Class 10th

Short Notes

## MATHEMATICS

### Chapter 13-STATISTICS

**Statistics**

Statistics may be defined as the science of collection, presentation, analysis and interpretation of numerical data.

**Measures of Central Tendency**

The commonly used measures of central tendency (or averages) are : (i) Arithmetic mean (AM) or simply mean (ii) Geometric Mean (iii) Harmonic Mean (iv) Median (v) Mode.

**1. Mean of Grouped Data:**

If $x_1, x_2, x_3, \ldots, x_n$ are n values of a variable X, then the arithmetic mean or simply mean of these values is denoted by $\bar{X}$ and is defined as

$$\bar{X} = \frac{x_1 + x_2 + x_3 + \ldots + x_n}{n} \text{ or, } \bar{X} = \frac{\sum_{i=1}^{n} x_i}{n}$$

Here, the symbol $\sum_{i=1}^{n} x_i$ denotes the sum $x_1 + x_2 + x_3 + \ldots + x_n$.

The arithmetic mean may be computed by any one of the following methods:

(i) Direct method,

(ii) Short-cut method,

(iii) Step-Deviation method.

**1.1 Direct Method:**

If a variate $X$ takes values $x_1, x_2, \ldots, x_n$ with corresponding frequencies $f_1, f_2, f_3, \cdots, f_n$ respectively, then arithmetic mean of these values is given by

$$\bar{X} = \frac{f_1x_1 + f_2x_2 + \cdots + f_nx_n}{f_1 + f_2 + \cdots + f_n}$$

$$\text{or, } \bar{X} = \frac{\sum_{i=1}^{n} f_i x_i}{N}, \text{ where } N = \sum_{i=1}^{n} f_i = f_1 + f_2 \cdots + f_n$$

**Example1:** Find the mean of the following distribution:

| | | | | | |
|---|---|---|---|---|---|
| x: | 4 | 6 | 9 | 10 | 15 |
| f: | 5 | 10 | 10 | 7 | 8 |

**Solution:** Calculation of Arithmetic Mean

| $x_i$ | $f_i$ | $f_ix_i$ |
|---|---|---|
| 4 | 5 | 20 |
| 6 | 10 | 60 |
| 9 | 10 | 90 |
| 10 | 7 | 70 |
| 15 | 8 | 120 |
| | $N = \Sigma f_i = 40$ | $\Sigma f_i x_i = 360$ |

$$\therefore \quad \text{Mean } = \bar{X} = \frac{\sum f_i x_i}{\sum f_i} = \frac{360}{40} = 9$$

## 1.2 Short-Cut Method

Let $x_1, x_2 \ldots, x_n$ be values of a variable $X$ with corresponding frequencies $f_1, f_2, f_3, \ldots, f_n$ respectively. Taking deviations about an arbitrary point 'A'(where A= assumed mean), we have

$d_1 = x_i - A, i = 12.3, \ldots, n$

$$\Rightarrow \bar{X} = A + \frac{1}{N}\sum_{i=1}^{n} f_i d_i$$

Finding AM by using the above formula is known as the short-cut method.

**Example2:** The following table shows the weights of 12 students:

| Weight (in kg): | 67 | 70 | 72 | 73 | 75 |
|---|---|---|---|---|---|
| Number of students: | 4 | 3 | 2 | 2 | 1 |

Find the mean weight by using short-cut method

**Solution:** Let the assumed mean be A = 72

**Calculation of mean**

| Weight in kg ($x_i$) | Number of Students $f_i$ | $d_i = x_i - A = x_i - 72$ | $f_i d_i$ |
|---|---|---|---|
| 67 | 4 | –5 | –20 |
| 70 | 3 | –2 | –6 |
| 72 | 2 | 0 | 0 |
| 73 | 2 | 1 | 2 |
| 75 | 1 | 3 | 3 |
| | $N = \Sigma f_i = 12$ | | $\sum f_i d_i = -21$ |

We have,

$N = 12, \Sigma f_i d_i = -21,$ and $A = 72$

$$\therefore \text{ Mean} = A + \frac{1}{N}\left(\sum f_i d_i\right)$$

$$\Rightarrow \text{ Mean} = 72 + \frac{(-21)}{12} = 72 - \frac{7}{4}$$

$$\Rightarrow \text{ Mean} = \frac{288-7}{4} = \frac{281}{4} = 70.25 \text{ kg}$$

Hence, mean weight $= 70.25$ kg .

