Student Information

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Answer 1

a)
$F(t_A) = \frac{t_A}{100}$
$F(t_B) = \frac{t_B}{100}$
$F(t_A, t_B) = \frac{t_B * t_A}{10000}$
we should take derivation for both t_A, t_B for $f(t_A, t_B)$
$f(t_A, t_B) = \frac{1}{10000}$

b) As stated in the question, they are distributed uniformly, and we can use uniform distribution formulas. The probability density function of uniform distribution is $\frac{1}{b-a}$ where b = 100 and a = 0. Integrating the equation will give us its cdf. We integrate the equation for the time. Doing so, our cdf is $\frac{x}{100}$ where x is the total time before the response. F(30)-F(0) is the A's probability and F(60)-F(40) is the B's probability. Calculating appropriately, A's probability is $\frac{3}{10}$ and B's probability is $\frac{2}{10}$ we should multiply the outputs we find in order to calculate the probability of these two events happening together. $\frac{2}{10} * \frac{3}{10} = \frac{6}{100}$ is the answer.

c) Let's first think about what values can time of B take for different A values. For A = 5, B can take the values between 0 and 15, For A = 91, B can take all values except 100ms. If we make a function that gives max B values, It would be like f(A) = A + 10. And B cannot exceed 100ms. So for A $\in$ [0,1), B is [0,11) interval. For So for A $\in$ [1,2), B is [0,12) (It is not important that 12 is included or not since this is a continuous distribution). If we multiply the probability of A and B for all 1-length intervals of A. We will find the probability. We can generalize the probability as: P(x)=$\frac{1}{100} * f(A)$ if we calculate it for all 1-length intervals of A, which gives us: $\frac{1}{100} * \frac{5950}{100}$ = 0.595.

d) We can use a similar approach we made in part c. But this time, the values of B are bounded from the bottom as well. For example, For tA = 55ms, tB is between [35,75] The values of A between 20 and 80 the probability of not failing is fixed and

0.4 ((tA+20-tA+20)/100 = 0.4). We should now calculate $\frac{1}{100} * \frac{4}{10} * 60$ **since there are 60 different values if we take the intervals starting as** $A \in [20, 21)$ **and increasing by 1 both intervals until** $A \in [79, 80)$**. Of course, we can calculate the sum using integral as well, but since the distribution is uniform, it is okay to sum independently. Now we have** $\frac{1}{100} * \frac{4}{10} * 60$**, and we need the edge cases** $A < 20$ **and** $A > 80$**. We can see when A goes 0 from 20 or 80 to 100, the interval of B decreases by the same amount that A changes. We can calculate these probabilities as** $\frac{2}{100} * \Sigma_{i=20}^{40} \frac{i}{100}$ **where** $\frac{2}{100}$ **denotes both** $A < 20$ **and** $A > 80$ **cases and summation part denotes different values of probability of not crashing for B. If we combine two equations we get 0.36.**

