CLASS 10th

MATHEMATICS

Short Note

Chapter 2 – POLYNOMIALS

- **Polynomials:** A polynomial is an expression consisting of constants, variables and exponents. Its mathematical form is-
  $f(x) = a_nx^n + a_{n-1}x^{n-1} + \ldots + a_1x + a_0,\ a_n \neq 0$ is called a polynomial in variable x.
  where x: variable
  a: Real number
  n: whole number
  For example:
  P(x) = 3x –2 is a polynomial in variable x.

  Degree of a Polynomial: The highest power of x in a polynomial f(x) is called the degree of the polynomial f(x)
  - Following are forms of various degree polynomials:

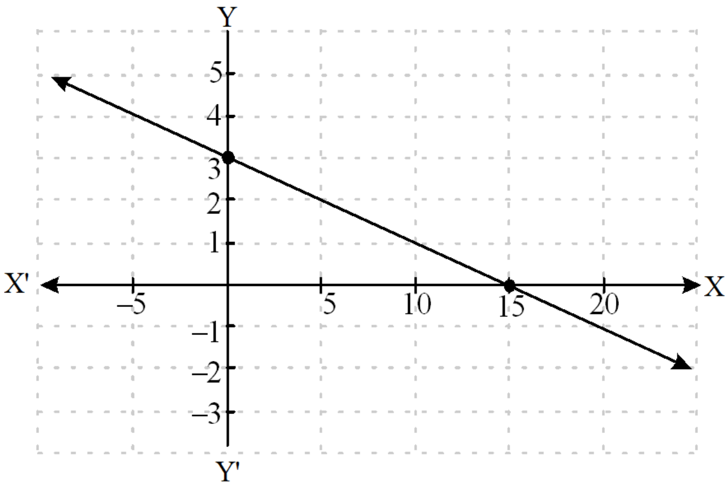
Degree	Name of the polynomial	Form of the polynomial
0	Constant polynomial	$f(x)$ = a, a is a constant
1	Linear polynomial	$f(x) = ax + b,\ a \neq 0$
2	Quadratic polynomial	$f(x) = ax^2 + bx + c,\ a \neq 0$
3	Cubic polynomial	$f(x) = ax^3 + bx^2 + cx + d,\ a \neq 0$
4	Biquadratic polynomial	$f(x) = ax^4 + bx^3 + cx^2 + dx + e,\ a \neq 0$

- **Value of Polynomial:** Let p(y) is a polynomial in y and α could be any real number, then the value calculated after putting the value y = α in p(y) is the final value of p(y) at y = α. This shows that p(y) at y = α is represented by p (α).

- **Zero of a Polynomial:** If the value of p(y) at y = k is 0, that is p (k) = 0 then y = k will be the zero of that polynomial p(y).

- **Geometrical meaning of the Zeroes of a Polynomial:** Zeroes of the polynomials are the x coordinates of the points where the graph of that polynomial intersects the x-axis.

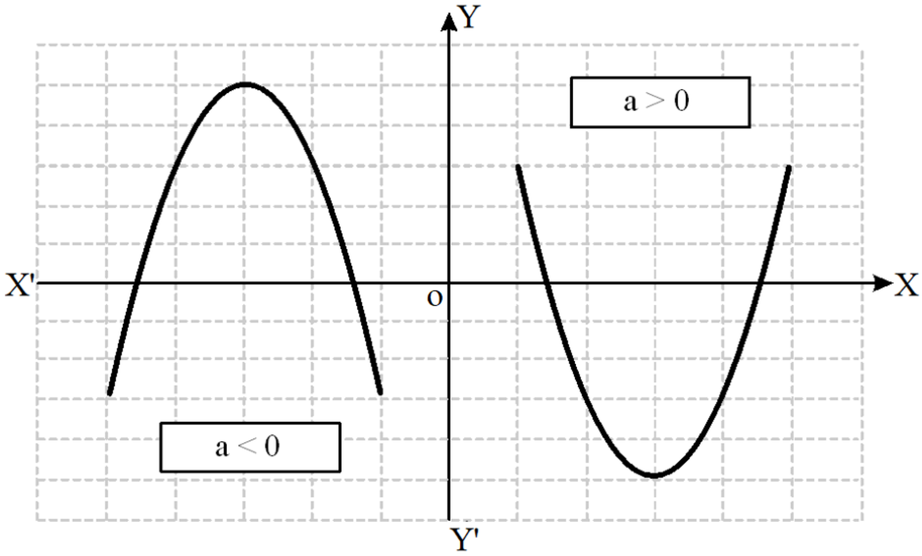
- **Graph of a Linear Polynomial:**



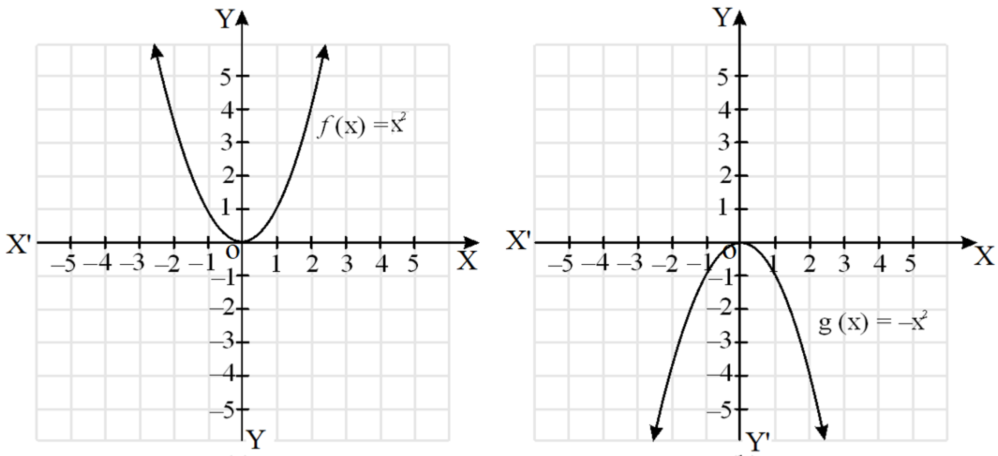
Graph of a linear polynomial is a straight line which intersects the x-axis at one point only, so a linear polynomial has degree 1.

