NCERT Exemplar Solutions

Class 12 – Mathematics

Chapter 7 – Integrals

Objective Type Questions

Question 1.48: $\int \frac{\cos 2x - \cos 2\theta}{\cos x - \cos \theta} dx$ is equal to

(a) $2(\sin x + x\cos\theta) + C$

(b) $2(\sin x - x\cos\theta) + C$

(c) $2(\sin x + 2x\cos\theta) + C$

(d) $2(\sin x - 2x\cos\theta) + C$

Solution 1.48: (a)

Let $I = \int \frac{\cos 2x - \cos 2\theta}{\cos x - \cos \theta} dx$

$$= \int \frac{2\cos^2 x - 1 - 2\cos^2\theta + 1}{\cos x - \cos\theta} dx \quad \left[\because \cos 2A = 2\cos^2 A - 1\right]$$

$$= 2\int \frac{\cos^2 x - \cos^2\theta}{\cos x - \cos\theta} dx$$

$$= 2\int \frac{(\cos x + \cos\theta)(\cos x - \cos\theta)}{\cos x - \cos\theta} dx \quad \left[\because A^2 - B^2 = (A+B)(A-B)\right]$$

$$= 2\int (\cos x + \cos\theta)\, dx$$

$$= 2(\sin x + x\cos\theta) + C \qquad \left[\because \int \cos x\, dx = \sin x + C.\right]$$

Question 1.49: $\frac{dx}{\sin(x-a)\sin(x-b)}$ is equal to

(a) $\sin(b-a)\log\left|\frac{\sin(x-b)}{\sin(x-a)}\right| + C$

(b) $\cos ec(b-a)\log\left|\dfrac{\sin(x-b)}{\sin(x-a)}\right|+C$

(c) $\cos ec(b-a)\log\left|\dfrac{\sin(x-b)}{\sin(x-a)}\right|+C$

(d) $\sin(b-a)\log\left|\dfrac{\sin(x-b)}{\sin(x-a)}\right|+C$

Solution 1.49: (c)

Let $I=\int\dfrac{dx}{\sin(x-a)\sin(x-b)}$

$$=\frac{1}{\sin(b-a)}\int\frac{\sin(b-a)}{\sin(x-a)\sin(x-b)}dx$$

$$=\frac{1}{\sin(b-a)}\int\frac{\sin(x-a-x+b)}{\sin(x-a)\sin(x-b)}dx$$

$$=\frac{1}{\sin(b-a)}\int\frac{\sin\{(x-a)-(x-b)\}}{\sin(x-a)\sin(x-b)}dx$$

$$=\frac{1}{\sin(b-a)}\int\frac{\sin(x-a)\cos(x-b)-\cos(x-a)\sin(x-b)}{\sin(x-a)\sin(x-b)}dx$$

$$=\frac{1}{\sin(b-a)}\int\frac{\sin(x-a)\cos(x-b)}{\sin(x-a)\sin(x-b)}-\frac{\cos(x-a)\sin(x-b)}{\sin(x-a)\sin(x-b)}dx$$

$$=\frac{1}{\sin(b-a)}\int\frac{\cos(x-b)}{\sin(x-b)}-\frac{\cos(x-a)}{\sin(x-a)}dx$$

$$=\frac{1}{\sin(b-a)}\left[\log\left|\sin(x-b)\right|-\log\left|\sin(x-a)\right|\right]+C$$

$$=\operatorname{cosec}(b-a)\log\left|\frac{\sin(x-b)}{\sin(x-a)}\right|+C$$

Question 1.50: $\int\tan^{-1}\sqrt{x}\,dx$ is equal to

(a) $(x+1)\tan^{-1}\sqrt{x}-\sqrt{x}+C$

(b) $x\tan^{-1}\sqrt{x}-\sqrt{x}+C$

(c) $\sqrt{x}-x\tan^{-1}\sqrt{x}+C$

(d) $\sqrt{x}-(x+1)\tan^{-1}\sqrt{x}+C$

Solution 1.50: (a)

