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Answer 1

Base case $n = 1$
$6^2 - 1 = 35$ is divisible by both 5 and 7
Inductive Case:
for $k; k \in N^+$, assume $6^{2k} - 1 = 35t$
$6^{2k+2} - 1 = 36 * 6^{2k} - 1 = 36 * (35t + 1) - 1$
$36*(35t+1)-1 = 36*35t+35$ since both elements have 35 in multiplication, the result is divisible by 35, means both 5 and 7. It is true for all k values by induction.

Answer 2

Basis cases:
$H_0 = 1 \leq 9^0$
$H_1 = 5 \leq 9^1$
$H_2 = 7 \leq 9^2$
Inductive Case:
For $0 \leq j \leq k$ and $k \geq 3$
$H_j \leq 9^j$ (assumed)
$8H_k + 8H_{k-1} + 9H_{k-2} = H_{k+1}$ and $8H_k \leq 8 * 9^k$, $8H_{k-1} \leq 8 * 9^{k-1}$, $9H_{k-1} \leq 9^{k-2}$
summing the right hand sides:
$8 * 9^k + 8 * 9^{k-1} + 9^{k-1} = 9^{k+1}$
$H_{k+1} \leq 9^{k+1}$
Therefore, by strong induction rule, statement is true for all integers in domain of n.

Answer 3

First let's compute bits with consecutive 1's. Starting from first position:
1111XXXX and X can be 1 or 0. There are $2^4 = 16$ possibilities.
For 2nd position:
X1111XXX and the very first X must be 0 otherwise it can be same with first situation for some X values. There are $2^3 = 8$ possibilities.
For 3rd position:
XX1111XX and the second X must be 0 for same reason. There are $2^3 = 8$ possibilities.
For 4th position:
XXX1111X and the third X must be 0 for the same reason. There are $2^3 = 8$ possibilities.

For 5th position:
XXXX1111 and the last X must be 0 for the same reason. There are $2^3 = 8$ possibilities.
all possibilities can be shown as:
1111XXXX
01111XXX
X01111XX
XX01111X
XXX01111
note that zeros are in different positions for different cases, so it is impossible to contain same sequences. There are 16+8+8+8+8=48 different possibilities for consecutive 1's.
If we repeat the process for 4 consecutive 0's. There are 48 cases for consecutive 0's as well.
But in this case 11110000 and 00001111 will be counted twice. If we substitute them, there will be 48+48-2=94 different sequence contains either 4 consecutive 0's or 4 consecutive 1's.

Answer 4

First let's pick a star. There are $\binom{10}{1}$ ways to do it.
Then lets pick planets. $\binom{80}{8}$ for non-habitables and $\binom{20}{2}$ for habitable ones.
There will be 6 different formations. Consider H as habitable and N for non-habitable.
1-)HNNNNNNHNN
2-)NHNNNNNNHN
3-)NNHNNNNNNH
4-)HNNNNNNNHN
5-)NHNNNNNNNH
6-)HNNNNNNNNH
Also non-habitable planets can form 8! different ways. Habitable planets can form 2! different ways. Putting all together:
$\binom{10}{1} * \binom{80}{8} * \binom{20}{2} * 6 * 8! * 2!$ is the number of possibilities.

Answer 5

a) Let's say the number of different step combinations = S_n. Last jump can be 1, 2 or 3 steps long. Therefore: For $n \geq 4$
$S_n = S_{n-1} + S_{n-2} + S_{n-3}$ is the function.

b) $S_0 = 1$; $S_1 = 1$; $S_2 = 2$

c) $S_9 = S_8 + S_7 + S_6$
$S_8 = S_7 + S_6 + S_5$
$S_7 = S_6 + S_5 + S_4$
$S_6 = S_5 + S_4 + S_3$
$S_5 = S_4 + S_3 + S_2$

$S_4 = S_3 + S_2 + S_1$
$S_3 = S_2 + S_1 + S_0$
$S_2 = S_1 + S_0 + S_{-1}$
$S_1 = S_0 + S_{-1} + S_{-2}$
$S_1 = 1; S_2 = 2; S_3 = 4; S_4 = 7; S_5 = 13; S_6 = 24; S_7 = 44; S_8 = 81$
$S_9 = 81 + 44 + 24 = 149$