## 1.3 Step-Deviation Method

Sometimes, during the application of the short-cut method for finding AM, the deviations $d_i$ are divisible by a common number h (say). In such a case the arithmetic is reduced to a great extent by taking

$$u_i = \frac{x_i - A}{h}; i = 1, 2, 3, \cdots, n$$

$$\bar{X} = A + h\left\{\frac{1}{N}\sum_{i=1}^{n} f_i u_i\right\}$$

**Example3:** Find the mean wage from the following data:

| Wage (in ₹) | 800 | 820 | 860 | 900 | 920 | 980 | 1000 |
|---|---|---|---|---|---|---|---|
| No. of workers: | 7 | 14 | 19 | 25 | 20 | 10 | 5 |

**Solution:** Let the assumed mean be A = 900 and h = 20.

**Calculation of mean**

| Wage (in ₹) $x_i$ | No. of workers: $f_i$ | $d_i = x_i - A$ $= x_i - 900$ | $u_i = \frac{x_i - 900}{20}$ | $f_iu_i$ |
|---|---|---|---|---|
| 800 | 7 | –100 | –5 | –35 |
| 820 | 14 | –80 | –4 | –56 |
| 860 | 19 | –40 | –2 | –38 |
| 900 | 25 | 0 | 0 | 0 |
| 920 | 20 | 20 | 1 | 20 |
| 980 | 10 | 80 | 4 | 40 |
| 1000 | 5 | 100 | 5 | 25 |
| | $N = \sum f_i = 100$ | | | $\Sigma f_i u_i = -44$ |

We have,

$N = 100, \Sigma f_i u_i = -44, A = 900$ and $h = 20$.

$$\therefore \text{Mean} = \bar{X} = A + h\left(\frac{1}{N}\Sigma f_i u_i\right)$$

$$\Rightarrow \bar{X} = 900 + 20 \times \frac{-44}{100} = 900 - 8.8 = 891.2$$

Hence, mean wage = ₹ 891.2

**1.4 Arithmetic mean of a continuous Frequency Distribution:**

The values of $x_1, x_2, x_3, \cdots, x_n$ are taken as the mid-points or class-marks of the various classes. It should be noted that the mid-value or class-marks of a class interval is equal to $\frac{1}{2}$ (lower limit + upper limit).

**Example4:** Find the mean of the following frequency distribution:

| Class-interval | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 |
|---|---|---|---|---|---|
| No. of workers f: | 7 | 10 | 15 | 8 | 10 |

**Solution:**

**Calculation of mean**

| Class-interval | Mid-values($x_i$) | Frequency $f_i$ | $d_i = x_i - 25$ | $u_i = \frac{x_i - 25}{10}$ | $f_iu_i$ |
|---|---|---|---|---|---|
| 0-10 | 5 | 7 | –20 | –2 | –14 |
| 10-20 | 15 | 10 | –10 | –1 | –10 |
| 20-30 | 25 | 15 | 0 | 0 | 0 |
| 30-40 | 35 | 8 | 10 | 1 | 8 |
| 40-50 | 45 | 10 | 20 | 2 | 20 |
| | | $N = \sum f_i = 50$ | | | $\Sigma f_i u_i = 4$ |

We have,

$A = 25, h = 10, N = 50$ and $\Sigma f_i u_i = 4$

$$\Rightarrow \text{Mean} = A + h\left\{\frac{1}{N}\Sigma f_i u_i\right\}$$

$$\Rightarrow \text{Mean} = 25 + 10 \times \frac{4}{50} = 25.8$$

## 2. Median

Median of individual observations. If $x_1, x_2, x_3, \cdots, x_n$ are $n$ values of a variable $X$, then to find the median we use the following algorithm.

