NCERT Exemplar Solutions

Class 12 – Mathematics

Chapter 6 – Application of Derivatives

Objective Type Questions

Question 35:

If the sides of an equilateral triangle are increasing at the rate of 2 cm/s then the rate at which the area increases, when side is 10 cm, is

(a) $10\text{cm}^2/\text{s}$

(b) $\sqrt{3}\ \text{cm}^2/\text{s}$

(c) $10\sqrt{3}\ \text{cm}^2/\text{s}$

(d) $\frac{10}{3}\text{cm}^2/\text{s}$

Solution: (c)

Let us suppose the side of an equilateral triangle be x cm.

Now, area of equilateral triangle, $A = \frac{\sqrt{3}}{4}x^2$ …(1)

On differentiating (1) w.r.t. t, we get

$$\frac{dA}{dt} = \frac{\sqrt{3}}{4}\cdot 2x\cdot\frac{dx}{dt}$$

$$= \frac{\sqrt{3}}{4}\cdot 2\cdot 10\cdot 2 \quad \left[\because x = 10\,\text{cm and}\,\frac{dx}{dt} = 2\text{ cm/s}\right]$$

$$= 10\sqrt{3}\ \text{cm}^2/\text{s}$$

Question 36:

A ladder, 5 m long, standing on a horizontal floor, leans against a vertical wall. If the top of the ladder slides downwards at the rate of 10 cm/s, then the rate at which the angle between the floor and the ladder is decreasing when lower end of ladder is 2 m from the wall is

(a) $\frac{1}{10}\text{rad/s}$

(b) $\frac{1}{20}$ rad/s

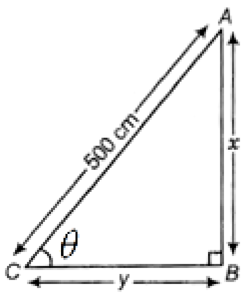
(c) 20 rad/s

(d) 10 rad/s

Solution: (b)

Let us suppose the angle between the ladder and the floor be θ.

Consider, a triangle ABC in which $AB = x$ cm and $BC = y$ cm



We are given the length of the ladder as 5 m or 500 cm.

$$\therefore \sin\theta = \frac{x}{500} \text{ and } \cos\theta = \frac{y}{500}$$

$$\Rightarrow x = 500 \sin\theta$$

$$\Rightarrow \frac{dx}{dt} = 500 \cdot \cos\theta \cdot \frac{d\theta}{dt}$$

$$\Rightarrow 10 = 500 \cdot \cos\theta \cdot \frac{d\theta}{dt} \qquad \left[\because \frac{dx}{dt} = 10 \text{ cm/s}\right]$$

$$\Rightarrow \frac{d\theta}{dt} = \frac{1}{50\cos\theta}$$

$$\Rightarrow \frac{d\theta}{dt} = \frac{1}{50 \times \frac{y}{500}} \qquad \left[\because \cos\theta = \frac{y}{500}\right]$$

$$\Rightarrow \frac{d\theta}{dt} = \frac{10}{y} \text{ rad/s}$$

$$\Rightarrow \frac{d\theta}{dt} = \frac{10}{200} \text{ rad/s} \qquad [\because y = 2 \text{ m or } 200 \text{ cm}]$$

$$\Rightarrow \frac{d\theta}{dt} = \frac{1}{20} \text{ rad/s}$$

Question 37:

The curve $y = x^{1/5}$ has at (0, 0)

(a) a vertical tangent (parallel to Y-axis)

(b) a horizontal tangent (parallel to X-axis)

(c) an oblique tangent

(d) no tangent

Solution: (a)

We are given that, $y = x^{1/5}$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{5}x^{\frac{1}{5}-1} \qquad \left[\because \frac{d}{dx}\left(x^n\right) = nx^{n-1}\right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{5}x^{-4/5}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{5x^{4/5}}$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{(0,0)} = \frac{1}{5(0)^{4/5}} = \infty$$

So, the curve $y = x^{1/5}$ has a vertical tangent at (0, 0), which is parallel to Y-axis.