Answer 2

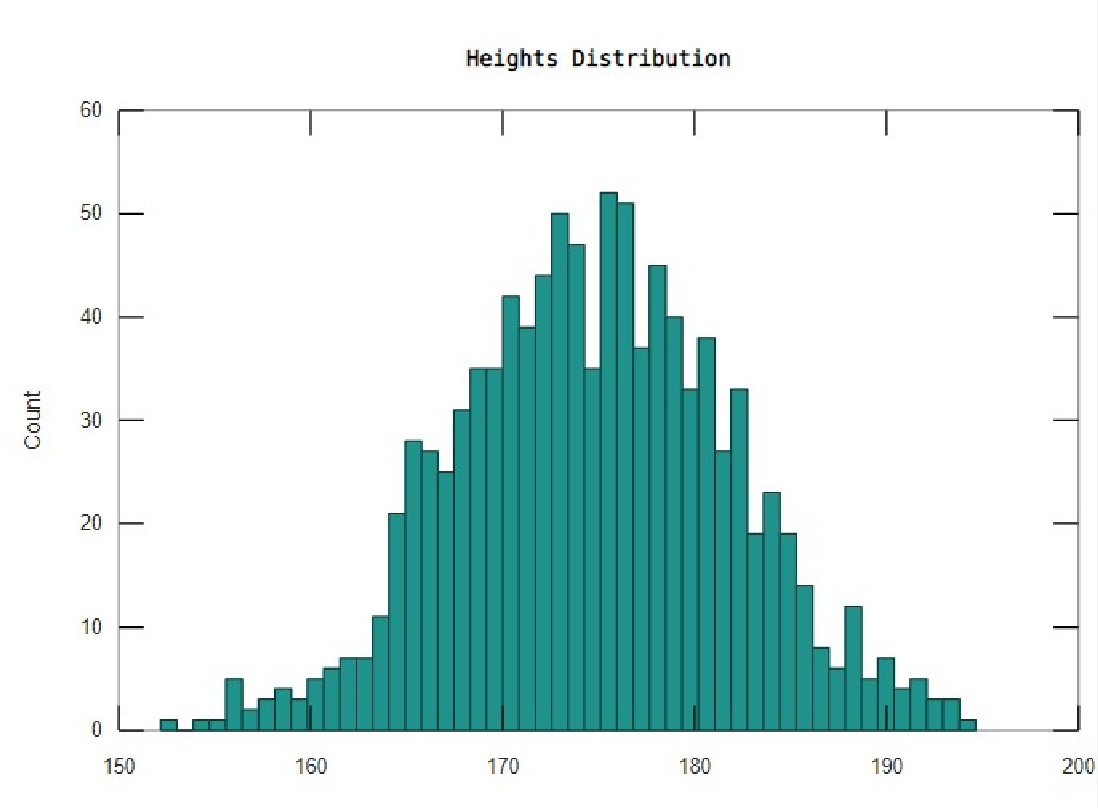
a) Since there is big data we can use the central limit theorem. The distribution will act like the normal distribution. First, we have to calculate the mean and standard deviation to normalize our data and use the normal distribution table. Our mean value equals the frequent shoppers' rate and 0.6, then we can find std dev = $(p*(1-p))^{0,5} =$ **0.49. Now we can standardize and use the normal distribution table. z = (**$0.65 * 150$**-**$150 * mean$**)/(standard dev.**$\sqrt{150}$**) = (97.5-90)/(6) = 1,25. We are asked to find less or equal to 0.65, so we should look at the 1.25 and right side of the table, namely** $1 - \Phi(1.25)$ **or we can directly check** $\Phi(-1, 25)$**, they are the same. From the table from the book, 0.1056 is the answer.**

b) We will apply the same process, but this time our mean value is 0.1. Our new standard deviation is $0.1 * (1 - 0.1)^{0.5}$ **= 0.3. If we standardize now, z = (**$0.15 * 150$**-**$150 * 0.1$**)/(**$\sqrt{150} *$**) = 2.04. Let's find this data from table** $\Phi(2.04) = 0.9793$ **is our answer.**

Answer 3

We will apply the same operation in Q2, our mean is 175, the standard deviation is 7, we find the probability of less than 180 cm and less than 170 cm then subtract them in order to find the desired interval. Standardizing for 180 cm, z = (180-175)/7 = 0.71 and $\Phi(0.71) = 0.7611$. For 170cm, z = (170-175)/7 = -0.71 and $\Phi(-0.71) = 0.2389$. If we subtract these values, 0.7611-0.2389 = 0.5222.

Answer 4




```
n = 1000;
mean = 175;
ro = 7;
heights = normrnd(mean, ro, n, 1);

hist(heights, 50)
xlabel('cm')
ylabel('Count')
title('Heights Distribution')
```

