NCERT Exemplar Solutions
Class 12 – Mathematics
Chapter 8 – Application of Integrals

Objective Type Questions

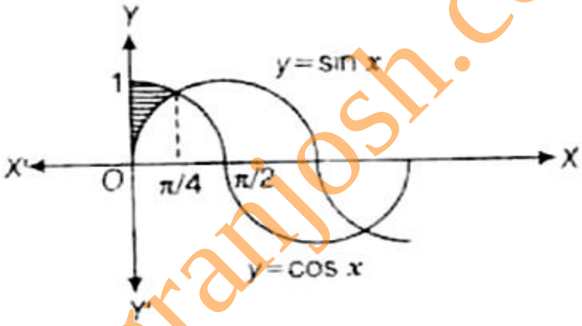
Question 24. The area of the region bounded by the *Y*-axis $y = \cos x$ and $y = \sin x$, where $0 \le x \le \frac{\pi}{2}$, is

(a) $\sqrt{2}$ sq units (b) $\left(\sqrt{2}+1\right)$ sq units

(c) $\left(\sqrt{2}-1\right)$ sq units (d) $\left(2\sqrt{2}-1\right)$ sq units

Solution. (c)

Explanation: We have, *Y*-axis *i.e.*, $x = 0$, $y = \cos x$ and $y = \sin x$, where $0 \le x \le \frac{\pi}{2}$



Question 25. The area of the region bounded by the curve $x^2 = 4y$ and the straight line $x = 4y - 2$ is

(a) $\frac{3}{8}$ sq unit (b) $\frac{5}{8}$ sq unit

(c) $\frac{7}{8}$ sq unit (d) $\frac{9}{8}$ sq units

Solution. (d)

Explanation: Given equation of curve is $x^2 - 4y$ and the straight line $x = 4y - 2$.

For intersection point, put $x = 4y - 2$ in equation of curve, we get

$$(4y - 2)^2 = 4y$$

$\Rightarrow \quad 16y^2 - + 4 - 16y = 4y$

$\Rightarrow \quad 16y^{\cdot} - 20y + 4 = 0$

$\Rightarrow \quad 4y^2 - 4y + 1 = 0$

$\Rightarrow \quad 4y^2 - 4y - y + 1 = 0$

$\Rightarrow \quad 4y(y-1) - 1(y-1) = 0$

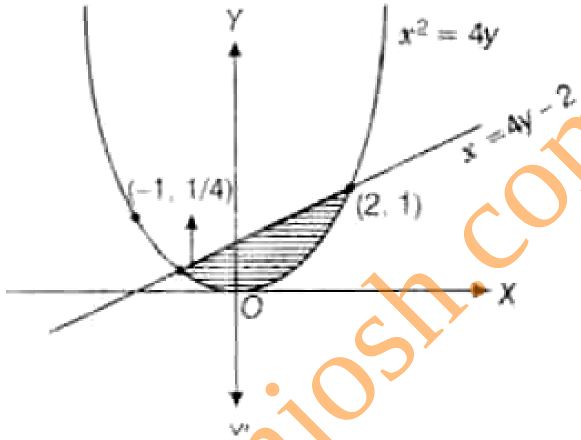
$\Rightarrow \quad (4y-1)(y-1) = 0$

For $y = 1, x = \sqrt{4 \cdot 1} = 2$ [since, negative value does not satisfy the equation of line]

For $y = \frac{1}{4}, x = \sqrt{4 \cdot \frac{1}{4}} = -1$ [positive value does not satisfy the equation of line]

So, the points of intersection are (2, 1) and $\left(-1, \frac{1}{4}\right)$

Graphs for the curve $x^2 = 4y$ and the straight line $x = 4y - 2$ are as shown below:



$\therefore$ Required area of shaded region $= \int_{-1}^{2}\left(\frac{x+2}{4}\right)dx - \int_{-1}^{2}\frac{x^2}{4}dx$

$$= \frac{1}{4}\left[\frac{x^2}{2} + 2x\right]_{-1}^{2} - \frac{1}{4}\left|\frac{x^3}{3}\right|^{2}$$

$$= \frac{1}{4}\left[\frac{4}{2} + 4 - \frac{1}{2} + 2\right] - \frac{1}{4}\left[\frac{8}{3} + \frac{1}{3}\right]$$

$$= \frac{1}{4}\cdot\frac{15}{2} - \frac{1}{4}\cdot\frac{9}{3} = \frac{45-18}{24}$$

$$= \frac{27}{24} = \frac{9}{8} \text{ sq units}$$

Question 26. The area of the region bounded by the curve $y = \sqrt{16 = x^2}$ and X-axis is

(a) 8π sq units (b) $20\,\pi$ sq units

(c) 16π sq units (d) $256\,\pi$ sq units

Solution. (a)

Explanation: Given equation of curve is $y = \sqrt{16 = x^2}$ and the equation of line is X-axis *i.e.*, $y = 0$.

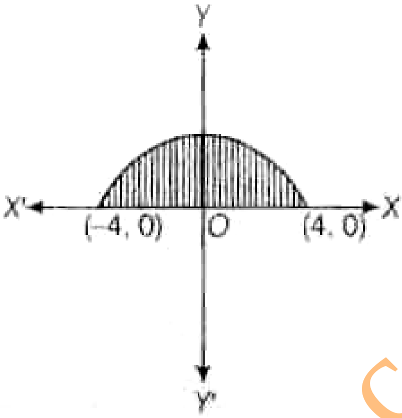
$\therefore \quad \sqrt{16 - x^2} = 0$

$\Rightarrow \quad 16 - x^2 = 0$

$\Rightarrow \quad x^2 = 16$

$\Rightarrow \quad x = \pm 4$

So, the points of intersection are (4, 0) and (–4, 0)



$\therefore$ Required area of shaded region, $A = \int_{-4}^{4} \left(16 - x^2\right)^{1/2} dx$

$$= \int_{-4}^{4} \sqrt{\left(4^2 - x^2\right)} dx$$

$$= \left[\frac{x}{2}\sqrt{4^2 - x^2} + \frac{4^2}{2}\sin^{-1}\frac{x}{4} \right]_{-4}^{4}$$

$$= \left[\frac{4}{2}\sqrt{4^2 - 4^2} + 8\sin^{-1}\frac{4}{4} \right] - \left[-\frac{4}{2}\sqrt{4^2 - (-4)^2} + 8\sin^{-1}\left(-\frac{4}{4}\right) \right]$$

$$= \left[2 \cdot 0 + 8 \cdot \frac{\pi}{2} - 0 + 8 \cdot \frac{\pi}{2} \right] = 8\pi \text{ sq units}$$

Question 27. Area of the region in the first quadrant enclosed by the X-axis, the line $y = x$ and the circle $x + y = 32$ is

(a) 16π sq units (b) 4π sq units

(c) 32π sq units (d) 24π sq units

Solution. (b)

Explanation: We have, $y = 0$, $y = x$ and the circle $x^2 + y^2 = 32$ in first quadrant.

Since, $x^2 + (x)^2 = 32 \qquad [\because y = x]$

$\Rightarrow \quad 2x^2 = 32$

$\Rightarrow \quad x = \pm 4$

So, the points of intersection of circle $x^2 + y^2 = 32$ and line $y = x$ are (4, 4) or (– 4, 4),

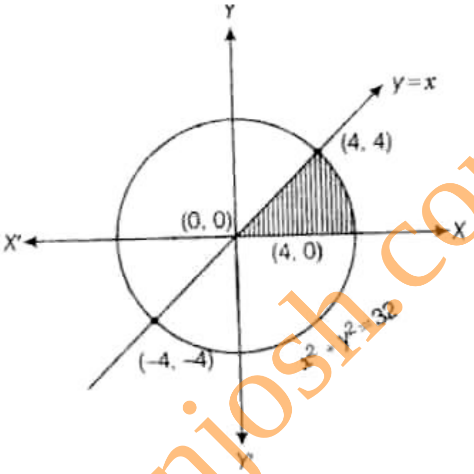
And $\quad x^2 + y^2 = \left(4\sqrt{2}\right)^2$

Since, $\quad y = 0$

$x^2 + (0)^2 = 32$

$\Rightarrow \qquad x = \pm 4\sqrt{2}$

So, the circle intersects the X-axis at $\left(\pm 4\sqrt{2}.0\right)$.