### Algorithm

Step I: Arrange the observations $x_1, x_2, \cdots, x_n$ in ascending or descending order of magnitude.

Step II: Determine the total number of observations, say, n

Step III: If $n$ is odd, then median is the value of $\left(\frac{n+1}{2}\right)^{th}$ observation.

If $n$ is even, then median is the AM of the values of $\left(\frac{n}{2}\right)^{th}$ and $\left(\frac{n}{2}+1\right)^{th}$ observations.

**Example5:**

(i) The following are the marks of 9 students in a class. Find the median.
34, 32, 48, 38, 24, 30, 27, 21, 35

(ii) Find the median of the daily wages of ten workers from the following data:
20, 25, 17, 18, 8, 15, 22, 11, 9, 14.

**Solution:**

(i) Arranging the data in ascending order of magnitude, we have
21, 24, 27, 30, 32, 34, 35, 38, 48

Since there are 9 i.e., an odd number of items. Therefore, median is the value of $\left(\frac{9+1}{2}\right)^{th}$ observation i.e., 32.

(ii) Arranging the wages in ascending order of magnitude, we have 8, 9, 11, 14, 15, 17, 18, 20, 22, 25

Since there are 10 observations Therefore, median is the arithmetic mean of $\left(\frac{10}{2}\right)^{th}$ and $\left(\frac{10}{2}+1\right)^{th}$ observations.

Hence, Median $= \frac{15+17}{2} = 16$

### 2.1 Median of Discrete Frequency Distribution

In case of a discrete frequency distribution $\frac{x_i}{f_i}; i = 1, 2, \cdots, n$ we calculate the median by using the following algorithm.

### Algorithm

**Step I:** Find the cumulative frequencies (c.f.)

**Step II:** Find $\frac{N}{2}$, where $N = \sum_{i=1}^{n} f_i$

**Step III:** The cumulative frequency (c.f) just greater than $\frac{N}{2}$ and determine the corresponding value of the variable.

**Step IV:** The value obtained in step III is the median.

**Example6:** Obtain the median for the following frequency distribution:

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| x: | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| f: | 8 | 10 | 11 | 16 | 20 | 25 | 15 | 9 | 6 |

**Solution:**

**Calculation of median**

| x | f | cf |
|---|---|---|
| 1 | 8 | 8 |
| 2 | 10 | 18 |
| 3 | 11 | 29 |
| 4 | 16 | 45 |
| 5 | 20 | 65 |
| 6 | 25 | 90 |
| 7 | 15 | 105 |
| 8 | 9 | 114 |
| 9 | 6 | 120 |
| | N = 120 | |

$\Rightarrow \frac{N}{2} = 60$

65 > 60

Median=5

## 2.2 Median of a grouped or continuous frequency Distribution

**Algorithm**

**Step I:** Obtain the frequency distribution.

**Step II:** Prepare the cumulative frequency column and obtain $N = \sum f_i$.

**Step III:** Find $\frac{N}{2}$.

**Step IV:** See the cumulative frequency just greater than $\frac{N}{2}$ and determine the corresponding class. This class is known as the median class.