Let $I=\int \underset{II}{1}\cdot \underset{I}{\tan^{-1}\sqrt{x}}\,dx$

$$=\tan^{-1}\sqrt{x}\cdot x-\int \frac{1}{(1+x)}\cdot\frac{1}{2\sqrt{x}}x\,dx \qquad \left[\because \int \mathrm{I}\cdot\mathrm{II}\,dx=\mathrm{I}\int \mathrm{II}dx-\int\left(\frac{d}{dx}\mathrm{I}\int \mathrm{II}dx\right)dx\right]$$

$$=x\tan^{-1}\sqrt{x}-\frac{1}{2}\int\frac{1}{\sqrt{x}(1+x)}x\,dx$$

$$=x\tan^{-1}\sqrt{x}-\frac{1}{2}\int\frac{\sqrt{x}}{(1+x)}dx$$

Put $x=t^2 \Rightarrow dx=2t\,dt$

$$\therefore I=x\tan^{-1}\sqrt{x}-\int\frac{t^2}{1+t^2}dt$$

$$=x\tan^{-1}\sqrt{x}-\int\frac{t^2+1-1}{1+t^2}dt$$

$$=x\tan^{-1}\sqrt{x}-\int 1-\frac{1}{1+t^2}dt$$

$$=x\tan^{-1}\sqrt{x}-\int\left(1-\frac{1}{1+t^2}\right)dt$$

$$=x\tan^{-1}\sqrt{x}-t+\tan^{-1}t+C \qquad \left[\because \int\frac{1}{a+x^2}dx=\tan^{-1}x+C\right]$$

$$=x\tan^{-1}\sqrt{x}-\sqrt{x}+\tan^{-1}\sqrt{x}+C \qquad \left[\begin{array}{l}\because t^2=x\\ \Rightarrow t=\sqrt{x}\end{array}\right]$$

$$=(x+1)\tan^{-1}\sqrt{x}-\sqrt{x}+C$$

Question 1.51: $\int \frac{x^9}{(4x^2+1)^6}dx$ is equal to

(a) $\frac{1}{5x}\left(4+\frac{1}{x^2}\right)^{-5}+C$

(b) $\frac{1}{5}\left(4+\frac{1}{x^2}\right)^{-5}+C$

(c) $\frac{1}{10x}(1+4)^{-5}+C$

(d) $\frac{1}{10}\left(\frac{1}{x^2}+4\right)^{-5}+C$

Solution 1.51: (d)

Let $I=\int \frac{x^9}{(4x^2+1)^6}dx=\int \frac{x^9}{x^{12}\left(4+\frac{1}{x^2}\right)^6}dx$

$=\int \frac{dx}{x^3\left(4+\frac{1}{x^2}\right)^6}$

Put $4+\frac{1}{x^2}=t$

$\Rightarrow \frac{-2}{x^3}dx=dt$

$\Rightarrow \frac{1}{x^3}dx=-\frac{1}{2}dt$

Substituting $4+\frac{1}{x^2}=t$ and $\frac{1}{x^3}dx=-\frac{1}{2}dt$ in I, we get

$\therefore I=-\frac{1}{2}\int \frac{dt}{t^6}=-\frac{1}{2}\left[\frac{t^{-6+1}}{-6+1}\right]+C$

$$= \frac{1}{10} t^{-5} + C$$

$$= \frac{1}{10}\left(4 + \frac{1}{x^2}\right)^{-5} + C \qquad \left[\because t = 4 + \frac{1}{x^2}\right]$$

Question 1.52: If $\int \frac{dx}{(x+2)(x^2+1)} = a\log\left|1+x^2\right| + b\tan^{-1}x + \frac{1}{5}\log\left|x+2\right| + C$, then

(a) $a = \frac{-1}{10}, b = \frac{-2}{5}$

(b) $a = \frac{1}{10}, b = -\frac{2}{5}$

(c) $a = \frac{-1}{10}, b = \frac{2}{5}$

(d) $a = \frac{1}{10}, b = \frac{2}{5}$

Solution 1.52: (c)

We are given that, $\int \frac{dx}{(x+2)(x^2+1)} = a\log\left|1+x^2\right| + b\tan^{-1}x + \frac{1}{5}\log\left|x+2\right| + C$

Let $I = \int \frac{dx}{(x+2)(x^2+1)}$

Use partial fraction form, $\frac{1}{(x-a)(x^2+bx+c)} = \frac{A}{(x-a)} + \frac{Bx+C}{(x^2+bx+c)}$

$$\frac{1}{(x+2)(x^2+1)} = \frac{A}{(x+2)} + \frac{Bx+C}{(x^2+1)}$$

$$1 = A(x^2+1) + (Bx+C)(x+2)$$

$$1 = Ax^2 + A + Bx^2 + 2Bx + Cx + 2C$$

$$1 = (A+B)x^2 + (2B+C)x + A + 2C$$

$$A+B=0, A+2C=1, 2B+C=0$$

Solving, we get

$A=\frac{1}{5}, B=-\frac{1}{5}$ and $C=\frac{2}{5}$

$$\therefore \int \frac{dx}{(x+2)(x^2+1)}=\frac{1}{5}\int \frac{1}{x+2}dx+\int \frac{-\frac{1}{5}x+\frac{2}{5}}{x^2+1}$$

$$=\frac{1}{5}\int \frac{1}{x+2}dx-\frac{1}{5}\int \frac{x}{1+x^2}dx+\frac{1}{5}\int \frac{2}{1+x^2}dx$$

$$=\frac{1}{5}\log|x+2|-\frac{1}{10}\log\left|1+x^2\right|+\frac{2}{5}\tan^{-1}x+C$$

