Class 10th

Mathematics

Short Note

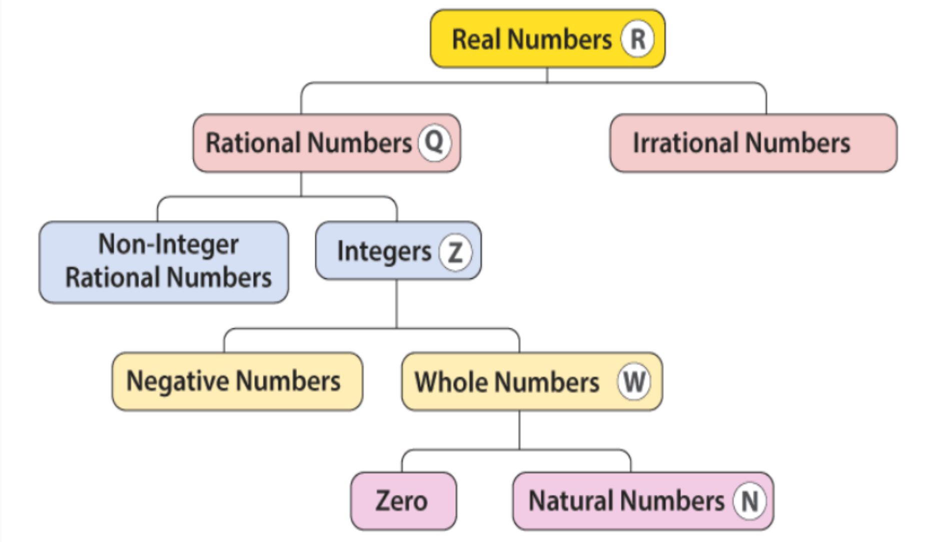
Chapter 1- Real Numbers

■ **Real Numbers :**

➢ Real numbers constitute the union of all rational and irrational numbers.

➢ Any real number can be plotted on the number line.

➢ We can say that any number is a real number, except for complex numbers. Examples of real numbers include $-5, \frac{2}{3}, 3.75, \sqrt{7}$ and so on.



■ **Euclid's division lemma :** Given positive integers a and b there exist whole number q and r satisfying $a = bq + r$, $0 \le r < b$.

Euclid's Division Algorithm: In order to compute the HCF of two positive integers, say a and b, with a > b by using Euclid's algorithm we follow the following steps:

STEP I: Apply Euclid's division lemma to a and b and obtain whole numbers q_1 and r_1 such that $a = bq_1 + r_1$, $0 \le r_1 < b$.

STEP II: If $r_1 = 0$, b is the HCF of a and b

STEP III: If $r_1 \ne 0$, apply Euclid's division lemma to b and r_1 and obtain two whole numbers q_1 and r_2 such that $b = q_1 r_1 + r_2$.

STEP IV : If $r_2 = 0$, then r_1 is the HCF of a and b.

STEP V : If $r_2 \ne 0$, then apply Euclid's division lemma to r_1 and r_2 continue the above process till the remainder r_n is zero. The divisor at this stage i.e. r_{n-1}, or the non-zero remainder at the previous stage, is the HCF of a and b.

The Fundamental Theorem of Arithmetic :

Every composite number can be expressed (factorized) as a product of primes, and this factorization is unique, apart from the order in which the prime factors occur.

- **Example 1:** $60 = 2 \times 2 \times 3 \times 5$ OR, $60 = 2 \times 3 \times 2 \times 5$
- **Example 2 :** Consider the numbers 4^n , where n is a natural number. Check whether there is any value of n for which 4^n ends with the digit zero.

Solution : If the number 4^n, for any n, were to end with the digit zero, then it would be divisible by 5. That is, the prime factorization of 4^n would contain the prime 5. This is not possible because $4^n = 2^{2n}$; so the only prime in the factorization of 4^n is 2. So, the uniqueness of the Fundamental Theorem of Arithmetic guarantees that there are no other primes in the factorization of 4^n. So, there is no natural number n for which 4^n ends with the digit zero.

Prime Factorisation :

- Prime Factorisation is the method of expressing a natural number as a product of prime numbers.
- **Example:** $30 = 2 \times 3 \times 5$ is the prime factorization of 30.