**Step V:** Use the following formula:

Median $= l + \left\{\frac{\frac{N}{2} - F}{f}\right\} \times h$

where,

$l$ = lower limit of the median class

f = frequency of the median class

h = width (size) of the median class

F = Cumulative frequency of the class preceding the median class $N = \sum f_i$

**Example7:** Calculate the median from the following distribution:

| Class | 5-10 | 10-15 | 15-20 | 20-25 | 25-30 | 30-35 | 35-40 | 40-45 |
|---|---|---|---|---|---|---|---|---|
| Frequency | 5 | 6 | 15 | 10 | 5 | 4 | 2 | 2 |

**Solution:**

| Class | Frequency | Cumulative frequency |
|---|---|---|
| 5-10 | 5 | 5 |
| 10-15 | 6 | 11 |
| 15-20 | 15 | 26 |
| 20-25 | 10 | 36 |
| 25-30 | 5 | 41 |
| 35-35 | 4 | 45 |
| 35-40 | 2 | 47 |
| 40-45 | 2 | 49 |
| | N = 49 | |

We have, N = 49

$$\frac{N}{2}=\frac{49}{2}=24.5$$

The cumulative frequency just greater than $\frac{N}{2}$ is 26 and the corresponding class is 15-20. This, 15-20 is the median class such that

$l=15, f=15, F=11$ and $h=5$

$$\therefore \text{ Median } = l+\frac{\frac{N}{2}-F}{f}\times h=15+\frac{24.5-11}{15}\times 5=15+\frac{13.5}{3}=19.5$$

**3. Computation of Mode of A Series of Individual Observations**

In order to compute the mode of a series of individual observations, we first convert it into a discrete series frequency distribution by preparing a frequency table. From the frequency table, we identify the value having maximum frequency. The value of variable so obtained is the mode or modal value.

**Example7**: Find the mode of the following data:

120, 110, 130, 110, 120, 140, 130, 120, 140, 120

**Solution:** Let us first form the frequency table for the given data as given below:

| Value $x_i$: | 110 | 120 | 130 | 140 |
|---|---|---|---|---|
| Frequency $f_i$: | 2 | 4 | 2 | 2 |

We observe that the value 120 has the maximum frequency, so Mode = 120.

**3.1 Computation of Mode for a Continuous Frequency Distribution.**

In case of a grouped or continuous frequency distribution with equal class intervals, we use the following algorithm to compute the mode.

**Algorithm**

**Step I:** Obtain the continuous frequency distribution.

**Step II:** Determine the class of maximum frequency either by inspection or by grouping method. This class is called the modal class.

**Step III:** Obtain the values of the following from the frequency distribution:

$l$ = lower limit of the modal class, f = frequency of the modal class

h = width of the modal class,

$f_1 =$ frequency of the class preceding the modal class.

$f_2 =$ frequency of the class following the modal class.

**Step IV:** Substitute the values obtained in step III in the following formula:

$$\text{Mode}=l+\frac{f-f_1}{2f-f_1-f_2}\times h$$

**Example8:** Compute the mode for the following frequency distribution:

| Size of items: | 0-4 | 4-8 | 8-12 | 12-16 | 16-20 | 20-24 | 24-28 | 28-32 | 32-36 | 36-40 |
|---|---|---|---|---|---|---|---|---|---|---|
| Frequency: | 5 | 7 | 9 | 17 | 12 | 10 | 6 | 3 | 1 | 0 |

**Solution:** Here, the maximum frequency is 17 and the corresponding class is 12-16. So, 12-16 is the modal class such that $l=12, h=4, f=17, f_1=9$ and $f_2=12$.

$$\therefore \text{Mode}=l+\frac{f-f_1}{2f-f_1-f_2}\times h$$

$$\text{Mode}=12+\frac{17-9}{34-9-12}\times 4=12+\frac{8}{13}\times 4=12+\frac{32}{13}=12+2.46=14.46$$

**4. Relationship Among Mean, Median and Mode**

Mode =3 Median -2 Mean

**5. Cumulative Frequency Polygon Curve (An Ogive)**

Two methods of construction a frequency polygon and an ogive.

(i) Less than method

(ii) More than method

**5.1 Less than Method**

To construct a cumulative frequency polygon and an ogive by less than method, we use the following algorithm

**Algorithm**:

**Step I:** Start with the upper limits of class intervals and add class frequencies to obtain the cumulative frequency distribution.