Question 38:

The equation of normal to the curve $3x^2 - y^2 = 8$ which is parallel to the line $x + 3y = 8$ is

(a) $3x - y = 8$

(b) $3x + y + 8 = 0$

(c) $x + 3y \pm 8 = 0$

(d).$x + 3y = 0$

Solution: (c)

We are given the equation of the line as $x + 3y = 8$

$\Rightarrow 3y = 8 - x$

$$\Rightarrow y = -\frac{x}{3} + \frac{8}{3}$$

We have, $3x^2 - y^2 = 8$ …(1)

On differentiating (1) w.r.t x, we get

$$6x - 2y\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{6x}{2y} = \frac{3x}{y}$$

Therefore, the slope of the curve is $\frac{3x}{y}$

Now, slope of normal to the curve = $-\frac{1}{\left(\frac{dy}{dx}\right)}$

$$= -\frac{1}{\left(\frac{3x}{y}\right)} = -\frac{y}{3x}$$

Now, the slope of the line is $-\frac{1}{3}$ which should be equal to slope of the equation of normal to the curve $3x^2 - y^2 = 8$.

$$\therefore -\left(\frac{y}{3x}\right) = -\frac{1}{3}$$

$\Rightarrow -3y = -3x$

$\Rightarrow y = x$

On substituting the value of the given equation of the curve, we get

$3x^2 - x^2 = 8$

$$\Rightarrow x^2 = \frac{8}{2}$$

$\Rightarrow x = \pm 2$

Substituting $x = 2$ in $3x^2 - y^2 = 8$, we get

$3(2)^2 - y^2 = 8$

$\Rightarrow y^2 = 4$

$\Rightarrow y = \pm 2$

Substituting $x = -2$ in $3x^2 - y^2 = 8$, we get

$3(-2)^2 - y^2 = 8$

$\Rightarrow y = \pm 2$

So, the points at which normal is parallel to the given line are $(\pm 2, \pm 2)$, Hence, the equation of normal at (+ 2, ± 2) is

$$y-(\pm 2)=-\frac{1}{3}[x-(\pm 2)]$$

$$\Rightarrow 3[y-(\pm 2)]=-[x-(\pm 2)]$$

$\Rightarrow x + 3y \pm 8 = 0$

Question 39:

If the curve $ay + x^2 = 7$ and $x^3 = y$, cut orthogonally at (1, 1), then the value of a is

(a) 1

(b) 0

(c) −6

(d) 6

Solution: (d)

We are given that, $ay + x^2 = 7$ and $x^3 = y$ cut orthogonally at (1, 1).

On differentiating w.r.t x, we get

$$a\cdot\frac{dy}{dx}+2x=0 \text{ and } 3x^2=\frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx}=-\frac{2x}{a} \text{ and } \frac{dy}{dx}=3x^2$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{(1,1)}=\frac{-2}{a}=m_1 \text{ and } \left(\frac{dy}{dx}\right)_{(1,1)}=3.1=3=m_2$$

Since, the curves cut orthogonally at (1, 1)

$m_1 \cdot m_2 = -1$

$$\Rightarrow \left(\frac{-2}{a}\right)\cdot 3=-1$$

$\Rightarrow a = 6$

Question 40:

If $y = x^4 - 10$ and x changes from 2 to 1.99, then what is the change in y ?

(a) 0.32

(b) 0.03.2

(c) 5.68

(d) 5.968

Solution: (a)

We have, $y = x^4 - 10$

$$\Rightarrow \frac{dy}{dx} = 4x^3 \qquad \left[\because \frac{d}{dx}\left(x^n\right) = nx^{n-1}\right]$$

We are also given that x changes from 2 to 1.99

$\therefore \Delta x = 2.00 - 1.99 = 0.01$

Now, $\Delta y = \frac{dy}{dx} \times \Delta x$

$= 4x^3 \times \Delta x$

$= 4 \times 2^3 \times 0.01 \qquad [\because x = 2 \text{ and } \Delta x = 0.01]$

$= 0.32$

Thus, the approximate change in y is 0.32.

