 1.32: (b)

We are given that, aRb if $a \geq b$

$\Rightarrow aRa \Rightarrow a \geq a$ which is true.

For relation aRb to be symmetric, we must have $a \geq b$ and $b \geq a$ which can't be possible.

Hence, R is not symmetric.

For relation aRb to be transitive, we must have aRb and bRc

$\Rightarrow a \geq b$ and $b \geq c$

$\Rightarrow a \geq c$

Hence, R is transitive

Question 1.33:

If $A = \{1,2,3\}$ and consider the relation $R = \{(1,1),(2,2),(3,3),(1,2),(2,3),(1,3)\}$

Then, R is

(a) reflexive but not symmetric

(b) reflexive but not transitive

(c) symmetric and transitive

(d) neither symmetric nor transitive

Solution 1.33: (a)

We are given that, $A = \{1, 2, 3\}$ and $R = \{(1,1), (2, 2), (3,3), (1, 2), (2, 3), (1, 3)\}$

Since, $(1,1), (2, 2), (3, 3) \in R$

Hence, R is reflexive

Now, $(1, 2) \in R$ but $(2,1) \notin R$

Hence, R is not symmetric

Since, $(1, 2) \in R$ and $(2, 3) \in R$

Also, $(1, 3) \in R$

Hence, R is transitive.

Question 1.34:

The identity element for the binary operation * defined on $Q - \{0\}$ as $a * b = \frac{ab}{2} \forall a, b \in Q - \{0\}$ *is*

(a) 1

(b) 0

(c) 2

(d) None of these

Solution 1.34: (c)

We are given that, $a * b = \frac{ab}{2}, \forall a, b \in Q - \{0\}$

Let us suppose e be the identity element for *.

$\therefore\ a * e = \frac{ae}{2}$

$\Rightarrow a = \frac{ae}{2}$

$\Rightarrow e = 2$

Question 1.35:

If the set a contains 5 elements and the set b contains 6 elements, then the number of one-one and onto mappings from a to b is

(a) 720

(b) 120

(c) 0

(d) None of these

Solution 1.35: (c)

Since, the number of elements in B is more than A.

Hence, there cannot be any one-one and onto mapping from A to B.

Question 1.36:

If $a = \{1, 2, 3,..., n\}$ and $b = \{a, b\}$. Then, the number of surjections from a into b is

(a) ${}^n p_2$

(b) $2^n - 2$

(c) 2^n -1

(d) None of these

Solution 1.36: (d)

We are given that, $a = \{1,2, 3\ldots.n\}$ and $b = \{a, b\}$.

If a and b are two non-empty finite sets containing m and n elements respectively, then the number of surjection from a into b is given by

${}^n C_m \times m!, \text{if } n \geq m$

$0, \text{if } n < m$

Here, $m = 2$

Hence, the number of surjection from a into b is ${}^n C_2 \times 2! = \frac{n!}{2!(n-2)!} \times 2!$

Question 1.37:

If f:$R \to R$ be defined by $f(x) = \frac{1}{x,}$, $\forall\ x \in R$ Then, f is

(a) one-one

(b) onto

(c) bijective

(*d*) *f* is not defined

Solution 1.37: (d)

We are given that, $f(x) = \frac{1}{x}, \forall x \in R$

If $x = 0$, then $f(x) = \infty$ or not defined.

Hence, $f(x)$ is not defined function.

Question 1.38:

If $f: R \to R$ be defined by f(x)) = $3x^2$-5 and $g: R \to R$ by $g(x) = \frac{x}{x^2+1}$. Then, gof is

(*a*) $\frac{3x^2-5}{9x^4-30x^2+26}$

(*b*) $\frac{3x^2-5}{9x^4-6x^2+26}$

(*c*) $\frac{3x^2}{x^4+2x^2-4}$

(*d*) $\frac{3x^2}{9x^4+30x^2-2}$

Solution 1.37: (a)

We are given that, $f(x) = 3x^2-5$ and $g(x) = \frac{x}{x^2+1}$

Now, $gof = g\{f(x)\}$

$= g(3x^2-5) \qquad \left[\because f(x) = 3x^2-5\right]$

$= \frac{3x^2-5}{(3x^2-5)^2+1}$

$= \frac{3x^2-5}{9x^4-30x^2+25+1}$

$= \frac{3x^2-5}{9x^4-30x^2+26}$

Question 1.39:

Which of the following functions from Z into Z are bijections?

(a) $f(x) = x^3$

(b) $f(x) = x + 2$

(c) $f(x) = 2x + 1$

(d) $f(x) = x^2 + 1$

Solution 1.39: (b)

Consider, the second option i.e., $f(x) = x + 2$

Now, $f(x_1) = f(x_2)$

$\Rightarrow x_1 + 2 = x_2 + 2$

$\Rightarrow x_1 = x_2$

Hence, $f(x) = x + 2$ is one-one function.

Now, let us suppose, $y = x + 2$

$x = y - 2 \in Z, \forall y \in x$

Hence, $f(x)$ is one-one and onto.

