Class 10th

Mathematics

Short Note

TRIGONOMETRY

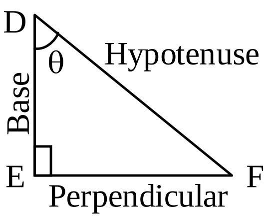
- Trigonometry is the branch of mathematics in which we study the relationships between the sides and the angles of a triangle.

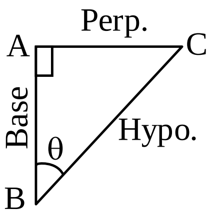
TRIGONOMETRIC RATIOS:-

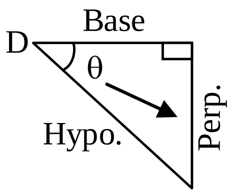
The ratio of sides of a right angle triangle with respect to acute angles are called "Trigonometric ratios of the angle".

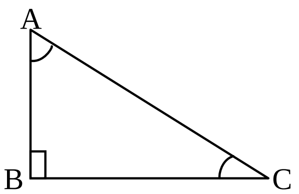
RIGHT ANGLE TRIANGLE:-

1. A triangle having one angle equal to 90º is called right angle triangle.
2. The sum of other two acute (Less than 90º) angles is 90º. (or both acute angles are complementary)
3. The side opposite to 90º, is called hypotenuse, it is longest side in triangle.
4. The side opposite to given acute angle is perpendicular and side adjacent to the angle is base.





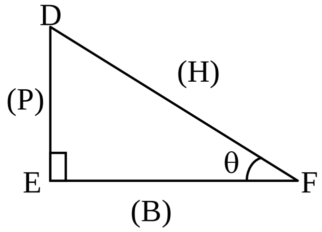




	Hypotenuse	Perpendicular	Base
for ∠A	AC	BC	AB
for ∠C	AC	AB	BC

TRIGONOMETRIC RATIOS:-

TRICK : Some People Have
Curly Brown Hair
To Produce Beauty.



sin θ	= P/H = DE/DF
cos θ	= B/H = EF/DF
tan θ	= P/B = DE/EF
cot θ	= B/P = EF/DE
sec θ	= H/B = DF/EF
cosec θ	= H/P = DF/DE

By above table $\sin\theta = \dfrac{1}{\text{cosec}\theta}$, $\cos\theta = \dfrac{1}{\sec\theta}$,

$\tan\theta = \dfrac{1}{\cot\theta}$

Points To be Remember :

1. The values of sin θ & cos θ are always less than or equal to 1 & greater than or equal to –1.
2. Value of tan θ & cot θ lie between – ∞ to + ∞.
3. sin A, cos A, etc. are not product of sin and A.
4. $(\sin A)^2 \neq \sin A^2$ etc.

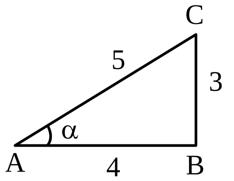
◈ $\sin^2\theta = (\sin\theta)^2$
$\cos^2\theta = (\cos\theta)^2$
$\tan^2\theta = (\tan\theta)^2$
$\text{cosec}^2\theta = (\text{cosec}\,\theta)^2$
$\sec^2\theta = (\sec\theta)^2$
$\cot^2\theta = (\cot\theta)^2$

If $\sec\alpha = \dfrac{5}{4}$, evaluate $\dfrac{1-\tan\alpha}{1+\tan\alpha}$.

Since $\sec\alpha = \dfrac{\text{Hypotenuse}}{\text{Base}} = \dfrac{5}{4}$, so we draw a right triangle ABC, right angled at B such that

Hypotenuse = AC = 5 units,

Base = AB = 4 units, and ∠BAC = α.



By Pythagoras theorem, we have

$AC^2 = AB^2 + BC^2$

$\Rightarrow 5^2 = 4^2 + BC^2$

$\Rightarrow BC^2 = 5^2 - 4^2 = 9$

$\Rightarrow BC = \sqrt{9} = 3$

$\therefore \tan\alpha = \dfrac{BC}{AB} = \dfrac{3}{4}$

Now, $\dfrac{1-\tan\alpha}{1+\tan\alpha} = \dfrac{1-\frac{3}{4}}{1+\frac{3}{4}} = \dfrac{\frac{1}{4}}{\frac{7}{4}} = \dfrac{1}{7}$.