Question 41:

The equation of tangent to the curve $y(1 + x^2) = 2 - x$, where it crosses X-axis, is

(a) $x + 5y = 2$

(b) $x - 5y = 2$

(c) $5x - y = 2$

(d) $5x + y = 2$

Solution: (a)

We have, $y(1 + x^2) = 2 - x$ …(1)

$$\Rightarrow y \cdot (0 + 2x) + (1 + x^2) \cdot \frac{dy}{dx} = 0 - 1 \qquad [\because (fg)' = fg' + gf']$$

$$\Rightarrow 2xy + (1 + x^2)\frac{dy}{dx} = -1$$

$$\Rightarrow \frac{dy}{dx} = \frac{-1 - 2xy}{1 + x^2} \qquad \text{…(ii)}$$

Since, the given curve passes through X-axis *i.e.*, $y = 0$.

$\therefore 0(1 + x^2) = 2 - x \qquad$ [Using (1)]

$\Rightarrow x = 2$

So, the curve passes through the point (2, 0).

$$\text{Now, } \left(\frac{dy}{dx}\right)_{(2,0)} = \frac{-1-2\times 0}{1+2^2} = -\frac{1}{5}$$

Hence, the slope of tangent to the curve is $-\frac{1}{5}$

Therefore, the equation of tangent of the curve passing through (2, 0) is given by

$$y - 0 = -\frac{1}{5}(x-2)$$

$$\Rightarrow 5y = -x + 2$$

$\Rightarrow 5y + x = 2$

Question 42:

The points at which the tangents to the curve $y = x^3 - 12x + 18$ are parallel to Y-axis are

(a) (2, –2), (–2,–34)

(b) (2, 34), (–2, 0)

(c) (0, 34), (–2, 0)

(d) (2, 2), (–2, 34)

Solution: (d)

We have, $y = x^3 - 12x + 18$

$$\Rightarrow \frac{dy}{dx} = 3x^2 - 12 \qquad \left[\because \frac{d}{dx}\left(x^n\right) = nx^{n-1}\right]$$

So, the slope of line parallel to the X-axis is given by $\left(\frac{dy}{dx}\right) = 0$

$\therefore\ 3x^2 - 12 = 0$

$\Rightarrow x^2 = 4$

$\Rightarrow x = \pm 2$

Substituting $x = 2$ in $y = x^3 - 12x + 18$, we get

$y = 2^3 - 12 \times 2 + 18 = 2$

Substituting $x = -2$ in $y = x^3 - 12x + 18$, we get

$y = (-2)^3 - 12(-2) + 18 = 34$

Thus, the points are (2, 2) and (– 2, 34).

Question 43:

The tangent to the curve $y = e^{2x}$ at the point (0, 1) meets X-axis at

(a) (0, 1)

(b) $\left(-\frac{1}{2}, 0\right)$

(c) (2, 0)

(d) (0, 2)

Solution: (b)

We have, $y = e^{2x}$

$\Rightarrow \frac{dy}{dx} = 2e^{2x}$

Since, it passes through the point (0, 1).

$\therefore \left(\frac{dy}{dx}\right)_{(0,1)} = 2 \cdot e^{2 \cdot 0} = 2 = \text{Slope of tangent to the curve}$

The equation of tangent is given by

$y - 1 = 2(x - 0)$

$\Rightarrow y - 1 = 2x$

$\Rightarrow y = 2x + 1$

We are given that, the tangent to curve $y = e^{2x}$ at the point (0, 1) meets X-axis *i.e.*, $y = 0$.

$\therefore 0 = 2x + 1$

$\Rightarrow x = -\frac{1}{2}$

Thus, the required point is $\left(\frac{-1}{2}, 0\right)$.

Question 44:

The slope of tangent to the curve $x = t^2 + 3t - 8$ and $y = 2t^2 - 2t - 5$ at the point (2, –1) is

(a) $\frac{22}{7}$

(b) $\frac{6}{7}$

(c) $-\frac{6}{7}$

(d) –6

Solution: (b)

We have, $x = t^2 + 3t - 8$ and $y = 2t^2 - 2t - 5$

$$\Rightarrow \frac{dx}{dt} = 2t + 3 \text{ and } \frac{dx}{dt} = 4t - 2$$

$$\text{Now, } \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{4t-2}{2t+3} \qquad \left[\because \frac{dy}{dt} = 4t - 2 \text{ and } \frac{dx}{dt} = 2t + 3\right] \qquad (1)$$

