Class 10th

Mathematics

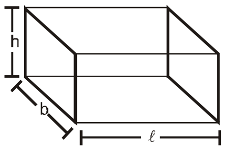
Short Note

SURFACE AREA & VOLUME

Some Solid Figures:-

(1) Cuboid :

If ℓ, b and h denote respectively the length, breadth and height of a cuboid, then :



(i) Total surface area of the cuboid (T.S.A.) = 2 (ℓb+bh+ℓh) sq. unit

(ii) Volume of the cuboid

= Area of the base × height = ℓbh cubic unit

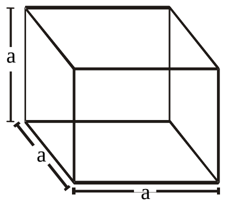
(iii) Diagonal of the cuboid or longest rod

$$= \sqrt{\ell^2 + b^2 + h^2} \text{ unit.}$$

(iv) Area of four walls of a room

$$= 2(\ell + b)\, h \text{ sq. unit}$$

(2) Cube : If the length of each edge of a cube is 'a' units, then :



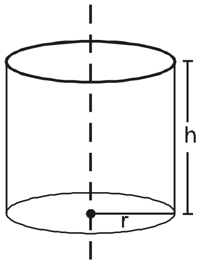
(i) Total surface area of the cube (T.S.A.) = $6a^2$ sq. unit.

(4) Lateral surface area (L.S.A.) = $4a^2$ sq. unit.

(4) Volume of the cube = a^3 cubic unit.

(iv) Diagonal of the cube = $\sqrt{3}\,a$ unit.

(3) Cylinder : If r and h denote respectively the radius of the base and height of a right circular cylinder, then :



(4) Area of each circular end = πr^2 sq. unit

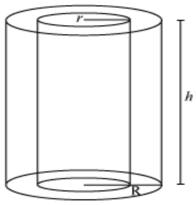
(ii) Curved surface area (C.S.A.)

= circumference × height = $2\pi rh$ sq. units

(4) Total surface area (T.S.A.) = $2\pi r(h + r)$ sq. unit.

(iv) Volume = Area of the base × height = $\pi r^2 h$ cu. Unit.

(4) Hollow cylinder : If R and r (R > r) denote respectively the external and internal radii of a hollow right circular cylinder, then :



(i) Area of each circular end = $\pi(R^2 - r^2)$ sq. unit.

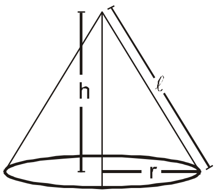
(ii) Curved surface area (C.S.A.) = $2\pi(R + r)h$ sq. unit.

(iii) Total surface area (T.S.A.) =

$2\pi(R + r)(R + h - r)$ sq. unit.

(iv) Volume of material = $\pi h(R^2 - r^2)$ cubic unit

(5) Cone : If r, h and ℓ denote respectively the radius of base, height and slant height of a right circular cone, then :



(5) $\ell^2 = r^2 + h^2$

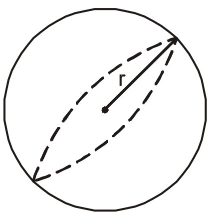
(i) Curved surface area (C.S.A.) = $\pi r\ell$ sq. unit.

(ii) Total surface area (T.S.A.) = $\pi r^2 + \pi r\ell$

$= \pi r(\ell + r)$ sq. unit.

(iii) Volume = $\frac{1}{3}\pi r^2 h$ cubic unit

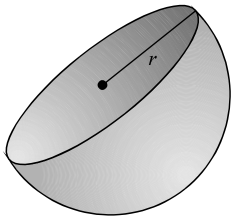
(6) Sphere : For a sphere of radius r, we have :



(i) Surface area = $4\pi r^2$ sq. unit.

(ii) Volume = $\frac{4}{3}\pi r^3$ cubic unit.

