NCERT Exemplar Solutions

Class 12 – Mathematics

Chapter 3 – Matrices

Objective Type Questions

Question 53: The matrix $P = \begin{bmatrix} 0 & 0 & 4 \\ 0 & 4 & 0 \\ 4 & 0 & 0 \end{bmatrix}$ is a

(a) square matrix

(b) diagonal matrix

(c) unit matrix

(d) None of these

Solution: *(a)*

We know that, a square matrix has equal number of rows and columns.

Therefore, P is a square matrix.

Question 54: Total number of possible matrices of order 3x3 with each entry 2 or 0 is

(a) 9

(b) 27

(c) 81

(d) 512

Solution: *(d)*

Total number of possible matrices of order 3×3 with each entry 2 or 0 is 2^9 *i.e.*, 512.

Question 55: $\begin{bmatrix} 2x+y & 4x \\ 5x-7 & 4x \end{bmatrix} = \begin{bmatrix} 7 & 7y-13 \\ y & x+6 \end{bmatrix}$, then the value of $x + y$ is

(a) $x = 3, y=1$

(b) $x = 2, y=3$

(c) $x = 2\,y = 4$

(d) $x = 3, y=3$

Solution: *(b)*

We have, $\begin{bmatrix} 2x+y & 4x \\ 5x-7 & 4x \end{bmatrix} = \begin{bmatrix} 7 & 7y-13 \\ y & x+6 \end{bmatrix}$

$\therefore 4x = x + 6$

$\Rightarrow 2x = 6$

$\Rightarrow x = 2$

Also, $4x = 7y - 13$

$\Rightarrow 8 = 7y - 13 \quad (\because x = 2)$

$\Rightarrow 7y = 21 \Rightarrow y = 3$

$\therefore\ x + y = 2 + 3 = 5$

Question 56: If $A = \frac{1}{\pi}\begin{bmatrix} \sin^{-1}(x\pi) & \tan^{-1}\left(\frac{x}{\pi}\right) \\ \sin^{-1}\left(\frac{x}{\pi}\right) & \cot^{-1}(\pi x) \end{bmatrix}$ and $B = \frac{1}{\pi}\begin{bmatrix} -\cos^{-1}(x\pi) & \tan^{-1}\left(\frac{x}{\pi}\right) \\ \sin^{-1}\left(\frac{x}{\pi}\right) & \tan^{-1}(\pi x) \end{bmatrix}$ then $A - B$ is

(a) I

(b) 0

(c) $2I$

(d) $\frac{1}{2}I$

Solution: *(d)*

We have, $A = \frac{1}{\pi}\begin{bmatrix} \sin^{-1}(x\pi) & \tan^{-1}\left(\frac{x}{\pi}\right) \\ \sin^{-1}\left(\frac{x}{\pi}\right) & \cot^{-1}(\pi x) \end{bmatrix} = \begin{bmatrix} \frac{1}{\pi}\sin^{-1} x\pi & \frac{1}{\pi}\tan^{-1}\frac{x}{\pi} \\ \frac{1}{\pi}\sin^{-1}\frac{x}{\pi} & \frac{1}{\pi}\cot^{-1}\pi x \end{bmatrix}$ and

$$B = \frac{1}{\pi}\begin{bmatrix} -\cos^{-1}(x\pi) & \tan^{-1}\left(\frac{x}{\pi}\right) \\ \sin^{-1}\left(\frac{x}{\pi}\right) & \tan^{-1}(\pi x) \end{bmatrix} = \begin{bmatrix} \frac{1}{\pi}\cos^{-1} x\pi & \frac{1}{\pi}\tan^{-1}\frac{x}{\pi} \\ \frac{1}{\pi}\sin^{-1}\frac{x}{\pi} & \frac{1}{\pi}\tan^{-1}\pi x \end{bmatrix}$$

$$\therefore A - B = \begin{bmatrix} \frac{1}{\pi}(\sin^{-1} x\pi + \cos^{-1} x\pi) & \frac{1}{\pi}\left(\tan^{-1}\frac{x}{\pi} - \tan^{-1}\frac{x}{\pi}\right) \\ \frac{1}{\pi}\left(\sin^{-1}\frac{x}{\pi} - \sin^{-1}\frac{x}{\pi}\right) & \frac{1}{\pi}\cot^{-1} x\pi + \tan^{-1} x\pi \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{\pi}\left(\frac{\pi}{2}\right) & 0 \\ 0 & \frac{1}{\pi}\left(\frac{\pi}{2}\right) \end{bmatrix} \quad \begin{bmatrix} \because \sin^{-1}x + \cos^{-1}x = \frac{\pi}{2} \\ \text{and } \tan^{-1}x + \cot^{-1}x = \frac{\pi}{2} \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \frac{1}{2}I$$

Question 57: If A and B are two matrices of the order $3 \times m$ and $3 \times n$, respectively and $m = n$, then order of matrix $(5A - 2B)$ is

(a) $m \times 3$

(b) 3×3

(c) $m \times n$

(d) $3 \times n$

Solution: (*d*)

We are given that, the order of the matrices A and B are $3 \times m$ and $3 \times n$ respectively. Now, If $m = n$, then A and B have same orders as $3 \times n$ each, so the order of $(5A - 2B)$ should be same as $3 \times n$.