Comparing $\frac{1}{5}\log|x+2|-\frac{1}{10}\log\left|1+x^2\right|+\frac{2}{5}\tan^{-1}x+C$ and $a\log\left|1+x^2\right|+b\tan^{-1}x+\frac{1}{5}\log|x+2|+C$, we get

$a=\frac{-1}{10}$ and $b=\frac{2}{5}$

Question 1.53: $\int \frac{x^3}{x+1}$ is equal to

(a) $x+\frac{x^2}{2}+\frac{x^3}{3}-\log|1-x|+C$

(b) $x+\frac{x^2}{2}-\frac{x^3}{3}-\log|1-x|+C$

(c) $x+\frac{x^2}{2}-\frac{x^3}{3}-\log|1+x|+C$

(d) $x-\frac{x^2}{2}+\frac{x^3}{3}-\log|1+x|+C$

Solution 1.53: (d)

Let $I=\int \frac{x^3}{x+1}dx$

We know that, $\frac{x^3}{x+1}$ is an improper fraction.

To convert it into proper fraction, we have to divide numerator by denominator.

After performing long division, we get

$$\frac{x^3}{x+1}=(x^2-x+1)-\frac{1}{(x+1)}$$

$$\therefore I=\int\left((x^2-x+1)-\frac{1}{(x+1)}\right)dx$$

$$=\frac{x^3}{3}-\frac{x^2}{2}+x-\log|x+1|+C$$

Question 1.54:

$\int\frac{x+\sin x}{1+\cos x}dx$ is equal to

(a) $\log|1+\cos x|+C$

(b) $\log|x+\sin x|+C$

(c) $x-\tan\frac{x}{2}+C$

(d) $x\cdot\tan\frac{x}{2}+C$

Solution 1.54: (d)

Let $I=\int\frac{x+\sin x}{1+\cos x}dx$

$$=\int\frac{x}{1+\cos x}dx+\int\frac{\sin x}{1+\cos x}dx$$

$$=\int\frac{x}{2\cos^2 x/2}dx+\int\frac{2\sin x/2\cos x/2}{2\cos^2 x/2}dx \qquad [\because \sin 2A=2\sin A\cos A \text{ and } 1+\cos 2A=2\cos^2 A]$$

$$=\frac{1}{2}\int \underset{I}{x}\,\underset{II}{\sec^2} x/2\,dx+\int\tan x/2\,dx$$

$$= \frac{1}{2}\left[2x\tan\frac{x}{2} - \int 2\tan\frac{x}{2}\,dx + C\right] + \int \tan\frac{x}{2}\,dx \qquad \left[\because \int \mathrm{I}\cdot\mathrm{II}\,dx = \mathrm{I}\int \mathrm{II}dx - \int\left(\frac{d}{dx}\mathrm{I}\int \mathrm{II}dx\right)dx\right]$$

$$= x\tan\frac{x}{2} - \int \tan\frac{x}{2}\,dx + \int \tan\frac{x}{2}\,dx + C$$

$$= x\tan\frac{x}{2} + C$$

Question 1.55:

If $\dfrac{x^3dx}{\sqrt{1+x^2}} = a(1+x^2)^{3/2} + b\sqrt{1+x^2} + C$, then

(a) $a = \dfrac{1}{3}, b = 1$

(b) $a = \dfrac{-1}{3}, b = 1$

(c) $a = \dfrac{-1}{3}, b = -1$

(d) $a = \dfrac{1}{3}, b = -1$

Solution 1.55: (d)

Let $I = \displaystyle\int \frac{x^3}{\sqrt{1+x^2}}\,dx$

$$= \int \frac{x^2 \cdot x}{\sqrt{1+x^2}}\,dx$$

Put $1 + x^2 = t^2$

$\Rightarrow 2x\,dx = 2t\,dt$

$\Rightarrow x\,dx = t\,dt$

Substituting $1+x^2 = t^2$ and $x\,dx = t\,dt$ in I, we get

$$I=\int \frac{t(t^2-1)}{t}dt \quad \left[\begin{array}{l}\because 1+x^2=t^2\\ \Rightarrow x^2=t^2-1\end{array}\right]$$

$$=\int\left(t^2-1\right)dt$$

$$=\frac{t^3}{3}-t+C$$

$$=\frac{1}{3}(1+x^2)^{3/2}-\sqrt{1+x^2}+C \qquad \left[\begin{array}{l}\because t^2=1+x^2\\ \Rightarrow t=\sqrt{1+x^2}\end{array}\right]$$