Method of Finding LCM :

As we know, the smallest of the common multiples of two or more numbers is called the lowest common multiple (LCM).

Example: To find the Least Common Multiple (L.C.M) of 90 and 210

1. $90 = 2 \times 3 \times 3 \times 5$

   $210 = 2 \times 3 \times 5 \times 7$

2. LCM of 90 and $210 = 2 \times 3 \times 3 \times 5 \times 7$ which is 630

Method of Finding HCF :

We know that the greatest number that divides each of the given numbers without leaving any remainder is the highest common factor (HCF) of two or more given numbers.
HCF can be found using two methods – Prime factorisation and Euclid's division algorithm.

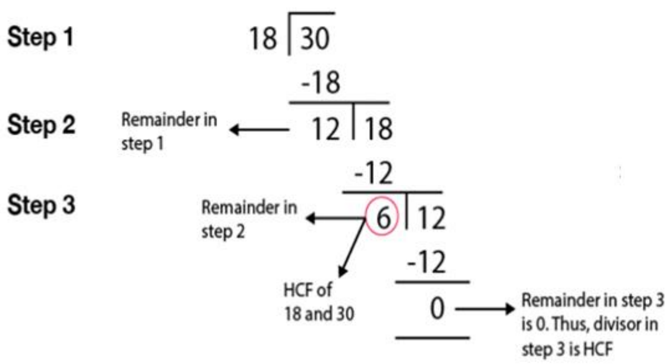
- **Example –** To find the HCF of 20 and 24

  $20 = 2 \times 2 \times 5$ and $24 = 2 \times 2 \times 2 \times 3$

- The factor common to 20 and 24 is 2×2, which is 4, which in turn is the HCF of 20 and 24.

■ **Euclid's Division Algorithm :**

- It is the repeated use of Euclid's division lemma to find the HCF of two numbers.
- **Example:** To find the HCF of 18 and 30



- The required HCF is **6**.

Product of Two Numbers = HCF × LCM of the Two Numbers :

- For any **two** positive integers a and b,

  HCF (a, b) × LCM $(a, b) = a \times b$

- **Example –** For 15 and 20, the HCF is 5 and the LCM is 60

  HCF (a, b) × LCM $(a, b) = a \times b$

  $5 \times 60 = 15 \times 20$

  Thus $300 = 300$

Note: The above relationship, however, doesn't hold true for 3 or more numbers.

■ HCF, LCM OF THREE INTEGERS :

HCF (p, q, r) × LCM $(p, q, r) \neq p \times q \times r$, where p, q, r are positive integers. However, the following results hold good for three numbers p, q, and r :

$$\text{LCM}(p, q, r) = \frac{(p \times q \times r) \times HCF(p,q,r)}{HCF(p,q) \times HCF(q,r) \times HCF(p,r)}$$

$$\text{HCF}(p, q, r) = \frac{p \times q \times r \times LCM(p,q,r)}{LCM(p,q) \times LCM(q,r) \times LCM(p,r)}$$

Rational numbers:-

- A number which can be expressed in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

  Example –: $\frac{2}{5}, \frac{-1}{4}, 0.\overline{16}, 2.1\overline{3}$

Irrational Numbers :

The number which are not rational is called irrational numbers.

Example – $\sqrt{2}$, π , e, $\sqrt{3}+1$

Theorem: Let p be a prime number. If p divides a^2, then p divides a, where a is a positive integer

Determining the nature of the decimal expansion of Rational Number:

(1) Let x be a rational number whose decimal expansion terminates. Then x can be expressed in the form $\frac{p}{q}$, where p & q are co-prime, and the prime factorization of q is of the form $2^m \times 5^n$, where m, n are non-negative integers.

(2) Let $x = \frac{p}{q}$ be a rational number, such that the prime factorization of q is of the form $2^m \times 5^n$, where m, n are non-negative integers. Then, x has a terminating decimal expansion which terminates after k places of decimals, where k is the larger of m & n.

(3) Let $x = \frac{p}{q}$ be a rational number, such that the prime factorization of q is not the form of $2^m \times 5^n$, where m, n are non-negative integers. Then, x has a non-terminating repeating decimal expansion.