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Answer 1

**a) Since f_1 has a domain of R, and its output is always non-negative because of $\forall x(f_1(x) = x^2 \geq 0)$, there is unmatched elements in co-domain R, ($f_1(x) < 0$ is not possible) so f_1 is not surjective.
Also f_1 is not injective because we can pick up two different x values that satisfies $f_1(x_1) = f_1(x_2)$ such that $f_1(3) = f_1(-3) = 9$**

**b) Assume that we picked an arbitrary a and b, (a≠b) and $f_2(a) = f_2(b)$ which is $a^2 = b^2$ if there is different a and b values we can say it is injective. $a^2 - b^2 = 0$ so $(a-b)*(a+b) = 0$ which says whether $a = -b$ or $a = b$. We said $a \neq b$ and b cannot equals $-a$ because of the domain is restricted by non-negative real numbers, a must be equal to b which leads us to conclude that the function is injective in its domain.
In order to check whether f_2 is surjective or not, we can pick a negative real number as output, then $f_2(x) = x^2 = -t$, ($t > 0$ and $t \in R$) there is no such $x \in R^+$ satisfies this condition, so f_2 is not surjective.**

**c) Assume that we picked an arbitrary c and d, ($c \neq d$) and $f_3(c) = f_3(d)$ which is $c^2 = d^2$ if there is different c and d values we can say it is injective. $c^2 - d^2 = 0$ so $(c-d)*(c+d) = 0$ which says whether $c = -d$ or $c = d$. We said $c \neq d$ and c can be equal to $-d$ because of the domain includes both positive and negative real number. This leads us to conclude f_3 is not injective.
For checking surjectiveness, say that $f_3(x) = x^2 = y$. We can say $x^2 = y$ and $x = \pm\sqrt{y}$ there is always a $\sqrt{y}$ satisfies this equation due to its domain.**

**d) Assume that we picked an arbitrary a and b, (a≠b) and $f_2(a) = f_2(b)$ which is $a^2 = b^2$ if there is different a and b values we can say it is injective. $a^2 - b^2 = 0$ so $(a-b)*(a+b) = 0$ which says whether $a = -b$ or $a = b$. We said $a \neq b$ and b cannot equals $-a$ because of the domain, a must be equal to b which leads us to conclude that the function is injective in its domain.
For checking surjectiveness, say that $f_3(x) = x^2 = y$. We can say $x^2 = y$ and $x = \pm\sqrt{y}$ there is always a $\sqrt{y}$ satisfies this equation due to its domain.**

Answer 2

a) Consider a function $f : Z \implies R$. Pick a $x_0 \in Z$ and $\epsilon > 0$ by definition. Then choose a $\delta < 1$ i.e $\delta = 0.3$. Let $x \in \epsilon$ and suppose that $\|x - x_0\| < \delta = 0.3$. Then since the

only integer within 0.3 distance of x_0 is itself, we must have $x = x_0$. Thus $f(x) = f(x_0)$ and $\|f(x) - f(x_0)\| = 0$ which is always less than ϵ. This shows that f is continuous at x_0. Since x_0 is arbitrary, the difference must be 0 in the whole domain which is the definition of continuous function.

b) If we pick up an arbitrary δ i.e $\delta = 0.33$ it satisfies exists condition(1). And the statement says that if (1), then for all $\epsilon \in R^+$, $\epsilon > \|f(x) - f(x_0)\|$ because of ϵ can be any positive number and $\|f(x) - f(x_0)\| < \epsilon$, $\|f(x) - f(x_0)\|$ must be equal to zero. Since x_0 is arbitrary, the difference must be 0 in the whole domain which is the definition of constant function.
But if $\|f(x) - f(x_0)\|$ is false, then (1) must be false as well. This implies that if $\|f(x) - f(x_0)\| \neq 0$ there is no such δ that satisfies the condition and function becomes non-continuous.

Answer 3

a) If we denote the cartesian product as an infinite table

A_{11}	A_{21}	A_{31}	...
A_{12}	A_{22}	A_{32}	...
...	...	...	...

We can draw a pattern which is called Cantor's diagonalization. $A_{11} \implies A_{12} \implies A_{21} \implies A_{31} \implies$ and so on. It creates a constant line of countable sets which is countable because we can map Z^+ with sets.

b) Assume that the very first element of that infinite sequence of infinite 1 and 0's(sets) start with 000000.. and the second set is 1100110.. and the third one is 0110101.., and so on. There is always a set of 1 and 0's which is not included in this sequence. To show that, if we consider first element of our new list, we can pick the opposite of the first sequence's first element, in this case number 1, it is already different from first set. Then do it for second element of second set and our new set is 10 now. we can go on like this and crate a brand new set which is unique.

