Class 10th

Short Notes

MATHEMATICS

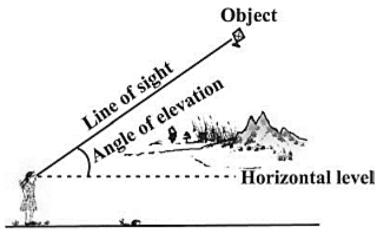
Chapter 9-SOME APPLICATIONS OF TRIGONOMETRY

1. The line of sight:

The line of sight is the line drawn from the eye of an observer to the point in the object viewed by the observer.

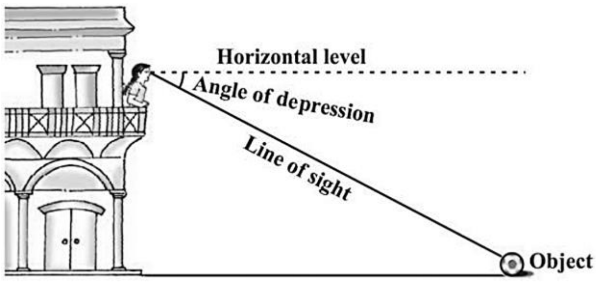
2. The angle of elevation:

The angle of elevation of an object viewed, is the angle formed by the line of sight with the horizontal when it is above the horizontal level, i.e., the case when we raise our head to look at the object.



3. The angle of depression:

The angle of depression of an object viewed, is the angle formed by the line of sight with the horizontal when it is below the horizontal level, i.e., the case when we lower our head to look at the object.



4. Heights and Distances

(i) Heights here refers to the distance above the ground, and the distances are most probably the horizontal distances from the observer to the building.

(ii) The height or length of an object or the distance between two distant objects can be determined with the help of trigonometric ratios.

$$\sin\theta = \frac{\text{Perpendicular distance}}{\text{length of the line of sight}}$$

$$\tan\theta = \frac{\text{Perpendicular distance}}{\text{Horizontal distance}}$$

$$\cos\theta = \frac{\textit{Horizontal}\ \text{distance}}{\text{length of the line of sight}}$$

Understanding the Problem Statements

Step-1: Always, look upon the numbers given in the question, they might be angles or distances.

Step-2: Then, always draw a basic diagram of the given scenario. Marking the observer, line of sight, angle of elevation, and angle of depression.

Step-3: The height or length of an object or the distance between two distant objects can be determined with the help of trigonometric ratios.

$sin\,\theta = \frac{\text{Perpendicular distance}}{\text{length of the line of sight}}$

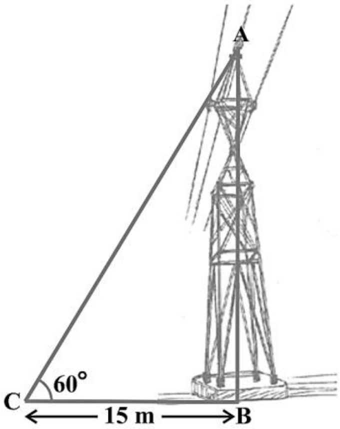
$tan\,\theta = \frac{\text{Perpendicular distance}}{\text{Horizontal distance}}$

$cos\,\theta = \frac{Horizontal\text{ distance}}{\text{length of the line of sight}}$

Example 1:

A tower stands vertically on the ground. From a point on the ground, which is 15m away from the foot of the tower, the angle of elevation of the top of the tower is found to be 60°. Find the height of the tower?

Solution:



$tan\,60° = \frac{AB}{BC}$

$\sqrt{3} = \frac{AB}{15}$

AB = $15\sqrt{3}$

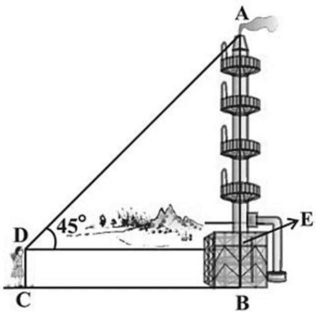
Hence, the height of the tower is $15\sqrt{3}\ m$.