From the figure area of shaded region $= \int_0^4 x dx + \int_4^{4\sqrt{2}} \sqrt{\left(4\sqrt{2}\right)^2 = x^2}\, dx$

$$= \left|\frac{x^2}{2}\right|_0^4 + \left|\frac{x}{2}\sqrt{\left(4\sqrt{2}\right)^2 = x^2} + \frac{\left(4\sqrt{2}\right)^2}{2}\sin^{-1}\frac{x}{4\sqrt{2}}\right|_4^{4\sqrt{2}}$$

$$= \frac{16}{2} + \left[\frac{4\sqrt{2}}{2}\cdot 0 + 1\sin^{-1}\frac{\left(4\sqrt{2}\right)}{\left(4\sqrt{2}\right)} - \frac{4}{2}\sqrt{(4\sqrt{2})^2 - 16} - 16\sin^{-1}\frac{4}{4\sqrt{2}}\right]$$

$$= 8 + \left[16\cdot\frac{\pi}{2} - 2\cdot\sqrt{16} - 16\cdot\frac{\pi}{4}\right]$$

$$= 8 + [8\pi - 8 - 4\pi] = 4\pi \text{ sq units}$$

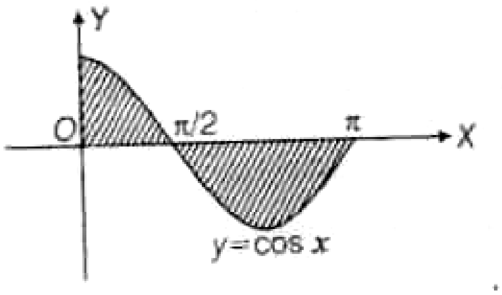
Question 28. Area of the region bounded by the curve $y = \cos x$ between $x = 0$ and $x = \pi$ is

(a) 2 sq units $\qquad$ (b) 4 sq units

(c) 3 sq units $\qquad$ (d) 1 sq units

Solution. (a)

Explanation: Graph for the curve $y = \cos x$ between $x = 0$ and $x = \pi$ is as below:



Required area enclosed by the curve $y = \cos x$, $x = 0$ and $x = \pi$ is

$$A = \int_0^{\pi/2} \cos x\, dx + \left| \int_{\pi/2}^{\pi} \cos x\, dx \right|$$

$$= \left[\sin\frac{\pi}{2} - \sin 0 \right] + \left[\sin\frac{\pi}{2} - \sin \pi \right]$$

$= 1 + 1 = 2$sq units

Question 29. The area of the region bounded by parabola $y^2 = x$ and the straight line $2y = x$ is

(a) $\frac{4}{3}$ sq units (b) 1 sq unit

(c) $\frac{2}{3}$ sq unit (d) $\frac{1}{3}$ sq unit

Solution. (a)

Explanation: Solving $y^2 = x$ and $2y = x$, we get:

$$\left(\frac{x}{2}\right)^2 = x$$

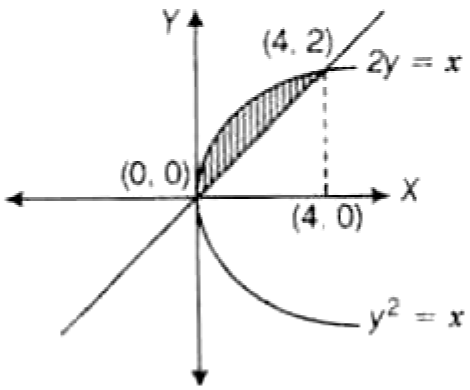
$\Rightarrow \quad x^2 = 4^{3/2}$

$\Rightarrow \quad x(x - 4) = 0$

$\Rightarrow \quad x = 4, 0$

When $x = 0$, $y = 0$ and when $x = 4$, $y = 2$

So, the intersection points are (0, 0) and (4, 2).



