NCERT Exemplar Solutions

Class 12 – Mathematics

Chapter 1 – Relations and Functions

Short Answer Type Questions

Question 1.1:

Let $a = \{a, b, c)$ and the relation R be defined on a as follows

$$R = \{(a,a),(b,c),(a,b)\}$$

Then, write minimum number of ordered pairs to be added in R to make R reflexive and transitive.

Solution 1.1:

Consider, the given relation, $R = \{(a,a),(b,c),(a,b)\}$

For R to be reflexive, we will have add (b,b) and (c,c).

To make R is transitive we must add (a,c) to R.

Hence, the minimum number of ordered pair to be added are $(b,b),(c,c),(a,c)$.

Question 1.2:

Let d be the domain of the real valued function f defined by $f(x) = \sqrt{25-x^2}$. Then, write D.

Solution 1.2:

Consider the given function, $f(x) = \sqrt{25-x^2}$

For $f(x)$ to be real, the term inside the square root can't be negative

i.e., $25-x^2 \geq 0$

$\Rightarrow x^2 \leq 25$

$\Rightarrow 5 \leq x \leq -5$

Therefore, the domain of the function, $f(x)$ is given by $D = [-5, 5]$

Question 1.3:

If $f, g : R \rightarrow R$ be defined by $f(x) = 2x+1$ and $g(x) = x^2 - 2, \forall x \in R,$ respectively. Then, find gof.

Solution 1.3:

We are given that, $f(x) = 2x+1$ and $g(x) = x^2 - 2, \forall x \in R$

Now, $gof = g\{f(x)\}$

$= g(2x+1) \quad [\because f(x) = 2x+1]$

$= (2x+1)^2 - 2 \quad [\because g(x) = x^2 - 2]$

$= 4x^2 + 4x + 1 - 2$

$= 4x^2 + 4x - 1$

Question 1.4:

Let $f : R \to R$ be the function defined by $f(x) = 2x - 3, \forall x \in R$. Write f^{-1}.

Solution 1.4:

We are given that, $f(x) = 2x - 3, \forall x \in R$

Let us suppose, $y = 2x - 3$

$\Rightarrow 2x = y + 3$

$\Rightarrow x = \dfrac{y+3}{2}$

$\Rightarrow f^{-1}(y) = \dfrac{y+3}{2} \quad \begin{bmatrix} \because f(x) = y \\ \Rightarrow f^{-1}(y) = x \end{bmatrix}$

Replace y by x on both sides.

$\Rightarrow f^{-1}(x) = \dfrac{x+3}{2}$

Question 1.5:

If $A = \{a, b, c, d\}$ and the function $f = \{(a,b), (b,d), (c,a), (d,c)\}$, write f^{-1}

Solution 1.5:

We are given that, $f = \{(a,b), (b,d), (c,a), (d,c)\}$

An inverse relation is the set of ordered pairs obtained by interchanging the first and second elements of each pair in the original relation.

$\therefore f^{-1} = \{(b,a), (d,b), (a,c), (c,d)\}$

Question 1.6:

If $f : R \to R$ is defined by $f(x) = x^2 - 3x + 2$, write $f\{f(x)\}$.

Solution 1.6:

We are given that, $f(x) = x^2 - 3x + 2$

Now, $f\{f(x)\} = f(x^2 - 3x + 2)$

$= (x^2 - 3x + 2)^2 - 3(x^2 - 3x + 2) + 2$

$= x^4 + 9x^2 + 4 - 6x^3 - 12x + 4x^2 - 3x^2 + 9x - 6 + 2$

$\left[\because (a+b+c)^2 = (a^2 + b^2 + c^2 + 2ab + 2bc + 2ca)\right]$

$= x^4 - 6x^3 + 10x^2 - 3x$

$\therefore f\{f(x)\} = x^4 - 6x^3 + 10x^2 - 3x$

Question 1.7:

Is $g = \{(1,1),(2,3),(3,5),(4,7)\}$ *a* function? If *g* is described by $g(x) = \alpha x + \beta$, then what value should be assigned to α and β?