**Step II:** Mark upper class limits along X-axis on a suitable scale.

**Step III:** Mark cumulative frequencies along Y-axis on a suitable scale.

**Step IV:** Plot the points ($x_i$, $f_i$), where $x_i$ is the upper limit of a class and $f_i$ is corresponding cumulative frequency.

**Step V:** Join the points obtained in step IV by a free hand smooth curve to get the ogive and to get the cumulative frequency polygon join the points obtained in step IV by line segments.

**5.2 More than Method**

To construct a cumulative frequency polygon and an ogive by more than method, we use the following algorithm.

**Algorithm**

**STEP I:** Start with the lower limits of the class intervals and from the total frequency subtract the frequency of each class to obtain the cumulative frequency distribution.

**STEP II:** Mark the lower-class limits along X-axis on a suitable scale.

**STEP III:** Mark the cumulative frequencies along $Y$-axis on a suitable scale.

**STEP IV:** Plot the points ($x_i$, $f_i$), where $x_i$ i3s the lower limit of a class and $f_i$ is the corresponding cumulative frequency.

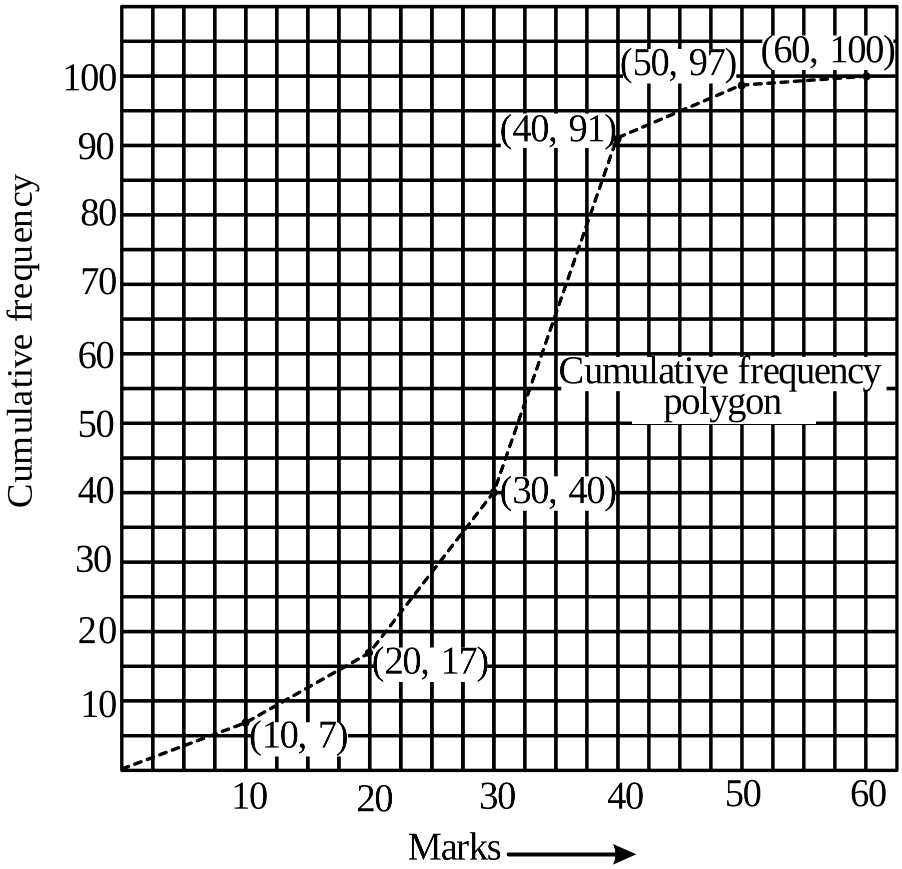
**STEP V:** Join the points obtained in step IV by a free hand smooth curve to get the ogive and to get the cumulative frequency polygon join these points by line segments.