Since, the curve passes through the point (2. – 1)

$\therefore$ $2 = t^2 - 3t - 8$ and $-1 = 2t^2 - 2t - 5$

$\Rightarrow t^2 + 3t - 10 = 0$ and $2t^2 - 2t - 4 = 0$

$\Rightarrow t^2 + 3t - 10 = 0$ and $t^2 - t - 2 = 0$

$\Rightarrow t^2 + 5t - 2t - 10 = 0$ and $t^2 + t - 2t - 2 = 0$

$\Rightarrow t(t + 5) - 2(t + 5) = 0$ and $t(t + 1) - 2(t + 1) = 0$

$\Rightarrow (t - 2)(t + 5) = 0$ and $(t - 2)(t + 1) = 0$

$\Rightarrow t = 2, -5$ and $t = -1, 2$

$\therefore$ $t = 2$

Therefore, the slope of the tangent is given by

$$\left(\frac{dy}{dx}\right)_{at\ t=2} = \frac{4\times2-2}{2\times2+3} = \frac{6}{7} \qquad \text{[Using (1)]}$$

Question 45:

Two curves $x^3 - 3xy^2 + 2 = 0$ and $3x^2y - y^3 - 2 = 0$ intersect at an angle of

(a) $\frac{\pi}{4}$

(b) $\frac{\pi}{3}$

(c) $\frac{\pi}{2}$

(d) $\frac{\pi}{6}$

Solution: (c)

We have, $x^3 - 3xy^2 + 2 = 0$ and $3x^2y - y^3 - 2 = 0$

$$\Rightarrow 3x^2 - 3\left[x \cdot 2y\frac{dy}{dx} + y^2 \cdot 1\right] + 0 = 0 \text{ and } 3\left[x^2\frac{dy}{dx} + y \cdot 2x\right] - 3y^2\frac{dy}{dx} - 0 = 0$$

$$\Rightarrow 6xy\frac{dy}{dx} + 3y^2 = 3x^2 \text{ and } 3y^2\frac{dy}{dx} = 3x^2\frac{dy}{dx} + 6xy$$

$$\Rightarrow \frac{dy}{dx} = \frac{3x^2 - 3y^2}{6xy} \text{ and } \frac{dy}{dx} = \frac{6xy}{3y^2 - 3x^2}$$

$$\Rightarrow \left(\frac{dy}{dx}\right) = \frac{3(x^2 - y^2)}{6xy} \text{ and } \left(\frac{dy}{dx}\right) = \frac{-6xy}{3(x^2 - y^2)}$$

$$\Rightarrow m_1 = \frac{x^2 - y^2}{2xy} \text{ and } m_2 = \frac{-2xy}{x^2 - y^2}$$

$$\therefore\ m_1m_2 = \frac{x^2 - y^2}{2xy} \cdot \frac{-(2xy)}{x^2 - y^2} = -1$$

Since, the product of the slopes is -1.

Hence, both the curves are intersecting at right angle *i.e.*, making $\frac{\pi}{2}$ with each other.

Question 46:

The interval on which the function $f(x) = 2x^3 + 9x^2 + 12x - 1$ is decreasing is

(a) $[-1, \infty)$

(b) $[-2, -1]$

(c) $(-\infty, -2]$

(d) $[-1, 1]$

Solution: (b)

We have, $f(x) = 2x^3 + 9x^2 + 12x - 1$

$\Rightarrow f'(x) = 6x^2 + 18x + 12$

$= 6(x^2 + 3x + 2)$

$= 6(x + 2)(x + 1)$

For decreasing, $f'(x) \le 0$

From the above number line, we can conclude that, $f'(x)$ is decreasing in $[-2, -1]$.

Question 47:

If $f: R \to R$ be defined by $f(x) = 2x + \cos x$, then f

(a) has a minimum at $x = \pi$

(b) has a maximum at $x = 0$

(c) is a decreasing function

(d) is an increasing function

Solution: (d)

We have, $f(x) = 2x + \cos x$

$\Rightarrow f'(x) = 2 + (-\sin x)$

$= 2 - \sin x$

Since, the maximum value of $\sin x$ is 1.