Solution 1.7:

We are given that, $g = \{(1,1),(2,3),(3,5),(4,7)\}$.

Since, in *g*, each element of domain has unique image.

Hence, g is *a* function.

Consider, $g(x) = \alpha x + \beta$

Substituting $x = 1$, we get

$g(1) = \alpha + \beta$

$\Rightarrow \alpha + \beta = 1$

$\Rightarrow \alpha = 1 - \beta \qquad \ldots(1)$

Again, substituting $x = 2$, we get

$g(2) = 2\alpha + \beta$

$\Rightarrow 2\alpha + \beta = 3 \qquad \ldots(2)$

Substituting $\alpha = 1 - \beta$ in (2), we get

$2(1-\beta) + \beta = 3$

$\Rightarrow \beta = -1$

Substituting $\beta = -1$ in (1), we get

$\alpha = -1$

$\therefore \alpha = 2, \beta = -1$

Question 1.8:

Are the following set of ordered pairs functions? If so examine whether the mapping is injective or surjective.

(i) $\{(x, y): x$ is a person, y is the mother of $x\}$.

(ii) $\{(a, b): a$ is a person, b is an ancestor of $a\}$.

Solution 1.8:

(i) The set of ordered pairs given here represents a function.

Here, the images of distinct elements of x under f are not distinct, so it is not injective but it is surjective.

(ii) Since, each element of domain does not have a unique image.

Therefore, the set of ordered pairs given here does not represent function.

Question 1.9:

If the mappings f and g are given by $f = \{(1,2),(3,5),(4,1)\}$ and $g = \{(2,3),(5,1),(1,3)\}$, write fog.

Solution 1.9:

We are given that, $f = \{(1,2),(3,5),(4,1)\}$ and $g = \{(2,3),(5,1),(1,3)\}$

Now, the domain of g is {2, 5, 1}

We know that, $fog\left(x\right) = f\left\{g\left(x\right)\right\}$

$\therefore\ fog(2) = f\{g(2)\} = f(3) = 5$

$\therefore\ fog(5) = f\{g(5)\} = f(1) = 2$

$\therefore\ fog(1) = f\{g(1)\} = f(3) = 5$

Therefore, $fog = \{(2,5),(5,2),(1,5)\}$

Question 1.10:

Let c be the set of complex numbers. Prove that the mapping $f: C \to R$ given by $f(z) = |z|, \forall z \in C$, is neither one-one nor onto.

Solution 1.10:

We are given that, $f(z) = |z|, \forall z \in C$

For $z = 1$, we have

$f(1) = |1| = 1$

For $z = -1$, we have

$f(-1) = |-1| = 1$

$\therefore f(1) = f(-1)$

But, $1 \neq -1$

Therefore, $f(z)$ is not one-one.

Also, $f(z)$ is not onto as we do not have any pre-image for any negative element of R under the mapping $f(z)$.

Question 1.11:

Let the function $f : R \to R$ be defined by $f(x) = \cos x, \forall x \in R$. Show that f is neither one-one nor onto.

Solution 1.11:

We are given, $f(x) = \cos x, \forall x \in R$

For $\frac{\pi}{2}$, we have

$$f\left(\frac{\pi}{2}\right) = \cos\frac{\pi}{2} = 0$$

For $-\frac{\pi}{2}$, we have

$$f\left(\frac{-\pi}{2}\right) = \cos\frac{\pi}{2} = 0$$

$$\therefore f\left(\frac{\pi}{2}\right) = f\left(\frac{-\pi}{2}\right)$$

But, $\frac{\pi}{2} \neq \frac{-\pi}{2}$

Hence, $f(x)$ is not one-one.

We also know that, there is not any pre-image for any real number which does not belong to the range of cosine function i.e., $[-1, 1]$,

Question 1.12:

Let $X = \{1, 2, 3\}$ and $Y = \{4, 5\}$. Find whether the following subsets of $X \times Y$ are functions from X to Y or not.