TRIGONOMETRIC RATIO (T.R.) OF SOME SPECIFIC ANGLES:-

The angles 0°, 30°, 45°, 60°, 90° are angles for which we have values of T.R.

$\angle A$	0°	30°	45°	60°	90°
sin A	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos A	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan A	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
cot A	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
sec A	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
cosec A	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

- $\sin\theta \uparrow$ when $\theta \uparrow$, $0^\circ \le \theta \le 90^\circ$
- $\cos\theta \downarrow$ when $\theta \uparrow$, $0^\circ \le \theta \le 90^\circ$
- $\tan\theta$, $\cot\theta$ are not defined for $\theta = 90^\circ$ & 0° respectively.
- $\operatorname{cosec}\theta$, $\sec\theta$ are not defined when $\theta = 0^\circ$ & 90° respectively.
- $\sin\theta = \cos\theta$ for only $\theta = 45^\circ$
- $180^\circ = \pi^c$

$$30^\circ = \left(\frac{\pi}{6}\right)^c;\quad 45^\circ = \left(\frac{\pi}{4}\right)^c;\quad 60^\circ = \left(\frac{\pi}{3}\right)^c;\quad 90^\circ = \left(\frac{\pi}{2}\right)^c$$

TRIGONOMETRIC RATIOS OF COMPLEMENTARY ANGLES:-

We know complementary angles are pair of angles whose sum is 90°

$\sin(90^\circ - \theta) = \cos\theta$, $\cot(90^\circ - \theta) = \tan\theta$

$\cos(90^\circ - \theta) = \sin\theta$, $\sec(90^\circ - \theta) = \operatorname{cosec}\theta$

$\tan(90^\circ - \theta) = \cot\theta$, $\operatorname{cosec}(90^\circ - \theta) = \sec\theta$

Without using trigonometric tables, evaluate the following :

(i) $\frac{\cos 37^\circ}{\sin 53^\circ}$ **(ii)** $\frac{\sin 41^\circ}{\cos 49^\circ}$ **(iii)** $\frac{\sin 30^\circ 17'}{\cos 59^\circ 43'}$

(i) We have

$$\frac{\cos 37^\circ}{\sin 53^\circ} = \frac{\cos(90^\circ - 53^\circ)}{\sin 53^\circ} = \frac{\sin 53^\circ}{\sin 53^\circ} = 1 \qquad [\because \cos(90^\circ - \theta) = \sin\theta]$$

(ii) We have,

$$\frac{\sin 41^\circ}{\cos 49^\circ} = \frac{\sin(90^\circ - 49^\circ)}{\cos 49^\circ} = \frac{\cos 49^\circ}{\cos 49^\circ} = 1 \qquad [\because \sin(90^\circ - \theta) = \cos\theta]$$

(iii) We have,

$$\frac{\sin 30^\circ 17'}{\cos 59^\circ 43'} = \frac{\sin(90^\circ - 59^\circ 43')}{\cos 59^\circ 43'} = \frac{\cos 59^\circ 43'}{\cos 59^\circ 43'} = 1.$$

TRIGONOMETRIC IDENTITIES

(1) $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$; $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

(2) $\sin^2\theta + \cos^2\theta = 1$

(i) $\sin^2\theta = 1 - \cos^2\theta$ (ii) $\cos^2\theta = 1 - \sin^2\theta$

(3) $1 + \tan^2\theta = \sec^2\theta$

(i) $\sec^2\theta - 1 = \tan^2\theta$ (ii) $\sec^2\theta - \tan^2\theta = 1$

(ii) $\tan^2\theta - \sec^2\theta = -1$

(4) $1 + \cot^2\theta = \mathrm{cosec}^2\theta$

(i) $\mathrm{cosec}^2\theta - 1 = \cot^2\theta$

(ii) $\mathrm{cosec}^2\theta - \cot^2\theta = 1$

(iii) $\cot^2\theta - \mathrm{cosec}^2\theta = -1$