Homework 5 . Predicate Logic: Syntax and Semantics

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Submit your solution in PDF file (and Latex source file if you use Latex) to the folder of "assignment" of Piazza by **11:59pm Sun Nov 19.** (If you need more time, no later than 3pm Monday Nov 20.).

1. Consider the following sentence.

   Every number greater than or equal to 4 can be written as the sum of two prime numbers.

   (a) Write *a language* (as defined in Def 2.1) such that some formula of the language can be used to represent the sentence above.

   Answer:

   Variables: x, y, z

   Constants: 4

   Connectives: $\vee, \wedge, \rightarrow, \iff, \neg$

   Quantifiers: $\forall, \exists$

   Predicate Symbols: P, R, S

   Function Symbols: f

   Punctuation: $, .)($

(b) Write a formula of your language that should reflect the meaning of the sentence above.

Answer:

$((\forall z)(P(z,4)))$ where z is a number.

$((\forall x)(\forall y)(R(x) \wedge R(y))$ where $R(x) and R(y)$ are primes.

$((\forall x)(\forall y)(R(x) \wedge R(y)) \rightarrow (S(f(x,y),z) \wedge P(z,4)))$

(c) In terms of your language, write two example terms, two example atomic formulas, and two example formulas.

Answer:

Terms: x, y, z

Atomic Formulas: $S(f(x,y),z), R(x), R(y), P(z,4)$

Formulas: $((\forall x)(\forall y)(R(x) \wedge R(y))), (S(f(x,y),z) \wedge P(z,4)), ((\forall x)(\forall y)(R(x) \wedge R(y)) \rightarrow (S(f(x,y),z) \wedge P(z,4)))$

2. Write a formula to represent the following information. Your formula should be as close as possible to the intended meaning of these sentences.

(a) *There is a mother to all children.*
Answer: $((\exists m)(\forall c)Mother(m,c)$

(b) ALL ITEMS NOT AVAILABLE AT ALL STORES.
Answer: $((\forall i)(\forall s)notAvailable(i,s))$

Note, the sentence above is a disclaimer in the weekly flyer of specials of a grocery store chain.

3. Given the language defined in Def 2.1, which of the following are formulas by Def 2.5 (i.e., follow the definition strictly)?

   (a) $f(x, c)$

   (b) $R(c, f(d, z))$

   (c) $\forall x(P(x))$

   (d) $((\exists x)(((\forall y)P(z)) \to R(x, y)))$

   Answer:

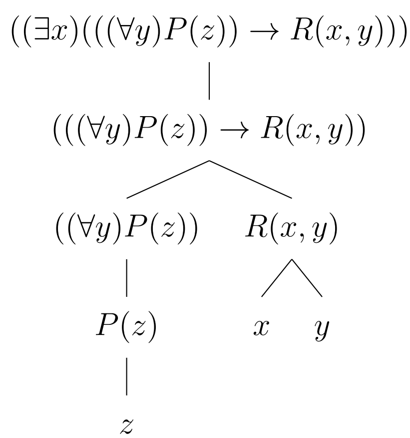
   (b) $R(c, f(d, z))$ and (d) $((\exists x)(((\forall y)P(z)) \to R(x, y)))$ are the formulas defined by Def 2.5.

4. Given

$$((\exists x)(((\forall y)P(z)) \to R(x, y))),$$

   (a) Draw its formation tree.

   Answer:



   (b) List two of its subformulas.

   Answer:

   i. $((\forall y)P(z))$
   ii. $P(z)$
   iii. $R(x, y)$
   iv. $(((\forall y)P(z)) \to R(x, y))$
   v. $((\exists x)(((\forall y)P(z)) \to R(x, y)))$

5. Which of the following terms are substitutable for x in the corresponding formulas?

   (a) $f(z, y)$ in $((\exists y)(P(y) \wedge R(x, z)))$.

   Answer: Not Substitutable.
   Because after substitution the variables of term are getting bounded.

   (b) $g(f(z, y), a)$ in $((\exists x)(P(x) \wedge R(x, y)))$.

   Answer: Not substitutable.
   Because X is not a free variable and it is bounded by $\exists x$.

6. Let $A = \{1, 2\}$. a) List all functions from A to A in the form of sets of pair. For example, one function is $\{(1, 1), (2, 2)\}$. If we let the function be named g. Then in the example function, $g(1)$ is 1 and $g(2)$ is 2. b) List all unary relations on A. Your relations must be represented as sets.

   Answer:

   (a) List all functions from A to A in the form of sets of pair
      i. g = $\{(1, 1), (2, 2)\}$. where g(1) = 1 and g(2) = 2
      ii. g = $\{(1, 2), (2, 1)\}$. where g(1) = 2 and g(2) = 1

   (b) List all unary relations on A. Your Relations must be represented as sets.
      i. $\{\}$
      ii. $\{1\}$
      iii. $\{2\}$
      iv. $\{1, 2\}$

7. Prove that $\mathcal{A} \models \neg\exists x\varphi(x)$ if and only if $\mathcal{A} \models \forall x\neg\varphi(x)$. You have to follow the proof format we used earlier. Follow the working backward methodology. You have to be able to apply the definitions.

   Answer:

   (a) Assume that the sentence φ of language L satisfies structure $\mathcal{A}$ for $L^{\mathcal{A}}$ to define $\mathcal{A} \models \varphi$ for sentences φ of L. (From Definition 4.3)

   (b) $\mathcal{A} \models \neg\exists x\varphi(x)$ for some ground term t, A$\neg\varphi$ i.e., A $\iff$ $\varphi(t)$. (From Definition 4.3 (ii) and (vii))

   (c) $\mathcal{A} \models \forall x\neg\varphi(x)$ for some ground term t, A $\models \neg\varphi(t)$ (i.e., $\mathcal{A} \models \varphi(t)$). (From Definition 4.3 (ii) and (viii))

   (d) we can conclude that; $\mathcal{A} \models \neg\exists x\varphi(x) \iff \mathcal{A} \models \forall x\neg\varphi(x)$. (By (2) and (3))