Question 58: If $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, then A^2 is equal to

(a) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Solution: *(d)*

We have, $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$\because A^2 = A \cdot A$

$= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Question 59: If matrix $A = [a_{ij}]_{2\times2}$, where $a_{ij} = 1$, if $i \neq j$

= 0 and if $i = j$, then , A^2 is equal to

(a) I

(b) A

(c) 0

(d) None of these

Solution: *(a)*

We have, $A = [a_{ij}]_{2\times2}$, where $a_{ij} = 1$, if $i \neq j$ and $a_{ij} = 0$ and if $i = j$

$\therefore A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

Now,

$A^2 = A.A$

$= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= I$

Question 60: The matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ is a

(a) identity matrix

(b) symmetric matrix

(c) skew-symmetric matrix

(d) None of these

Solution: *(b)*

We have, $A=\begin{bmatrix}1 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 4\end{bmatrix}$

$\Rightarrow A'=\begin{bmatrix}1 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 4\end{bmatrix}$

$=A$

Since, $A' = A$

Thus, A is a symmetric matrix.

Question 61: The matrix $\begin{bmatrix}0 & -5 & 8\\ 5 & 0 & 12\\ -8 & -12 & 0\end{bmatrix}$ is a

(a) diagonal matrix

(b) symmetric matrix

(c) skew-symmetric matrix

(d) scalar matrix

Solution: *(c)*

We have, $B=\begin{bmatrix}0 & -5 & 8\\ 5 & 0 & 12\\ -8 & -12 & 0\end{bmatrix}$

$\Rightarrow B'=\begin{bmatrix}0 & 5 & -8\\ -5 & 0 & -12\\ 8 & 12 & 0\end{bmatrix}$

$=-\begin{bmatrix}0 & -5 & 8\\ 5 & 0 & 12\\ -8 & -12 & 0\end{bmatrix}$

$=-B$

Since, $B' = -B$,

Thus, B is a skew-symmetric matrix.

Question 62: If A is matrix of order $m \times n$ and B is a matrix such that AB' and $B'A$ are both defined, then order of matrix B is

(a) $m \times m$

(b) $n \times n$

(c) $n \times m$

(d) $m \times n$

Solution: *(d)*

We are given that, the order of matrix A is $m \times n$.

Let us suppose the order of the matrix B is $p \times q$.

Therefore, the order of the matrix B' will be $q \times p$.

Since, AB' is defined.

$\therefore n = q$

Again, BA is also defined,
$\therefore p = m$

Hemce, the order of the matrix B is $m \times n$

Question 63: If A and B are matrices of same order, then $(AB' - BA')$ is a

(a) skew-symmetric matrix

(b) null matrix

(c)symmetric matrix

(d) unit matrix

Solution: *(a)*

We have matrices A and B of same order.

Let us suppose $Q = (AB' - BA')$

$\therefore Q' = (AB' - BA')'$

$= (AB')' - (BA')'$

$= (B')'(A)' - (A')'B'$
$= BA' - AB'$

$= -(AB' - BA')$

$= -Q$

Since, $Q = -Q$

Hence, $(AB' - BA')$ is a skew-symmetric matrix.

Question 64: If A is a square matrix such that $A^2 = I$, then $(A - I)^3 + (A + I)^3 - 7A$ is equal to

(a) A

(b) $I - A$

(c) $J + A$

(d) $3A$

Solution: (a)

We have, $A^2 = I$

Now, $(A - I)^3 + (A + I)^3 - 7A = [(A - I) + (A + I)]\,[(A - I)^2 + (A + I)^2 - (A - I)(A + I)] - 7A$

$[\therefore\ a^3 + b^3 = (a + b)\,(a^2 + b^2 - ab)]$

$= [(2A)\,\{A^2 + I^2 - 2AI + A^2 + I^2 + 2AI - (A^2 - I^2)\}] - 7A$

$= [(2A)\,\{AI + I^2 - 2AI + AI + I^2 + 2AI - AI + I^2)\}] - 7A \qquad [\because A^2 = AI]$

$= 2A\,[I + I^2 + I + I^2 - I + I^2] - 7A$

$= 2A\,[5I - I] - 7A$

$= 8AI - 7AI \qquad [\because A = AI]$

$= AI$

$= A$

Question 65: For any two matrices A and B, we have

(a) $AB = BA$

(b) $AB \neq BA$

(c) $AB = O$

(d) None of these

Solution: *(d)*

For any two matrices A and S, we may have $AB = BA = I$, $AB \neq BA$ and $AB = O$ but it is not always true.

Question 66: On using elementary column operations $C_2 \to C_2 - 2C_1$ in the following matrix equation $\begin{bmatrix} 1 & -3 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$ we have

(a) $\begin{bmatrix} 1 & -5 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -2 & 2 \end{bmatrix}\begin{bmatrix} 3 & -5 \\ 2 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & -5 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 3 & -5 \\ -0 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & -5 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -2 & 4 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & -5 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 3 & -5 \\ 2 & 0 \end{bmatrix}$

Solution: ***(c)***

We have, $\begin{bmatrix} 1 & -3 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$

On using $C_2 \to C_2$ - $2C_1$, we get

$$\begin{bmatrix} 1 & -5 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 3 & -5 \\ 2 & 0 \end{bmatrix}$$

Question 67: On using elementary row operation $R_1 \to R_1 - 3R_2$ in the following

$$\begin{bmatrix} 4 & 2 \\ 3 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$$

(a) $\begin{bmatrix} -5 & -7 \\ 3 & 3 \end{bmatrix} = \begin{bmatrix} 1 & -7 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} -5 & -7 \\ 3 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} -1 & -3 \\ 1 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} -5 & -7 \\ 3 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 1 & -7 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} 4 & 2 \\ -5 & -7 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -3 & -3 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$

Solution: (b)

We have, $\begin{bmatrix} 4 & 2 \\ 3 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$

Using elementary row operation $R_1 \to R_1 - 3R_2$, we get

$$\begin{bmatrix} -5 & -7 \\ 3 & 3 \end{bmatrix} = \begin{bmatrix} 1 & -7 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} -1 & -3 \\ 1 & 1 \end{bmatrix}$$