**Example9:** Draw an ogive and the cumulative frequency polygon for the following frequency distribution by less than method.

| **Marks:** | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 | 50-60 |
|---|---|---|---|---|---|---|
| **Number of students:** | 7 | 10 | 23 | 51 | 6 | 3 |

**Solution:**

| **Marks** | **No. of Students** | **Marks less then** | **Cumulative Frequency** |
|---|---|---|---|
| 0-10 | 7 | 10 | 7 |
| 10-20 | 10 | 20 | 17 |
| 20-30 | 23 | 30 | 40 |
| 30-40 | 51 | 40 | 91 |
| 40-50 | 6 | 50 | 97 |
| 50-60 | 3 | 60 | 100 |

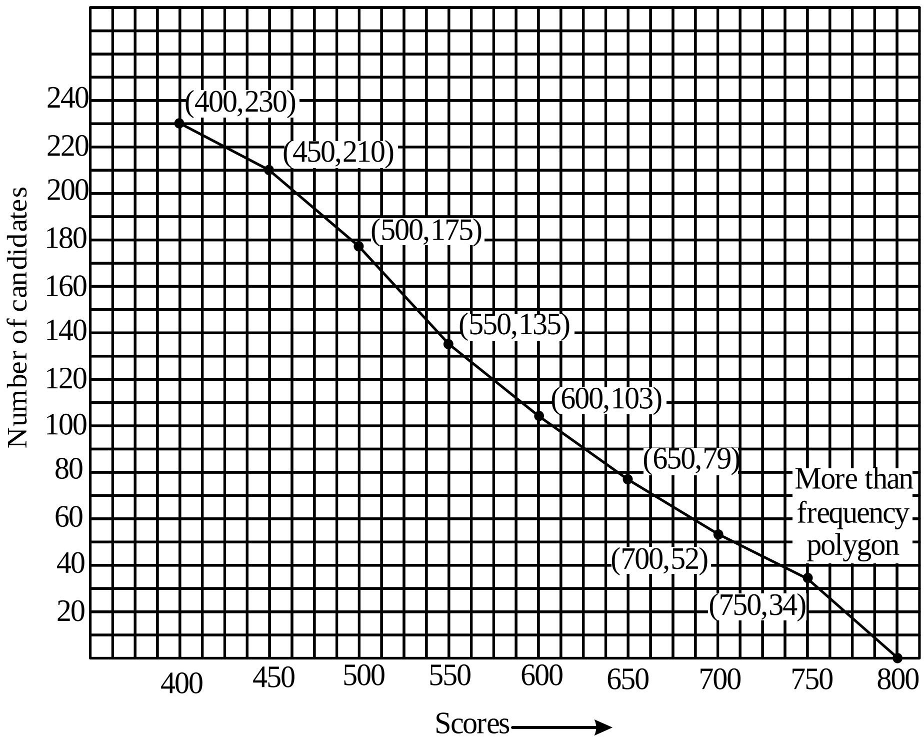



**Example10:** The frequency distribution of scores obtained by 230 candidates in a medical entrance test is as follows.

| **Scores:** | 400-450 | 450-500 | 500-550 | 550-600 | 600-650 | 650-700 | 700-750 | 750-800 |
|---|---|---|---|---|---|---|---|---|
| **Number of students:** | 20 | 35 | 40 | 32 | 24 | 27 | 18 | 24 |

Draw cumulative frequency curve by more than method.

**Solution:**



| Scores | Number of Candidates | Scores more than | Cumulative Frequency |
|---|---|---|---|
| 400-450 | 20 | 400 | 230 |
| 450-500 | 35 | 450 | 210 |
| 500-550 | 40 | 500 | 175 |
| 550-600 | 32 | 550 | 135 |
| 600-650 | 24 | 600 | 103 |
| 650-700 | 27 | 650 | 79 |
| 700-750 | 18 | 700 | 52 |
| 750-800 | 34 | 750 | 34 |


**PW Web/App** - https://smart.link/7wwosivoicgd4

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