Thus required area of shaded region,

$$A = \int_0^4 \left[\sqrt{x} - \frac{x}{2}\right] dx$$

$$= \left[\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} - \frac{1}{2}\cdot\frac{x^2}{2}\right]_0^4 = \left[2\cdot\frac{x^{3/2}}{3} - \frac{x^2}{4}\right]_0^4$$

$$= \frac{2}{3}4^{3/2} - \frac{16}{4} - \frac{2}{3}\cdot 0 + \frac{1}{4}\cdot 0$$

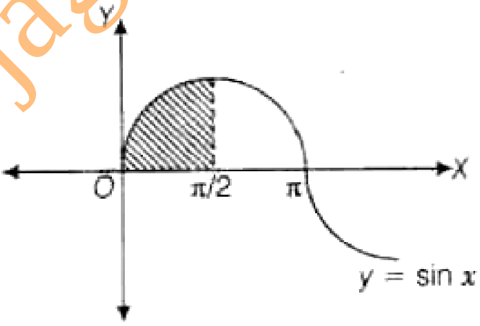
$$= \frac{16}{3} - \frac{16}{4} = \frac{64-48}{12} = \frac{16}{12} = \frac{4}{3} \text{ sq units}$$

Question 30. The area of the region bounded by the curve y = sin x between the ordinates x = 0, x = $\frac{\pi}{2}$ and the X-axis is

(a) 2 sq units
(b) 4 sq units
(c) 3 sq units
(d) 1 sq unit

Solution. (d)

Explanation: Graph for y = sin x; $0 \le x \le \frac{\pi}{2}$ is shown below:



Thus, required area of the shaded region, $A = \int_0^{\pi/2} \sin x dx$

$$= -[\cos x]_0^{\pi/2} = -\left[\cos\frac{\pi}{2} - \cos o\right]$$

$$= -[0-1] = 1 \text{ sq unit}$$

Question 31. The area of the region bounded by the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ is

(a) 20π sq units (b) $20\pi^2$ sq units

(c) $1\ 6\pi^2$ sq units (d) 25π sq units

Solution. (a)

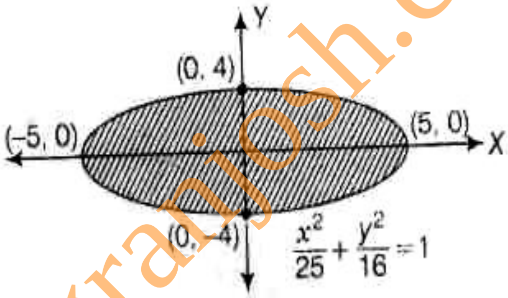
Explanation: We have, $\frac{x^2}{5^2}+\frac{y^2}{4^2}=1$ which is an ellipse with $a = \pm 5$ and $b = \pm 4$.

$$\Rightarrow \quad \frac{y^2}{4^2}=1-\frac{x^2}{5^2}$$

$$\Rightarrow \quad y^2=16\left(1-\frac{x^2}{25}\right)$$

$$\Rightarrow \quad y=\sqrt{\frac{16}{25}\left(25-x^2\right)}$$

$$\Rightarrow \quad y=\frac{4}{5}\sqrt{\left(5^2-x^2\right)}$$



$\therefore$ Area enclosed by ellipse, $A=2\cdot\frac{4}{5}\int_{-5}^{5}\sqrt{\left(5^2-x^2\right)}\,dx$

$$= 2\cdot\frac{8}{5}\int_0^5\sqrt{5^2-x^2}\,dx$$

$$= 2\cdot\frac{8}{5}\left[\frac{x}{2}\sqrt{5^2-x^2}+\frac{5^2}{2}\sin^{-1}\frac{x}{5}\right]_0^5$$

$$= 2\cdot\frac{8}{5}\left[\frac{5}{2}\sqrt{5^2-5^2}+\frac{5^2}{2}\sin^{-1}\frac{5}{5}-0-\frac{25}{2}\cdot 0\right]$$

