Class 10th

Mathematics

Short Notes

Chapter 5- Arithmetic Progressions

1. **Sequence:-**

   A sequence is an arrangement of number or objects in definite order.

2. **Arithmetic Progression:-**

   (i) An arithmetic progression is a list of numbers in which each term is obtained by adding a fixed number to the preceding term except the first term

   (ii) A sequence $a_1, a_2, a_3, \ldots, a_n$ is called an arithmetic progression, if there exists a constant 'd' such that

   $a_2 - a_1 = d, a_3 - a_2 = d, a_4 - a_3 = d, \ldots, a_{n+1} - a_n = d$ and so on.

   The constant 'd' is called the common difference.

   (iii) If 'a' is the first term and 'd' the common difference of an AP, then the $A.P.$ is

   $a, a + d, a + 2d, a + 3d, a + 4d, \ldots\ldots$

3. **General term of an A.P.:-**

   The nth term 'a_n' of an A.P. with first term 'a' and common difference 'd' is given by

   $a_n = a + (n-1)\,d$

4. **n^{th} Term of an A.P. from the end:-**

   (i) Let there be an $A.P.$ with first term 'a' and common difference d. If there are m terms in the AP, then

   n^{th} term from the end $= (m - n + 1)^{th}$ term from the beginning

   $$= a + (m - n)\,d$$

   Also,

   (ii)

   n^{th} term from the end $=$ Last term $+ (n - 1)\,(-d)$

   $$= l - (n - 1)\,d, \text{ where } l \text{ denotes the last term.}$$

5. **Selection of terms in an A.P. :-**

   Various terms is an $A.P.$ can be chosen is the following manner:

Number of terms	Terms	Common difference
3	$a - d, a, a, + d$	d
4	$a - 3d, a - d, a + d, a + 3d$	$2d$
5	$a - 2d, a - d, a, a + d, a + 2d$	d
6	$a - 5d, a - 3d, a - d, a + d, a + 3d, a + 5d$	$2d$

6. **Sum of first n terms of an A.P:-**

   The sum of n terms of an A.P with first term 'a' and common difference 'd' is given by

   $$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

   Also, $S_n = \frac{n}{2}\{a + l\}$, where l = last term = $a + (n-1)d$

7. **Middle Term(s) of a finite A.P.:**

   Let there be a finite A.P. with first term a, common difference d and number of terms n.

   Case1: If n is odd, then $\left(\frac{n+1}{2}\right)^{th}$ term is the middle term and given by $a + \left(\frac{n+1}{2} - 1\right)d$.

   Case2: If n is even, then $\left(\frac{n}{2}\right)^{th}$ and $\left(\frac{n}{2} + 1\right)^{th}$ are middle terms and given by $a + \left(\frac{n}{2} - 1\right)d$ and $a + \left(\frac{n}{2} + 1 - 1\right)d$ respectively.

11. **Arithmetic mean (AM):-**

    If a, b, c are in $A.P$. Then,

    $\Rightarrow 2b = a + c$

    $\Rightarrow b = \frac{a+c}{2}$

    Thus, A.M. between a and c is $\frac{a+c}{2}$.

12. **Relation between S_n and a_n :-**

    If the sum S_n of n terms of a sequence is given, then n^{th} term a_n of the sequence can be determined by using the following formula:

    $a_n = S_n - S_{n-1}$

13. **Sum of some special sequences: -**

- Sum of first 'n' natural numbers

  $$\sum_{k=1}^{n} k = 1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}$$

- Sum of squares of the first 'n' natural numbers

  $$\sum_{k=1}^{n} k^2 = 1^2 + 2^2 + 3^3 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

- Sum of cubes of first 'n' natural numbers

  $$\sum_{k=1}^{n} k^3 = 1^3 + 2^3 + 3^3 + \ldots + n^3 = \left(\frac{n^2(n+1)^2}{4}\right)$$

- Sum of first 'n' odd natural numbers

  $$\sum_{k=1}^{n} (2k-1) = 1 + 3 + 5 + \ldots + (2n-1) = n^2$$