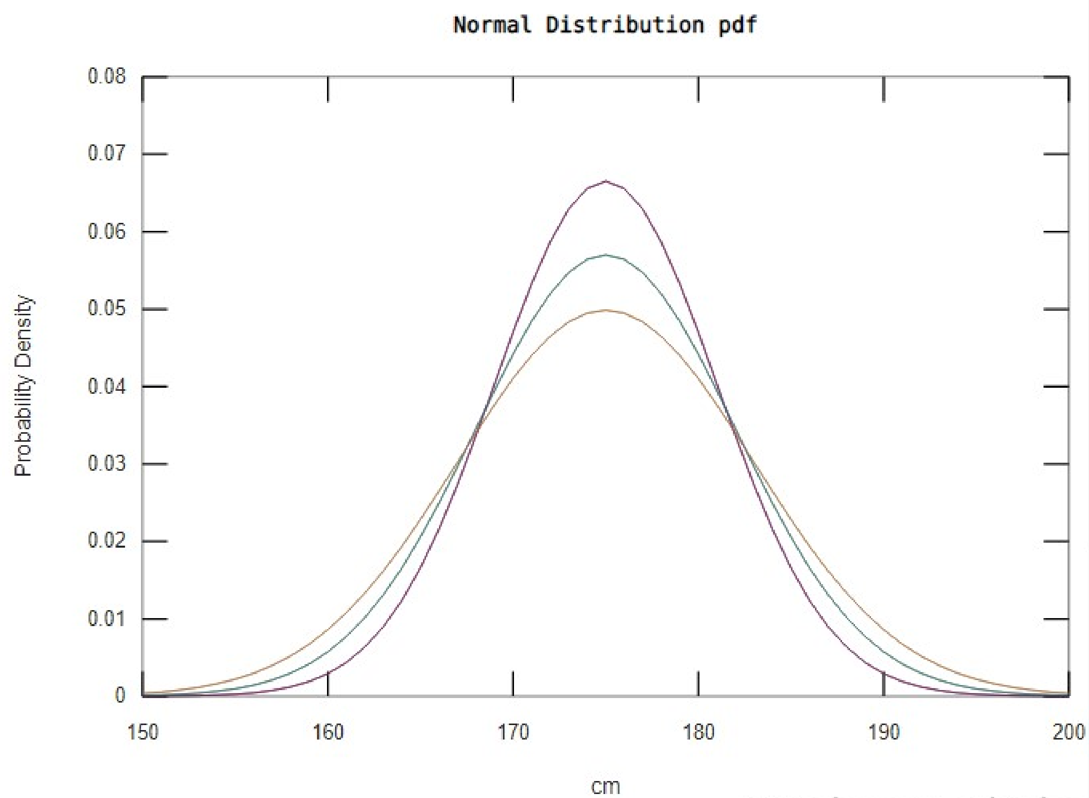
This is part a

Most people are around 175 cm since it is the mean and the standard deviation is not too high.

```
mean = 175;
rolist = [6 7 8];

for i = 1:3
  ro = rolist(i);
  y = normpdf(150:200, mean, ro);
  plot(150:200, y);
  hold on;
end

xlabel('cm');
ylabel('Probability Density');
title('Normal Distribution pdf');
```



As seen in the graph, as the standard deviation increases, the peak of the graph decreases and it becomes more stable. Purple is standard dev=5, yellow is standard dev = 7.

```
Having minimum 45 percent of adults with heights between 170
and 180 cm: 0.96
Having minimum 50 percent of adults with heights between 170
and 180 cm: 0.75
Having minimum 55 percent of adults with heights between 170
and 180 cm: 0.27
```

```
n = 1000;
count45 = 0;
count50 = 0;
count55 = 0;
for i = 1:n

  heights = normrnd(175, 7, 150, 1);
  count = sum(heights >= 170 );
  count2 = sum (heights >= 180);
  count=count-count2;
  nihai = count/150;
  if nihai >= 0.45
    count45 = count45 + 1;
  end
  if nihai >= 0.50
    count50 = count50 + 1;
  end
  if nihai >= 0.55
    count55 = count55 + 1;
  end
end

prob_45 = count45/n;
prob_50 = count50/n;
prob_55 = count55/n;

fprintf("Having minimum 45 percent of adults with heights between 170 and 180 cm: %.2f\n", prob_45);
fprintf("Having minimum 50 percent of adults with heights between 170 and 180 cm: %.2f\n", prob_50);
fprintf("Having minimum 55 percent of adults with heights between 170 and 180 cm: %.2f\n", prob_55);

```

If we look at the output, when the percentage is lower the probability increases because it is more possible since distributed normally.