Hence, $f'(x) > 0, \forall x$

Thus, $f'(x)$ is an increasing function.

Question 48:

If $y = x(x-3)^2$ decreases for the values of x given by

(a) $1 < x < 3$

(b) $x < 0$

(c) $x > 0$

(d) $0 < x < \frac{3}{2}$

Solution: (a)

We have, $y = x(x-3)^2$

$$\Rightarrow \frac{dy}{dx} = x \cdot 2(x-3) \cdot 1 + (x-3)^2 \cdot 1 \qquad \left[\begin{array}{l} \because (fg)' = gf' + fg' \\ \frac{d}{dx}(x^n) = nx^{n-1} \end{array}\right]$$

$= 2x^2 - 6x + x^2 + 9 - 6x$

$= 3x^2 - 12x + 9$

$= 3(x^2 - 3x - x + 3)$

$= 3\,(x-3)\,(x-1)$

From the above number line, we can conclude that, $y = x(x-3)^2$ is decreasing in (1, 3).

Question 49:

The function $f(x) = 4\sin^3 x - 6\sin^2 x + 12\sin x + 100$ is strictly

(a) increasing in $\left(\pi, \frac{3\pi}{2}\right)$

(b) decreasing in $\left(\frac{\pi}{2}, \pi\right)$

(c) decreasing in $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

(d) decreasing in $\left[0, \frac{\pi}{2}\right]$

Solution: (b)

We have, $f(x) = 4\sin^3 x - 6\sin^2 x + 12\sin x + 100$

$\Rightarrow f'(x) = 12\sin^2 x \cdot \cos x - 12\sin x \cdot \cos x + 12\cos x$

$= 12\cos x\,[\sin^2 x - \sin x + 1]$

Now, $1 - \sin x \geq 0$ and $\sin^2 x \geq 0$

$\therefore \sin^2 x + 1 - \sin x \geq 0$

Hence, $f'(x) > 0$, when $\cos x > 0$ *i.e.*, $x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$.

So, $f(x)$ is increasing when $x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ and $f'(x) < 0$, when $\cos x < 0$ *i.e.*, $x \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

Hence, $f(x)$ is decreasing when $x \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

Since, $\left(\frac{\pi}{2}, \pi\right) \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

Hence, $f(x)$ is decreasing in $\left(\frac{\pi}{2}, \pi\right)$.

Question 50:

Which of the following functions is decreasing on $\left(0, \frac{\pi}{2}\right)$?

(a) $\sin 2x$

(b) $\tan x$

(c) $\cos x$

(d) $\cos 3x$

Solution: (c)

Consider, $f(x) = \cos x$

$\Rightarrow f'(x) = -\sin x$

In the interval $\left(0, \frac{\pi}{2}\right)$ $\sin x$ is positive.

Hence, $f'(x) < 0$ in $\left(0, \frac{\pi}{2}\right)$

Hence, $\cos x$ is decreasing in $\left(0, \frac{\pi}{2}\right)$

Question 51:

The function $f(x) = \tan x - x$

(a) always increases

(b) always decreases

(c) never increases

(d) sometimes increases and sometimes decreases

Solution: (a)

We have, $f(x) = \tan x - x$

$\therefore f'(x) = \sec^2 x - 1$

Since, $f'(x) > 0, \forall x \in R$

Hence, $f(x)$ always increases.

Question 52:

If x is real then the minimum value of $x^2 - 8x + 17$ is

(a) -1

(b) 0

(c) 1

(d) 2

Solution: *(c)*

Let us suppose, $f(x) = x^2 - 8x + 17$

$\Rightarrow f'(x) = 2x - 8$

Find the critical points by equating $f'(x)$ to 0.

$\therefore f'(x) = 0$

$\Rightarrow 2x - 8 = 0$

$\Rightarrow x = 4$

Therefore, the minimum value of $f(x)$ at $x = 4$ is given by

$f(4) = 4 \times 4 - 8 \times 4 + 17 = 1$

Question 53:

The smallest value of polynomial $x^3 - 18x^2 + 96x$ in $[0, 9]$ is

(a) 126

(b) 0

(c) 135

(d) 160

Solution: (b)

Let us suppose, $f(x) = x^3 - 18x^2 + 96x$

$\Rightarrow f'(x) = 3x^2 - 36x + 96 \quad \left[\because \frac{d}{dx}\left(x^n\right) = nx^{n-1}\right]$

Find the critical points by equating $f'(x)$ to 0.