Comparing $\frac{1}{3}(1+x^2)^{3/2}-\sqrt{1+x^2}+C$ and $a(1+x^2)^{3/2}+b\sqrt{1+x^2}+C$, we get

$a=\frac{1}{3}$ and $b=-1$

Question 1.56: $\int_{-\pi/4}^{\pi/4}\frac{dx}{1+\cos 2x}$ is equal to

(a) 1

(b) 2

(c) 3

(d) 4

Solution 1.56: (a)

We have, $\int_{-\pi/4}^{\pi/4}\frac{dx}{1+\cos 2x}=\int_{-\pi/4}^{\pi/4}\frac{dx}{2\cos^2 x}$ $\quad\left[\because 1+\cos 2A=2\cos^2 A\right]$

$$=\frac{1}{2}\int_{-\pi/4}^{\pi/4}\sec^2 x\,dx$$

$$=\int_0^{\pi/4}\sec^2 x\,dx \qquad \int_{-a}^{a} f(x)dx=\begin{cases}\int_0^a f(x), \text{ if } f(x) \text{ is even}\\ 0, \text{ if } f(x) \text{ is odd}\end{cases}$$

$$=[\tan x]_0^{\pi/4}=1$$

$$=\tan\frac{\pi}{4}-\tan 0$$

$$=1$$

Question 1.57: $\int_0^{\pi/2} \sqrt{1-\sin 2x}dx$ is equal to

(a) $2\sqrt{2}$

(b) $2\left(\sqrt{2}+1\right)$

(c) 2

(d) $2\left(\sqrt{2}-1\right)$

Solution 1.57: (d)

Let $I = \int_0^{\pi/2} \sqrt{1-\sin 2x}dx$

$1-\sin 2x = 0$
$\Rightarrow \sin 2x = 1$
$\Rightarrow 2x = \sin\frac{\pi}{2}$
$\Rightarrow x = \frac{\pi}{4}$

Therefore, break the limits of the integral as given below.

$$I = \int_0^{\pi/4} \sqrt{1-\sin 2x}dx + \int_{\pi/4}^{\pi/2} \sqrt{1-\sin 2x}dx$$
$$= \int_0^{\pi/4} \sqrt{\cos^2 x + \sin^2 x - 2\sin x \cos x}dx + \int_{\pi/4}^{\pi/2} \sqrt{\cos^2 x + \sin^2 x - 2\sin x \cos x}dx$$
$$= \int_0^{\pi/4} \sqrt{(\cos x - \sin x)^2}dx + \int_{\pi/4}^{\pi/2} \sqrt{(\cos x - \sin x)^2}dx$$

$$= [\sin x + \cos x]_0^{\pi/4} + [\sin x + \cos x]_{\pi/4}^{\pi/2}$$

$$= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - 0 - 1 + \left(-0 - 1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)$$

$$= 2\sqrt{2} - 2 = 2(\sqrt{2}-1)$$

Question 1.58: $\int_0^{\pi/2} \cos x e^{\sin x} dx$ is equal to

(a) $e+1$

(b) $e-1$

(c) e

(d) – e

Solution 1.58: (b)

Let $I=\int \cos x e^{\sin x} dx$

Put $\sin x = t$

$\Rightarrow \cos x\, dx = dt$

$\therefore I=\int e^{t} dt$

$=e^{t}$

$=e^{\sin x} \qquad [\because t=\sin x]$

Now,

$$\int_0^{\pi/2} \cos x e^{\sin x} dx = \left[e^{\sin x}\right]_0^{\frac{\pi}{2}}$$

$$=e^{\sin\frac{\pi}{2}} - e^{\sin 0}$$

$$=e-1$$

Question 1.59: $\int \frac{x+3}{(x+4)^2} e^{x} dx$ is equal to

(a) $e^{x}\left(\frac{1}{x+4}\right)+C$

(b) $e^{-x}\left(\frac{1}{x+4}\right)+C$

(c) $e^{-x}\left(\frac{1}{x-4}\right)+C$

(d) $e^{2x}\left(\frac{1}{x-4}\right)+C$

Solution 1.59: (a)

Let $I=\int \frac{x+3}{(x+4)^2}e^x dx$

$$=\int \frac{e^x}{(x+4)}-\int \frac{e^x}{(x+4)^2}dx$$

$$=\int e^x\left(\frac{1}{(x+4)}-\frac{1}{(x+4)^2}\right)dx$$

$$=e^x\left(\frac{1}{x+4}\right)+C \quad [\because \int e^x\{f(x)+f'(x)\}dx=e^x f(x)+C]$$