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Answer 1

First, let's draw an adjacency and incidence matrices of the graph.

$$\begin{bmatrix} 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{bmatrix}$$

Where indexes go a to e from left to right and from up to down. This is the adjacency matrix.

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Is the incidence matrix representation.

a) Sum of all degrees = Sum of all vertex adjacents. For vertex a, count of adjacents = 3
$b \implies 3$
$c \implies 3$
$d \implies 2$
$e \implies 3$
$3 + 3 + 3 + 2 + 3 = 14$ is the sum of all degrees of all nodes.

b)We can observe that the sum of non-zero entries is 14 from the adjacency matrix.
c)We can observe that the sum of non-zero entries is 21 from the incidence matrix.

d)No. There are no such 4 vertices that form a complete graph.
e)G is not bipartite. From the theorem, it is impossible to paint vertices in two different colors and no two adjacents aren't the same color.
f) There are 7 undirected edges in the graph. Either of them could have two different ways. So, 2^7 total different directional graphs underlying in the graph G.
g) *abcde* is the longest path.
h) Since the graph is connected. There is only one connected component.
i) There is not an Euler circuit in G because G includes vertices with odd-numbered adjacents.
j) Since G does not have an Euler circuit, and G does not have exactly two vertices with odd-numbered adjacents, there is not an Euler path in G.
k) Yes. *abcdea* is a Hamilton circuit.
l) Yes. *abcde* is a Hamilton path.

Answer 2

Both graphs G and H have 5 vertices and 5 edges. They also have 5 vertices with 2 edges. If we take the symmetric of vertex e from the a-d axis, vertex c from the d-b axis and so on we can obtain the same shape with graph H. So H and G are isomorphic. Also, their adjacency matrix representations are same.

Answer 3

First, we will keep a list that represents the distances with respect to s and another for a previous list to reach that vertex.
All the distances before visiting the vertex are infinity D=Distance and P=Prev. So:
D=$(s,0),(u,\infty),(w,\infty),(v,\infty),(y,\infty),(x,\infty),(z,\infty),(t,\infty)$
P=$(s,-),(u,-),(w,-),(v,-),(y,-),(x,-),(z,-),(t,-)$
We will traverse the graph and not visit the visited vertices. If we find a shorter path we will change the distance. We will traverse starting from vertex s.
The adjacents of vertex s are w, u and v, and their distance in the list (∞) is bigger than 3,4, and 5, so we change the distance between them as follows:
D=$(s,0),(u,4),(w,3),(v,5),(y,\infty),(x,\infty),(z,\infty),(t,\infty)$

P=$(s,-),(u,s),(w,s),(v,s),(y,-),(x,-),(z,-),(t,-)$
We repeat the operation for the closest vertex adjacent to s. We add the distance value to the new vertex by adding it the distance of its previous. Closest is the w in this case.
D=$(s,0),(u,4),(w,3),(v,5),(y,\infty),(x,11),(z,15),(t,\infty)$
P=$(s,-),(u,s),(w,s),(v,s),(y,-),(x,w),(z,w),(t,-)$
Note that we didn't change the bigger or equal total distance (v, u and s).
Repeat for u.
D=$(s,0),(u,4),(w,3),(v,5),(y,11),(x,11),(z,15),(t,\infty)$
P=$(s,-),(u,s),(w,s),(v,s),(y,u),(x,w),(z,w),(t,-)$
Repeat for v.
D=$(s,0),(u,4),(w,3),(v,5),(y,11),(x,7),(z,15),(t,\infty)$
P=$(s,-),(u,s),(w,s),(v,s),(y,u),(x,v),(z,w),(t,-)$
We changed the distance and prev of x because there is a smaller distance if we go from v instead of w.
Repeat for x
D=$(s,0),(u,4),(w,3),(v,5),(y,8),(x,7),(z,13),(t,\infty)$
P=$(s,-),(u,s),(w,s),(v,s),(y,x),(x,v),(z,x),(t,-)$ (y,z changed)
Repeat for y
D=$(s,0),(u,4),(w,3),(v,5),(y,8),(x,7),(z,12),(t,17)$
P=$(s,-),(u,s),(w,s),(v,s),(y,x),(x,v),(z,y),(t,y)$(t added, z changed)
Repeat for z
D=$(s,0),(u,4),(w,3),(v,5),(y,8),(x,7),(z,12),(t,15)$
P=$(s,-),(u,s),(w,s),(v,s),(y,x),(x,v),(z,y),(t,z)$(t changed)
Repeat for t
D=$(s,0),(u,4),(w,3),(v,5),(y,8),(x,7),(z,12),(t,15)$
P=$(s,-),(u,s),(w,s),(v,s),(y,x),(x,v),(z,y),(t,z)$(nothing changed)
We traversed across every vertex and tried every possible way. In this graph, using the previous list, we can see the shortest path is s-v-x-y-z-t. And the distance is 15.

Answer 4

a)
Using Prim's algorithm: I pick a for the base point.
Choice 1 is {a,b} We keep choosing the smallest edges among what we have reached.

Choice 2 is {c,b}
Choice 3 is {f,c}
Choice 4 is {d,c}
Choice 5 is {k,d}
Choice 6 is {j,f}
Choice 7 is {g,f}
Choice 8 is {i,f}
Choice 9 is {h,i}
Choice 10 is {e,f}
This is the correct order by using Prim's algorithm.
b)

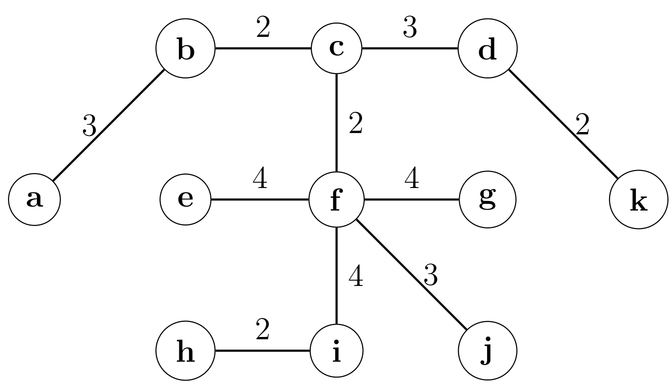

Figure 1: Tree.

c) The minimum spanning tree is not unique because the tree could be formed in different forms such as {g,j} edge can be replaced to {f,g}.