Example 2:

An observer 1.5m tall is 28.5m away from a chimney. The angle of elevation of the top of the chimney from her eyes is 45°. What is the height of the chimney?

Solution:

Here, AB is the chimney, CD the observer and ∠ADE the angle of elevation (see Fig.). In this case, ADE is a triangle, right-angled at E and we are required to find the height of the chimney.



AB = AE + BE = AE + 1.5

and DE = CB = 28.5m

To determine AE, we choose a trigonometric ratio, which involves both AE and DE.

Now, $tan\,45° = \frac{\text{AE}}{\text{DE}}$

i.e., $1 = \frac{\text{AE}}{28.5}$

Therefore, AE = 28.5m

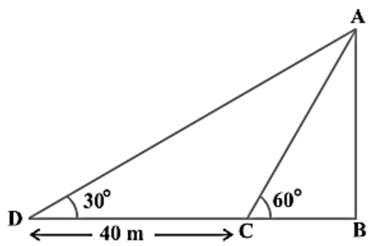
So the height of the chimney (AB) = (28.5 + 1.5) m = 30m.

Example 3:

The shadow of a tower standing on a level ground is found to be 40m longer when the Sun's altitude is 30° than when it is 60°. Find the height of the tower.

Solution:

In Fig., AB is the tower and BC is the length of the shadow when the Sun's altitude is 60°, i.e., the angle of elevation of the top of the tower from the tip of the shadow is 60° and DB is the length of the shadow, when the angle of elevation is 30°.



Now, let AB be h m and BC be x m. According to the question, DB is 40m longer than BC.

So, DB = (40 + x) m

Now, we have two right triangles ABC and ABD.

In ΔABC, tan 60° = $\frac{\text{AB}}{\text{BC}}$

or, $\sqrt{3} = \frac{h}{x}$ …(i)

In ΔABD, tan 30° = $\frac{\text{AB}}{\text{BD}}$

i.e., $\frac{1}{\sqrt{3}} = \frac{h}{x+40}$ …(ii)

h = $x\sqrt{3}$

From (i), we have h = $x\sqrt{3}$

Putting this value in (ii), we get $\left(x\sqrt{3}\right)\sqrt{3}$ = x + 40, i.e., 3x = x + 40

i.e., x = 20

So, h = $20\sqrt{3}$ [from (i)]

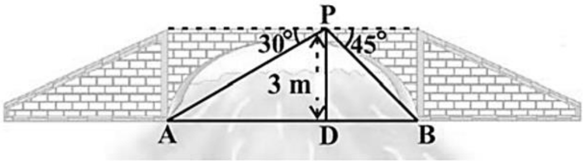
Therefore, the height of the tower is $20\sqrt{3}m$.

Example 4:

From a point on a bridge across a river, the angles of depression of the banks on opposite sides of the river are 30° and 45°, respectively. If the bridge is at a height of 3m from the banks, find the width of the river.

Solution:

In Fig., A and B represent points on the bank on opposite sides of the river, so that AB is the width of the river. P is a point on the bridge at a height of 3m, i.e., DP = 3m. We are interested to determine the width of the river, which is the length of the side AB of the Δ APB.



Now, AB = AD + DB

In right ΔAPD, ∠A = 30°.

So, $tan\, 30° = \frac{\text{PD}}{\text{AD}}$

i.e., $\frac{1}{\sqrt{3}} = \frac{3}{\text{AD}}$ or $\text{AD} = 3\sqrt{3}\ m$

Also, in right ΔPBD, ∠B = 45°. So, BD = PD = 3 m.

Now, AB = BD + AD = 3 + $3\sqrt{3}$ = $3\left(1+\sqrt{3}\right)m$.

Therefore, the width of the river is $3\left(\sqrt{3}+1\right)m$.

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