Graph of Quadratic Polynomial:

Case 1: When the graph cuts the x-axis at the two points then these two points are the two zeroes of that quadratic polynomial.

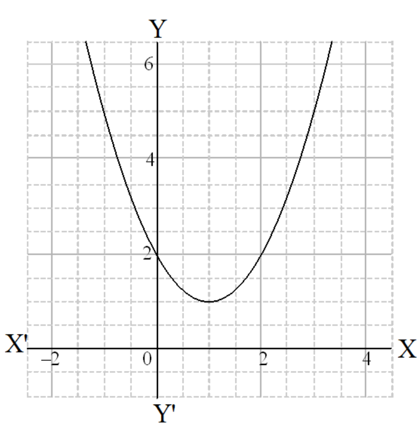


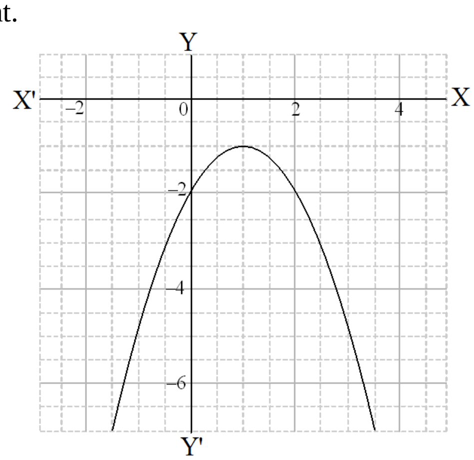
Case 2: When the graph touches the x-axis at only one point then that particular point is the zero of that quadratic polynomial and the equation is in the form of a perfect square.



(a) (b)

Case 3: When the graph does not intersect the x-axis at any point i.e. the graph is either completely above the x-axis or below the x-axis then that quadratic polynomial has no zero as it is not intersecting the x-axis at any point.





Hence the quadratic polynomial can have either two zeroes, one zero or no zero. Or you can say that it can have maximum two zero only.

Relationship between Zeroes and Coefficients of a Polynomial:

- If α and β are the zeros of a quadratic polynomial $f(x) = ax^2 + bx + c$, $a \neq 0$

$$\alpha + \beta = -\frac{b}{a} = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

$$\alpha\beta = \frac{c}{a} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

- If α, β, γ are the zeros of a cubic polynomial $f(x) = ax^3 + bx^2 + cx + d$, $a \neq 0$

$$\alpha + \beta + \gamma = -\frac{b}{a} = -\frac{\text{Coefficient of } x^2}{\text{Coefficient of } x^3}$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a} = \frac{\text{Coefficient of } x}{\text{Coefficient of } x^3}$$

$$\alpha\beta\gamma = -\frac{d}{a} = -\frac{\text{Constant term}}{\text{Coefficient of } x^3}$$

- If α, β, γ, δ are the zeros of a biquadratic polynomial $f(x) = ax^4 + bx^3 + cx^2 + dx + e$, $a \neq 0$

$$\alpha + \beta + \gamma + \delta = -\frac{b}{a} = -\frac{\text{Coefficient of } x^3}{\text{Coefficient of } x^4}$$

$$(\alpha + \beta)(\gamma + \delta) + \alpha\beta + \gamma\beta = \frac{c}{a} = \frac{\text{Coefficient of } x^2}{\text{Coefficient of } x^4}$$

$$(\alpha + \beta)\gamma\delta + \alpha\beta(\gamma + \delta) = -\frac{d}{a} = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^4}$$

$$\alpha\beta\gamma\delta = \frac{e}{a} = \frac{\text{Constant terms}}{\text{Coefficient of } x^4}$$

REMARKS:

- If α and β are the zeros of a quadratic polynomial f(x). Then, the polynomial f(x) is given by

  $f(x) = k\{x^2 - (\alpha + \beta)x + \alpha\beta\}$

  or,

  $f(x) = k\{x^2 -$ (Sum of the zeros) $x +$ Product of the zeros$\}$, where k is any non-zero real number.

- It follows from the above discussion that a cubic polynomial having α, β and γ as its zeros is given by

  $f(x) = k(x - \alpha)(x - \beta)(x - \gamma)$

  or,

  $f(x) = k\{x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma\}$, where k is any non-zero real number.

Division Algorithm for Polynomial

If $f(x)$ and $g(x)$ are any two polynomials with $g(x) \neq 0$, then we can find polynomials $q(x)$ and $r(x)$ such that

$f(x) = g(x) \times q(x) + r(x)$, $r(x) = 0$ or degree of $r(x)$ < degree of $g(x)$.

where,

$f(x)$ is the dividend

$g(x)$ is the divisor

$q(x)$ is the quotient

$r(x)$ is the remainder