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Answer 1

$G(x) = \sum_{k=0}^{\infty} a_k x^k$
$a_0 = a_1 = 1 \implies$ Initial values
and $a_k = 3a_{k-1} + 4a_{k-2} \implies$ Recurrence relation
$\sum_{k=2}^{\infty} a_k x^k = 3\sum_{k=2}^{\infty} a_{k-1}x^k + 4\sum_{k=2}^{\infty} a_{k-2}x^k$
$G(x) = \sum_{k=2}^{\infty}(3a_{k-1}x^k + 4a_{k-2}x^k) = \sum_{k=2}^{\infty}(x3a_{k-1}x^{k-1} + x^2 4a_{k-2}x^{k-2})$
$G(x) - 3xG(x) - 4x^2G(x) = 0$
$G(x) - a_0 - xa_1 = 3x(G(x) - 1) + 4x^2G(x)$
$G(x) - 1 - x = 3xG(x) - 3x + 4x^2G(x)$
$G(x) = \dfrac{1-2x}{1-3x-4x^2} = \dfrac{2x-1}{(4x-1)(x+1)} = \dfrac{A}{4x-1} + \dfrac{B}{x+1}$
$A + 4B = 2,\ -A + B = 1 \implies A = -0.4,\ B = 0.6$
$G(x) = \dfrac{-0.4}{4x-1} + \dfrac{0.6}{x+1}$
$a_k = 0.4 * 4^n + (-0.6)^n$

Answer 2

a)

$G(x) = \sum_{n=0}^{\infty} a_n x^n = \dfrac{1}{1-x}$
Sequence goes like $2 + 5x + 11x^2 + 29x^3 + 83x^4 ... = G(x)$
$2 + 2x + 2x^2 + 2x^3 ... = 2 * \dfrac{1}{1-x}$
$\sum_{n=0}^{\infty} 3^n x^n = 1 + 3x + 9x^2 + 27x^3 + 81x^4 ... = \dfrac{1}{1-3x}$
The sum of these sequences: $3 + 5x + 11x^2 + 29x^3 .... = G(x) + 1$
$G(x)$ can be written as $\dfrac{1}{1-3x} + \dfrac{2}{1-x} - 1$

b)

We can split up the generating function as follows and convert them to sequences:
$G(x) = \dfrac{-2}{x-1} - \dfrac{5}{2x-1}$
$a_{n1} = \dfrac{2}{1-x} = 2\sum_{n=0}^{\infty} x^n$; $a_{n1} = 2$ and

$a_{n2} = \dfrac{5}{1-2x} = 5\sum_{n=0}^{\infty} 2^n x^n$; $a_{n2} = 5 * 2^n$
$a_n = a_{n1} + a_{n2} = 2 + 5 * 2^n$

Answer 3

a)

For an equivalence relation, R must be reflexive. It says if and only if there exists a right triangle whose has all edges $a, b, n \in Z$. If we try 3,3,n we obtain $n = (9+9)^{0.5} \notin Z$. So, R is not an equivalence relation.

b)

For an equivalence relation, R should satisfy reflexive, symmetric and transitive conditions.

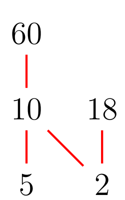
Reflexive condition:
$(x_1, y_1)R(x_1, Y_1) = 2x_1 + y_1$ satisfies reflexive condition.

Symmetric test:
If $(x_1, y_1)R(x_2, y_2)$ then $2x_1 + y_1 = 2x_2 + y_2$ and vice versa is $(x_2, y_2)R(x_1, y_1)$: $2x_2 + y_2 = 2x_1 + y_1$. We can observe that R is symmetric.

Transitive test:
Assume that $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ are three different input sets. Assume $(x_1, y_1)R(x_2, y_2)$ is true. It is possible if $(x_1, y_1) = (x_2, y_2)$. Also $(x_2, y_2)R(x_3, y_3)$ is true if and only if $(x_2, y_2) = (x_3, y_3)$ which shows $(x_1, y_1) = (x_3, y_3)$ and satisfies the transition condition.
It satisfies all three conditions, so R is an equivalence relation.

$2 * 1 - 2 = 0$.
Equivalence class of (1,-2) is all numbers that $(a, 2a); a \in R$
It forms a linear graph whose slope is -2 and passes from origin. $y = -2x$ is its equation.

Answer 4



Hasse diagram

$$\begin{bmatrix} 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Matrix representation

$R^{(s)} = R \cup R^{-1}$

R = {(2, 2),(2, 10),(2, 18),(2, 60),(5, 5),(5, 10),(5, 60),(10, 10),(10, 60),(18, 18),(60, 60)}

$R^{(-1)} = \{(2,2), (10,2), (18,2), (60,2), (5,5), (10,5), (60,5), (10,10), (60,10), (18,18), (60,60)\}$

$R^{(s)} = \{(2,2), (2,10), (2,18), (2,60), (5,5), (5,10), (5,60), (10,10), (10,60), (18,18), (60,60),$
$(10,2), (18,2)(60,2), (10,5), (60,5), (60,10)\}$

$R_s \wedge (x, y) \notin R$:

{(10,2), (18,2), (60,2), (10,5), (60,5), (60,10)}

$$\begin{bmatrix} 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \end{bmatrix}$$

There is no way to reach total ordering just by removing one element. However, by removing 2 and 18 we can reach total ordering. We can add any integer that is divisible by 60.