$\therefore f'(x) = 0$

$\Rightarrow 3x^2 - 36x + 96 = 0$

$\Rightarrow 3(x^2 - 12x + 32) = 0$

$\Rightarrow (x - 8)(x - 4) = 0$

$\Rightarrow x = 8, 4 \in [0, 9]$

Now,

$f(0) = 0^3 - 18 \cdot 0^2 + 96 \cdot 0 = 0$

$f(4) = 4^3 - 18{\cdot}4^2 + 96{\cdot}4 = 160$

$f(8) = 8^3 - 18{\cdot}8^2 + 96{\cdot}8 = 128$

$f(9) = 9^3 - 18{\cdot}9^2 + 96{\cdot}9 = 135$

Thus, we conclude that absolute minimum value of f in [0, 9] is 0 occurring at $x = 0$.

Question 54:

The function $f(x) = 2x^3 - 3x^2 - 12x + 4$, has

(a) two points of local maximum

(b) two points of local minimum

(c) one maxima and one minima

(d) no maxima or minima

Solution:*(c)*

We have, $f(x) = 2x^3 - 3x^2 - 12x + 4$

$\Rightarrow f'(x) = 6x^2 - 6x - 12$

$\Rightarrow f'(x) = 6(x^2 - x - 2)$

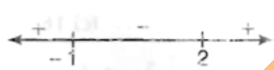
$\Rightarrow f'(x) = 6(x + 1)(x - 2)$

Find the critical points by equating $f'(x)$ to 0.

$\therefore f'(x) = 0$

$\Rightarrow 6(x + 1)(x - 2) = 0$

$\Rightarrow x = -1$ and $x = +2$



From the above number line, we can conclude that, $x = -1$ is point of local maxima and $x = 2$ is point of local minima.

Thus, $f(x)$ has one maxima and one minima.

Question 55:

The maximum value of $\sin x \cos x$ is

(a) $\dfrac{1}{4}$

(b) $\dfrac{1}{2}$

(c) $\sqrt{2}$

(d) $2\sqrt{2}$

Solution: (b)

Let us suppose, $f(x)=\sin x\cdot\cos x=\frac{1}{2}\sin 2x$

$\Rightarrow f'(x)=\frac{1}{2}\cdot\cos 2x\cdot 2=\cos 2x$

Find the critical points by equating $f'(x)$ to 0.

$\therefore f'(x)=0$

$\Rightarrow \cos 2x=0$

$\Rightarrow \cos 2x=\cos\frac{\pi}{2}$

$\Rightarrow x=\frac{\pi}{4}$

Also, $f''(x)=-\sin 2x\cdot 2=-2\sin 2x$

$\therefore \left[f''(x)\right]_{\text{at } x=\pi/4}=-2\sin 2\cdot\frac{\pi}{4}=-2\sin\frac{\pi}{2}=-2<0$

Therefore, at $x=\frac{\pi}{4}$, $f(x)$ is maximum and $\frac{\pi}{4}$ is point of maxima.

$f\left(\frac{\pi}{4}\right)=\frac{1}{2}\sin 2\cdot\frac{\pi}{4}=\frac{1}{2}$

Question 56:

At $x=\frac{5\pi}{6}$, $f(x)=2\sin 3x+3\cos 3x$ is 6

(a) maximum

(b) minimum

(c) zero

(d) neither maximum nor minimum

Solution: (d)

We have, $f(x) = 2\sin 3x + 3\cos 3x$

$\Rightarrow f'(x) = 2 \cdot \cos 3x \cdot 3 + 3(-\sin 3x)$

$f'(x) = 6\cos 3x - 9\sin 3x$ ...(1)

Now, $f''(x) = -18\sin 3x - 27\cos 3x$

$= -9(2\sin 3x + 3\cos 3x)$

$\therefore f'\left(\frac{5\pi}{6}\right) = 6\cos\left(3 \cdot \frac{5\pi}{6}\right) - 9\sin\left(3 . \frac{5\pi}{6}\right)$

$= 6\cos\frac{5\pi}{2} - 9\sin\frac{5\pi}{2}$

$= 6\cos\left(2\pi + \frac{\pi}{2}\right) - 9\sin\left(2\pi + \frac{\pi}{2}\right)$

$= 0 - 9 \neq 0$

So, $x = \frac{5\pi}{6}$ cannot be point of maxima or minima.