(i) $f = \{(1,4), (1,5), (2,4), (3,5)\}$

(ii) $g = \{(1,4),(2,4),(3,4)\}$

(iii) $h = \{(1,4),(2,5),(3,5)\}$

(iv) $k = \{(1,4),(2,5)\}$

Solution 1.12:

We are given that, $X = \{1,2,3\}$ and $Y = \{4,5\}$

Now, $X \times Y = \{(1,4),(1,5),(2,4),(2,5),(3,4),(3,5)\}$

(i) $f = \{(1,4),(1,5),(2,4),(3,5)\}$

Since, 1 has two images i.e., 4 and 5.

Hence, *f* is not *a* function.

(ii) $g = \{(1,4),(2,4),(3,4)\}$

Since, the domain of *g* has unique image.

Hence, *g* is *a* function.

(iii) $h = \{(1,4),(2,5),(3,5)\}$

Since, the domain of *h* has unique image.

Hence, *h* is *a* function.

(iv) $k = \{(1,4),(2,5)\}$

Since, the element 3 has not any image in *k*.

Hence, *k* is not *a* function.

Question 1.13:

If functions $f : A \rightarrow B$ and $g : B \rightarrow A$ satisfy $gof = I_A$, then show that *f* is one-one and *g* is onto.

Solution 1.13:

Consider, $gof = I_A$

$\Rightarrow gof\{f(x_1)\} = gof\{f(x_2)\}$

$\Rightarrow g(x_1) = g(x_2)$ $[\because gof = I_A]$

$\Rightarrow x_1 = x_2$

Therefore, *f* is one-one and *g* is onto.

Question 1.14:

Let $f: R \to R$ be the function defined by $f(x) = \frac{1}{2-\cos x}, \forall x \in R$ Then, find the range of f

Solution 1.14:

We are given that, $f(x) = \frac{1}{2-\cos x}, \forall x \in R$

Let us suppose, $y = \frac{1}{2-\cos x}$

$\Rightarrow 2y - y\cos x = 1$

$\Rightarrow y\cos x = 2y - 1$

$\Rightarrow \cos x = \frac{2y-1}{y} = 2 - \frac{1}{y}$

$\Rightarrow \cos x = 2 - \frac{1}{y}$

We know that, the range of cosine function is $[-1, 1]$

$\therefore -1 \le \cos x \le 1$

$\Rightarrow \ -1 \le 2 - \frac{1}{y} \le 1 \qquad \left[\because \cos x = 2 - \frac{1}{y}\right]$

$\Rightarrow -3 \le -\frac{1}{y} \le -1$

$\Rightarrow \ 1 \le \frac{1}{y} \le 3$

$\Rightarrow \frac{1}{3} \le y \le 1$

Therefore, the range of y is $\left[\frac{1}{3}, 1\right]$.

Question 1.15:

Let n be a fixed positive integer. Define a relation R in Z as follows $\forall a, b \in Z, aRb$ if and only if $a-b$ is divisible by n. Show that R is an equivalence relation.

Solution 1.15:

I. Reflexive

$aRa \Rightarrow (a-a)$ is divisible by n, which is true for any integer a as '0' is divisible by n.
Hence, it is reflexive.

II. Symmetric

aRb

$\Rightarrow a-b$ is divisible by n.

$\Rightarrow -b+a$ is divisible by n.

$\Rightarrow -(b-a)$ is divisible by n.

$\Rightarrow (b-a)$ is divisible by n.

Hence, R is symmetric.

III. Transitive

Let aRb and bRc

$\Rightarrow (a-b)$ is divisible by n and $(b-c)$ is divisible by n

$\Rightarrow (a-b)+(b-c)$ is divisibly by n

$\Rightarrow (a-c)$ is divisible by n

$\Rightarrow aRc$

Hence, R is transitive.

A relation which is symmetric, transitive and reflexive is known as an equivalence relation.

Therefore, R is an equivalence relation.