Question 1.40:

If $f: R \rightarrow R$ be the functions defined by $f(x) = x^3 + 5$, then $f^{-1}(x)$ is

(a) $(x+5)^{\frac{1}{3}}$

(b) $(x-5)^{\frac{1}{3}}$

(c) $(5-x)^{\frac{1}{3}}$

(d) $5-x$

Solution 1.40: (b)

We are given that, $f(x) = x^3 + 5$

Let us suppose, $y = x^3 + 5$

$\Rightarrow x^3 = y - 5$

$\Rightarrow x = (y-5)^{\frac{1}{3}}$

$\Rightarrow f^{-1}(y) = (y-5)^{\frac{1}{3}} \qquad \left[\begin{array}{l} \because f(x) = y \\ \Rightarrow x = f^{-1}(y) \end{array}\right]$

$\Rightarrow f^{-1}(x) = (x-5)^{\frac{1}{3}}$

Question 1.41:

If f:$A \rightarrow b$ and g :$B \rightarrow c$ be the bijective functions, then $(gof)^{-1}$ is

(a) $f^{-1}og^{-1}$

(b) fog

(c) $g^{-1}of^{-1}$

(d) gof

Solution 1.41: (a)

We are given that, $f : A \rightarrow B$ and $g : B \rightarrow C$ are the bijective functions

$\therefore (gof)^{-1} = f^{-1}og^{-1}$

Question 1.42:

If $f : R - \left\{\frac{3}{5}\right\} \rightarrow R$ be defined by $f(x) = \frac{3x+2}{5x-3}$, the

(**a**) $f^{-1}(x) = f(x)$

(b) $(fof)x = -x$

(**c**) $g^{-1}of^{-1}$

(d) gof

Solution 1.42: (d)

We are given that, $f(x) = \frac{3x+2}{5x-3}$

Let us suppose, $y = \frac{3x+2}{5x-3}$

$$\Rightarrow y(5x-3) = 3x+2$$
$$\Rightarrow 5xy - 3y = 3x + 2$$
$$\Rightarrow x(3-5y) = -3y-2$$
$$\Rightarrow x = \frac{3y+2}{5y-3}$$
$$\Rightarrow f^{-1}(y) = \frac{3y+2}{5y-3} \qquad \begin{bmatrix} \because f(x) = y \\ \Rightarrow x = f^{-1}(y) \end{bmatrix}$$
$$\Rightarrow f^{-1}(x) = \frac{3x+2}{5x-3}$$
$$\therefore f^{-1}(x) = f(x)$$

Question 1.43:

If: $[0,1] \to [0,1]$ be defined by $\begin{cases} x, \\ 1-x, \end{cases}$ if x is rational then $(fof)x$ is

If x is irrational

(*a*) constant

(*b*) 1 + *x*

(*c*) *x*

(*d*) None of these

Solution 1.43: (c)

We are given that, $f:[0,1] \to [0,1]$ be defined by $f(x) = \begin{cases} x, \text{ if } x \text{ is rational} \\ 1-x, \text{ if } x \text{ is irrational} \end{cases}$

Now, $(fof)x = f(f(x))$

$= x$

Question 1.44:

If $f:[2,\infty) \to R$ be the function defined by $f(x) = x^2 - 4x + 5$, then the range of *f* is

(*a*) R

(*b*) $[1, \infty)$

(c) $[4, \infty)$

(d) $[5, \infty)$

Solution 1.44: (b)

We are given that, $f(x) = x^2 - 4x + 5$

Let us suppose, $y = x^2 - 4x + 5$

$\Rightarrow y = x^2 - 4x + 4 + 1 = (x-2)^2 + 1$

$\Rightarrow (x-2)^2 = y - 1$

$\Rightarrow x - 2 = \sqrt{y-1}$

$\Rightarrow x = 2 + \sqrt{y-1}$

For the function to be real, the term inside the square root must be greater than or equal to zero.

$\therefore y - 1 \geq 0$

$\Rightarrow y \geq 1$

Therefore, the range of *f* is $(1, \infty)$.

Question 1.45:

If $f : N \to R$ be the function defined by $f(x) = \frac{2x-1}{2}$ and $g : Q \to R$ be another function defined by $g(x) = x + 2$. Then, $(gof)\frac{3}{2}$ is

(a) 1

(b) 1

(c) $\frac{7}{2}$

(d)None of these

Solution 1.45: (d)

We are given that, $f(x) = \frac{2x-1}{2}$ and $g(x) = x + 2$

Now, $(gof)(x) = g\{f(x)\}$

$$\therefore (gof)\frac{3}{2} = g\left[f\left(\frac{3}{2}\right)\right]$$
$$= g\left(\frac{2\times\frac{3}{2}-1}{2}\right)$$
$$= g(1)$$
$$= 1+2$$
$$= 3$$

Question 1.46:

If $f : R \to R$ be defined by $f(x) = \begin{cases} 2x : x > 3 \\ x^2 : 1 < x \le 3 \\ 3x : x \le 1 \end{cases}$ Then, $f(-1) + f(2) + f(4)$ is

(*a*) 9

(*b*) 14

(*c*) 5

(*d*) None of these

Solution 1.46: (a)

We are given that, $f(x) = \begin{cases} 2x : x > 3 \\ x^2 : 1 < x \le 3 \\ 3x : x \le 1 \end{cases}$

Now, $f(-1) + f(2) + f(4) = 3(-1) + (2)^2 + 2\times 4$

$= -3+4+8$

$= 9$

Question 1.47:

If $f : R \to R$ *be* given by $f(x) = \tan x$, then $f^{-1}(1)$ is

(*a*) $\dfrac{\pi}{4}$

(*b*) $\left\{n\pi + \frac{\pi}{4} : n \in Z\right\}$

(*c*) Does not exist

(*d*) None of these

Solution 1.47: (a)

We are given that, $f(x) = \tan x$

Let us suppose, $y = \tan x$

$\Rightarrow \; x = \tan^{-1} y$

$$\Rightarrow f^{-1}(y) = \tan^{-1} y \qquad \left[\begin{array}{l} \because f(x) = y \\ \Rightarrow x = f^{-1}(y) \end{array}\right]$$

$\Rightarrow f^{-1}(x) = \tan^{-1} x$

$\Rightarrow f^{-1}(1) = \tan^{-1} 1$

$$= \tan^{-1} \tan \frac{\pi}{4} = \frac{\pi}{4} \qquad \left[\because \tan \frac{\pi}{4} = 1\right]$$