(7) Hemisphere :



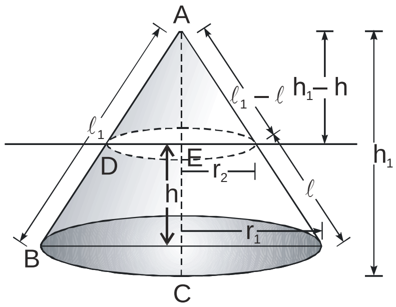
Curved surface area (C.S.A.) = $2\pi r^2$ sq. unit.

Total surface area (T.S.A.) = $3\pi r^2$ sq. unit.

Volume = $\frac{2}{3}\pi r^3$ cubic unit.

- **FRUSTUM OF A CONE:-**

If a right circular cone is cut off along a plane parallel to its base, the portion of the cone between the plane and base of the cone is known as **'Frustum of Cone'.**



(i) Volume of Frustum

$$= \frac{1}{3}\pi h\left[r_1^2 + r_2^2 + r_1 r_2\right] \text{ cubic unit.}$$

(ii) Curved Surface Area of Frustum

= $\pi\ell(r_1 + r_2)$ sq. unit.

(iii) Total surface area = $\pi\{(r_1 + r_2)\,\ell + r_1^2 + r_2^2\}$ sq. unit.

(iv) Slant height of the frustum = $\sqrt{h^2 + (r_1 - r_2)^2}$ unit.

(v) Height of the cone of which the frustum is a part = $\frac{hr_1}{r_1 - r_2}$ unit.

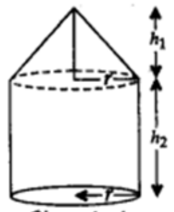
(vi) Slant height of the cone of which the frustum is a part = $\frac{\ell r_1}{r_1 - r_2}$ unit

(vii) Volume of the frustum

= $\frac{h}{3}\left\{A_1 + A_2 + \sqrt{A_1 A_2}\right\}$ cu. unit, where A_1 and A_2 denote the areas of circular bases of the frustum.

Surface Area And Volume Of Combinations:-

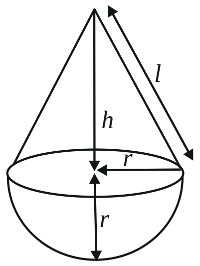
Cone on a Cylinder.



Circus tent

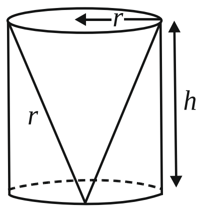
r = radius of cone & cylinder;
h_1 = height of cone
h_2 = height of cylinder
Total Surface area = Curved surface area of cone + Curved surface area of cylinder + area of circular base
$= \pi rl + 2\pi rh_2 + \pi r^2$;
Slant height, I $= \sqrt{r^2 + h_1^2}$
Total Volume = Volume of cone + Volume of cylinder
$= \frac{1}{3}\pi r^2 h_1 + \pi r^2 h_2$

Cone on a Hemisphere:-



h = height of cone;
I = slant height of cone $= \sqrt{r^2 + h^2}$
r = radius of cone and hemisphere
Total Surface area = Curved surface area of cone + Curved surface area of hemisphere $= \pi rl + 2\pi r^2$
Volume = Volume of cone + Volume of hemisphere $= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$

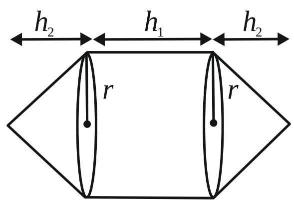
Conical Cavity in a Cylinder:-



r = radius of cone and cylinder;
h = height of cylinder and conical cavity;
I = Slant height
Total Surface area = Curved surface area of cylinder + Area of the bottom face of cylinder + Curved surface area of cone $= 2\pi rh + \pi r^2 + \pi rl$

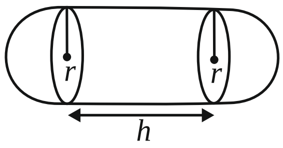
Volume = Volume of cylinder - Volume of cone $= \pi r^2 h - \frac{1}{3}\pi r^2 h = \frac{2}{3}\pi r^2 h$

