Homework 6 . Predicate Logic: PROLOG program. Resolution Proofs.

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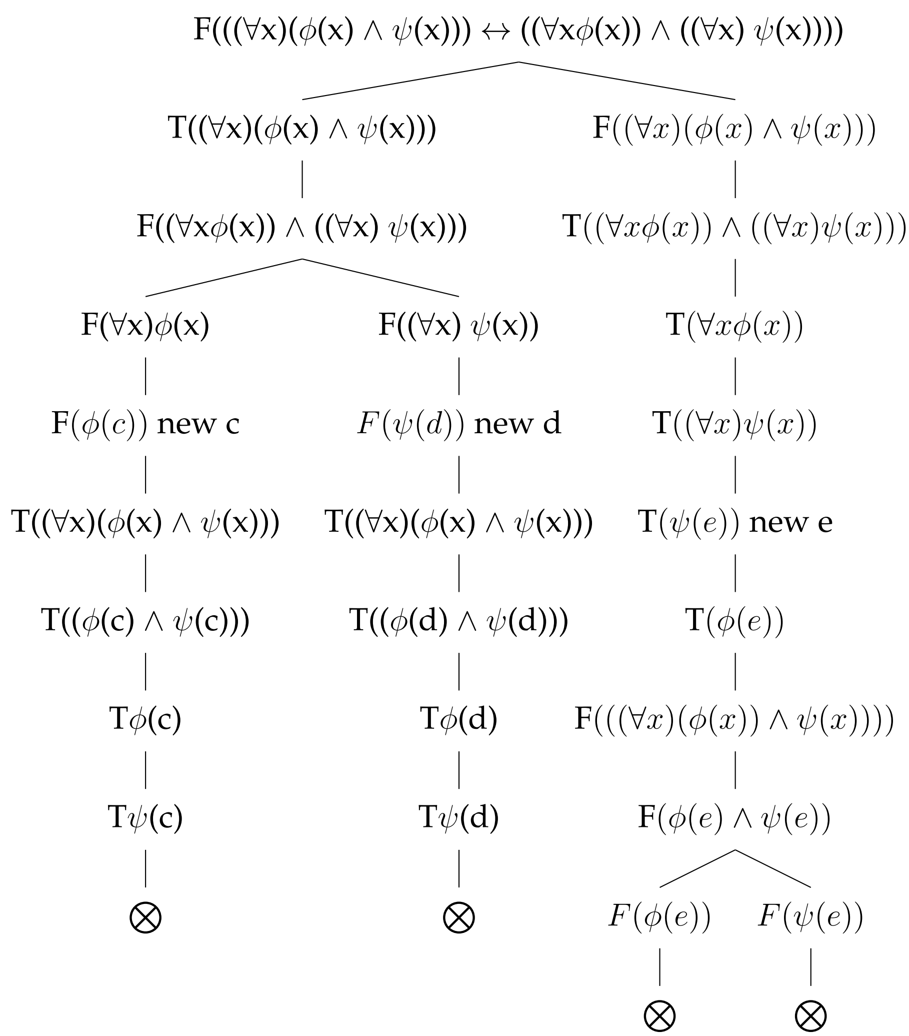
Submit your solution in PDF file (and Latex source file if you use Latex) to blackboard by **11:59pm Sunday, Dec 3** .

1. **(Tableau Proof). Write a tableau proof for**

$$((\forall x)(\phi(x) \wedge \psi(x))) \leftrightarrow ((\forall x \phi(x)) \wedge ((\forall x)\psi(x)))$$

Proof:



2. **Find the Prenex normal form of the following formula.**

$$\forall x \exists y P(x, y) \vee \neg \exists x \forall y Q(x, y).$$

Ans:
Prenex normal form :
Here we can move all quantifiers to the beginning of the formula keeping its "satisfiability" unchanged, then remove all existence quantifiers by Skolemization, and finally change the part (without quantifiers) to a CNF.
Moving quantifiers to the beginning :

$$\forall x \exists y P(x, y) \vee \neg \exists x \forall y Q(x, y).$$
$$\forall u[\exists y P(u, y) \vee \neg \exists x \forall y Q(x, y)]$$
$$\forall u \exists v[P(u, v) \vee \neg \exists x \forall y Q(x, y)]$$
$$\forall u \exists v[P(u, v) \vee \forall x \neg \forall y Q(x, y)]$$
$$\forall u \exists v[P(u, v) \vee \forall x \exists y \neg Q(x, y)]$$
$$\forall u \exists v \forall w[P(u, v) \vee \exists y \neg Q(w, y)]$$
$$\forall u \exists v \forall w \exists z[P(u, v) \vee \neg Q(w, z)]$$

3. **Find the Skolemization of the following sentence.**

$$\forall x \forall y \forall z \exists w \varphi(x, w, y, z).$$

Ans:
Skolemization :
Removing existence quantifiers
After applying Skolemization to the given question

$$\forall x \forall y \forall z \exists w \varphi(x, w, y, z).$$

$$\forall x \forall y \forall z \varphi(x, f(x, y, z), y, z).$$

4. **Let $\mathcal{L}$ consist of the constant c, function symbol $f/1$ and the unary predicate $R/1$.**

   (a) **What is the Herbrand universe for $\mathcal{L}$?**
   Ans:
   $U(L)$: $\{c, f(c), f(f(c)), f(f(f(c))), ...\}$

   (b) **Give infinitely many Herbrand structures for $\mathcal{L}$. When defining/inventing a relation on the Herbrand universe, you may use the enumeration methods. For example, one binary relation on the Herbrand universe could be $\{\}$ or $\{(c, f(c)\}$ or $\{(c, f(c)), (f(c), f(f(c))), \ldots\}$.**
   Ans:
   $U(L)$: c, f(c), f(f(c)), f(f(f(c)))...