Answer 4

a)
We will compute $\sqrt{n} * log(n)$ and $log(n)^2$
$$\frac{log(n)^2}{\sqrt{n} * log(n)} = \frac{log(n)}{\sqrt{n}}$$

$\lim_{n\to+\infty} \frac{log(n)}{\sqrt{n}}$ **by L'Hospital's rule,** $\lim_{n\to+\infty} \frac{log(n)}{\sqrt{n}} = \lim_{n\to+\infty} \frac{\frac{1}{n} * \frac{1}{ln(10)}}{\frac{1}{2\sqrt{n}}} = \lim_{n\to+\infty} \frac{2}{\sqrt{n}} =$

0
$\sqrt{n} * log(n)$ **is the Big-O of** $log(n)^2$

b)
We will compute $\sqrt{n} * log(n)$ **and** n^{50}
$n^{50} = (\sqrt{n})^{100}$
$\lim_{n\to+\infty} \dfrac{\sqrt{n} * log(n)}{(\sqrt{n})^{100}} = \lim_{n\to+\infty} \dfrac{log(n)}{(\sqrt{n})^{99}}$

by L'Hospital's rule, $\lim_{n\to+\infty} \dfrac{\frac{1}{n} * \frac{1}{ln(10)}}{49.5 * n^{48.5}} = \lim_{n\to+\infty} \dfrac{1}{49.5 * n^{49.5}} = 0$
n^{50} **is the Big-O of** $\sqrt{n} * log(n)$

c)
We will compute n^{50} **and** $n^{51} + n^{49}$
$\lim_{n\to+\infty} \dfrac{n^{50}}{n^{51} + n^{49}} = \lim_{n\to+\infty} \dfrac{n^{50}}{n^{50} * (n + \frac{1}{n})} = \lim_{n\to+\infty} \dfrac{1}{n + \frac{1}{n}} = 0$
$n^{51} + n^{49}$ **is a Big-O of** n^{50}

d)
We will compute $n^{51} + n^{49}$ **and** 2^n**. If we differentiate both** $n^{51} + n^{49}$ **and** 2^n **51 times (L'Hospital's rule by 51 times), We end up with** $\lim_{n\to+\infty} \dfrac{51! + 49! + \frac{1}{n^2}}{ln(2)^{51} * 2^n} = 0$
So 2^n **is a Big-O of** $n^{51} + n^{49}$

e)
We will compute 2^n **and** 5^n
$\lim_{n\to+\infty} (\frac{2}{5})^n$ **By p-test this statement converges to 0.**
5^n **is a Big-O of** 2^n**.**

f) We will compute 5^n **and** $(n!)^2$**. We will use ratio test (studied in Calculus II) which states** $\lim_{n\to+\infty} \dfrac{\frac{5^{n+1}}{((n+1)!)^2}}{\frac{5^n}{(n!)^2}}$ **converges if the result is different from 1 and** ∞

$\lim_{n\to+\infty} \dfrac{\frac{5^{n+1}}{((n+1)!)^2}}{\frac{5^n}{(n!)^2}} = \lim_{n\to+\infty} \dfrac{5}{(n+1)^2} = 0$ **So** $(n!)^2$ **is Big-O of** 5^n

Answer 5

a) $134 - 94 * 1 = 40$
$94 - 40 * 1 = 54$
$54 - 40 * 1 = 14$
$40 - 14 * 1 = 26$
$26 - 14 * 1 = 12$
$14 - 12 * 1 = 2$
So the gcd of 134-94 is 2.

b) Suppose a **is an integer and** $a > 5$ **if** a **is odd it is denoted by** $a = 2n + 1$ **and** $n \geq 3$**. So** $a - 3$ **is even. Apply Goldbach's conjecture and we get** $a - 3 = p_1 + p_2$ **which states** $a - 3$ **is the sum of 2 prime numbers. And this leads us to** $2 * n + 1 = p_1 + p_2 + 3$ **where both** $p_1, p_2, 3$ **are primes.**
If a is even $a = 2 * n$ **and** $n \geq 3$ **and** $a - 2 = 2 * (n - 1)$ **so** $a - 2$ **is even, too. Like upper part, apply Goldbach's conjecture and we can rewrite as** $2 * n = p_1 + p_2 + 2$ **where both** $p_1, p_2, 2$ **are prime numbers.**