Cones on Either Sides of the Cylinder:-

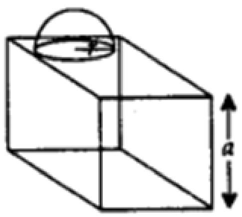


r = radius of cylinder and cone;
h_1 = height of the cylinder
h_2 = height of cones
Slant height of cone, $I = \sqrt{h_2^2 + r^2}$
Surface area = Curved surface area of 2 cones + Curved surface area of cylinder $= 2\pi rl + 2\pi rh_1$

Cylinder with Hemispherical Ends.:-

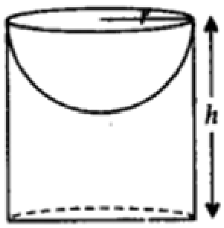


r = radius of cylinder and hemispherical ends;
h = height of cylinder
Total surface area = Curved surface area of cylinder + Curved surface area of 2 hemispheres $= 2\pi rh + 4\pi r^2$
Volume = Volume of cylinder + Volume of 2 hemispheres $= \pi r^2 h + \frac{4}{3}\pi r^3$



a = side of cubee
r = radius of hemisphere.
Surface area = Surface area of cube - Area of hemisphere face + Curved surface area of hemisphere $= 6a^2 - \pi r^2 + 2\pi r^2 = 6a^2 + \pi r^2$
Volume = Volume of cube + Volume of hemisphere $= a^3 + \frac{4}{3}\pi r^3$

Hemispherical Cavity in a Cylinder :-



r = radius of hemisphere;
h = height of cylinder
Total surface area = Curved surface area of cylinder + Surface area of base + Curved surface area of hemisphere

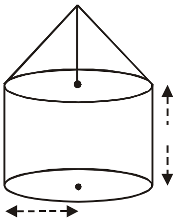
$= 2\pi rh + \pi r^2 + 2\pi r^2 = 2\pi rh + 3\pi r^2$

Volume = Volume of cylinder - Volume of hemisphere $= \pi r^2 h - \frac{2}{3}\pi r^3$

❖ EXAMPLES ❖

A circus tent is in the shape of a cylinder, surmounted by a cone then total inner curved surface area of the tent.

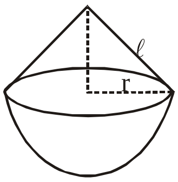
According to the given statement, the rough sketch of the circus tent will be as shown:



= C.S.A. of cylindrical portion + C.S.A. of the conical portion

Formula for the total surface area of each figure given bellow :

(i)

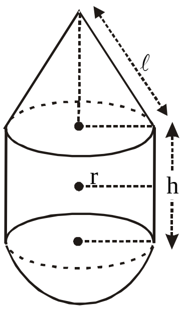


Required surface area

= C.S.A. of the hemisphere + C.S.A. of the cone

$= 2\pi r^2 + \pi r\ell = \pi r\,(2r + \ell)$ sq. unit

(ii)



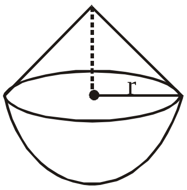
Required surface area

= C.S.A. of the hemisphere

+ C.S.A. of the cylinder + C.S.A. of the cone

$= 2\pi r^2 + 2\pi rh + \pi r\ell = \pi r\,(2r + 2h + \ell)$ sq. unit

(iii)



If slant height of the given cone be ℓ

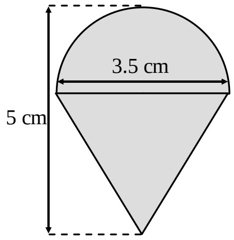
$= \ell^2 = h^2 + r^2 \quad \Rightarrow \quad \ell = \sqrt{h^2 + r^2}$

And, required surface area

$= 2\pi r^2 + \pi r\ell = \pi r\,(2r + \ell)$

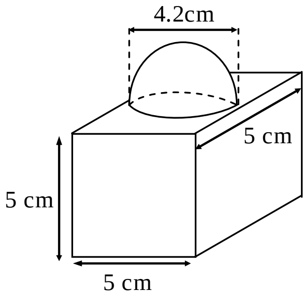
$= \pi r \left(2r + \sqrt{h^2 + r^2}\right)$ sq. unit

In given figure the top is shaped like a cone surmounted by a hemisphere



TSA of the top = CSA of hemisphere + CSA of cone

The decorative block shown in figure is made of two solids - a cube and a hemisphere.