$c^A = c$
$f^A(c) = f(c)$
$f^A(f^A(c)) = f(f(c))$
$f^A(f^A(f^A(c))) = f(f(f(c)))$
$f^A(f^A(f^A(f^A(c)))) = f(f(f(f(c))))$
......
......

(c) **Give a Herbrand model of $\forall x R(f(x))$.**
Ans:
Herbrand model of $\forall x R(f(x))$.
$U(L)$: $c, f(c), f(f(c)), \ldots$.
C^A = c.
$R = \{c\}$
f^A(X) = c
$\forall c R(f(c))$

5. **Following the algorithm in the slides, list the major steps to find a most general unifier for the following expressions.**

$$\{Q(h(x,y),w), Q(h(g(v),a), f(v)), Q(h(g(v),a), f(b))\}.$$

The unify algorithm accepts term equations only. So, to unify the expressions above, the initial term equations are

$$Q(h(x,y),w) = Q(h(g(v),a), f(v))$$

(making the first two expressions identical) and

$$Q(h(g(v),a), f(v)) = Q(h(g(v),a), f(b))$$

(making the second expression and the third one identical and thus all expressions are identical and thus unified.)
Ans:
Finding the Most general unifier for the given expression :
$S = \{Q(h(x,y),w),\ Q(h(g(v),a), f(v)),\ Q(h(g(v),a), f(b))\}$

STEP 1 : D(S_0) = h(x,y) , h(g(v),a) , 0 = x/g(v),
Then we get
$S_0 = \{Q(h(g(v),y),w),\ Q(h(g(v),a), f(v)),\ Q(h(g(v),a), f(b))\}$

STEP 2 : D(S_1) = h(g(v), y), h(g(v),a), 1 =y/a,
Then we get
$S_1 = \{Q(h(g(v),a),w),\ Q(h(g(v),a), f(v)),\ Q(h(g(v),a), f(b))\}$

STEP 3 : D(S_2) = w,f(v),f(b), 2 =w/f(v),
Then we get
$S_2 = \{Q(h(g(v), a), f(v)),\ Q(h(g(v), a), f(v)),\ Q(h(g(v), a), f(b))\}$

STEP 4 : D(S_3) = f(v),f(v),f(b), 3 =v/b,
Then we get
$S_3 = \{Q(h(g(b), a), f(b)),\ Q(h(g(b), a), f(b)),\ Q(h(g(b), a), f(b))\}$

S3 is a singleton and the most general unifier for S is

$$x/g(v), y/a, w/f(v), v/b = x/g(b), y/a, w/f(b), v/b$$

6. **State the Herbrand theorem.**
   Ans:
   Herbrand's theorem:

   Let S = φ_i (x_1 , ... ,x_n) be a set of open formulas of a language L.
   Either

   (i) S has an Herbrand model or

   (ii) S is unsatisfiable and, in particular, there are finitely many ground instances of elements of S whose conjunction is unsatisfiable.

   The latter case (ii) is equivalent to

   (ii′) There are finitely many ground instances of the negations of formulas of S whose disjunction is valid.
   (As we may view these ground in- stances as built from propositional letters, the disjunction being valid is equivalent to its being a truth-functional tautology′)

7. **Find a resolvent for**
   $$\{P(x, y), P(y, z)\}, \{\neg P(u, f(u))\}$$

   where x, y, z, u **are variables.**
   Ans:
   $C_1 = \{P(x, y), P(y, z)\}$, $C_2 = \{\neg P(u, f(u))\}$
   mgu = $\{y/u, z/f(u)\}$
   $C_1 = \{P(x, u), P(u, f(u))\}$
   $C_2 = \{\neg P(u, f(u))\}$
   Resolvent: $C_3 = \{P(x, u)\}$

8. **Translate the following formulas into a set S of clauses:**

- $\forall x \forall y(above(x, y) \wedge on(y, z) \rightarrow above(x, z))$
- $\forall x \forall y(on(x, y) \rightarrow above(x, y))$
- $on(a, b)$
- $on(b, c)$

Write a resolution tree proof of clause $\{above(a, c)\}$ from S. Indicate the literals being resolved on and the substitutions being made to do the resolution. You may refer to Figure 35 of the textbook.

Ans:

Resolution tree proof

In casual form, we derive C_5 from $S = \{C_1, C_2, C_3, C_4\}$ where
$C_1 = \{\neg above(x, y), \neg on(y, z), above(x, z)\}$
$C_2 = \{\neg on(x, y), above(x, y)\}$
$C_3 = \{on(a, b)\}$
$C_4 = \{on(b, c)\}$
$C_5 = \{above(a, c)\}$

$\{above(a,c)\}$=C_5

$\{\neg on(b,z),above(a,z)\}(substitute\{z/c\})$ $\quad$ $\{on(b,c)\}$=C_4

$\{\neg on(y,z),above(x,z),\neg on(x,y)\}(substitute\{x/a,y/b\})$ $\quad$ $\{on(a,b)\}$=C_3

$\{\neg above(x,y),\neg on(y,z),above(x,z)\}$ =C_1 $\quad$ $\{\neg on(x,y),above(x,y)\}$=$C_2$