Class 10th

MATHEMATICS

CIRCLES

Introduction to Circles

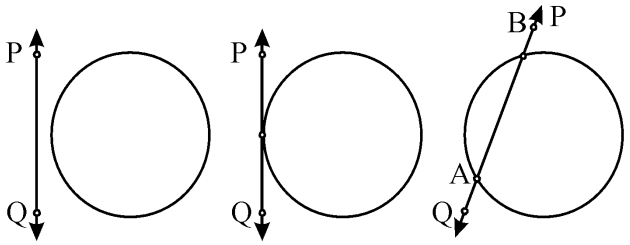
Circle and line in a plane

For a circle and a line on a plane, there can be three possibilities.

i) they can be **non-intersecting**

ii) they can have a **single common point**: in this case, the line touches the circle.

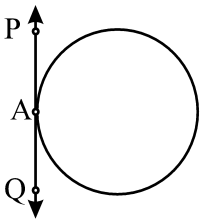
ii) they can have **two common points**: in this case, the line cuts the circle.



(i) Non intersection, (ii) Touching, (iii) Intersecting

Tangent

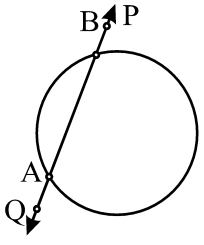
A **tangent to a circle** is a line that touches the circle at exactly one point. For every point on the circle, there is a unique tangent passing through it.



Tangent

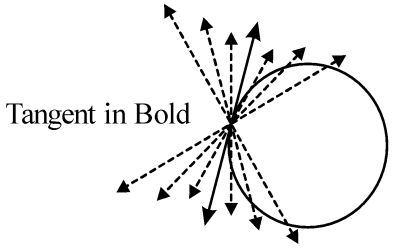
Secant

A **secant to a circle** is a line that has two points in common with the circle. It cuts the circle at two points, forming a chord of the circle.



Secant

Tangent as a special case of Secant

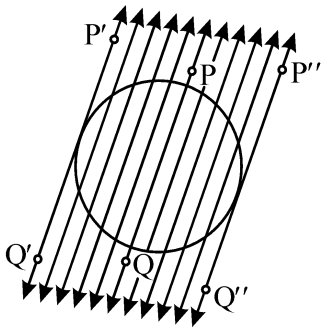


Tangent as a special case of Secant

The tangent to a circle can be seen as a special case of the secant when the two endpoints of its corresponding chord coincide.

Two parallel tangents at most for a given secant

For every given secant of a circle, there are exactly two tangents which are parallel to it and touch the circle at two diametrically opposite points.

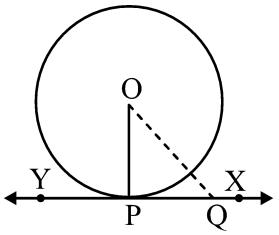


Parallel tangents

Theorems

Tangent perpendicular to the radius at the point of contact

Theorem: The theorem states that "the tangent to the circle at any point is the perpendicular to the radius of the circle that passes through the point of contact".

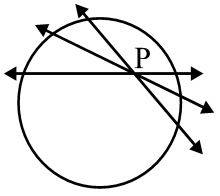


Tangent and radius

Here, O is the centre and OP⊥XY.

The number of tangents drawn from a given point

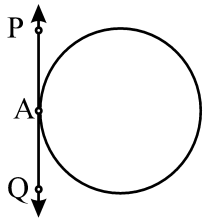
i) If the point is in an interior region of the circle, any line through that point will be a secant. So, no tangent can be drawn to a circle which passes through a point that lies inside it.



No tangent can be drawn to a circle from a point inside it

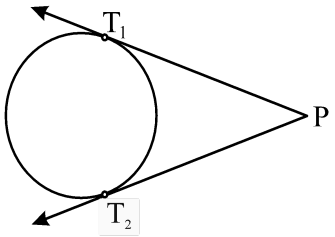
AB is a secant drawn through the point S

ii) When a point of tangency lies on the circle, there is **exactly one tangent** to a circle that passes through it.



A tangent passing through a point lying on the circle

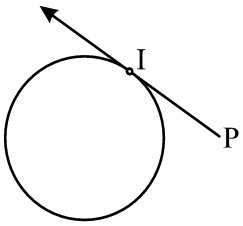
iii) When the point lies outside of the circle, there are accurately two tangents to a circle through it



Tangents to a circle from an external point

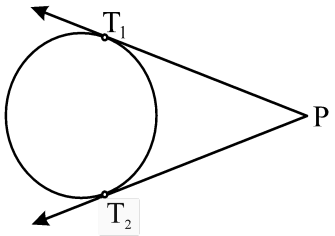
Length of a tangent

The length of the tangent from the point (Say P) to the circle is defined as the segment of the tangent from the external point P to the point of tangency I with the circle. In this case, PI is the tangent length.



Lengths of tangents drawn from an external point

Theorem: Two tangents are of equal length when the tangent is drawn from an external point to a circle.



Tangents to a circle from an external point

$PT_1 = PT_2$

Theorem 10.1: The tangent at any point of a circle is **perpendicular** to the radius through the **point of contact**.
Theorem 10.2: The lengths of tangents drawn from an external point to a circle are equal.

Interesting facts about Circles and their properties are listed below:

- In two concentric circles, the chord of the larger circle, which touches the smaller circle, is bisected at the point of contact.
- The tangents drawn at the ends of a diameter of a circle are parallel.
- The perpendicular at the point of contact to the tangent to a circle passes through the centre.

- The angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segment joining the points of contact at the centre.

[4]