Hence, $f(x)$ at $x = \frac{5\pi}{6}$ is neither maximum nor minimum.

Question 57:

The maximum slope of curve $y = -x^3 + 3x^2 + 9x - 27$ is

(a) 0

(b) 12

(c) 16

(d) 32

Solution: (b)

We have, $y = -x^3 + 3x^2 + 9x - 27$

$\Rightarrow \frac{dy}{dx} = -3x^2 + 6x + 9 =$ Slope of the curve

$\Rightarrow \frac{d^2y}{dx^2} = -6x + 6 = -6(x - 1)$

Find the critical points by equating $\frac{d^2y}{dx^2}$ to 0.

$\therefore \frac{d^2y}{dx^2} = 0$

$\Rightarrow -6(x - 1) = 0$

$\Rightarrow x = 1$

Now, $\frac{d^3y}{dx^3} = -6 < 0$

So, the maximum slope of given curve is at $x = 1$

$\therefore \left(\frac{dy}{dx}\right)_{(x=1)} = -3.1^2 + 6.1 + 9 = 12$

Question 58:

The function $f(x) = x^x$ has a stationary point at

(a) $x = e$

(b) $x = \frac{1}{e}$

(c) $x = 1$

(d) $x = \sqrt{e}$

Solution: (b)

We have, $f(x) = x^x$

Let us suppose $y = x^x$

Taking logarithm on both sides, we get

$\log y = x \log x$

$\therefore \frac{1}{y} \cdot \frac{dy}{dx} = x \cdot \frac{1}{x} + \log x \cdot 1 \qquad [\because (fg)' = fg' + gf']$

$\Rightarrow \frac{dy}{dx} = (1 + \log x) \cdot x^x$

Find the critical points by equating $\frac{dy}{dx}$ to 0.

$\therefore \frac{dy}{dx} = 0$

$\Rightarrow (1 + \log x)\, x^x = 0$

$\Rightarrow \log x = -1$ as $x^x \neq 0$

$\Rightarrow \log x = \log e^{-1}$

$\Rightarrow x = e^{-1}$

$\Rightarrow x = \frac{1}{e}$

Hence, $f(x)$ has a stationary point at $x = \frac{1}{e}$.

Question 59:

The maximum value of $\left(\frac{1}{x}\right)^x$ is

(a) e

(b) e^e

(c) $e^{1/e}$

(d) $\left(\frac{1}{e}\right)^{1/e}$

Solution:*(c)*

Let us suppose $y = \left(\frac{1}{x}\right)^x$

$$\Rightarrow \log y = x \cdot \log \frac{1}{x}$$

$$\therefore \frac{1}{y} \cdot \frac{dy}{dx} = x \cdot \frac{1}{\frac{1}{x}}\left(-\frac{1}{x^2}\right) + \log \frac{1}{x} \cdot 1$$

$$\Rightarrow \frac{1}{y} \cdot \frac{dy}{dx} = -1 + \log \frac{1}{x}$$

$$\Rightarrow \frac{dy}{dx} = \left(\log \frac{1}{x} - 1\right) y$$

$$\Rightarrow \frac{dy}{dx} = \left(\log \frac{1}{x} - 1\right) \cdot \left(\frac{1}{x}\right)^x \qquad \left[\because y = \left(\frac{1}{x}\right)^x\right]$$

Find the critical points by equating $\frac{dy}{dx}$ to 0.

$$\therefore \frac{dy}{dx} = 0$$

$$\Rightarrow \log\frac{1}{x} = 1 = \log e$$

$$\Rightarrow \frac{1}{x} = e$$

$$\therefore \ x = \frac{1}{e}$$

Hence, the maximum value of $f\left(\frac{1}{e}\right) = (e)^{1/e}$.