1. **Maximum and Minimum Value** Let f be a function define on an interval I. Then,

   (i) f is said to have a **maximum value** in I, if there exists a point c in I such that $f(c) > f(x), \forall\ x \in I$. The number $f(c)$ is called the maximum value of f in I and the point c is called a **point of maximum value** of f in I.

   (ii) f is said to have a **minimum value** in I, if there exists a point c in I such that $f(c) < f(x)$, $\forall x \in I$. The number $f(c)$ is called the minimum value of f in I and the point c is called a **point of minimum value** of f in I.

   (iii) f is said to have an extreme value in I, if there exists a point c in I such that $f(c)$ is either a maximum value or a minimum value of f in I. The number $f(c)$ is called an extreme value of f in I and the point c is called an extreme point.

2. **Local Maxima and Local Minima**

   (i) A function $f(x)$ is said to have a local maximum value at point $x = a$, if there exists a neighbourhood $(a-\delta, a+\delta)$ of a such that $f(x) < f(a), \forall\ x \in (a-\delta, a+\delta),\ x \neq a$. Here, $f(a)$ is called the local maximum value of $f(x)$ at the point $x = a$.

   (ii) A function $f(x)$ is said to have a local minimum value at point $x = a$, if there exists a neighbourhood $(a-\delta, a+\delta)$ of a such that $f(x) > f(a), \forall\ x \in (a-\delta, a+\delta),\ x \neq a$. Here, $f(a)$ is called the local minimum value of $f(x)$ at $x = a$.

3. The points at which a function changes its nature from decreasing to increasing or *vice-versa* are called turning points.

NOTE

(i) Through the graphs, we can even find maximum/minimum value of a function at a point at which it is not even differentiable.

(ii) Every monotonic function assumes its maximum/minimum value at the end points of the domain of definition of the function.

4. Every continuous function on a closed interval has a maximum and a minimum value.

5. Let f be a function defined on an open interval I. Suppose $c \in I$ is any point. If f has a local maxima or a local minima at $x = c$, then either $f'(c) = 0$ or f is not differentiable at c.

6. **Critical Point** A point c in the domain of a function f at which either $f'(c) = 0$ or f is not differentiable, is called a critical point of f.

7. **First Derivative Test** Let f be a function defined on an open interval I and f be continuous of a critical point c in I. Then,

(i) if $f'(x)$ changes sign from positive to negative as x increases through c, then c is a point of **local maxima.**

(ii) if $f'(x)$ changes sign from negative to positive as x increases through c, then c is a point of **local minima.**

(iii) if $f'(x)$ does not change sign as x increases through c, then c is neither a point of local maxima nor a point of local minima. Such a point is called a **point of inflection.**

8. **Second Derivative Test** Let $f(x)$ be a function defined on an interval I and $c \in I$. Let f be twice differentiable at c. Then,

(i) $x = c$ is a point of local maxima, *if* $f'(c) = 0$ and $f''(c) < 0$.

(ii) $x = c$ is a point of local minima, if $f'(c) = 0$ and $f''(c) > 0$.

(iii) the test fails, if $f'(c) = 0$ and $f''(c) = 0$.

NOTE

(i) If the test fails, then we go back to the first derivative test and find whether a is a point of local maxima, local minima or a point of inflection.

(ii) If we say that f is twice differentiable at a, then it means second order derivative exist at a.

9. **Absolute Maximum Value** Let $f(x)$ be a function defined in its domain say $Z \subset R$. Then, $f(x)$ is said to have the maximum value at a point $a \in Z$, if $f(x) \leq f(a), \forall\, x \in Z$.

10. **Absolute Minimum Value** Let $f(x)$ be a function defined in its domain say $Z \subset R$. Then, $f(x)$ is said to have the minimum value at a point $a \in Z$, if $f(x) \geq f(a), \forall\, x \in Z$.

NOTE Every continuous function defined in a closed interval has a maximum or a minimum value which lies either at the end points or at the solution of $f'(x) = 0$ or at the point, where function is not differentiable.

11. Let f be a continuous function on an interval $I = [a, b]$. Then, f has the absolute maximum value and f attains it atleast once in I. Also, f has the absolute minimum value and attains it atleast once in I.