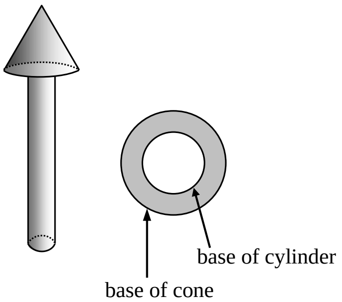
The total surface area of the cube = 6 × (edge).

Note that the part of the cube where the hemisphere is attached is not included in the surface area.

So, the surface area of the block

= TSA of cube – Base area of hemisphere + CSA of hemisphere

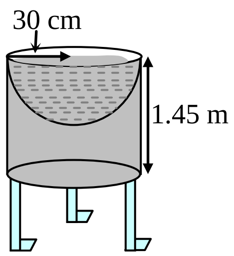
A wooden toy rocket is in the shape of a cone mounted on a cylinder, as shown in figure.



Let radius of cone be r, slant height of cone be ℓ, height of cone be h, radius of cylinder by r' and height of cylinder by h'. Here, the conical portion has its circular base resting on the base of the cylinder, but the base of the cone is larger than the base of the cylinder. So a part of the base of the cone (a ring) is to be painted. So, the area to be painted orange

= CSA of the cone + base area of the cone – base area of the cylinder

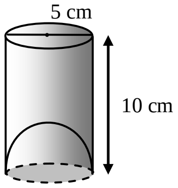
A bird bath for garden is in the shape of a cylinder with a hemispherical depression at one end (see figure).



Let h be height of the cylinder and r the common radius of the cylinder and hemisphere. Then, the total surface area of the bird-bath

= CSA of cylinder + CSA of hemisphere + bottom of cylinder

A juice seller was serving his customers using glasses as shown in figure. The inner diameter of the cylindrical glass is given, but the bottom of the glass had a hemispherical raised portion which reduced the capacity of the glass. Then the apparent capacity of the glass and its actual capacity.



The actual capacity of the glass is less by the volume of the hemisphere at the base of the glass. So, the actual capacity of the glass = apparent capacity of glass – volume of the hemisphere

Conversion of Solid from One Shape to Another:

When a solid is converted into another solid of a different shape (by melting or casting), the volume remains constant.

Surface Areas and Volumes Formulas

Shape	Parameters	Surface Area (Square units)	Volume (Cubic units)
Cuboid	Length = l Breadth = b Height = h	TSA = 2(lb + bh + lh) LSA = 2h(l + b)	V = l × b × h
Cube	Length = Breadth = Height = l	TSA = $6l^2$ LSA = $4l^2$	V = l^3
Cylinder	Radius = r Height = h	CSA = 2π × r × h TSA = 2πr(h + r)	V = $\pi r^2 h$
Cone	Radius = r Height = h Slant Height = l	CSA = πrl TSA = πr(l + r)	V = $(1/3)\pi r^2 h$
Sphere	Radius = r	CSA = TSA = $4\pi r^2$	V = $(4/3)\pi r^3$
Hemisphere	Radius = r	CSA = $2\pi r^2$ TSA = $3\pi r^2$	V = $(2/3)\pi r^3$
Frustum	Radius of top circular part = r_1 Radius of bottom circular part = r_2 Height = h Slant height = l	CSA = $\pi(r_1+r_2)l$ TSA = $\pi(r_1 + r_2)l + \pi(r_1^2 + r_2^2)$	V = $(1/3)\pi h(r_1^2 + r_2^2 + r_1 r_2)$