$$= 2\cdot\frac{8}{5}\left[\frac{25}{2}\cdot\frac{\pi}{2}\right]$$

$$= \frac{16}{5}\cdot\frac{25\pi}{4}$$

$$= 20\ \pi \text{ sq units}$$

Question 32. The area of the region bounded by the circle $x^2 + y^2 = 1$ is

(a) 2π sq units (b) π sq units

(c) 3π sq units (d) 4π sq units

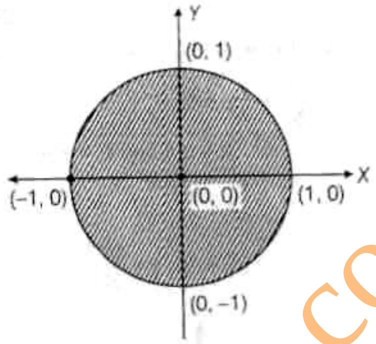
Solution. (b)

Explanation: Here, $x^2 + y^2 = 1^2$ is a circle with centre at (0, 0)

$\Rightarrow \quad y^2 = 1 - x^2$

$\Rightarrow \quad y = \sqrt{1 = x^2}$

Graph for the circle $x^2 + y^2 = 1^2$ is shown below:



$\therefore$ Area enclosed by circle $= 2\int_{-1}^{1}\sqrt{1^2 - x^2}\,dx = 2\cdot 2\int_{0}^{1}\sqrt{1^2 - x^2}\,dx$

$$= 2\cdot 2\left[\frac{x}{2}\sqrt{1^2 - x^2} + \frac{1^2}{2}\sin^{-1}\frac{x}{1}\right]_0^1$$

$$= 4\left[\frac{1}{2}\cdot 0 + \frac{1}{2}\cdot\frac{\pi}{2} - 0 - \frac{1}{2}\cdot 0\right]$$

$$= 4\cdot\frac{\pi}{4} = \pi \text{ sq units}$$

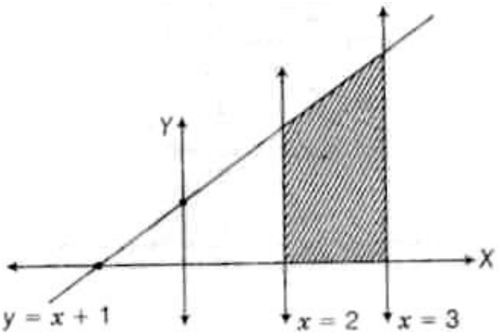
Question 33. The area of the region bounded by the curve $y = x + 1$ and the lines $x = 2$, $x - 3$, is

(a) $\frac{7}{2}$ sq units (b) $\frac{9}{2}$ sq units

(c) $\frac{11}{2}$ sq units (d) $\frac{13}{2}$ sq units

Solution. (a)

Explanation: Graph for given functions is given below:



From figure, required area of shaded region, $A = \int_2^3 (x+1)\,dx = \left[\frac{x^2}{2} + x\right]_2^3$

$$= \left[\frac{9}{2} + 3 - \frac{4}{2} - 2\right] = \left[\frac{5}{2} + 1\right] = \frac{7}{2} \text{ sq units}$$

Question 34. The area of the region bounded by the curve $x = 2y + 3$ and the lines $y = 1$, $y = 1$ is

(a) 4 sq units

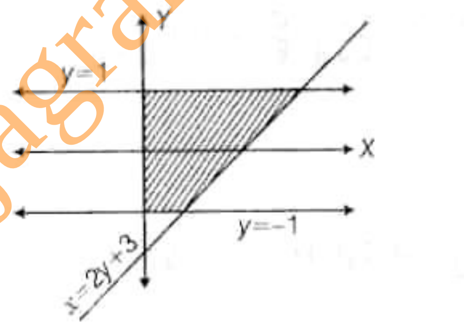
(b) $\frac{3}{2}$ sq units

(c) 6squnits

(d) 8 sq units

Solution. (c)

Explanation: Graph for given functions is given below:



From figure, required area of shaded region $= \int_{-1}^{1} (2y+3)\,dy$

$$= \left[\frac{2y^2}{2} + 3y\right]_{-1}^{1}$$

$$= \left[y^2 + 3y\right]_{-1}^{1}$$

$$= [1 + 3 - 1 + 3] = 6 \text{ sq units}$$