Quiz 4. Tableau Proof from Premises.

Ayyappa Reddy - R11888944

- This would be counted as 30% of HW3. Please submit your solution in PDF from Latex i.e.,post your solution to "assignment" folder with summary line starting with Quiz 4 to Piazza by 11:59pm Oct 3. If you need more time, post before 11:59am Oct 4. No late submission would be accepted. Write TA if needed. You are expected to spend significant work on this. It may take 1 to 2 hours for those who know all materials from the videos very well..

Q1. (10) Tableau proof from premises:

1) Write the definition of the tableau proof from premises.

Answer:

A finite tableau from premises is a binary tree structure, τ, that adheres to the following inductive criteria:

(i) If τ is a finite tableau obtained from premises Σ, and α is an atomic proposition in Σ, then creating a tableau, τ_α, by extending every non contradictory path in τ (that does not already contain τ_α) with τ_α results in another finite tableau from Σ.

(ii) If τ is a finite tableau derived from premises Σ, P is a path within τ, Σ' is an entry in τ found on path P, and τ' is constructed by adding a unique atomic tableau with a root entry E to τ at the end of path P, then τ' is also a finite tableau from Σ then the newly generated tableau is also a finite tableau.

Concept Names: Tableau, Premises, Finite Tableau, Atomic Tableau, Path, Entry.
Parameters: Σ (Set of Propositions), τ (Tableau), α (Atomic Proposition), P (Path), Σ' (Entry).

2) Compare yours with the one in the book, write down all major differences and how you address them in terms of what you have learned (e.g., you should have learned how to give concept name and parameters clearly when defining a concept, and give concept name and arguments clearly when using a concept).

Answer:

Major Differences:

a) Naming of the concepts are differently named compare to the text book definition here using terms like "Premises," "Atomic tableau," "Finite tableau," "Entry," and "Path," while maintaining the core concepts.

b) When it compares to the definition in the material, the key parameters such as Σ set of propositions, τ Finite tableau, $T\alpha$ Atomic proposition, P Path, and Σ' Entry are consistent. These are the major variations in parameters that i have found.

c) From lecture notes given by the professor, it makes minor difference with the definitions in terms of phrasing and relating the words that which may affect the readability but not the fundamental meaning of the definition.

Q2. (10) Formulate the soundness and completeness result about tableau proof from premises.

Answer:

Soundness Result of Tableau Proof from premises:

Soundness Result: "If α is tableau provable from a set of premises Σ, then α is logically valid with respect to Σ."

if $\Sigma \vdash_{\text{Tableau}} \alpha$, then $\Sigma \vDash \alpha$.

Here,

- $\Sigma \vdash_{\text{Tableau}} \alpha$ denotes that α is tableau provable from the set of premises Σ.
- $\Sigma \vDash \alpha$ denotes that α is logically valid with respect to Σ.

The proof of soundness typically involves showing that if there is a tableau proof of from , then any assignment that satisfies all the formulas in also satisfies . This ensures that the tableau system is sound, and it doesn't derive conclusions that are not logically valid based on the premises.

Completeness Result of Tableau Proof from Premises:

Completeness Result: If $\vDash\alpha$, then there exists a tableau proof for α from premises.

$$\Sigma \vDash \alpha \Rightarrow \Sigma \vdash \alpha$$

Here's a breakdown:

- $\Sigma \vDash \alpha$ signifies that α is semantically valid with respect to Σ
- $\Sigma \vdash \alpha$ denotes that α is tableau provable from the set of premises Σ.

The completeness result states that if a formula α is semantically valid, then it is tableau provable. In other words, if α is true in all possible models, then there exists a tableau proof for α using premises.

Q3. (10) Recall the intuition for Lemma 5.2 for soundness of tableau proof. For tableau proof from premises:

1) Formulate similar intuition:

Answer:

Intuition: From the context of a tableau proof from premises, this lemma 5.2 suggest that, Let there be a finite tableau produced in accordance with the inductive definition for tableaux as stated in Definition 4.1. It is obtained from the premises.

A valuation that agrees with the root entry, indicated as 0, is written as $V(0)$. Let V be such a valuation.

There is then a path P in the set P, starting at 0 and written as $[0, E_1, E_2, \ldots, E_i]$, where E_i represents an entry in the set P along the path.

It is true that $V(E_i)$ is consistent with E_i for each entry E_i in the path P, which can be stated as:

$$\forall i,\ V(E_i)\ is\ consistent\ with\ E_i,$$

which can be expressed as:

$$\forall i, V(E_i) \vDash E_i.$$

2) Formulate a precise statement (following Lemma 5.2) for your intuition.

Answer: This precise statement captures the essence of the lemma, ensuring that, Let τ denote a finite tableau created in accordance with Definition 4.1 and derived from premises.

Indicated as $V(0)$, V is a value that agrees with the root entry 0 of τ.

P is a path inside of τ that begins at 0 and is represented as $[0, E_1, E_2, \ldots, E_i]$, where E_i stands for an entry on the path.

The symbol i denotes the fact that this condition is true for each entry E_i along the path P.

The tableau respects the premises and maintains consistency with the valuation V throughout the path, as indicated by the condition $V(E_i) \models E_i$, which specifies that the valuation V aligns with each entry E_i.

3) Compare your formulation. Write done the differences and show clearly how you would address each difference (most time, you may revise your old writing in terms of what is given in book or lecture notes).

Answer: After comparing to textbook, material and lecture notes, that which provided from the professor, I don't have a major changes but definitely I found some minor and similar changes that are follows:

a) To represent the Notation of Tableau, I have used this τ .

b) where as in the statement of valuation agreement, I have used this "V(τ_0)to denote.

c) And in defining the path notation, I have represented it as P=[τ_0, E_1, E_2, ... , E_i].

d) In defining the Condition, I found this was one of the major difference i wrote very significantly like this " $V(E) \models E$ " to express the valuation agreement but in the material the concept name "agrees with" is used.

e) whereas in the consistency and respect for premises both the formulations express the idea that the tableau maintains consistency with the valuation and respects the premises. There are no significant differences in this regard.